Naval Architecture & Ocean Engineering

[2008][01-1]

Engineering Mathematics 2

September, 2008

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Department of Naval Architecture and Ocean Engineering,
Seoul National University of College of Engineering

Naval Architecture & Ocean Engineering

Mathematical Modeling & Linearization

Why Mathematics?

Why Mathematics?

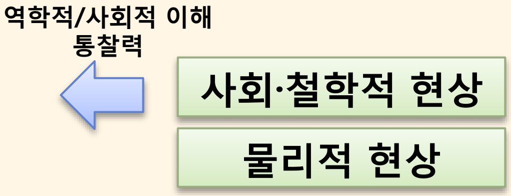
사회·철학적 현상

Why Mathematics?

사회·철학적 현상

물리적 현상

Why Mathematics?



역학적/사회적 이해
통찰력

사회·철학적 현상

물리적 현상

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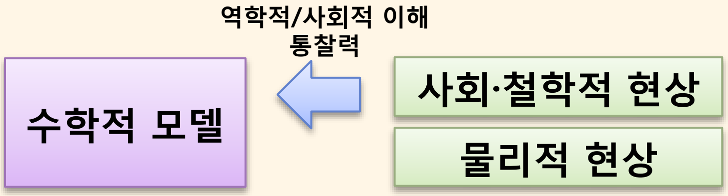

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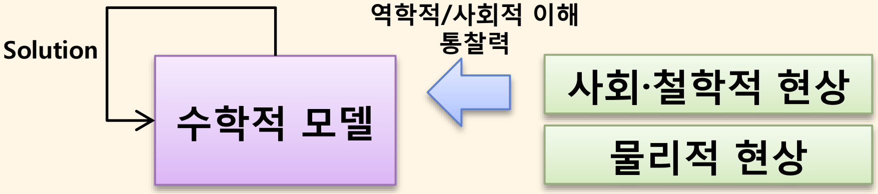

Solution

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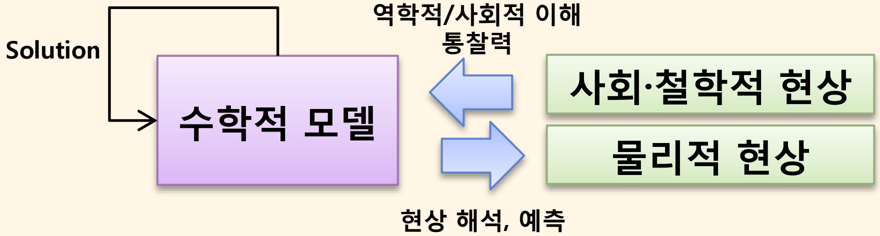

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현상 해석, 예측

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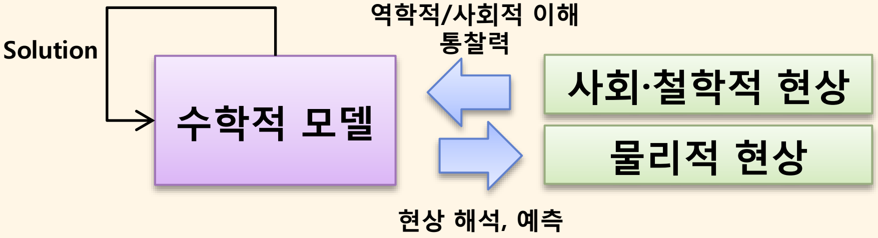

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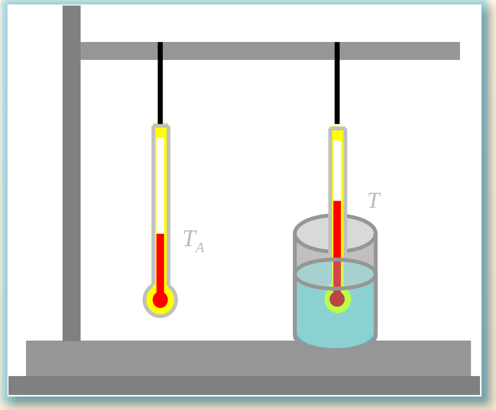

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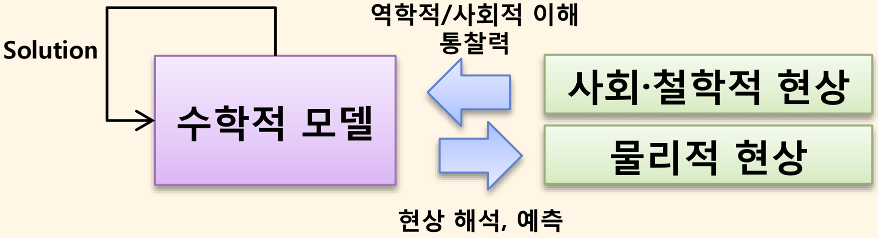

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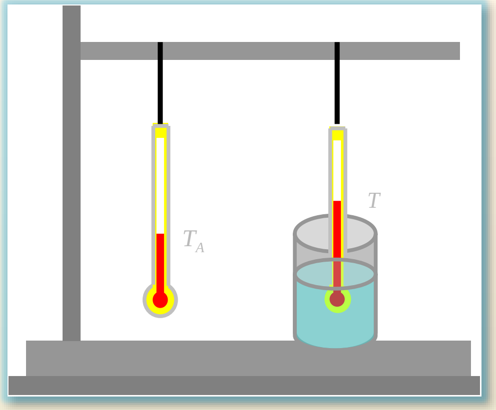

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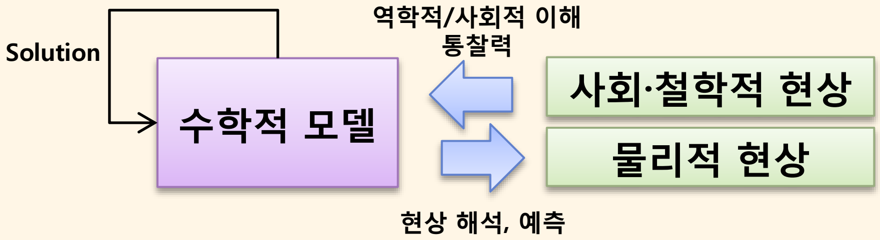

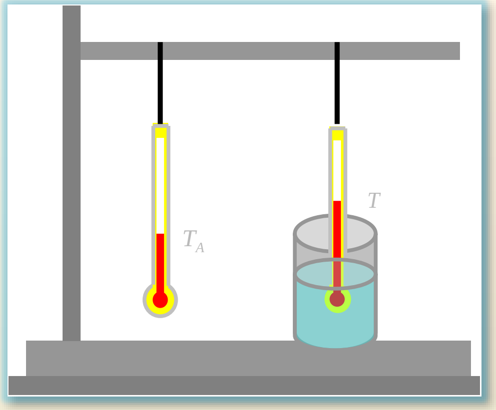

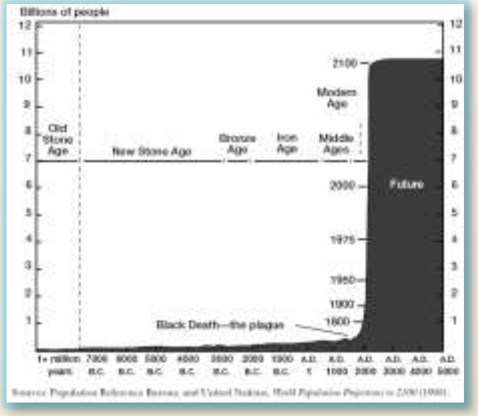

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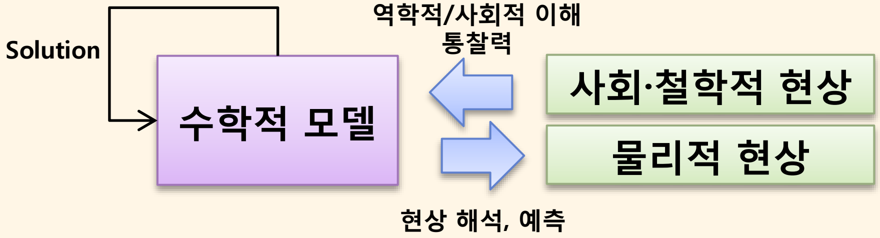

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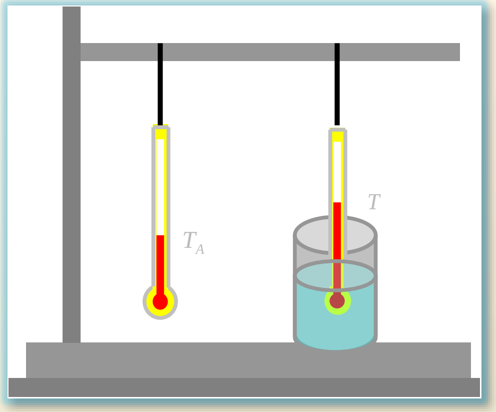
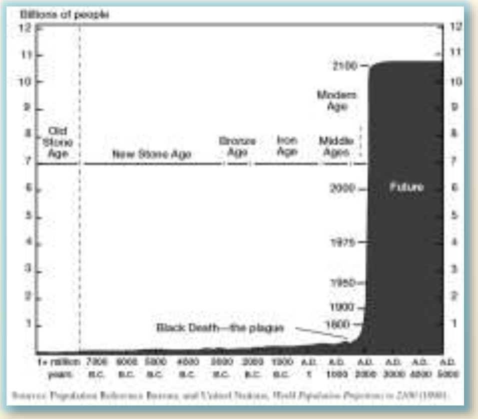
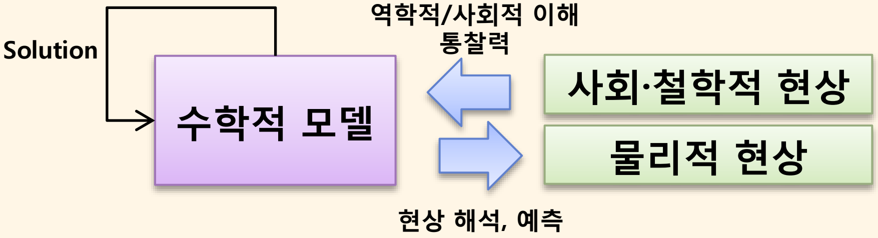




Source: Population Reference Bureau and United Nations, *World Population Projections to 2100* (1998).

Why Mathematics?



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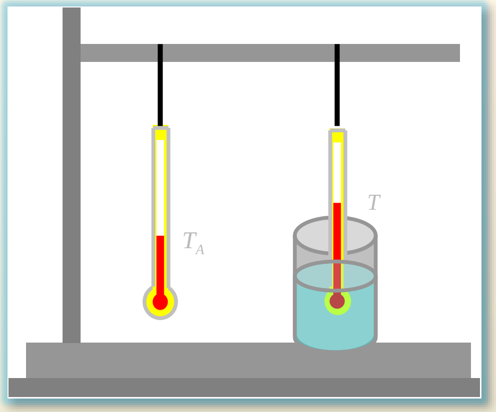
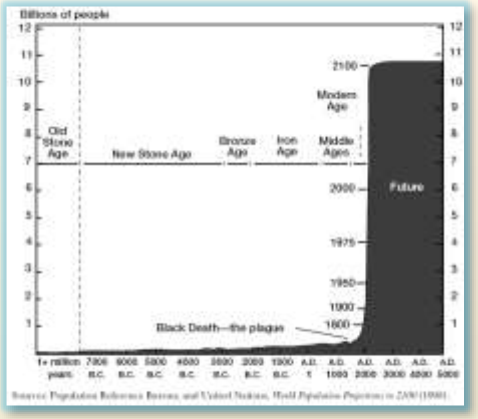
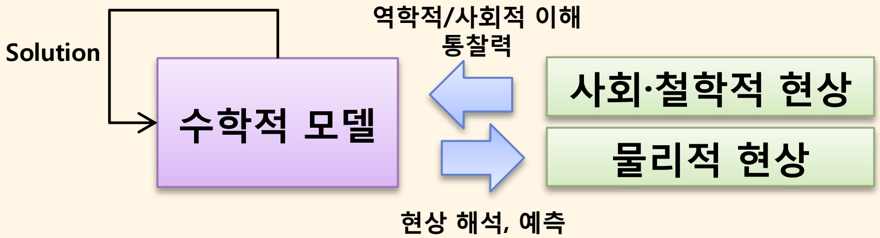
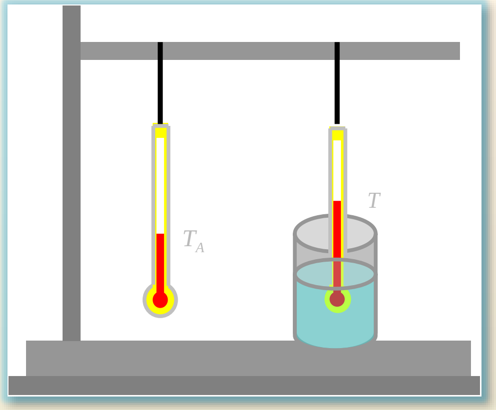


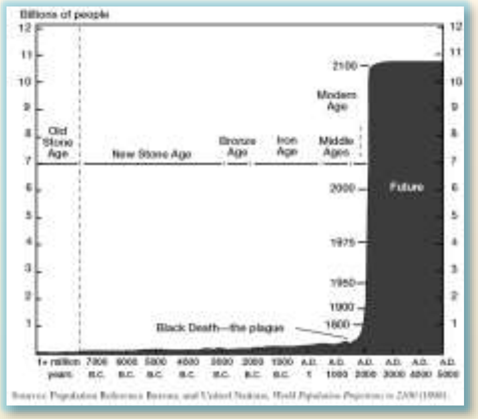


Changing **rate** of water temperature $\propto$ Temperature difference between water and outside

Why Mathematics?







Changing **rate** of water temperature $\propto$ Temperature difference between water and outside

$$\frac{dT(t)}{dt} = k(T - T_A), k < 0$$

T : Water temperature

T_A :Outside temperature (constant)

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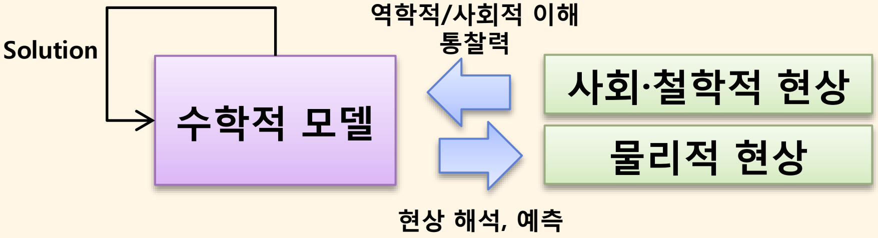

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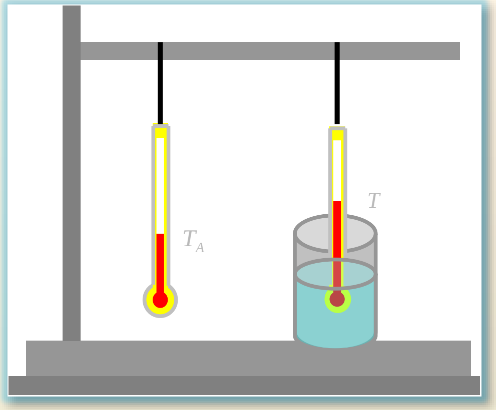

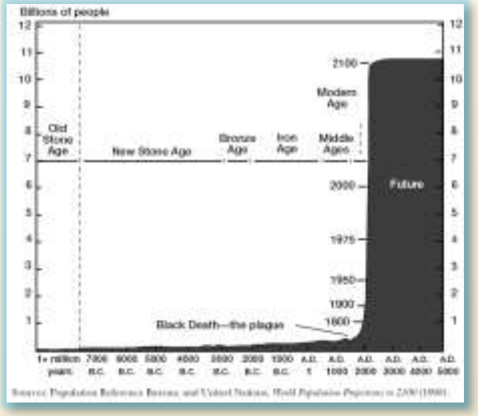

Changing **rate** of water temperature ∝ Temperature difference between water and outside

Increasing **rate** of population ∝ Present Population

$$\frac{dT(t)}{dt} = k(T - T_A), k < 0$$

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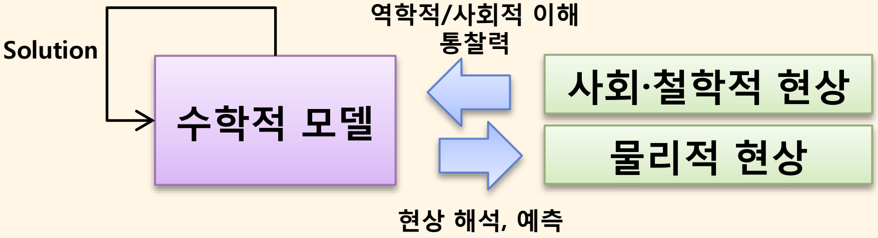

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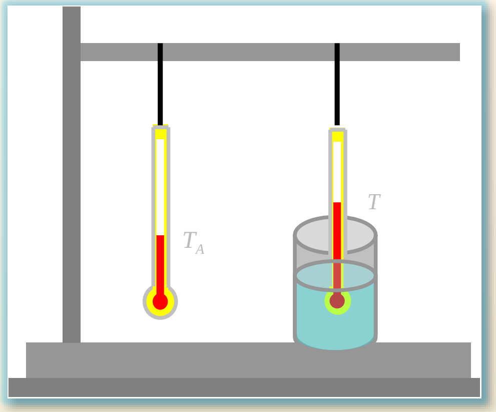

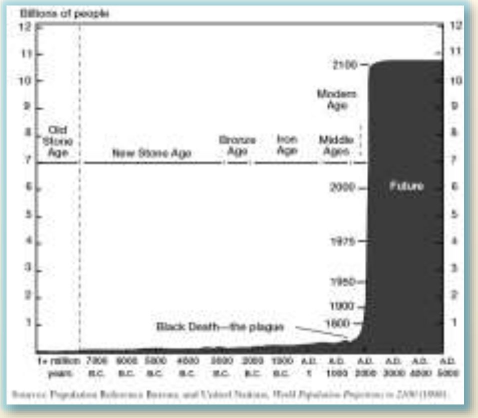

Changing **rate** of water temperature $\propto$ Temperature difference between water and outside

Increasing **rate** of population $\propto$ Present Population

$$\frac{dT(t)}{dt} = k(T - T_A), k < 0$$

T : Water temperature
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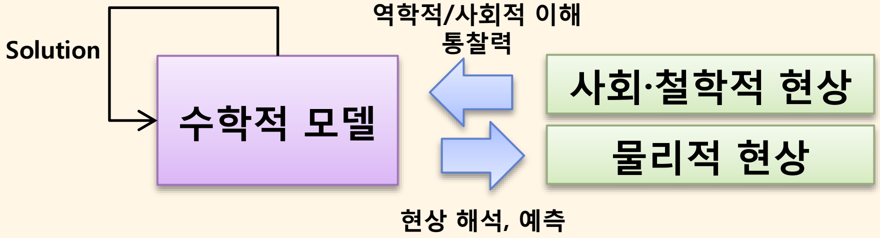
$$\frac{dy(t)}{dt} = k \cdot y(t)$$

y : population
t : time
k : proportional constant

Why Mathematics?



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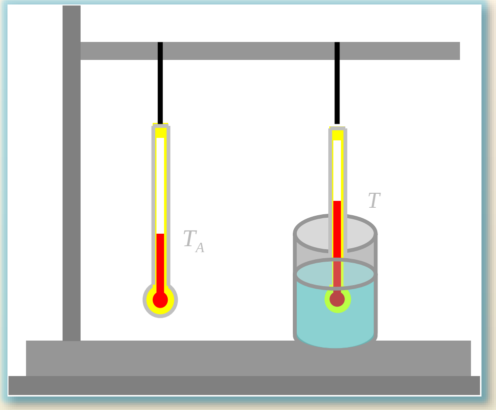

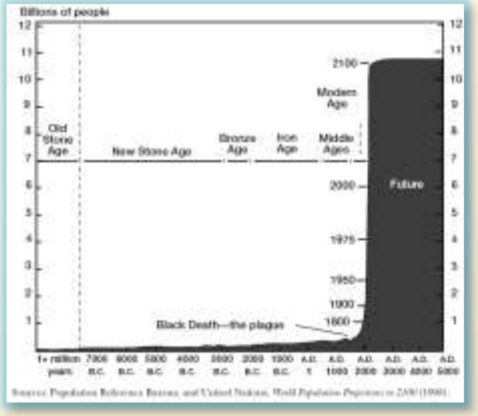

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<Newton's law of cooling>

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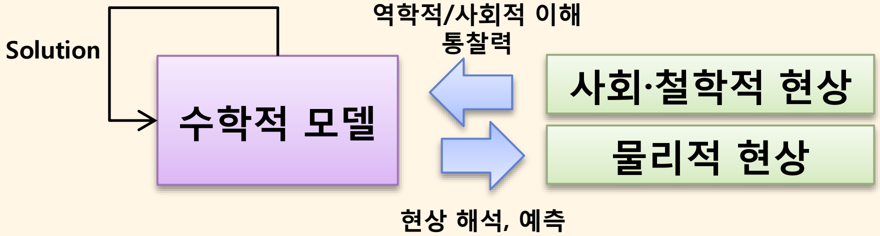

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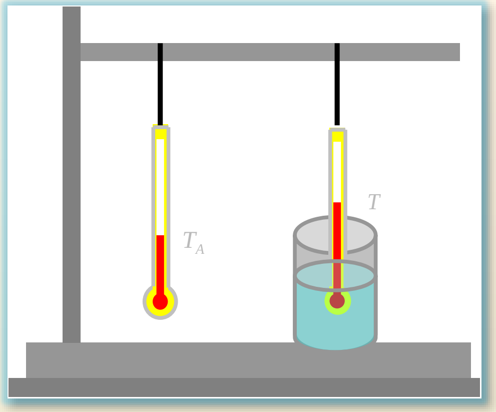

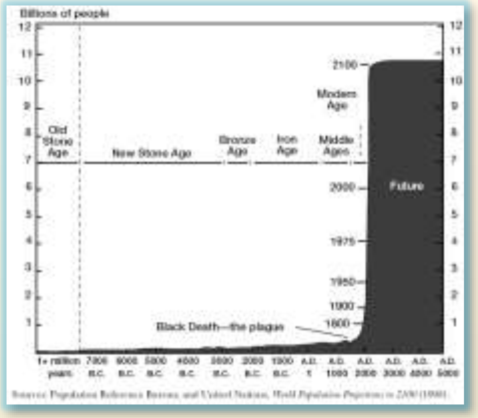

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<Newton’s law of cooling>

<Malthus Law>

Linear O.D.E

Linear O.D.E

Linear Model
(Linear Equation)

Linear/Nonlinear O.D.E

Nonlinear Model
(Nonlinear Equation)

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Linear Model
(Linear Equation)

Linear/Nonlinear O.D.E

Nonlinear Model
(Nonlinear Equation)

$$ex)\ mz'' + cz' + kz = F_0 \cos \omega t$$

z_1 z_2

Try:
$z = e^{\lambda t}$

-Basis
Linearly Independent

Linear O.D.E

Linear Model
(Linear Equation)

Linear/Nonlinear O.D.E

Nonlinear Model
(Nonlinear Equation)

$$ex)\ mz'' + cz' + kz = F_0 \cos \omega t$$

$$ex)\ mz'' + cz' + kz = 0 \quad z_h$$

General Solution
- Homogeneous

$$z_h = c_1 z_1 + c_2 z_2$$

-Linear Combination
-Superposition

z_1 z_2

Try:
$z = e^{\lambda t}$

-Basis
Linearly Independent

Linear O.D.E

Linear Model (Linear Equation)

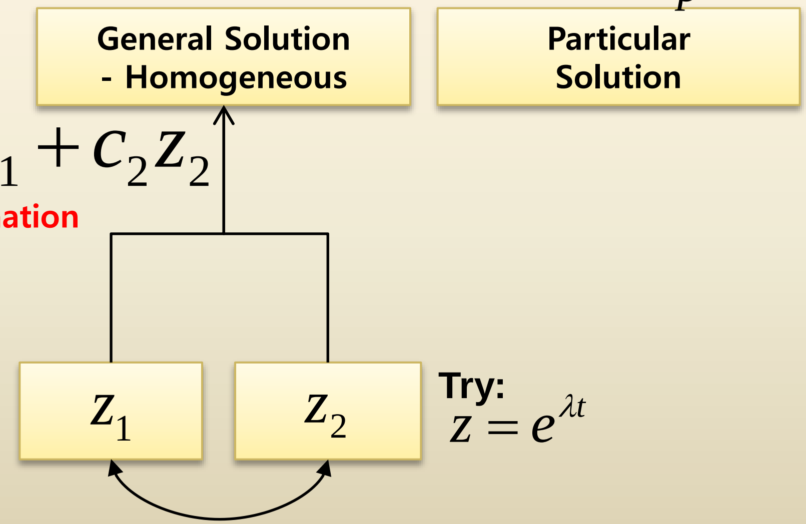
Linear/Nonlinear O.D.E

Nonlinear Model (Nonlinear Equation)

$$ex)\ mz'' + cz' + kz = F_0 \cos \omega t$$

$ex)\ mz'' + cz' + kz = 0$ z_h

z_p



General Solution - Homogeneous

Particular Solution

$$z_h = c_1 z_1 + c_2 z_2$$

-Linear Combination
-Superposition

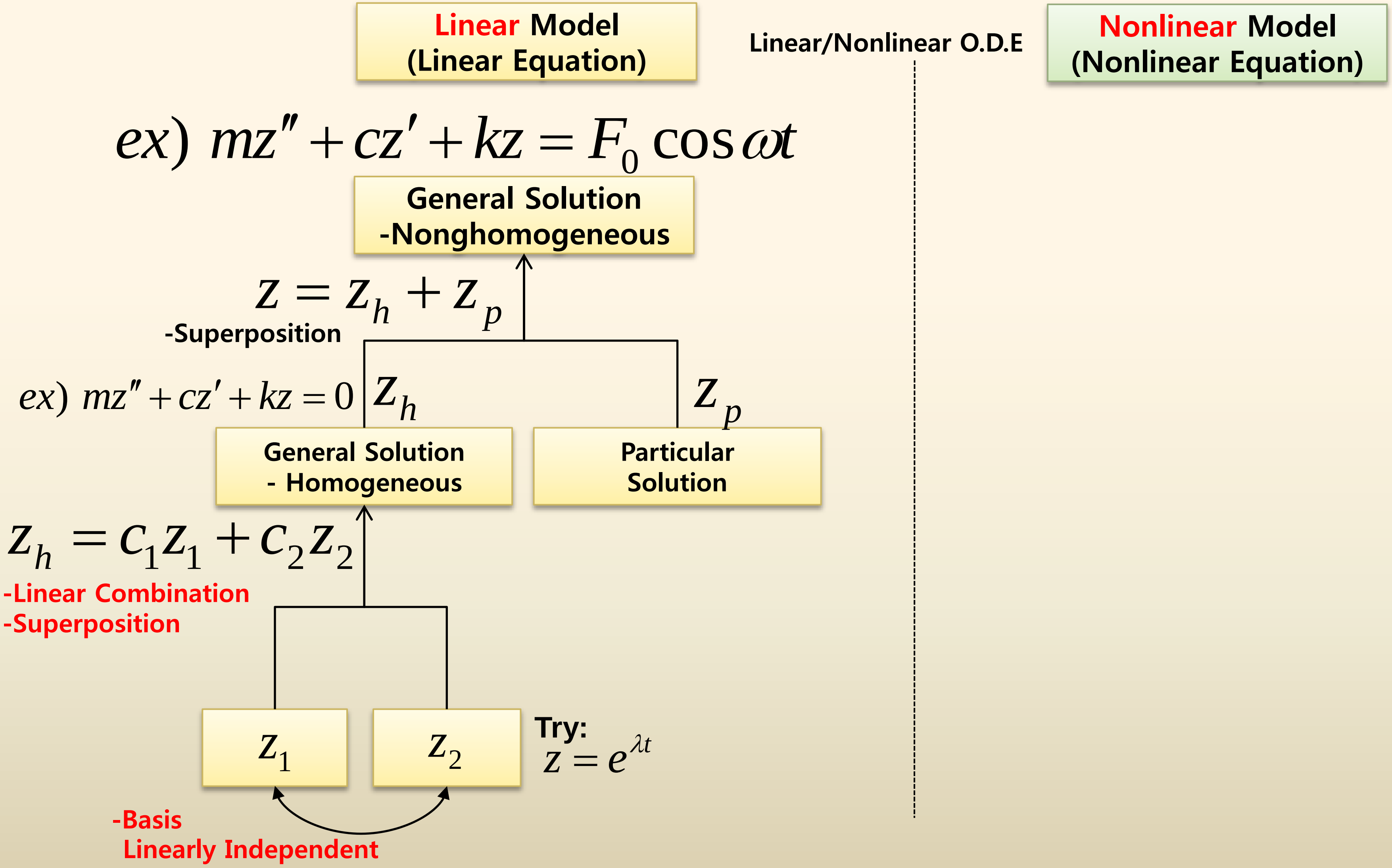
z_1

z_2

Try:
$z = e^{\lambda t}$

-Basis
Linearly Independent

Linear O.D.E



Linear Model (Linear Equation)

