[2008] [06-2]

Planning Procedure of Naval Architecture & Ocean Engineering

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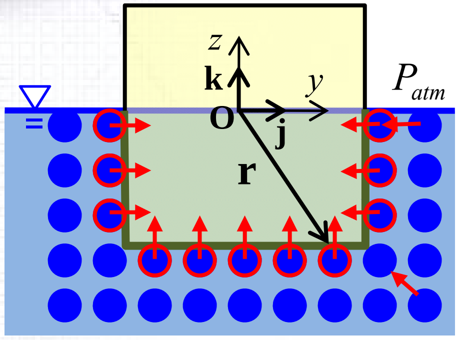
Part 1. Stability & Trim

[06-2] Pressure integration technique (2)

유체 중에 잠긴 물체가 받는 힘과 모멘트



S_B : 침수표면

V : 침수부피

유체 입자가 주위 유체 입자에 작용하는 정적인 압력

Bernoulli Eq.

$$\rho\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left|\nabla\Phi\right|^2+\rho g\,z+\bigcirc\!\!\!\!\!P=C$$

$$\bigcirc\!\!\!\!\!P=-\cancel{\rho\frac{\partial\Phi}{\partial t}}-\cancel{\frac{1}{2}\left|\nabla\Phi\right|^2}\underbrace{-\rho g\,z+C}_{\rightarrow P_{Static}}$$

정수 ➡ 유체의 속도 = 0

$$P_{Static}=-\rho gz+\bigcirc\!\!\!\!\!C\rightarrow P_{atm}$$

⇩

$$\underbrace{P_{Static}}_{\rightarrow P_{Fluid}(z)+\cancel{P_{atm}}}=-\rho gz+\cancel{P_{atm}}$$

⇩

$$P_{Fluid}(z)=-\rho gz$$

유체 입자들이 유체 중에 잠긴 물체에 작용하는 정적인 힘과 모멘트

$$\mathbf{F}=\iint_{S_B}(-\rho gz)d\mathbf{S}=-\rho g\iint_{S_B}z(\mathbf{n}dS)=-\rho g\iint_{S_B}\mathbf{n}zdS$$

$$\mathbf{M}=\iint_{S_B}\mathbf{r}\times(-\rho gz)d\mathbf{S}=-\rho g\iint_{S_B}\mathbf{r}\times z(\mathbf{n}dS)=-\rho g\iint_{S_B}(\mathbf{r}\times\mathbf{n})zdS$$

Divergence Theorem ⇨

$$\mathbf{F}=\mathbf{k}\rho g\iiint_V dV$$

$$\mathbf{M}=\rho g\iiint_V[\mathbf{i}y-\mathbf{j}x]dV$$

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선박의 자세와 힘, 모멘트와의 관계

– Pressure Integration Technique[1)]

1) J.N.Newman, Marine Hydrodynamics, 1977, pp.290~295

선박이 유체 중에서 받는 정적인 힘과 모멘트

$\mathbf{F} = \mathbf{k}\rho g \iiint_{V(\xi_3,\xi_4,\xi_5)} dV$	$\mathbf{M} = \rho g \iiint_{V(\xi_3,\xi_4,\xi_5)} [\mathbf{i}y - \mathbf{j}x] dV$
⇩	⇩
$\mathbf{F} = \mathbf{F}(\xi_3,\xi_4,\xi_5)$	$\mathbf{M} = \mathbf{M}(\xi_3,\xi_4,\xi_5)$

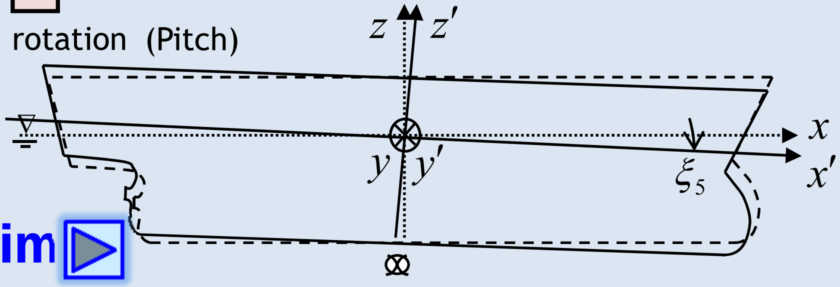
(Q) 침수부피(중심)가 변하는 자세는?

대경사 경우 미소경사 경우

$$\mathbf{F} \approx \mathbf{k}\rho g \xi_5 L_{WP}(z')$$
$$\mathbf{M} \approx \mathbf{i}\rho g \xi_5 I_P + \mathbf{j}(-\rho g \xi_5 V_0 z'_{B0} - \rho g \xi_5 I_L + mgz'_G \xi_5)$$

⇧

y 축 rotation (Pitch)

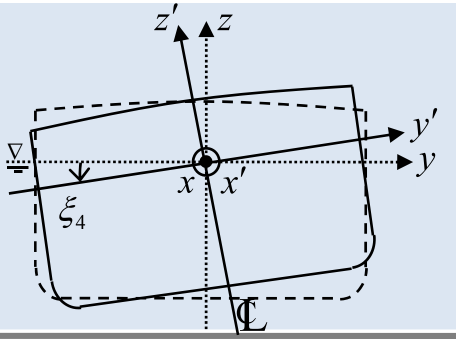


Trim ▶

$$\mathbf{F} \approx -\mathbf{k}\rho g \xi_4 T_{WP}(z')$$
$$\mathbf{M} \approx \mathbf{i}(-\rho g \xi_4 V_0 z'_{B0} - \rho g \xi_4 I_T + mgz'_G \xi_4) + \mathbf{j}\rho g \xi_4 I_P$$

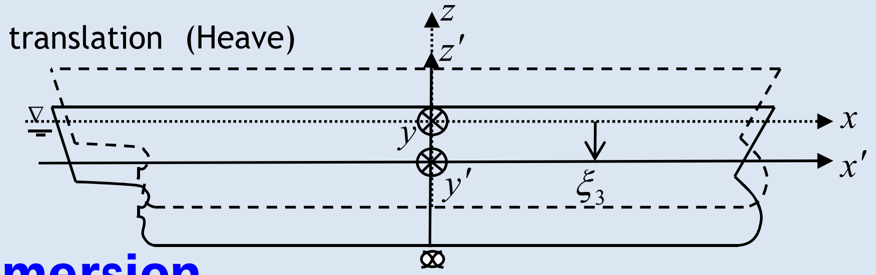
⇧

x 축 rotation (Roll)



Heel ▶

z 축 translation (Heave)

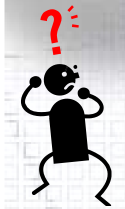


Immersion

⇩

$$\mathbf{F} \approx -\mathbf{k}\rho g \xi_3 A_{WP}(z')$$
$$\mathbf{M} \approx -\mathbf{i}\rho g \xi_3 T_{WP}(z') + \mathbf{j}\rho g \xi_3 L_{WP}(z')$$

복원력(모멘트)의 선형화 (Taylor series expansion) (1)



초기 자세(ξ_3,ξ_4,ξ_5)에서

복원력(F), 횡 방향 복원 모멘트(M_T), 종 방향 복원 모멘트(M_L)를 알고 있을 때,

미소 변화된 자세($\xi_3+\Delta\xi_3$, $\xi_4+\Delta\xi_4$, $\xi_5+\Delta\xi_5$)에서의

복원력(F), 횡 방향 복원 모멘트(M_T), 종 방향 복원 모멘트(M_L)는?

$$F(\xi_3,\xi_4,\xi_5) \qquad \Rightarrow ? \qquad F(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)$$

$$M_T(\xi_3,\xi_4,\xi_5) \qquad \qquad M_T(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)$$

$$M_L(\xi_3,\xi_4,\xi_5) \qquad \qquad M_L(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)$$

Taylor series expansion

$$F(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=F(\xi_3,\xi_4,\xi_5)+\frac{\partial F}{\partial \xi_3}\Delta\xi_3+\frac{\partial F}{\partial \xi_4}\Delta\xi_4+\frac{\partial F}{\partial \xi_5}\Delta\xi_5\cancel{+\cdots} \quad \text{선형화}$$

$$M_T(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=M_T(\xi_3,\xi_4,\xi_5)+\frac{\partial M_T}{\partial \xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial \xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial \xi_5}\Delta\xi_5\cancel{+\cdots} \quad \text{선형화}$$

$$M_L(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=M_L(\xi_3,\xi_4,\xi_5)+\frac{\partial M_L}{\partial \xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial \xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial \xi_5}\Delta\xi_5\cancel{+\cdots} \quad \text{선형화}$$

복원력(모멘트)의 선형화 (Taylor series expansion) (2)

- ✓ Taylor series 1**차항까지 전개**

$$F(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=F(\xi_3,\xi_4,\xi_5)+\frac{\partial F}{\partial\xi_3}\Delta\xi_3+\frac{\partial F}{\partial\xi_4}\Delta\xi_4+\frac{\partial F}{\partial\xi_5}\Delta\xi_5$$

$$M_T(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=M_T(\xi_3,\xi_4,\xi_5)+\frac{\partial M_T}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial\xi_5}\Delta\xi_5$$

$$M_L(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)=M_L(\xi_3,\xi_4,\xi_5)+\frac{\partial M_L}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial\xi_5}\Delta\xi_5$$

⇩

$$F(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)-F(\xi_3,\xi_4,\xi_5)=\frac{\partial F}{\partial\xi_3}\Delta\xi_3+\frac{\partial F}{\partial\xi_4}\Delta\xi_4+\frac{\partial F}{\partial\xi_5}\Delta\xi_5$$

$$M_T(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)-M_T(\xi_3,\xi_4,\xi_5)=\frac{\partial M_T}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial\xi_5}\Delta\xi_5$$

$$M_L(\xi_3+\Delta\xi_3,\xi_4+\Delta\xi_4,\xi_5+\Delta\xi_5)-M_L(\xi_3,\xi_4,\xi_5)=\frac{\partial M_L}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial\xi_5}\Delta\xi_5$$

⇩

$$\Delta F=\frac{\partial F}{\partial\xi_3}\Delta\xi_3+\frac{\partial F}{\partial\xi_4}\Delta\xi_4+\frac{\partial F}{\partial\xi_5}\Delta\xi_5$$

$$\Delta M_T=\frac{\partial M_T}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_T}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_T}{\partial\xi_5}\Delta\xi_5$$

$$\Delta M_L=\frac{\partial M_L}{\partial\xi_3}\Delta\xi_3+\frac{\partial M_L}{\partial\xi_4}\Delta\xi_4+\frac{\partial M_L}{\partial\xi_5}\Delta\xi_5$$

복원력(모멘트)의 선형화 (Taylor series expansion) (3)

✓ Taylor series 1차항까지 전개

$$\Delta F = \frac{\partial F}{\partial \xi_3}\Delta\xi_3 + \frac{\partial F}{\partial \xi_4}\Delta\xi_4 + \frac{\partial F}{\partial \xi_5}\Delta\xi_5$$

$$\Delta M_T = \frac{\partial M_T}{\partial \xi_3}\Delta\xi_3 + \frac{\partial M_T}{\partial \xi_4}\Delta\xi_4 + \frac{\partial M_T}{\partial \xi_5}\Delta\xi_5$$

$$\Delta M_L = \frac{\partial M_L}{\partial \xi_3}\Delta\xi_3 + \frac{\partial M_L}{\partial \xi_4}\Delta\xi_4 + \frac{\partial M_L}{\partial \xi_5}\Delta\xi_5$$

변수 3개, 식 3개

✓ Matrix로 표현 $\mathbf{b} = \mathbf{A}\mathbf{x}$

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial \xi_3} & \frac{\partial F}{\partial \xi_4} & \frac{\partial F}{\partial \xi_5} \\ \frac{\partial M_T}{\partial \xi_3} & \frac{\partial M_T}{\partial \xi_4} & \frac{\partial M_T}{\partial \xi_5} \\ \frac{\partial M_L}{\partial \xi_3} & \frac{\partial M_L}{\partial \xi_4} & \frac{\partial M_L}{\partial \xi_5} \end{pmatrix} \begin{pmatrix} \Delta\xi_3 \\ \Delta\xi_4 \\ \Delta\xi_5 \end{pmatrix}$$

1. 자세의 변화량이 주어져 있을 때,
   힘(모멘트)의 변화량을 구하는 경우

Find Given Given

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial \xi_3} & \frac{\partial F}{\partial \xi_4} & \frac{\partial F}{\partial \xi_5} \\ \frac{\partial M_T}{\partial \xi_3} & \frac{\partial M_T}{\partial \xi_4} & \frac{\partial M_T}{\partial \xi_5} \\ \frac{\partial M_L}{\partial \xi_3} & \frac{\partial M_L}{\partial \xi_4} & \frac{\partial M_L}{\partial \xi_5} \end{pmatrix} \begin{pmatrix} \Delta\xi_3 \\ \Delta\xi_4 \\ \Delta\xi_5 \end{pmatrix}$$

$\mathbf{b} = \mathbf{A}\mathbf{x}$ 를 풀면 됨

2. 힘(모멘트)의 변화량이 주어져 있을 때,
   자세의 변화량을 구하는 경우

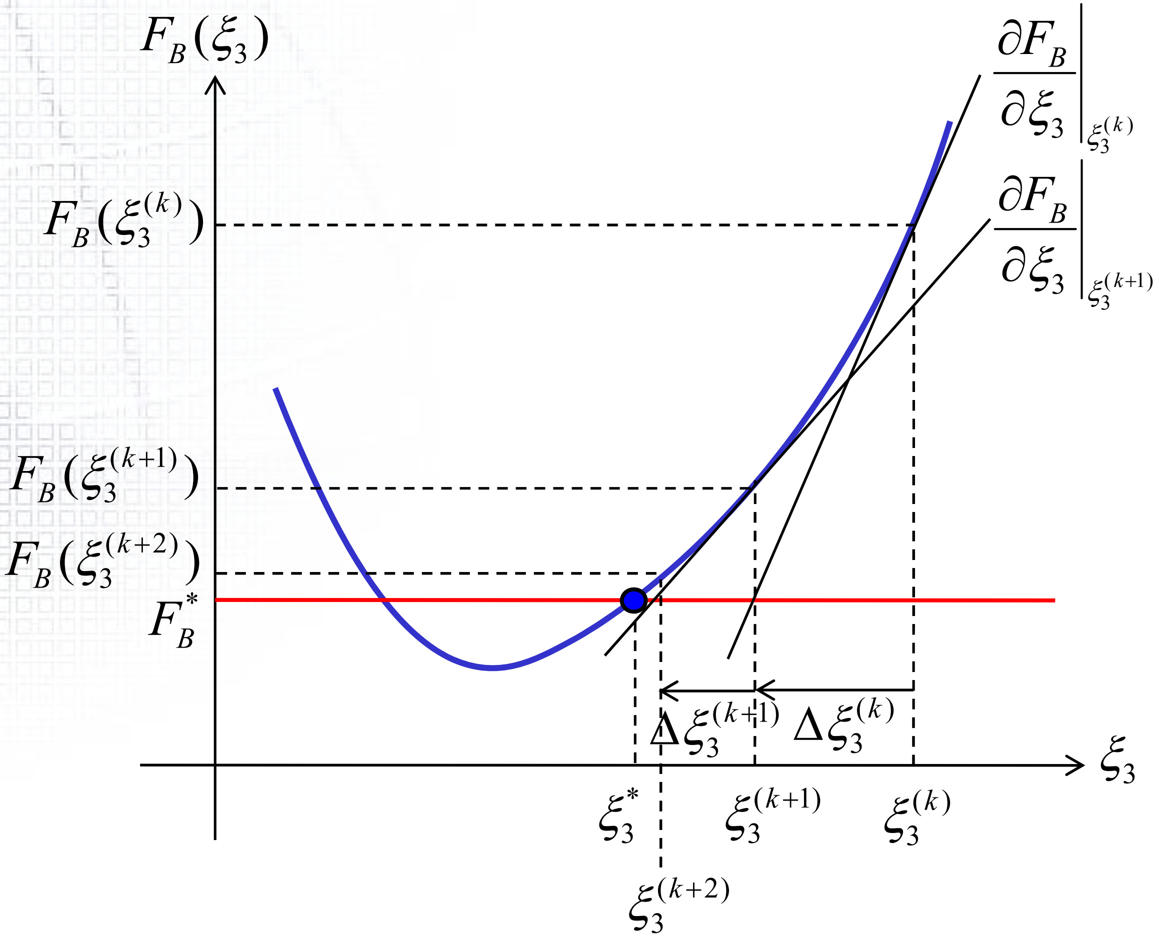
Given Given Find

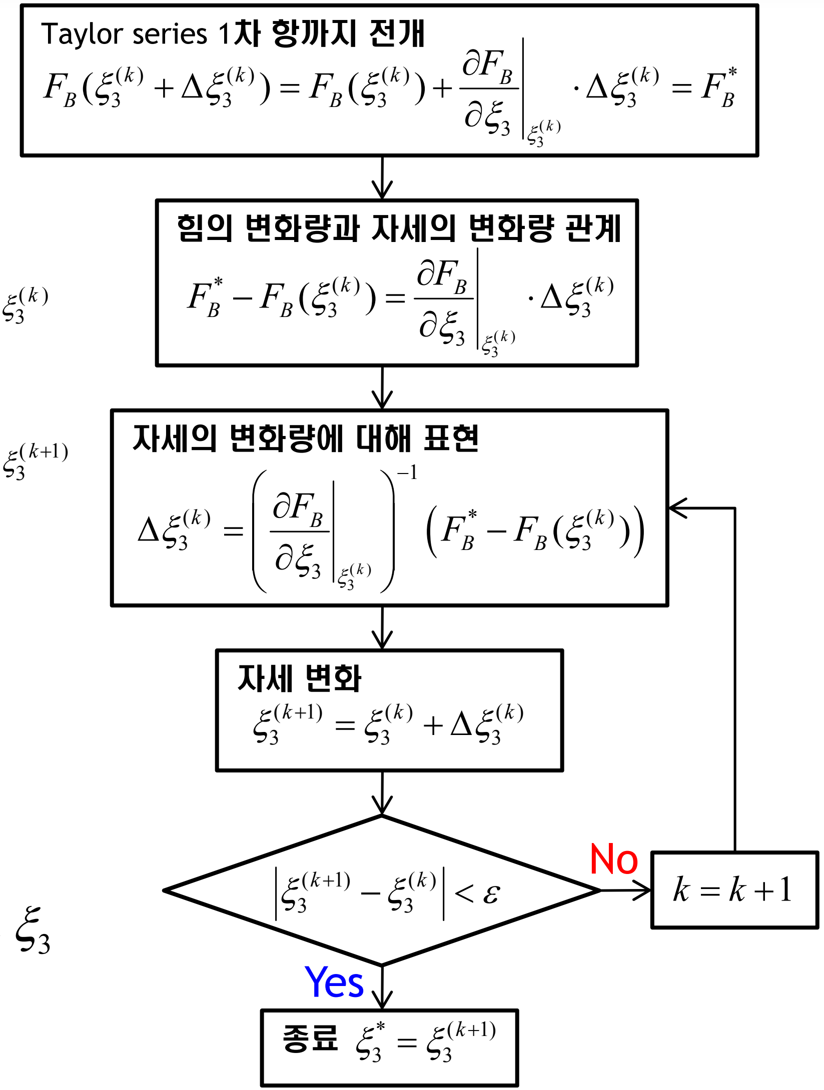
$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial \xi_3} & \frac{\partial F}{\partial \xi_4} & \frac{\partial F}{\partial \xi_5} \\ \frac{\partial M_T}{\partial \xi_3} & \frac{\partial M_T}{\partial \xi_4} & \frac{\partial M_T}{\partial \xi_5} \\ \frac{\partial M_L}{\partial \xi_3} & \frac{\partial M_L}{\partial \xi_4} & \frac{\partial M_L}{\partial \xi_5} \end{pmatrix} \begin{pmatrix} \Delta\xi_3 \\ \Delta\xi_4 \\ \Delta\xi_5 \end{pmatrix}$$

$\mathbf{x} = \mathbf{A}^{-1}\mathbf{x}$ 를 풀면 됨

※ A가 선형화되어 있기 때문에
반복 계산(iteration)을 해야 함

목표 힘(F^*)에 대한 선박의 자세 계산 (1 변수 예)





Taylor series 1차 항까지 전개

$$F_B(\xi_3^{(k)}+\Delta\xi_3^{(k)}) = F_B(\xi_3^{(k)}) + \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} = F_B^*$$

힘의 변화량과 자세의 변화량 관계

$$F_B^* - F_B(\xi_3^{(k)}) = \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)}$$

자세의 변화량에 대해 표현

$$\Delta\xi_3^{(k)} = \left(\left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}}\right)^{-1} \left(F_B^* - F_B(\xi_3^{(k)})\right)$$

자세 변화

$$\xi_3^{(k+1)} = \xi_3^{(k)} + \Delta\xi_3^{(k)}$$

$$\left|\xi_3^{(k+1)} - \xi_3^{(k)}\right| < \varepsilon$$

No

$$k = k+1$$

Yes

종료 $\xi_3^* = \xi_3^{(k+1)}$

목표 힘(F^*, M_T^*, M_L^*)에 대한 선박의 자세 계산

힘의 변화량과 자세의 변화량 관계

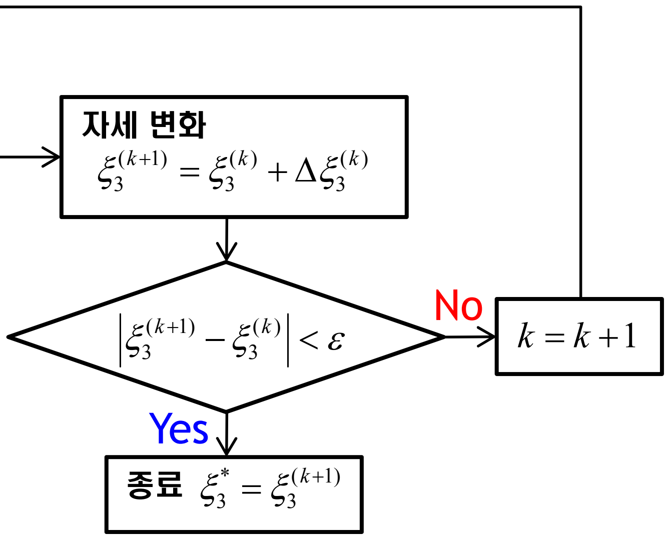
$$F_B^* - F_B(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) = \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} + \left.\frac{\partial F_B}{\partial \xi_4}\right|_{\xi_4^{(k)}} \cdot \Delta\xi_4^{(k)} + \left.\frac{\partial F_B}{\partial \xi_5}\right|_{\xi_5^{(k)}} \cdot \Delta\xi_5^{(k)}$$

$$M_T^* - M_T(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) = \left.\frac{\partial M_T}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} + \left.\frac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} \cdot \Delta\xi_4^{(k)} + \left.\frac{\partial M_T}{\partial \xi_5}\right|_{\xi_5^{(k)}} \cdot \Delta\xi_5^{(k)}$$

$$M_L^* - M_L(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) = \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} \cdot \Delta\xi_3^{(k)} + \left.\frac{\partial M_L}{\partial \xi_4}\right|_{\xi_4^{(k)}} \cdot \Delta\xi_4^{(k)} + \left.\frac{\partial M_L}{\partial \xi_5}\right|_{\xi_5^{(k)}} \cdot \Delta\xi_5^{(k)}$$

자세의 변화량에 대해 표현

$$\begin{bmatrix} \Delta\xi_3^{(k)} \\ \Delta\xi_4^{(k)} \\ \Delta\xi_5^{(k)} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial F_B}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial F_B}{\partial \xi_4}\right|_{\xi_4^{(k)}} & \left.\frac{\partial F_B}{\partial \xi_4}\right|_{\xi_4^{(k)}} \\ \left.\frac{\partial M_T}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} & \left.\frac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} \\ \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\frac{\partial M_L}{\partial \xi_3}\right|_{\xi_3^{(k)}} \end{bmatrix}^{-1} \begin{bmatrix} F_B^* - F_B(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) \\ M_T^* - M_T(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) \\ M_L^* - M_L(\xi_3^{(k)}, \xi_4^{(k)}, \xi_5^{(k)}) \end{bmatrix}$$



자세 변화

$$\xi_3^{(k+1)} = \xi_3^{(k)} + \Delta\xi_3^{(k)}$$

$$\left|\xi_3^{(k+1)} - \xi_3^{(k)}\right| < \varepsilon$$

$$k = k + 1$$

종료 $\xi_3^* = \xi_3^{(k+1)}$

1) Söding, H., “Naval Architecture Calculation”, WEGEMT, E2 Computer Aided Ship Design, pp.74-95, 1978

미소 자세와 미소 힘, 모멘트와의 관계식1)

$$\begin{pmatrix} \Delta F \\ \Delta M_T \\ \Delta M_L \end{pmatrix} = \boxed{\begin{pmatrix} \left.\dfrac{\partial F}{\partial \xi_3}\right|_{\xi_3^{(k)}} & \left.\dfrac{\partial F}{\partial \xi_4}\right|_{\xi_3^{(k)}} & \left.\dfrac{\partial F}{\partial \xi_5}\right|_{\xi_3^{(k)}} \\ \left.\dfrac{\partial M_T}{\partial \xi_3}\right|_{\xi_4^{(k)}} & \left.\dfrac{\partial M_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} & \left.\dfrac{\partial M_T}{\partial \xi_5}\right|_{\xi_4^{(k)}} \\ \left.\dfrac{\partial M_L}{\partial \xi_3}\right|_{\xi_5^{(k)}} & \left.\dfrac{\partial M_L}{\partial \xi_4}\right|_{\xi_5^{(k)}} & \left.\dfrac{\partial M_L}{\partial \xi_5}\right|_{\xi_5^{(k)}} \end{pmatrix}} \begin{pmatrix} \Delta \xi_3 \\ \Delta \xi_4 \\ \Delta \xi_5 \end{pmatrix}$$

