1. Shallow Foundations

1) General

* Shallow Foundations: Foundations that transmit structural loads to the near-surface soils. (Spread footing foundation + Mat foundation)

\[ \frac{D_f}{B} \leq 1 \text{ (by Terzaghi)} \rightarrow \text{Later } \frac{D_f}{B} \leq 3 \sim 4 \]

* Requirements to satisfactory foundations
  i) Safe against shear failure (bearing capacity failure).
  ii) Should not undergo excessive displacements.
     (settlements \( \leftrightarrow \text{differential settlements} \))
  iii) Consideration on the any future influences which could adversely affect its performance. (frost action, scouring of pier foundations of bridge, …)

* Types of shallow foundation
  ➢ Spread footings
    - Spread footing foundations: An enlargement at the bottom of a column or bearing wall that spreads the applied structural loads over a sufficiently large soil area.
    i) Square spread footings : Supporting a single centrally-supported column.
    ii) Rectangular spread footings : In cases that obstructions prevent
construction of a square footing with a sufficiently large base area and large moment loads are present.

iii) Circular spread footings: Supporting a single centrally-supported column, but less common than square footing. (flagpoles).

iv) Continuous spread footings (Strip footings): Used to support bearing walls.

v) Combined footing: When columns are located too close together for each to give its own footing.

vi) Strap footing with a grade beam: Provides the necessary moment resistance in the exterior footing with eccentric load and a more rigid foundation system.

![Spread footing shapes](image-url)
- Mat foundations (Raft foundations): A very large spread footing that usually encompasses the entire footprint of the structure.

- The advantages of the mat foundation over individual spread footings
  i) Spreads the structure load over a larger area, thus reduces bearing pressure.
  ii) Provides much more structural rigidity and thus reduces the potential for excessive differential settlements.
  iii) Is easier to waterproof.
  iv) Has a greater weight and thus is able to resist greater uplift pressure.
2) Bearing Pressure

* Bearing Pressure: the contact forces per unit area along the bottom of the footing.

* The actual distribution of the bearing pressure depends on:
  - Structural rigidity of the footing.
  - Stress-strain properties of the soil.
  - Eccentricity of the applied load.
  - Magnitude of the applied moment.
  - Roughness of the bottom of the footing.

* Perfectly flexible footings

\[
\begin{align*}
\text{flexible footing on clay} & \quad \text{flexible footing on sand} \\
\rightarrow & \quad \text{Bend but maintain a uniform bearing pressure.}
\end{align*}
\]

* Perfectly rigid footings

\[
\begin{align*}
\text{rigid footing on clay} & \quad \text{rigid footing on sand} \\
\rightarrow & \quad \text{Settle uniformly but have variations in the bearing pressure.} \\
& \quad \text{Close to behaviors of the real footing. (especially for spread footing but not for raft foundation)}
\end{align*}
\]
For bearing capacity and settlement analysis, we assume the uniform pressure with rigid footings. (Error is not significant for spread footing.)

* The footing subjected to eccentric and/or moment loads.
  → The bearing pressure is biased toward one side.

< Distribution of bearing pressure along the base of spread footing subjected to eccentric and/or moment loads: (a) actual distribution (b) simplified distribution >
Presumptive Bearing Pressures

- Presumptive bearing pressures are allowable bearing pressures based on experience and expressed as a function of soil type. (An attempt to control excessive settlements.)
  → Give quick reference values of foundation design. *(Tales 6.1)*
  → Useful for small structure at sites with good soils (Column Loads < 200kN).

- Presumptive bearing pressures vary considerably, due to differing degree of conservatism and reflecting the subsurface conditions in the area where the code is used.