Linear/Nonlinear O.D.E

Nonlinear Model (Nonlinear Equation)

$$ex)\ mz'' + cz' + kz = F_0 \cos \omega t$$

General Solution -Nonghomogeneous

$$z = z_h + z_p$$

-Superposition

$$ex)\ mz'' + cz' + kz = 0$$

z_h

z_p

General Solution - Homogeneous

Particular Solution

$$z_h = c_1 z_1 + c_2 z_2$$

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Try:
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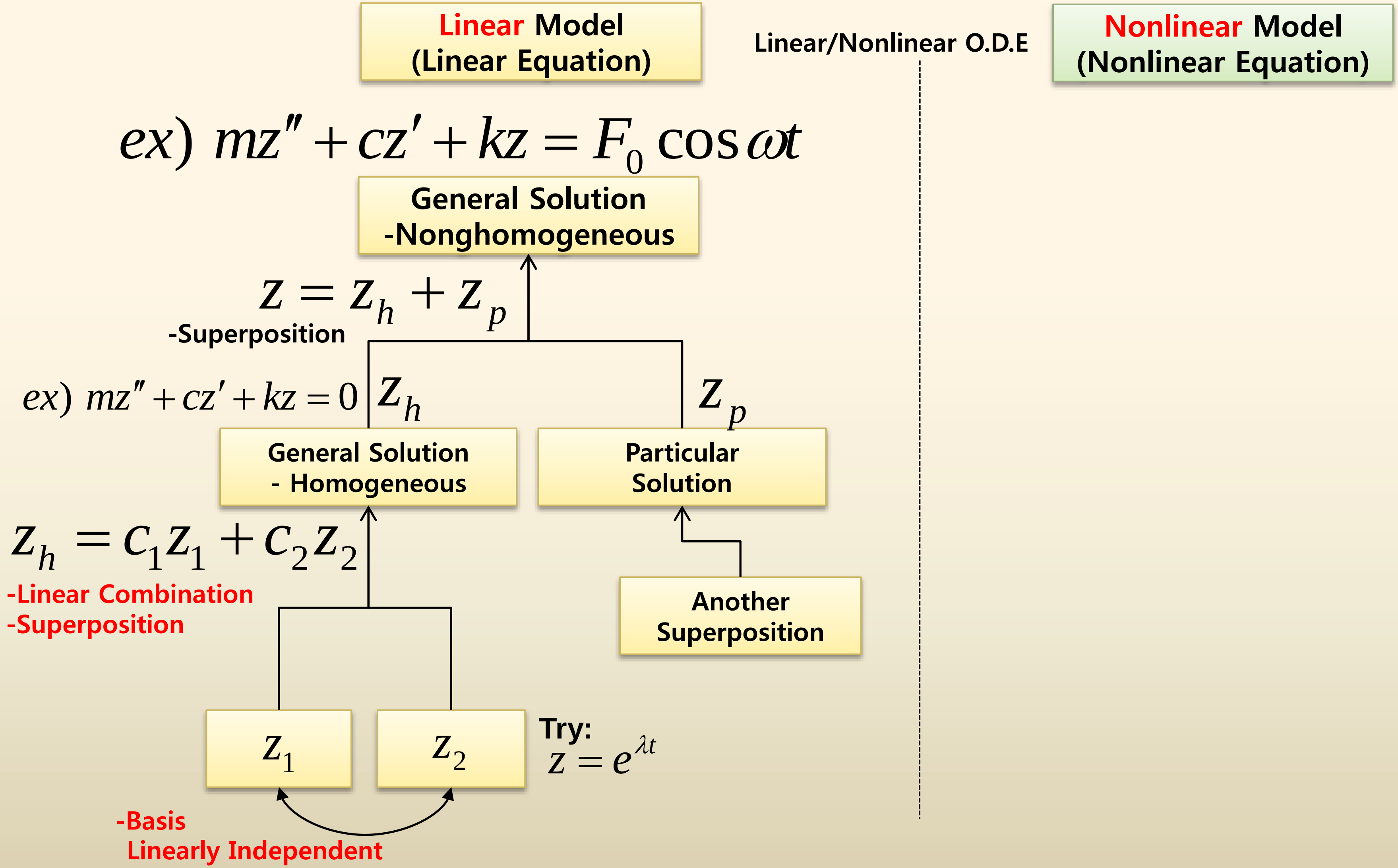
Linear O.D.E



Linear Model (Linear Equation)

Linear/Nonlinear O.D.E

Nonlinear Model (Nonlinear Equation)

$$ex)\ mz'' + cz' + kz = F_0 \cos \omega t$$

General Solution -Nonghomogeneous

$$z = z_h + z_p$$

-Superposition

$$ex)\ mz'' + cz' + kz = 0$$

z_h

z_p

General Solution - Homogeneous

Particular Solution

$$z_h = c_1 z_1 + c_2 z_2$$

-Linear Combination
-Superposition

Another Superposition

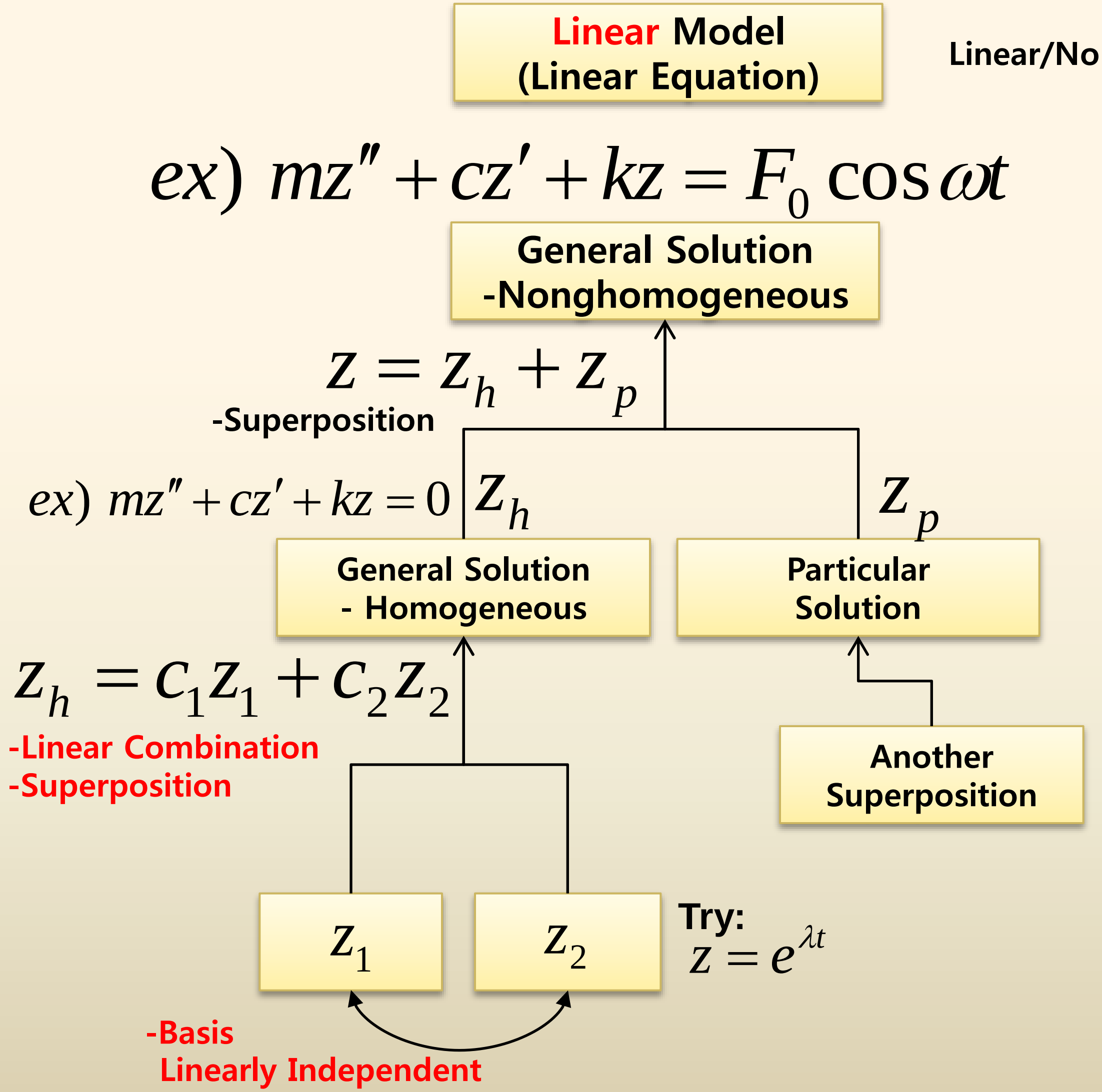
z_1

z_2

Try:
$z = e^{\lambda t}$

-Basis
Linearly Independent

Linear O.D.E



Linear Model
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$$ex)\ mz'' + cz' + kz = F_0 \cos \omega t$$

General Solution
-Nonghomogeneous

$$z = z_h + z_p$$

-Superposition

$$ex)\ mz'' + cz' + kz = 0$$

z_h

z_p

General Solution
- Homogeneous

Particular
Solution

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-Linear Combination
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Another
Superposition

z_1

z_2

Try:
$z = e^{\lambda t}$

-Basis
Linearly Independent

Linear/Nonlinear O.D.E

Nonlinear Model
(Nonlinear Equation)

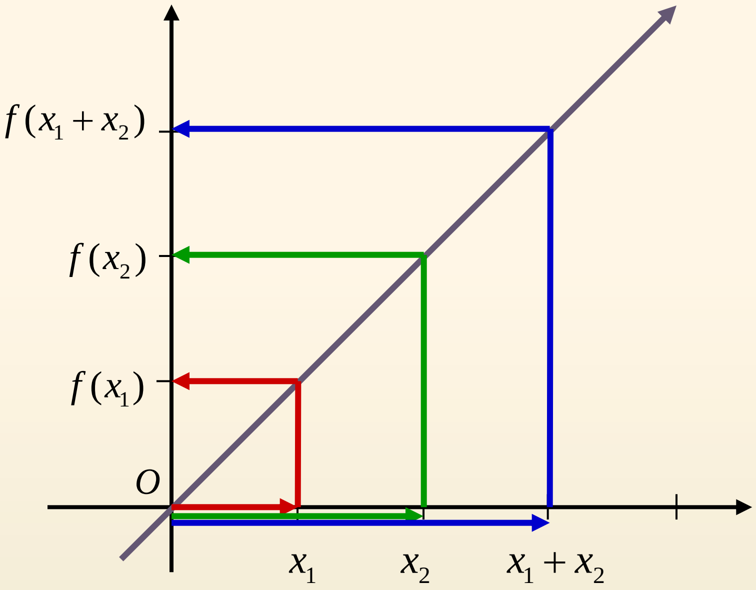
$$ex)\ mz'' + kz + k_1 z^3 = 0$$

$$ex)\ (y'')^2 - y^2 = 0$$

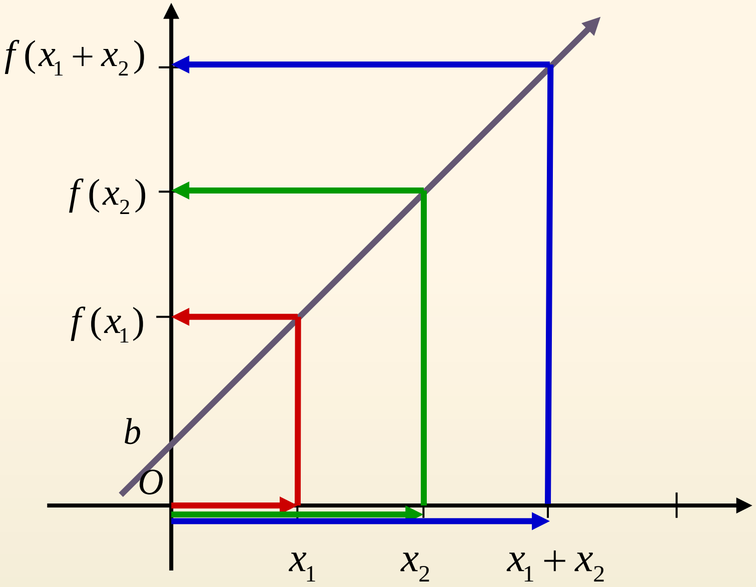
-Superposition?

Linearity, Superposition

$$f(x) = mx$$

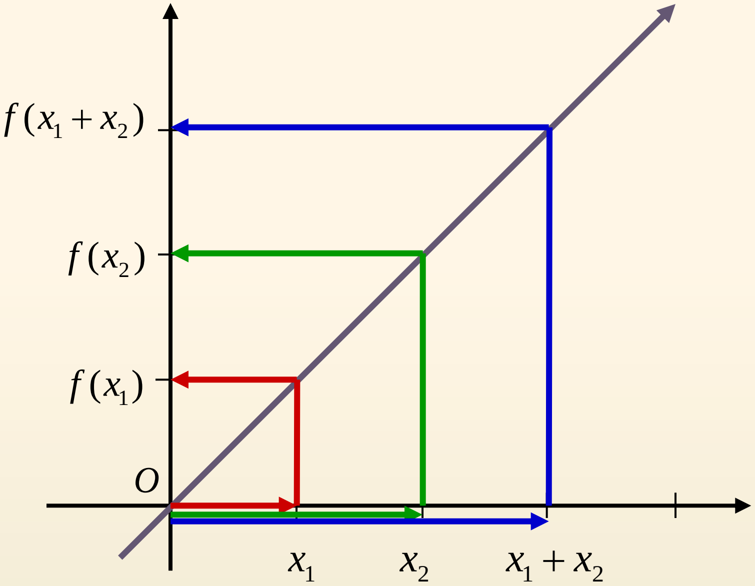


$$f(x) = mx + b$$



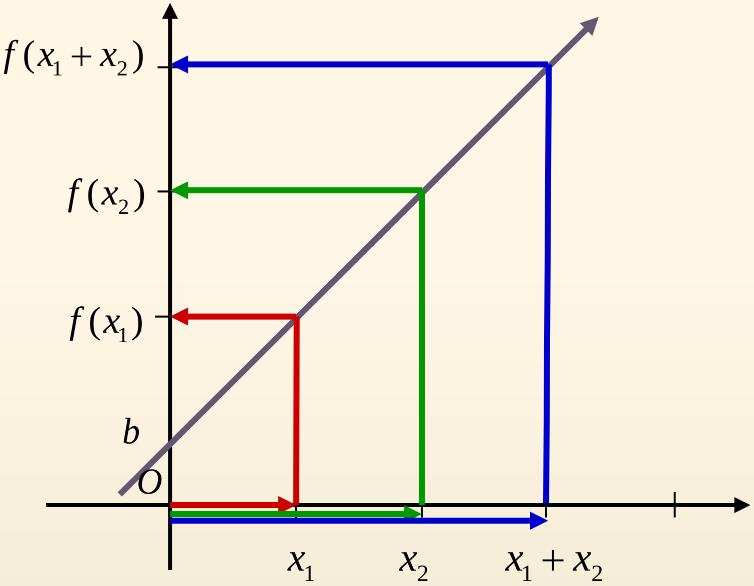
Linearity, Superposition

$$f(x)=mx$$



$$f(x_1)=mx_1,\quad f(x_2)=mx_2$$

$$f(x)=mx+b$$



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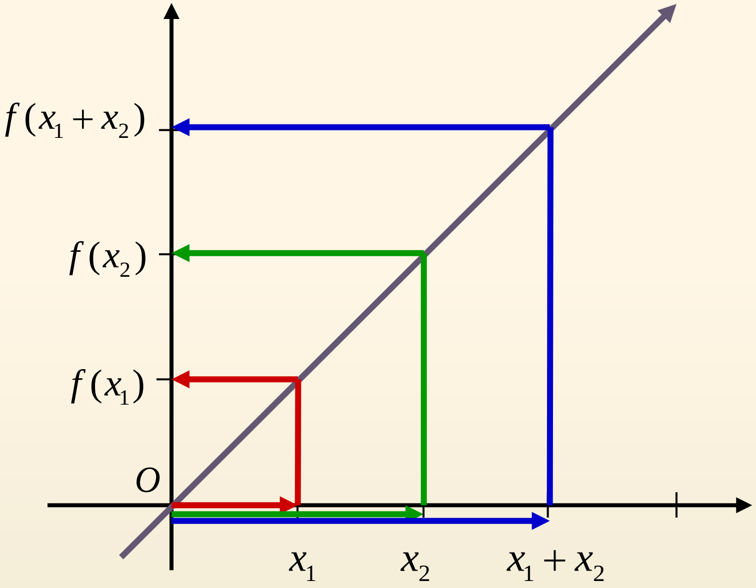
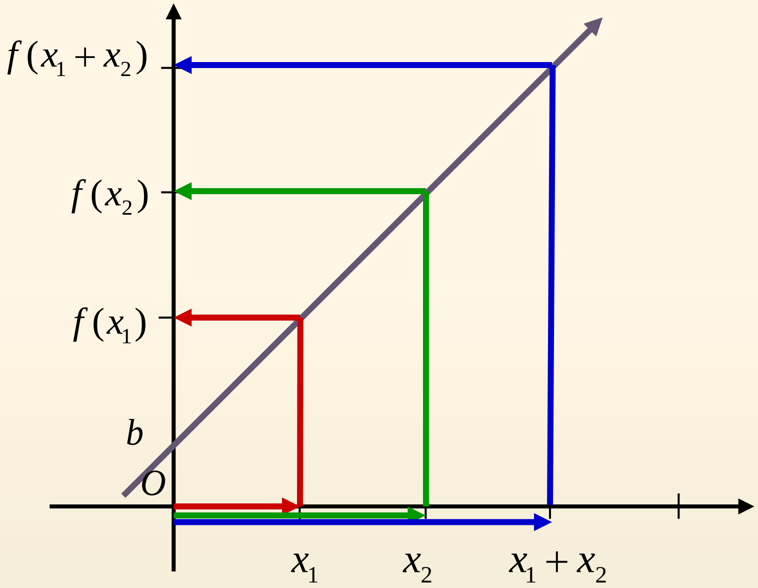


$$f(x_1)=mx_1,\quad f(x_2)=mx_2$$

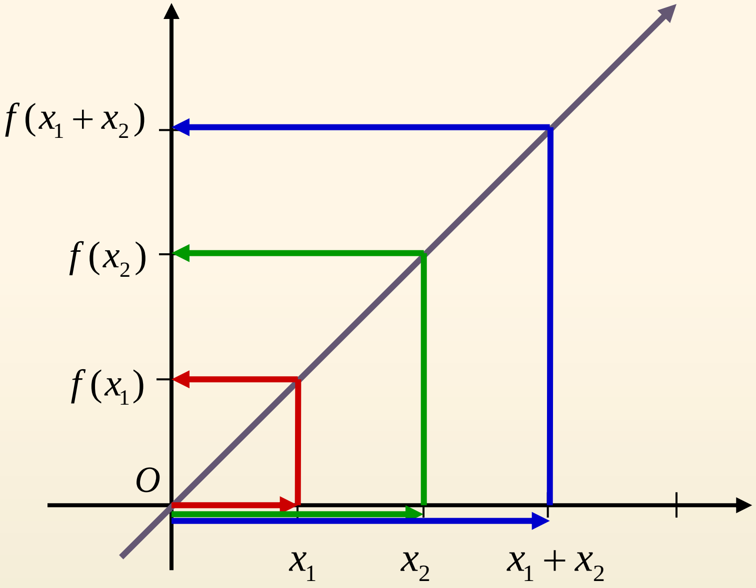
$$f(x_1+x_2)=m(x_1+x_2)$$

$$f(x)=mx+b$$



Linearity, Superposition

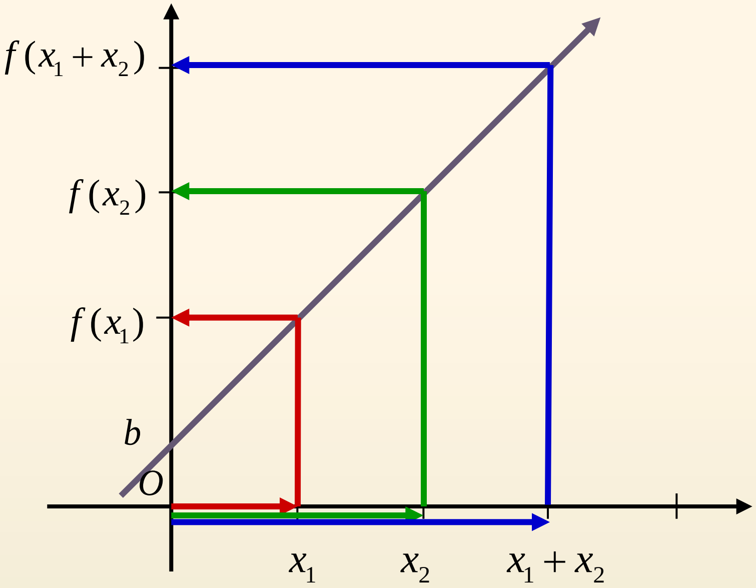
$$f(x) = mx$$



$$f(x_1) = mx_1, \quad f(x_2) = mx_2$$

$$\begin{aligned} f(x_1+x_2) &= m(x_1+x_2) \\ &= mx_1 + mx_2 \\ &= f(x_1) + f(x_2) \end{aligned}$$

$$f(x) = mx + b$$



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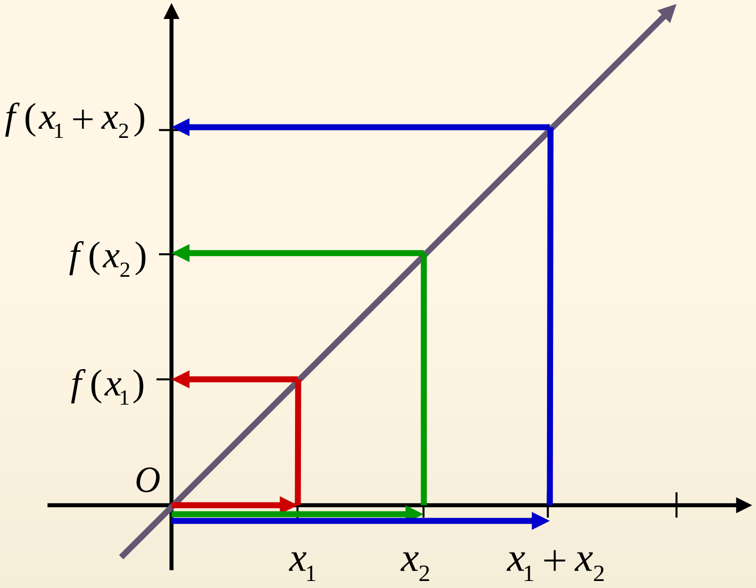
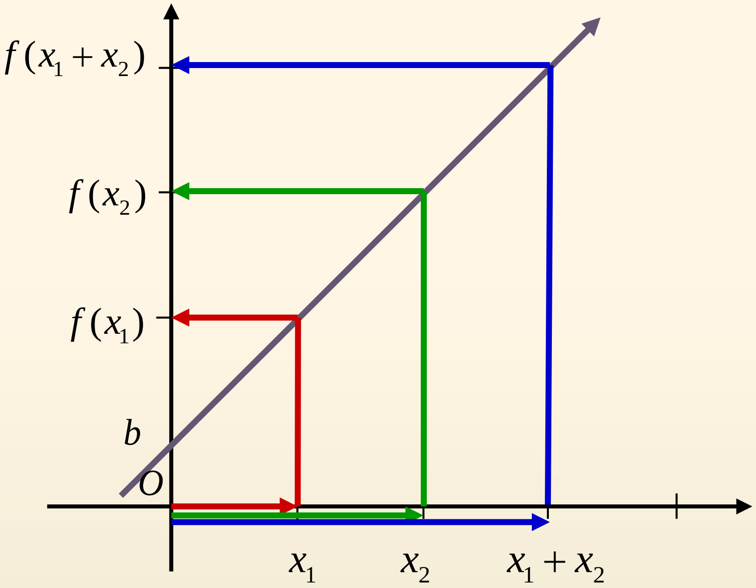


$$f(x_1)=mx_1,\quad f(x_2)=mx_2$$

$$\begin{aligned} f(x_1+x_2)&=m(x_1+x_2)\\ &=mx_1+mx_2\\ &=f(x_1)+f(x_2)\end{aligned}$$