Jacobian matrix

(자세 변화량을 힘과 모멘트의 변화량으로 변환)

$$= \begin{pmatrix} -\rho g A_{WP}(\xi_3^{(k)}) & -\rho g T_{WP}(\xi_3^{(k)}) & \rho g L_{WP}(\xi_3^{(k)}) \\ -\rho g T_{WP}(\xi_4^{(k)}) & -\rho g I_T(\xi_4^{(k)}) - \rho g V(\xi_4^{(k)}) z_B^{(k)} + mg \cdot z_G^{(k)} & \rho g I_P(\xi_4^{(k)}) \\ \rho g L_{WP}(\xi_5^{(k)}) & \rho g I_P(\xi_5^{(k)}) & -\rho g I_L(\xi_5^{(k)}) - \rho g V(\xi_5^{(k)}) z_B^{(k)} + mg \cdot z_G^{(k)} \end{pmatrix} \cdot \begin{pmatrix} \Delta \xi_3 \\ \Delta \xi_4 \\ \Delta \xi_5 \end{pmatrix}$$

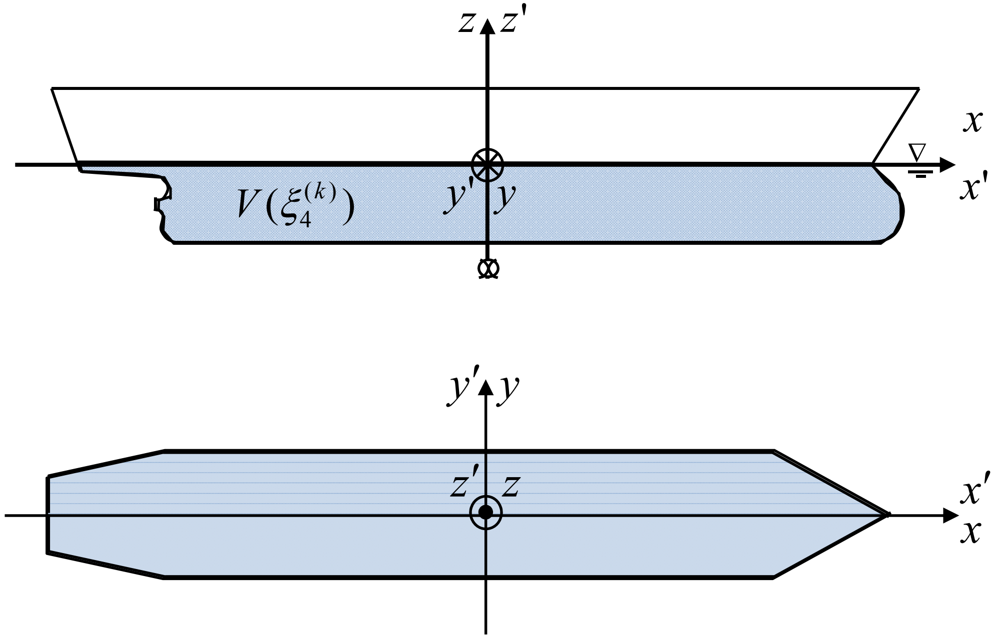
Linked Slides

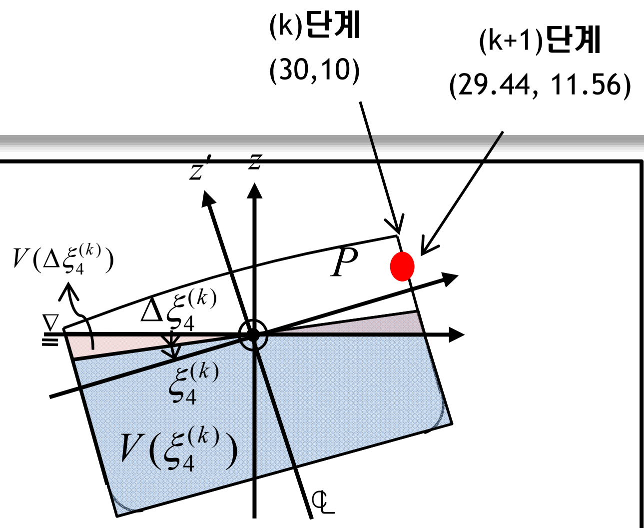
- Pressure Integration Technique을 이용한 부력, 횡 방향 모멘트, 종 방향 모멘트 계산

J.N.Newman, Marine Hydrodynamics, 1977, pp.290~295

A dvanced
S hip
D esign
A utomation
L aboratory

Heel에 의한 좌표계 변환





$$x_P^{(k)} = x_P^{(k)}$$

$$y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)}$$

$$z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)}$$

ex) 배가 x축 중심으로 회전하고 있다. (k)단계에서 (k+1)단계로 넘어갈 때, $\Delta\xi_4^{(k)} = \pi / 60(rad)$ 만큼 회전한다고 한다. (k)단계에서 (30,10)의 위치는 (k+1)단계에서 어떻게 보이는가?

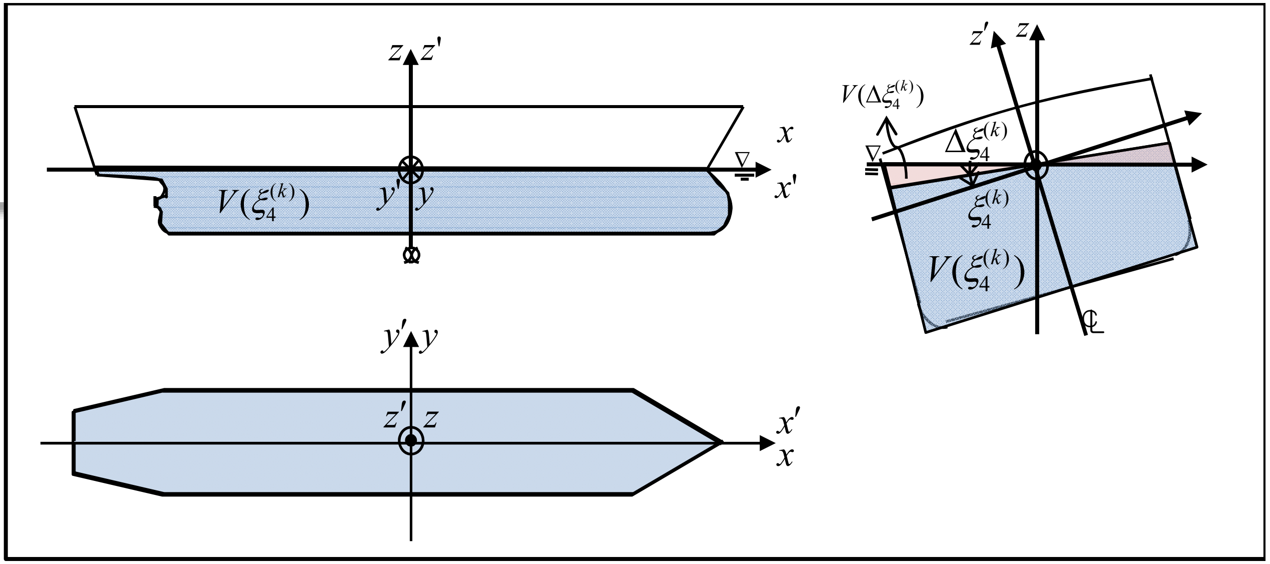
(회전변환)

sol) 좌표의 회전 변환에 의해

$$\begin{bmatrix} y_P^{(k+1)} \\ z_P^{(k+1)} \end{bmatrix} = \begin{bmatrix} \cos\Delta\xi_4^{(k)} & -\sin\Delta\xi_4^{(k)} \\ \sin\Delta\xi_4^{(k)} & \cos\Delta\xi_4^{(k)} \end{bmatrix} \begin{bmatrix} y_P^{(k)} \\ z_P^{(k)} \end{bmatrix} = \begin{bmatrix} y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ y_P^{(k)}\sin\Delta\xi_4^{(k)} + z_P^{(k)}\cos\Delta\xi_4^{(k)} \end{bmatrix}$$

$$= \begin{bmatrix} 30\cos 3^\circ - 10\sin 3^\circ \\ 30\sin 3^\circ + 10\cos 3^\circ \end{bmatrix} = \begin{bmatrix} 29.44 \\ 11.56 \end{bmatrix}$$

Heel에 의한 힘 (부력) (1)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

(k)번째 상태의 부피

$$\mathbf{F}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \mathbf{k}\rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} dV = \mathbf{k}\rho g \left\{ \boxed{\iiint_{V(\xi_4^{(k)})} dV} + \iiint_{V(\Delta\xi_4^{(k)})} dV \right\}$$

$$\iiint_{V(\xi_4^{(k)})} dV = \iiint_{V(\xi_4^{(k)})} dxdydz = V(\xi_4^{(k)})$$

Heel에 의한 힘 (부력) (2)

좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

변화된 부피

$$\mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{k}\rho g \iiint\limits_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint\limits_{V(\xi_4^{(k)})} dV + \boxed{\iiint\limits_{V(\Delta\xi_4^{(k)})} dV} \right\}$$

적분하기 편리하게 적분 순서를 변경

$$\iiint\limits_{V(\Delta\xi_4^{(k)})} dV = \iiint\limits_{V(\Delta\xi_4^{(k)})} dxdydz = \iiint\limits_{V(\Delta\xi_4^{(k)})} dzdydx$$

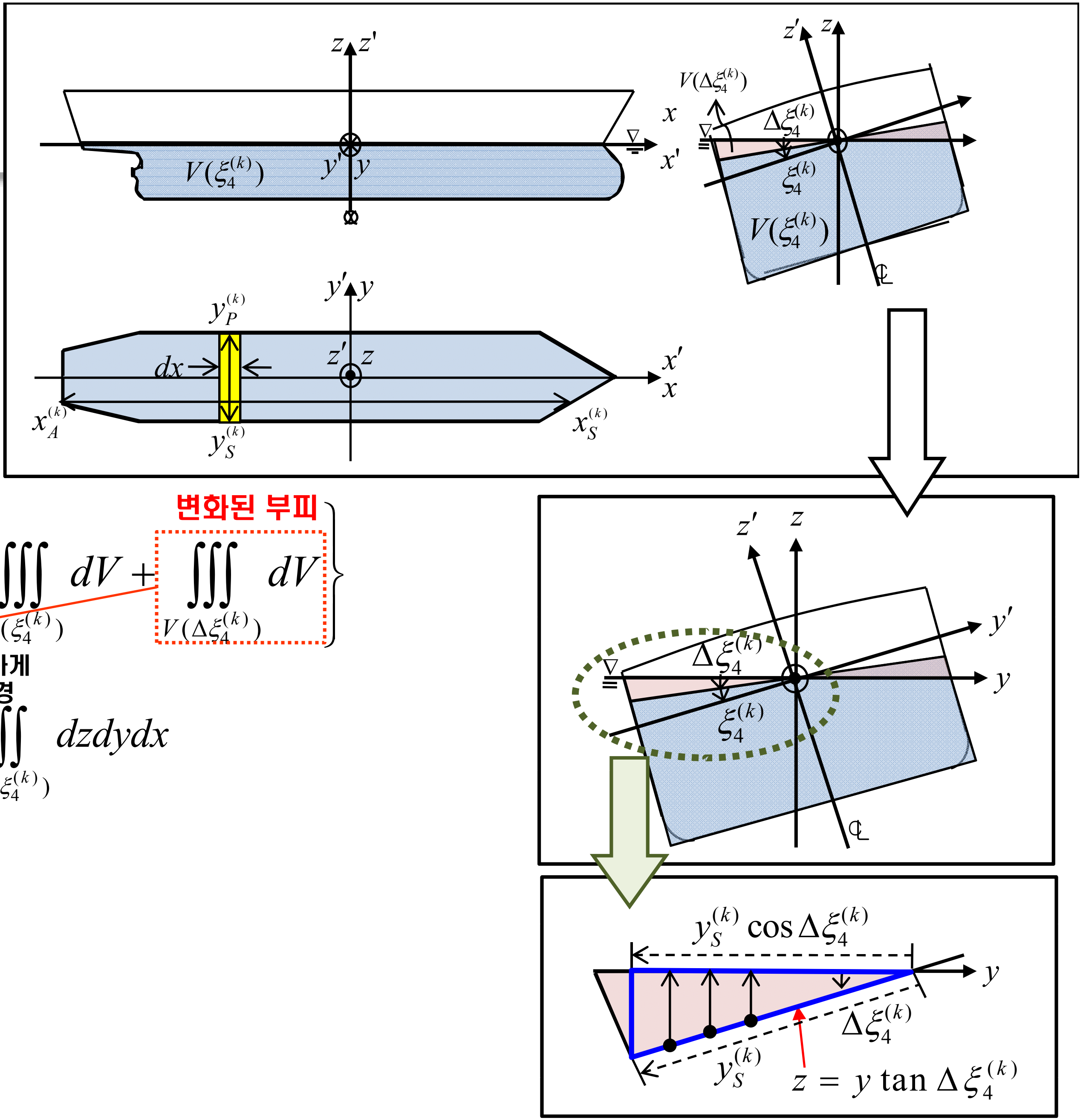
만약 $\Delta\xi_4^{(k)}$ 가 작다면,

우현만 보면,

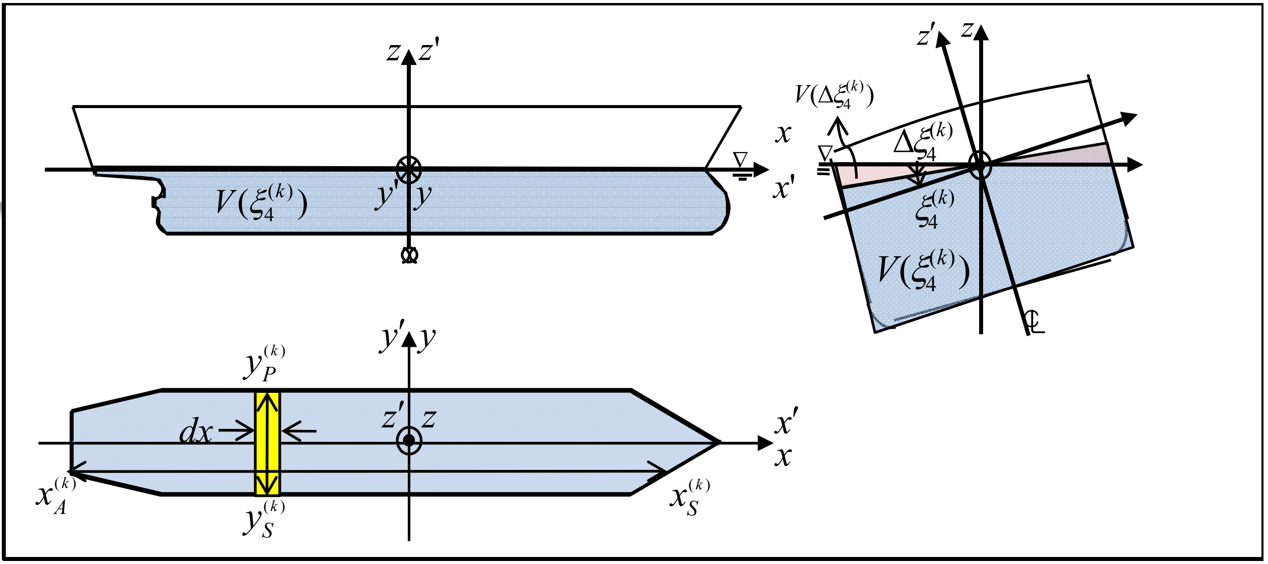
$$\int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} \int_{y\tan\Delta\xi_4^{(k)}}^{0} dzdydx$$

좌현도 이와 같이,

$$\int_{x_A^{(k)}}^{x_F^{(k)}} \int_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \int_{0}^{y\tan\Delta\xi_4^{(k)}} dzdydx$$



Heel에 의한 힘 (부력) (3)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \cos\Delta\xi_4^{(k)} - z_P^{(k)} \sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_4^{(k)} + y_P^{(k)} \sin\Delta\xi_4^{(k)} \end{pmatrix}$$

(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

변화된 부피

$$\mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{k}\rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_4^{(k)})} dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} dV} \right\}$$

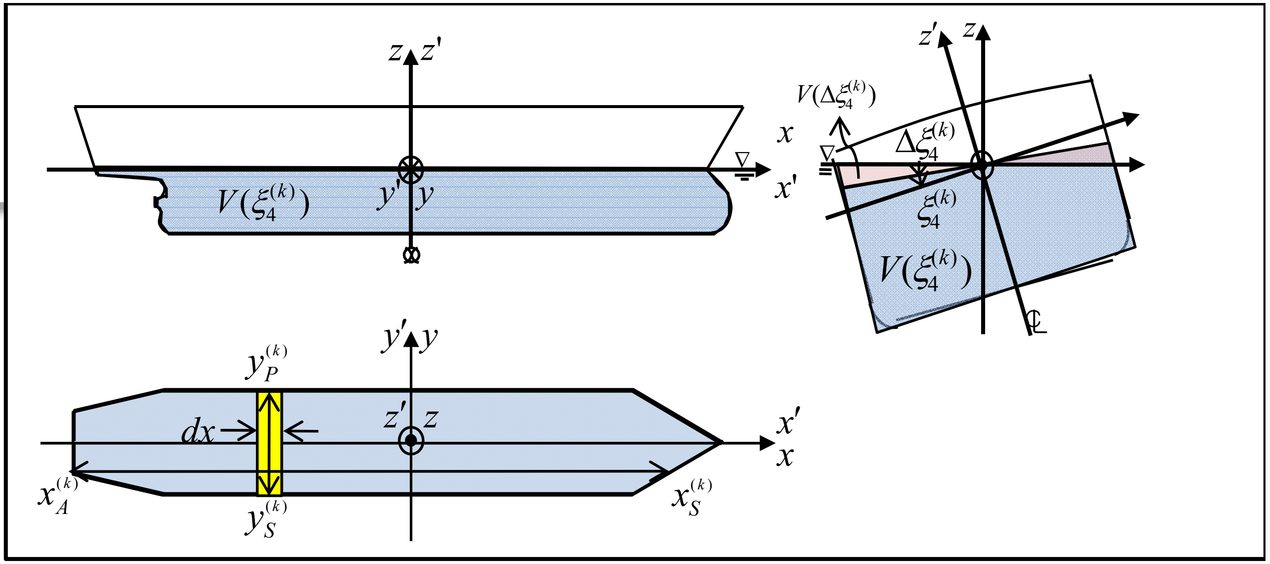
만약 $\Delta\xi_4^{(k)}$ 가 작다면,

$$\iiint_{V(\Delta\xi_4^{(k)})} dV \approx \int_{x_A^{(k)}}^{x_F^{(k)}} \left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} \int_{y\tan\Delta\xi_4^{(k)}}^{0} dzdy - \int_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \int_{0}^{y\tan\Delta\xi_4^{(k)}} dzdy \right) dx$$

$$= \int_{x_A^{(k)}}^{x_F^{(k)}} \left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} \left(-y\tan\Delta\xi_4^{(k)}\right) dy - \int_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \left(y\tan\Delta\xi_4^{(k)}\right) dy \right) dx$$

$$= \int_{x_A^{(k)}}^{x_F^{(k)}} \left(\frac{-y^2\tan\Delta\xi_4^{(k)}}{2} \Bigg|_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} - \frac{y^2\tan\Delta\xi_4^{(k)}}{2} \Bigg|_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \right) dx$$

Heel에 의한 힘 (부력) (4)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

변화된 부피

$$\mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{k}\rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_4^{(k)})} dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} dV} \right\}$$

$$\iiint_{V(\Delta\xi_4^{(k)})} dV = \int_{x_A^{(k)}}^{x_F^{(k)}} \left(\left. \frac{-y^2 \tan\Delta\xi_4^{(k)}}{2} \right|_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} - \left. \frac{y^2 \tan\Delta\xi_4^{(k)}}{2} \right|_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \right) dx$$

$$= \cos^2\Delta\xi_4^{(k)} \tan\Delta\xi_4^{(k)} \int_{x_A^{(k)}}^{x_F^{(k)}} \left(\frac{\left(y_S^{(k)}\right)^2}{2} - \frac{\left(y_P^{(k)}\right)^2}{2} \right) dx = -\cos^2\Delta\xi_4^{(k)} \tan\Delta\xi_4^{(k)} \int_{x_A^{(k)}}^{x_F^{(k)}} \underline{\frac{1}{2}\left(\left(y_P^{(k)}\right)^2 - \left(y_S^{(k)}\right)^2 \right) dx}$$

$$\underbrace{\left(\frac{y_P^{(k)} + y_S^{(k)}}{2} \right)}_{\text{횡 방향 중심}} \times \underbrace{\left(y_P^{(k)} - y_S^{(k)} \right) dx}_{\text{수선면적}}$$

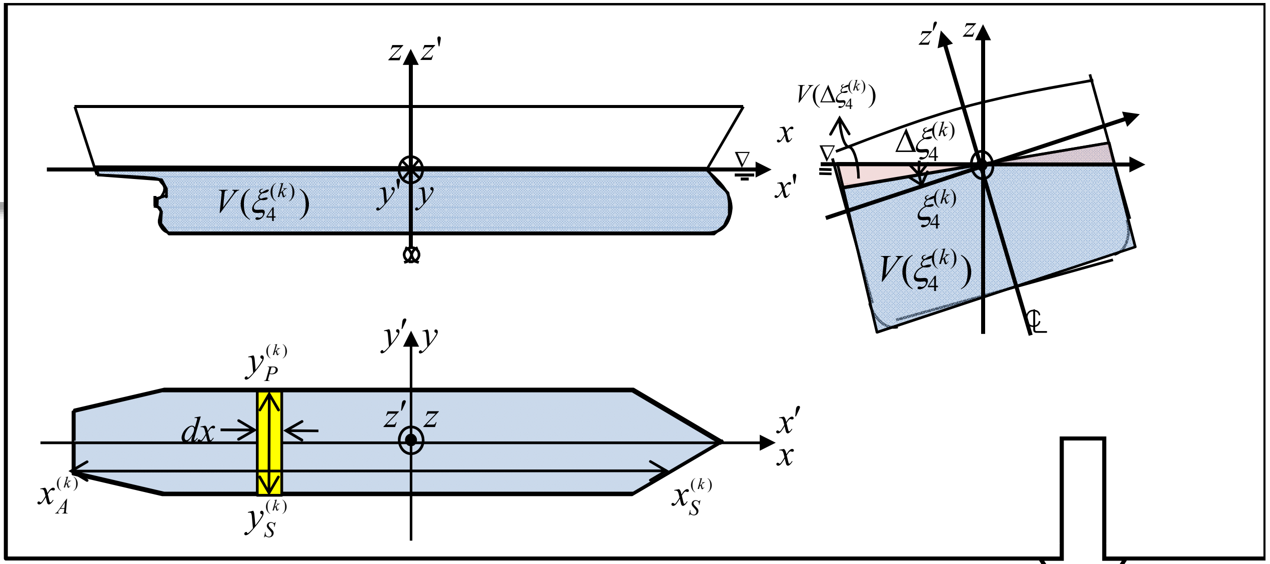
$$= -\cos^2\Delta\xi_4^{(k)} \tan\Delta\xi_4^{(k)} \underline{T_{WP}\left(\xi_4^{(k)}\right)}$$

(k)번째 상태의 수선면 횡 방향 1차 모멘트

만약 $\Delta\xi_4^{(k)}$ 가 작다면,

$$\approx -\Delta\xi_4^{(k)} T_{WP}\left(\xi_4^{(k)}\right)$$

Heel에 의한 힘 (부력) (5)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \cos\Delta\xi_4^{(k)} - z_P^{(k)} \sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_4^{(k)} + y_P^{(k)} \sin\Delta\xi_4^{(k)} \end{pmatrix}$$

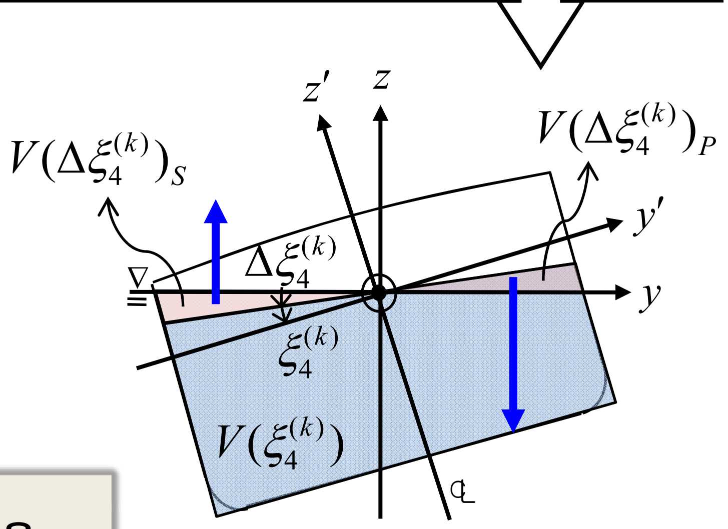
(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

변화된 부피

$$\mathbf{F}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \mathbf{k}\rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_4^{(k)})} dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} dV} \right\}$$

$$\iiint_{V(\Delta\xi_4^{(k)})} dV \approx \underbrace{-\underbrace{\Delta\xi_4^{(k)}}_{(+)} \underbrace{T_{WP}\left(\xi_4^{(k)}\right)}_{(+)}}_{(-)}$$

(k)번째 상태의 수선면 횡 방향 1차 모멘트



$$V(\Delta\xi_4^{(k)})_S < V(\Delta\xi_4^{(k)})_P$$

부호 검증

$T_{WP}(\xi_4^{(k)})$ 가 (+)이면, 수선면 중심이 좌현쪽. 즉 좌현쪽의 수선면적이 더 넓음.

