[2008] [06-1]

Planning Procedure of Naval Architecture & Ocean Engineering

October, 2008

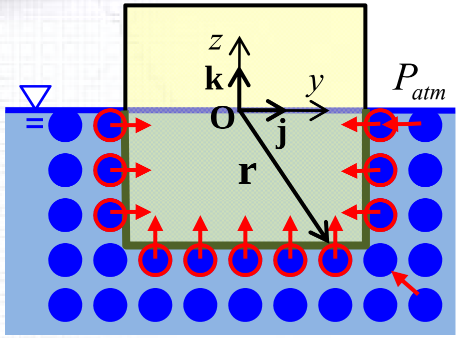
Prof. Kyu-Yeul Lee

Department of Naval Architecture and Ocean Engineering,
Seoul National University of College of Engineering

Part 1. Stability & Trim

[06-1] Pressure integration technique (1)

유체 중에 잠긴 물체가 받는 힘과 모멘트



S_B : 침수표면

V : 침수부피

유체 입자가 주위 유체 입자에 작용하는 정적인 압력

Bernoulli Eq.

$$\rho\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left|\nabla\Phi\right|^2+\rho g\,z+P=C$$

$$P=\cancel{-\rho\frac{\partial\Phi}{\partial t}}\cancel{-\frac{1}{2}\left|\nabla\Phi\right|^2}\underbrace{-\rho g\,z+C}_{P_{Static}}$$

정수 ➡ 유체의 속도 = 0

$$P_{Static}=-\rho gz+C \rightarrow P_{atm}$$

$$P_{Static}=-\rho gz+\cancel{P_{atm}}$$

$$\rightarrow P_{Fluid}(z)+\cancel{P_{atm}}$$

$$P_{Fluid}(z)=-\rho gz$$

유체 입자들이 유체 중에 잠긴 물체에 작용하는 정적인 힘과 모멘트

$$\mathbf{F}=\iint_{S_B}(-\rho gz)d\mathbf{S}=-\rho g\iint_{S_B}z(\mathbf{n}dS)=-\rho g\iint_{S_B}\mathbf{n}zdS$$

$$\mathbf{M}=\iint_{S_B}\mathbf{r}\times(-\rho gz)d\mathbf{S}=-\rho g\iint_{S_B}\mathbf{r}\times z(\mathbf{n}dS)=-\rho g\iint_{S_B}(\mathbf{r}\times\mathbf{n})zdS$$

Divergence Theorem

$$\mathbf{F}=\mathbf{k}\rho g\iiint_V dV$$

$$\mathbf{M}=\rho g\iiint_V[\mathbf{i}y-\mathbf{j}x]dV$$

선박의 자세와 힘, 모멘트와의 관계

– Pressure Integration Technique[1]

1) J.N.Newman, Marine Hydrodynamics, 1977, pp.290~295

선박이 유체 중에서 받는 정적인 힘과 모멘트

$$\mathbf{F} = \mathbf{k}\rho g \iiint_{V(\xi_3,\xi_4,\xi_5)} dV \quad \Downarrow \quad \mathbf{F} = \mathbf{F}(\xi_3, \xi_4, \xi_5)$$

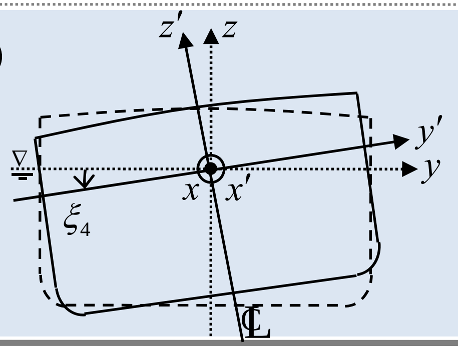
$$\mathbf{M} = \rho g \iiint_{V(\xi_3,\xi_4,\xi_5)} [\mathbf{i}y - \mathbf{j}x] dV \quad \Downarrow \quad \mathbf{M} = \mathbf{M}(\xi_3, \xi_4, \xi_5)$$

(Q) 침수부피(중심)가 변하는 자세는?

대경사 경우 / 미소경사 경우

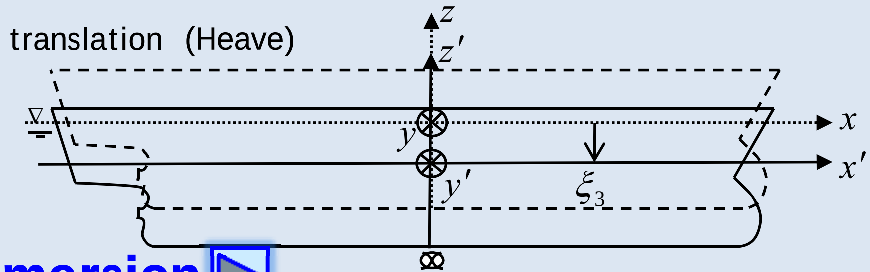
$$\mathbf{F} \approx -\mathbf{k}\rho g \xi_4 T_{WP}(z')$$
$$\mathbf{M} \approx \mathbf{i}(-\rho g \xi_4 V_0 z'_{B0} - \rho g \xi_4 I_T + mgz'_G \xi_4) + \mathbf{j}\rho g \xi_4 I_P$$

x 축 rotation (Roll)



Heel

z 축 translation (Heave)

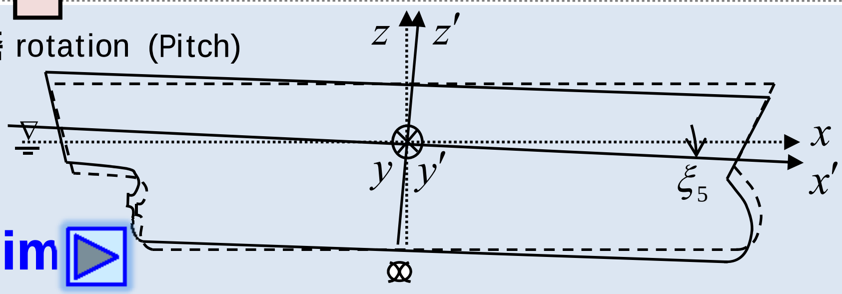


Immersion

$$\mathbf{F} \approx -\mathbf{k}\rho g \xi_3 A_{WP}(z')$$
$$\mathbf{M} \approx -\mathbf{i}\rho g \xi_3 T_{WP}(z') + \mathbf{j}\rho g \xi_3 L_{WP}(z')$$

$$\mathbf{F} \approx \mathbf{k}\rho g \xi_5 L_{WP}(z')$$
$$\mathbf{M} \approx \mathbf{i}\rho g \xi_5 I_P + \mathbf{j}(-\rho g \xi_5 V_0 z'_{B0} - \rho g \xi_5 I_L + mgz'_G \xi_5)$$

y 축 rotation (Pitch)



Trim

복원력(모멘트)의 선형화 (Taylor series expansion) (1)

초기 자세(ξ_3, ξ_4, ξ_5)에서

복원력(F), 횡 방향 복원 모멘트(M_T), 종 방향 복원 모멘트(M_L)를 알고 있을 때,

미소 변화된 자세($\xi_3 + \Delta\xi_3$, $\xi_4 + \Delta\xi_4$, $\xi_5 + \Delta\xi_5$)에서의

복원력(F), 횡 방향 복원 모멘트(M_T), 종 방향 복원 모멘트(M_L)는?

$$F(\xi_3, \xi_4, \xi_5) \qquad F(\xi_3 + \Delta\xi_3, \xi_4 + \Delta\xi_4, \xi_5 + \Delta\xi_5)$$

$$M_T(\xi_3, \xi_4, \xi_5) \quad \xrightarrow{?} \quad M_T(\xi_3 + \Delta\xi_3, \xi_4 + \Delta\xi_4, \xi_5 + \Delta\xi_5)$$

$$M_L(\xi_3, \xi_4, \xi_5) \qquad M_L(\xi_3 + \Delta\xi_3, \xi_4 + \Delta\xi_4, \xi_5 + \Delta\xi_5)$$

Taylor series expansion

$$F(\xi_3 + \Delta\xi_3, \xi_4 + \Delta\xi_4, \xi_5 + \Delta\xi_5) = F(\xi_3, \xi_4, \xi_5) + \frac{\partial F}{\partial \xi_3}\Delta\xi_3 + \frac{\partial F}{\partial \xi_4}\Delta\xi_4 + \frac{\partial F}{\partial \xi_5}\Delta\xi_5 + \cancel{\cdots}$$

선형화

$$M_T(\xi_3 + \Delta\xi_3, \xi_4 + \Delta\xi_4, \xi_5 + \Delta\xi_5) = M_T(\xi_3, \xi_4, \xi_5) + \frac{\partial M_T}{\partial \xi_3}\Delta\xi_3 + \frac{\partial M_T}{\partial \xi_4}\Delta\xi_4 + \frac{\partial M_T}{\partial \xi_5}\Delta\xi_5 + \cancel{\cdots}$$

선형화

$$M_L(\xi_3 + \Delta\xi_3, \xi_4 + \Delta\xi_4, \xi_5 + \Delta\xi_5) = M_L(\xi_3, \xi_4, \xi_5) + \frac{\partial M_L}{\partial \xi_3}\Delta\xi_3 + \frac{\partial M_L}{\partial \xi_4}\Delta\xi_4 + \frac{\partial M_L}{\partial \xi_5}\Delta\xi_5 + \cancel{\cdots}$$

선형화

복원력(모멘트)의 선형화 (Taylor series expansion) (2)

✓ Taylor series 1차항까지 전개

$$F(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=F(\xi_3,\xi_4,\xi_5)+\frac{\partial F}{\partial\xi_3}\Delta\xi_3+\frac{\partial F}{\partial\xi_4}\Delta\xi_4+\frac{\partial F}{\partial\xi_5}\Delta\xi_5$$

$$M_T(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=M_T(\xi_3,\xi_4,\xi_5)+\frac{\partial M_T}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial\xi_5}\Delta\xi_5$$

$$M_L(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=M_L(\xi_3,\xi_4,\xi_5)+\frac{\partial M_L}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial\xi_5}\Delta\xi_5$$

$$F(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)-F(\xi_3,\xi_4,\xi_5)=\frac{\partial F}{\partial\xi_3}\Delta\xi_3+\frac{\partial F}{\partial\xi_4}\Delta\xi_4+\frac{\partial F}{\partial\xi_5}\Delta\xi_5$$

$$M_T(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)-M_T(\xi_3,\xi_4,\xi_5)=\frac{\partial M_T}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial\xi_5}\Delta\xi_5$$

$$M_L(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)-M_L(\xi_3,\xi_4,\xi_5)=\frac{\partial M_L}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial\xi_5}\Delta\xi_5$$

$$\Delta F=\frac{\partial F}{\partial\xi_3}\Delta\xi_3+\frac{\partial F}{\partial\xi_4}\Delta\xi_4+\frac{\partial F}{\partial\xi_5}\Delta\xi_5$$

$$\Delta M_T=\frac{\partial M_T}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial\xi_5}\Delta\xi_5$$

$$\Delta M_L=\frac{\partial M_L}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial\xi_5}\Delta\xi_5$$

복원력(모멘트)의 선형화 (Taylor series expansion) (3)

✓ Taylor series 1차항까지 전개

$$\Delta F = \frac{\partial F}{\partial \xi_3}\Delta\xi_3 + \frac{\partial F}{\partial \xi_4}\Delta\xi_4 + \frac{\partial F}{\partial \xi_5}\Delta\xi_5$$

$$\Delta M_T = \frac{\partial M_T}{\partial \xi_3}\Delta\xi_3 + \frac{\partial M_T}{\partial \xi_4}\Delta\xi_4 + \frac{\partial M_T}{\partial \xi_5}\Delta\xi_5$$

$$\Delta M_L = \frac{\partial M_L}{\partial \xi_3}\Delta\xi_3 + \frac{\partial M_L}{\partial \xi_4}\Delta\xi_4 + \frac{\partial M_L}{\partial \xi_5}\Delta\xi_5$$

변수 3개, 식 3개

✓ Matrix로 표현 $\mathbf{b} = \mathbf{A}\mathbf{x}$

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial \xi_3} & \frac{\partial F}{\partial \xi_4} & \frac{\partial F}{\partial \xi_5} \\ \frac{\partial M_T}{\partial \xi_3} & \frac{\partial M_T}{\partial \xi_4} & \frac{\partial M_T}{\partial \xi_5} \\ \frac{\partial M_L}{\partial \xi_3} & \frac{\partial M_L}{\partial \xi_4} & \frac{\partial M_L}{\partial \xi_5} \end{pmatrix} \begin{pmatrix} \Delta\xi_3 \\ \Delta\xi_4 \\ \Delta\xi_5 \end{pmatrix}$$

1. **자세**의 변화량이 주어져 있을 때, **힘(모멘트)**의 변화량을 구하는 경우

Find, Given, Given

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial \xi_3} & \frac{\partial F}{\partial \xi_4} & \frac{\partial F}{\partial \xi_5} \\ \frac{\partial M_T}{\partial \xi_3} & \frac{\partial M_T}{\partial \xi_4} & \frac{\partial M_T}{\partial \xi_5} \\ \frac{\partial M_L}{\partial \xi_3} & \frac{\partial M_L}{\partial \xi_4} & \frac{\partial M_L}{\partial \xi_5} \end{pmatrix} \begin{pmatrix} \Delta\xi_3 \\ \Delta\xi_4 \\ \Delta\xi_5 \end{pmatrix}$$