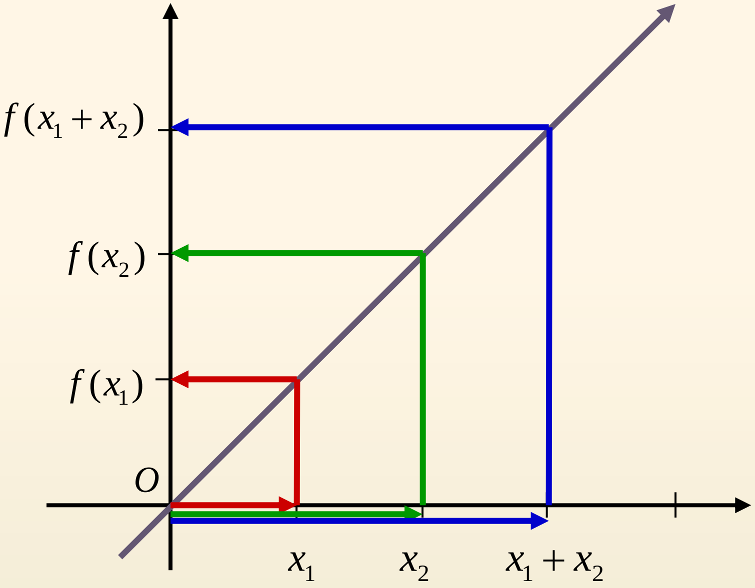
$$f(x)=mx+b$$



$$f(x_1)=mx_1+b,\quad f(x_2)=mx_2+b$$

Linearity, Superposition

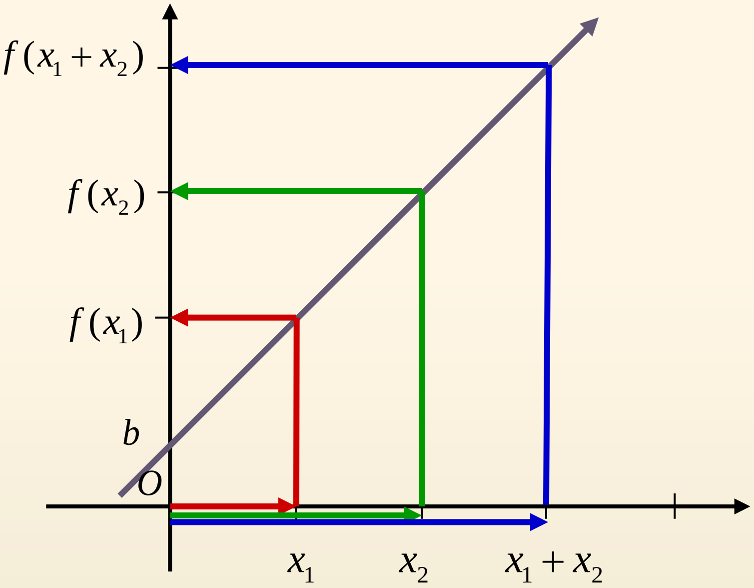
$$f(x) = mx$$



$$f(x_1) = mx_1, \quad f(x_2) = mx_2$$

$$f(x_1+x_2) = m(x_1+x_2)$$
$$= mx_1 + mx_2$$
$$= f(x_1) + f(x_2)$$

$$f(x) = mx + b$$

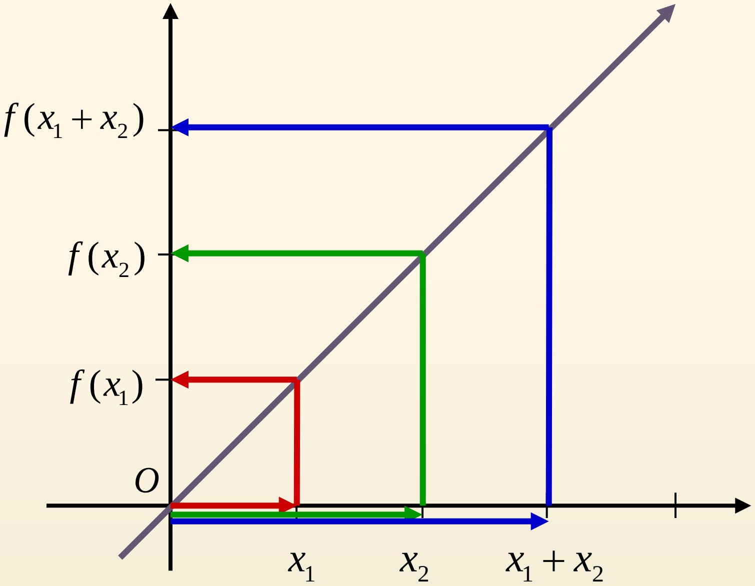


$$f(x_1) = mx_1 + b, \quad f(x_2) = mx_2 + b$$

$$f(x_1+x_2) = m(x_1+x_2) + b$$

Linearity, Superposition

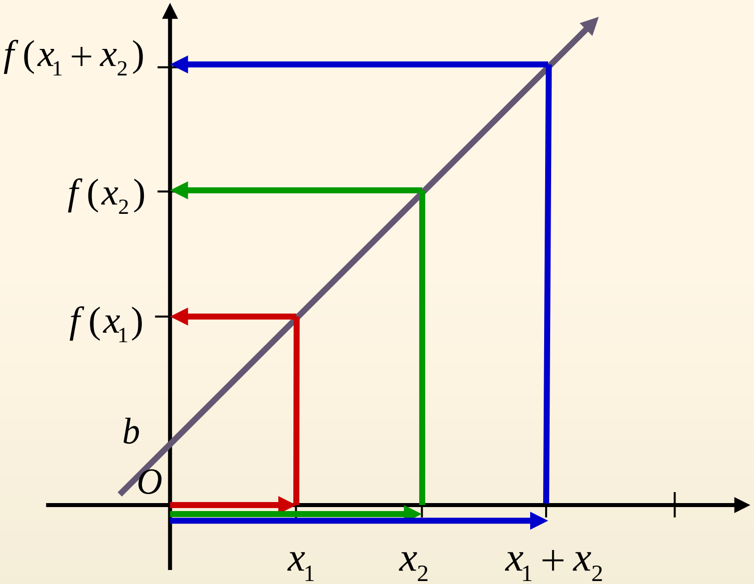
$$f(x) = mx$$



$$f(x_1) = mx_1, \quad f(x_2) = mx_2$$

$$\begin{aligned} f(x_1+x_2) &= m(x_1+x_2) \\ &= mx_1 + mx_2 \\ &= f(x_1) + f(x_2) \end{aligned}$$

$$f(x) = mx + b$$

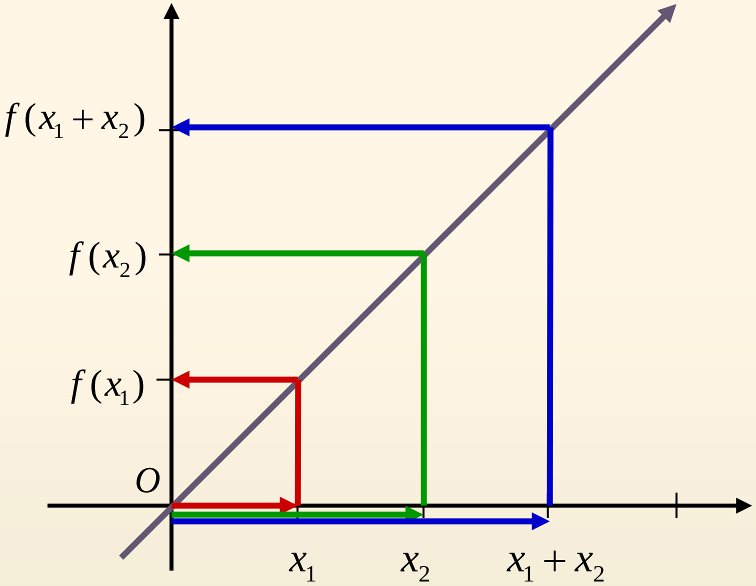


$$f(x_1) = mx_1 + b, \quad f(x_2) = mx_2 + b$$

$$\begin{aligned} f(x_1+x_2) &= m(x_1+x_2) + b \\ &= mx_1 + mx_2 + b \\ &\neq f(x_1) + f(x_2) \end{aligned}$$

Linearity, Superposition

$$f(x)=mx$$

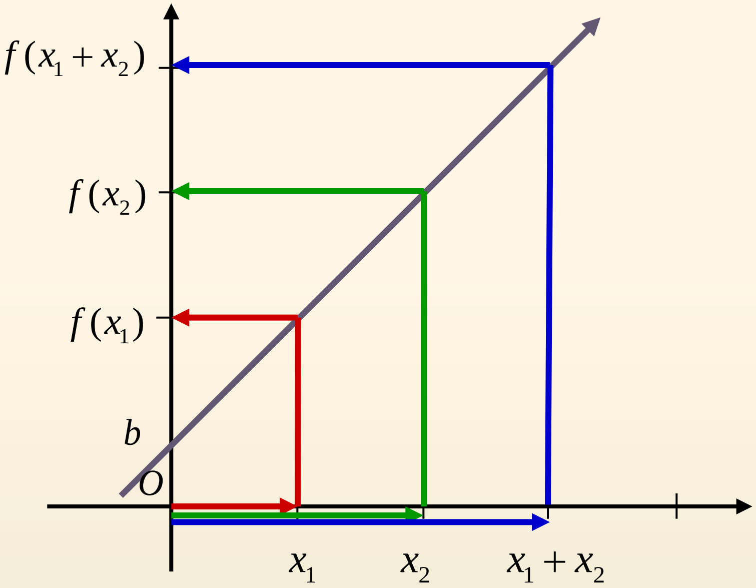


$$f(x_1)=mx_1,\quad f(x_2)=mx_2$$

$$\begin{aligned} f(x_1+x_2) &= m(x_1+x_2) \\ &= mx_1+mx_2 \\ &= f(x_1)+f(x_2) \end{aligned}$$

Linearity, Superposition

$$f(x)=mx+b$$



$$f(x_1)=mx_1+b,\quad f(x_2)=mx_2+b$$

$$\begin{aligned} f(x_1+x_2) &= m(x_1+x_2)+b \\ &= mx_1+mx_2+b \\ &\neq f(x_1)+f(x_2) \end{aligned}$$

Linearity, Superposition

If a function $f(x)$ has linearity,

the function $f(x)$ has these 2 characters.

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

Homogeneity

$$(b = 0)$$

$$f(ax_1) = af(x_1)$$

Additivity

$$(a = b = 1)$$

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

x_1, x_2 : independent variable

Linear O.D.E

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

- **Linear O.D.E.**

The dependent variable y and all its derivatives y', y'',…,$y^{(n)}$ are of the first degree, that is the power of each term involving y is 1
The coefficients a_0 ,…,a_n of y', y'',…,$y^{(n)}$ depend at most on the independent variable x

Linear O.D.E

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

- **Linear O.D.E.**

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The coefficients a$_0$,…,a$_n$ of y′, y″,…,y$^{(n)}$ depend at most on the independent variable x

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$$

Linear O.D.E

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$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$$

$$ex)\ my'' + cy' + ky = f(x)$$

$$y'' + \alpha y = 0$$

$$x^2y'' + xy' - \alpha y = 0$$

$$xy'' + y' + \alpha^2 y = 0$$

$$(1 - x^2)y'' - 2xy + n(n+1)y = 0$$

$$where, y = y(x), y' = \frac{dy}{dx}$$

$$m, c, k = \text{constant}$$

$$n = 0, 1, 2\ldots$$

Nonlinear O.D.E

$$f(ax_1 + bx_2) \neq af(x_1) + bf(x_2)$$

- linear O.D.E.

The dependent variable y and all its derivatives y′, y″,…,$y^{(n)}$ are of the first degree, that is the power of each term involving y is 1
The coefficients a_0 ,…,a_n of y′, y″,…,$y^{(n)}$ depend at most on the independent variable x

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$$

- **Nonlinear O.D.E.**

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Nonlinear O.D.E

$$f(ax_1 + bx_2) \neq af(x_1) + bf(x_2)$$

- linear O.D.E.

The dependent variable y and all its derivatives y′, y″,…,$y^{(n)}$ are of the first degree, that is the power of each term involving y is 1
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- **Nonlinear O.D.E.**

$$(1-y)y' + 2y = e^x$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Nonlinear O.D.E

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$$a_n(x)y^{(n)}+a_{n-1}(x)y^{(n-1)}+\cdots+a_2(x)y''+a_1(x)y'+a_0(x)y=g(x)$$

- **Nonlinear O.D.E.**

$$(1-\boxed{y)y'}+2y=e^x$$

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\cdots$$

Nonlinear O.D.E

$$f(ax_1+bx_2) \neq af(x_1)+bf(x_2)$$

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The dependent variable y and all its derivatives y′, y″,…,$y^{(n)}$ are of the first degree, that is the <u>power of each term involving y is 1</u>
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- **Nonlinear O.D.E.**

$$(1-\boxed{y)y'}+2y=e^x \qquad y''+\sin y=e^x$$

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\cdots$$

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$$a_n(x)y^{(n)}+a_{n-1}(x)y^{(n-1)}+\cdots+a_2(x)y''+a_1(x)y'+a_0(x)y=g(x)$$

- **Nonlinear O.D.E.**

$$(1-\boxed{y)y'}+2y=e^x \qquad y''+\boxed{\sin y}=e^x \qquad y^{(4)}+y^2=e^x$$

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\cdots$$

Nonlinear O.D.E

$$f(ax_1+bx_2)\neq af(x_1)+bf(x_2)$$

- linear O.D.E.

The dependent variable y and all its derivatives y′, y″,…,$y^{(n)}$ are of the first degree, that is the <u>power of each term involving y is 1</u>
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- **Nonlinear O.D.E.**

$$(1-\boxed{y)y'}+2y=e^x \qquad y''+\boxed{\sin y}=e^x \qquad y^{(4)}+\boxed{y^2}=e^x$$

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\cdots$$

Nonlinearity

Nonlinearity of the nature

Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model**

Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

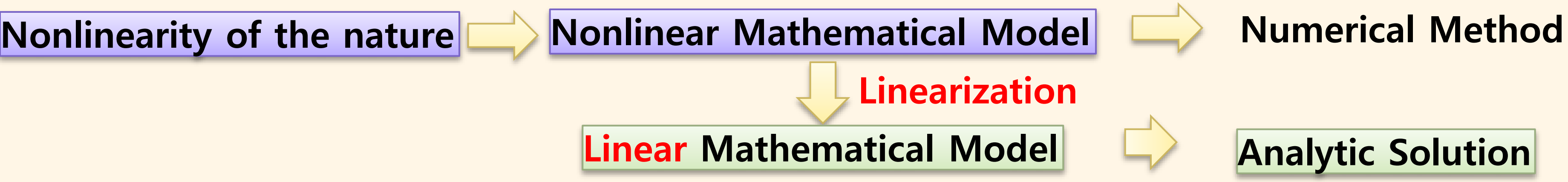
Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model

Nonlinearity

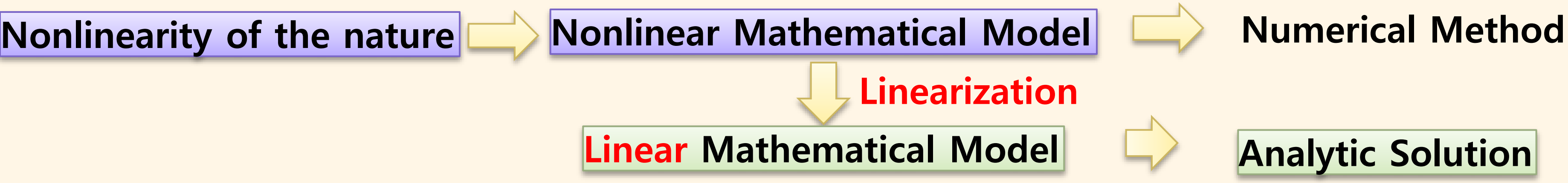


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Nonlinearity



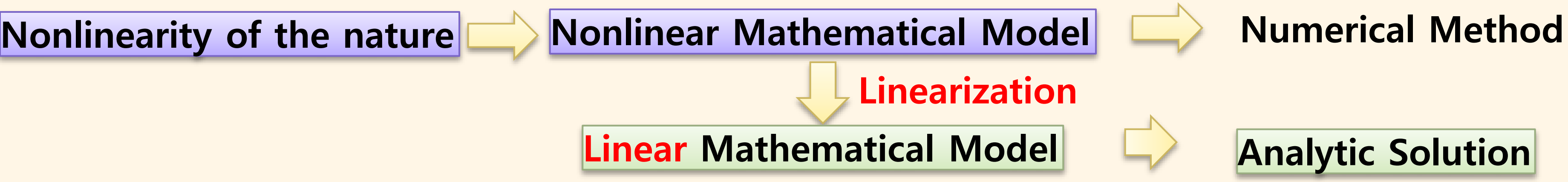
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

How to linearize the nonlinear model?

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series

How to linearize the nonlinear model?

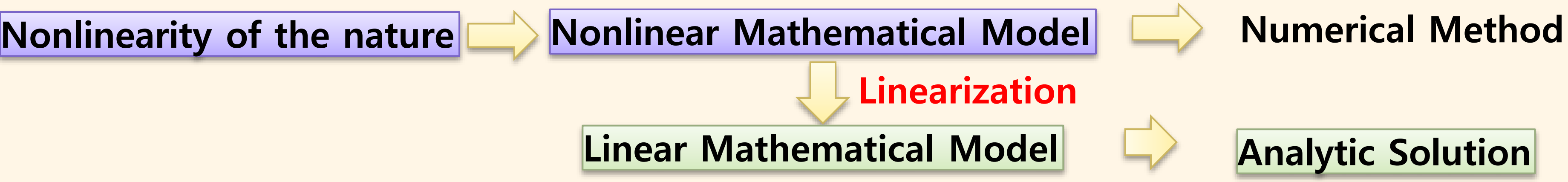
Taylor Series : 미분 가능한 어떤 함수를 다항식의 형태로 근사하는 방법.
n≥0 인 정수 n에 대하여, x=x*인 지점에서 n번 미분 가능한 함수 f는 아래와 같이 나타낼 수 있다

주어진 점 x^* 에서 *f*(*x*)의 Taylor Series

$$f(x) = f(x^*) + \frac{df(x^*)}{dx}(x-x^*) + \frac{1}{2}\frac{d^2 f(x^*)}{dx^2}(x-x^*)^2 + R$$

나머지항(Remainder)
: x가 x^* 에 충분히
가까우면 그 값이 매우 작음

Nonlinearity



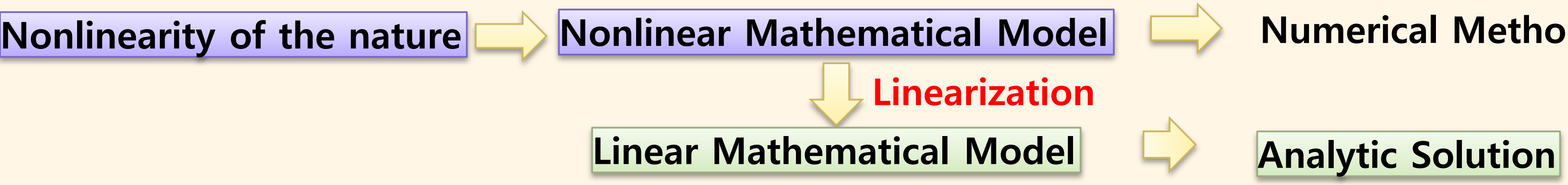
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

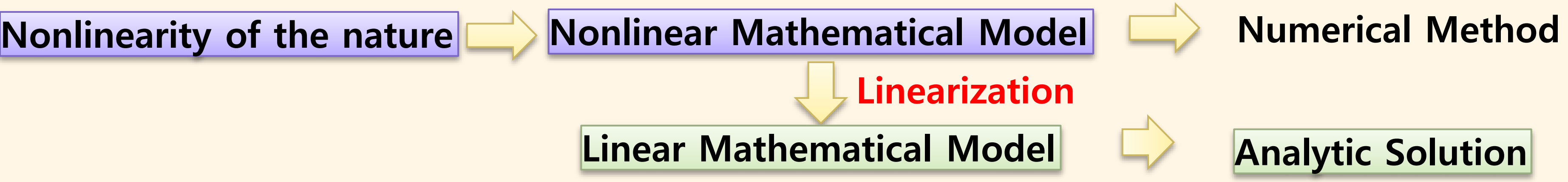
Linear Mathematical Model → Analytic Solution

Taylor Series

Given : $x^*, f(x^*),\quad x^*$ **에서의 *i*차 미분 계수** $\left(\dfrac{\partial^i f(x^*)}{\partial x^i}\right)$

Find : $f(x^* + \Delta x)$

Nonlinearity


Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

Taylor Series

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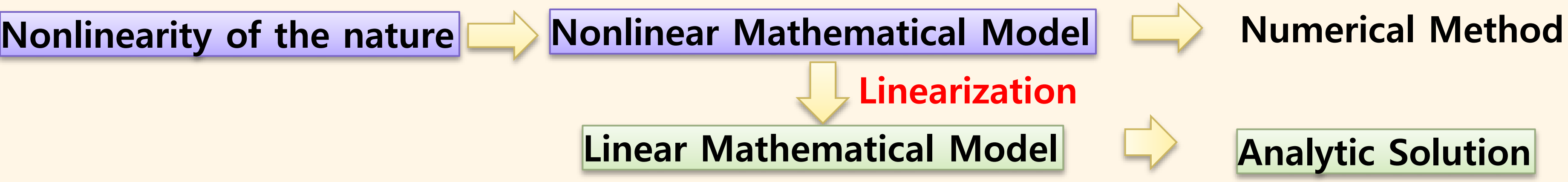
Find : $f(x^* + \Delta x)$

※ **1변수 함수의** Taylor Series Expansion

$$f(x^* + \Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \ldots$$

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ **Linearization**

Linear Mathematical Model → Analytic Solution

Taylor Series

Given : $x^*, f(x^*),\quad x^*$ 에서의 *i*차 미분 계수 $\left(\dfrac{\partial^i f(x^*)}{\partial x^i}\right)$

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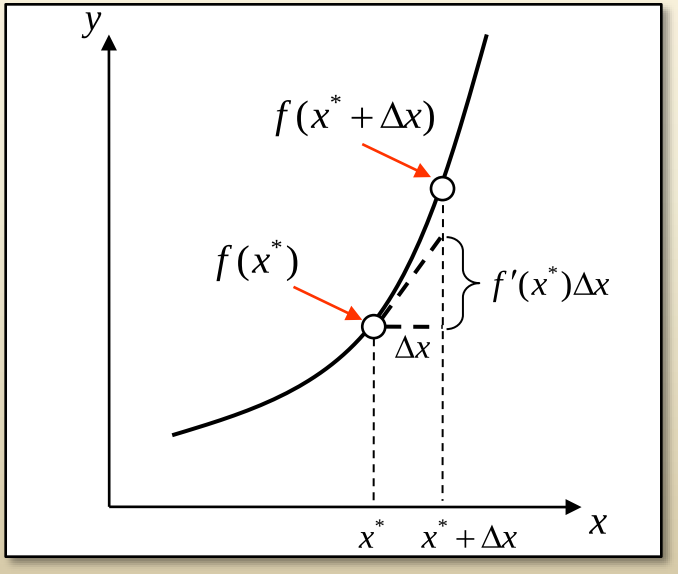
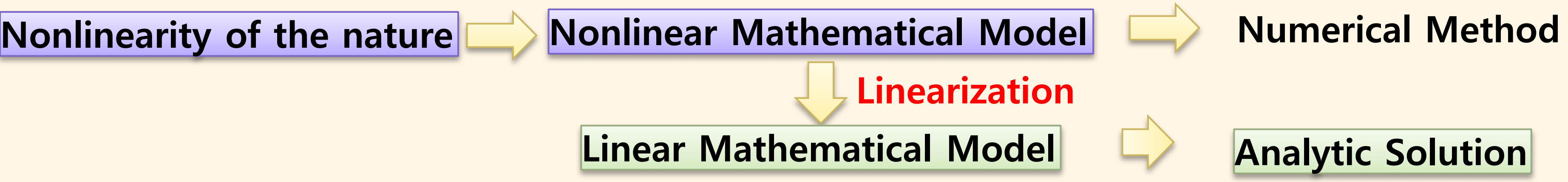


Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

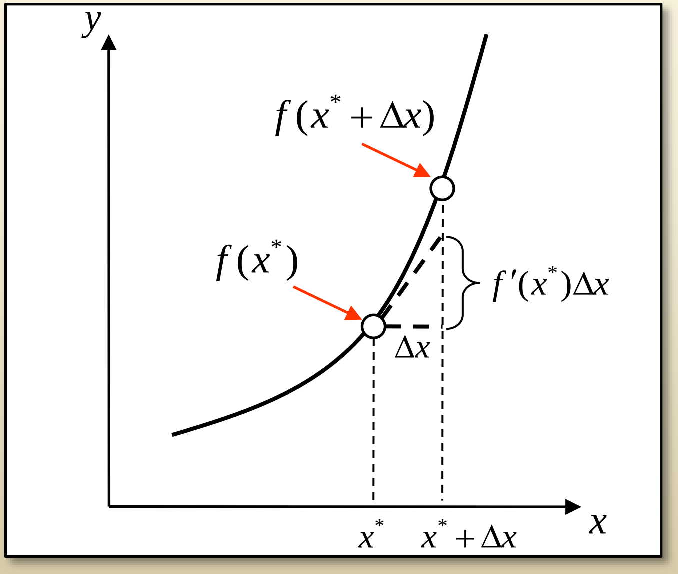
Taylor Series

Given : $x^*, f(x^*)$, x^* 에서의 *i*차 미분 계수 $\left(\dfrac{\partial^i f(x^*)}{\partial x^i}\right)$

Find : $f(x^* + \Delta x)$

※ 1변수 함수의 Taylor Series Expansion

$$f(x^* + \Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \ldots$$



Maclaurin Series

$$\text{let } x^* \rightarrow 0,\ \Delta x \rightarrow x$$

$$f(x) = f(0) + f'(0)\Delta x + \frac{1}{2} f''(0)\Delta x^2 + \ldots$$

Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → **Numerical Method**

Linearization

Linear Mathematical Model → **Analytic Solution**

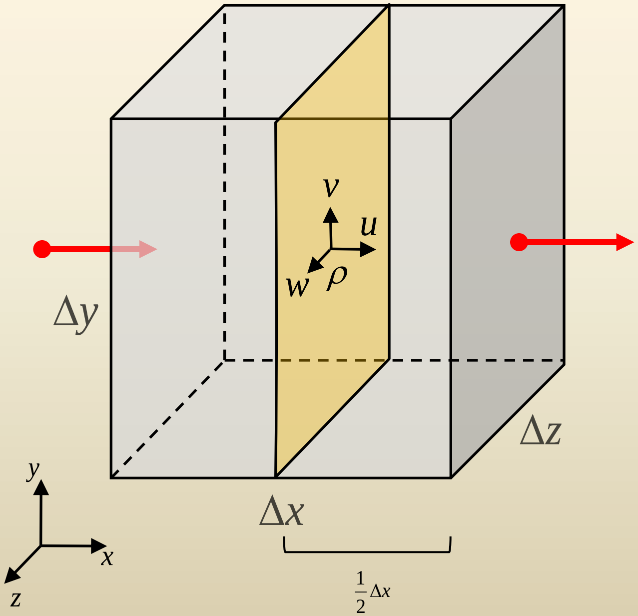
Taylor Series

$$f(x^* + \Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \ldots$$

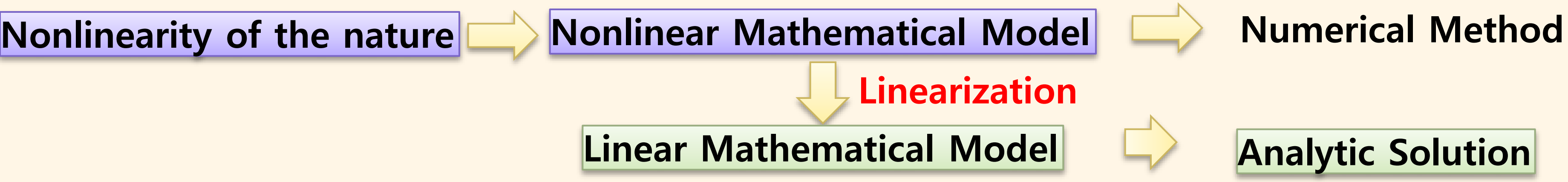
Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \ldots$$

Ex) Continuity Equation



Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

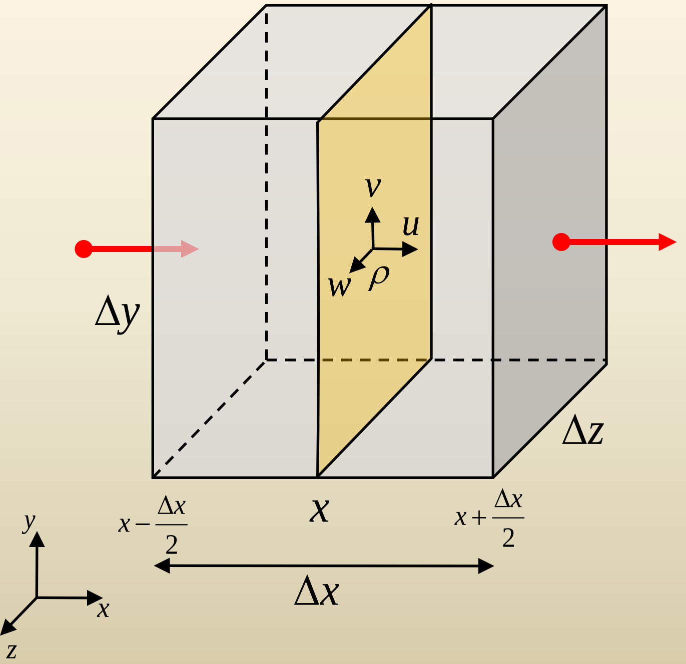
Taylor Series

$$f(x^*+\Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \dots$$

Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \dots$$

Ex) Continuity Equation*



✓**Given :** $\rho(x,y,z)u(x,y,z)$ and $\frac{\partial(\rho u)}{\partial x}$ at (x,y,z)