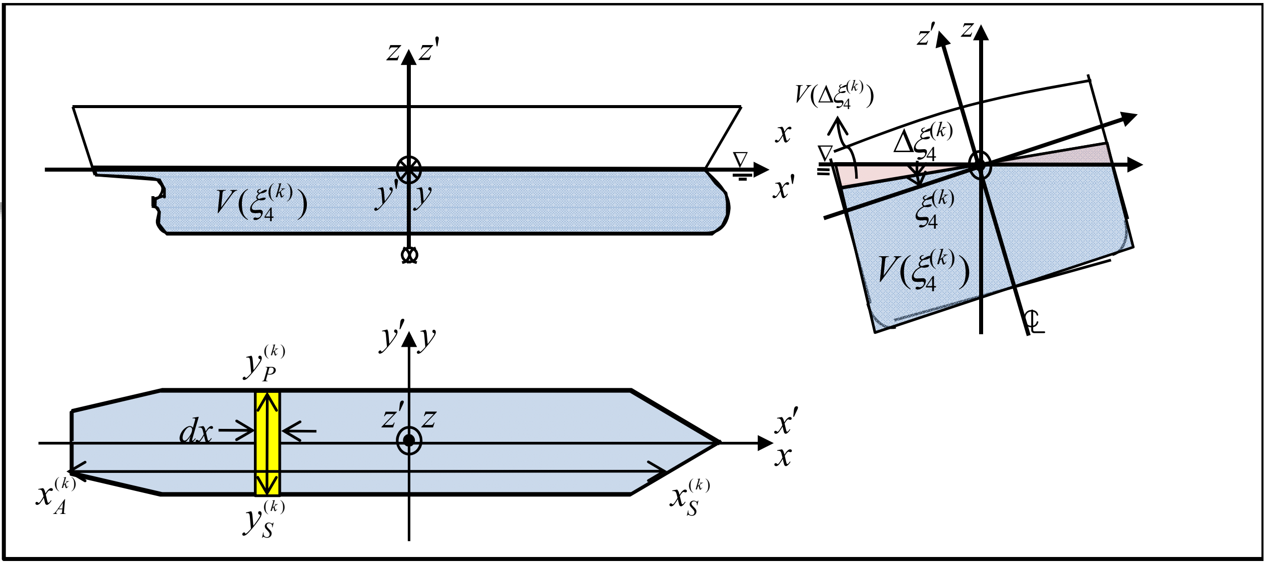
⇩

$\Delta\xi_4^{(k)}$ 이 (+)이면, 좌현쪽이 떠오르고(부력 (-)), 우현이 가라앉음(부력 (+))

⇩

좌현쪽의 떠오르는 부피가 우현의 가라앉는 부피보다 큼. ⇨ 부력의 합은 (-)

Heel에 의한 힘 (부력) (6)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)})$$

$$= \mathbf{k}\rho g \iiint_{V(\xi_4^{(k)} + \Delta\xi_4^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_4^{(k)})} dV + \iiint_{V(\Delta\xi_4^{(k)})} dV \right\}$$

$$\approx \boxed{\mathbf{k}\rho g(V(\xi_4^{(k)})} - \Delta\xi_4^{(k)} \underline{T_{WP}(\xi_4^{(k)})})$$

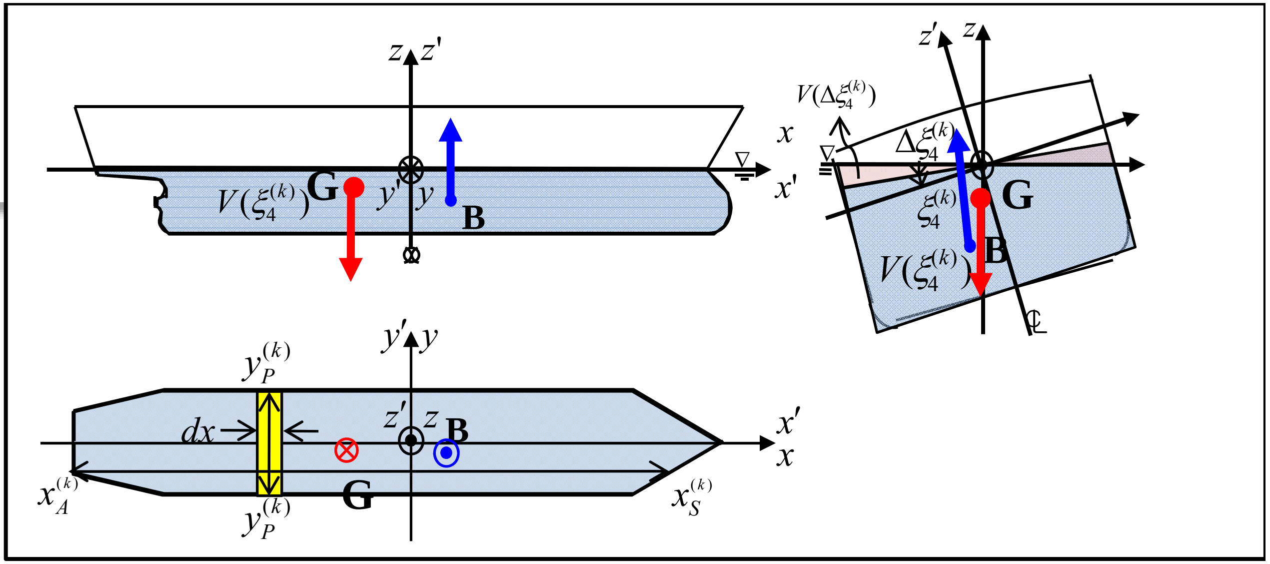
(k)번째 상태의
수선면 횡 방향 1차 모멘트

대입

$$\boxed{\mathbf{F}_B(\xi_4^{(k)})} = \mathbf{k}\rho g \iiint_{V(\xi_4^{(k)})} dV = \mathbf{k}\rho g V(\xi_4^{(k)})$$

$$\mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{F}_B(\xi_4^{(k)}) - \mathbf{k}\Delta\xi_4^{(k)}\rho g T_{WP}(\xi_4^{(k)})$$

Heel에 의한 힘 (부력) (7)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \cos\Delta\xi_4^{(k)} - z_P^{(k)} \sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_4^{(k)} + y_P^{(k)} \sin\Delta\xi_4^{(k)} \end{pmatrix}$$

✓ **부력**

$$\mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{F}_B(\xi_4^{(k)}) - \mathbf{k}\Delta\xi_4^{(k)} \rho g T_{WP}(\xi_4^{(k)})$$

✓ **중력**

$$\mathbf{F}_G(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = -\mathbf{k}mg = \mathbf{F}_G(\xi_4^{(k)})$$

✓ **선박이 받는 힘**

$$\mathbf{F}(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{F}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) + \mathbf{F}_G(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{F}_B(\xi_4^{(k)}) + \mathbf{F}_G(\xi_4^{(k)}) - \mathbf{k}\Delta\xi_4^{(k)} \rho g T_{WP}(\xi_4^{(k)})$$

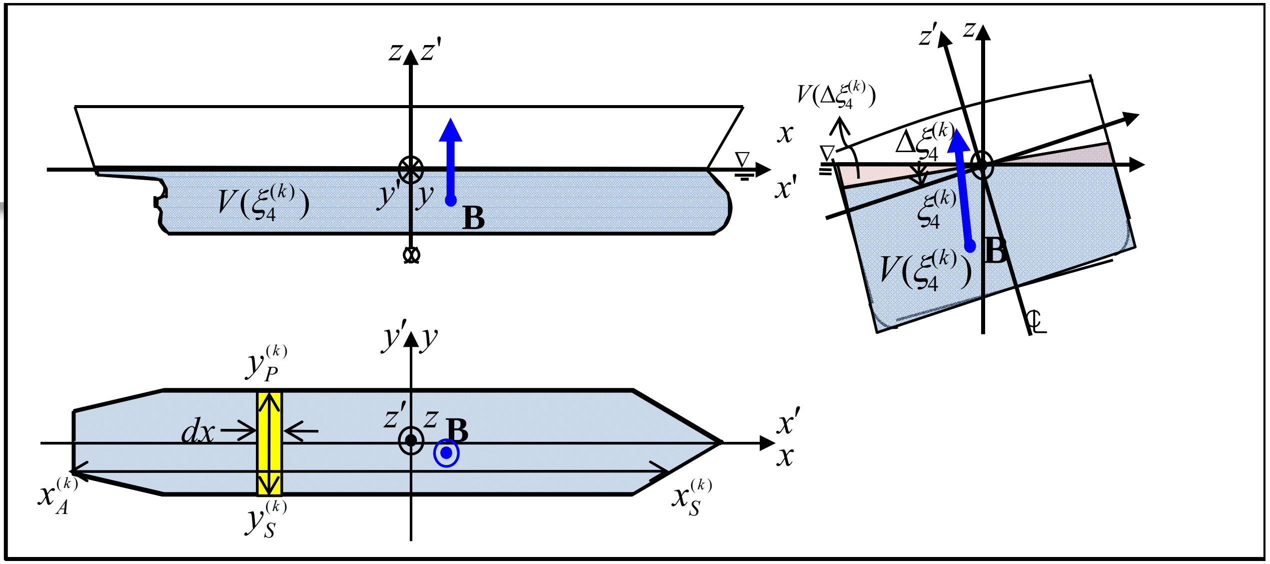
$$\mathbf{F}(\xi_4^{(k)} + \Delta\xi_4^{(k)}) - \mathbf{F}(\xi_4^{(k)}) = -\mathbf{k}\Delta\xi_4^{(k)} \rho g T_{WP}(\xi_4^{(k)})$$

$$\Delta\mathbf{F}(\xi_4^{(k)}) = \mathbf{k}\boxed{\left\{-\rho g T_{WP}(\xi_4^{(k)})\right\}}\Delta\xi_4^{(k)}$$

$$\Delta\mathbf{F}(\xi_4^{(k)}) = \mathbf{k}\boxed{\left.\frac{\partial \mathbf{F}}{\partial \xi_4}\right|_{\xi_4^{(k)}}}\Delta\xi_4^{(k)}$$

$$\left.\frac{\partial \mathbf{F}}{\partial \xi_4}\right|_{\xi_4^{(k)}} = -\rho g T_{WP}(\xi_4^{(k)})$$

Heel에 의한 모멘트 (1)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g \left\{ \boxed{\iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} + \iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV \right\}$$

(k)번째 상태의 부피

$$\iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dxdydz = \mathbf{i}\iiint_{V(\xi_4^{(k)})} ydxdydz - \mathbf{j}\iiint_{V(\xi_4^{(k)})} xdxdydz$$

$$= \mathbf{i}V(\xi_4^{(k)})y_B^{(k+1)} - \mathbf{j}V(\xi_4^{(k)})x_B^{(k+1)}$$

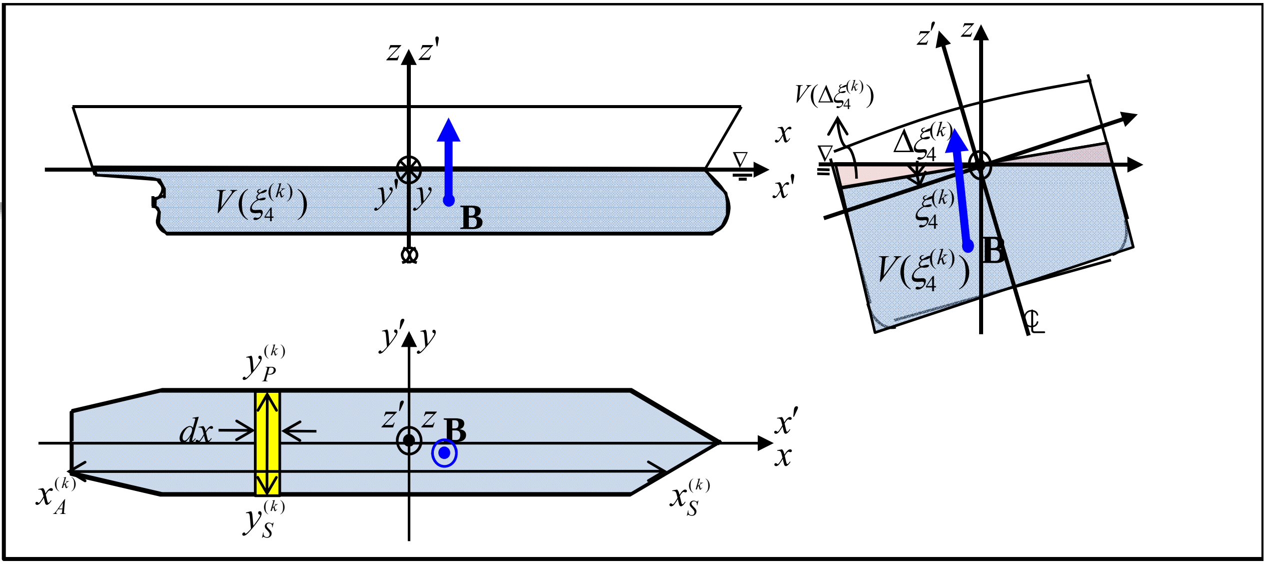
$$= \mathbf{i}V(\xi_4^{(k)})\left(y_B^{(k)}\cos\Delta\xi_4^{(k)} - z_B^{(k)}\sin\Delta\xi_4^{(k)}\right) - \mathbf{j}V(\xi_4^{(k)})x_B^{(k)}$$

$$= \mathbf{i}\left(V(\xi_4^{(k)})y_B^{(k)}\cos\Delta\xi_4^{(k)} - V(\xi_4^{(k)})z_B^{(k)}\sin\Delta\xi_4^{(k)}\right) - \mathbf{j}V(\xi_4^{(k)})x_B^{(k)}$$

만약 $\Delta\xi_4^{(k)}$ **가 작다면,**

$$\approx \mathbf{i}\left(V(\xi_4^{(k)})y_B^{(k)} - \Delta\xi_4^{(k)}V(\xi_4^{(k)})z_B^{(k)}\right) - \mathbf{j}V(\xi_4^{(k)})x_B^{(k)}$$

Heel에 의한 모멘트 (2)



좌표 변환

$$
\begin{pmatrix}
x_P^{(k)} = x_P^{(k)} \\
y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\
z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)}
\end{pmatrix}
$$

$$
\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} \right\}
$$

변화된 부피

변화된 부피의 횡 방향 1차 모멘트

$$
\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dxdydz = \boxed{\mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz
$$

만약 $\Delta\xi_4^{(k)}$가 작다면,

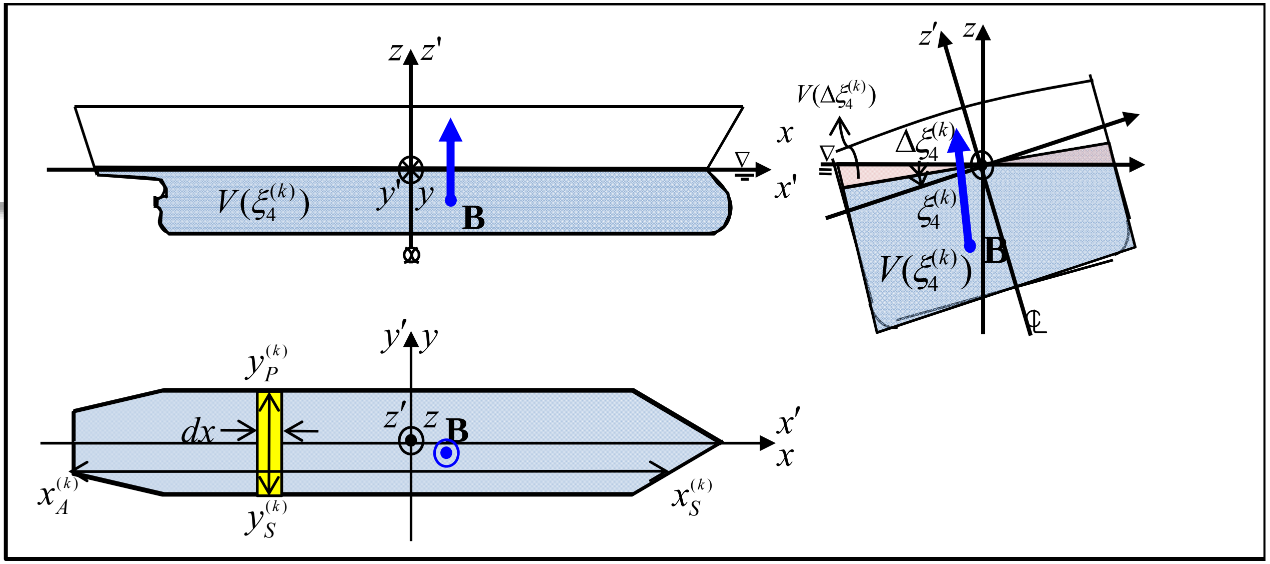
$$
\mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \approx \mathbf{i}\int_{x_A^{(k)}}^{x_F^{(k)}} \left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} \int_{y\tan\Delta\xi_4^{(k)}}^{0} ydzdy - \int_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \int_{0}^{y\tan\Delta\xi_4^{(k)}} ydzdy \right) dx
$$

$$
= \mathbf{i}\int_{x_A^{(k)}}^{x_F^{(k)}} \left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} \left(yz\Big|_{y\tan\Delta\xi_4^{(k)}}^{0} \right) dy - \int_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \left(yz\Big|_{0}^{y\tan\Delta\xi_4^{(k)}} \right) dy \right) dx
$$

$$
= \mathbf{i}\int_{x_A^{(k)}}^{x_F^{(k)}} \left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0} \left(-y^2\tan\Delta\xi_4^{(k)} \right) dy - \int_{0}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \left(y^2\tan\Delta\xi_4^{(k)} \right) dy \right) dx
$$

$$
= \mathbf{i}\int_{x_A^{(k)}}^{x_F^{(k)}} \int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} \left(-y^2\tan\Delta\xi_4^{(k)} \right) dydx
$$

Heel에 의한 모멘트 (3)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

변화된 부피의 횡 방향 1차 모멘트

$$\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dxdydz = \boxed{\mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz$$

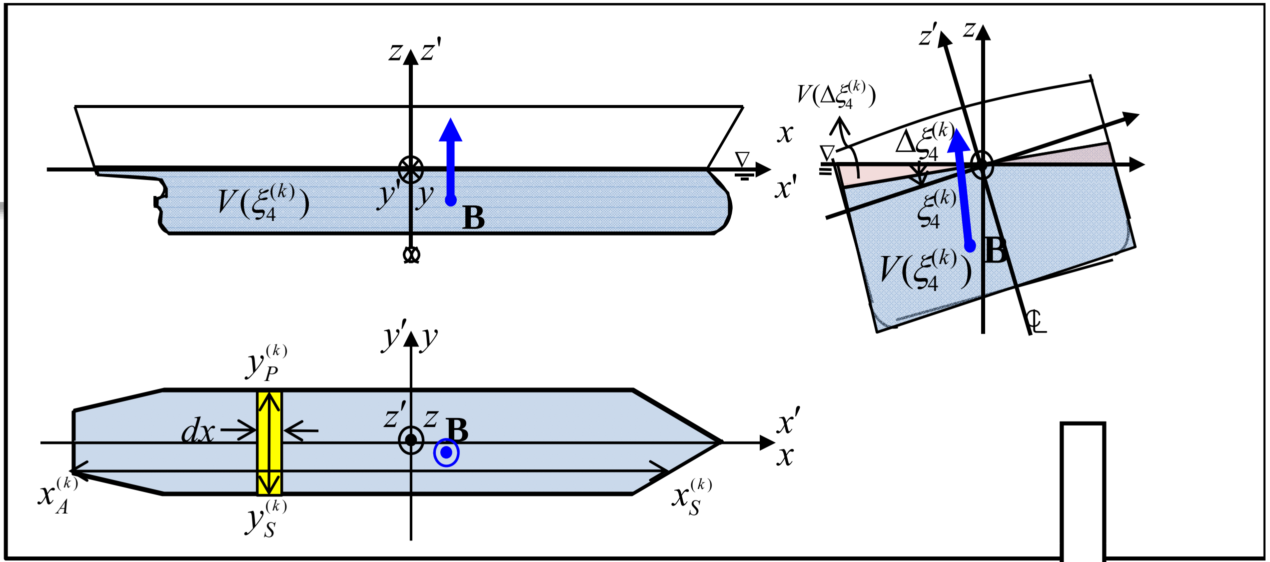
$$\mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \approx \mathbf{i}\int_{x_A^{(k)}}^{x_F^{(k)}}\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\left(-y^2\tan\Delta\xi_4^{(k)}\right)dydx$$

$$= \mathbf{i}\int_{x_A^{(k)}}^{x_F^{(k)}}\left(-\frac{y^3}{3}\tan\Delta\xi_4^{(k)}\Big|_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\right)dx = -\mathbf{i}\tan\Delta\xi_4^{(k)}\cos^3\Delta\xi_4^{(k)}\underbrace{\int_{x_A^{(k)}}^{x_F^{(k)}}\frac{1}{3}\left(\left(y_P^{(k)}\right)^3-\left(y_S^{(k)}\right)^3\right)dx}_{\text{수선면의 횡 방향 2차 모멘트}}$$

$$= -\mathbf{i}\tan\Delta\xi_4^{(k)}\cos^3\Delta\xi_4^{(k)}I_T(\xi_4^{(k)}) \approx -\mathbf{i}\Delta\xi_4^{(k)}I_T(\xi_4^{(k)})$$

만약 $\Delta\xi_4^{(k)}$ 가 작다면,

Heel에 의한 모멘트 (4)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} \right\}$$

변화된 부피

변화된 부피의 횡 방향 1차 모멘트

$$\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dxdydz = \boxed{\mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz$$

$$\approx \underbrace{\underbrace{-\mathbf{i}\Delta\xi_4^{(k)}}_{(+)}\underbrace{I_T(\xi_4^{(k)})}_{(+)}}_{(-)}$$

수선면의 횡 방향 2차 모멘트

부호 검증

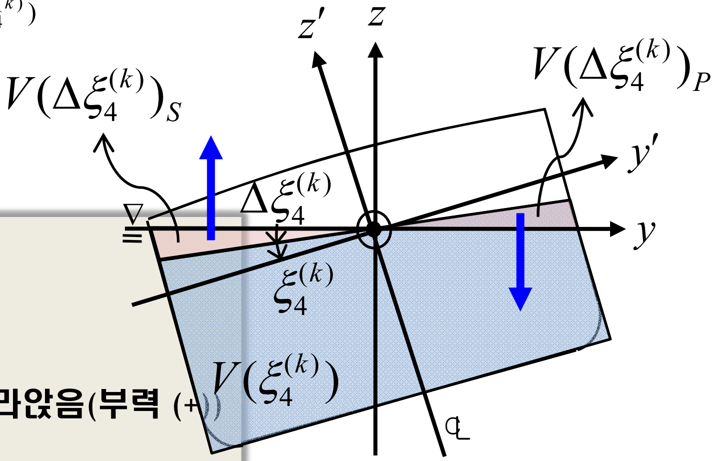
I_T 는 항상 (+)

⇩

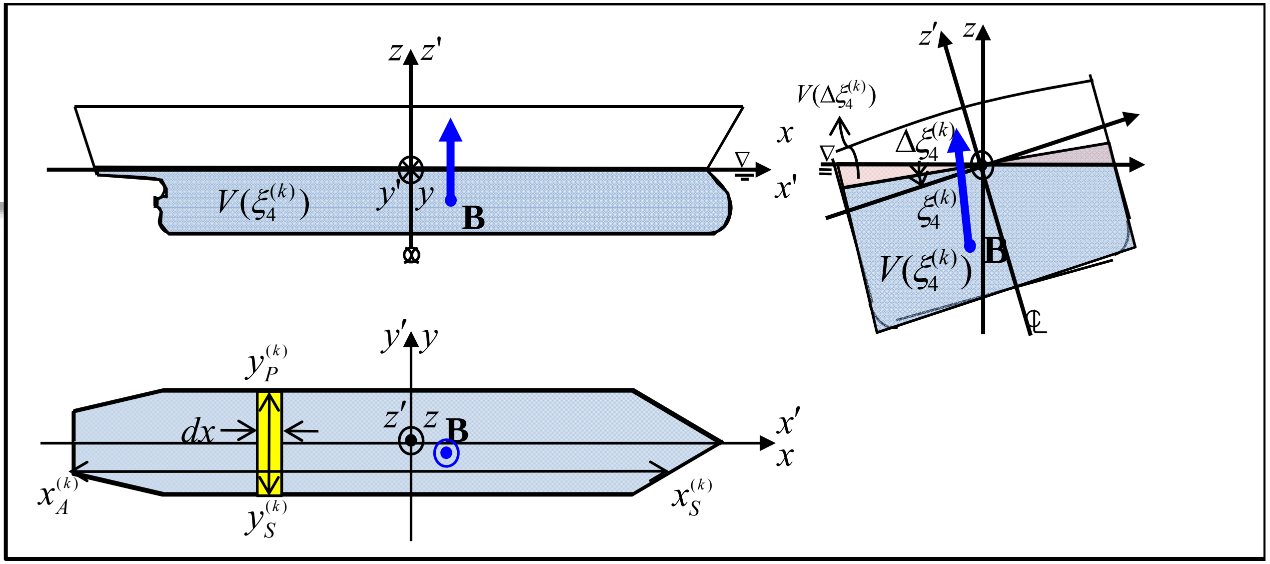
ξ_4 이 (+)이면, 좌현쪽이 떠오르고(부력 (-)), 우현이 가라앉음(부력 (+))

⇩

x 축에 대한 (-)의 모멘트



Heel에 의한 모멘트 (5)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dxdydz = \mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \boxed{-\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz}$$