$\mathbf{b} = \mathbf{A}\mathbf{x}$ 를 풀면 됨

2. **힘(모멘트)**의 변화량이 주어져 있을 때, **자세**의 변화량을 구하는 경우

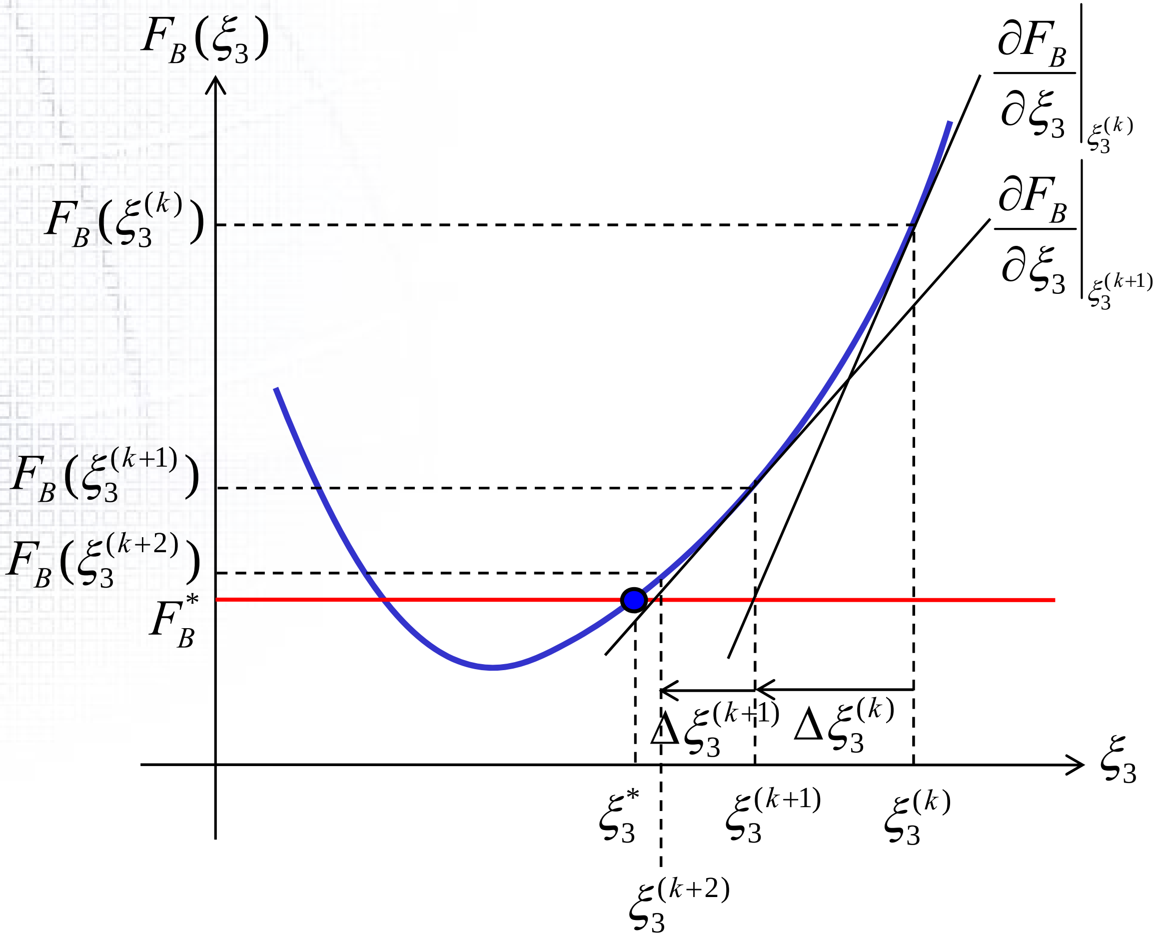
Given, Given, Find

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial \xi_3} & \frac{\partial F}{\partial \xi_4} & \frac{\partial F}{\partial \xi_5} \\ \frac{\partial M_T}{\partial \xi_3} & \frac{\partial M_T}{\partial \xi_4} & \frac{\partial M_T}{\partial \xi_5} \\ \frac{\partial M_L}{\partial \xi_3} & \frac{\partial M_L}{\partial \xi_4} & \frac{\partial M_L}{\partial \xi_5} \end{pmatrix} \begin{pmatrix} \Delta\xi_3 \\ \Delta\xi_4 \\ \Delta\xi_5 \end{pmatrix}$$

$\mathbf{x} = \mathbf{A}^{-1}\mathbf{x}$ 를 풀면 됨

※ **A가 선형화되어 있기 때문에**
반복 계산(iteration)을 해야 함

목표 힘(F^*)에 대한 선박의 자세 계산 (1 변수 예)



Taylor series 1차 항까지 전개

$$F_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = F_B(\xi_3^{(k)}) + \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} = F_B^*$$

↓

힘의 변화량과 자세의 변화량 관계

$$F_B^* - F_B(\xi_3^{(k)}) = \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)}$$

↓

자세의 변화량에 대해 표현

$$\Delta\xi_3^{(k)} = \left(\left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}}\right)^{-1} \left(F_B^* - F_B(\xi_3^{(k)})\right)$$

↓

자세 변화

$$\xi_3^{(k+1)} = \xi_3^{(k)} + \Delta\xi_3^{(k)}$$

↓

$$\left|\xi_3^{(k+1)} - \xi_3^{(k)}\right| < \varepsilon$$

No → $k = k+1$ → (자세의 변화량에 대해 표현 단계로)

Yes ↓

종료 $\xi_3^* = \xi_3^{(k+1)}$

목표 힘(F^*, M_T^*, M_L^*)에 대한 선박의 자세 계산

힘의 변화량과 자세의 변화량 관계

$$F_B^* - F_B(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) = \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} + \left.\frac{\partial F_B}{\partial \xi_4}\right|_{\xi_4^{(k)}} \cdot \Delta\xi_4^{(k)} + \left.\frac{\partial F_B}{\partial \xi_5}\right|_{\xi_5^{(k)}} \cdot \Delta\xi_5^{(k)}$$

$$M_T^* - M_T(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) = \left.\frac{\partial M_T}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} + \left.\frac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} \cdot \Delta\xi_4^{(k)} + \left.\frac{\partial M_T}{\partial \xi_5}\right|_{\xi_5^{(k)}} \cdot \Delta\xi_5^{(k)}$$

$$M_L^* - M_L(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) = \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} + \left.\frac{\partial M_L}{\partial \xi_4}\right|_{\xi_4^{(k)}} \cdot \Delta\xi_4^{(k)} + \left.\frac{\partial M_L}{\partial \xi_5}\right|_{\xi_5^{(k)}} \cdot \Delta\xi_5^{(k)}$$

자세의 변화량에 대해 표현

$$\begin{bmatrix} \Delta\xi_3^{(k)} \\ \Delta\xi_4^{(k)} \\ \Delta\xi_5^{(k)} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial F_B}{\partial \xi_4}\right|_{\xi_4^{(k)}} & \left.\frac{\partial F_B}{\partial \xi_4}\right|_{\xi_4^{(k)}} \\ \left.\frac{\partial M_T}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} & \left.\frac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} \\ \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} \end{bmatrix}^{-1} \begin{bmatrix} F_B^* - F_B(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) \\ M_T^* - M_T(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) \\ M_L^* - M_L(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) \end{bmatrix}$$

자세 변화

$$\xi_3^{(k+1)} = \xi_3^{(k)} + \Delta\xi_3^{(k)}$$

$$\left|\xi_3^{(k+1)} - \xi_3^{(k)}\right| < \varepsilon$$

No: $k = k + 1$

Yes: **종료** $\xi_3^* = \xi_3^{(k+1)}$

미소 자세와 미소 힘, 모멘트와의 관계식[1)]

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \boxed{\begin{pmatrix} \left.\dfrac{\partial F}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\dfrac{\partial F}{\partial \xi_4}\right|_{\xi_3^{(k)}} & \left.\dfrac{\partial F}{\partial \xi_5}\right|_{\xi_3^{(k)}} \\ \left.\dfrac{\partial M_T}{\partial \xi_3}\right|_{\xi_4^{(k)}} & \left.\dfrac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} & \left.\dfrac{\partial M_T}{\partial \xi_5}\right|_{\xi_4^{(k)}} \\ \left.\dfrac{\partial M_L}{\partial \xi_3}\right|_{\xi_5^{(k)}} & \left.\dfrac{\partial M_L}{\partial \xi_4}\right|_{\xi_5^{(k)}} & \left.\dfrac{\partial M_L}{\partial \xi_5}\right|_{\xi_5^{(k)}} \end{pmatrix}} \begin{pmatrix} \Delta \xi_3 \\ \Delta \xi_4 \\ \Delta \xi_5 \end{pmatrix}$$

Jacobian matrix

(자세 변화량을 힘과 모멘트의 변화량으로 변환)

$$= \begin{pmatrix} -\rho g A_{WP}(\xi_3^{(k)}) & -\rho g T_{WP}(\xi_3^{(k)}) & \rho g L_{WP}(\xi_3^{(k)}) \\ -\rho g T_{WP}(\xi_4^{(k)}) & -\rho g I_T(\xi_4^{(k)}) - \rho g V(\xi_4^{(k)}) z_B^{(k)} + mg \cdot z_G^{(k)} & \rho g I_P(\xi_4^{(k)}) \\ \rho g L_{WP}(\xi_5^{(k)}) & \rho g I_P(\xi_5^{(k)}) & -\rho g I_L(\xi_5^{(k)}) - \rho g V(\xi_5^{(k)}) z_B^{(k)} + mg \cdot z_G^{(k)} \end{pmatrix} \cdot \begin{pmatrix} \Delta \xi_3 \\ \Delta \xi_4 \\ \Delta \xi_5 \end{pmatrix}$$

Linked Slides

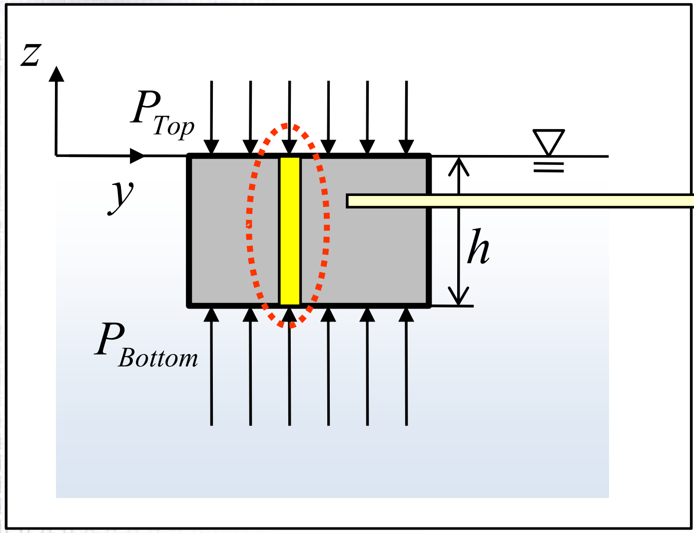
압력

$$P_{Static} = P_{atm} - \rho g z$$

$$P_{Fluid} = -\rho g z$$

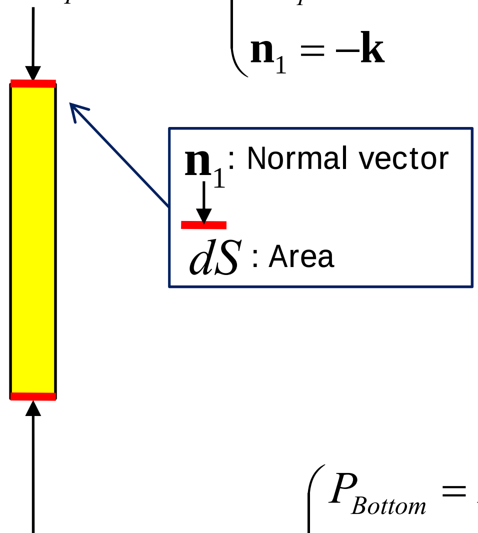
※ 압력(Pressure) : 단위 면적에 **수직**으로 작용하는 힘
즉, 힘을 구하기 위해서는 압력에 면적과
그 작용면의 법선 벡터(Normal Vector)를 곱해야 함

✓ 아래 물체에 작용하는 수직방향의 정적인 힘은?