✓**Find : 오른쪽 면을 통해 검사체적으로부터 빠져나간 유체의 질량**

$$\rho(x+\frac{\Delta x}{2},y,z)u(x+\frac{\Delta x}{2},y,z)\Delta y\Delta z$$

$$=\left[\rho(x,y,z)u(x,y,z)+\frac{\partial(\rho u)}{\partial x}\frac{\Delta x}{2}+\cdots\right]\Delta y\Delta z$$

$$=\left[\rho u+\frac{\partial(\rho u)}{\partial x}\frac{\Delta x}{2}+\cdots\right]\Delta y\Delta z \approx \left[\rho u+\frac{\partial(\rho u)}{\partial x}\frac{\Delta x}{2}\right]\Delta y\Delta z$$

✓ **단위 시간당 왼쪽 면을 통해 들어온 유체의 질량**

$$\rho(x-\frac{\Delta x}{2},y,z)u(x-\frac{\Delta x}{2},y,z)\Delta y\Delta z$$

$$=\left[\rho(x,y,z)u(x,y,z)+\frac{\partial(\rho u)}{\partial x}\left(-\frac{\Delta x}{2}\right)+\cdots\right]\Delta y\Delta z$$

$$=\left[\rho u+\frac{\partial(\rho u)}{\partial x}\left(-\frac{\Delta x}{2}\right)+\cdots\right]\Delta y\Delta z \approx \left[\rho u+\frac{\partial(\rho u)}{\partial x}\left(-\frac{\Delta x}{2}\right)\right]\Delta y\Delta z$$

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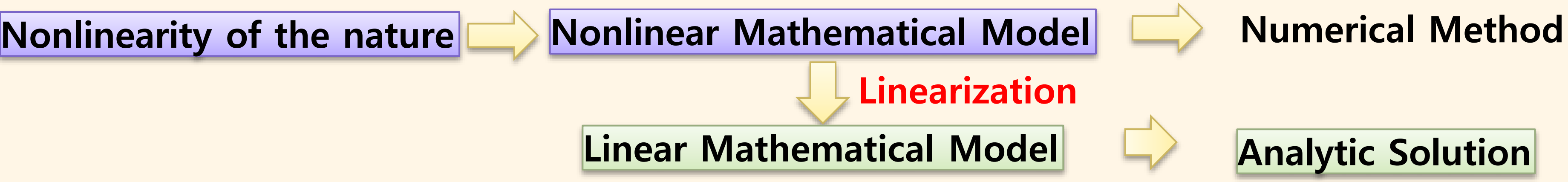
*Dean R.G., Dalrymple R.A., Water Wave Mechanics for Engineers and Scientists, Advanced Series on Ocean Engineering World Scientific, 1994, p9

Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

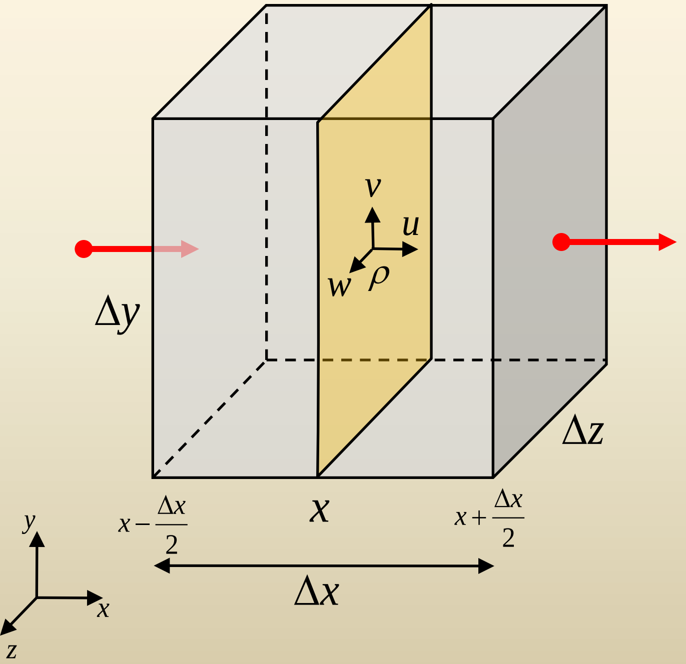
Taylor Series

$$f(x^*+\Delta x)=f(x^*)+f'(x^*)\Delta x+\frac{1}{2}f''(x^*)\Delta x^2+\dots$$

Maclaurin Series

$$f(x)=f(0)+f'(0)x+\frac{1}{2}f''(0)x^2+\dots$$

Ex) Continuity Equation*



✓ the net flux of mass into the cube in the x direction

(+ : mass flow rate in)

$$\left[\rho u+\frac{\partial(\rho u)}{\partial x}\left(-\frac{\Delta x}{2}\right)\right]\Delta y\Delta z-\left[\rho u+\frac{\partial(\rho u)}{\partial x}\frac{\Delta x}{2}\right]\Delta y\Delta z=-\frac{\partial(\rho u)}{\partial x}\Delta x\Delta y\Delta z$$

✓ the net flux of mass into the cube in the y direction $-\frac{\partial(\rho v)}{\partial x}\Delta x\Delta y\Delta z$

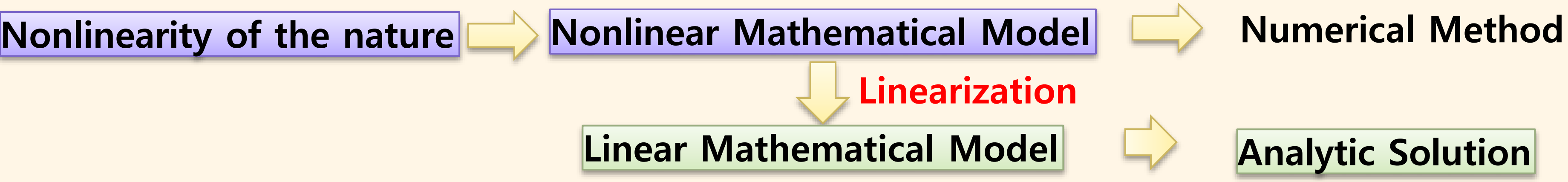
✓ the net flux of mass into the cube in the z direction $-\frac{\partial(\rho w)}{\partial x}\Delta x\Delta y\Delta z$

✓ the net rate of mass accumulation inside the control volume

$$-\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial x}+\frac{\partial(\rho w)}{\partial x}\right]\Delta x\Delta y\Delta z$$

*Dean R.G., Dalrymple R.A., Water Wave Mechanics for Engineers and Scientists, Advanced Series on Ocean Engineering World Scientific, 1994, p9

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series $$f(x^* + \Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \ldots$$

Maclaurin Series $$f(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \ldots$$

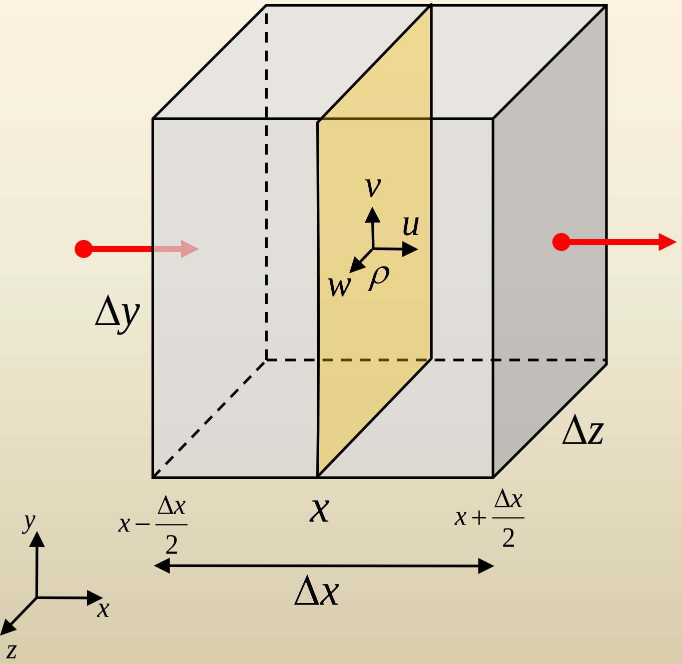
Ex) Continuity Equation*

✓**Given :** $\rho(t)$ and $\dfrac{\partial \rho}{\partial x}$ at (t)

✓**Find : the increase in mass for a time increment**

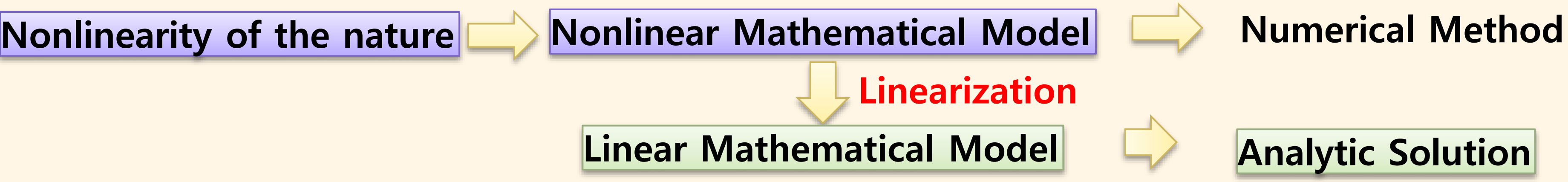
$$\left[\rho(t+\Delta t) - \rho(t)\right]\Delta x \Delta y \Delta z$$

$$= \left[\frac{\partial \rho}{\partial t}\Delta t + \cdots\right]\Delta x \Delta y \Delta z \approx \left[\frac{\partial \rho}{\partial t}\Delta t\right]\Delta x \Delta y \Delta z$$



*Dean R.G., Dalrymple R.A., Water Wave Mechanics for Engineers and Scientists, Advanced Series on Ocean Engineering World Scientific, 1994, p9

Nonlinearity


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → Analytic Solution

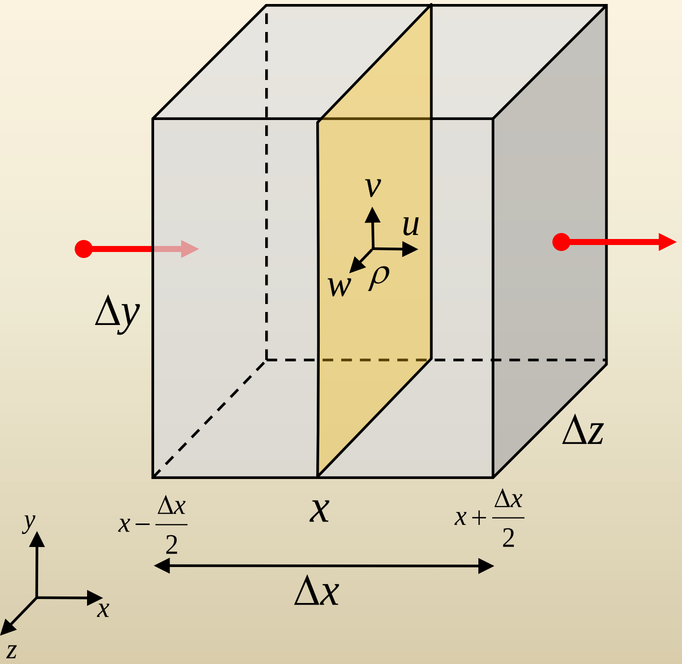
Taylor Series

$$f(x^* + \Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \ldots$$

Maclaurin Series

$$f(x) = f(0) + f'(0)\Delta x + \frac{1}{2} f''(0)\Delta x^2 + \ldots$$

Ex) Continuity Equation*



✓**Mass conservation**

: the increase in mass for a time increment

must be due to

the net inflow rate occurring over a time increment

$$\left[\frac{\partial\rho}{\partial t}\Delta t\right]\Delta x\Delta y\Delta z = -\left[\frac{\partial\rho}{\partial x}+\frac{\partial(\rho v)}{\partial x}+\frac{\partial(\rho w)}{\partial x}\right]\Delta x\Delta y\Delta z\Delta t$$

$$\therefore \frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial x}+\frac{\partial(\rho w)}{\partial x}=0$$

$$\searrow \quad \downarrow \quad \downarrow \quad \swarrow$$

assumed as given at first

but we need to find them in the result.

*Dean R.G., Dalrymple R.A., Water Wave Mechanics for Engineers and Scientists, Advanced Series on Ocean Engineering World Scientific, 1994, p9

Nonlinearity

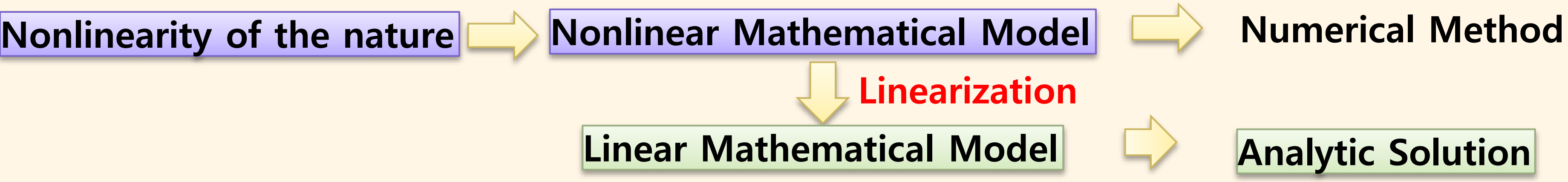
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series

Given : $\left(x_1^*, x_2^*\right)$, $f\,(x_1^*, x_2^*)$, x^* **에서의 미분 계수** $\left(\dfrac{\partial^{i+j} f\,(x_1^*, x_2^*)}{\partial x_1{}^i \partial x_2{}^j}\right)$

Find : $f\,(x_1^* + \Delta x_1, x_2^* + \Delta x_2)$

Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series

Given : (x_1^*, x_2^*), $f(x_1^*, x_2^*)$, x^* **에서의 미분 계수** $\left(\dfrac{\partial^{i+j} f(x_1^*, x_2^*)}{\partial x_1{}^i \partial x_2{}^j} \right)$

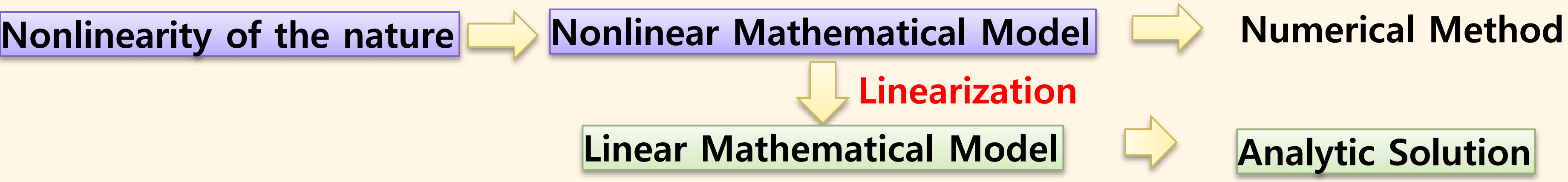
Find : $f(x_1^* + \Delta x_1, x_2^* + \Delta x_2)$

※ 2변수 함수의 Tayler Series Expansion

$$f(x_1^* + \Delta x_1, x_2^* + \Delta x_2) = f(x_1^*, x_2^*) + \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2$$

$$+ \frac{1}{2}\left(\frac{\partial^2 f}{\partial x_1^2}\Delta x_1{}^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}\Delta x_1 \Delta x_2 + \frac{\partial^2 f}{\partial x_2^2}\Delta x_2{}^2 \right) + \ldots$$

Nonlinearity


Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

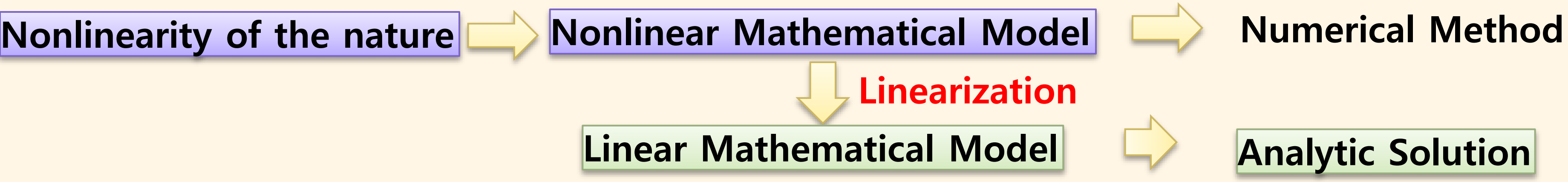
Linearization

Linear Mathematical Model

Analytic Solution

Taylor Series

Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

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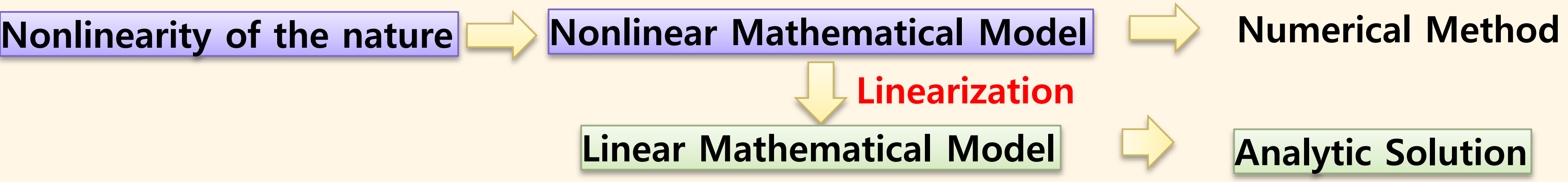
Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

Taylor Series

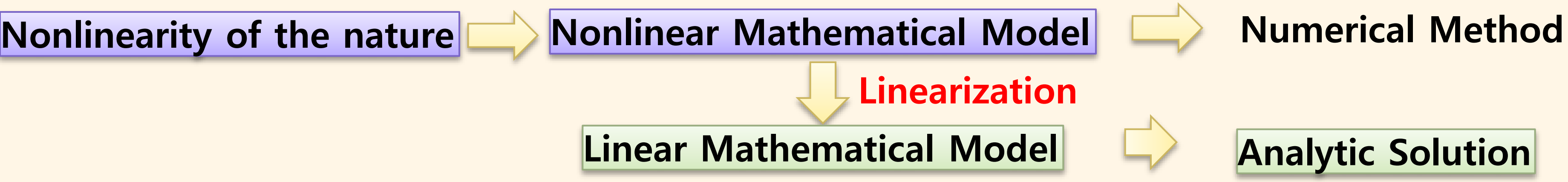
※ 2변수 함수의 Tayler Series Expansion

$$\nabla f(\mathbf{x}^*)^T(\mathbf{x}-\mathbf{x}^*) = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} \\ \dfrac{\partial f}{\partial x_2} \end{bmatrix}^T \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$f(x_1^* + \Delta x_1, x_2^* + \Delta x_2) = f(x_1^*, x_2^*) + \boxed{\frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2}$$

$$+\frac{1}{2}\left(\frac{\partial^2 f}{\partial x_1^2}\Delta x_1{}^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}\Delta x_1 \Delta x_2 + \frac{\partial^2 f}{\partial x_2^2}\Delta x_2{}^2\right) + \ldots$$

Nonlinearity



Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Taylor Series

※ 2변수 함수의 Tayler Series Expansion

$$\nabla f(\mathbf{x}^*)^T(\mathbf{x}-\mathbf{x}^*) = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} \\ \dfrac{\partial f}{\partial x_2} \end{bmatrix}^T \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$f(x_1^* + \Delta x_1, x_2^* + \Delta x_2) = f(x_1^*, x_2^*) + \boxed{\frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2}$$

$$+\frac{1}{2}\left(\boxed{\frac{\partial^2 f}{\partial x_1^2}\Delta x_1{}^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}\Delta x_1 \Delta x_2 + \frac{\partial^2 f}{\partial x_2^2}\Delta x_2{}^2}\right) + \ldots$$

$$\frac{1}{2}(\Delta\mathbf{x})^T\mathbf{H}(\mathbf{x}^*)(\Delta\mathbf{x}) = \frac{1}{2}\begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2}(\Delta x_1) + \dfrac{\partial^2 f}{\partial x_2 \partial x_1}(\Delta x_2) & \dfrac{\partial^2 f}{\partial x_1 \partial x_2}(\Delta x_1) + \dfrac{\partial^2 f}{\partial x_2^2}(\Delta x_2) \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \Delta x_1 & \Delta x_2 \end{bmatrix}\begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Taylor Series

※ 2변수 함수의 Tayler Series Expansion

$$\nabla f(\mathbf{x}^*)^T(\mathbf{x}-\mathbf{x}^*) = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} \\ \dfrac{\partial f}{\partial x_2} \end{bmatrix}^T \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$f(x_1^* + \Delta x_1, x_2^* + \Delta x_2) = f(x_1^*, x_2^*) + \boxed{\frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2}$$

$$+\frac{1}{2}\left(\boxed{\frac{\partial^2 f}{\partial x_1^2}\Delta {x_1}^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}\Delta x_1 \Delta x_2 + \frac{\partial^2 f}{\partial x_2^2}\Delta {x_2}^2}\right) + \ldots$$

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$$= \frac{1}{2}\begin{bmatrix} \Delta x_1 & \Delta x_2 \end{bmatrix}\begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$\boxed{\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}}$$

$$\therefore\ f(\mathbf{x}) = f(\mathbf{x}^*) + \nabla f(\mathbf{x}^*)^T(\Delta\mathbf{x}) + \frac{1}{2}(\Delta\mathbf{x})^T\mathbf{H}(\mathbf{x}^*)(\Delta\mathbf{x}) + R$$

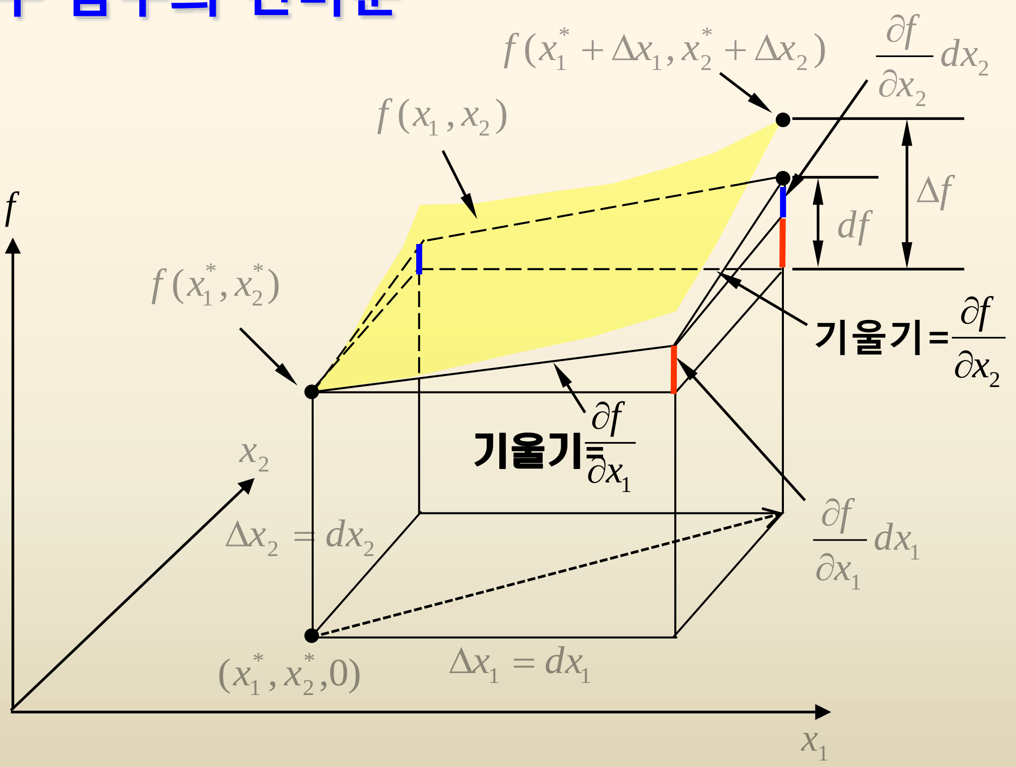
Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

2변수 함수의 전미분



주어진 것: $(x_1^*, x_2^*), f(x_1^*, x_2^*)$

실제 구해야 하는 것:

$$f(x_1^*+\Delta x_1, x_2^*+\Delta x_2)$$
$$= f(x_1^*, x_2^*) + \Delta f$$

근사적으로 구할 수 있는 것:

$$f(x_1^*, x_2^*) + df$$

$\Delta x_1,\ \Delta x_2$ 가 아주 작다면

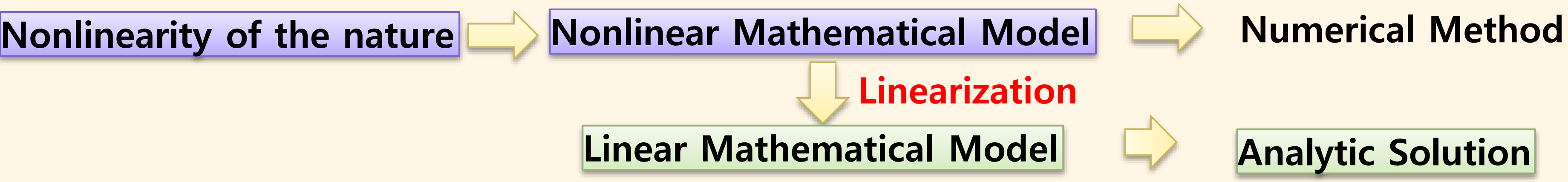
$\Delta f \cong df$ 라 볼 수 있음

x_2 방향의 변화량

$$df = \boxed{\frac{\partial f}{\partial x_1}dx_1} + \boxed{\frac{\partial f}{\partial x_2}dx_2}$$

x_1 방향의 변화량

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model → **Linearization** → Linear Mathematical Model → Analytic Solution

2변수 함수의 전미분

✓ 2변수 함수의 전미분

$$df = \boxed{\frac{\partial f}{\partial x_1} dx_1} + \boxed{\frac{\partial f}{\partial x_2} dx_2}$$