변화된 부피의 종 방향 1차 모멘트

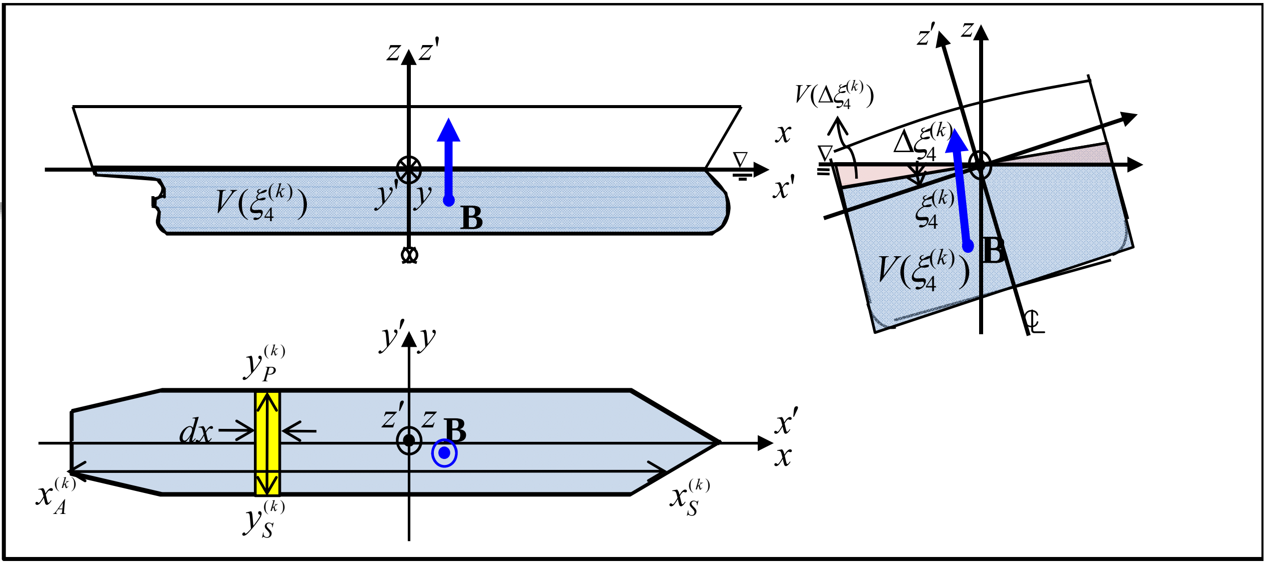
$$-\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz \approx -\mathbf{j}\int_{x_A^{(k)}}^{x_F^{(k)}}\left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0}\int_{y\tan\Delta\xi_4^{(k)}}^{0} xdzdy - \int_0^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\int_0^{y\tan\Delta\xi_4^{(k)}} xdzdy\right)dx$$

$$= -\mathbf{j}\int_{x_A^{(k)}}^{x_F^{(k)}}\left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0}\left(xz\Big|_{y\tan\Delta\xi_4^{(k)}}^{0}\right)dy - \int_0^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\left(xz\Big|_0^{y\tan\Delta\xi_4^{(k)}}\right)dy\right)dx$$

$$= -\mathbf{j}\int_{x_A^{(k)}}^{x_F^{(k)}}\left(\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{0}\left(-xy\tan\Delta\xi_4^{(k)}\right)dy - \int_0^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\left(xy\tan\Delta\xi_4^{(k)}\right)dy\right)dx$$

$$= -\mathbf{j}\int_{x_A^{(k)}}^{x_F^{(k)}}\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\left(-xy\tan\Delta\xi_4^{(k)}\right)dydx$$

Heel에 의한 모멘트 (6)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dxdydz = \mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \boxed{-\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz}$$

변화된 부피의 종 방향 1차 모멘트

$$-\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz = -\mathbf{j}\int_{x_A^{(k)}}^{x_F^{(k)}}\int_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}}\left(-xy\tan\Delta\xi_4^{(k)}\right)dydx = -\mathbf{j}\int_{x_A^{(k)}}^{x_F^{(k)}}\frac{-xy^2\tan\Delta\xi_4^{(k)}}{2}\Bigg|_{y_S^{(k)}\cos\Delta\xi_4^{(k)}}^{y_P^{(k)}\cos\Delta\xi_4^{(k)}} dx$$

$$= \mathbf{j}\tan\Delta\xi_4^{(k)}\cos^2\Delta\xi_4^{(k)}\int_{x_A^{(k)}}^{x_F^{(k)}}\frac{x}{2}\left(y_P^{(k)}\right)^2-\left(y_S^{(k)}\right)^2 dx = \mathbf{j}\tan\Delta\xi_4^{(k)}\cos^2\Delta\xi_4^{(k)}\int_{x_A^{(k)}}^{x_F^{(k)}} x\,\underbrace{\frac{y_P^{(k)}+y_S^{(k)}}{2}}_{\text{폭방향 중심}}\underbrace{\left(y_P^{(k)}-y_S^{(k)}\right)dx}_{\text{미소 면적}}$$

$$= \mathbf{j}\tan\Delta\xi_4^{(k)}\cos^2\Delta\xi_4^{(k)}I_P(\xi_4^{(k)}) \approx \mathbf{j}\Delta\xi_4^{(k)}I_P(\xi_4^{(k)})$$

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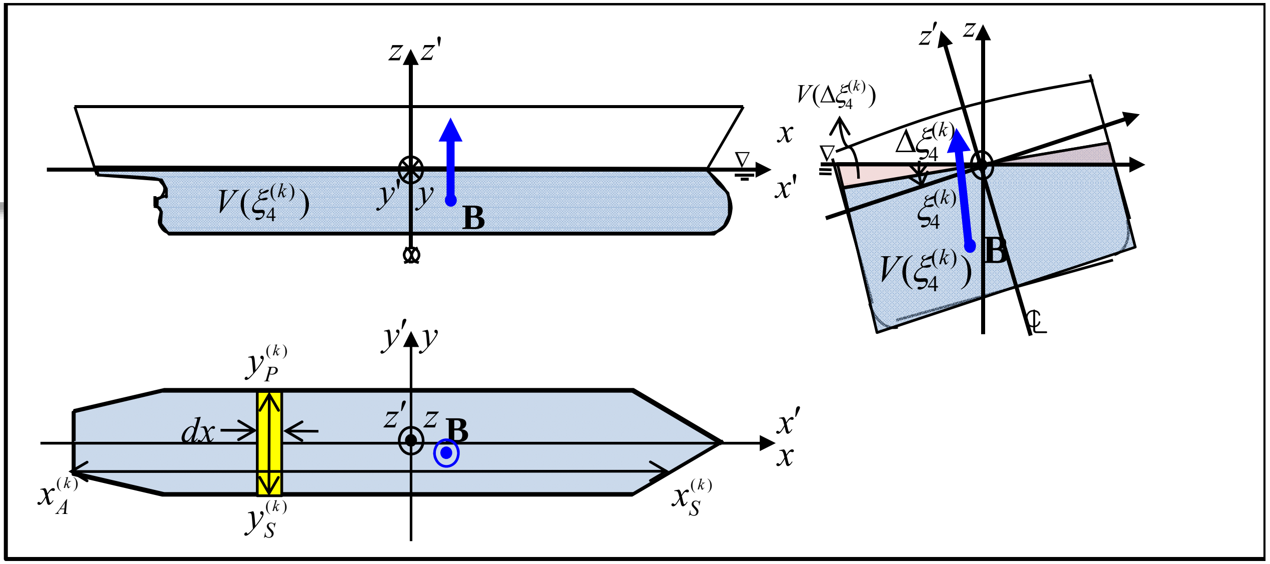
Heel에 의한 모멘트 (7)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} \right\}$$

변화된 부피

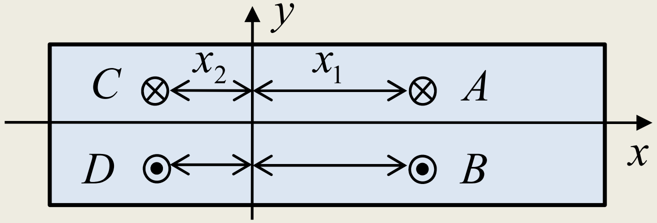
$$\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dxdydz = \mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \boxed{-\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz} \rightarrow \approx \mathbf{j}\Delta\xi_4^{(k)} I_P(\xi_4^{(k)})$$

변화된 부피의 종 방향 1차 모멘트

부호 검증

※ 좌우현 대칭

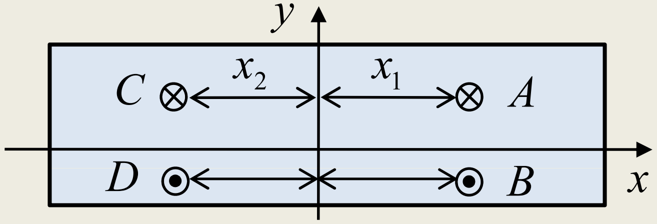
$A = B,\ C = D$



$$\mathbf{M}_{BL} = \mathbf{j}(x_1A - x_1B - x_2C + x_2D) = \mathbf{j}\cdot 0$$

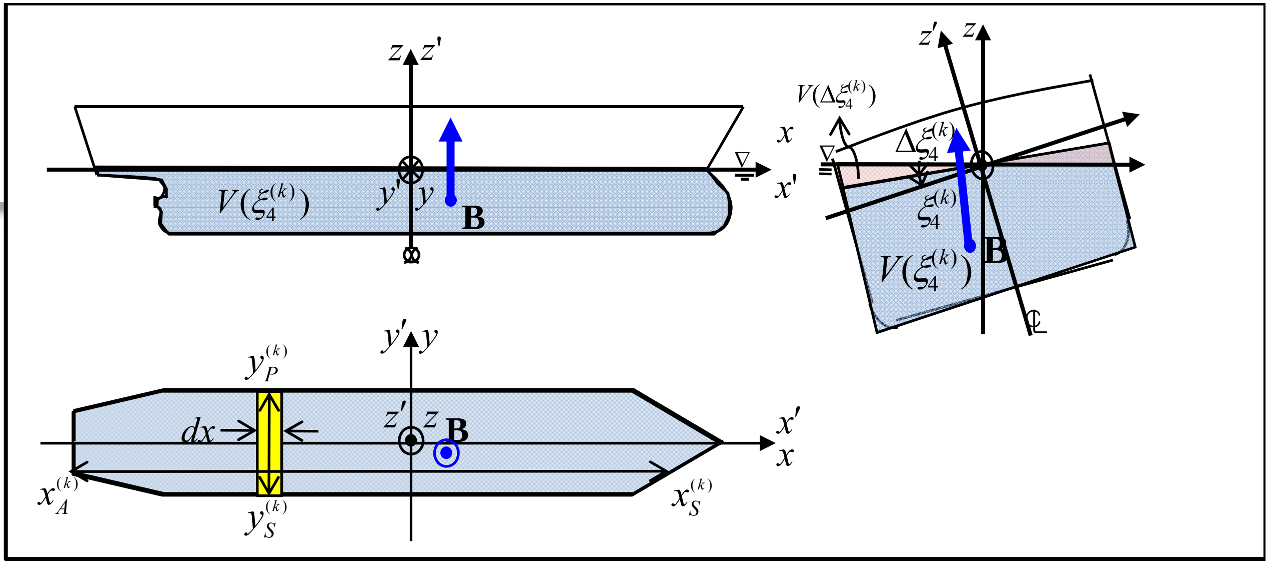
※ 선수미 대칭

$x_1 = x_2,\ A = C,\ B = D$



$$\mathbf{M}_{BL} = \mathbf{j}(x_1A - x_1B - x_2C + x_2D) = \mathbf{j}\cdot 0$$

Heel에 의한 모멘트 (8)



좌표 변환

$$\left(\begin{aligned} x_P^{(k)} &= x_P^{(k)} \\ y_P^{(k+1)} &= y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} &= z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{aligned}\right)$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

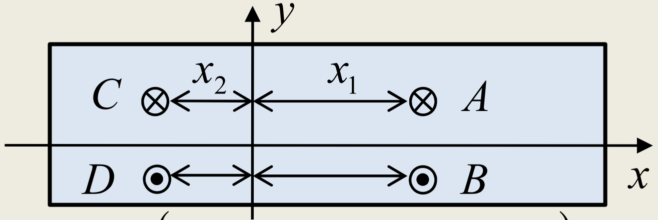
$$\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dxdydz = \mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \boxed{-\,\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz} \longrightarrow \approx \mathbf{j}\Delta\xi_4^{(k)} I_P(\xi_4^{(k)})$$

변화된 부피의 종 방향 1차 모멘트

부호 검증

I_P 가 (+)이면, 수선면 중심이 좌현(+) 선수(+)쪽

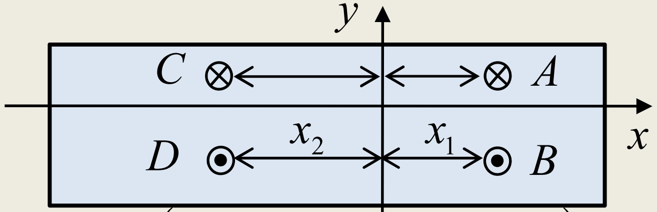
$$x_1 > x_2,\; A-B > C-D$$



$$\mathbf{M}_{BL} = \mathbf{j}\left(x_1A - x_1B - x_2C + x_2D\right) = \mathbf{j}\left(x_1\left(A-B\right) - x_2\left(C-D\right)\right) > \mathbf{j}\cdot 0$$

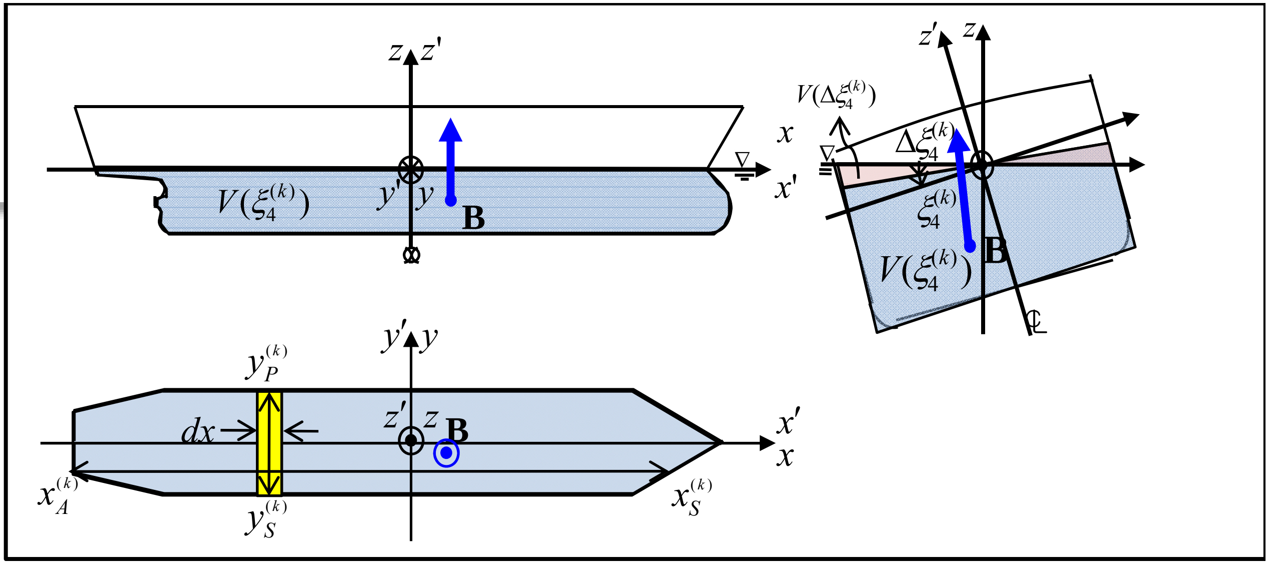
I_P 가 (+)이면, 수선면 중심이 우현(-) 선미(-)쪽

$$x_1 < x_2,\; D-C > B-A$$



$$\mathbf{M}_{BL} = \mathbf{j}\left(x_1A - x_1B - x_2C + x_2D\right) = \mathbf{j}\left(-x_1(B-A) + x_2(D-C)\right) > \mathbf{j}\cdot 0$$

Heel에 의한 모멘트 (9)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

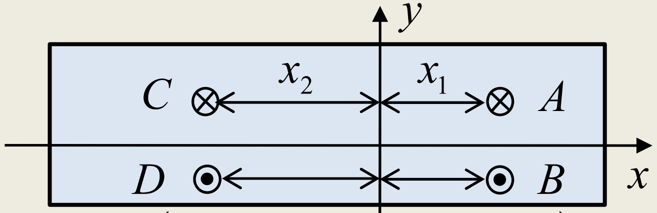
$$\iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \iiint_{V(\Delta\xi_4^{(k)})}[\mathbf{i}y-\mathbf{j}x]dxdydz = \mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} ydxdydz \boxed{-\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} xdxdydz} \rightarrow \approx \mathbf{j}\Delta\xi_4^{(k)} I_P(\xi_4^{(k)})$$

변화된 부피의 종 방향 1차 모멘트

부호 검증

I_P 가 (-)이면, 수선면 중심이 좌현(+) 선미(-)쪽

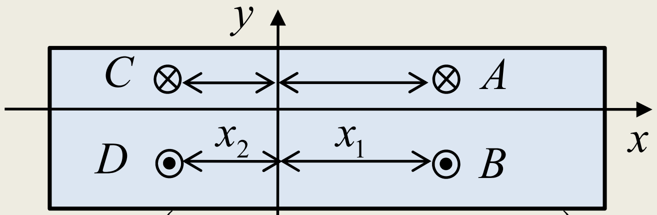
$$x_1 < x_2,\ A-B<C-D$$



$$\mathbf{M}_{BL} = \mathbf{j}(x_1A - x_1B - x_2C + x_2D) = \mathbf{j}(x_1(A-B) - x_2(C-D)) < \mathbf{j}\cdot 0$$

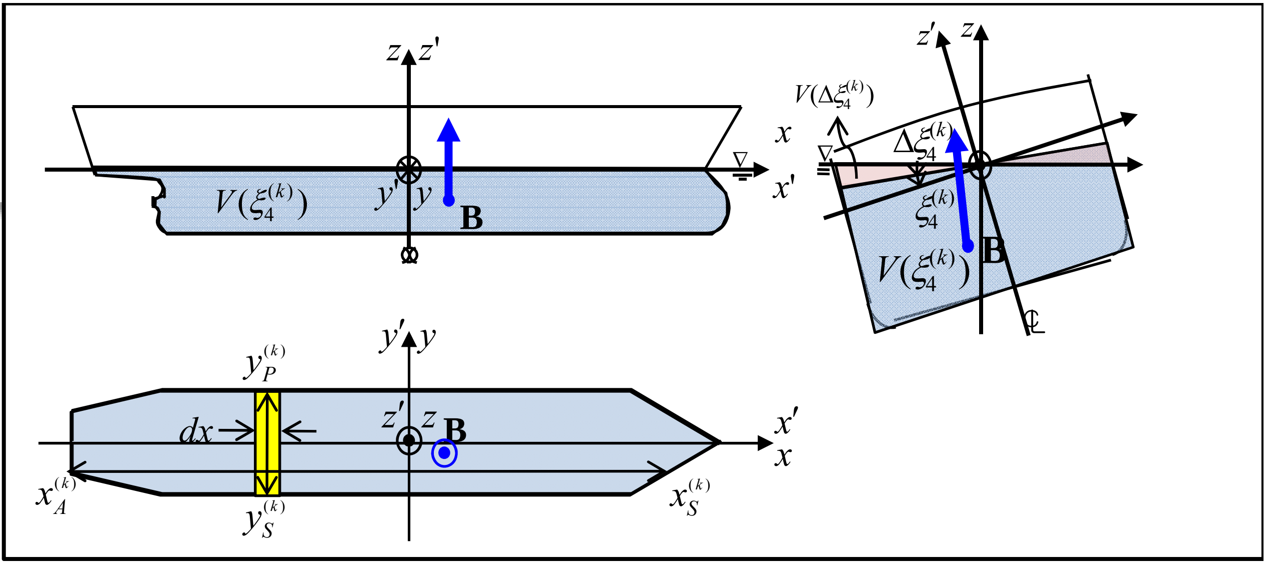
I_P 가 (-)이면, 수선면 중심이 우현(-) 선수(+)쪽

$$x_1 > x_2,\ D-C<B-A$$



$$\mathbf{M}_{BL} = \mathbf{j}(x_1A - x_1B - x_2C + x_2D) = \mathbf{j}(-x_1(B-A) + x_2(D-C)) < \mathbf{j}\cdot 0$$

Heel에 의한 모멘트 (10)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \rho g \iiint_{V(\xi_4^{(k)}+\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{ \boxed{\iiint_{V(\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} + \boxed{\iiint_{V(\Delta\xi_4^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} \right\}$$

(k)번째 상태의 부피 변화된 부피

$$\approx \mathbf{i}\left(V(\xi_4^{(k)})y_B^{(k)} - \Delta\xi_4^{(k)}V(\xi_4^{(k)})z_B^{(k)}\right) - \mathbf{j}V(\xi_4^{(k)})x_B^{(k)}$$

$$= \boxed{\mathbf{i}\iiint_{V(\Delta\xi_4^{(k)})} y\,dxdydz} - \boxed{\mathbf{j}\iiint_{V(\Delta\xi_4^{(k)})} x\,dxdydz}$$

변화된 부피의 횡 방향 1차 모멘트

$$\approx -\mathbf{i}\Delta\xi_4^{(k)} I_T(\xi_4^{(k)})$$

변화된 부피의 종 방향 1차 모멘트

$$\approx \mathbf{j}\Delta\xi_4^{(k)} I_P(\xi_4^{(k)})$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) \approx \mathbf{i}\rho g\left(V(\xi_4^{(k)})y_B^{(k)} - \Delta\xi_4^{(k)}V(\xi_4^{(k)})z_B^{(k)}\right)$$

$$-\mathbf{j}\rho g V(\xi_4^{(k)})x_B^{(k)} - \mathbf{i}\Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)}) + \mathbf{j}\Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)})$$

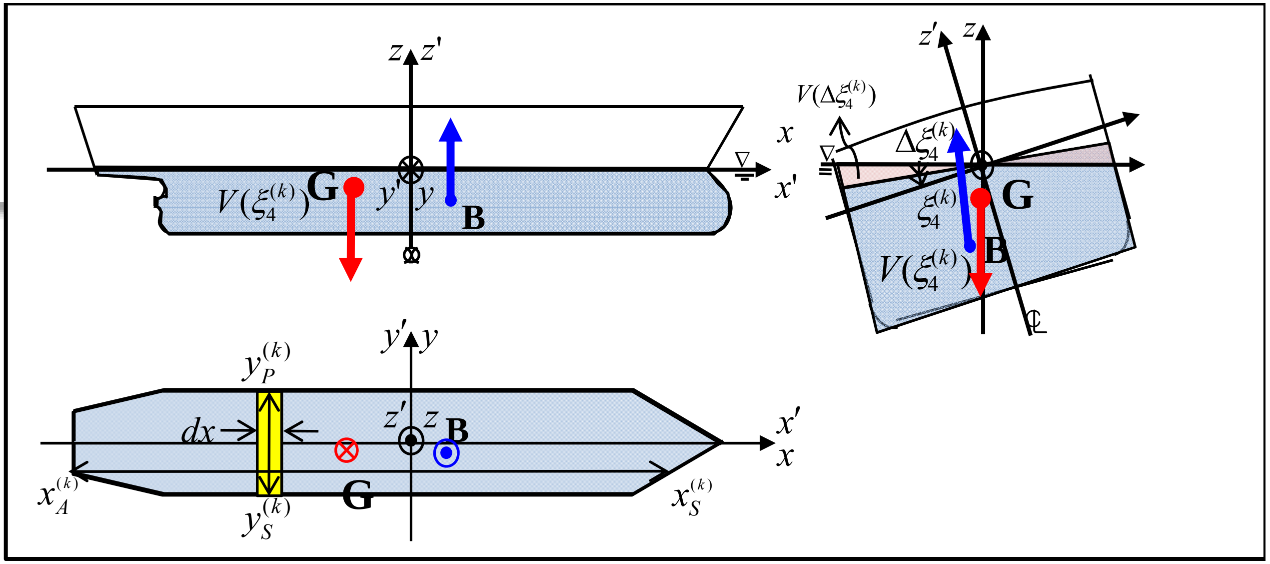
$$= \mathbf{i}\left\{\boxed{\rho g V(\xi_4^{(k)})y_B^{(k)}} - \Delta\xi_4^{(k)}\rho g V(\xi_4^{(k)})z_B^{(k)} - \Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)})\right\} + \mathbf{j}\left\{\boxed{-\rho g V(\xi_4^{(k)})x_B^{(k)}} + \Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)})\right\}$$

대입 대입

$$\boxed{\mathbf{M}_B(\xi_4^{(k)})} = \mathbf{i}\rho g V(\xi_4^{(k)})y_B^{(k)} - \mathbf{j}\rho g V(\xi_4^{(k)})x_B^{(k)}$$

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \mathbf{M}_B(\xi_4^{(k)}) - \mathbf{i}\left(\Delta\xi_4^{(k)}\rho g V(\xi_4^{(k)})z_B^{(k)} + \Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)})\right) + \mathbf{j}\Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)})$$

Heel에 의한 모멘트 (11)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

✓ 부력에 의한 모멘트

$$\mathbf{M}_B(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \mathbf{M}_B(\xi_4^{(k)}) - \mathbf{i}\left(\Delta\xi_4^{(k)}\rho g V(\xi_4^{(k)}) z_B^{(k)} + \Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)})\right) + \mathbf{j}\Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)})$$

✓ 중력에 의한 모멘트

$$\left(\mathbf{r}_G^{(k+1)} \times (-\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G^{(k+1)} & y_G^{(k+1)} & z_G^{(k+1)} \\ 0 & 0 & -1 \end{vmatrix} = -\mathbf{i}y_G^{(k+1)} + \mathbf{j}x_G^{(k+1)} \right)$$

$$\mathbf{M}_G(\xi_4^{(k)}+\Delta\xi_4^{(k)}) = \mathbf{r}_G^{(k+1)} \times (-\mathbf{k}mg) = -\mathbf{i}mg\cdot y_G^{(k+1)} + \mathbf{j}mg\cdot x_G^{(k+1)}$$