: 물체 윗면의 미소 면적에 작용하는 힘

$$dF_{Top} = P_{Top} \cdot \mathbf{n}_1 dS \quad \left(\begin{aligned} P_{Top} &= P_{atm} - \rho g \cdot 0 \\ \mathbf{n}_1 &= -\mathbf{k} \end{aligned} \right)$$



$$dF_{Bottom} = P_{Bottom} \cdot \mathbf{n}_2 dS \quad \left(\begin{aligned} P_{Bottom} &= P_{atm} - \rho g h \\ \mathbf{n}_2 &= \mathbf{k} \end{aligned} \right)$$

: 물체 아랫면의 미소 면적에 작용하는 힘

$$\begin{aligned} dF &= dF_{Top} + dF_{Bottom} \\ &= P_{Top} \cdot \mathbf{n}_1 dS + P_{Bottom} \cdot \mathbf{n}_2 dS \\ &= \cancel{P_{atm}(-\mathbf{k})dS} + (\cancel{P_{atm}} - \rho g h)\mathbf{k} dS \\ &= -\rho g h \mathbf{k} dS = \mathbf{k}(-\rho g h \cdot dS) \end{aligned}$$

: 대기압에 의한 힘이 서로 상쇄됨

✓ Bernoulli Equation : $\left(where\ P_{Static} = P_{atm} + P_{Fluid} \right)$

$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho |\nabla \Phi|^2 + \rho g z = P_{atm}$$

$$\rho \frac{\partial \Phi}{\partial t} + (P_{atm} + P_{Fluid}) + \frac{1}{2}\rho |\nabla \Phi|^2 + \rho g z = P_{atm}$$

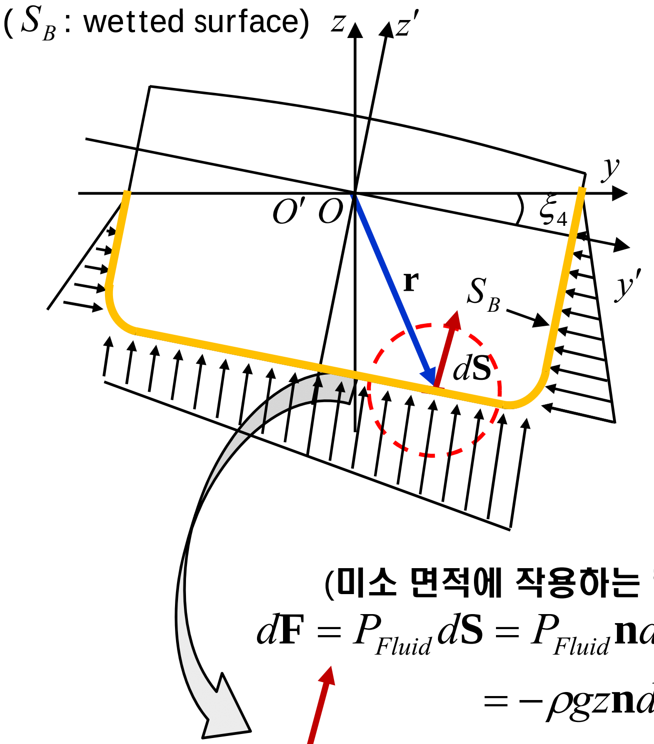
$$\rho \frac{\partial \Phi}{\partial t} + P_{Fluid} + \frac{1}{2}\rho |\nabla \Phi|^2 + \rho g z = 0$$

선박에 작용하는 정적인 힘과 모멘트

$$\boxed{P_{Fluid} = -\rho g z}$$

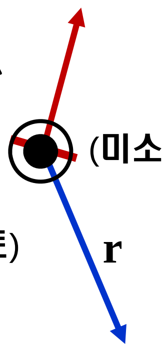
좌현으로 기울어진 상태 (횡 경사)
(선박을 선수에서 선미 방향으로 바라봄)

(S_B : wetted surface)



(미소 면적에 작용하는 힘)

$$d\mathbf{F} = P_{Fluid} d\mathbf{S} = P_{Fluid}\mathbf{n}dS$$
$$= -\rho g z \mathbf{n} dS$$



$d\mathbf{M} = \mathbf{r} \times d\mathbf{F}$
(미소면적에 작용하는 모멘트)

- Hydrostatic force
: **침수표면에 작용하는 모든 정적인 힘을 적분하여 구함**

✓ **미소 면적에 작용하는 힘 :**

$$d\mathbf{F} = P_{Fluid} \cdot d\mathbf{S} = P_{Fluid} \cdot \mathbf{n}dS = -\rho g z \cdot \mathbf{n}dS$$

✓ Total force :

$$\mathbf{F} = \iint_{S_B} P_{Fluid}\mathbf{n}dS \Rightarrow \boxed{\mathbf{F} = -\rho g \iint_{S_B} \mathbf{n} z dS}$$

- Hydrostatic Moment : **(모멘트)=(거리)** X **(힘)**

✓ **미소 면적에 작용하는 모멘트 :**

$$d\mathbf{M} = \mathbf{r} \times d\mathbf{F} = \mathbf{r} \times P_{Fluid}\mathbf{n}dS = (\mathbf{r} \times \mathbf{n}) P_{Fluid} dS$$

✓ Total moment :

$$\mathbf{M} = \iint_{S_B} P_{Fluid} (\mathbf{r} \times \mathbf{n}) dS \Rightarrow \boxed{\mathbf{M} = -\rho g \iint_{S_B} (\mathbf{r} \times \mathbf{n}) z dS}$$

왜 r이 먼저 오는가? (좌표축에서 양의 방향을 고려함)

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(pp.458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(pp.414~417)

Hydrostatic Force

✓ Hydrostatic force (Surface force)

$$\mathbf{F} = -\rho g \iint_{S_B} \mathbf{n} z dS$$

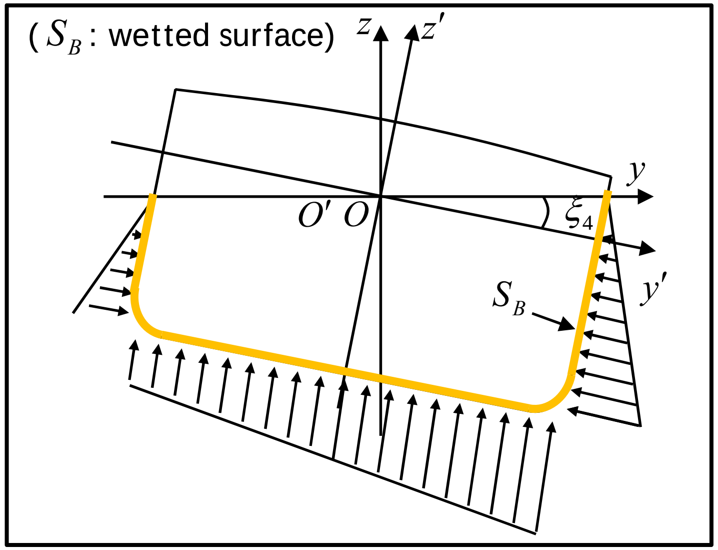
Divergence theorem[1] 사용하면,

$$\left(\iiint_V \nabla f dV = \iint_S f \cdot \mathbf{n} dA \right)$$

$$\mathbf{F} = \rho g \iiint_V \nabla z dV \qquad \left(\nabla z \overset{2)}{=} \frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} + \frac{\partial z}{\partial z}\mathbf{k} = \mathbf{k} \right)$$

$$= \mathbf{k} \rho g \iiint_V dV$$

$$= \mathbf{k} \rho g V(t)$$



$$\therefore \mathbf{F} = \mathbf{k} \rho g \iiint_V dV$$

: 유체에 잠긴 물체가 배제한(displaced) 유체의 무게만큼의 위쪽 방향으로 받는 힘(부력)
(Archimedes' Principle)

※ (−)부호가 사라진 이유
: Divergence theorem은 면의 외향 단위 벡터를 기준으로 한다.
부력 계산시 사용하는 Normal vector는 내향 단위 벡터이므로,
(−)를 곱한 뒤, Divergence theorem을 적용해야 한다.

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(pp.458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(pp.414~417)

Hydrostatic Moment

✓ Hydrostatic moment

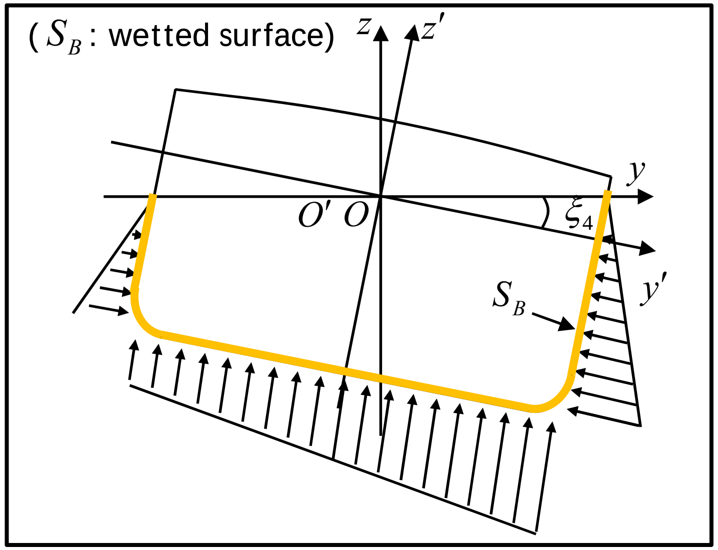
$$\mathbf{M} = -\rho g \iint_{S_B} (\mathbf{r}\times\mathbf{n}) z\,dS = \rho g \iint_{S_B} (\mathbf{n}\times\mathbf{r}) z\,dS$$

Divergence theorem[1] 사용하면,

$$\left(\iiint_V \nabla\times\mathbf{F}\,dV = \iint_S \mathbf{n}\times\mathbf{F}\,dA \right)$$

$$\mathbf{M} = -\rho g \iiint_V (\nabla\times\mathbf{r}) z\,dV$$

Normal Vector의 방향이 반대이므로
(-)부호를 붙여줌



$$\left(\nabla\times\mathbf{r}\,z \stackrel{2)}{=} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xz & yz & z^2 \end{vmatrix} = \mathbf{i}\left(\frac{\partial}{\partial y}z^2 - \frac{\partial}{\partial z}yz\right) + \mathbf{j}\left(\frac{\partial}{\partial z}xz - \frac{\partial}{\partial x}z^2\right) + \mathbf{k}\left(\frac{\partial}{\partial x}yz - \frac{\partial}{\partial y}xz\right) = -\mathbf{i}y + \mathbf{j}x \right)$$

$$\therefore \mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x]\,dV$$

← 의미 ?

Hydrostatic Moment (횡 경사)

✓ Hydrostatic moment

$$\mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV = \rho g \iiint_V [\mathbf{i}y - \mathbf{j}x] dV$$

$$= \rho g \iiint_V \mathbf{i} y dV - \rho g \iiint_V \mathbf{j} x dV = \mathbf{i}\boxed{\rho g V} y_B - \mathbf{j}\boxed{\rho g V} x_B = \mathbf{i}\boxed{M_{BT}} + \mathbf{j}\boxed{M_{BL}}$$