<table>
<thead>
<tr>
<th>Soil or Rock Classification</th>
<th>Allowable Bearing Pressure, lb/ft² (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uniform Code&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Massive crystalline bedrock</td>
<td>4,000 - 12,000</td>
</tr>
<tr>
<td></td>
<td>(200 - 600)</td>
</tr>
<tr>
<td>Sedimentary or foliated rock</td>
<td>2,000 - 6,000</td>
</tr>
<tr>
<td></td>
<td>(100 - 300)</td>
</tr>
<tr>
<td>Sandy gravel or gravel</td>
<td>2,000 - 6,000</td>
</tr>
<tr>
<td></td>
<td>(100 - 300)</td>
</tr>
<tr>
<td>Sand, silty sand, clayey sand, silty gravel, or clayey gravel</td>
<td>1,500 - 4,500</td>
</tr>
<tr>
<td></td>
<td>(75 - 225)</td>
</tr>
<tr>
<td>Clay, sandy clay, silty clay, or clayey silt</td>
<td>1,000 - 3,000</td>
</tr>
<tr>
<td></td>
<td>(50 - 150)</td>
</tr>
</tbody>
</table>

<sup>a</sup> The values in this table are for illustrative purposes only and are not a complete description of the code provisions. Portions of the table include the author’s interpretations to classify the presumptive bearing values into uniform soil groups. Refer to the individual codes for more details.

<sup>b</sup> The Uniform Building Code values in soil are intended to provide a factor of safety of at least 3 against a bearing capacity failure, and a total settlement of no more than 0.5 in (12 mm) (ICBO, 1991c).
3) Failure modes

- General shear failure
- Local shear failure
- Punching shear failure

![Failure modes diagram](image)
Failure modes → a function of relative density and relative depth ($D_t/B$)

$B^o = B$ for circular (diameter) or square ft.

$B^o = 2BL/(B+L)$ for rectangular ft.

Ultimate load occurs at 4 ∼ 10% of $B$ for general shear failure
15 ∼ 25% of $B$ for local-punching shear failure
4) General Bearing Capacity Equation

\[ q_u = \text{ultimate bearing capacity (stress)} \]
\[ q_a = \text{allowable bearing capacity} = \frac{q_u}{F.S}. \]

* Failure zones for strip footing.

I : Active Rankine Zone
II : Radial Zone
III : Passive Rankine Zone

\[ \alpha = \phi' \text{ for Terzaghi} \]
\[ = 45 + \phi' / 2 \Rightarrow \text{realistic value} \]
* Bearing Capacity Equation

\[ q_u = cN_c + qN_q + \frac{1}{2} \gamma B N_\gamma \]

- \( \gamma \): unit weight of soil
- \( c \): cohesion of soil
- \( q = \gamma D_t \)
- \( N_c, N_q, N_r \): Bearing capacity factors = \( f(\phi') \)
  - based on \( \alpha = \phi' \) (Terzaghi, Table 3.1 (p.129))
  - based on \( \alpha = 45 + \phi'/2 \) (Vesic, Table 3.4(p138))

* Assumptions

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8.
* General Bearing Capacity Equations (Bearing Capacity Factors and Other Influential Factors)

- To consider the influences of the shape of foundations, embedded depth, and inclined load, the following form of equation has been suggested.

\[ q_u = cN_cF_{cs}F_{cd}F_{cl} + qN_qF_{qs}F_{qd}F_{ql} + \frac{1}{2}\gamma BN_rF_{ps}F_{pd}F_{pl} \]

i) \( N_c, N_q, N_r \)

\( N_c \) by Prandtl (1921)
\( N_q \) by Reissner (1924)
\( N_r \) by Caquot and Kerisel (1953) and Vesic (1973)

<table>
<thead>
<tr>
<th>( \phi' )</th>
<th>( N_c )</th>
<th>( N_q )</th>
<th>( N_r )</th>
<th>( \phi' )</th>
<th>( N_c )</th>
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<td>7.07</td>
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<td>173.64</td>
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</tr>
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<td>48</td>
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<td>19.32</td>
<td>9.60</td>
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<td>266.89</td>
<td>319.07</td>
<td>762.89</td>
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<td>20.72</td>
<td>10.66</td>
<td>10.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ii) Shape factor (De Beer (1970))

\[
F_{cs} = 1 + \frac{B}{L} \frac{N_q}{N_c} \\
F_{qs} = 1 + \frac{B}{L} \tan \phi' \\
F_{ys} = 1 - 0.4 \frac{B}{L}
\]

where \( L \) = length of the foundation \((L > B)\)

iii) Depth factor (Hansen (1970))