*x*₂가 고정일 때 *x*₁의 변화에 따른 *f* 의 변화량

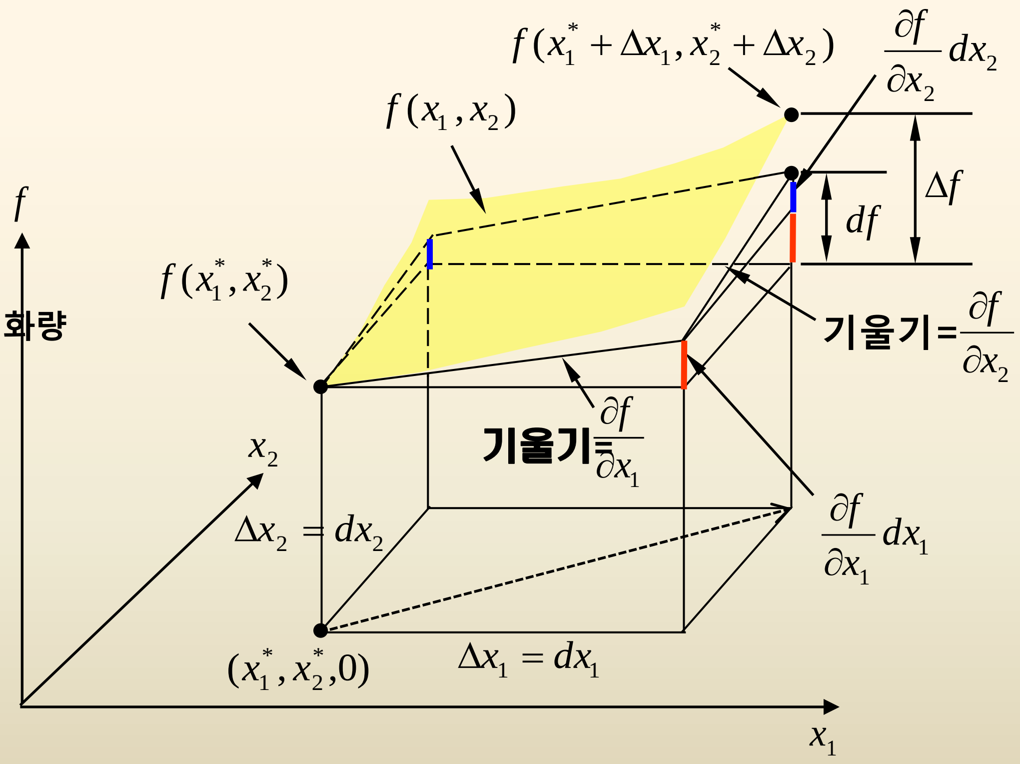
*x*₁가 고정일 때 *x*₂의 변화에 따른 *f* 의 변화량

$$\underbrace{f = \mathbf{V}}_{\text{속도}} \; , \; \underbrace{x_1 = t}_{\text{시간}} \; , \; \underbrace{x_2 = x}_{\text{변위}}$$

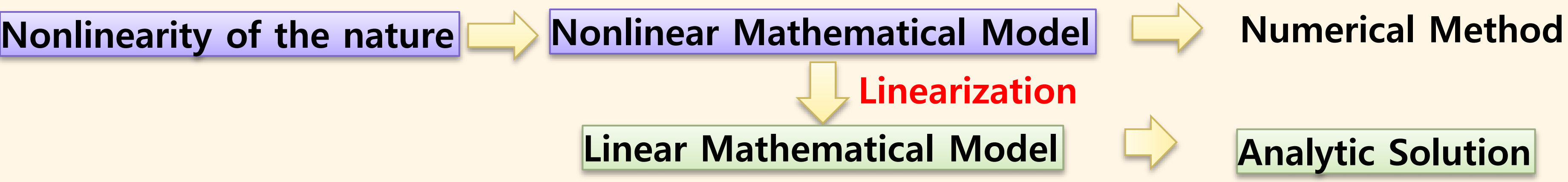
$$d\mathbf{V} = \boxed{\frac{\partial \mathbf{V}}{\partial t} dt} + \boxed{\frac{\partial \mathbf{V}}{\partial x} dx}$$

시간이 고정일 때, 위치 변화에 따른 속도 변화량

위치가 고정일 때, 시간 변화에 따른 속도 변화량



Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Taylor Series

$$f(x^* + \Delta x) = f(x^*) + f'(x^*)\Delta x + \frac{1}{2} f''(x^*)\Delta x^2 + \ldots$$

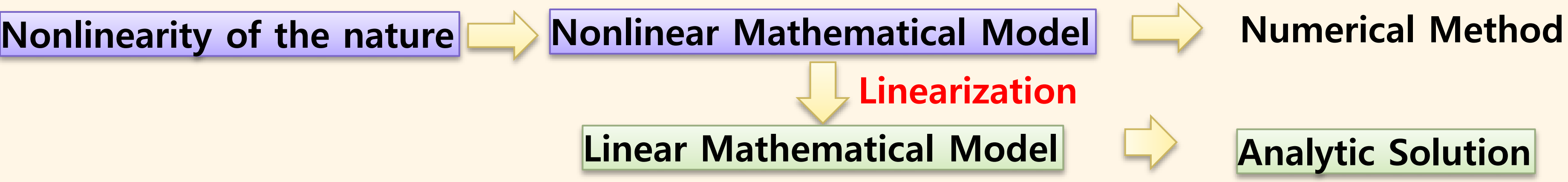
삼각함수 Taylor 전개

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \cdots = \sum_{n=0}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!} x^{2n-1}$$

Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

Taylor Series

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Nonlinearity

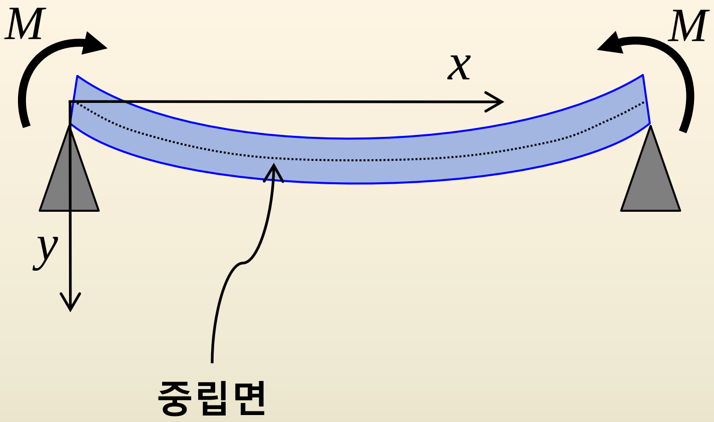
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ Linearization

Linear Mathematical Model → Analytic Solution

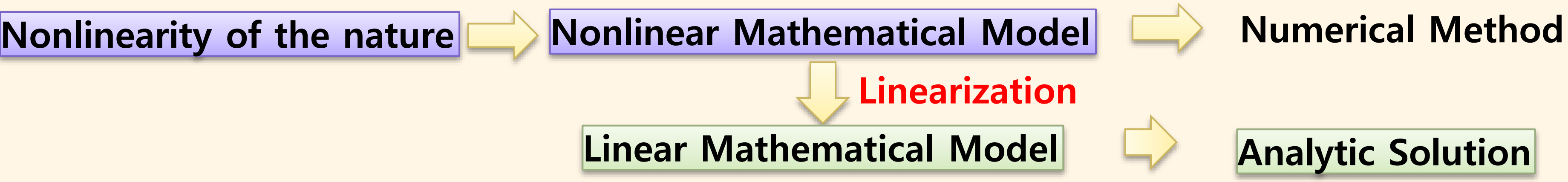
Ex) 탄성선의 미분 방정식

Deflection of a beam



* 중립면 : 보의 볼록 한 쪽의 재료는 늘어나고, 오목한 쪽의 재료는 줄어든다.
이 때, 보의 상면과 하면 사이의 어딘가는 길이가 변하지 않는 재료들의 층이 존재할 것이다.
그와 같은 섬유들이 이루는 면을 중립면이라 한다.

Nonlinearity



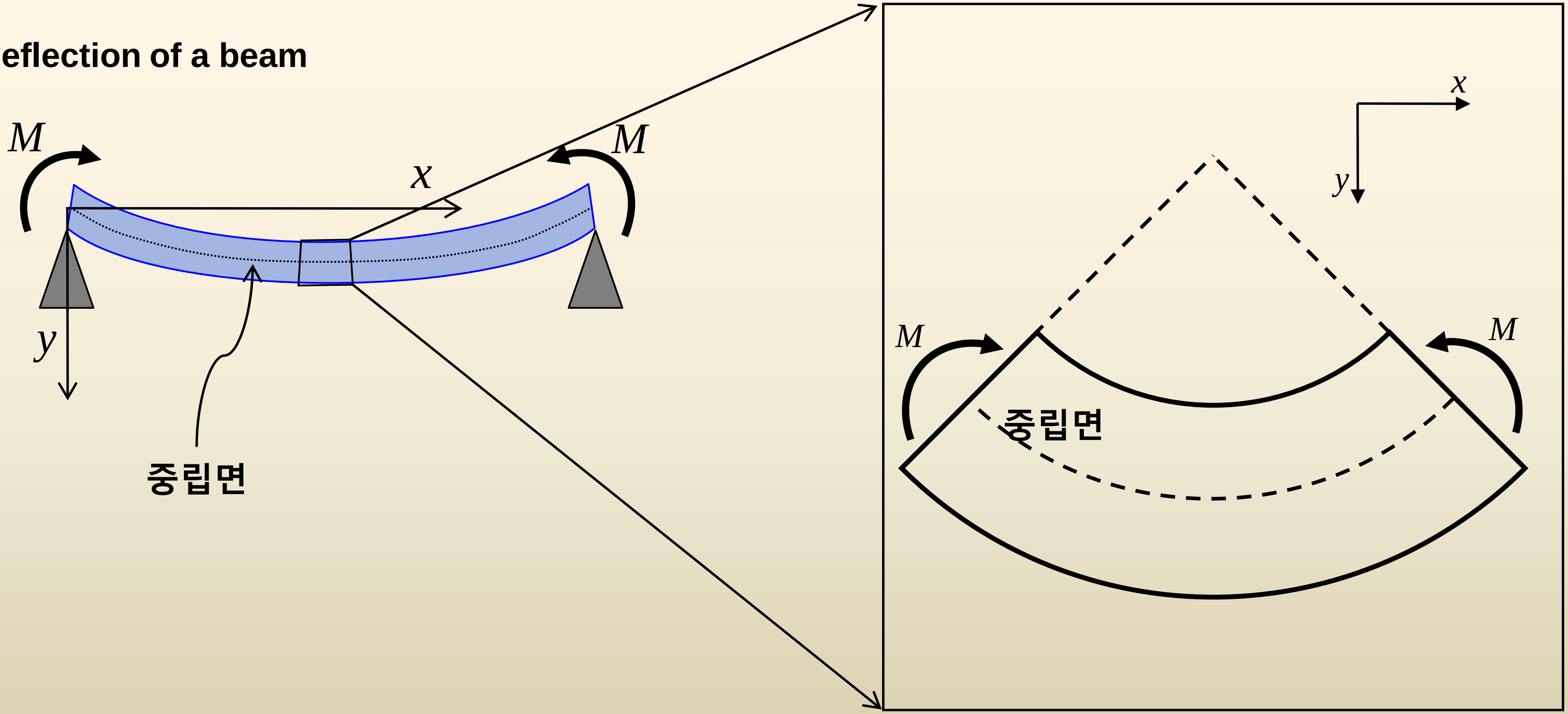
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) **탄성선의 미분 방정식**

Deflection of a beam



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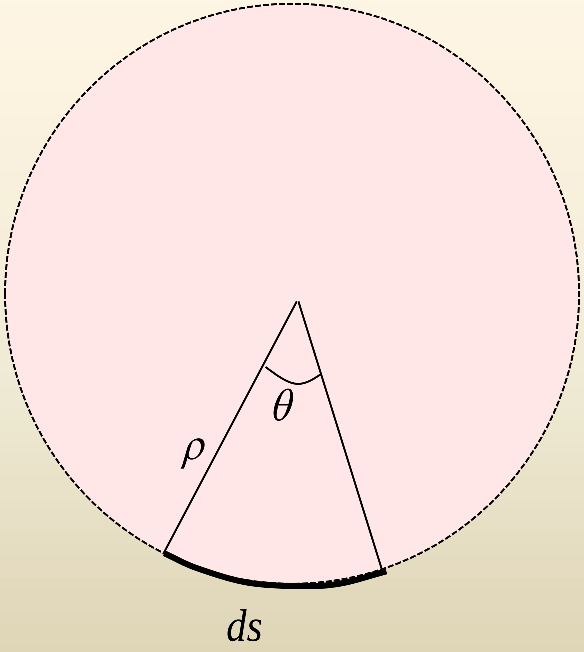
Nonlinearity

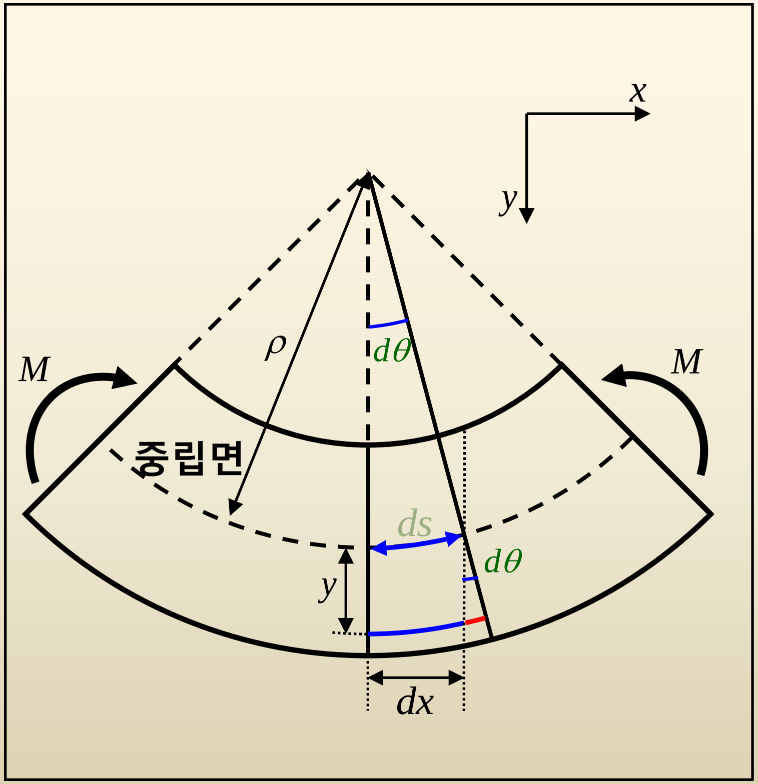
Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ Analytic Solution

Ex) 탄성선의 미분 방정식





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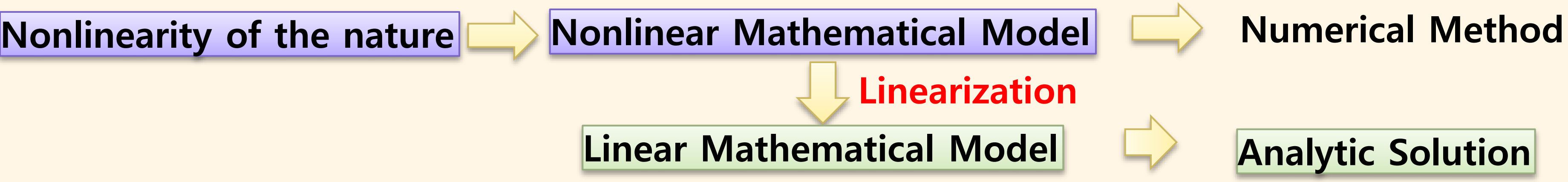
Nonlinearity



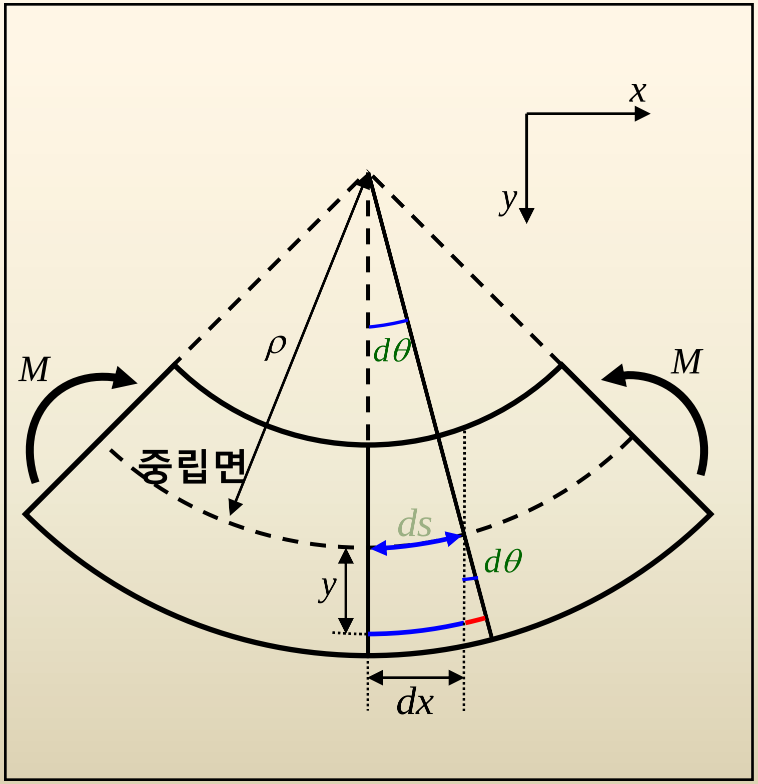
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) 탄성선의 미분 방정식

$$\sigma = E\varepsilon$$



Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

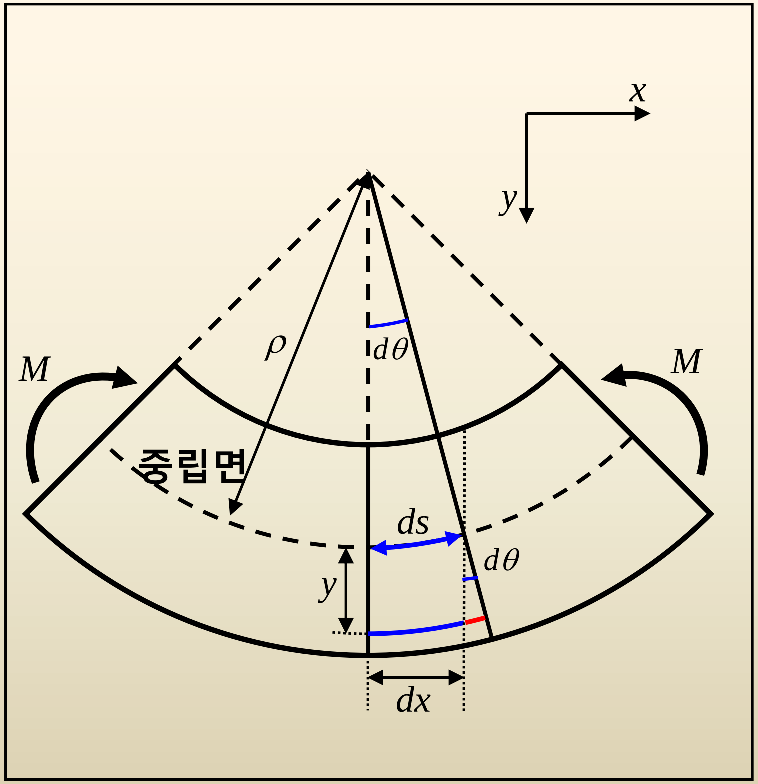
⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) 탄성선의 미분 방정식

$$\sigma = E\varepsilon$$

① $\rho \cdot d\theta = ds$ ⇨ $\dfrac{d\theta}{ds} = \dfrac{1}{\rho}$



Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

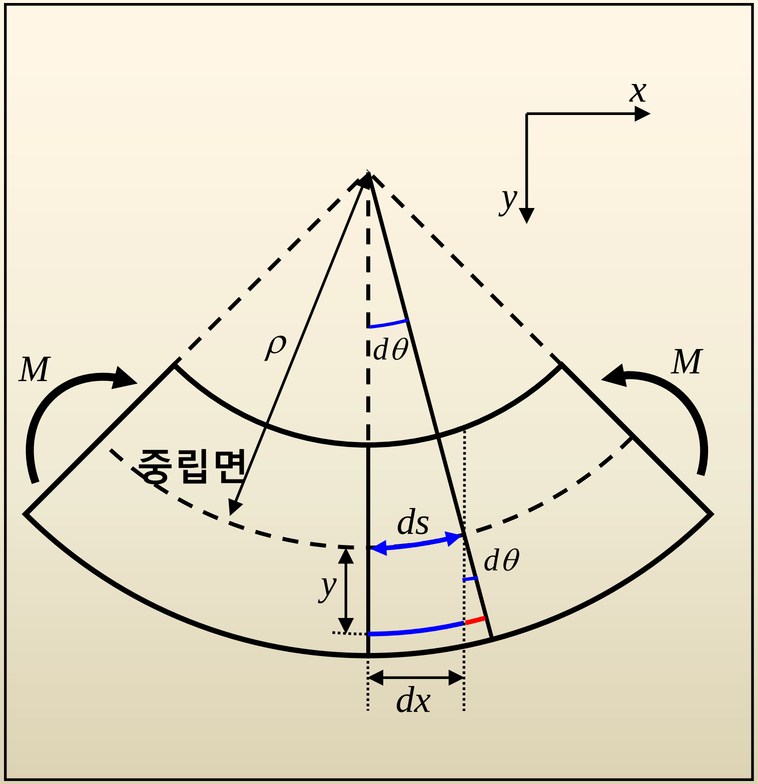
Linear Mathematical Model ⇨ **Analytic Solution**

Ex) **탄성선의 미분 방정식**

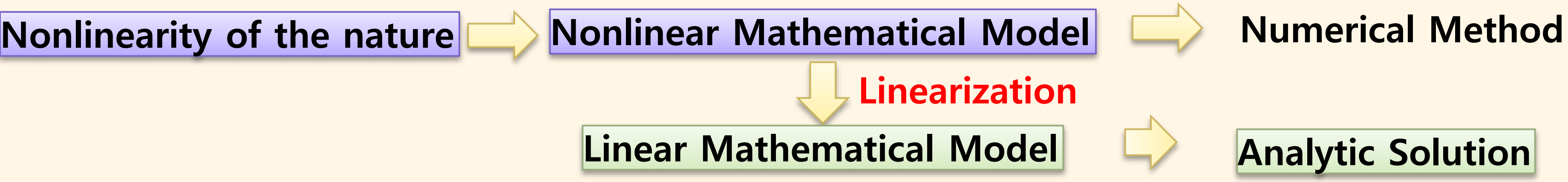
$$\sigma = E\varepsilon$$

① $\rho \cdot d\theta = ds$ ⇨ $\dfrac{d\theta}{ds} = \dfrac{1}{\rho}$

② 중립면에서 y만큼 떨어진 곳의 변형율 (ε)



Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

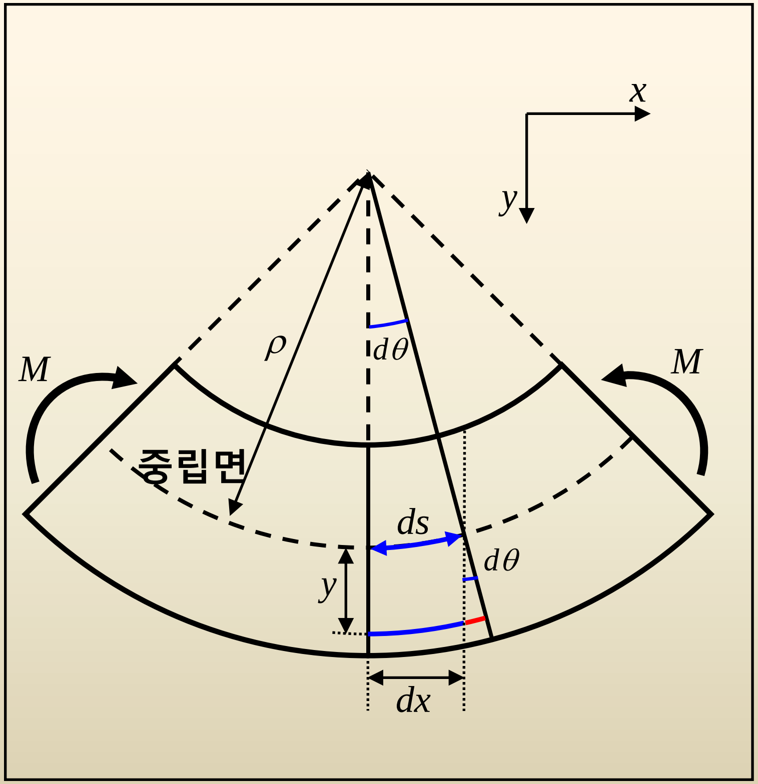
↓ **Linearization**

Linear Mathematical Model → **Analytic Solution**

Ex) 탄성선의 미분 방정식

$$\sigma = E\varepsilon$$

① $\rho \cdot d\theta = ds \Rightarrow \dfrac{d\theta}{ds} = \dfrac{1}{\rho}$



② 중립면에서 y만큼 떨어진 곳의 변형율 (ε)

ds : 원래 길이

Nonlinearity

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Linearization

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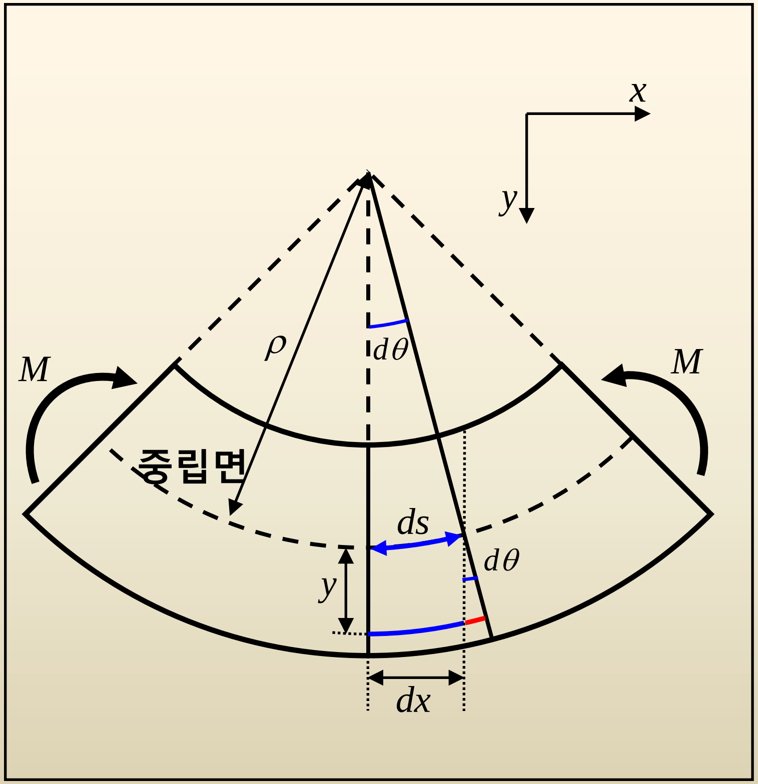

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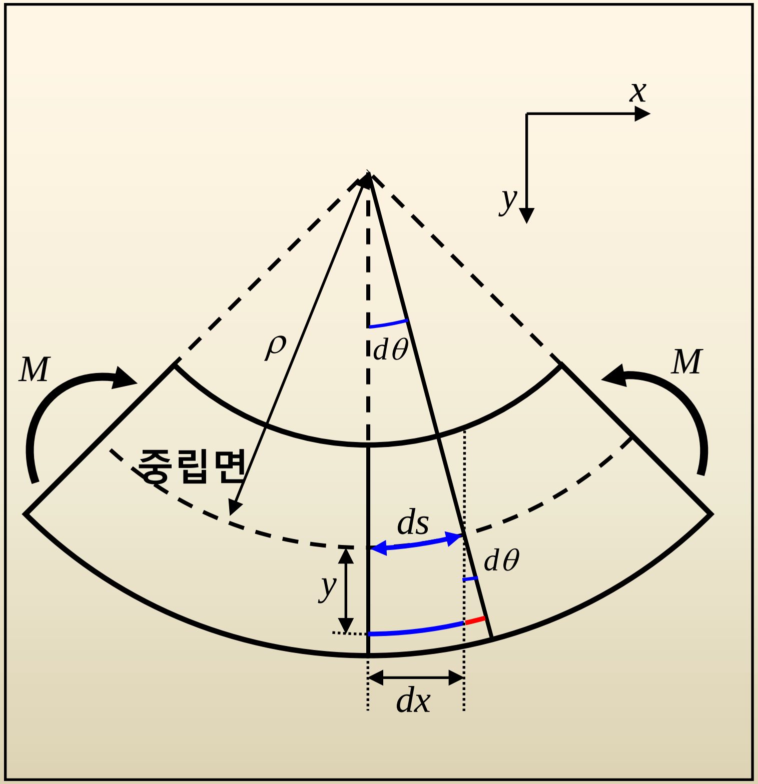