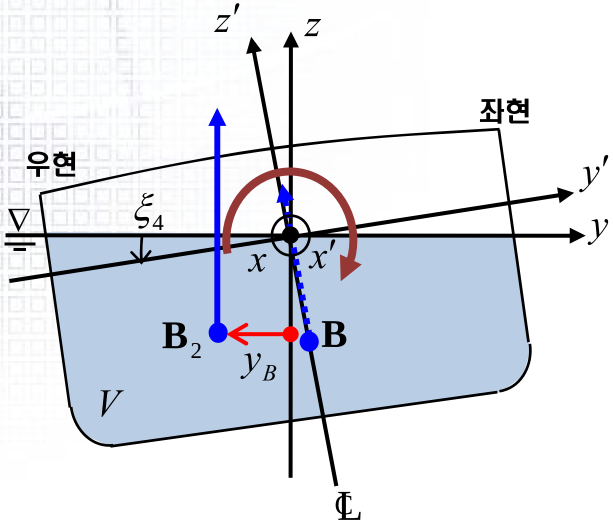
부력

부력에 의한 횡 방향 모멘트 | 부력에 의한 종 방향 모멘트

$$\left(M_{BL} = -\rho g V \cdot x_B\right)$$

부력의 횡 방향 중심 | 부력의 종 방향 중심

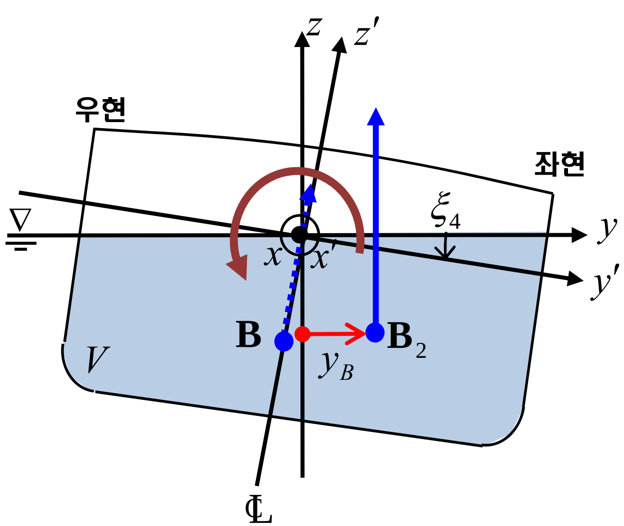
< 우현으로 기울었을 경우(+방향) >



$$\mathbf{M}_{BT} < 0$$

$y_B<0$ 이므로, 음의 모멘트

< 좌현으로 기울었을 경우(-방향) >



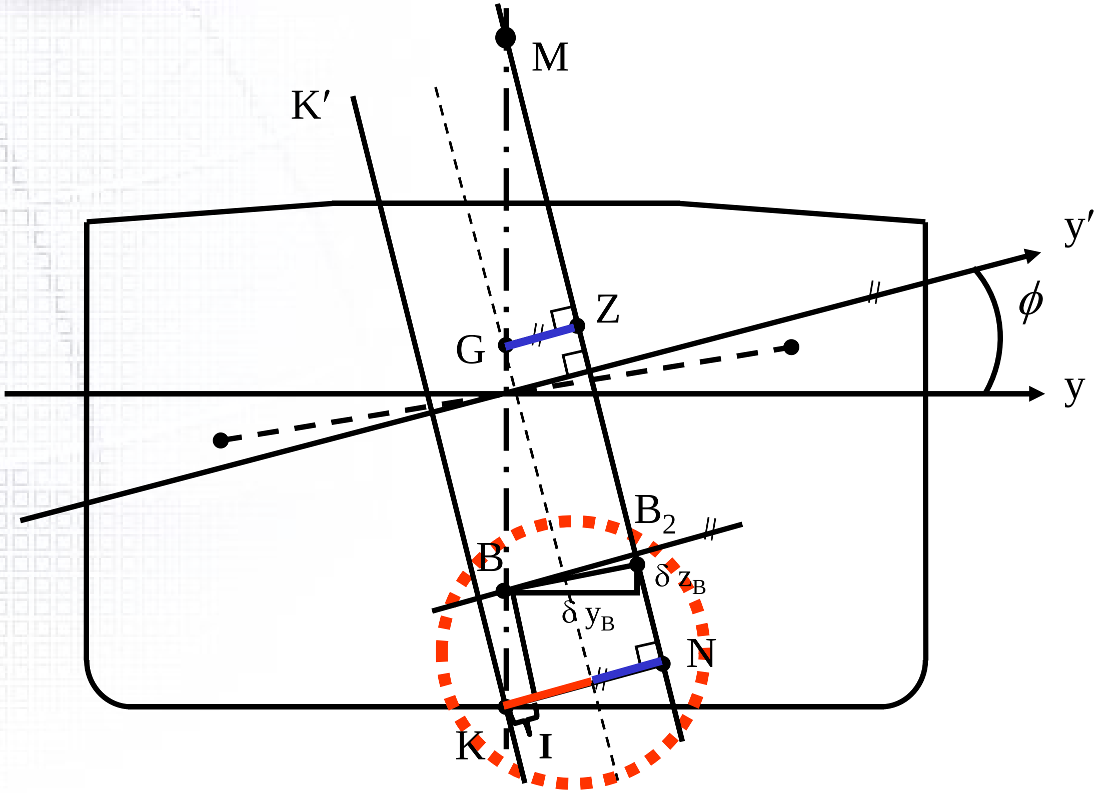
$$\mathbf{M}_{BT} > 0$$

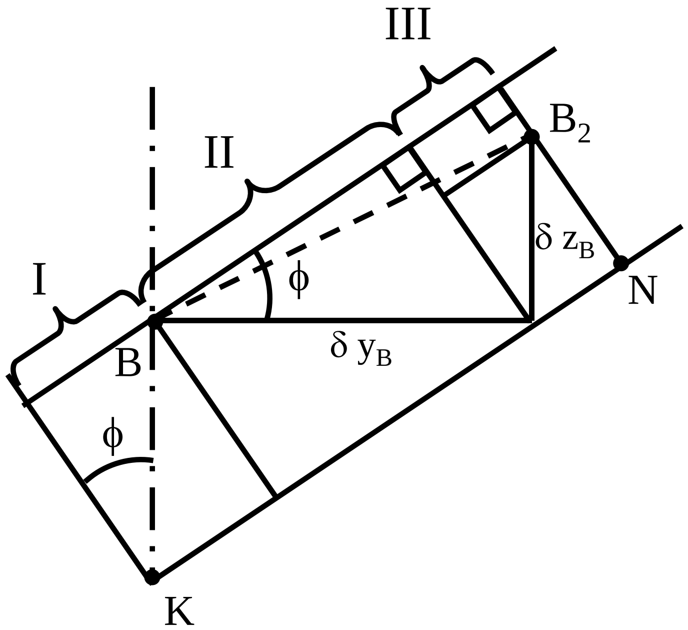
$y_B>0$ 이므로, 양의 모멘트

선박 안정론[1] (선박계산법)의 횡방향 부력 모멘트

1) 이규열, 노명일, 안재윤, 선박안정론, 5th, pp.96~97

부심 이동에 따른 복원 아암 *KN*





$$\mathbf{KN}(\phi) = I + II + III$$

$$= \mathbf{KB}\sin\phi + \delta y_B \cos\phi + \delta z_B \sin\phi$$

$$= \mathbf{KG}\sin\phi \;\; + \mathbf{GZ}$$

Hydrostatic Moment (횡 경사)

✓ Hydrostatic moment

$$\mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x]dV = \rho g \iiint_V [\mathbf{i}y - \mathbf{j}x]dV$$

$$= \rho g \iiint_V \mathbf{i}y dV - \rho g \iiint_V \mathbf{j}x dV = \mathbf{i}\boxed{\rho g V}\, y_B - \mathbf{j}\boxed{\rho g V}\, x_B = \mathbf{i}\boxed{M_{BT}} + \mathbf{j}\boxed{M_{BL}}$$

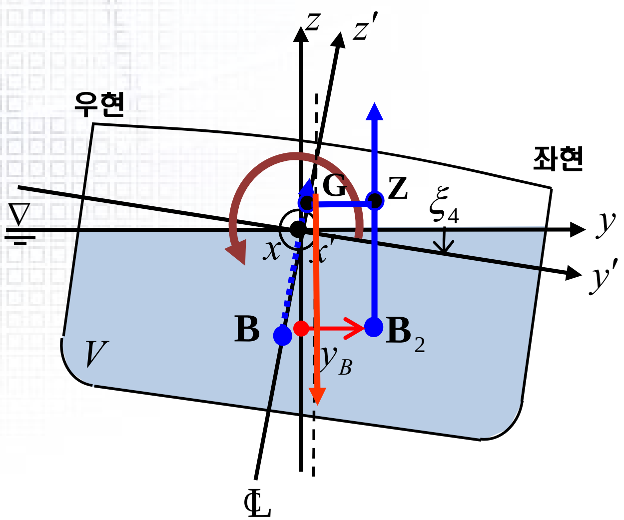
$$(M_{BL} = -\rho g V \cdot x_B)$$

부력

부력에 의한 **횡 방향 모멘트** | 부력에 의한 **종 방향 모멘트**

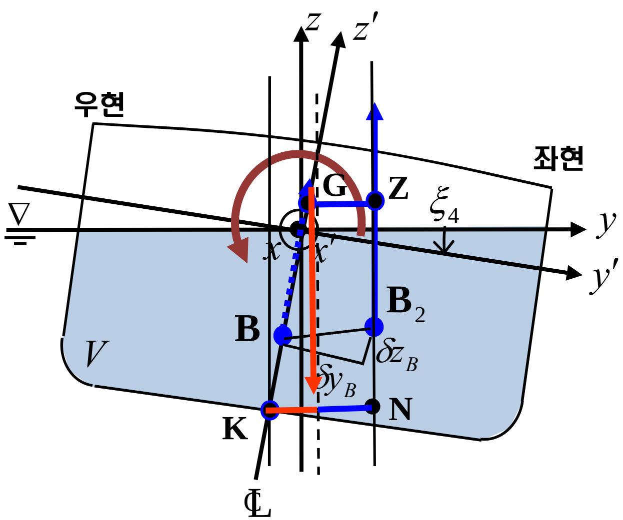
부력의 **횡 방향 중심** | 부력의 **종 방향 중심**

< 좌현으로 기울었을 경우(-방향) >



y_B : **수선면 고정 좌표계의 x축을 기준으로한 부력의 횡 방향 모멘트 암**

< 선박 안정론 > $KN = KG\sin\phi + GZ$



KN : **배 밑면의 중심 K를 통과하는 축을 기준으로한 부력의 횡 방향 모멘트 암**

(Q) **다른 것인가?**

(A) **중력에 의한 모멘트를 고려해 보면,모두 GZ가 횡 복원 모멘트 암이 된다.**

즉, 선박이 받는 횡 복원 모멘트는 동일하다.

Hydrostatic Moment (종 경사)

✓ Hydrostatic moment

$$\mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV = \rho g \iiint_V [\mathbf{i}y - \mathbf{j}x] dV$$

$$= \rho g \iiint_V \mathbf{i} y dV - \rho g \iiint_V \mathbf{j} x dV = \mathbf{i} \boxed{\rho g V} y_B - \mathbf{j} \boxed{\rho g V} x_B = \mathbf{i}\boxed{M_{BT}} + \mathbf{j}\boxed{M_{BL}}$$

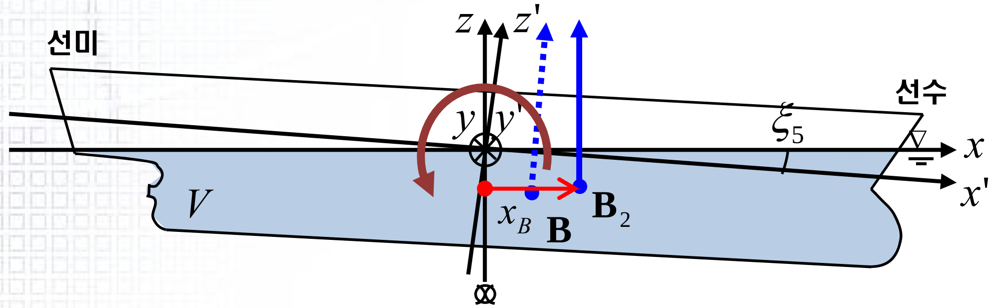
$$\left(M_{BL} = -\rho g V \cdot x_B\right)$$