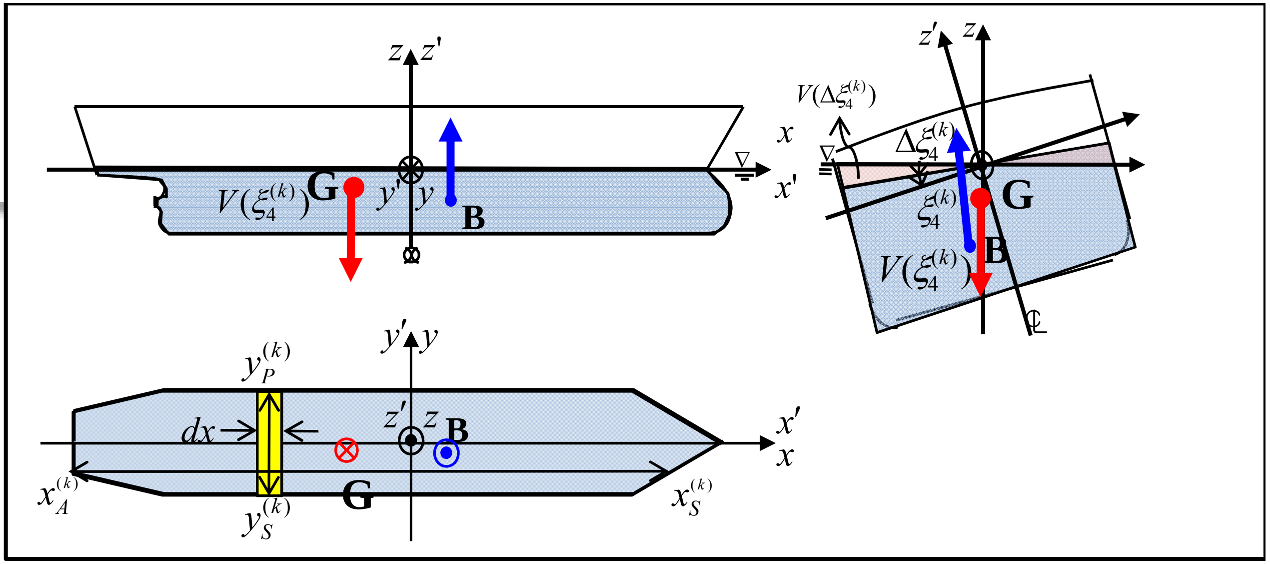
만약 $\Delta\xi_4^{(k)}$가 작다면,

$$= -\mathbf{i}mg\cdot\left(y_G^{(k)}\cos\Delta\xi_4^{(k)} - z_G^{(k)}\sin\Delta\xi_4^{(k)}\right) + \mathbf{j}mg\cdot x_G^{(k)} \approx \underline{-\mathbf{i}mg\cdot\left(y_G^{(k)}\right.} \left. - z_G^{(k)}\Delta\xi_4^{(k)}\right) \underline{+\mathbf{j}mg\cdot x_G^{(k)}}$$

$$= \mathbf{M}_G(\xi_4^{(k)}) + \mathbf{i}mg\cdot z_G^{(k)}\Delta\xi_4^{(k)}$$

$$\mathbf{M}_G(\xi_4^{(k)}) = -\mathbf{i}mg\cdot y_G^{(k)} + \mathbf{j}mg\cdot x_G^{(k)}$$

Heel에 의한 모멘트 (12)



좌표 변환

$$\begin{pmatrix} x_P^{(k)} = x_P^{(k)} \\ y_P^{(k+1)} = y_P^{(k)}\cos\Delta\xi_4^{(k)} - z_P^{(k)}\sin\Delta\xi_4^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_4^{(k)} + y_P^{(k)}\sin\Delta\xi_4^{(k)} \end{pmatrix}$$

✓ 부력에 의한 모멘트

$$\mathbf{M}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{M}_B(\xi_4^{(k)}) - \mathbf{i}\left(\Delta\xi_4^{(k)}\rho g V(\xi_4^{(k)}) z_B^{(k)} + \Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)})\right) + \mathbf{j}\Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)})$$

✓ 중력에 의한 모멘트

$$\mathbf{M}_G(\xi_4^{(k)} + \Delta\xi_4^{(k)}) = \mathbf{M}_G(\xi_4^{(k)}) + \mathbf{i}mg\cdot z_G^{(k)}\Delta\xi_4^{(k)}$$

✓ 선박이 받는 모멘트

$$\begin{aligned}\mathbf{M}(\xi_4^{(k)} + \Delta\xi_4^{(k)}) &= \mathbf{M}_B(\xi_4^{(k)} + \Delta\xi_4^{(k)}) + \mathbf{M}_G(\xi_4^{(k)} + \Delta\xi_4^{(k)}) \\ &= \mathbf{M}_B(\xi_4^{(k)}) + \mathbf{M}_G(\xi_4^{(k)}) - \mathbf{i}\left(\Delta\xi_4^{(k)}\rho g V(\xi_4^{(k)}) z_B^{(k)} + \Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)})\right) + \mathbf{j}\Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)}) + \mathbf{i}mg\cdot z_G^{(k)}\Delta\xi_4^{(k)}\end{aligned}$$

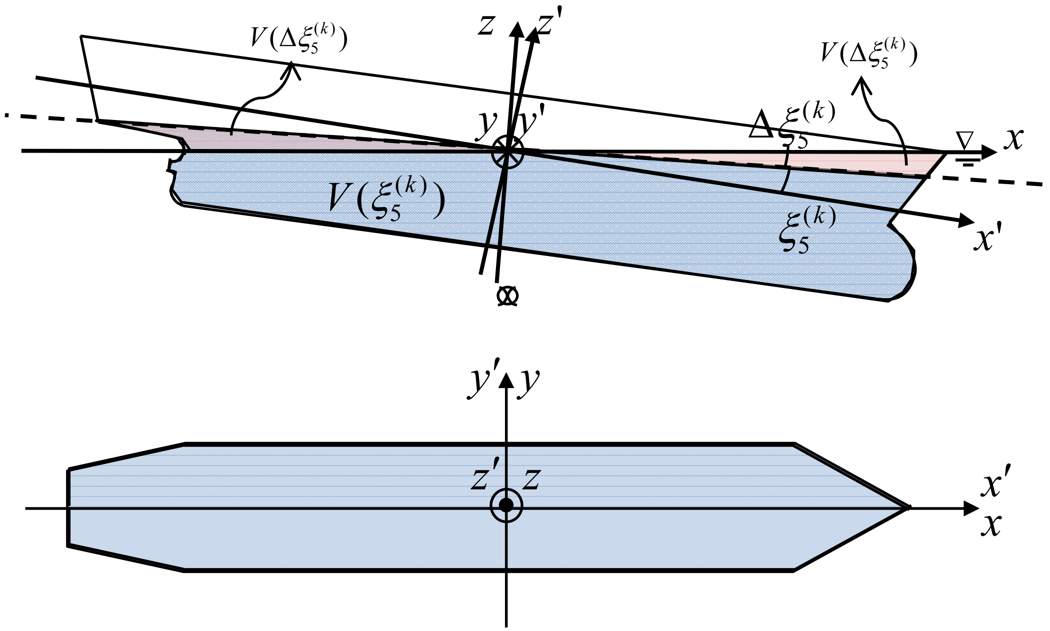
$$\mathbf{M}(\xi_4^{(k)} + \Delta\xi_4^{(k)}) - \mathbf{M}(\xi_4^{(k)}) = -\mathbf{i}\left(\Delta\xi_4^{(k)}\rho g V(\xi_4^{(k)}) z_B^{(k)} + \Delta\xi_4^{(k)}\rho g I_T(\xi_4^{(k)}) - mg\cdot z_G^{(k)}\Delta\xi_4^{(k)}\right) + \mathbf{j}\Delta\xi_4^{(k)}\rho g I_P(\xi_4^{(k)})$$

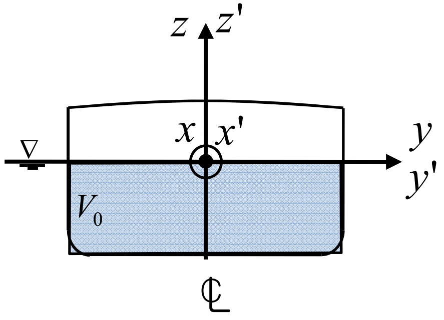
$$\Delta\mathbf{M} = -\mathbf{i}\left(\boxed{\rho g V(\xi_4^{(k)}) z_B^{(k)} + \rho g I_T(\xi_4^{(k)}) - mg\cdot z_G^{(k)}}\right)\Delta\xi_4^{(k)} + \mathbf{j}\boxed{\rho g I_P(\xi_4^{(k)})}\Delta\xi_4^{(k)}$$

$$\left.\frac{\partial \mathbf{M}_T}{\partial \xi_4}\right|_{\xi_4^{(k)}} = -\rho g V(\xi_4^{(k)}) z_B^{(k)} - \rho g I_T(\xi_4^{(k)}) + mg\cdot z_G^{(k)}$$

$$\left.\frac{\partial \mathbf{M}_L}{\partial \xi_4}\right|_{\xi_4^{(k)}} = \rho g I_P(\xi_4^{(k)})$$

Trim에 의한 좌표계 변환





$$x_P^{(k+1)} = x_P^{(k)} \cos\Delta\xi_5^{(k)} + z_P^{(k)} \sin\Delta\xi_5^{(k)}$$

$$y_P^{(k+1)} = y_P^{(k)}$$

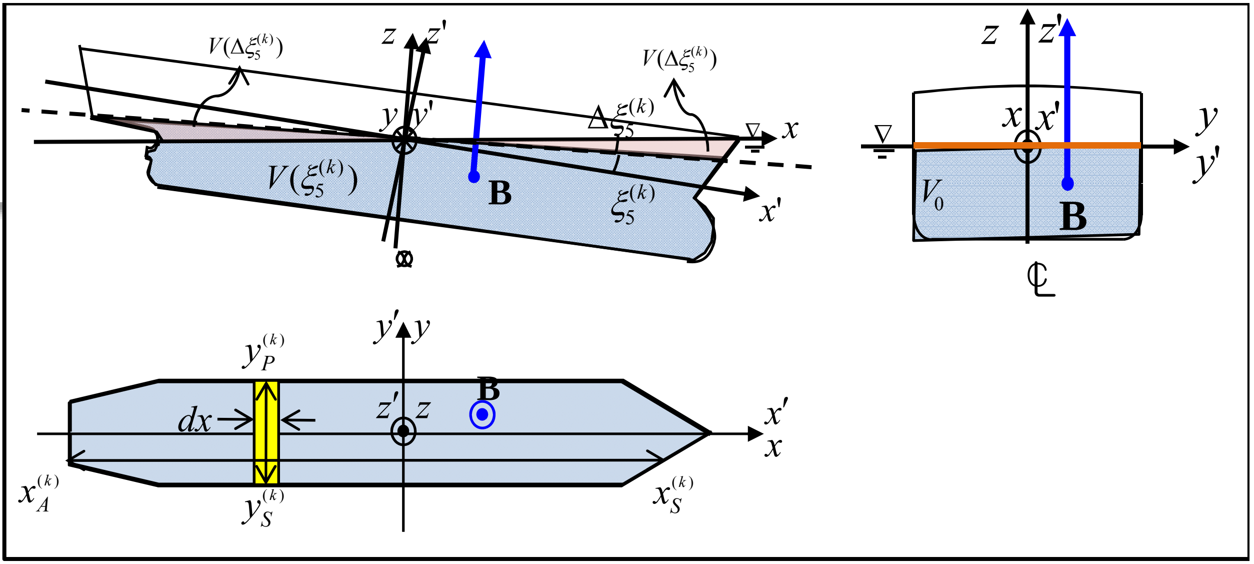
$$z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_5^{(k)} - x_P^{(k)} \sin\Delta\xi_5^{(k)}$$

ex) 배가 y축을 중심으로 회전하고 있다. (k)단계에서 (k+1)단계로 넘어갈 때, $\Delta\xi_5^{(k)} = \pi / 60(rad)$ 만큼 회전한다고 한다. (k)단계에서 (30,10)의 위치는 (k+1)단계에서 어떻게 보이는가?

✓ 좌표의 회전 변환에 의해

$$\begin{bmatrix} x_P^{(k+1)} \\ z_P^{(k+1)} \end{bmatrix} = \begin{bmatrix} \cos\Delta\xi_5^{(k)} & \sin\Delta\xi_5^{(k)} \\ -\sin\Delta\xi_5^{(k)} & \cos\Delta\xi_5^{(k)} \end{bmatrix} \begin{bmatrix} x_P^{(k)} \\ z_P^{(k)} \end{bmatrix} = \begin{bmatrix} x_P^{(k)} \cos\Delta\xi_5^{(k)} + z_P^{(k)} \sin\Delta\xi_5^{(k)} \\ -x_P^{(k)} \sin\Delta\xi_5^{(k)} + z_P^{(k)} \cos\Delta\xi_5^{(k)} \end{bmatrix}$$

Trim에 의한 힘(부력) (1)



좌표 변환

$$\begin{cases} x_P^{(k+1)} = x_P^{(k)} \cos\Delta\xi_5^{(k)} + z_P^{(k)} \sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_5^{(k)} - x_P^{(k)} \sin\Delta\xi_5^{(k)} \end{cases}$$

$$\mathbf{F}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

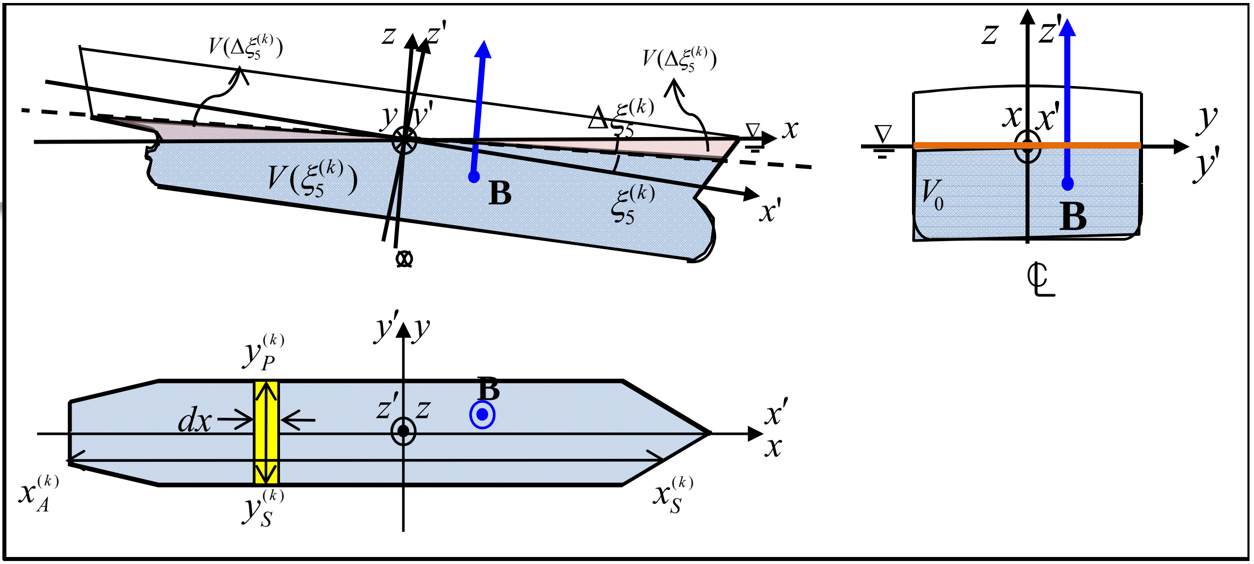
(k)번째 상태의 부피와 변화된 부피에 의한 힘으로 분리

(k)번째 상태의부피

$$= \mathbf{k}\rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} dV = \mathbf{k}\rho g \left\{ \boxed{\iiint_{V(\xi_5^{(k)})} dV} + \iiint_{V(\Delta\xi_5^{(k)})} dV \right\}$$

$$\iiint_{V(\xi_5^{(k)})} dV = \iiint_{V(\xi_5^{(k)})} dxdydz = V(\xi_5^{(k)})$$

Trim에 의한 힘(부력) (2)



좌표 변환

$$\left(\begin{aligned} x_P^{(k+1)} &= x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} &= y_P^{(k)} \\ z_P^{(k+1)} &= z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{aligned}\right)$$

$$\mathbf{F}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \mathbf{k}\rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_5^{(k)})} dV + \overset{\text{변화된 부피}}{\boxed{\iiint_{V(\Delta\xi_5^{(k)})} dV}} \right\}$$

$$\iiint_{V(\Delta\xi_5^{(k)})} dV = \iiint_{V(\Delta\xi_5^{(k)})} dxdydz \underset{\substack{\text{적분하기 편리하게} \\ \text{적분 순서를 변경}}}{=} \iiint_{V(\Delta\xi_5^{(k)})} dzdydx$$

만약 $\Delta\xi_5^{(k)}$ 가 작다면,

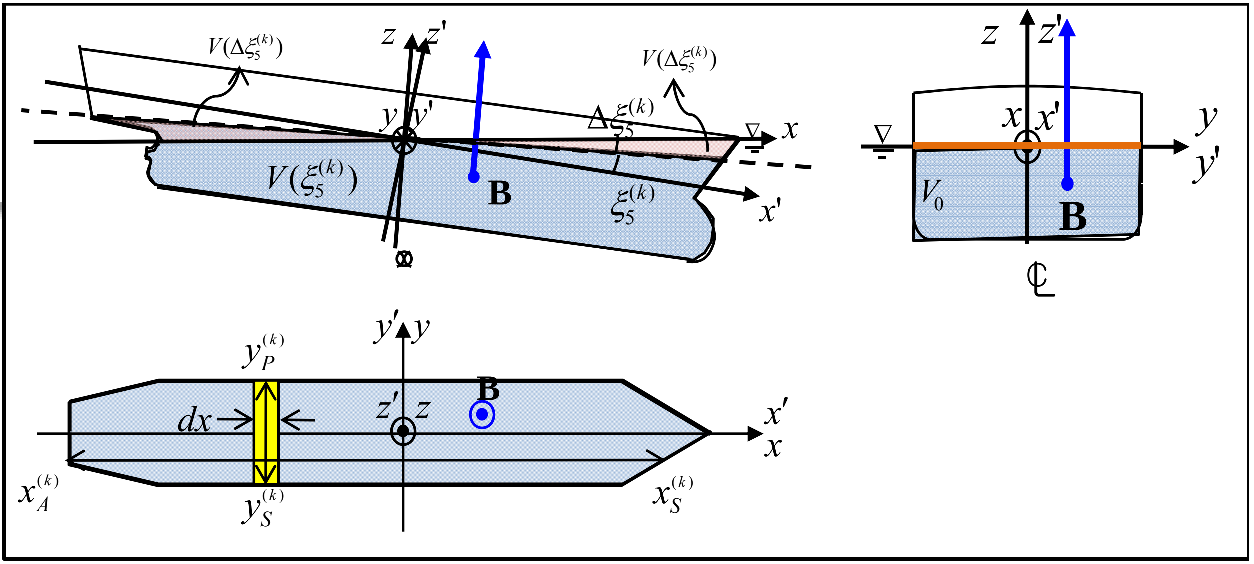
$$\approx -\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{0}^{-x\tan\Delta\xi_5^{(k)}} dzdydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{-x\tan\Delta\xi_5^{(k)}}^{0} dzdydx$$

$$= -\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} \left(-x\tan\Delta\xi_5^{(k)}\right) dydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \left(x\tan\Delta\xi_5^{(k)}\right) dydx = \int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} x\tan\Delta\xi_5^{(k)} dydx$$

$$= \int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} x\tan\Delta\xi_5^{(k)} \Big|_{y_S^{(k)}}^{y_P^{(k)}} dx = \tan\Delta\xi_5^{(k)} \int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \underbrace{x\left[y_P^{(k)} - y_S^{(k)}\right]dx}_{\text{수선면적}}$$

$$= \cos^2\Delta\xi_5^{(k)} \tan\Delta\xi_5^{(k)} \underbrace{L_{WP}(\xi_5^{(k)})}_{\text{수선면의 종 방향 1차 모멘트}} \approx \Delta\xi_5^{(k)} L_{WP}(\xi_5^{(k)})$$

Trim에 의한 힘(부력) (3)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{F}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \mathbf{k}\rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} dV = \mathbf{k}\rho g \left\{ \iiint_{V(\xi_5^{(k)})} dV + \overset{\text{변화된 부피}}{\boxed{\iiint_{V(\Delta\xi_5^{(k)})} dV}} \right\}$$

$$\iiint_{V(\Delta\xi_5^{(k)})} dV = \iiint_{V(\Delta\xi_5^{(k)})} dxdydz$$

$$\approx \underbrace{\underbrace{\Delta\xi_5^{(k)}}_{(+)} \underbrace{\overset{\text{수선면의 종 방향 1차 모멘트}}{\overline{L_{WP}(\xi_5^{(k)})}}}_{(+)}}_{(+)}$$

부호 검증

$L_{WP}(\xi_5^{(k)})$가 (+)이면, 수선면 중심이 선수쪽. 즉 선수쪽의 수선면적이 더 넓음.

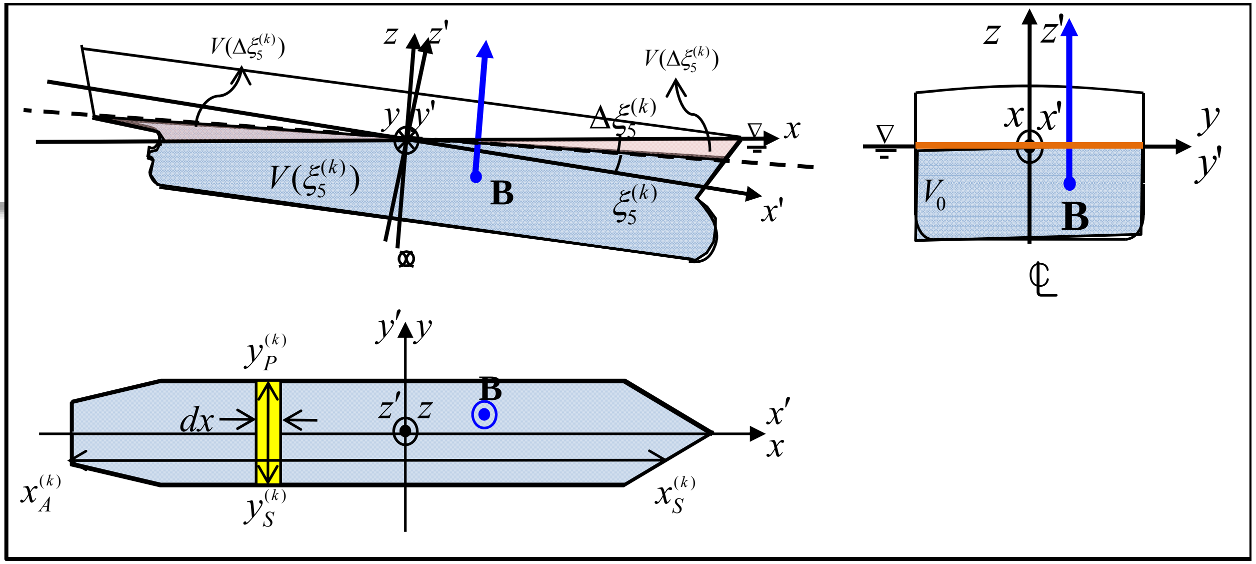
⇩

$\Delta\xi_5^{(k)}$ 이 (+)이면, 선수쪽이 가라앉고(부력 (+)), 선미쪽이 떠오름(부력 (-))

⇩

선수쪽의 가라앉는 부피가 선미쪽의 떠오르는 부피보다 큼. ⇨ 부력의 합은 (+)

Trim에 의한 힘(부력) (4)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{pmatrix}$$

$$\mathbf{F}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \mathbf{k}\rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} dV = \mathbf{k}\rho g\left\{\iiint_{V(\xi_5^{(k)})} dV + \iiint_{V(\Delta\xi_5^{(k)})} dV\right\}$$

$$\approx \boxed{\mathbf{k}\rho g(V(\xi_5^{(k)})} + \Delta\xi_5^{(k)}\underline{L_{WP}(\xi_5^{(k)})})$$

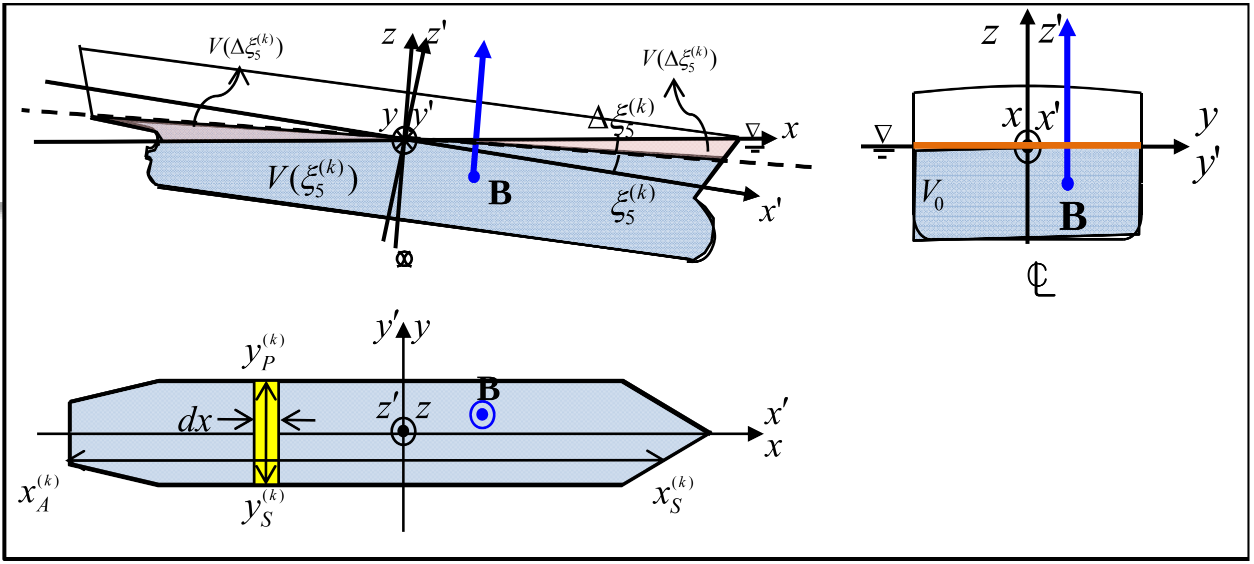
대입

수선면적의
종 방향 1차 모멘트

$$\boxed{\mathbf{F}_B(\xi_5^{(k)})} = \mathbf{k}\rho g V(\xi_5^{(k)})$$

$$\mathbf{F}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)}) = \mathbf{F}_B(\xi_5^{(k)}) + \mathbf{k}\rho g L_{WP}(\xi_5^{(k)})\Delta\xi_5^{(k)}$$