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⇩ Linearization

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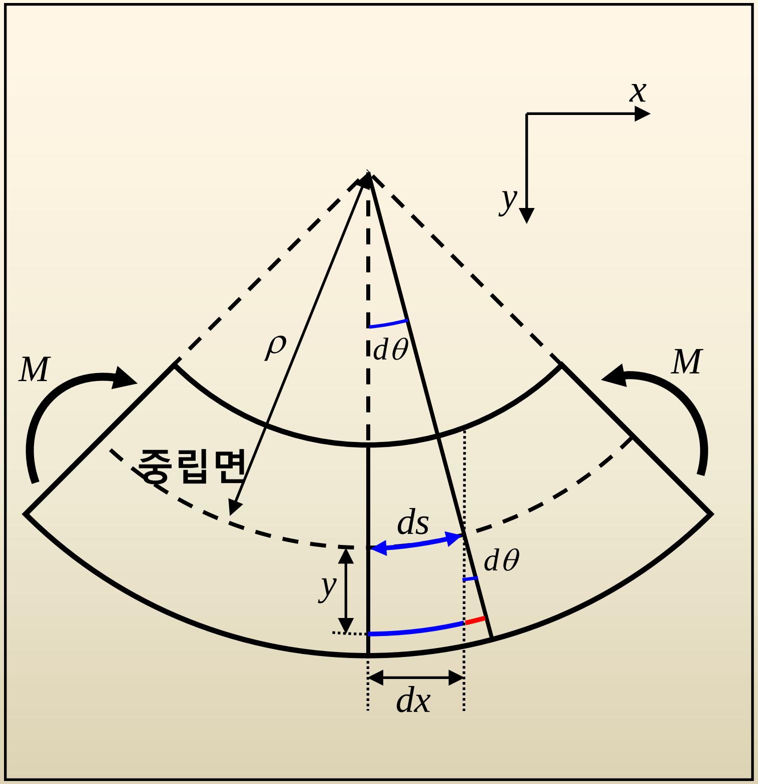

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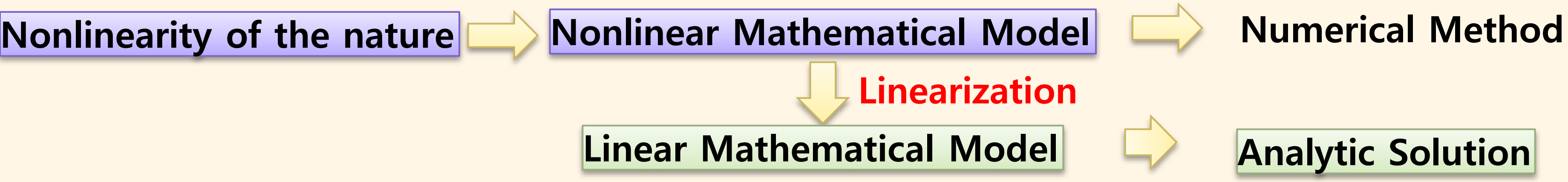
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Linearization

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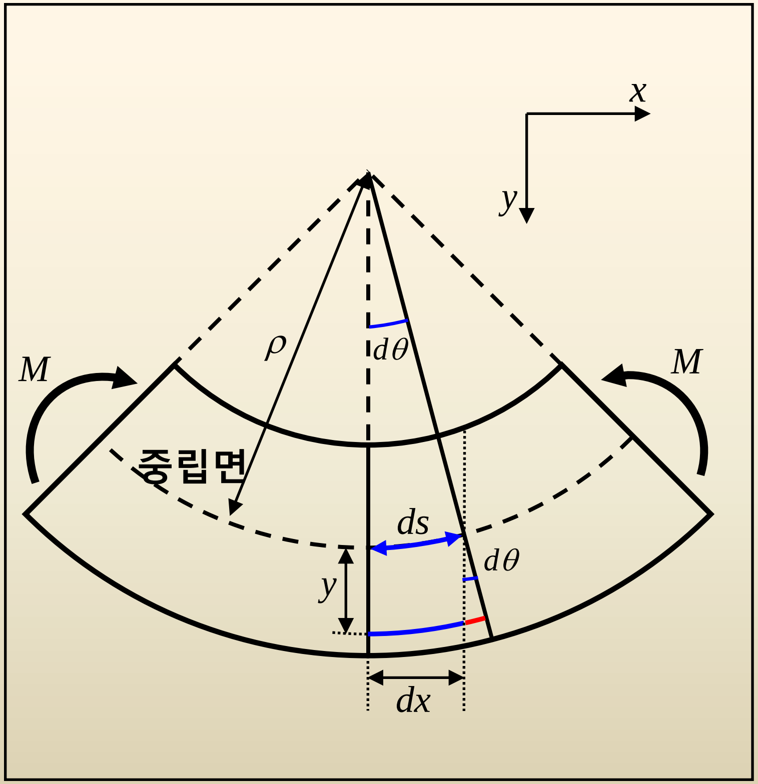

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Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

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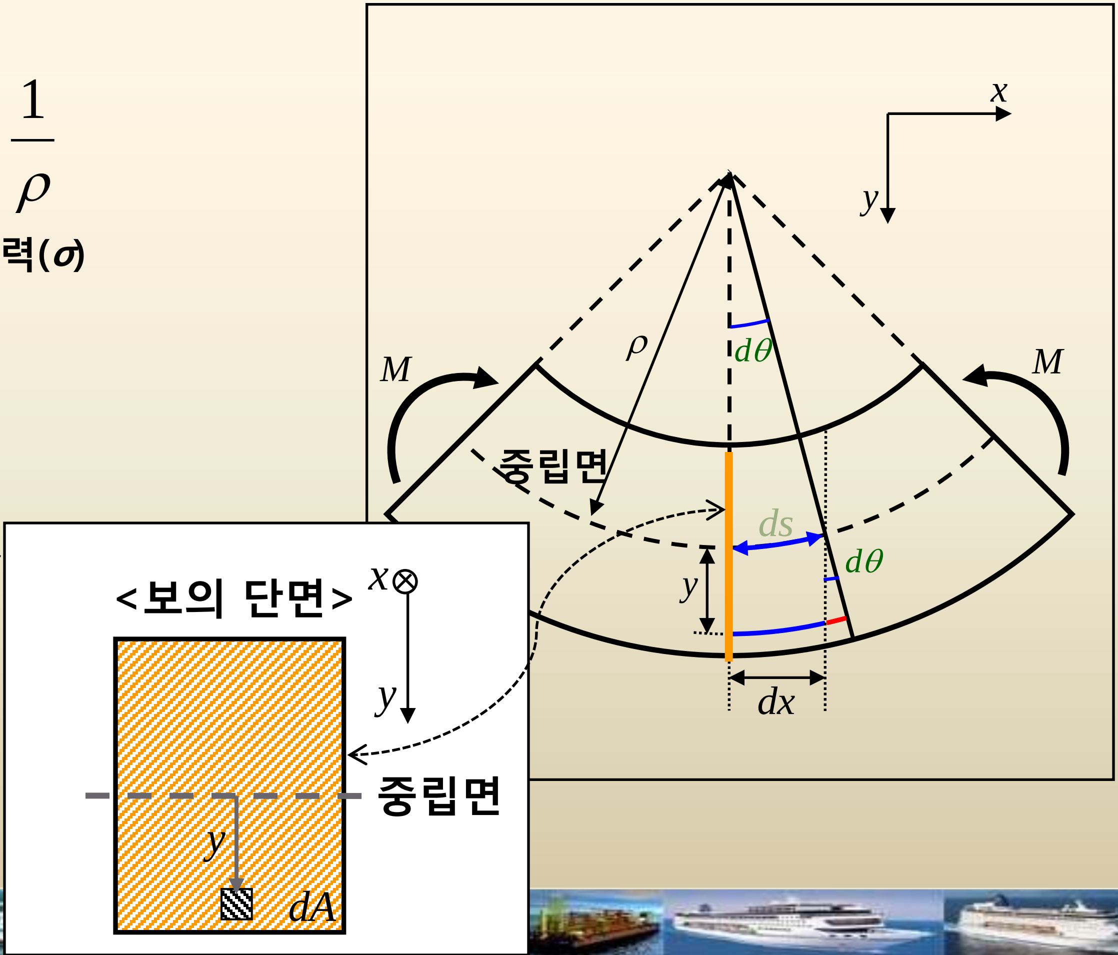
④ 미소면적에 작용하는 힘 :

$$dF = \sigma dA = E \cdot \frac{y}{\rho} dA$$



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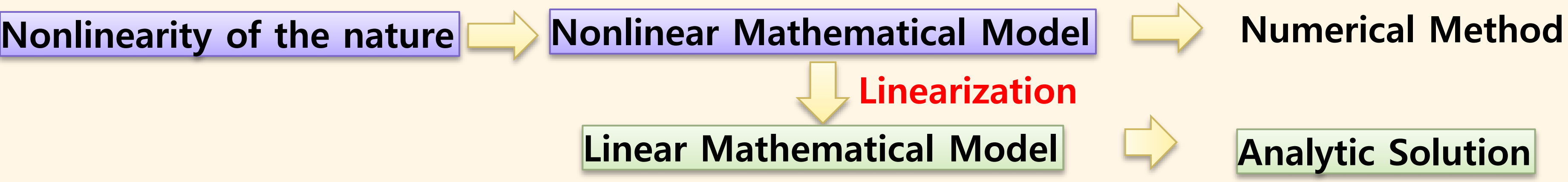

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⇓ Linearization

Linear Mathematical Model ⇒ **Analytic Solution**

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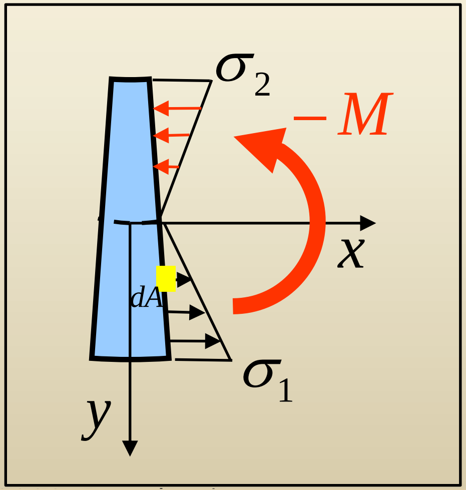
⑤ **미소면적에 작용하는 모멘트**



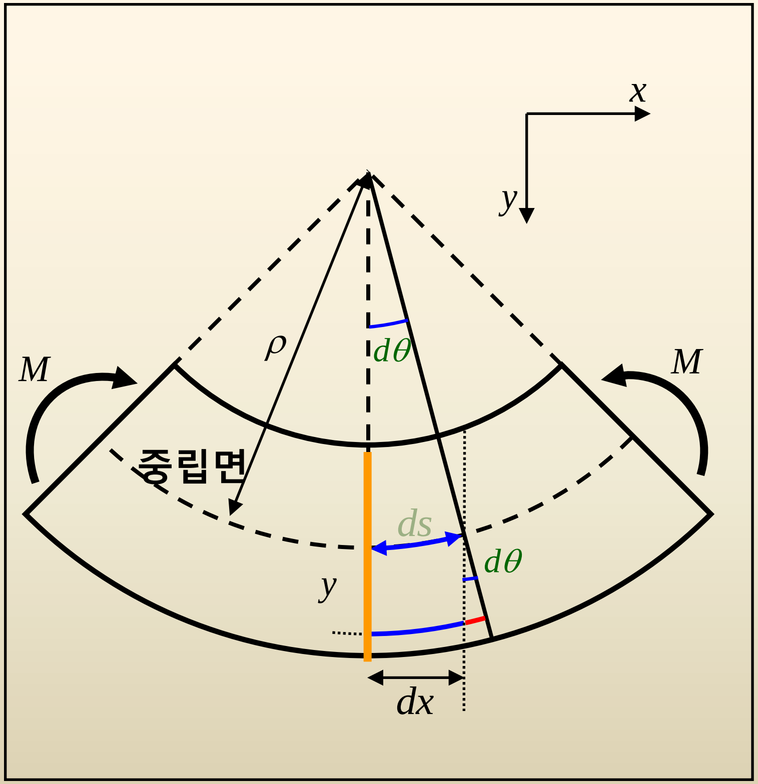
•응력이 양인 곳에서의 모멘트

$$\underset{(<0)}{dM} = -\ \underset{(>0)}{y}\ \underset{(>0)}{\sigma}\ \underset{(>0)}{dA}$$

•응력이 음인 곳에서의 모멘트

$$\underset{(<0)}{dM} = -\ \underset{(<0)}{y}\ \underset{(<0)}{\sigma}\ \underset{(>0)}{dA}$$

$$\therefore dM = -y\sigma dA$$



- 상면 '압축', 하면 '인장'
- 선박의 경우 sagging condition

Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

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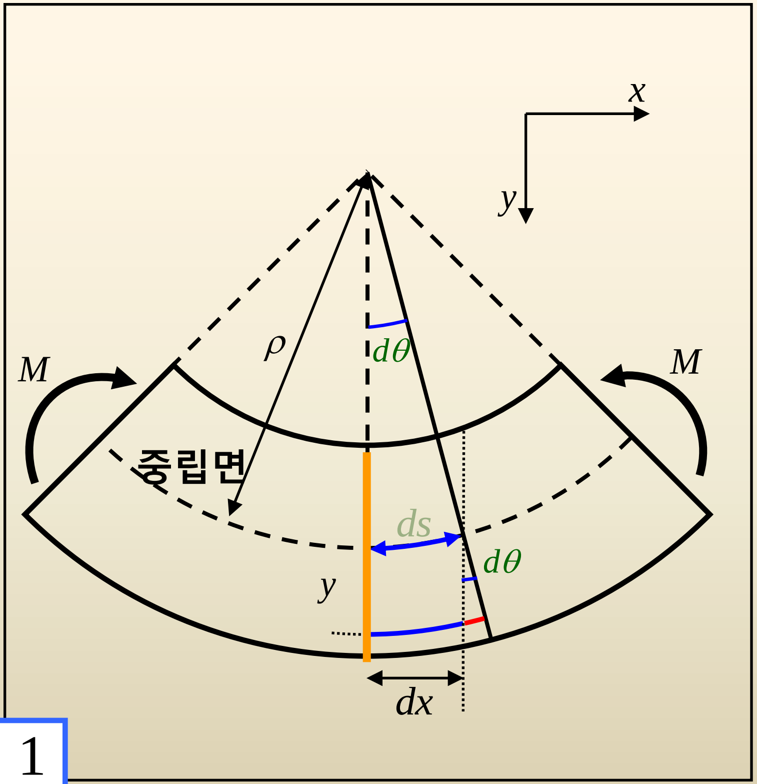
⑥ **단면에 작용하는 모멘트 :**

$$M = \int_A dM = -\int_A ydF$$

$$= -\int_A y \cdot E\frac{y}{\rho} dA = -\frac{E}{\rho}\int_A y^2 dA$$



Define $I = \int_A y^2 dA$ then, $M = -\dfrac{EI}{\rho} \Rightarrow \boxed{\dfrac{M}{EI} = -\dfrac{1}{\rho}}$

Nonlinearity

Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ Analytic Solution

Ex) 탄성선의 미분 방정식

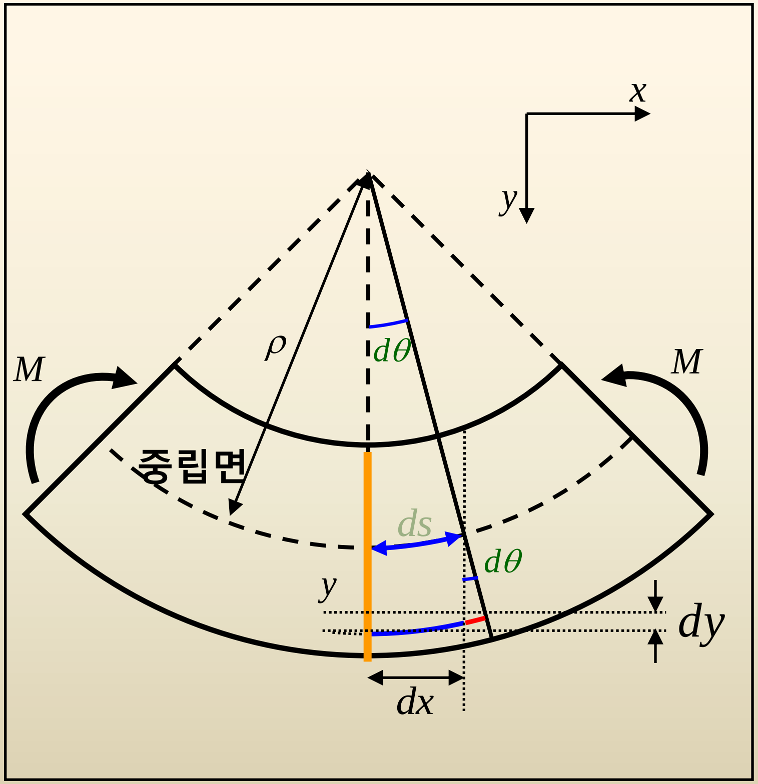
$$\rho \cdot d\theta = ds \Rightarrow \frac{d\theta}{ds} = \frac{1}{\rho}$$

$$dF = \sigma dA = E \cdot \frac{y}{\rho} dA$$

$$\Downarrow$$

$$dM = -y\sigma dA \Rightarrow \frac{M}{EI} = -\frac{1}{\rho}$$

$$\therefore \frac{d\theta}{ds} = -\frac{M}{EI}$$



Nonlinearity

Nonlinearity of the nature ⇒ **Nonlinear Mathematical Model** ⇒ Numerical Method

⇓ Linearization

Linear Mathematical Model ⇒ **Analytic Solution**

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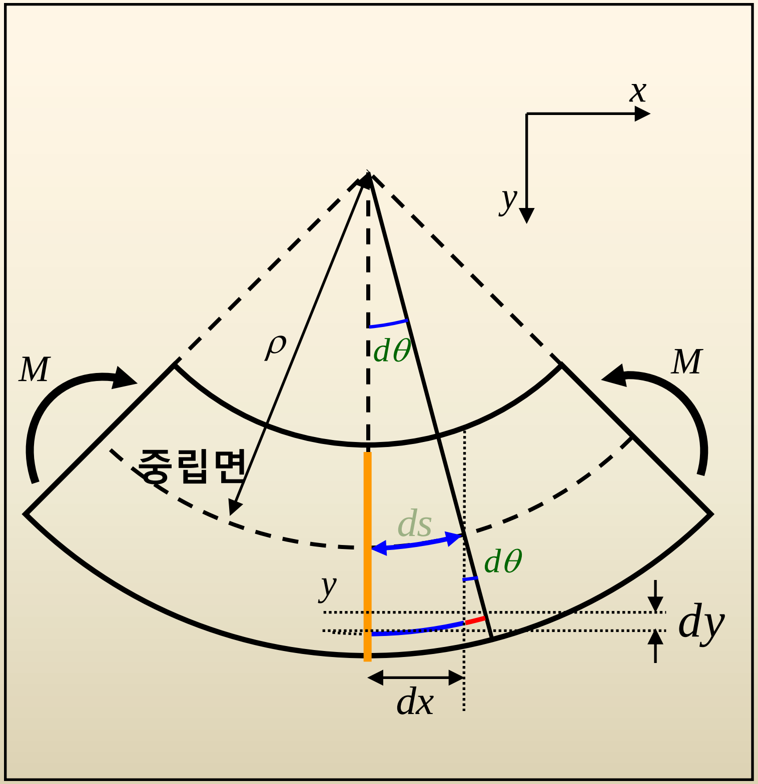

Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

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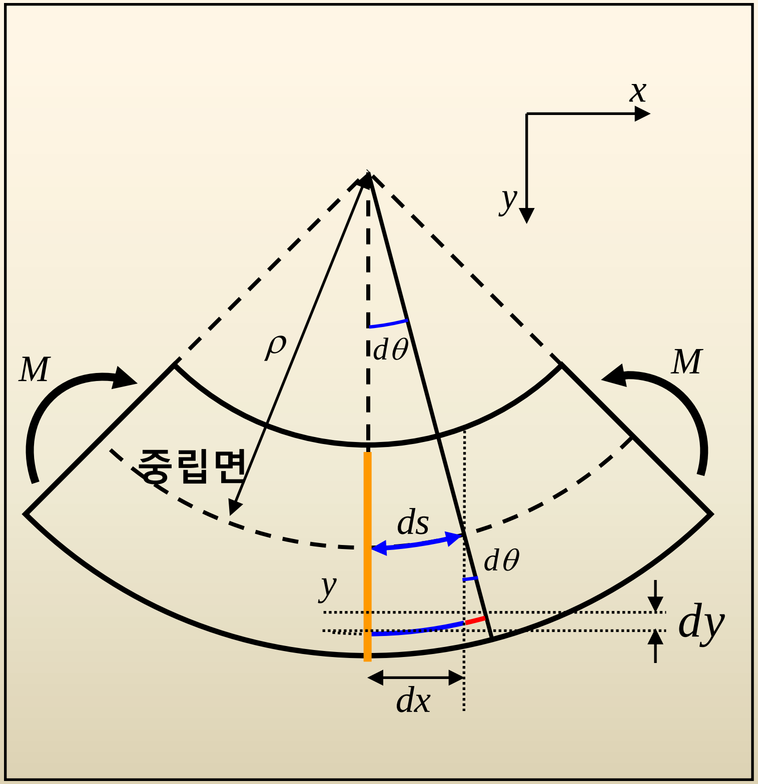
⑦ Assume that



Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

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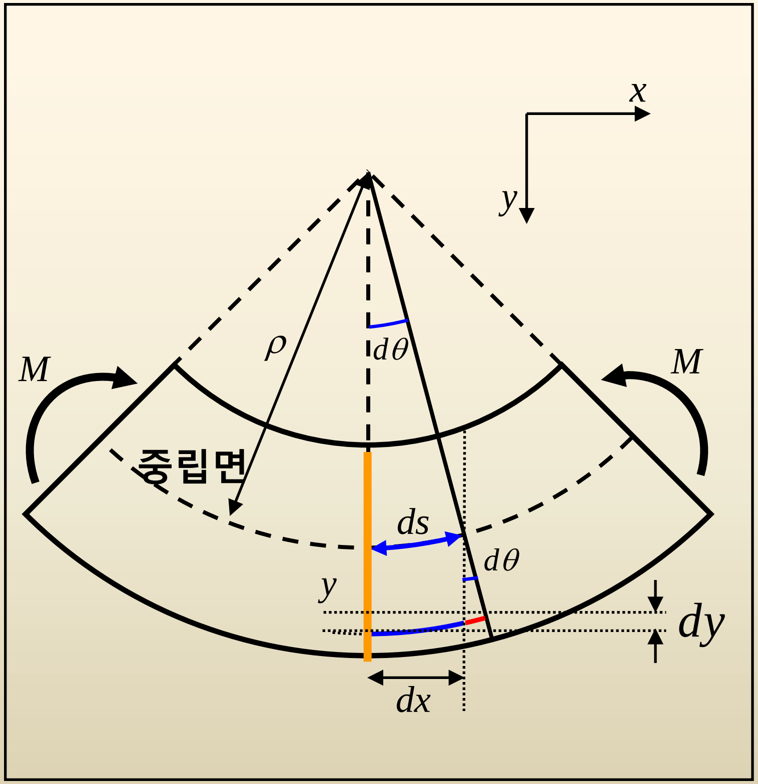
⑦ **Assume that**

$$ds \approx dx,\ \theta \approx \tan(\theta) = \frac{dy}{dx}$$



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Linear Mathematical Model → Analytic Solution

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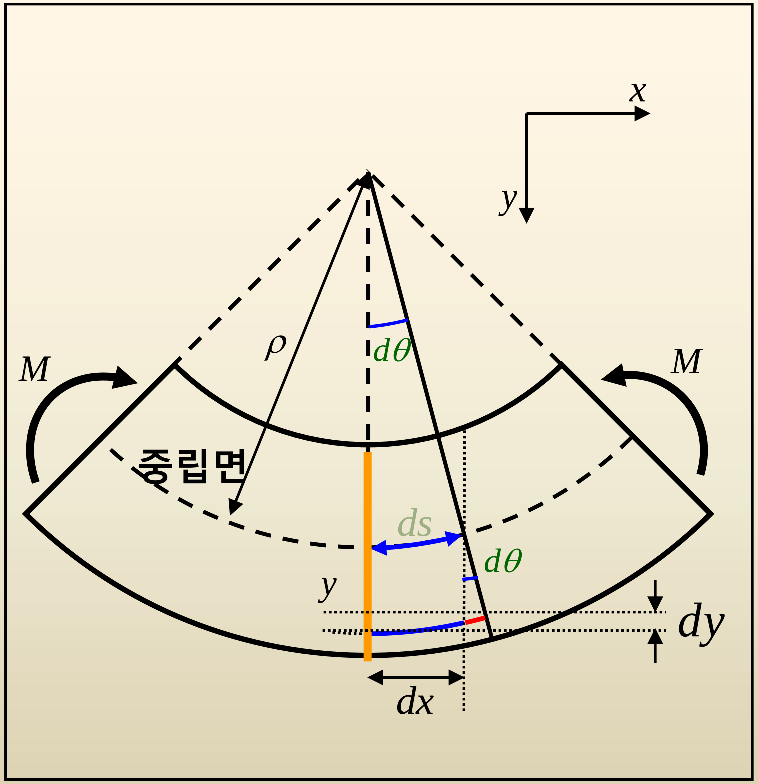

Nonlinearity

Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

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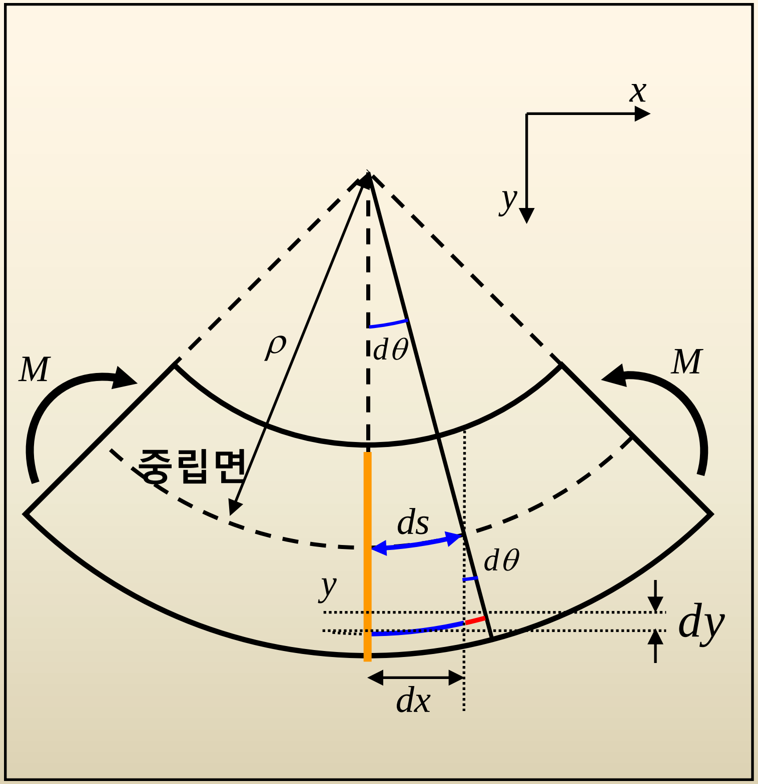