부력

부력에 의한 횡 방향 모멘트 | 부력에 의한 종 방향 모멘트

부력의 횡 방향 중심 | 부력의 종 방향 중심

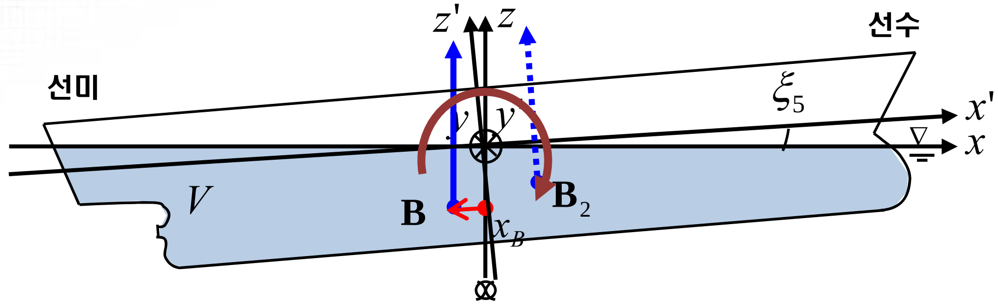
< 선수쪽으로 기울었을 경우(+방향) >



$\mathbf{M}_{BL} < 0$

x_B>0 이므로, 음의 모멘트

< 선미쪽으로 기울었을 경우(-방향) >



$\mathbf{M}_{BL} > 0$

x_B<0 이므로, 양의 모멘트

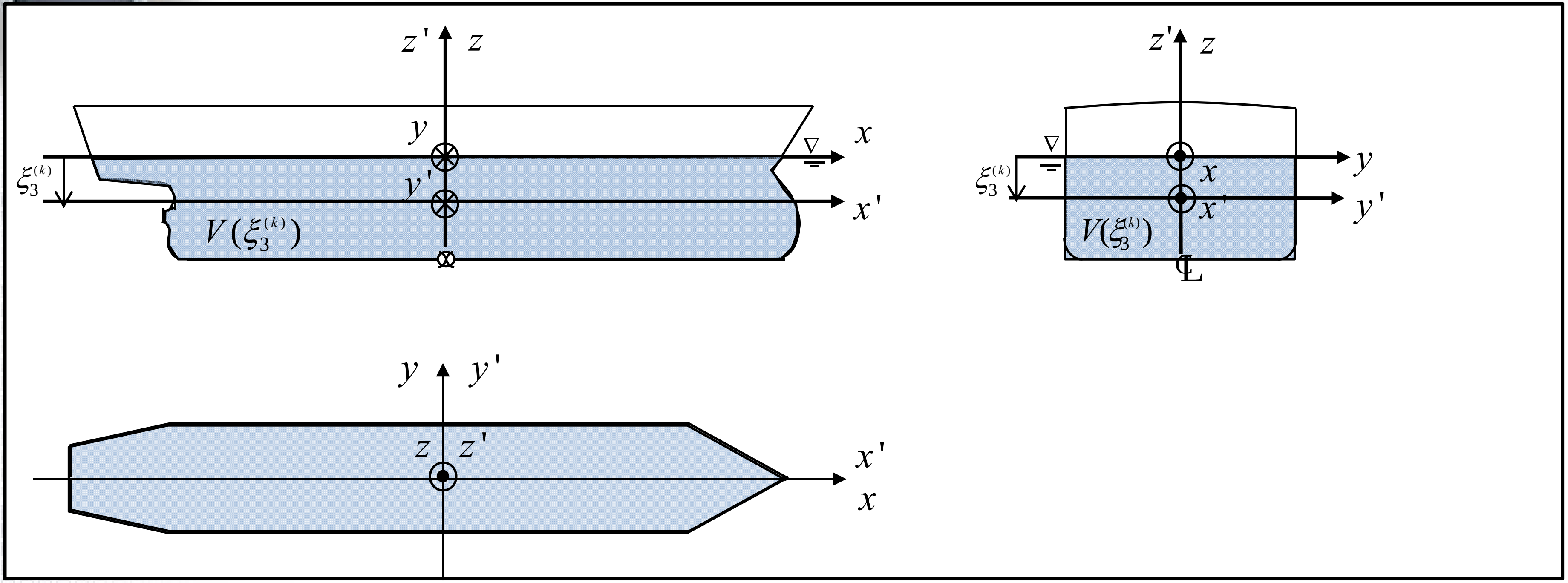
Linked Slides

- Pressure Integration Technique을 이용한 부력, 횡 방향 모멘트, 종 방향 모멘트 계산

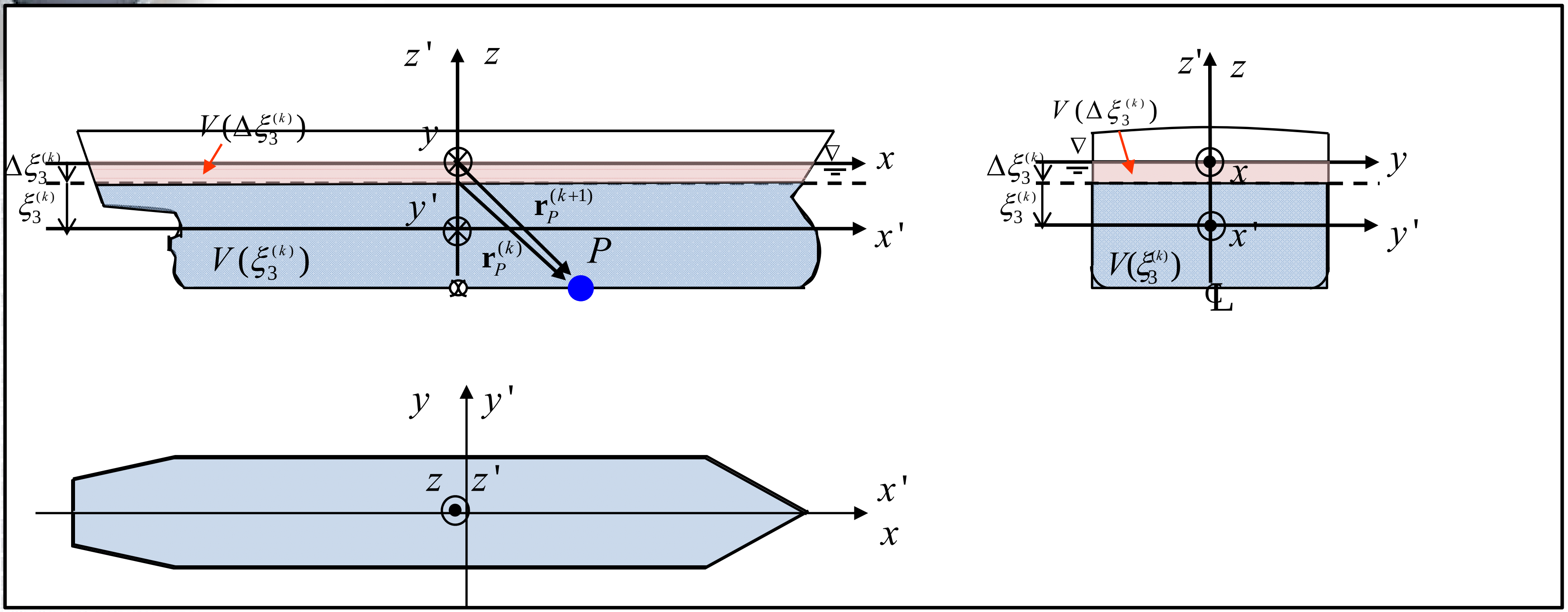
J.N.Newman, Marine Hydrodynamics, 1977, pp.290~295

Advanced
Ship
Design
Automation
Laboratory

Immersion에 의한 좌표계 변환



Immersion에 의한 좌표계 변환



ex) **선박이** immersion**하고 있다.** k**번째** immersion **상태에서 흘수가** 20 m **였다면,** k+1**번째 상태에서** 1m **가라 앉았을 때, 각각** k**번째 와** k+1**번째의 배 밑면의 좌표를 구하시오.**

sol) k**번째 에서는** $z_P^{(k)} = -20$

K+1**번째 에서는 배가 아래로** 1m **내려간 것이므로,** $z_P^{(k+1)} = -21$

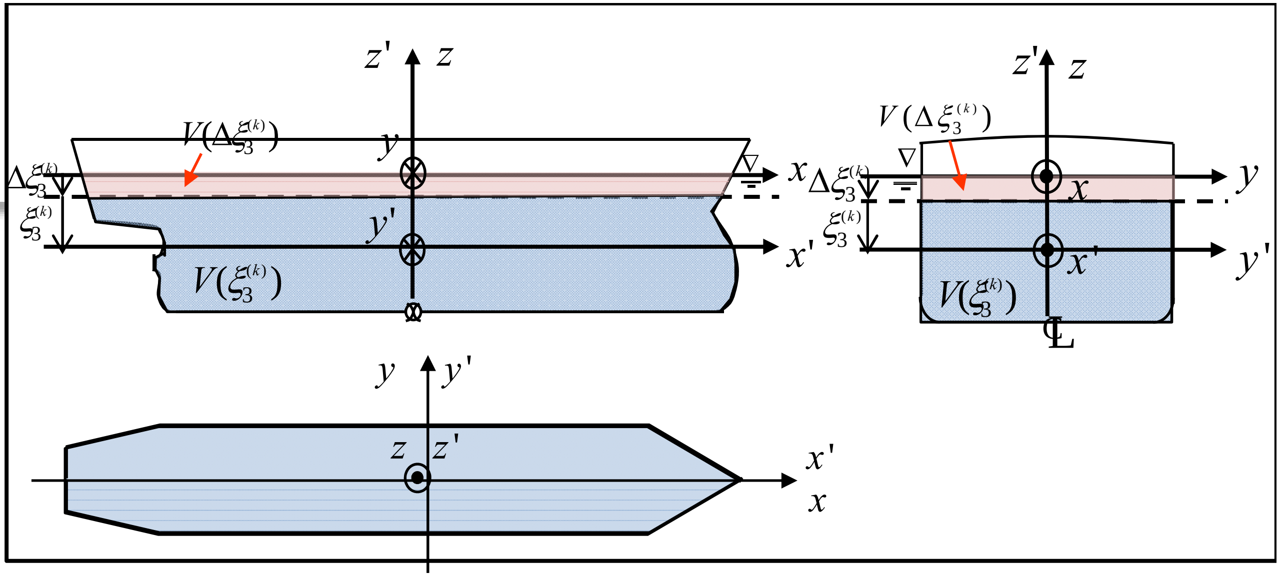
즉, 변위 $\Delta\xi_3^{(k)} = -1$**이라고 할 때,** $z_P^{(k)}$ **와** $z_P^{(k+1)}$ **사이에는 다음 관계가 성립한다.**

$$z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)}$$

$$\left(x_P^{(k+1)} = x_P^{(k)}, y_P^{(k+1)} = y_P^{(k)}\right)$$

x,y**는 좌표 동일함**

Immersion에 의한 힘 (부력) (1)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

$$\mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

(k)번째 상태의 부피

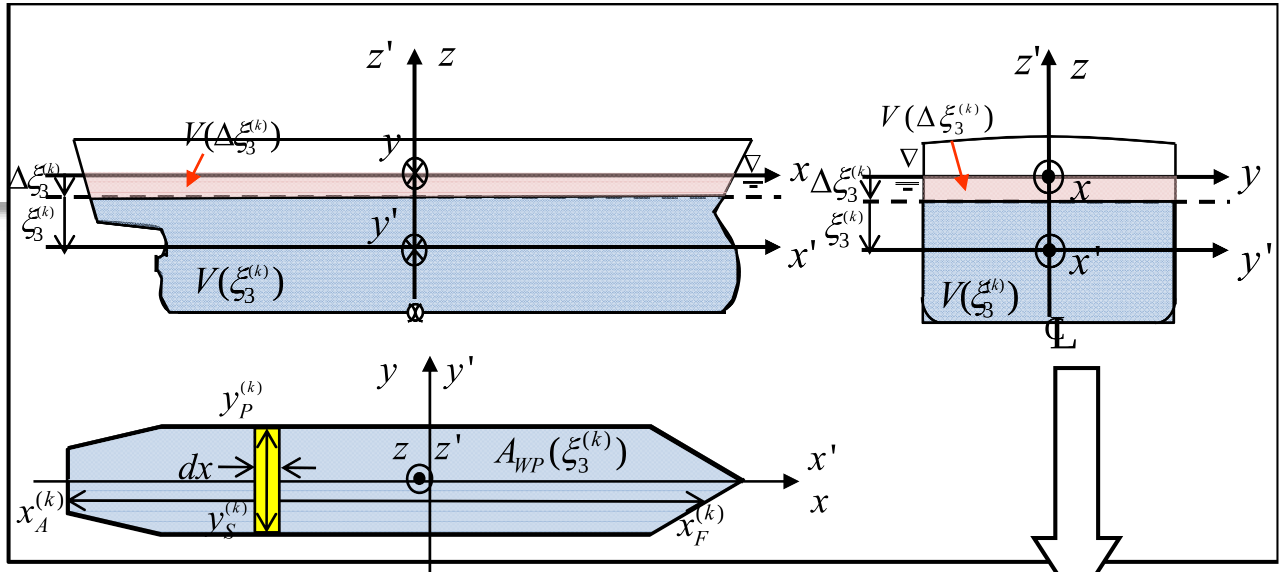
$$= \mathbf{k}\rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} dV = \mathbf{k}\rho g \left\{ \boxed{\iiint_{V(\xi_3^{(k)})} dV} + \iiint_{V(\Delta\xi_3^{(k)})} dV \right\}$$

$$\iiint_{V(\xi_3^{(k)})} dV = \iiint_{V(\xi_3^{(k)})} dxdydz = V(\xi_3^{(k)})$$

Immersion에 의한 힘 (부력) (2)

좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$



$$\mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

변화된 부피

$$= \mathbf{k}\rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_3^{(k)})} dV + \boxed{\iiint_{V(\Delta\xi_3^{(k)})} dV} \right\}$$

적분하기 편리하게 적분 순서를 변경

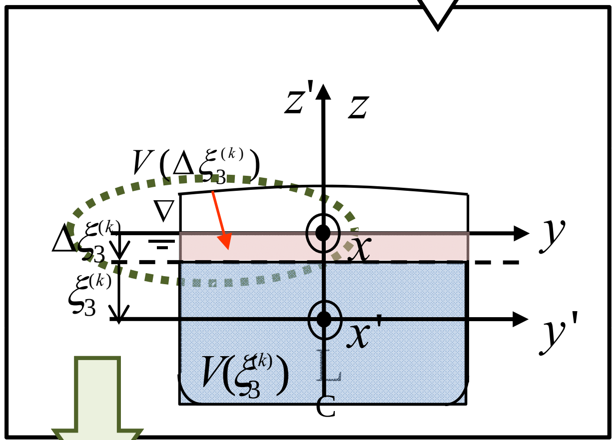
$$\iiint_{V(\Delta\xi_3^{(k)})} dV = \iiint_{V(\Delta\xi_3^{(k)})} dxdydz = \int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{z_L^{(k)}}^{z_U^{(k)}} dzdydx$$

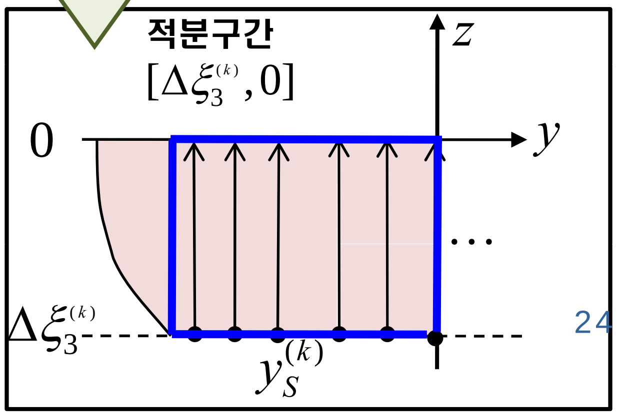
만약 $\Delta\xi_3^{(k)}$ 가 작다면,

$$\approx \int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{\Delta\xi_3^{(k)}}^{0} dzdydx = \int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} (-\Delta\xi_3^{(k)})\, dydx$$

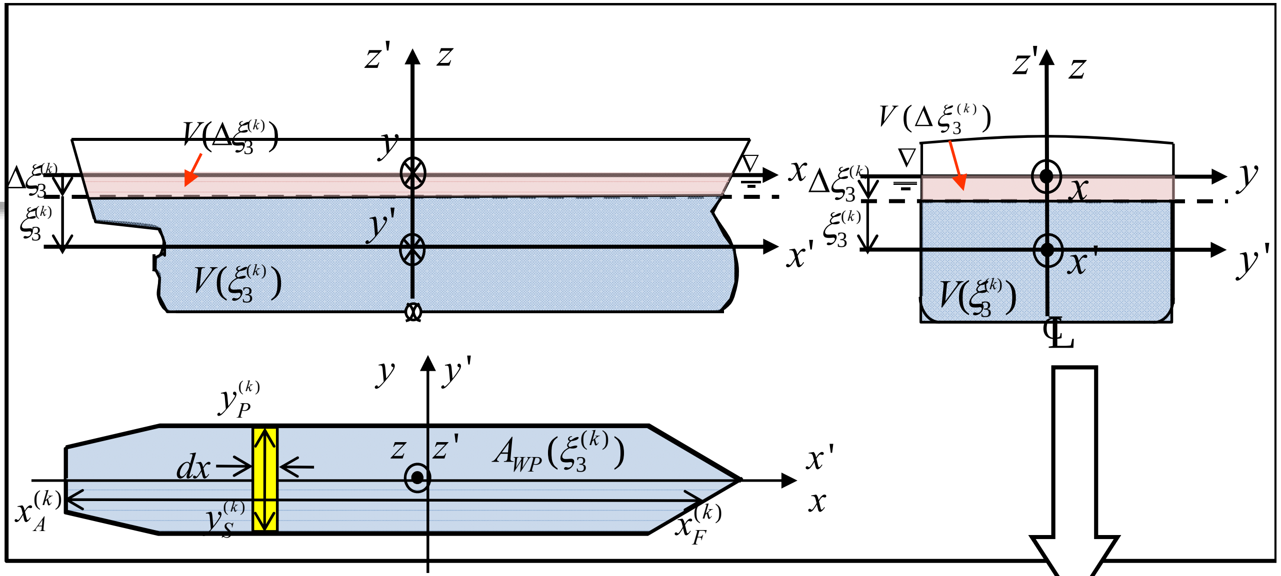
$$= \int_{x_A^{(k)}}^{x_F^{(k)}} (-\Delta\xi_3^{(k)}) \left[y_P^{(k)} - y_S^{(k)} \right] dx = -\Delta\xi_3^{(k)} \underbrace{\int_{x_A^{(k)}}^{x_F^{(k)}} \left[y_P^{(k)} - y_S^{(k)} \right] dx}_{\text{(k)번째 상태의 수선면적}}$$

$$= -\Delta\xi_3^{(k)} A_{WP}(\xi_3^{(k)})$$





Immersion에 의한 힘 (부력) (3)