Trim에 의한 힘(부력) (5)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{pmatrix}$$

- ✓ 부력

$$\mathbf{F}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{F}_B(\xi_5^{(k)}) + \mathbf{k}\rho g L_{WP}(\xi_5^{(k)})\Delta\xi_5^{(k)}$$

- ✓ 중력

$$\mathbf{F}_G(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = -\mathbf{k}mg = \mathbf{F}_G(\xi_5^{(k)})$$

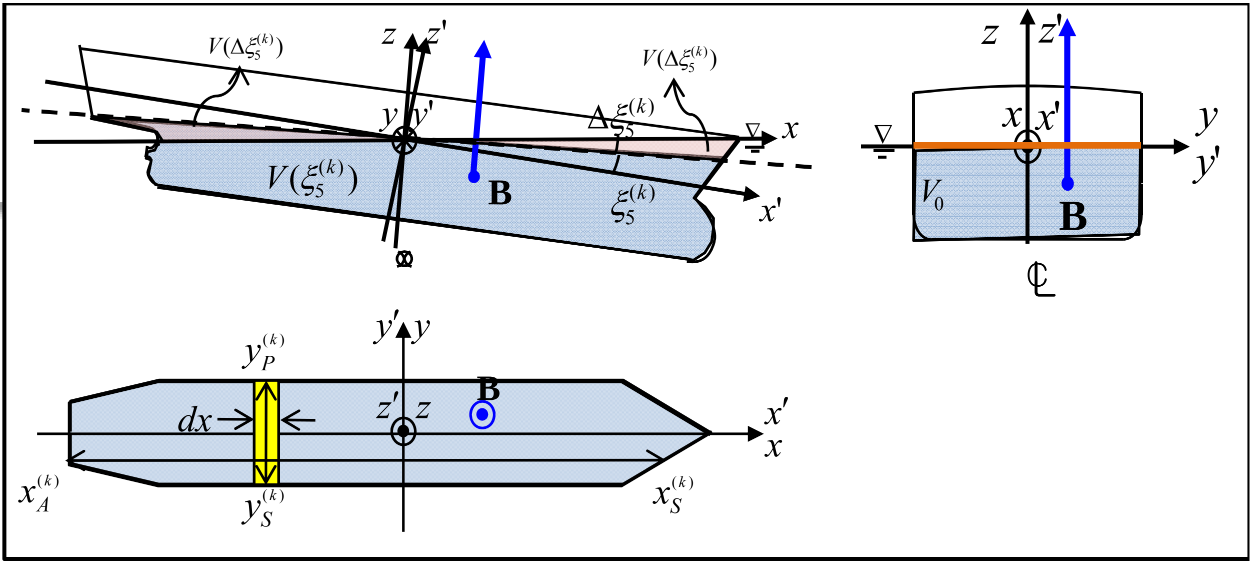
- ✓ 선박이 받는 힘

$$\begin{aligned}\mathbf{F}(\xi_5^{(k)}+\Delta\xi_5^{(k)}) &= \mathbf{F}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) + \mathbf{F}_G(\xi_5^{(k)}+\Delta\xi_5^{(k)}) \\ &= \mathbf{F}_B(\xi_5^{(k)}) + \mathbf{k}\rho g L_{WP}(\xi_5^{(k)})\Delta\xi_5^{(k)} + \mathbf{F}_G(\xi_5^{(k)}) \\ &= \mathbf{F}(\xi_5^{(k)}) + \mathbf{k}\rho g L_{WP}(\xi_5^{(k)})\Delta\xi_5^{(k)}\end{aligned}$$

$$\left.\frac{\partial \mathbf{F}}{\partial \xi_5}\right|_{\xi_5^{(k)}} = \rho g L_{WP}(\xi_5^{(k)})$$

$$\mathbf{F}(\xi_5^{(k)}+\Delta\xi_5^{(k)}) - \mathbf{F}(\xi_5^{(k)}) = \mathbf{k}\boxed{\rho g L_{WP}(\xi_5^{(k)})}\Delta\xi_5^{(k)} \qquad \Delta\mathbf{F}(\xi_5^{(k)}) = \mathbf{k}\boxed{\left.\frac{\partial \mathbf{F}}{\partial \xi_5}\right|_{\xi_5^{(k)}}}\Delta\xi_5^{(k)}$$

Trim에 의한 모멘트 (1)



좌표 변환

$$\begin{cases} x_P^{(k+1)} = x_P^{(k)} \cos\Delta\xi_5^{(k)} + z_P^{(k)} \sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_5^{(k)} - x_P^{(k)} \sin\Delta\xi_5^{(k)} \end{cases}$$

$$\mathbf{M}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x] dV = \rho g \left\{ \boxed{\iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x] dV} + \iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x] dV \right\}$$

(k)번째 상태의 부피

$$\iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x] dV = \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x] dxdydz$$

$$= \mathbf{i} \iiint_{V(\xi_5^{(k)})} y dxdydz - \mathbf{j} \iiint_{V(\xi_5^{(k)})} x dxdydz$$

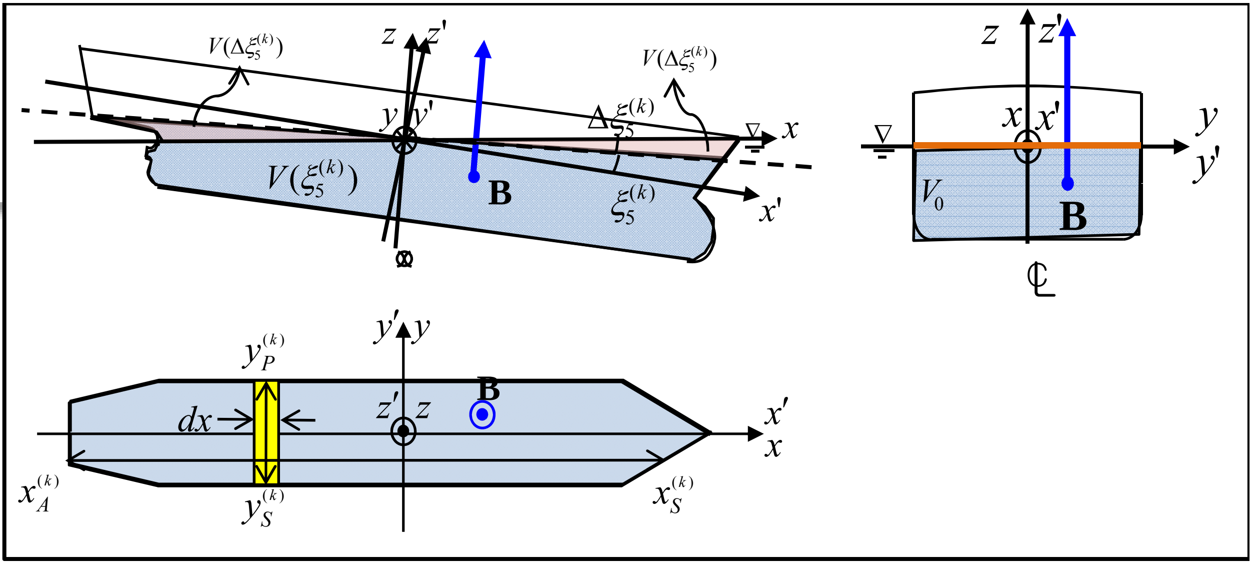
$$= \mathbf{i}V(\xi_5^{(k)}) y_B^{(k+1)} - \mathbf{j}V(\xi_5^{(k)}) x_B^{(k+1)}$$

$$= \mathbf{i}V(\xi_5^{(k)}) y_B^{(k)} - \mathbf{j}V(\xi_5^{(k)}) \{x_B^{(k)} \cos\Delta\xi_5^{(k)} + z_B^{(k)} \sin\Delta\xi_5^{(k)}\}$$

만약 $\Delta\xi_5^{(k)}$가 작다면,

$$= \mathbf{i}V(\xi_5^{(k)}) y_B^{(k)} - \mathbf{j}\left[V(\xi_5^{(k)}) x_B^{(k)} + \Delta\xi_5^{(k)} V(\xi_5^{(k)}) z_B^{(k)}\right]$$

Trim에 의한 모멘트 (2)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{M}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

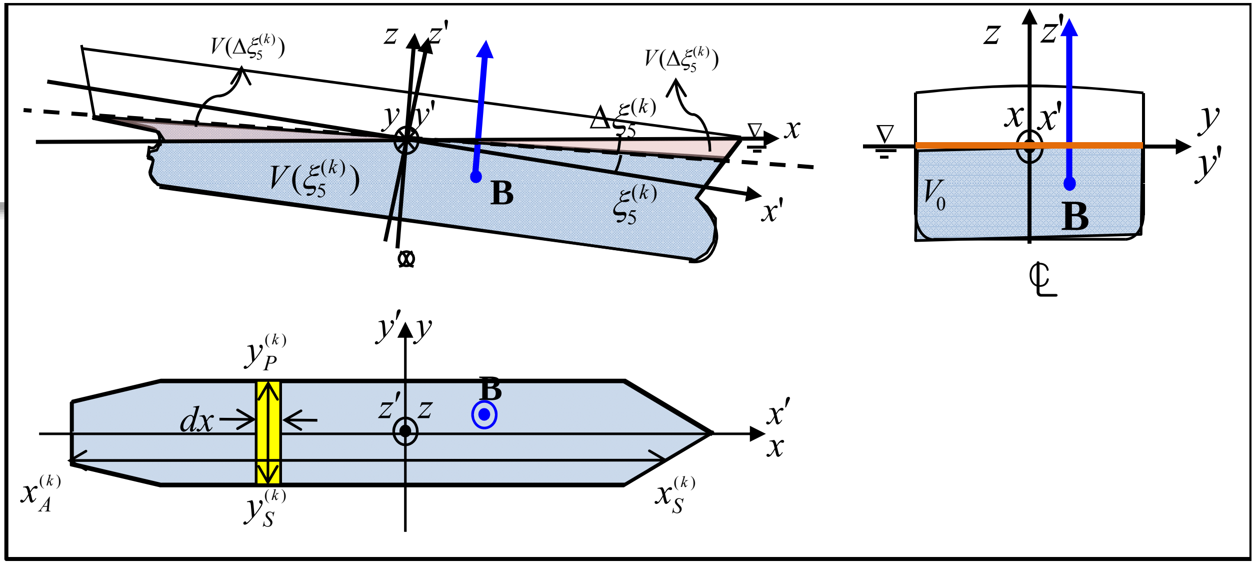
$$= \rho g \iiint\limits_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \rho g \left\{ \iiint\limits_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV + \boxed{\iiint\limits_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV} \right\}$$

변화된 부피

$$\iiint\limits_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \iiint\limits_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dxdydz$$

$$= \mathbf{i} \iiint\limits_{V(\Delta\xi_5^{(k)})} ydxdydz - \mathbf{j} \iiint\limits_{V(\Delta\xi_5^{(k)})} xdxdydz$$

Trim에 의한 모멘트 (3)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{M}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV} \right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \mathbf{i}\boxed{\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} xdxdydz$$

변화된 부피의 횡 방향 1차 모멘트

만약 $\Delta\xi_5^{(k)}$ 가 작다면,

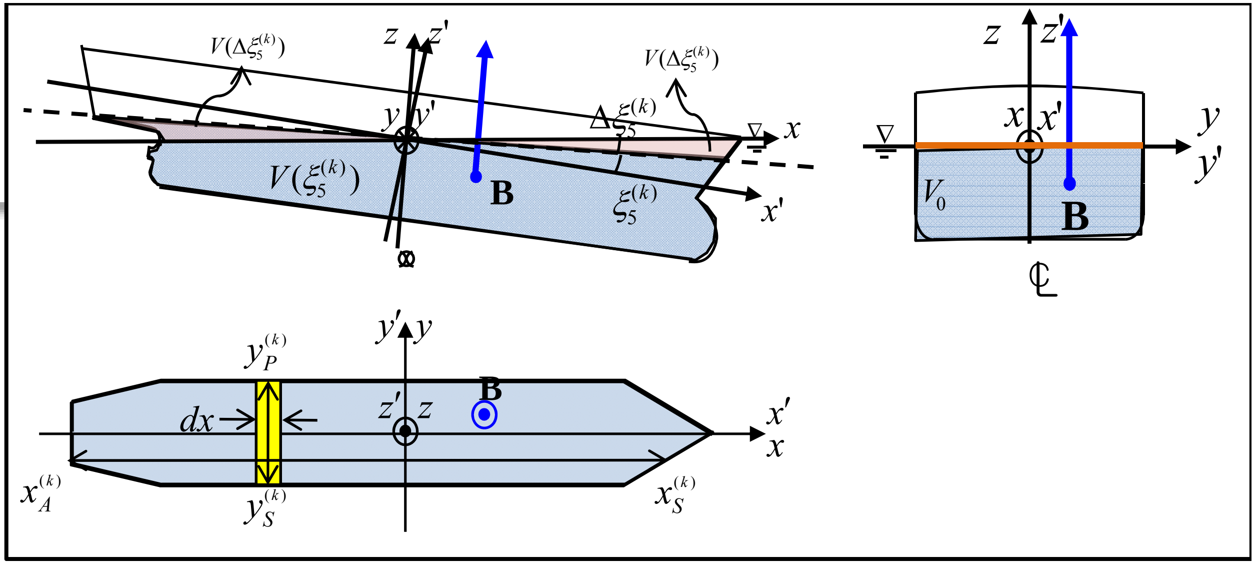
$$\mathbf{i}\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz \approx \mathbf{i}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{0}^{-x\tan\Delta\xi_5^{(k)}} ydzdydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{-x\tan\Delta\xi_5^{(k)}}^{0} ydzdydx \right]$$

$$= \mathbf{i}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} yz\Big|_{0}^{-x\tan\Delta\xi_5^{(k)}} dydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} yz\Big|_{-x\tan\Delta\xi_5^{(k)}}^{0} dydx \right]$$

$$= \mathbf{i}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} \left(-yx\tan\Delta\xi_5^{(k)}\right)dydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \left(yx\tan\Delta\xi_5^{(k)}\right)dydx \right]$$

$$= \mathbf{i}\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \left(yx\tan\Delta\xi_5^{(k)}\right)dydx = \mathbf{i}\left[\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \frac{y^2}{2}x\tan\Delta\xi_5^{(k)}\Bigg|_{y_S^{(k)}}^{y_P^{(k)}} dx \right]$$

Trim에 의한 모멘트 (4)



좌표 변환

$$\begin{cases} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{cases}$$

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{ \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} \right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \mathbf{i}\boxed{\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} xdxdydz$$

변화된 부피의 횡 방향 1차 모멘트

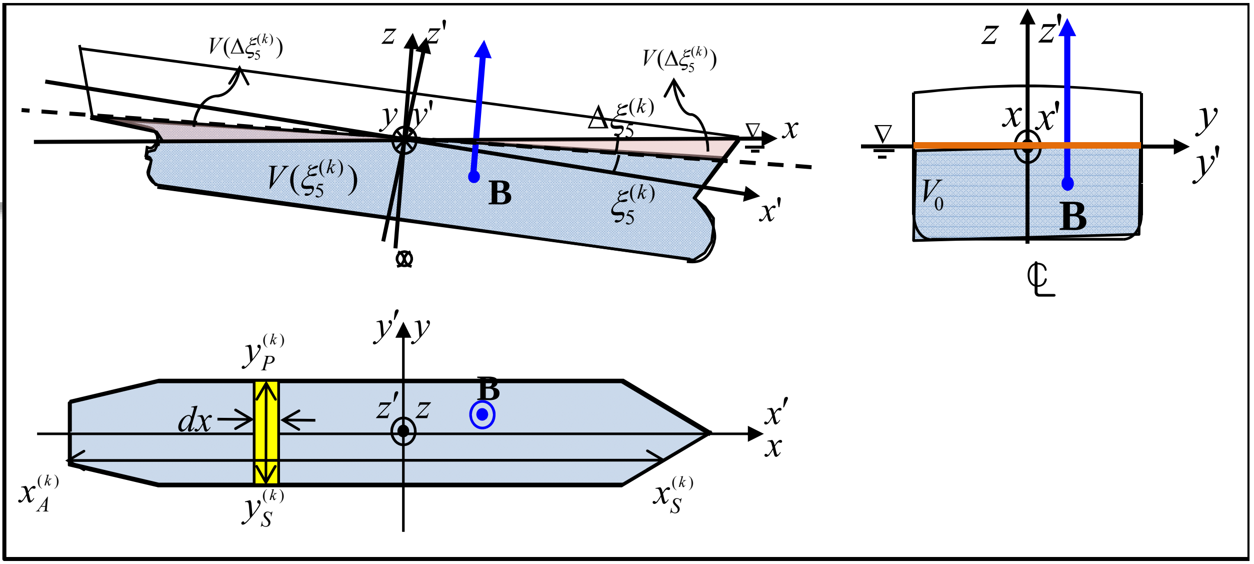
$$= \mathbf{i}\left[\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \frac{y^2}{2} x\tan\Delta\xi_5^{(k)} \Big|_{y_S^{(k)}}^{y_P^{(k)}} dx \right]$$

$$= \mathbf{i}\tan\Delta\xi_5^{(k)} \int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \frac{x}{2}\left[\left\{y_P^{(k)}\right\}^2 - \left\{y_S^{(k)}\right\}^2\right]dx = \mathbf{i}\tan\Delta\xi_5^{(k)} \int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} x\cdot \underbrace{\frac{\left[y_P^{(k)}+y_S^{(k)}\right]}{2}}_{\text{폭방향 중심}} \cdot \underbrace{\left[y_P^{(k)}-y_S^{(k)}\right]}_{\text{미소 면적}} dx$$

만약 $\Delta\xi_5^{(k)}$ 가 작다면,

$$= \mathbf{i}\cos^2\Delta\xi_5^{(k)}\tan\Delta\xi_5^{(k)} I_P(\xi_5^{(k)}) \approx \mathbf{i}\Delta\xi_5^{(k)} I_P(\xi_5^{(k)})$$

Trim에 의한 모멘트 (5)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

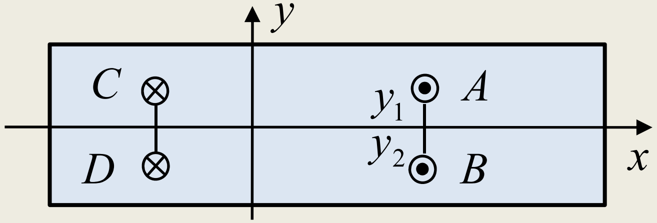
$$\iiint_{V(\Delta\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \mathbf{i}\boxed{\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} xdxdydz$$

변화된 부피의 횡 방향 1차 모멘트 → $\approx \mathbf{i}\Delta\xi_5^{(k)} I_P(\xi_5^{(k)})$

부호 검증

※ 좌우현 대칭

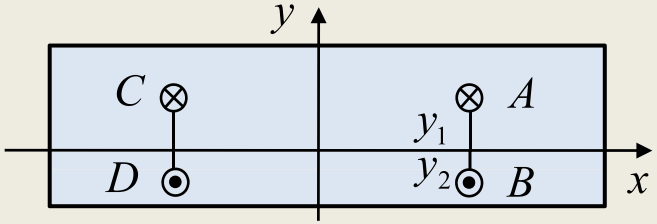
$y_1 = y_2,\ A=B,\ C=D$



$$\mathbf{M}_{BT} = \mathbf{i}(y_1A - y_2B - y_1C + y_2D) = \mathbf{i}\cdot 0$$

※ 선수미 대칭

$A=C,\ B=D$

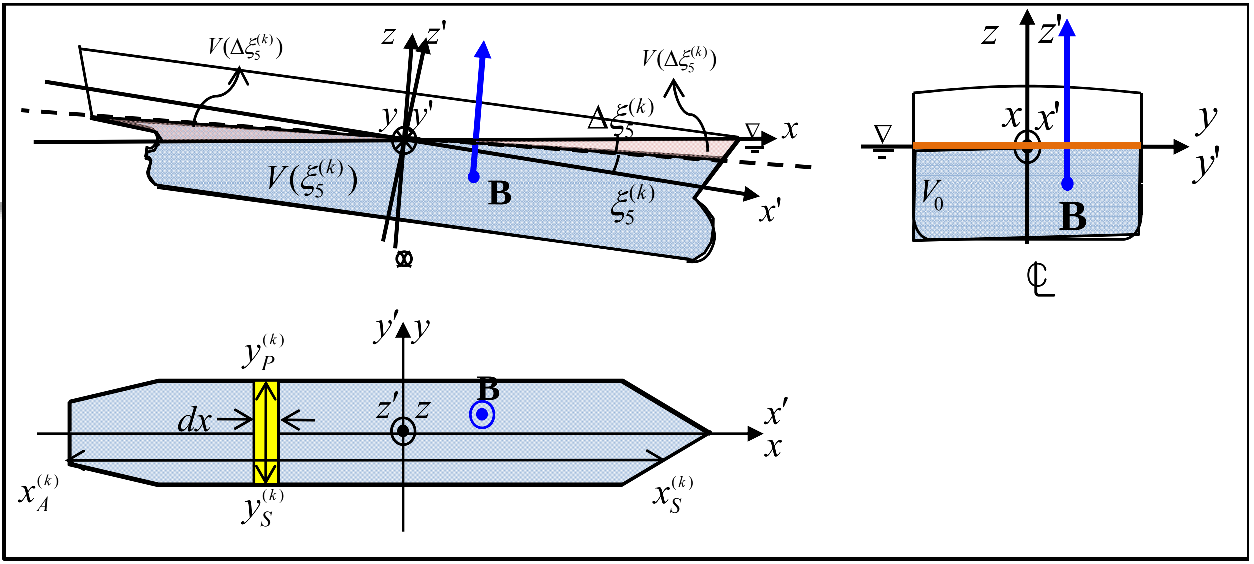


$$\mathbf{M}_{BT} = \mathbf{i}(y_1A - y_2B - y_1C + y_2D) = \mathbf{i}\cdot 0$$

Trim에 의한 모멘트 (6)

좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$



$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV = \rho g \left\{ \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV} \right\}$$

변화된 부피

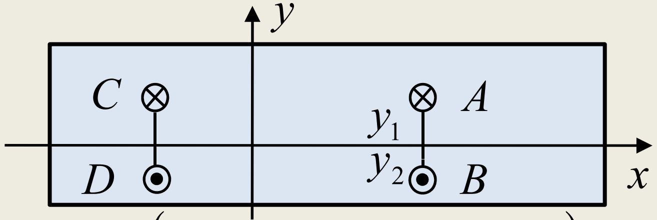
$$\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV = \mathbf{i}\boxed{\iiint_{V(\Delta\xi_5^{(k)})} y\,dxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} x\,dxdydz$$

변화된 부피의 횡 방향 1차 모멘트 → $\approx \mathbf{i}\Delta\xi_5^{(k)} I_P(\xi_5^{(k)})$

부호 검증

I_P 가 (+)이면, 수선면 중심이 좌현(+) 선수(+)쪽

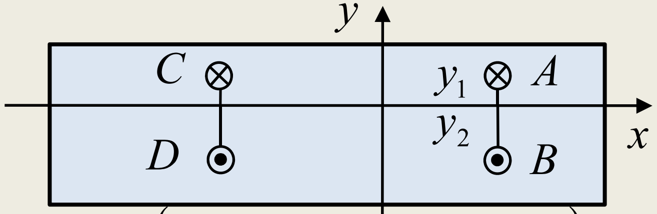
$$y_1 > y_2,\ A - C > B - D$$



$$\mathbf{M}_{BT} = \mathbf{i}\big(y_1A - y_2B - y_1C + y_2D\big) = \mathbf{i}\big(y_1(A-C) - y_2(B-D)\big) > \mathbf{i}\cdot 0$$

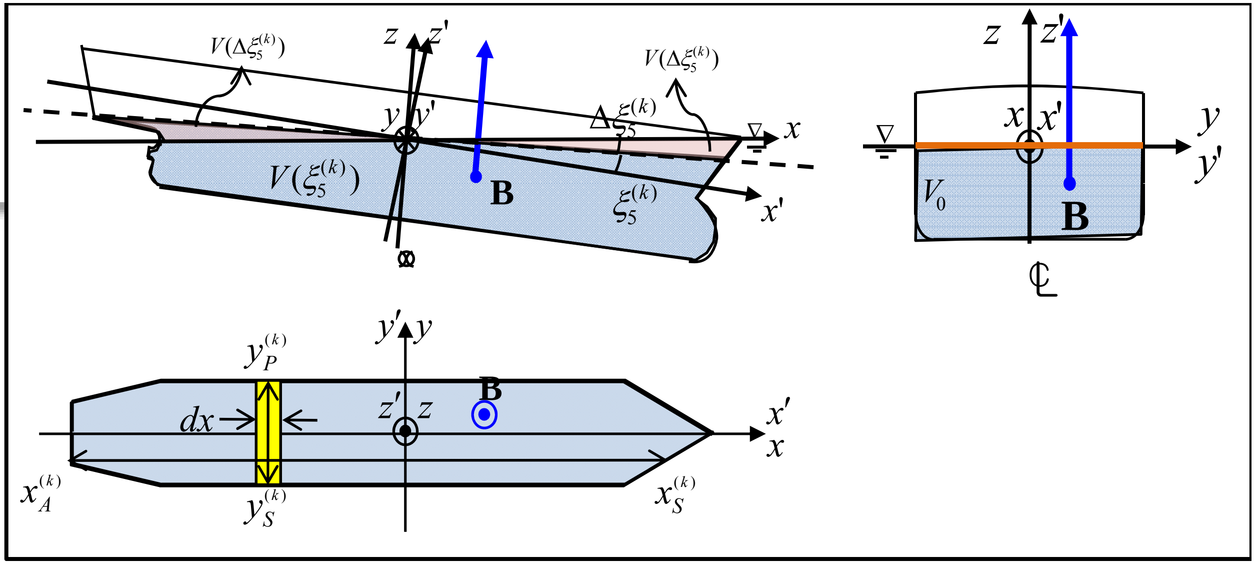
I_P 가 (+)이면, 수선면 중심이 우현(-) 선미(-)쪽

$$y_1 < y_2,\ D - B > C - A$$



$$\mathbf{M}_{BT} = \mathbf{i}\big(y_1A - y_2B - y_1C + y_2D\big) = \mathbf{i}\big(-y_1(C-A) + y_2(D-B)\big) > \mathbf{i}\cdot 0$$