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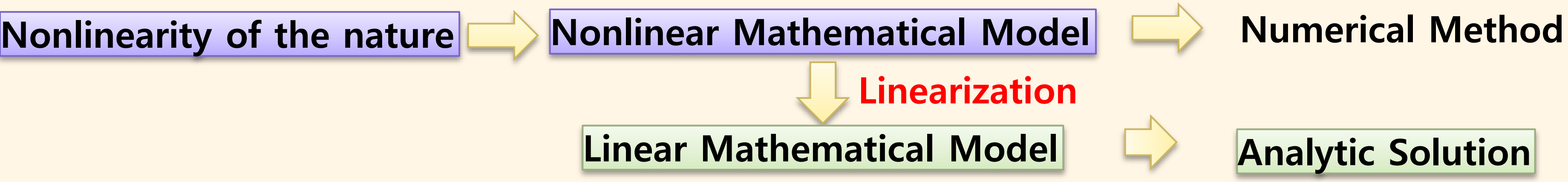

Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ Analytic Solution

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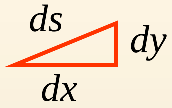
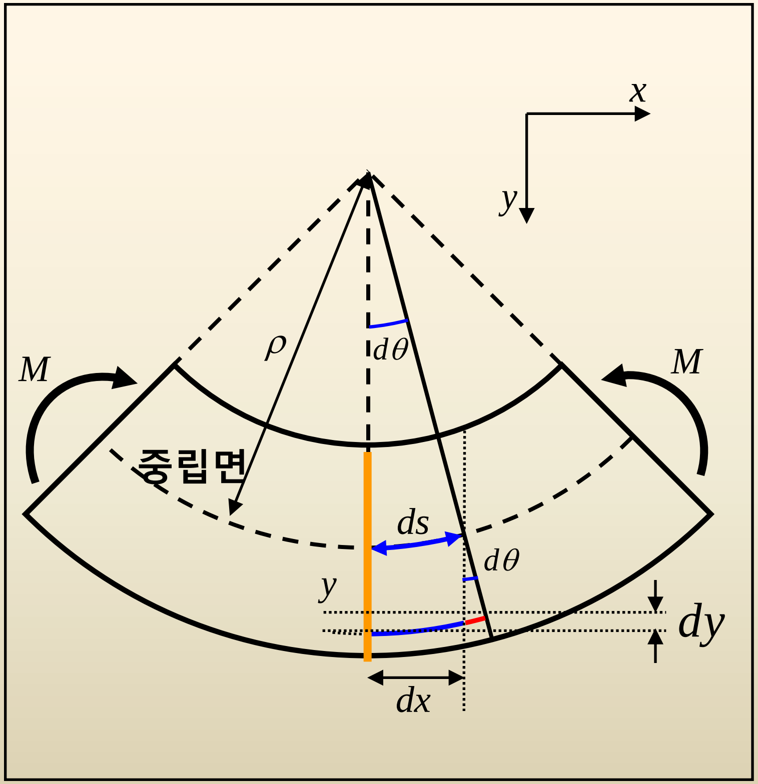


$$ds^2 = dx^2 + dy^2 \rightarrow ds = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$



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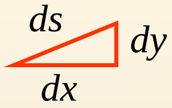

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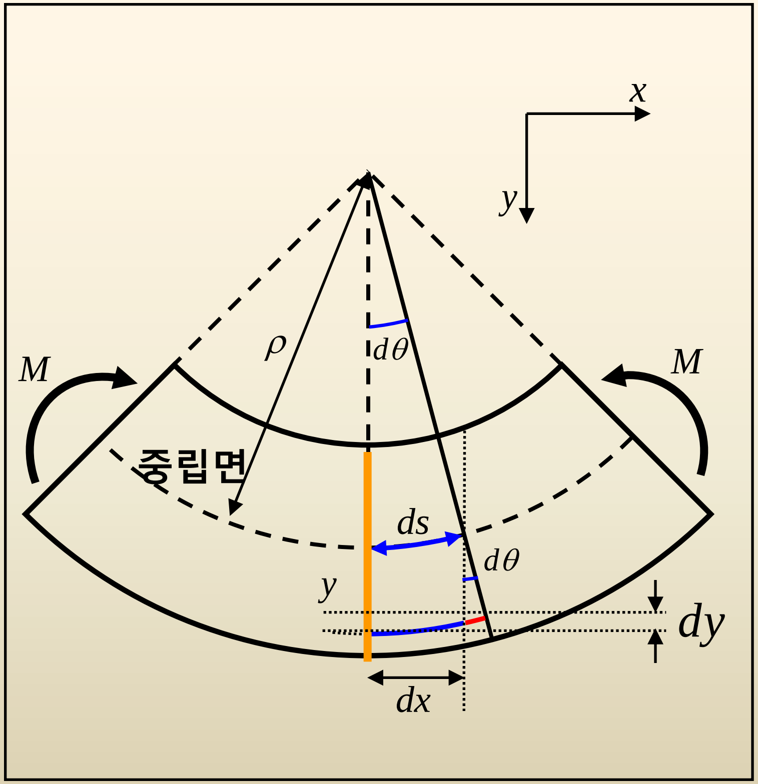

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

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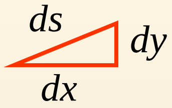

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$$f(z) = \sqrt{1+z}$$

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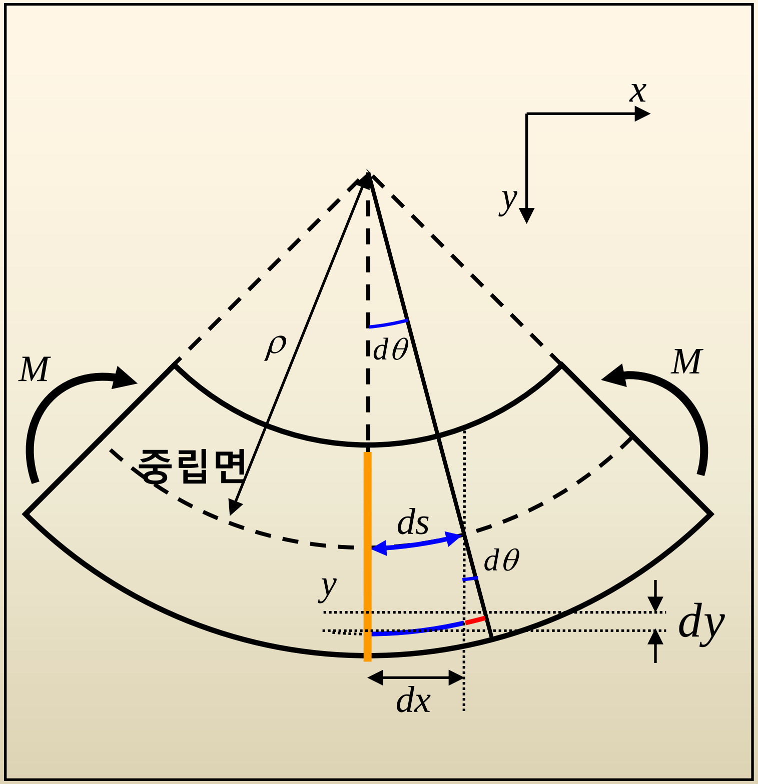

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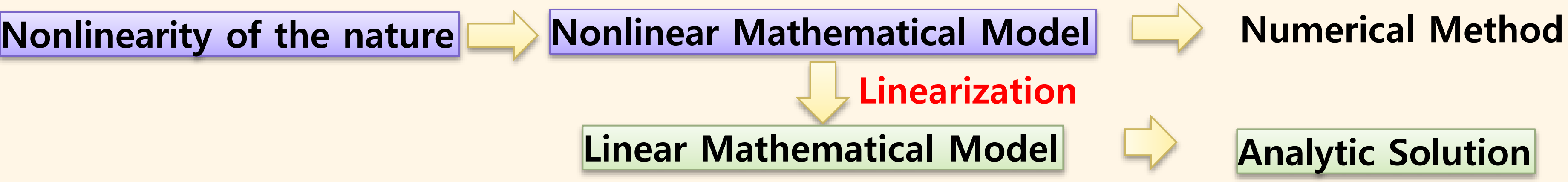

Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

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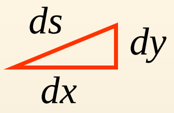

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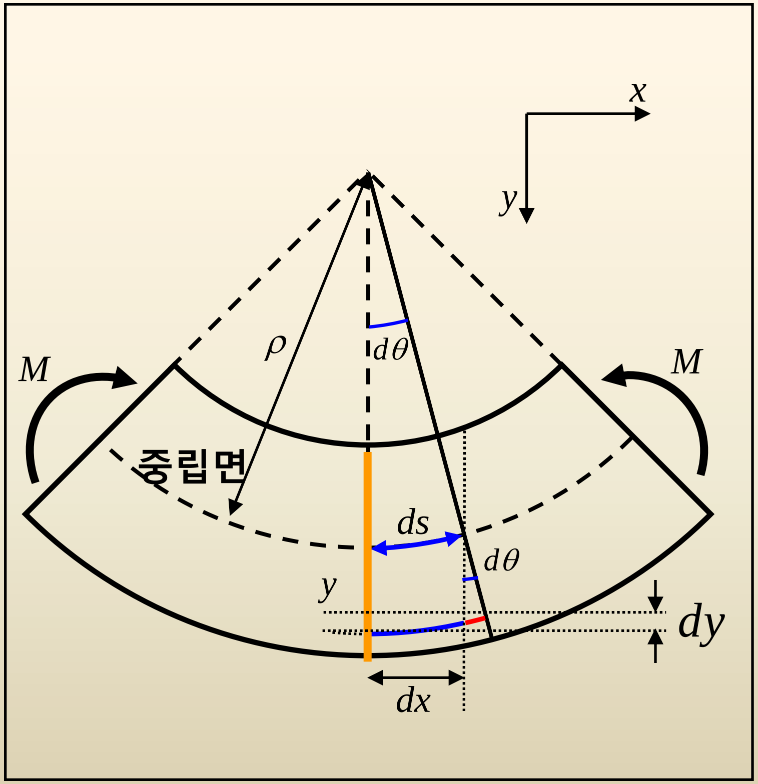

Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

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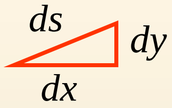

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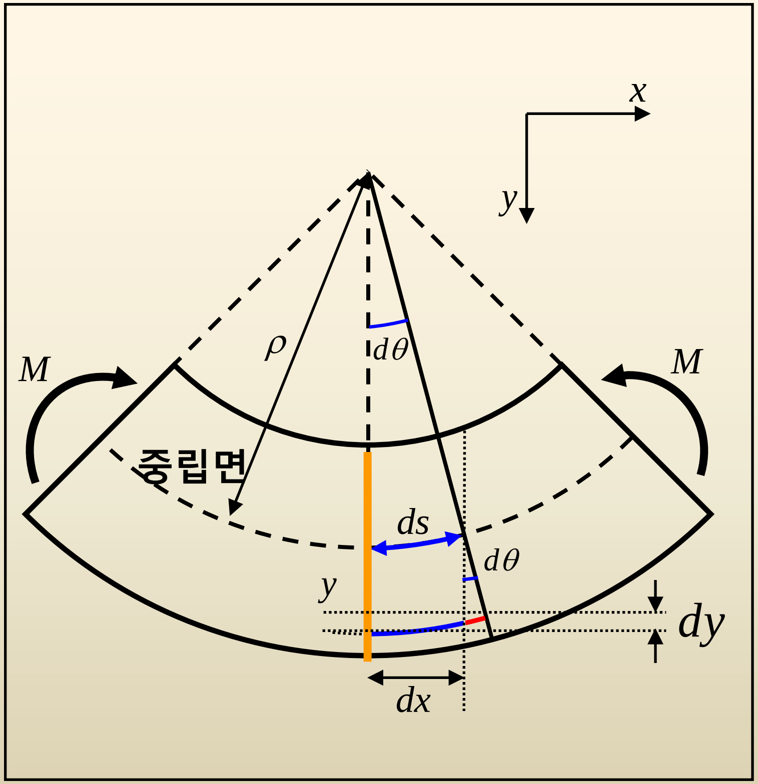

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Linearization

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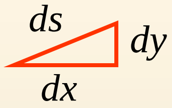

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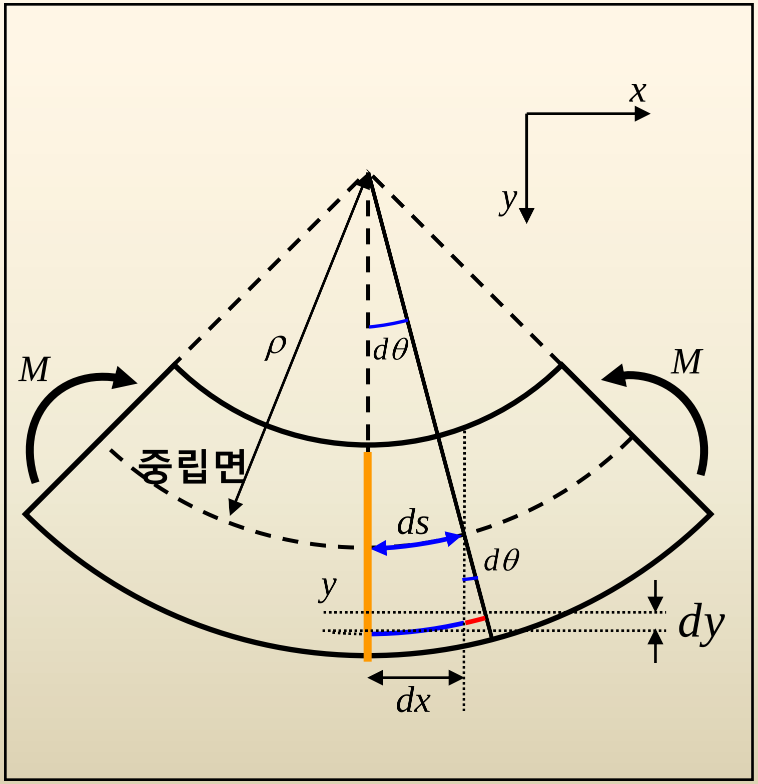
$$f''(0) = -\frac{1}{4}(1+z)^{-\frac{3}{2}}\Big|_{z=0} = -\frac{1}{4}$$

$$\boxed{\therefore f(z) = 1 + \frac{1}{2}z + \frac{1}{2}\left(-\frac{1}{4}\right)z^2 + \cdots}$$



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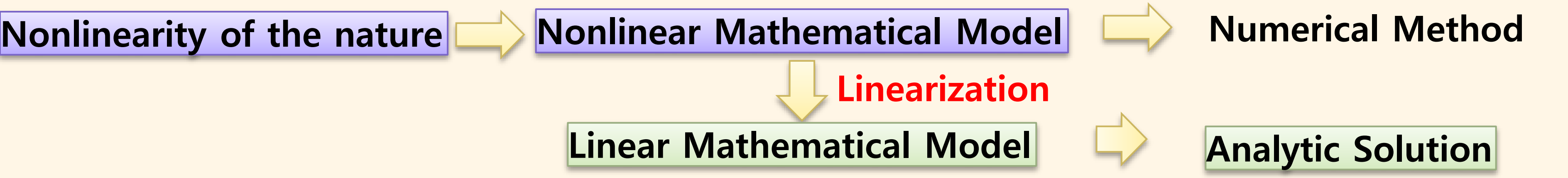

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → Analytic Solution

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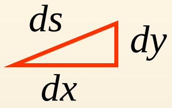

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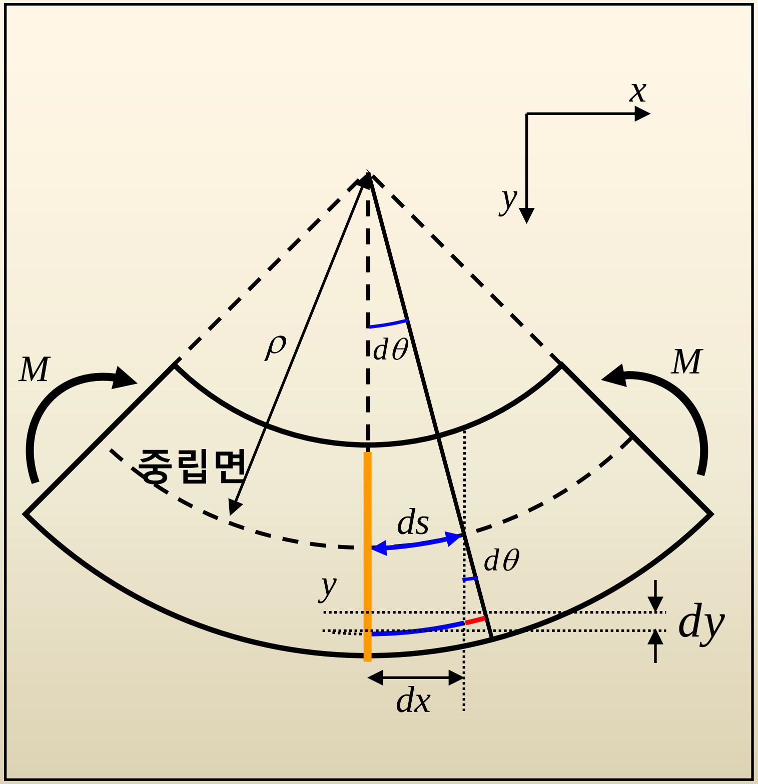
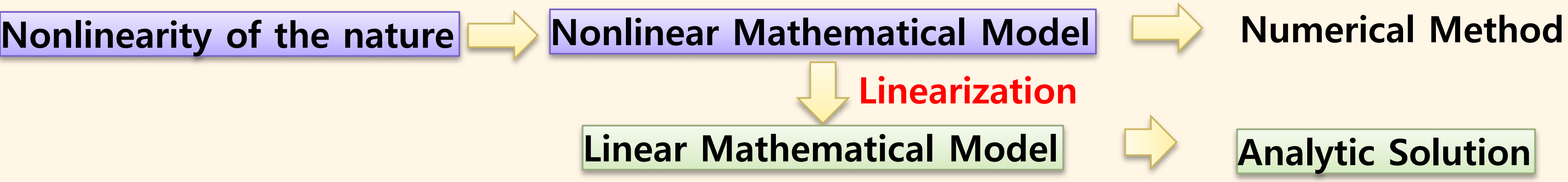


Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) 탄성선의 미분 방정식

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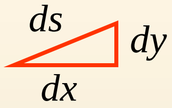

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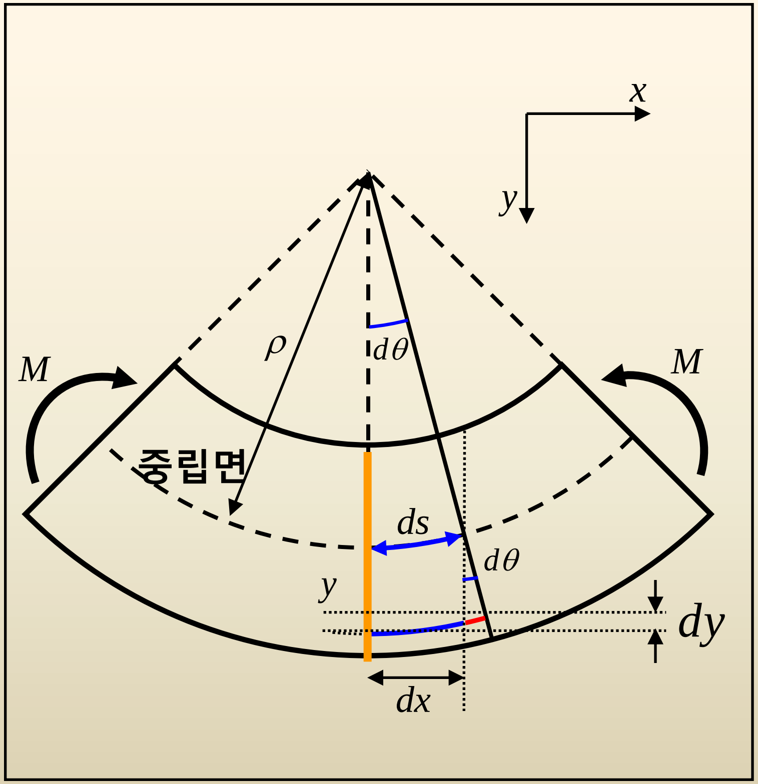

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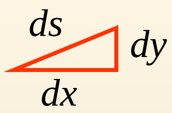

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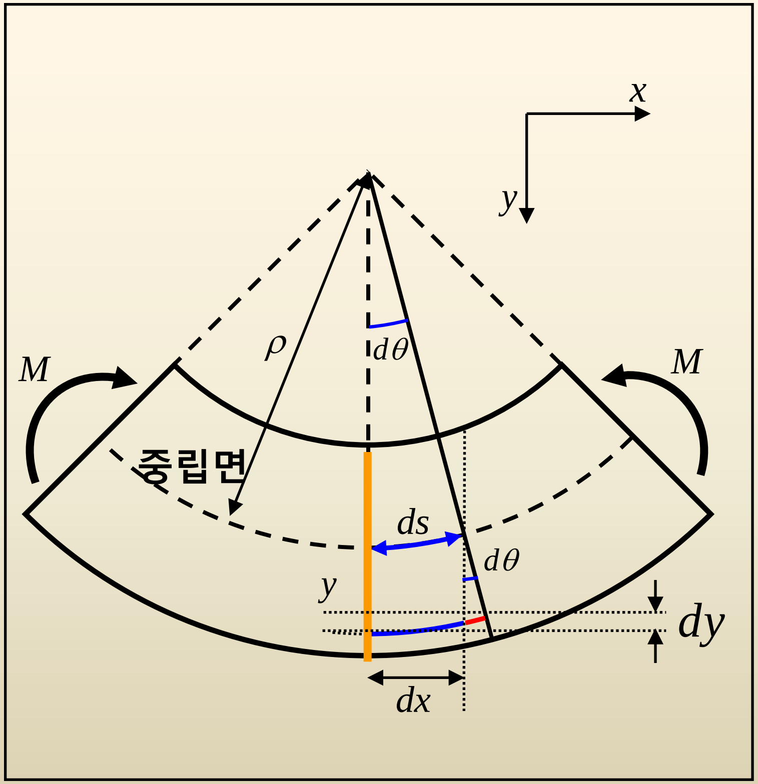

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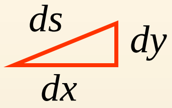

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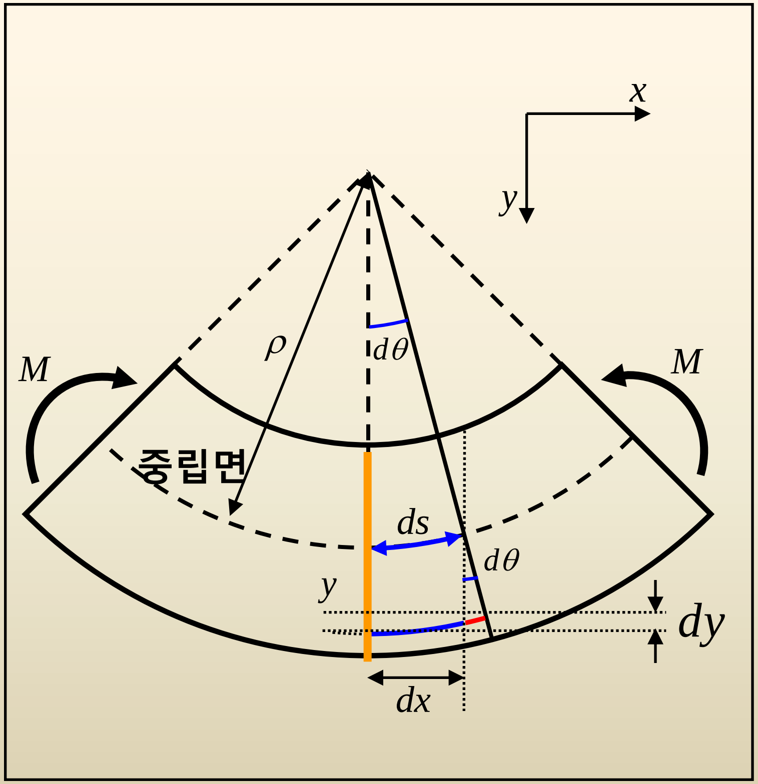
$$\therefore ds \approx dx$$



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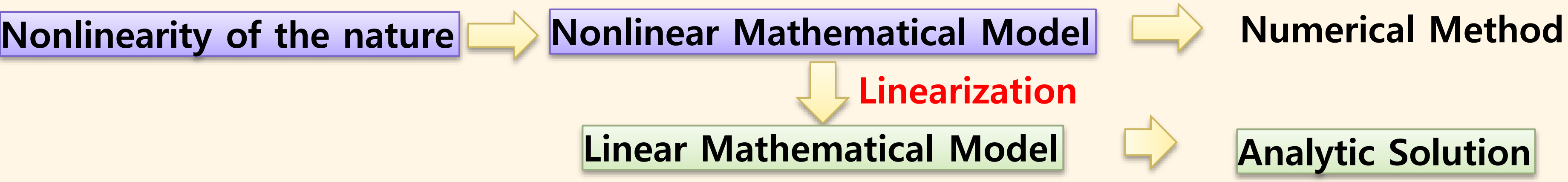

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

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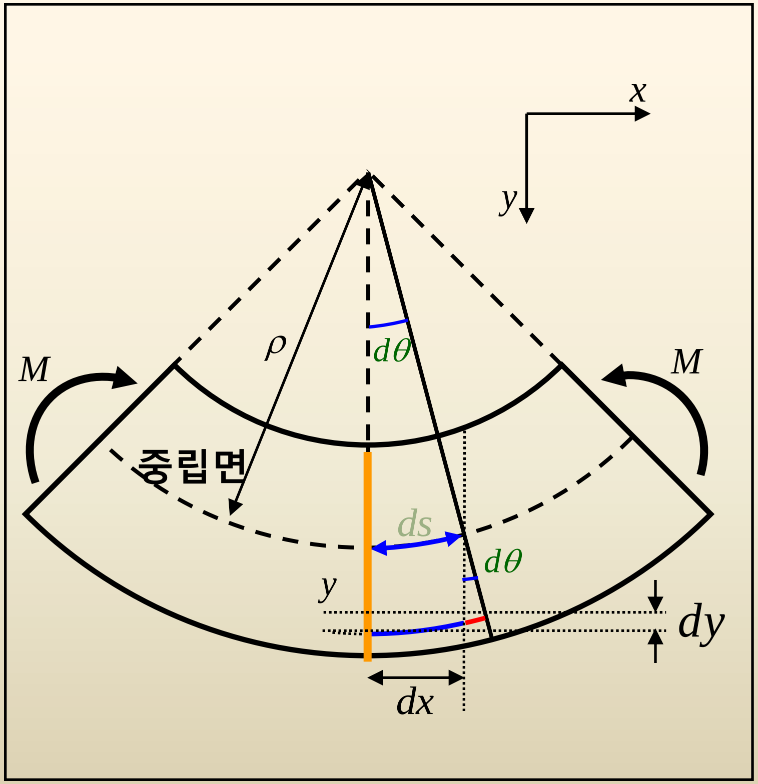