좌표 변환

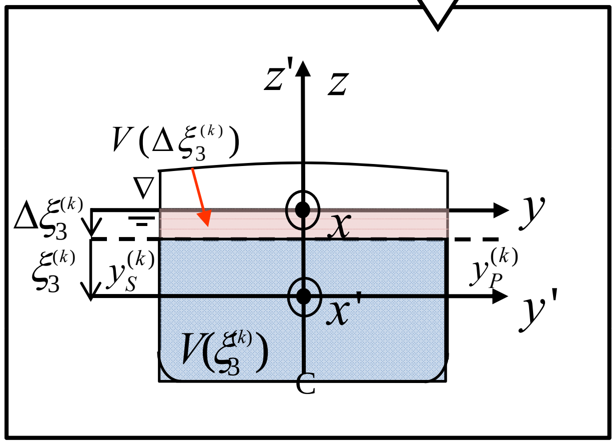
$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

$\mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$ **(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리**

변화된 부피

$$= \mathbf{k}\rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_3^{(k)})} dV + \boxed{\iiint_{V(\Delta\xi_3^{(k)})} dV} \right\}$$

$$\iiint_{V(\Delta\xi_3^{(k)})} dV \approx \underbrace{\underbrace{-\Delta\xi_3^{(k)}}_{(-)} \underbrace{A_{WP}(\xi_3^{(k)})}_{(+)}}_{(+)}$$



부호 검증

$A_{WP}(\xi_3^{(k)})$**는 항상 (+)**

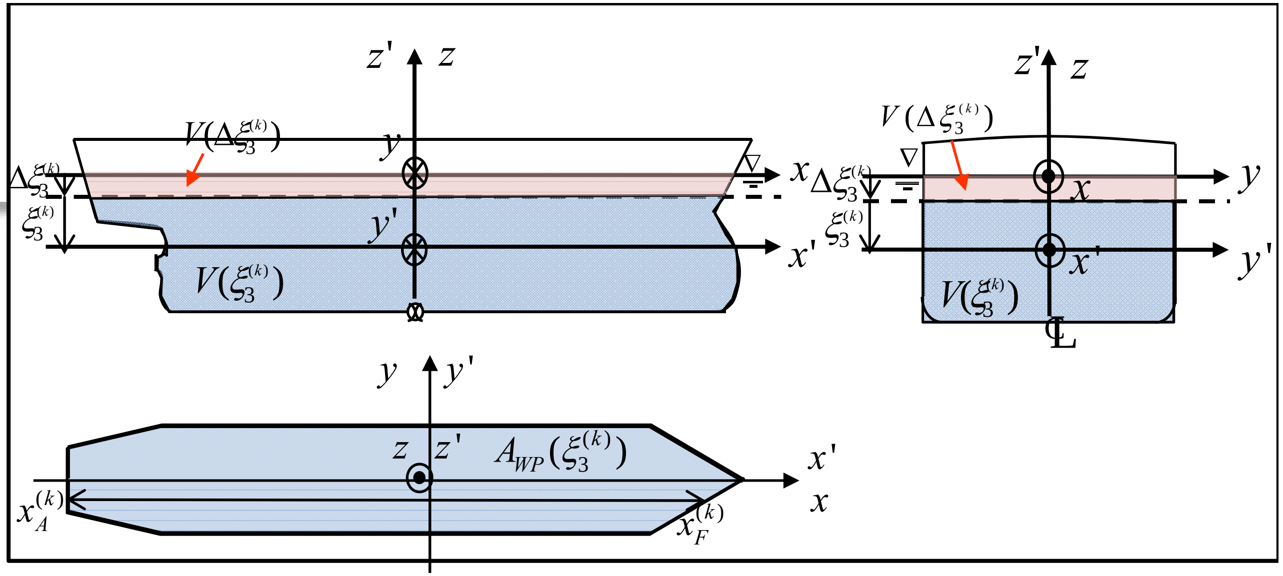
⇩

$\Delta\xi_3^{(k)}$**이 (-)이면, 가라앉는 것임.**

⇩

이 때 $\mathbf{F}$ 는 (+) ➡부력 증가

Immersion에 의한 힘 (부력) (4)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

$$\mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

$$= \mathbf{k}\rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_3^{(k)})} dV + \iiint_{V(\Delta\xi_3^{(k)})} dV \right\}$$

만약 $\Delta\xi_3^{(k)}$가 작다면,

$$\approx \boxed{\mathbf{k}\rho g(V(\xi_3^{(k)})} - \Delta\xi_3^{(k)} \underline{A_{WP}(\xi_3^{(k)})})$$

(k)번째 상태의 수선면적

$$\boxed{\mathbf{F}_B(\xi_3^{(k)})} = \mathbf{k}\rho g V(\xi_3^{(k)})$$

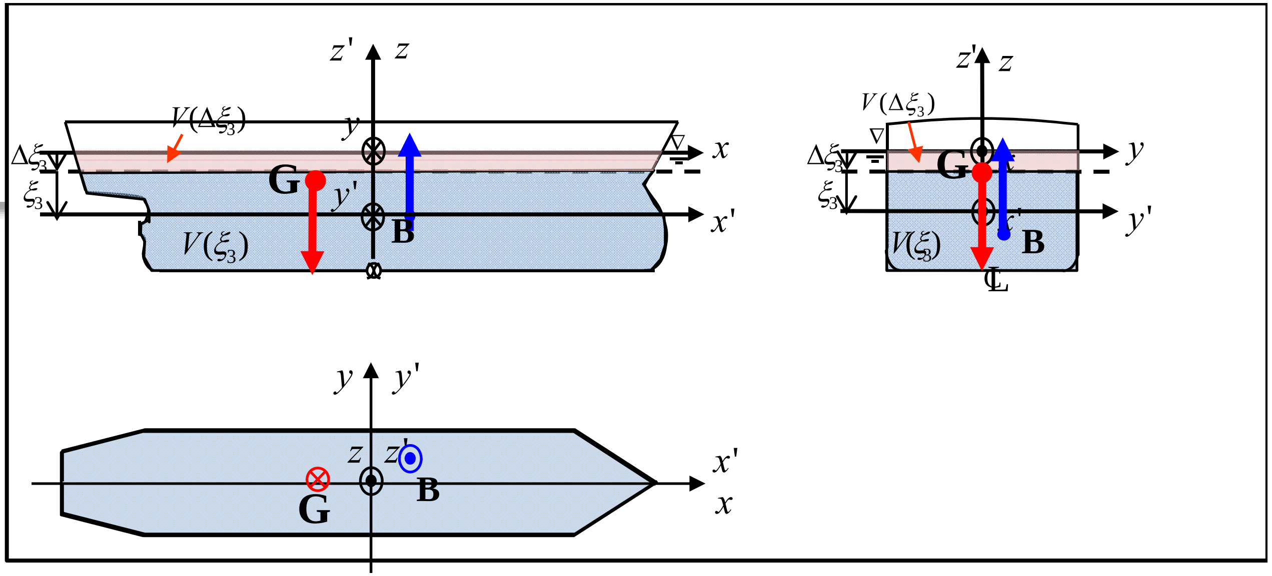
대입하면,

$$\mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{F}_B(\xi_3^{(k)}) - \mathbf{k}\rho g A_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

NAOE/SNU

@SDAL Advanced Ship Design Automation Lab. http://asdal.snu.ac.kr

Immersion에 의한 힘 (부력) (5)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

✓ 부력

$$\mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{F}_B(\xi_3^{(k)}) - \mathbf{k}\rho g A_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

✓ 중력

$$\mathbf{F}_G(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = -\mathbf{k}mg = \mathbf{F}_G(\xi_3^{(k)})$$

✓ 선박이 받는 힘

$$\mathbf{F}(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{F}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) + \mathbf{F}_G(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

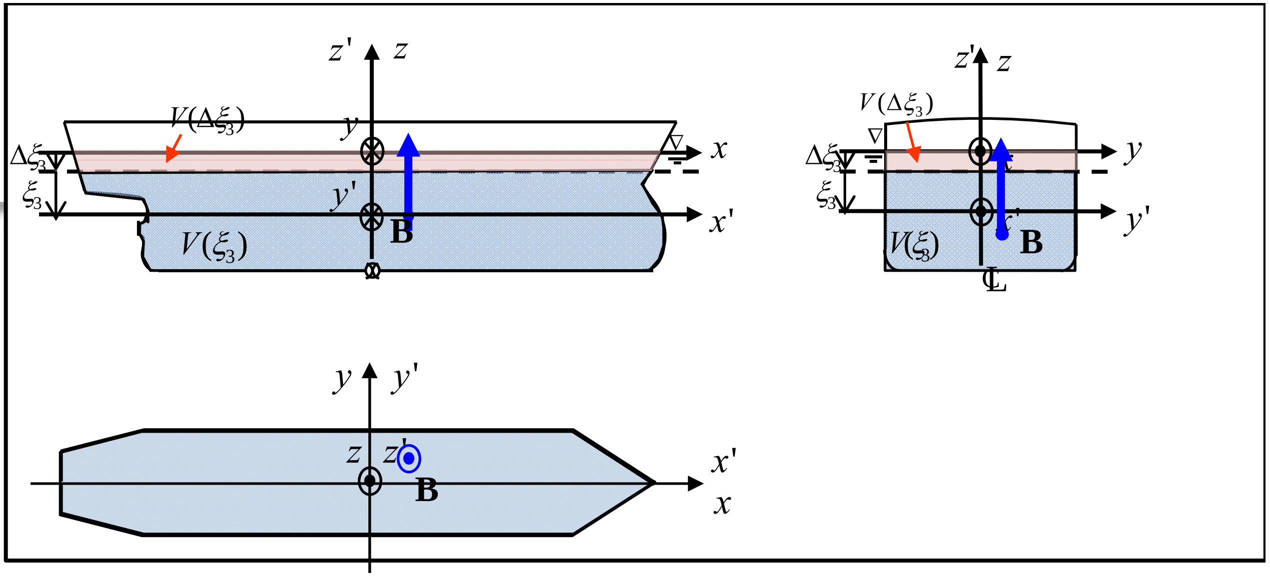
$$= \mathbf{F}_B(\xi_3^{(k)}) + \mathbf{F}_G(\xi_3^{(k)}) - \mathbf{k}\rho g A_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)} = \mathbf{F}(\xi_3^{(k)}) - \mathbf{k}\rho g A_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

$$\mathbf{F}(\xi_3^{(k)} + \Delta\xi_3^{(k)}) - \mathbf{F}(\xi_3^{(k)}) = \mathbf{k}\left\{\boxed{-\rho g A_{WP}(\xi_3^{(k)})}\Delta\xi_3^{(k)}\right\}$$

$$\Delta\mathbf{F}(\xi_3^{(k)}) = \mathbf{k}\boxed{\left.\frac{\partial F}{\partial \xi_3}\right|_{\xi_3^{(k)}}}\Delta\xi_3^{(k)}$$

$$\left.\frac{\partial F}{\partial \xi_3}\right|_{\xi_3^{(k)}} = -\rho g A_{WP}(\xi_3^{(k)})$$

Immersion에 의한 모멘트 (1)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

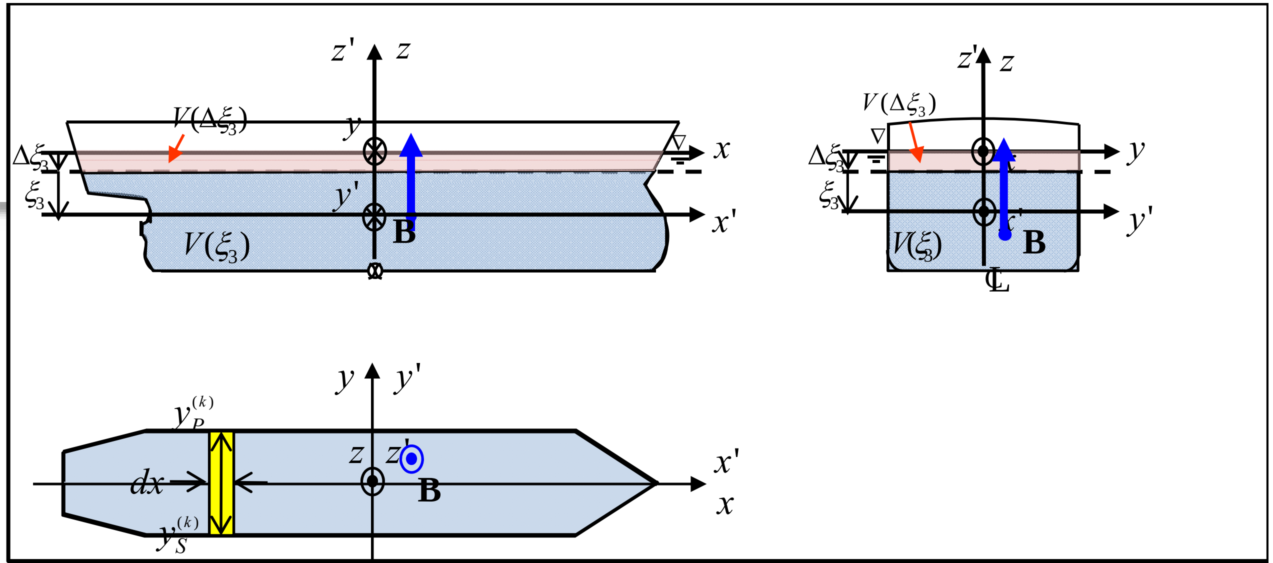
$$\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV = \rho g \left\{ \boxed{\iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV} + \iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV \right\}$$