Trim에 의한 모멘트 (7)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g\left\{\iiint_{V(\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV}\right\}$$

변화된 부피

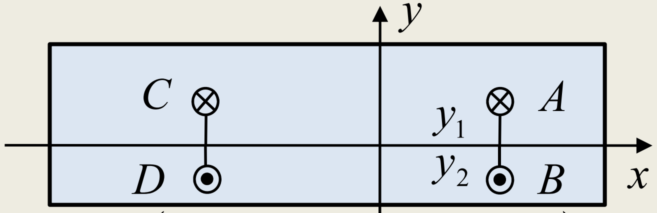
$$\iiint_{V(\Delta\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]dV = \mathbf{i}\boxed{\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz} - \mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} xdxdydz$$

변화된 부피의 횡 방향 1차 모멘트 → $\approx \mathbf{i}\Delta\xi_5^{(k)} I_P(\xi_5^{(k)})$

부호 검증

I_P 가 (-)이면, 수선면 중심이 좌현(+) 선미(-)쪽

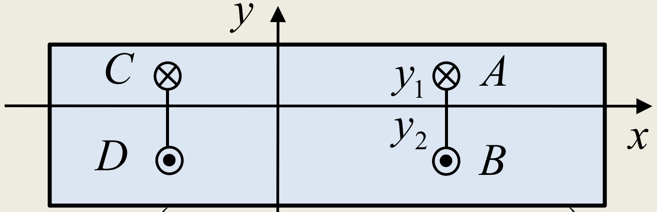
$y_1 > y_2,\ C-A > D-B$



$$\mathbf{M}_{BT} = \mathbf{i}(y_1A - y_2B - y_1C + x_2D)$$
$$= \mathbf{i}(-y_1(C-A) + y_2(D-B)) < \mathbf{i}\cdot 0$$

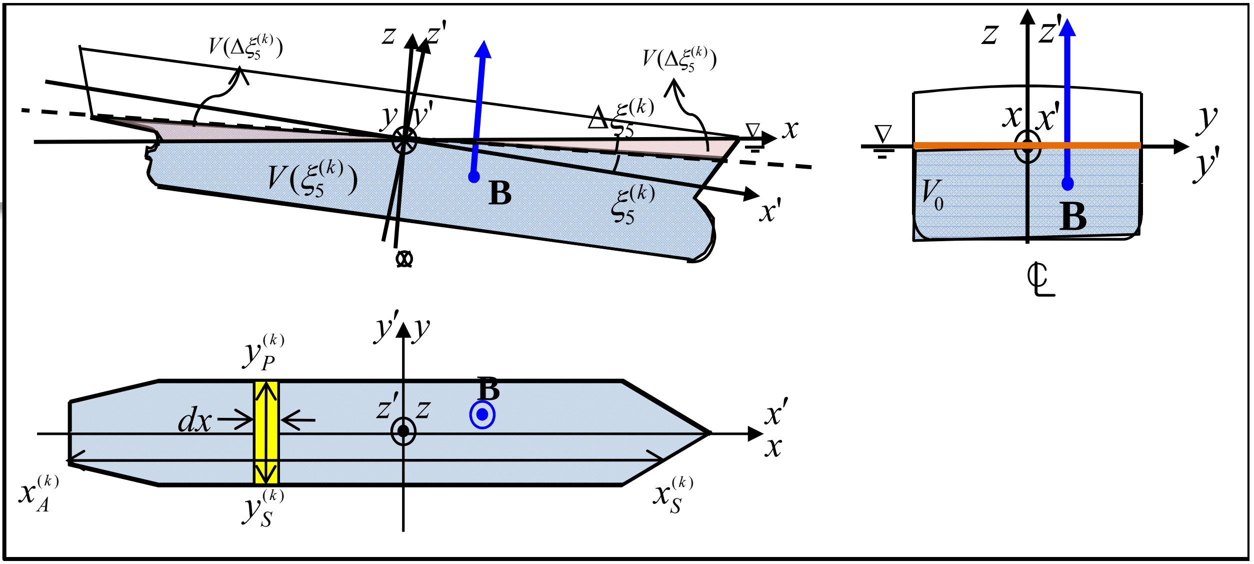
I_P 가 (-)이면, 수선면 중심이 우현(-) 선수(+)쪽

$y_1 < y_2,\ B-D > A-C$



$$\mathbf{M}_{BT} = \mathbf{i}(y_1A - y_2B - y_1C + y_2D)$$
$$= \mathbf{i}(y_1(A-C) - y_2(B-D)) < \mathbf{i}\cdot 0$$

Trim에 의한 모멘트 (8)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{M}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV} \right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]dV = \mathbf{i} \iiint_{V(\Delta\xi_5^{(k)})} ydxdydz \boxed{- \mathbf{j} \iiint_{V(\Delta\xi_5^{(k)})} xdxdydz}$$

변화된 부피의 종 방향 1차 모멘트

$$-\mathbf{j} \iiint_{V(\Delta\xi_5^{(k)})} xdxdydz$$

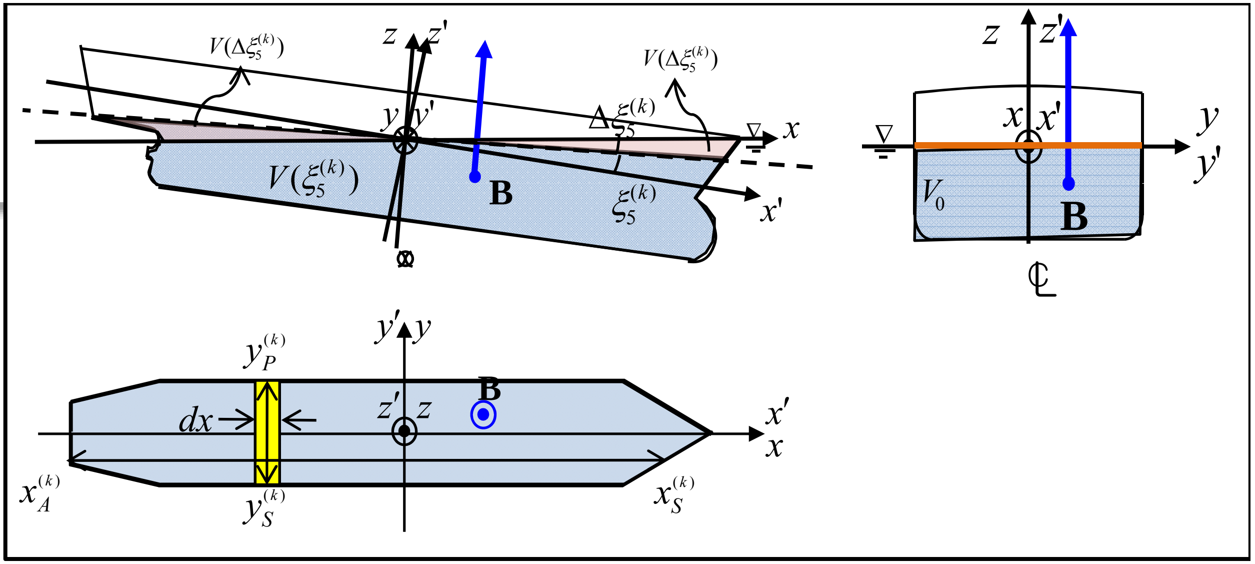
만약 $\Delta\xi_5^{(k)}$ 가 작다면,

$$\approx -\mathbf{j}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{0}^{-x\tan\Delta\xi_5^{(k)}} xdzdydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} \int_{-x\tan\Delta\xi_5^{(k)}}^{0} xdzdydx \right]$$

$$= -\mathbf{j}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} xz\Big|_{0}^{-x\tan\Delta\xi_5^{(k)}} dydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} xz\Big|_{-x\tan\Delta\xi_5^{(k)}}^{0} dydx \right]$$

$$= -\mathbf{j}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} (-x^2\tan\Delta\xi_5^{(k)})dydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} (x^2\tan\Delta\xi_5^{(k)})dydx \right]$$

Trim에 의한 모멘트 (9)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \rho g \left\{ \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV} \right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]dV = \mathbf{i}\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz \boxed{-\mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} xdxdydz}$$

변화된 부피의 종 방향 1차 모멘트

$$= -\mathbf{j}\left[-\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{0} \int_{y_S^{(k)}}^{y_P^{(k)}} (-x^2\tan\Delta\xi_5^{(k)})dydx + \int_{0}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} (x^2\tan\Delta\xi_5^{(k)})dydx \right]$$

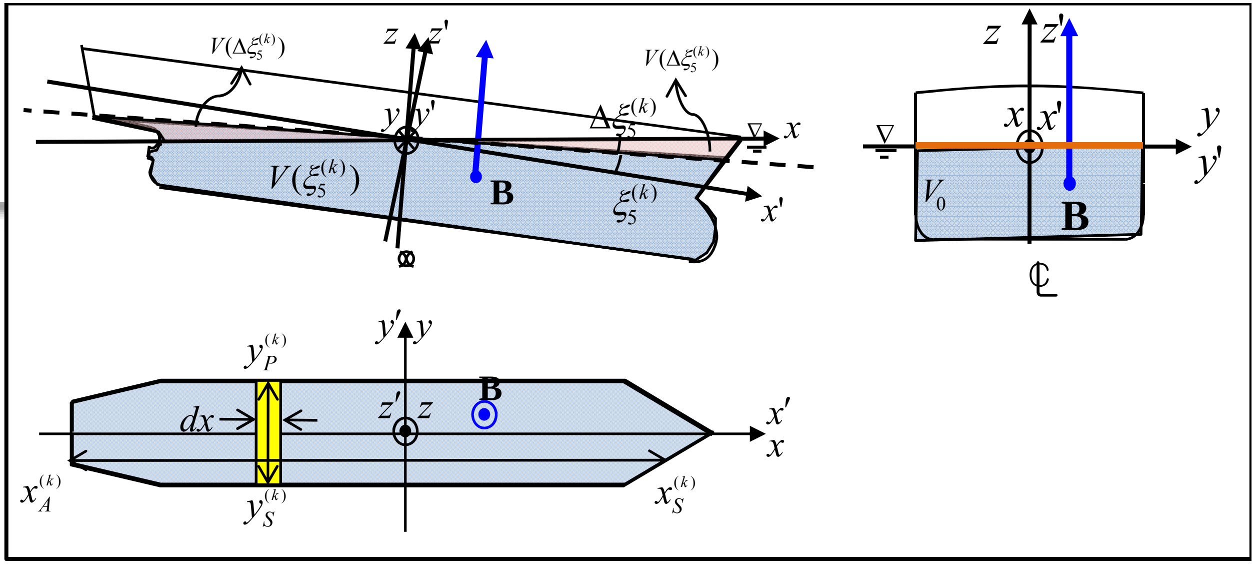
$$= -\mathbf{j}\left[\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \int_{y_S^{(k)}}^{y_P^{(k)}} (x^2\tan\Delta\xi_5^{(k)})dydx \right] = -\mathbf{j}\left[\int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} (x^2\tan\Delta\xi_5^{(k)})(y_P^{(k)}-y_S^{(k)})dx \right]$$

만약 $\Delta\xi_5^{(k)}$ 가 작다면,

$$= -\mathbf{j}\tan\Delta\xi_5^{(k)} \int_{x_A^{(k)}\cos\Delta\xi_5^{(k)}}^{x_F^{(k)}\cos\Delta\xi_5^{(k)}} \underline{x^2(y_P^{(k)}-y_S^{(k)})dx} = -\tan\Delta\xi_5^{(k)}\cos^3\Delta\xi_5^{(k)} I_L(\xi_5^{(k)}) \approx -\mathbf{j}\Delta\xi_5^{(k)} I_L(\xi_5^{(k)})$$

수선면의 종 방향 2차 모멘트

Trim에 의한 모멘트 (10)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)} \cos\Delta\xi_5^{(k)} + z_P^{(k)} \sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_5^{(k)} - x_P^{(k)} \sin\Delta\xi_5^{(k)} \end{array}\right)$$

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]\,dV = \rho g\left\{\iiint_{V(\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]\,dV + \boxed{\iiint_{V(\Delta\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]\,dV}\right\}$$

변화된 부피

$$\iiint_{V(\Delta\xi_5^{(k)})}[\mathbf{i}y-\mathbf{j}x]\,dV = \mathbf{i}\iiint_{V(\Delta\xi_5^{(k)})} ydxdydz \boxed{-\mathbf{j}\iiint_{V(\Delta\xi_5^{(k)})} xdxdydz}$$

변화된 부피의 종 방향 1차 모멘트

$$\approx -\underbrace{\mathbf{j}\underbrace{\Delta\xi_5^{(k)}}_{(+)}\underbrace{I_L(\xi_5^{(k)})}_{(+)}}_{(-)}$$

수선면의 종 방향 2차 모멘트

부호 검증

I_L 는 항상 (+)

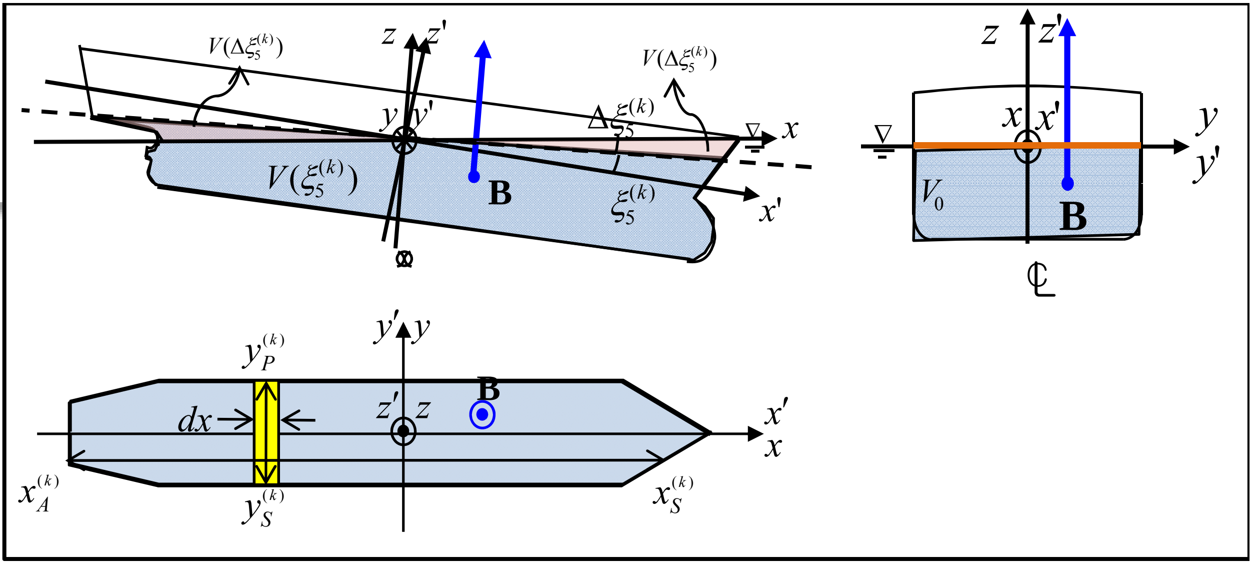
⇩

$\Delta\xi_5^{(k)}$ 가 (+)이면, 선수쪽이 가라앉고(부력 (+)), 선미쪽이 떠오름(부력 (-))

⇩

y 축에 대한 (-)의 모멘트

Trim에 의한 모멘트 (11)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)} \cos\Delta\xi_5^{(k)} + z_P^{(k)} \sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)} \cos\Delta\xi_5^{(k)} - x_P^{(k)} \sin\Delta\xi_5^{(k)} \end{pmatrix}$$

$$\mathbf{M}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV = \rho g \left\{ \boxed{\iiint_{V(\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV} + \boxed{\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV} \right\}$$

(k) 번째 상태의 부피　　변화된 부피

$$\approx \mathbf{i}V(\xi_5^{(k)})y_B^{(k)} - \mathbf{j}\left[V(\xi_5^{(k)})x_B^{(k)} + \Delta\xi_5^{(k)}V(\xi_5^{(k)})z_B^{(k)}\right]$$

$$\iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y - \mathbf{j}x]\,dV = \boxed{\mathbf{i} \iiint_{V(\Delta\xi_5^{(k)})} ydxdydz} \boxed{- \mathbf{j} \iiint_{V(\Delta\xi_5^{(k)})} xdxdydz}$$

변화된 부피의 횡 방향 1차 모멘트　　변화된 부피의 종 방향 1차 모멘트

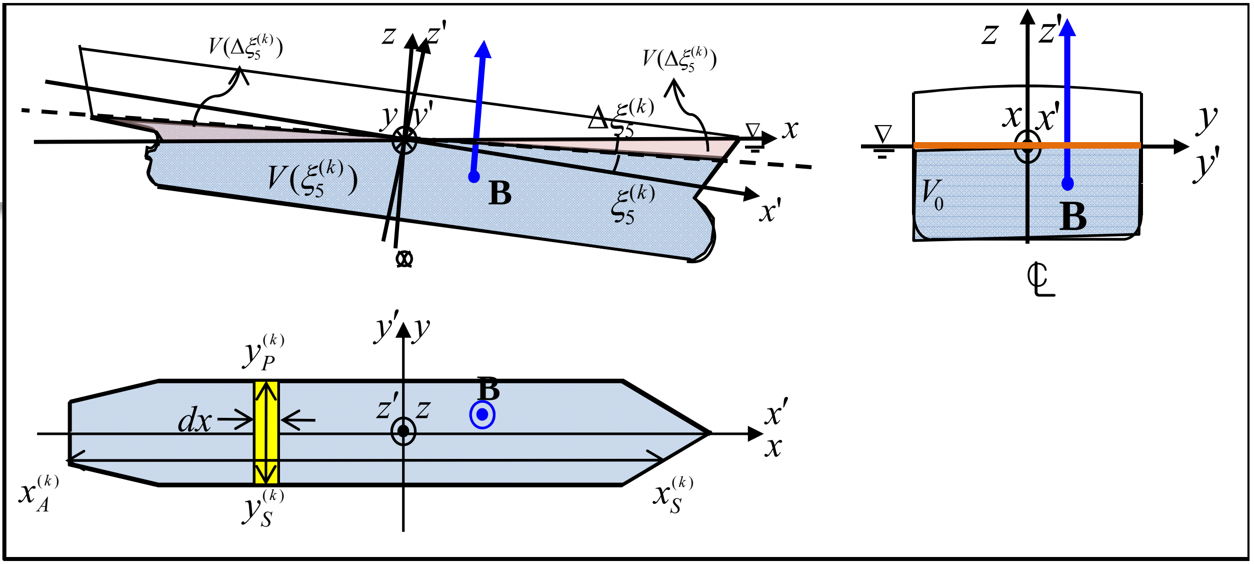
$$\approx \mathbf{i}\Delta\xi_5^{(k)} I_P \qquad \approx -\mathbf{j}\Delta\xi_5^{(k)} I_L$$

$$\mathbf{M}_B(\xi_5^{(k)} + \Delta\xi_5^{(k)})$$

$$= \rho g \left(\mathbf{i}V(\xi_5^{(k)})y_B^{(k)} - \mathbf{j}\left[V(\xi_5^{(k)})x_B^{(k)} + \Delta\xi_5^{(k)}V(\xi_5^{(k)})z_B^{(k)}\right] + \mathbf{i}\Delta\xi_5^{(k)} I_P - \mathbf{j}\Delta\xi_5^{(k)} I_L \right)$$

$$= \mathbf{i}\rho g \left[V(\xi_5^{(k)})y_B^{(k)} + \Delta\xi_5^{(k)} I_P \right] - \mathbf{j}\rho g \left[V(\xi_5^{(k)})x_B^{(k)} + \Delta\xi_5^{(k)}V(\xi_5^{(k)})z_B^{(k)} + \Delta\xi_5^{(k)} I_L \right]$$

Trim에 의한 모멘트 (12)



좌표 변환

$$\begin{cases} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{cases}$$

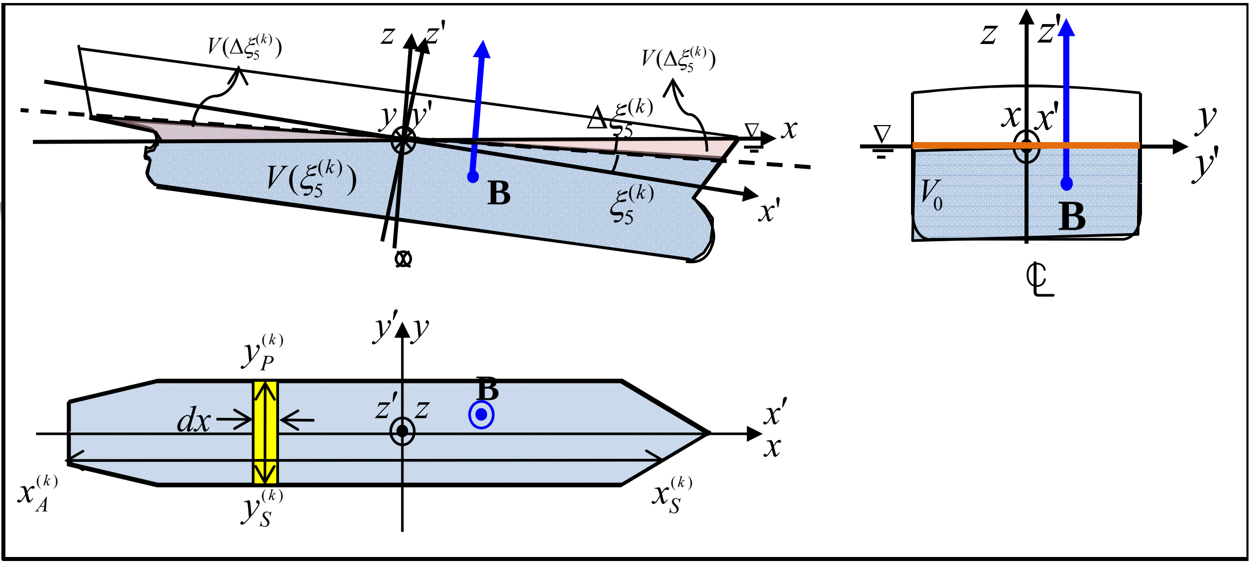
$$\begin{aligned} &\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) \\ &= \rho g \iiint_{V(\xi_5^{(k)}+\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]\,dV = \rho g\left\{ \iiint_{V(\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]\,dV + \iiint_{V(\Delta\xi_5^{(k)})} [\mathbf{i}y-\mathbf{j}x]\,dV \right\} \\ &= \mathbf{i}\rho g\left[V(\xi_5^{(k)})y_B^{(k)} + \Delta\xi_5^{(k)} I_P\right] - \mathbf{j}\rho g\left[V(\xi_5^{(k)})x_B^{(k)} + \Delta\xi_5^{(k)} V(\xi_5^{(k)}) z_B^{(k)} + \Delta\xi_5^{(k)} I_L\right] \\ &= \boxed{\mathbf{i}\rho g V(\xi_5^{(k)})y_B^{(k)} - \mathbf{j}\rho g V(\xi_5^{(k)})x_B^{(k)}} + \mathbf{i}\rho g I_P \Delta\xi_5^{(k)} - \mathbf{j}\rho g\left[V(\xi_5^{(k)}) z_B^{(k)}\Delta\xi_5^{(k)} + I_L\Delta\xi_5^{(k)}\right] \end{aligned}$$

대입

$$\boxed{\mathbf{M}_B(\xi_5^{(k)})} = \mathbf{i}\rho g V(\xi_5^{(k)})y_B^{(k)} - \mathbf{j}\rho g V(\xi_5^{(k)})x_B^{(k)}$$

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{M}_B(\xi_5^{(k)}) + \mathbf{i}\rho g I_P\Delta\xi_5^{(k)} - \mathbf{j}\rho g\left[V(\xi_5^{(k)}) z_B^{(k)}\Delta\xi_5^{(k)} + I_L\Delta\xi_5^{(k)}\right]$$

Trim에 의한 모멘트 (13)



좌표 변환

$$\left(\begin{array}{l} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{array}\right)$$

✓ 부력에 의한 모멘트

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{M}_B(\xi_5^{(k)}) + \mathbf{i}\rho g I_P \Delta\xi_5^{(k)} - \mathbf{j}\rho g\left[V(\xi_5^{(k)}) z_B^{(k)}\Delta\xi_5^{(k)} + I_L\Delta\xi_5^{(k)}\right]$$

✓ 중력에 의한 모멘트

$$\left(\mathbf{r}_G^{(k+1)}\times(-\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G^{(k+1)} & y_G^{(k+1)} & z_G^{(k+1)} \\ 0 & 0 & -1 \end{vmatrix} = -\mathbf{i}y_G^{(k+1)} + \mathbf{j}x_G^{(k+1)}\right)$$

$$\mathbf{M}_G(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{r}_G^{(k+1)}\times(-\mathbf{k}mg) = -\mathbf{i}mg\cdot y_G^{(k+1)} + \mathbf{j}mg\cdot x_G^{(k+1)}$$

$$= -\mathbf{i}mg\cdot y_G^{(k)} + \mathbf{j}mg\cdot\left(x_G^{(k)}\cos\Delta\xi_5^{(k)} + z_G^{(k)}\sin\Delta\xi_5^{(k)}\right) \approx \boxed{-\mathbf{i}mg\cdot y_G^{(k)} + \mathbf{j}mg\cdot x_G^{(k)}} + \mathbf{j}mg\cdot z_G^{(k)}\Delta\xi_5^{(k)}$$