For \( D_f / B \leq 1 \),

\[
F_{cd} = 1 + 0.4 D_f / B \\
F_{qcd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \frac{D_f}{B}
\]

For \( D_f / B > 1 \)

\[
F_{cd} = 1 + 0.4 \tan^{-1}(D_f / B) \\
F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \tan^{-1}\left(\frac{D_f}{B}\right)
\]

iv) Inclination Factor (Hanna and Meyerhof (1981))

\[
F_{ci} = F_{qi} = (1 - \frac{\beta}{90})^2 \\
F_{\beta} = (1 - \frac{\beta}{\phi})^2
\]

where \( \beta \) = inclination of the load on the foundation with respect to the vertical.
- For inclined load, we must check sliding failure in addition to bearing capacity.

For clays, \( s = \alpha s_s \) (generally \( \alpha \leq 1.0 \), but \( \alpha = 1.0 \) for \( s_u \leq 500 \text{lb/ft}^2 \))

For sands, \( s = \frac{P_v}{\text{(Area)}} \tan \phi_{\text{int}} \) (\( \phi_{\text{int}} = 2/3\phi \))

\[ P_{H} \leq s \times \frac{\text{(Area)}}{F.S.} \]
* Effect of water (influence on \( q \) and \( \gamma \) of bearing capacity equation)

I) \( D_1 < D_f \)

\[
q N_q = \left[ D_1 \gamma_t + D_2 (\gamma_{sat} - \gamma_w) \right] N_q
\]

\[
\frac{1}{2} \gamma BN_y = \frac{1}{2} \left( \gamma_{sat} - \gamma_w \right) BN_y
\]

II) \( 0 \leq d \leq B \)

\[
q N_q = \gamma_i D_f N_q
\]

\[
\frac{1}{2} \gamma BN_y = \frac{1}{2} \left( \gamma_{sat} - \gamma_w + \frac{d}{B} \left( \gamma_t - (\gamma_{sat} - \gamma_w) \right) \right) BN_y
\]

\( \gamma_t \): Total unit weight of soil  
\( \gamma_{sat} \): Saturated unit weight of soil  
\( \gamma_w \): Unit weight of water

III) \( d \geq B \)  No effect

Note) No seepage force
* Effect of Compressibility

For compressible soils ⇒ local shear failure

Ex) Loose sands ⇒ low relative density

- Shear (triaxial) test response and load-settlement curve for loose soil with those for dense sand

- Terzaghi Recommendations
  Use c*, φ* (reduced strength parameter) in bearing capacity equation;

\[
c* = \frac{2}{3} c' \\
\tan \phi* = \frac{2}{3} (\tan \phi')
\]
* Net ultimate bearing capacity

\[ q_u = \frac{Q}{A} + \gamma D_f \]
\[ \therefore q_{\text{net}} = \frac{Q}{A} = q_u - \gamma D_f \]

* Factor of Safety

i) \( q_{\text{all}} = \frac{q_u}{F.S.} \), \( q_{\text{all(net)}} = \frac{(q_u - q)}{F.S.} \)

ii) \( FS_{\text{shear}} \Rightarrow c_d = c / FS_{\text{shear}}, \phi_d = \phi / FS_{\text{shear}} \)
\[ q_{\text{all}} = c_d N_c F_{cs} F_{sd} F_{ci} + q N_q F_{qk} F_{qd} F_{qi} + 1/2 \gamma B N_f F_{pf} F_{qf} F_{qf} \]
\[ q_{\text{all(net)}} = q_{\text{all}} - q \]

\( F.S. = 3-4, \quad FS_{\text{shear}} = 1.4-1.6 \)
**Case history**

: Ultimate Bearing Capacity in Saturated Clay

- Comparison between measured and estimated bearing capacities on 5 different sizes of the foundations.