K번째 상태의 부피

$$\iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV = \iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dxdydz = \mathbf{i}\iiint_{V(\xi_3^{(k)})} y\,dxdydz - \mathbf{j}\iiint_{V(\xi_3^{(k)})} x\,dxdydz$$

$$= \mathbf{i}V(\xi_3^{(k)})y_B^{(k+1)} - \mathbf{j}V(\xi_3^{(k)})x_B^{(k+1)} = \mathbf{i}V(\xi_3^{(k)})y_B^{(k)} - \mathbf{j}V(\xi_3^{(k)})x_B^{(k)}$$

Immersion에 의한 모멘트 (2)



좌표 변환

$$
\begin{pmatrix}
x_P^{(k+1)} = x_P^{(k)} \\
y_P^{(k+1)} = y_P^{(k)} \\
z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)}
\end{pmatrix}
$$

$$\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

$$= \rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV} \right\}$$

$$\iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dxdydz$$

변화된 부피

만약 $\Delta\xi_3^{(k)}$가 작다면

$$\approx \int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{\Delta\xi_3^{(k)}}^{0} [\mathbf{i}y - \mathbf{j}x]dzdydx = \int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} [\mathbf{i}yz - \mathbf{j}xz]_{\Delta\xi_3^{(k)}}^{0} dydx = \int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \left\{\mathbf{i}y(-\Delta\xi_3^{(k)}) - \mathbf{j}x(-\Delta\xi_3^{(k)})\right\} dydx$$

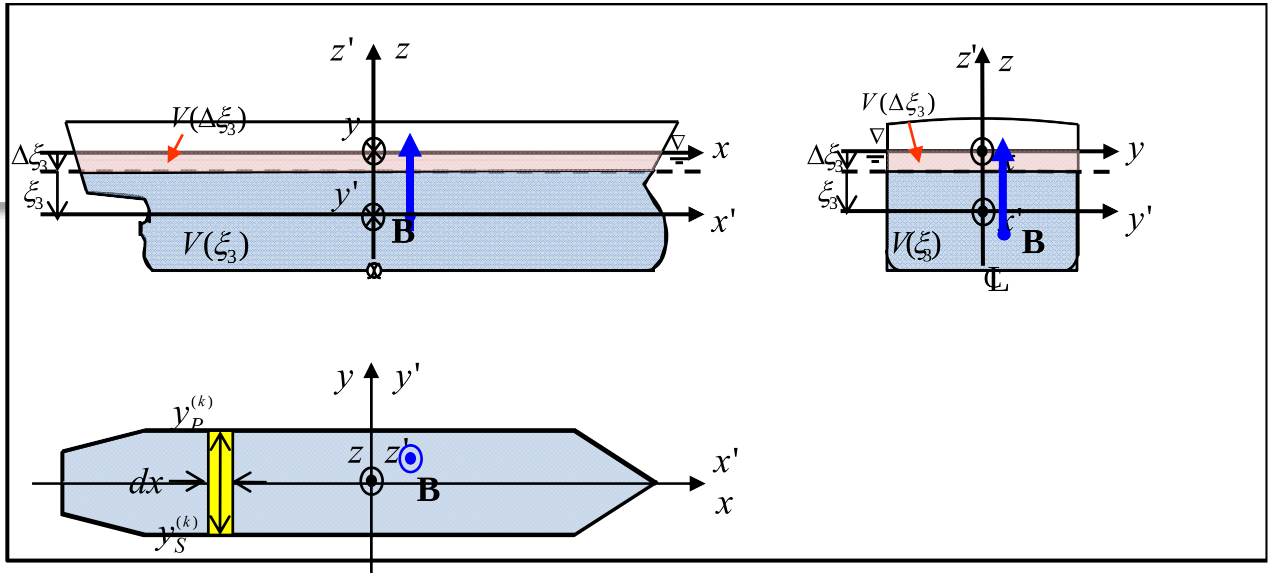
$$= -\Delta\xi_3^{(k)} \int_{x_A^{(k)}}^{x_F^{(k)}} \left\{ \mathbf{i}\left.\frac{y^2}{2}\right|_{y_S^{(k)}}^{y_P^{(k)}} - \mathbf{j}\left.xy\right|_{y_S^{(k)}}^{y_P^{(k)}} \right\} dx = -\Delta\xi_3^{(k)} \int_{x_A^{(k)}}^{x_F^{(k)}} \left\{ \mathbf{i}\frac{(y_P^{(k)})^2 - (y_S^{(k)})^2}{2} - \mathbf{j}x(y_P^{(k)} - y_S^{(k)}) \right\} dx$$

$$= -\Delta\xi_3^{(k)} \left\{ \int_{x_A^{(k)}}^{x_F^{(k)}} \mathbf{i}\underbrace{\frac{(y_P^{(k)})^2 - (y_S^{(k)})^2}{2}dx}_{\left[\frac{y_P^{(k)} + y_S^{(k)}}{2}\right]\times[y_P^{(k)} - y_S^{(k)}]dx} - \int_{x_A^{(k)}}^{x_F^{(k)}} \underbrace{\mathbf{j}x(y_P^{(k)} - y_S^{(k)})dx}_{x\times[y_P^{(k)} - y_S^{(k)}]dx} \right\} = -\Delta\xi_3^{(k)}(\mathbf{i}\underline{T_{WP}(\xi_3^{(k)})} - \mathbf{j}\underline{L_{WP}(\xi_3^{(k)})})$$

횡 방향 중심　　수선면적 | 종 방향 중심　　수선면적

수선면의 횡 방향 1차 모멘트 | 수선면의 종 방향 1차 모멘트

Immersion에 의한 모멘트 (3)



좌표 변환

$$
\begin{pmatrix}
x_P^{(k+1)} = x_P^{(k)} \\
y_P^{(k+1)} = y_P^{(k)} \\
z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)}
\end{pmatrix}
$$

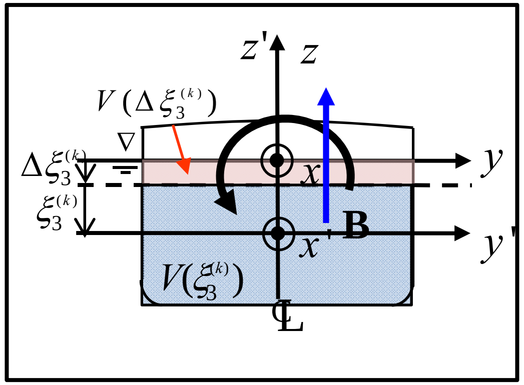
$$
\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})
= \rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\, dV = \rho g \left\{ \iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\, dV + \underbrace{\iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\, dV}_{\text{변화된 부피}} \right\}
$$

$$
\iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\, dV = \iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]\, dxdydz
$$

만약 $\Delta\xi_3^{(k)}$ 가 작다면,

수선면의 횡 방향 1차 모멘트 | 수선면의 종 방향 1차 모멘트

$$
\approx \underbrace{-\underbrace{\Delta\xi_3^{(k)}}_{(-)} (\mathbf{i}\underbrace{T_{WP}(\xi_3^{(k)})}_{(+)}}_{(+)} - \mathbf{j}L_{WP}(\xi_3^{(k)}))
$$



부호 검증

$T_{WP}(\xi_3^{(k)})$가 (+)이면, 수선면 중심이 좌현쪽

⇩

$\Delta\xi_3^{(k)}$ 이 (-)이면, 가라앉는 것임.

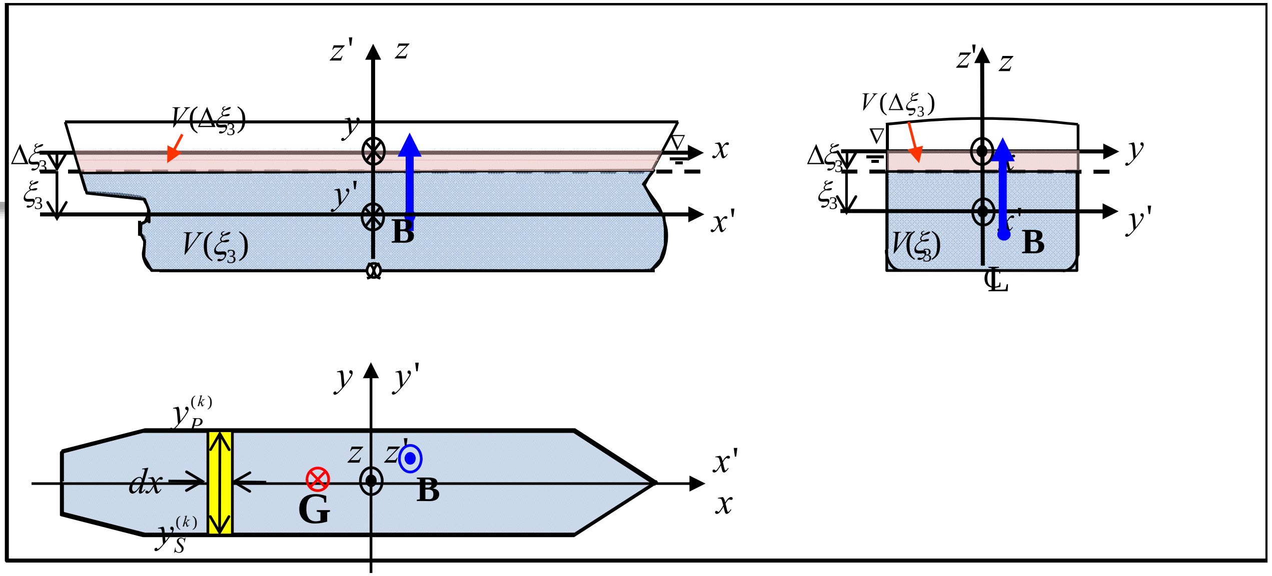
⇩

이 때, 수선면 중심이 좌현쪽이기 때문에,
부력 중심도 좌현쪽 (미소 자세)

⇩

x 축에 대한 (+)의 모멘트

Immersion에 의한 모멘트 (4)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

$$= \rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV} \right\}$$

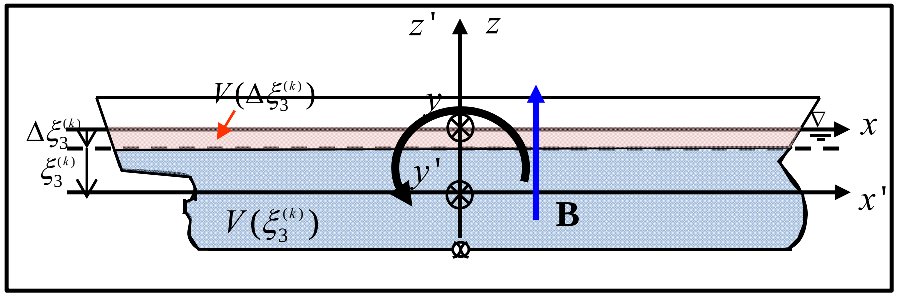
변화된 부피

$$\iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dxdydz$$

수선면의 횡 방향 1차 모멘트 | 수선면의 종 방향 1차 모멘트

만약 ξ_3가 작다면,

$$\approx \underbrace{\underbrace{-\Delta\xi_3^{(k)}}_{(-)}(\mathbf{i}T_{WP}(\xi_3^{(k)}) - \mathbf{j}\underbrace{L_{WP}(\xi_3^{(k)})}_{(+)})}_{(-)}$$



부호 검증

$L_{WP}(\xi_3^{(k)})$가 (+)이면, 수선면 중심이 선수쪽

⇩

$\Delta\xi_3^{(k)}$ 이 (-)이면, 가라앉는 것임.