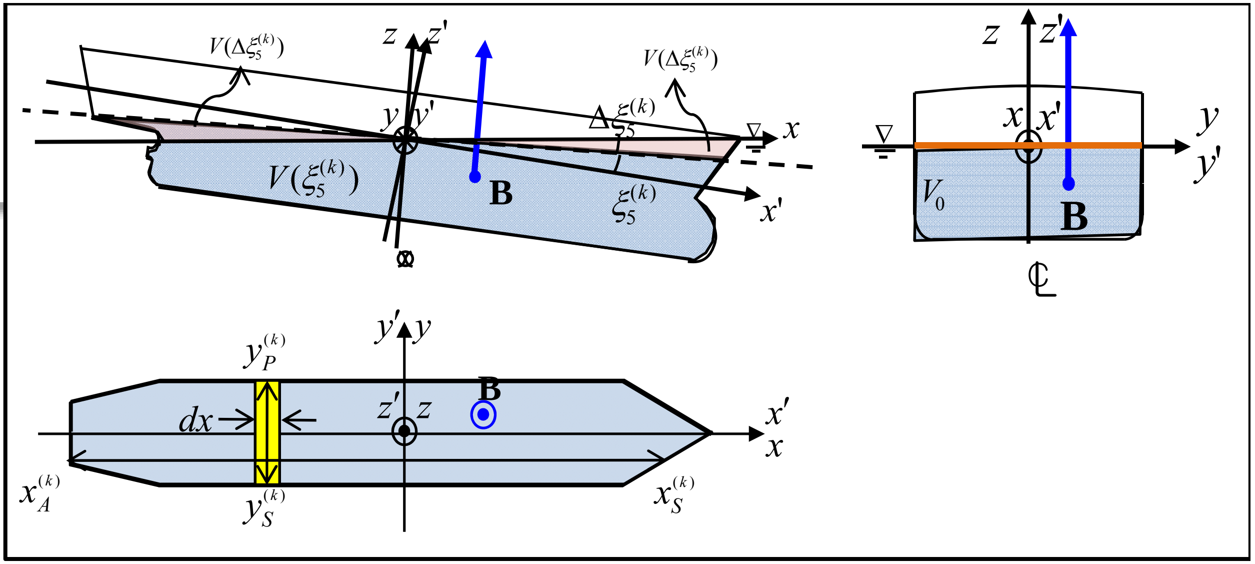
$$(\xi_5 << 1)$$

대입

$$= \mathbf{M}_G(\xi_5^{(k)}) + \mathbf{j}mg\cdot z_G^{(k)}\Delta\xi_5^{(k)}$$

$$\boxed{\mathbf{M}_G(\xi_5^{(k)})} = -\mathbf{i}mg\cdot y_G^{(k)} + \mathbf{j}mg\cdot x_G^{(k)}$$

Trim에 의한 모멘트 (14)



좌표 변환

$$\begin{pmatrix} x_P^{(k+1)} = x_P^{(k)}\cos\Delta\xi_5^{(k)} + z_P^{(k)}\sin\Delta\xi_5^{(k)} \\ y_P^{(k+1)} = y_P^{(k)} \\ z_P^{(k+1)} = z_P^{(k)}\cos\Delta\xi_5^{(k)} - x_P^{(k)}\sin\Delta\xi_5^{(k)} \end{pmatrix}$$

✓ 부력에 의한 모멘트

$$\mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{M}_B(\xi_5^{(k)}) + \mathbf{i}\rho g I_P \Delta\xi_5^{(k)} - \mathbf{j}\rho g\left[V(\xi_5^{(k)})z_B^{(k)}\Delta\xi_5^{(k)} + I_L\Delta\xi_5^{(k)}\right]$$

✓ 중력에 의한 모멘트

$$\mathbf{M}_G(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{M}_G(\xi_5^{(k)}) + \mathbf{j}mg\cdot z_G^{(k)}\Delta\xi_5^{(k)}$$

$$\left.\frac{\partial \mathbf{M}_T}{\partial \xi_5}\right|_{\xi_5^{(k)}} = \rho g I_P$$

$$\left.\frac{\partial \mathbf{M}_L}{\partial \xi_5}\right|_{\xi_5^{(k)}} = \rho g V(\xi_5^{(k)})z_B^{(k)} + \rho g I_L - mg\cdot z_G^{(k)}$$

✓ 선박이 받는 모멘트

$$\mathbf{M}(\xi_5^{(k)}+\Delta\xi_5^{(k)}) = \mathbf{M}_B(\xi_5^{(k)}+\Delta\xi_5^{(k)}) + \mathbf{M}_G(\xi_5^{(k)}+\Delta\xi_5^{(k)})$$

$$= \mathbf{M}_B(\xi_5^{(k)}) + \mathbf{M}_G(\xi_5^{(k)}) + \mathbf{i}\rho g I_P \Delta\xi_5^{(k)} - \mathbf{j}\rho g\left[V(\xi_5^{(k)})z_B^{(k)}\Delta\xi_5^{(k)} + I_L\Delta\xi_5^{(k)}\right] + \mathbf{j}mg\cdot z_G^{(k)}\Delta\xi_5^{(k)}$$

$$\mathbf{M}(\xi_5^{(k)}+\Delta\xi_5^{(k)}) - \mathbf{M}(\xi_5^{(k)}) = \mathbf{i}\left\{\boxed{\rho g I_P}\right\}\Delta\xi_5^{(k)} - \mathbf{j}\left\{\boxed{\rho g V(\xi_5^{(k)})z_B^{(k)} + \rho g I_L - mg\cdot z_G^{(k)}}\right\}\Delta\xi_5^{(k)}$$

$$\Delta\mathbf{M}(\xi_5^{(k)}) = \mathbf{i}\boxed{\left.\frac{\partial \mathbf{M}_T}{\partial \xi_5}\right|_{\xi_5^{(k)}}}\Delta\xi_5^{(k)} + \mathbf{j}\boxed{\left.\frac{\partial \mathbf{M}_L}{\partial \xi_5}\right|_{\xi_5^{(k)}}}\Delta\xi_5^{(k)}$$

Linked Slides

- 기하학적 접근에 따른 부력, 횡 방향 모멘트, 종 방향 모멘트의 변화

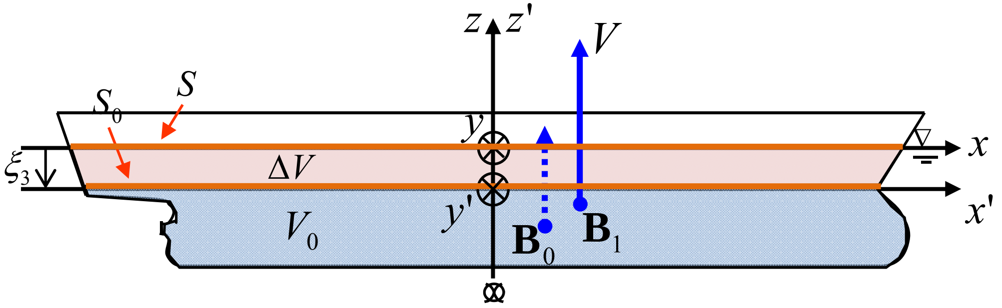
Ref) Edward V. Lewis , Principles of Naval Architecture: Stability and Strength, 1989

Ref) **이규열**, **노명일**, **안재윤**, **선박안정론**, 5th edition, 2003, pp.95~197

A dvanced
S hip
D esign
A utomation
L aboratory

선박 안정론 (선박계산법)의 1*cm* 침하 톤수(TPC)와 비교

< 선박 안정론 >



1*cm* **침하 톤수**(TPC; Tonnes per 1 *cm* Immersion)

- ξ_3 **가 작을 때** (1*cm*는 **매우 작은 값임**),

  수선면적의 변화가 거의 없다고 가정할 수 있음.

$$TPC \boxed{= \rho A_{WP}(z')/100}$$

< 적분 계산 >

$$\mathbf{F} = -\mathbf{k}\rho g \xi_3 A_{WP}(z')$$

단위 변환 (힘 ➡ 질량)

$$m = \frac{\mathbf{F}}{g} - \rho \xi_3 A_{WP}(z')$$

1*cm* Immersion**으로 환산**

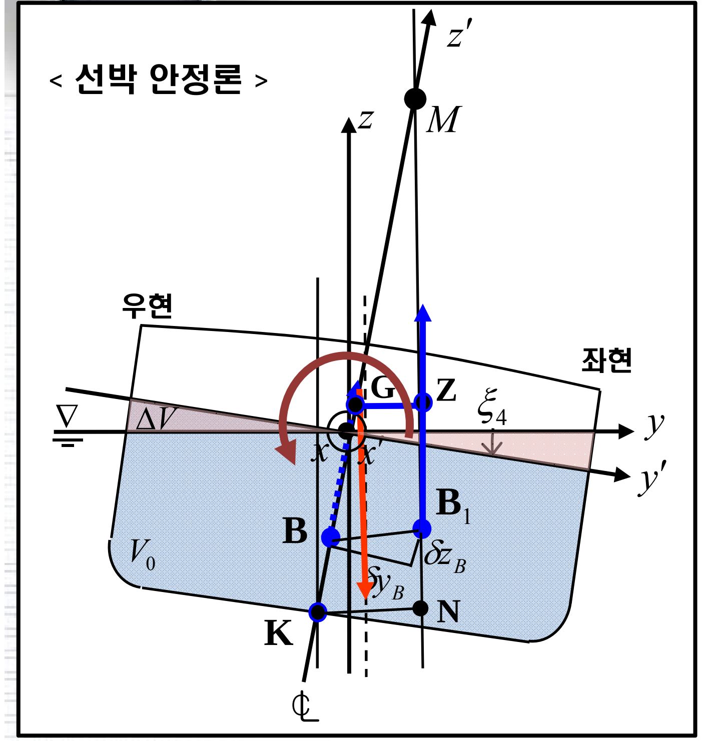
(양변에 $\frac{\xi_3}{100}$ 을 곱함)

$$m \cdot \frac{\xi_3}{100} = -\rho \xi_3 A_{WP}(z') \cdot \frac{\xi_3}{100}$$

$$\boxed{= -\rho A_{WP}(z')/100}$$

※ (-)**부호 차이 나는 이유** :ξ_3 **가 양일 때, 변위가 서로 반대로 정의됨**

선박 안정론 (선박계산법)의 횡 방향 모멘트와 비교



ξ_4가 작을 때 $(\Delta = \rho g V_0)$

$$\mathbf{M} = \mathbf{i}\Delta \cdot \mathbf{GZ}$$

$$= \mathbf{i}\Delta \cdot \mathbf{GM}_T \sin \xi_4$$

$$\boxed{= \mathbf{i}\Delta \cdot \xi_4 \mathbf{GM}_T}$$

< 적분 계산 >

$$\mathbf{M} = \mathbf{i}\left[-\rho g \xi_4 V_0 z'_{B0} - \rho g \xi_4 I_T + mgz'_G \xi_4\right] + \mathbf{j}\rho g \xi_4 I_P$$

좌우 대칭이라고 가정하면, $(I_P = 0)$

$$\mathbf{M} = \mathbf{i}\left[-\rho g \xi_4 V_0 z'_{B0} - \rho g \xi_4 I_T + mgz'_G \xi_4\right]$$

$(mg = \rho g V_0)$

$$= \mathbf{i}\left[-\rho g \xi_4 V_0 z'_{B0} - \rho g \xi_4 I_T + \rho g V_0 \cdot z'_G \xi_4\right]$$

$$= \mathbf{i}\rho g V_0 \cdot \xi_4 \left[-z'_{B0} - \frac{I_T}{V_0} + z'_G\right]$$

$$= \mathbf{i}\rho g V_0 \cdot (-\xi_4)\left[z'_{B0} + \frac{I_T}{V_0} - z'_G\right]$$

$(\Delta = \rho g V_0)$

$$= \mathbf{i}\Delta \cdot (-\xi_4)\left[z'_{B0} + \frac{I_T}{V_0} - z'_G\right]$$

$\left(\mathbf{BM}_T = \frac{I_T}{V_0}\right)$

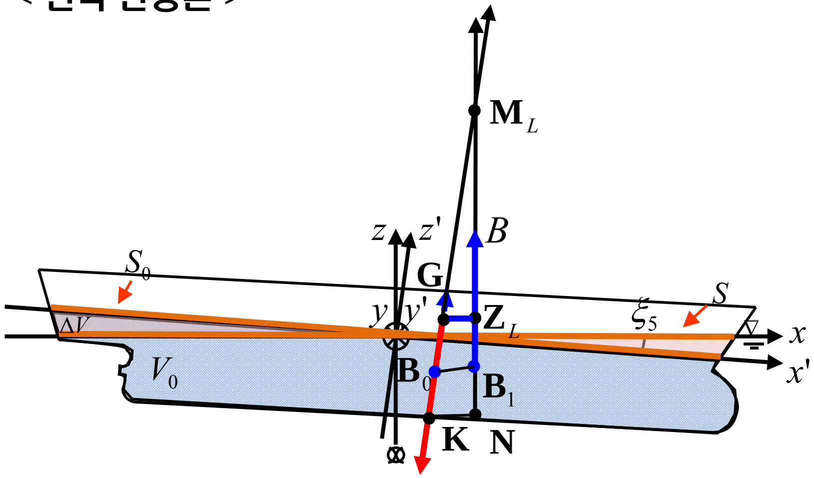
$$= \mathbf{i}\Delta \cdot (-\xi_4)\left[\mathbf{KB} + \mathbf{BM}_T - \mathbf{KG}\right]$$

$$\boxed{= \mathbf{i}\Delta \cdot (-\xi_4)\mathbf{GM}_T}$$

※ (-)부호 차이 나는 이유 : ξ_4 가 양일 때, 기우는 각도가 서로 반대로 정의됨

선박 안정론 (선박계산법)의 종 방향 모멘트와 비교

< 선박 안정론 >



ξ_5가 작을 때 $(\Delta = \rho g V_0)$

$$\mathbf{M} = \mathbf{j}\Delta \cdot \mathbf{GZ}_L$$
$$= \mathbf{j}\Delta \cdot \mathbf{GM}_L \sin\xi_5$$
$$\boxed{= \mathbf{j}\Delta \cdot \xi_5 \mathbf{GM}_L}$$

< 적분 계산 >

$$\mathbf{M} = \mathbf{i}\left[\rho g \xi_5 I_P\right] + \mathbf{j}\left[-\rho g \xi_5 V_0 z'_{B0} - \rho g \xi_5 I_L + mgz'_G \xi_5\right]$$

좌우 대칭이라고 가정하면, $(I_P = 0)$

$$\mathbf{M} = \mathbf{j}\left[-\rho g \xi_5 V_0 z'_{B0} - \rho g \xi_5 I_L + mgz'_G \xi_5\right]$$

$(mg = \rho g V_0)$

$$= \mathbf{j}\left[-\rho g \xi_5 V_0 z'_{B0} - \rho g \xi_5 I_T + \rho g V_0 z'_G \xi_5\right]$$
$$= \mathbf{j}\rho g V_0 \cdot \xi_5 \left[-z'_{B0} - \frac{I_L}{V_0} + z'_G\right]$$
$$= \mathbf{j}\rho g V_0 \cdot \xi_5 \left[-z'_{B0} - \frac{I_L}{V_0} + z'_G\right]$$

$(\Delta = \rho g V_0)$

$$= \mathbf{j}\Delta \cdot (-\xi_5)\left[z'_{B0} + \frac{I_L}{V_0} - z'_G\right]$$

$\left(\mathbf{BM}_L = \frac{I_L}{V_0}\right)$

$$= \mathbf{j}\Delta \cdot (-\xi_5)\left[\mathbf{KB}_L + \mathbf{BM}_L - \mathbf{KG}_L\right]$$
$$\boxed{= \mathbf{j}\Delta \cdot (-\xi_5)\mathbf{GM}_L}$$

※ (-)부호 차이 나는 이유 : ξ_5 가 양일 때, 기우는 각도가 서로 반대로 정의됨

Linked Slide

- 점과 좌표축의 회전 변환

Advanced
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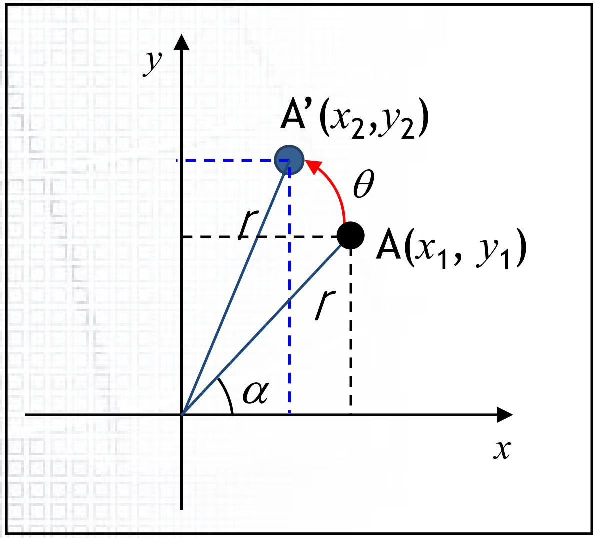
회전 변환 행렬 – 점의 회전 이동

- 좌표평면 위 한 점의 회전이동



① 삼각함수 합공식

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

② 점 A'의 좌표를 각으로 표현하면,

$$x_2=r\cos(\alpha+\theta)$$

$$y_2=r\sin(\alpha+\theta)$$

③ 삼각함수의 합공식으로 전개하면,

$$\begin{aligned}x_2&=r\cos\alpha\cos\theta-r\sin\alpha\sin\theta\\&=(r\cos\alpha)\cos\theta-(r\sin\alpha)\sin\theta\\&=x_1\cos\theta-y_1\sin\theta\end{aligned}$$

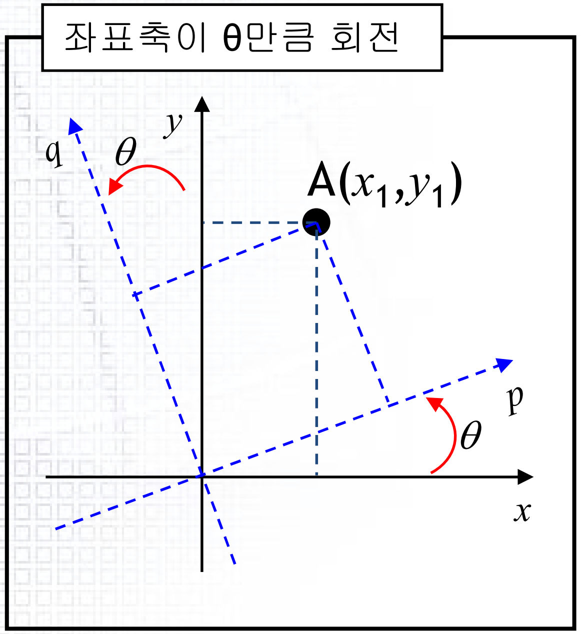
$$\begin{aligned}y_2&=r\sin\alpha\cos\theta+r\cos\alpha\sin\theta\\&=(r\sin\alpha)\cos\theta+(r\cos\alpha)\sin\theta\\&=y_1\cos\theta+x_1\sin\theta\end{aligned}$$

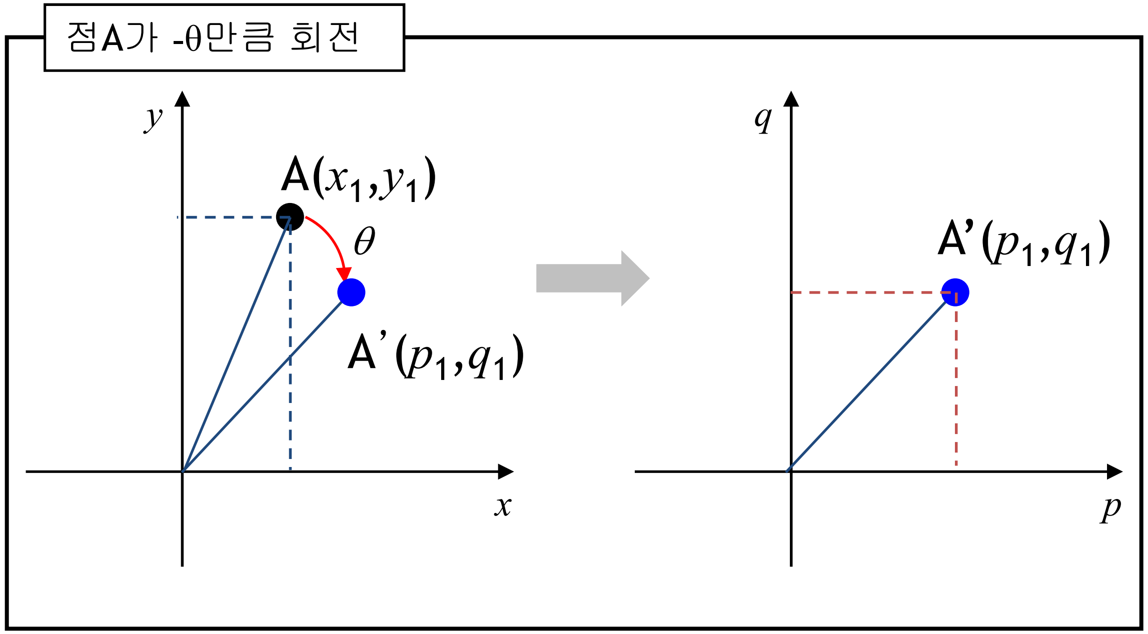
④ 행렬로 표현하면,

$$\boxed{\begin{bmatrix}x_2\\y_2\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x_1\\y_1\end{bmatrix}}$$

회전 변환 행렬 – 좌표축의 회전 이동

- **좌표축의 회전변환 : *xy*좌표계가 *pq*좌표계로 θ 만큼 회전하였을 때, 점 A의 좌표**





※ 점의 회전이동

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$(-\theta)$만큼 회전 변환한 점의 좌표

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Linked Slide

- 수선면 횡 방향 2차 모멘트

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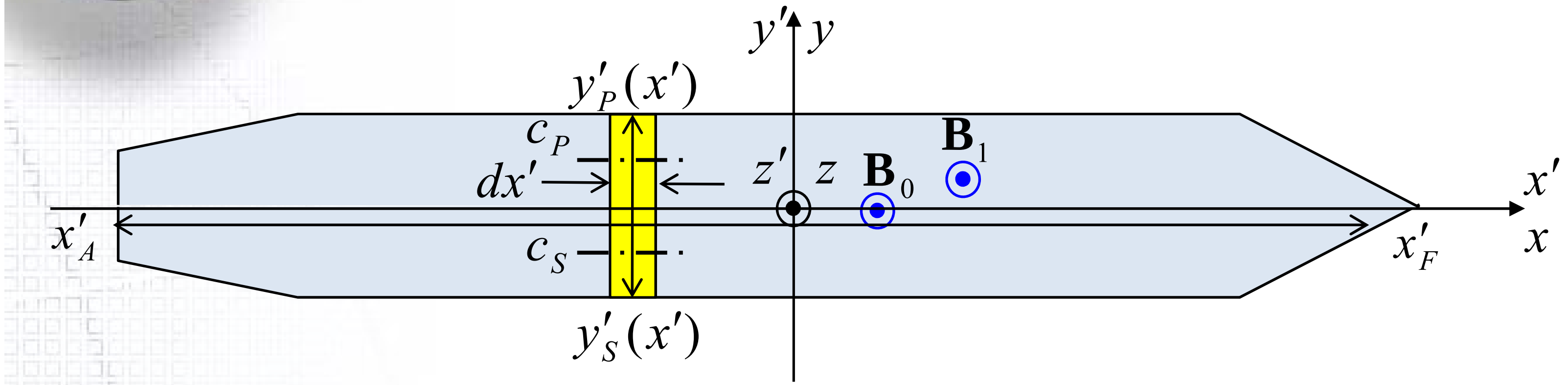
수선면적의 횡 방향 2차 모멘트



$$I_{c_P} = \frac{1}{12}(x'_F - x'_A)y'^{\,3}_P$$

(평행축 정리)

$$I_{x_P} = I_{c_P} + (x'_F - x'_A)y'_P\left(\frac{1}{2}y'_P\right)^2$$

$$= \frac{1}{12}(x'_F - x'_A)y'^{\,3}_P + (x'_F - x'_A)y'_P\left(\frac{1}{2}y'_P\right)^2$$

$$= \frac{1}{3}(x'_F - x'_A)y'^{\,3}_P \quad = \int_{x'_A}^{x'_F}\frac{y'^{\,3}_P}{3}dx'$$

$$I_{c_S} = \frac{1}{12}(x'_F - x'_A)(-y'_S)^3$$

(평행축 정리)

$$I_{x_S} = I_{c_S} + (x'_F - x'_A)(-y'_S)\left(\frac{1}{2}(-y'_S)\right)^2$$

$$= -\frac{1}{12}(x'_F - x'_A)y'^{\,3}_S - (x'_F - x'_A)y'_S\left(\frac{1}{2}y'_S\right)^2$$

$$= -\frac{1}{3}(x'_F - x'_A)y'^{\,3}_S \quad = -\int_{x'_A}^{x'_F}\frac{y'^{\,3}_S}{3}dx'$$

$$I_T = I_{x_P} + I_{x_S}$$

$$= \int_{x'_A}^{x'_F}\frac{y'^{\,3}_P}{3}dx' - \int_{x'_A}^{x'_F}\frac{y'^{\,3}_S}{3}dx' = \int_{x'_A}^{x'_F}\left(\frac{y'^{\,3}_P}{3} - \frac{y'^{\,3}_S}{3}\right)dx'$$