Brand et al. (1972) reported field-test results of soil explorations on small foundations in soft Bangkok clay (a deposit of marine clay) in Rangsit, Thailand. The results are shown in Figure 3.7. Because of the sensitivity of the clay, the laboratory test results for $c_u$ (unconfined compression and unconsolidated undrained triaxial) were rather scattered; however, better results were obtained for the variation of $c_u$ with depth from field vane shear tests, which showed that the average variations of the undrained cohesion were as follows:

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$c_u$ (kN/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1.5</td>
<td>≈35</td>
</tr>
<tr>
<td>1.5–2</td>
<td>Decreasing linearly from 35 to 24</td>
</tr>
<tr>
<td>2–8</td>
<td>≈24</td>
</tr>
</tbody>
</table>

Five small square foundations were tested for ultimate bearing capacity. The sizes of the foundations were 0.6 m × 0.6 m, 0.675 m × 0.675 m, 0.75 m × 0.75 m, 0.9 m × 0.9 m, and 1.05 m × 1.05 m. The depth of the bottom of the foundations was 1.5 m, measured from the ground surface. The load–settlement plots obtained from the bearing capacity tests are shown in Figure 3.8.

![Figure 3.8](image.png)
Analysis of the Field-Test Results

The ultimate loads, $Q_u$, obtained from each test are also shown in Figure 3.8. The ultimate load is defined as the point where the load displacement becomes practically linear. The failure in soil below the foundation is of the local shear type. Hence, we may apply Eq. (3.10):

$$q_u = 0.867c_uN'_c + qN'_q + 0.4\gamma BN'_\gamma$$

For $\phi = 0$, $c = c_u$ and, from Table 3.2, $N'_c = 5.7$, $N'_q = 1$, and $N'_\gamma = 0$. Thus, for $\phi = 0$,

$$q_u = 4.94c_u + q$$

(3.19)

Assuming that the unit weight of soil is about 18.5 kN/m$^3$, it follows that $q = D/\gamma = (1.5) (18.5) = 27.75$ kN/m$^3$. We can then assume average values of $c_u$: For depths of 1.5 m to 2.0 m, $c_u = (35 + 24)/2 = 29.5$ kN/m$^3$; for depths greater than 2.0 m, $c_u = 24$ kN/m$^3$. If we further assume that the undrained cohesion of clay at depth $= B$ below the foundation controls the ultimate bearing capacity, then

$$c_u(\text{average}) = \frac{(29.5)(2.0 - 1.5) + (24)[B - (2.0 - 1.5)]}{B}$$

(3.20)

The value of $c_u(\text{average})$ obtained for each foundation needs to be corrected in view of Eq. (2.19). Table 3.3 presents the details of other calculations and a comparison of the theoretical and field ultimate bearing capacities.

<table>
<thead>
<tr>
<th>$B$ (m)</th>
<th>$c_{u(\text{average})}$ (kN/m$^3$)</th>
<th>Plasticity index$^a$</th>
<th>Correction factor, $\lambda^c$</th>
<th>$c_{u(\text{corrected})}$ (kN/m$^3$)</th>
<th>$q_{u(\text{theory})}$ (kN/m$^3$)</th>
<th>$Q_u(\text{field})^f$ (kN)</th>
<th>$q_{u(\text{field})}$ (kN/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>28.58</td>
<td>40</td>
<td>0.84</td>
<td>24.01</td>
<td>146.4</td>
<td>60</td>
<td>166.6</td>
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<tr>
<td>0.675</td>
<td>28.07</td>
<td>40</td>
<td>0.84</td>
<td>23.58</td>
<td>144.2</td>
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<td>0.75</td>
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$^a$Eq. (3.20)

$^b$From Figure 3.7

$^c$From Eq. (2.28) [$\lambda = 1.7 - 0.54 \log \Pi$; Bjerrum (1972)]

$^d$Eq. (2.27)

$^e$Eq. (3.19)

$^f$Figure 3.8

$^g$q$_{u(\text{field})}/B^2$

Note that the ultimate bearing capacities obtained from the field are about 10% higher than those obtained from theory. One reason for such a difference is that the ratio $D/\gamma$ for the field tests varies from 1.5 to 2.5. The increase in the bearing capacity due to the depth of embedment has not been accounted for in Eq. (3.20).