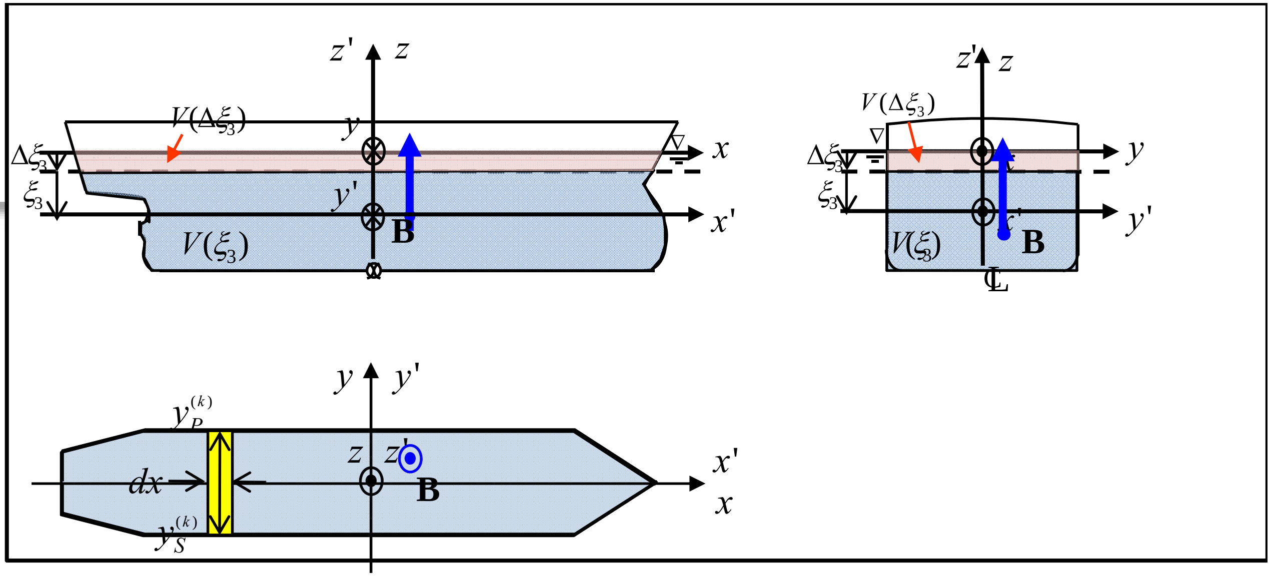
⇩

이 때, 수선면 중심이 선수쪽이기 때문에,
부력 중심도 선수쪽 (미소 자세)

⇩

y 축에 대한 (-)의 모멘트

Immersion에 의한 모멘트 (5)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

$$= \rho g \iiint_{V(\xi_3^{(k)}+\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV + \iiint_{V(\Delta\xi_3^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV \right\}$$

만약 $\Delta\xi_3^{(k)}$가 작다면,

$$\approx \boxed{\rho g \left\{ \left[\mathbf{i}V(\xi_3^{(k)})y_B^{(k)} - \mathbf{j}V(\xi_3^{(k)})x_B^{(k)} \right]\right.} - \Delta\xi_3^{(k)} \left. \left[\mathbf{i}\underline{T_{WP}(\xi_3^{(k)})} - \mathbf{j}\underline{L_{WP}(\xi_3^{(k)})} \right] \right\}$$

대입

수선면의 횡 방향 1차 모멘트

수선면의 종 방향 1차 모멘트

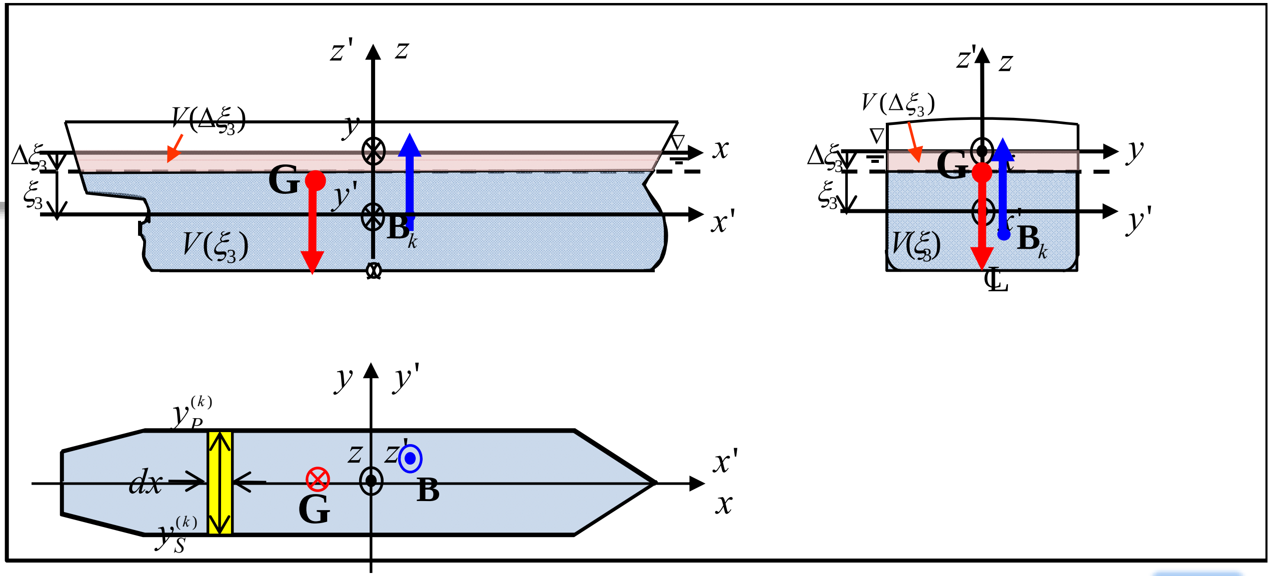
$$\boxed{\mathbf{M}_B(\xi_3^{(k)})} = \rho g \left\{ \mathbf{i}V(\xi_3^{(k)})y_B^{(k)} - \mathbf{j}V(\xi_3^{(k)})x_B^{(k)} \right\}$$

$$\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{M}_B(\xi_3^{(k)}) - \mathbf{i}\rho g T_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)} + \mathbf{j}\rho g L_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

NAOE/SNU

@SDAL Advanced Ship Design Automation Lab. http://asdal.snu.ac.kr

Immersion에 의한 모멘트 (6)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} + \Delta\xi_3^{(k)} \end{pmatrix}$$

✓ 부력에 의한 모멘트

$$\mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{M}_B(\xi_3^{(k)}) - \mathbf{i}\rho g T_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)} + \mathbf{j}\rho g L_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

✓ 중력에 의한 모멘트

$$\mathbf{M}_G(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{r}_G^{(k+1)} \times (-\mathbf{k}mg) = -\mathbf{i}mg \cdot y_G^{(k+1)} + \mathbf{j}mg \cdot x_G^{(k+1)}$$

$$= -\mathbf{i}mg \cdot y_G^{(k)} + \mathbf{j}mg \cdot x_G^{(k)} = \mathbf{M}_G(\xi_3^{(k)})$$

$$\left(\mathbf{r}_G^{(k+1)} \times (-\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G^{(k+1)} & y_G^{(k+1)} & z_G^{(k+1)} \\ 0 & 0 & -1 \end{vmatrix} = -\mathbf{i}y_G^{(k+1)} + \mathbf{j}x_G^{(k+1)} \right)$$

✓ 선박이 받는 모멘트

$$\mathbf{M}(\xi_3^{(k)} + \Delta\xi_3^{(k)}) = \mathbf{M}_B(\xi_3^{(k)} + \Delta\xi_3^{(k)}) + \mathbf{M}_G(\xi_3^{(k)} + \Delta\xi_3^{(k)})$$

$$= \underline{\mathbf{M}_B(\xi_3^{(k)}) + \mathbf{M}_G(\xi_3^{(k)})} - \mathbf{i}T_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)} + \mathbf{j}L_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

$$= \mathbf{M}(\xi_3^{(k)}) - \mathbf{i}T_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)} + \mathbf{j}L_{WP}(\xi_3^{(k)})\Delta\xi_3^{(k)}$$

$$\left.\frac{\partial \mathbf{M}_T}{\partial \xi_3}\right|_{\xi_3^{(k)}} = -\rho g T_{WP}(\xi_3^{(k)})$$

$$\left.\frac{\partial \mathbf{M}_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} = \rho g L_{WP}(\xi_3^{(k)})$$

$$\mathbf{M}(\xi_3^{(k)} + \Delta\xi_3^{(k)}) - \mathbf{M}(\xi_3^{(k)}) = \mathbf{i}\left\{\boxed{-\rho g T_{WP}(\xi_3^{(k)})}\Delta\xi_3^{(k)}\right\} + \mathbf{j}\left\{\boxed{\rho g L_{WP}(\xi_3^{(k)})}\Delta\xi_3^{(k)}\right\}$$

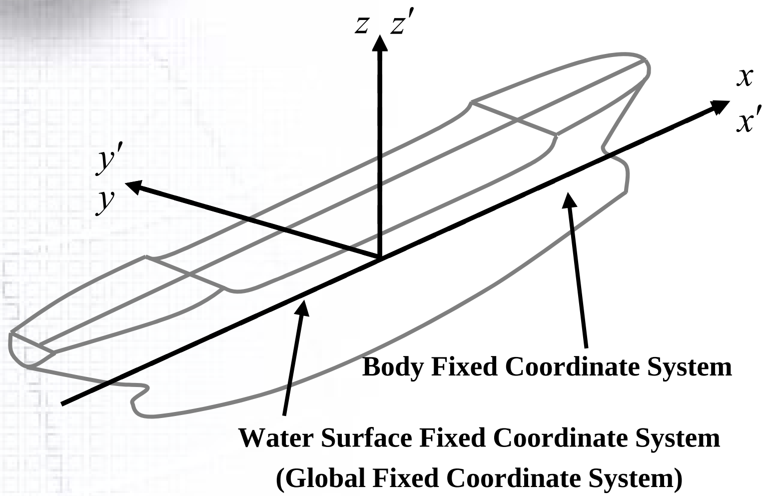
$$\Delta\mathbf{M} = \mathbf{i}\boxed{\left.\frac{\partial \mathbf{M}_T}{\partial \xi_3}\right|_{\xi_3^{(k)}}}\Delta\xi_3^{(k)} + \mathbf{j}\boxed{\left.\frac{\partial \mathbf{M}_L}{\partial \xi_3}\right|_{\xi_3^{(k)}}}\Delta\xi_3^{(k)}$$

Supplementary Slides

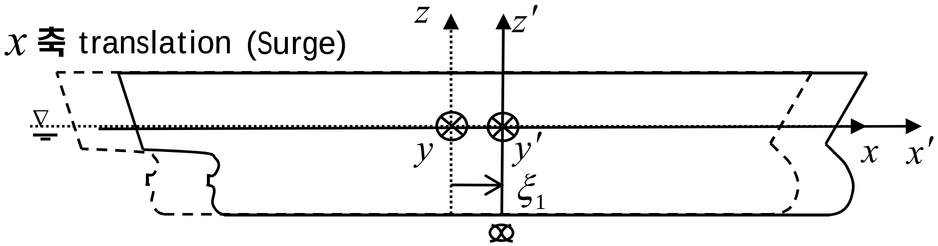
Coordinate System

x' 축 - 원점: Midship, (+): 선수
y' 축 - 원점: Centerline, (+): 좌현
z' 축 - 원점: 수선면, (+): 선박의 위

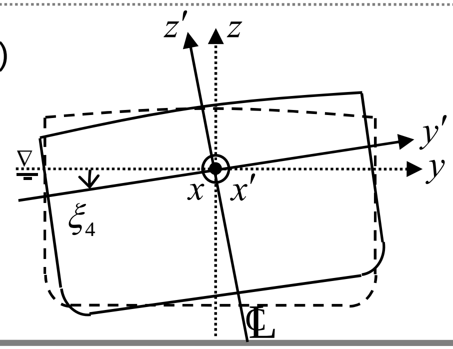
x 축 - 원점: Midship, (+): x'축을 포함하고 수선면과 직교인 평면과 수선면 사이의 교선
y축 - 원점: Centerline, (+): z축과 x축의 외적 방향
z 축 - 원점: 수면, (+): 수선면에 수직한 위 방향



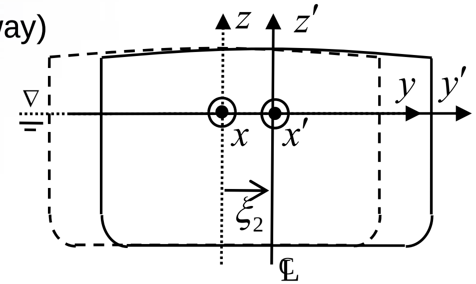
x 축 translation (Surge)



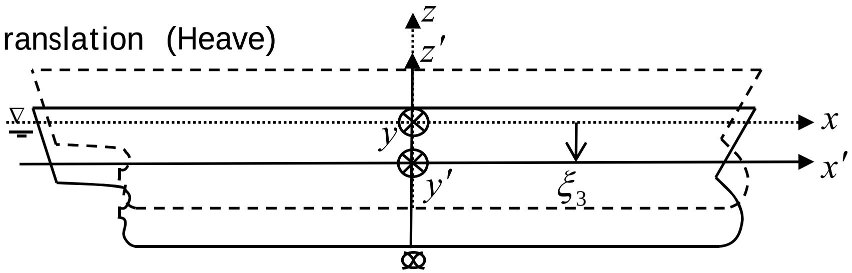
x 축 rotation (Roll)



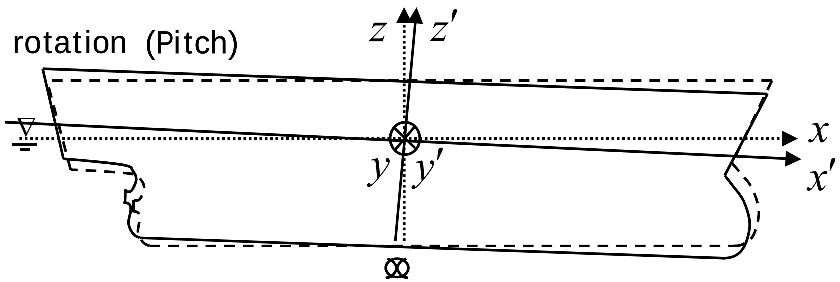
y 축 translation (Sway)



z 축 translation (Heave)



y 축 rotation (Pitch)



z 축 rotation (Yaw)