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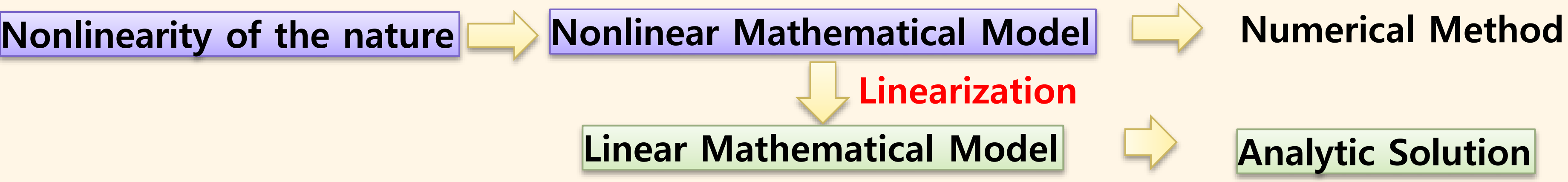

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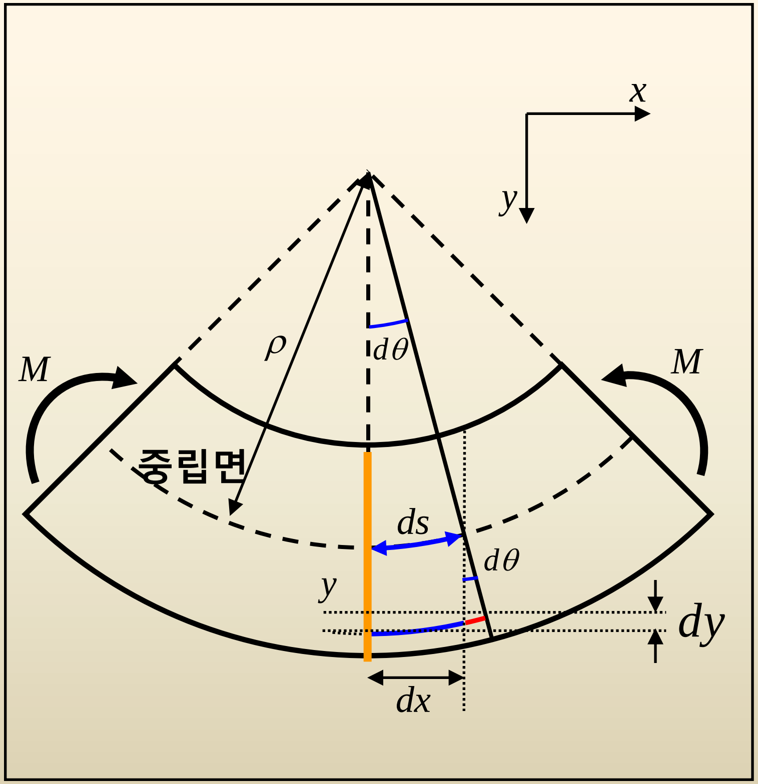

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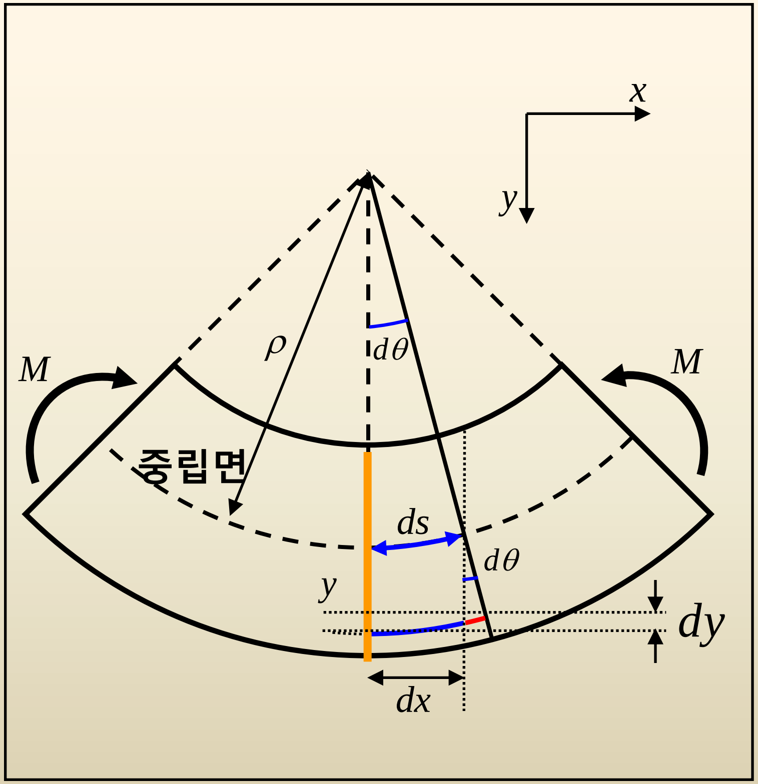

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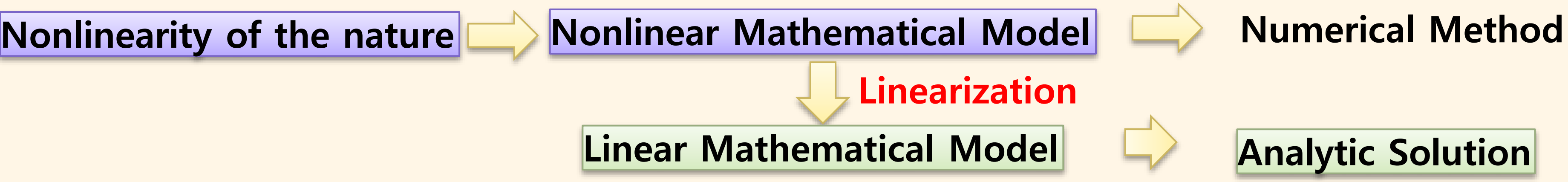

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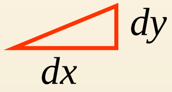
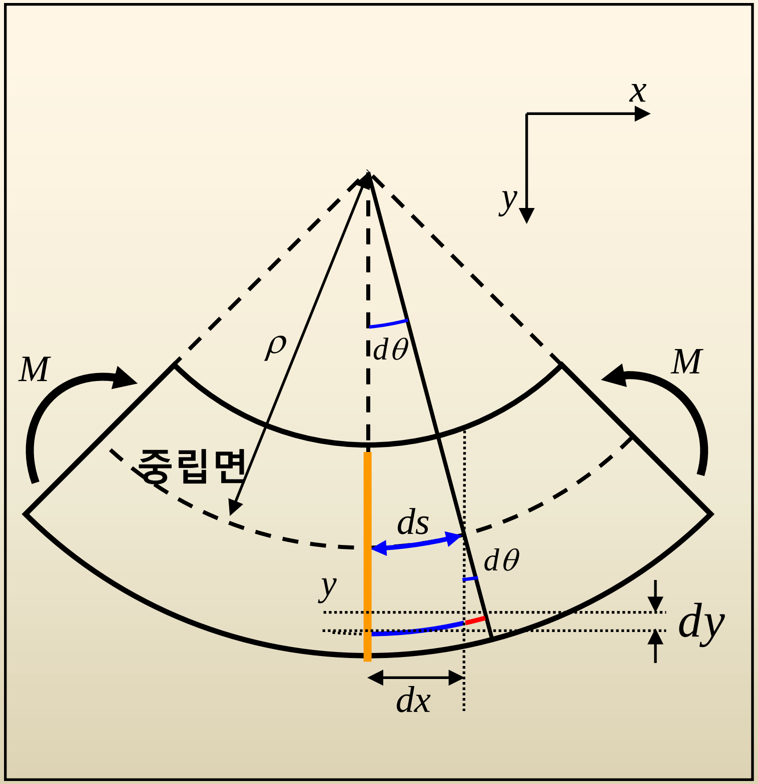


$$\boxed{\tan\theta = \theta} + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \cdots$$



Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

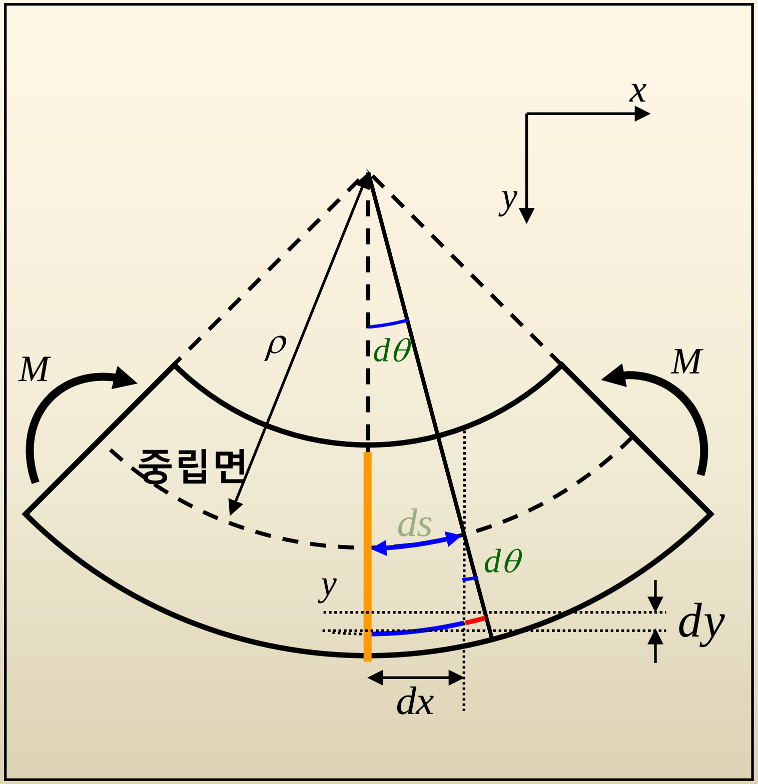
Ex) 탄성선의 미분 방정식

$$\rho \cdot d\theta = ds \Rightarrow \frac{d\theta}{ds} = \frac{1}{\rho}$$

$$dF = \sigma dA = E \cdot \frac{y}{\rho} dA$$

$$dM = -y\sigma dA \Rightarrow \frac{M}{EI} = -\frac{1}{\rho}$$

$$\therefore \frac{d\theta}{ds} = -\frac{M}{EI}$$



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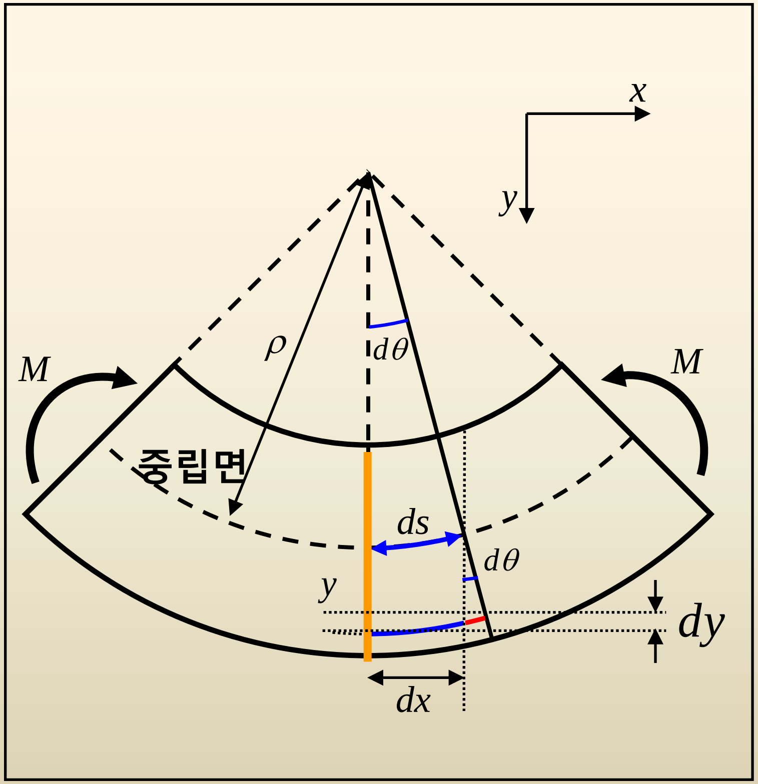

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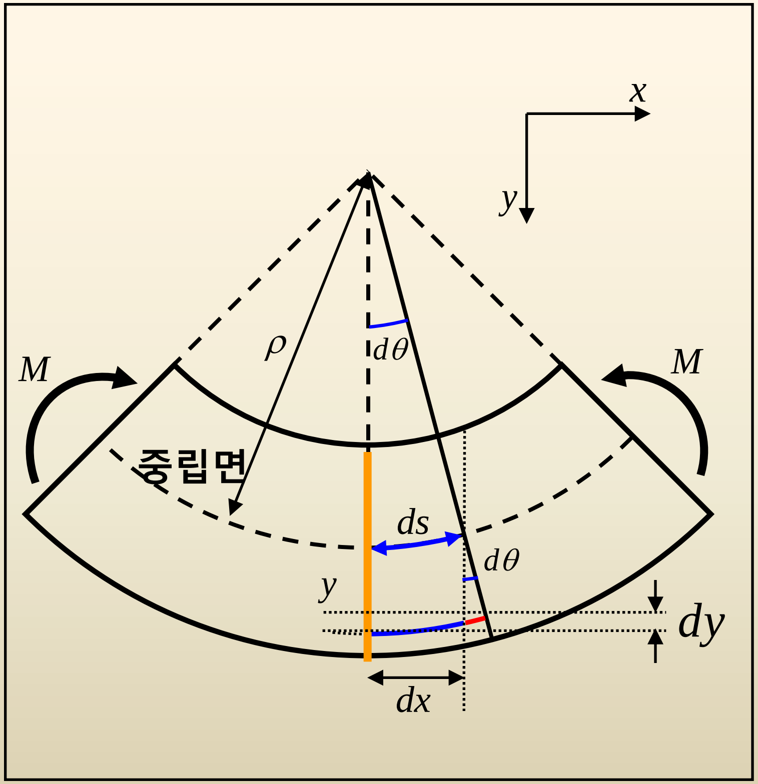

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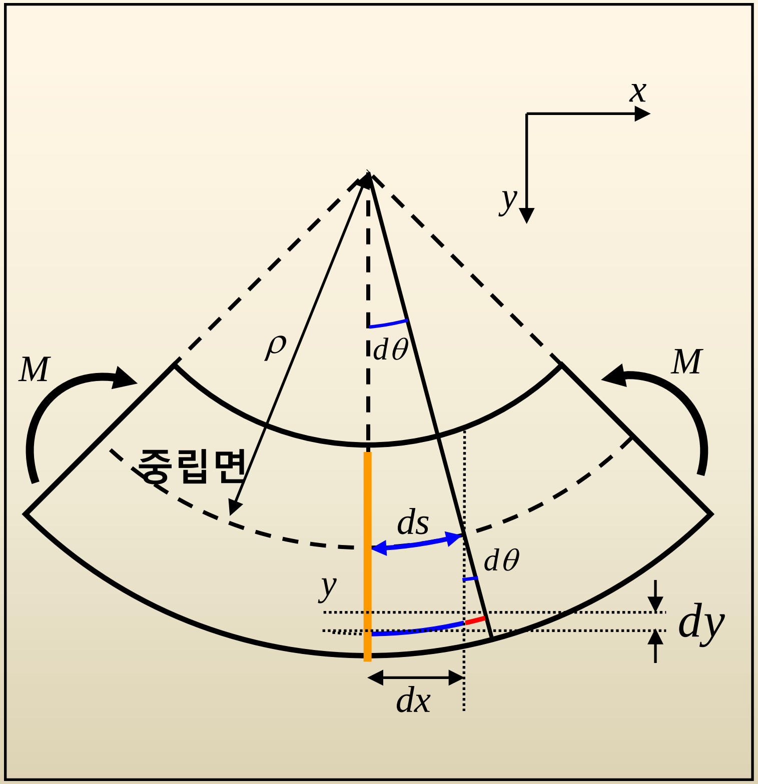

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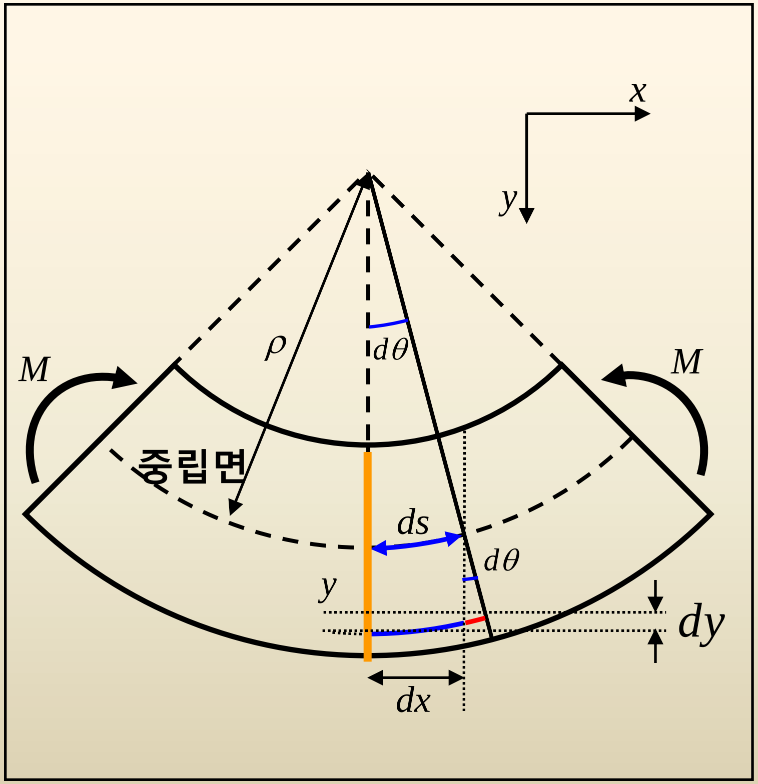

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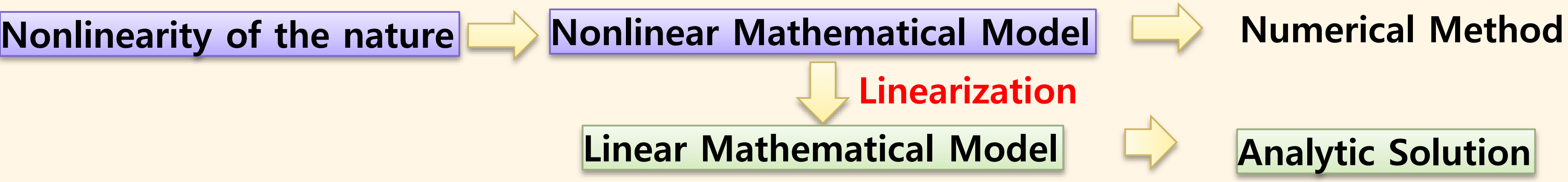

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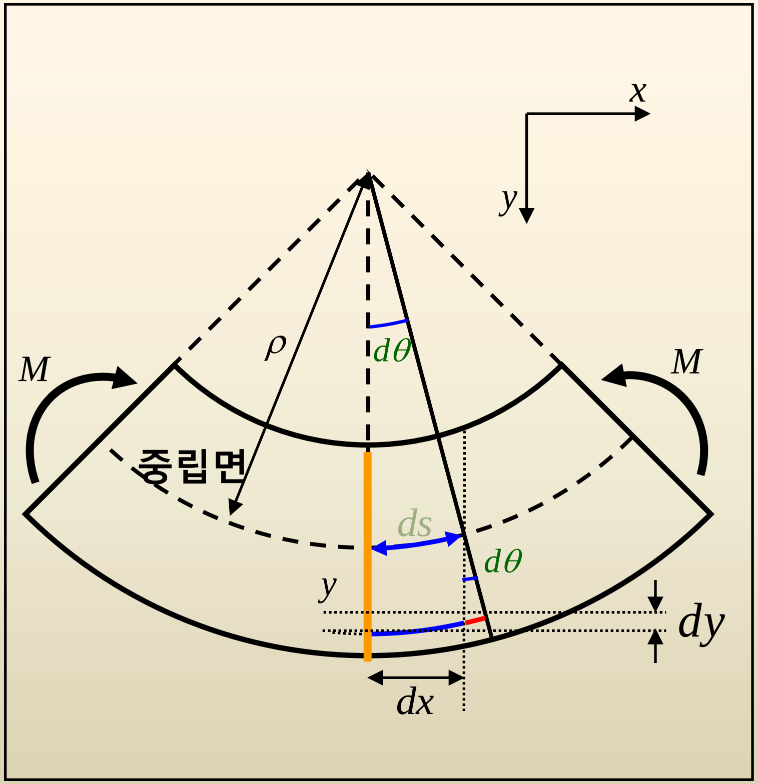

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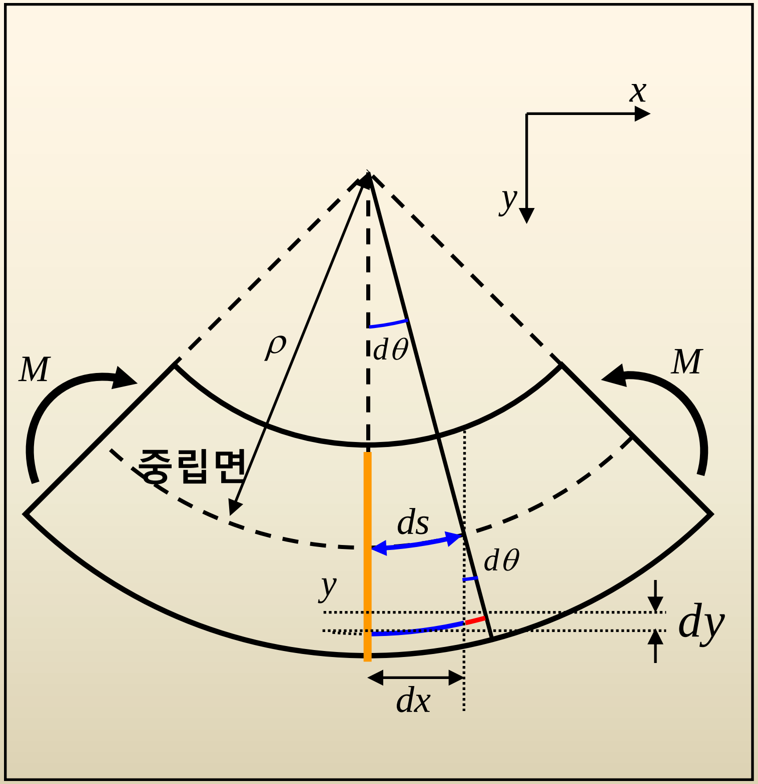

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Linear Mathematical Model → Analytic Solution

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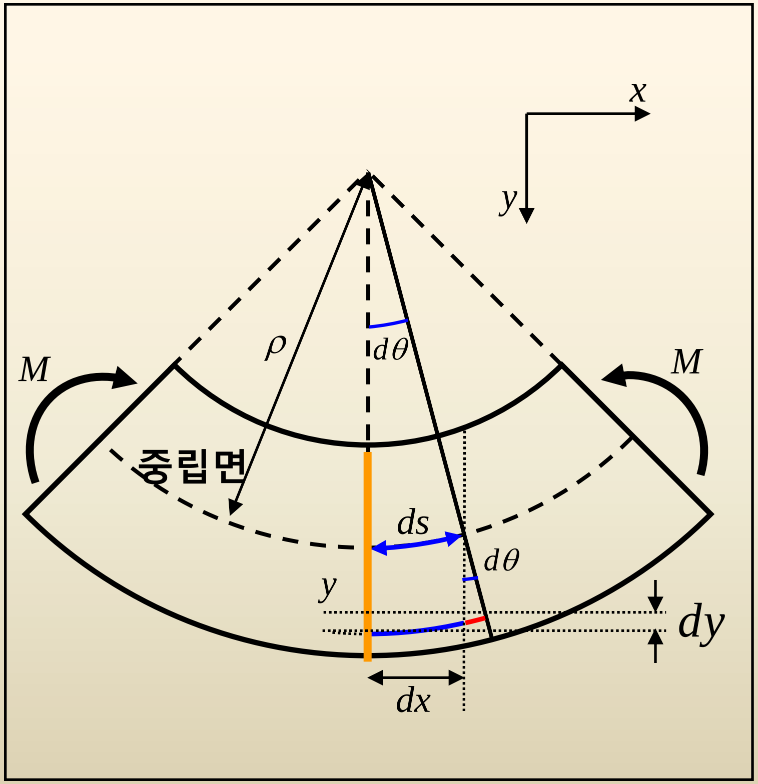

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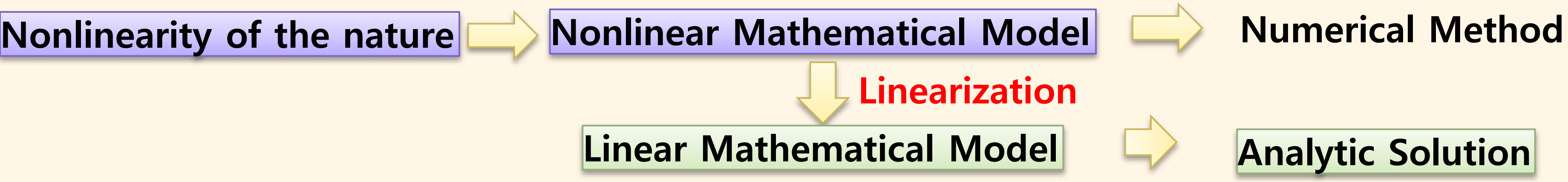


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

$$\boxed{\frac{d^2y}{dx^2}=-\frac{M}{EI}}$$

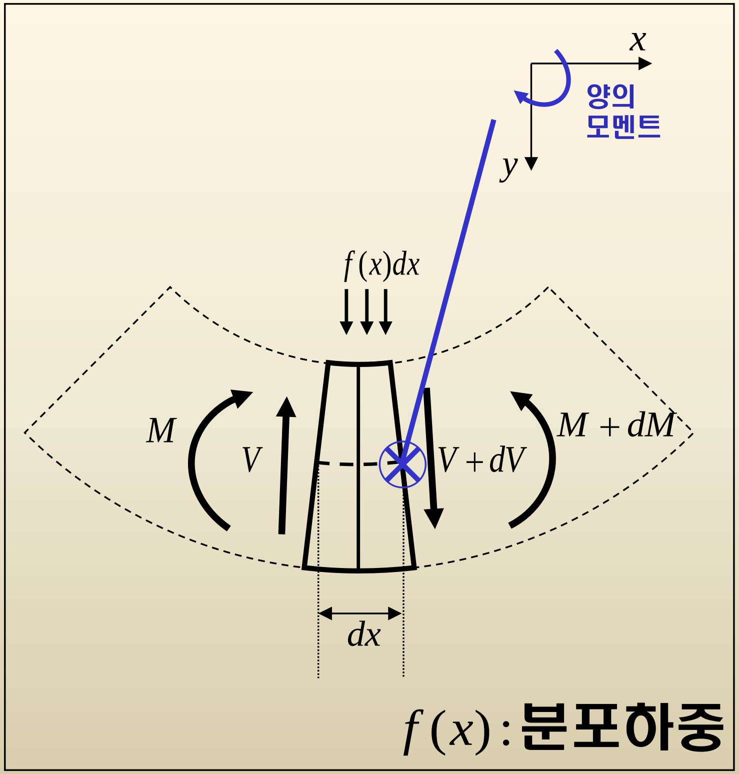
Ex) **분포하중 을 고려한 탄성선의 미분 방정식**

y 방향의 합력 :

$$(V+dV)-V+f(x)dx=0$$

$$\Rightarrow dV+f(x)dx=0$$

$$\Rightarrow \frac{dV}{dx}=-f(x)$$



모멘트 (파란색 축 기준):

$$-(M+dM)+M+Vdx-\cancel{f(x)dx\cdot\frac{1}{2}dx}=0$$

$$\Rightarrow dM-Vdx=0\quad ,\left(\because (dx)^2\approx 0\right)$$

$$\Rightarrow \frac{dM}{dx}=V(x)$$

Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) 분포하중 을 고려한 탄성선의 미분 방정식

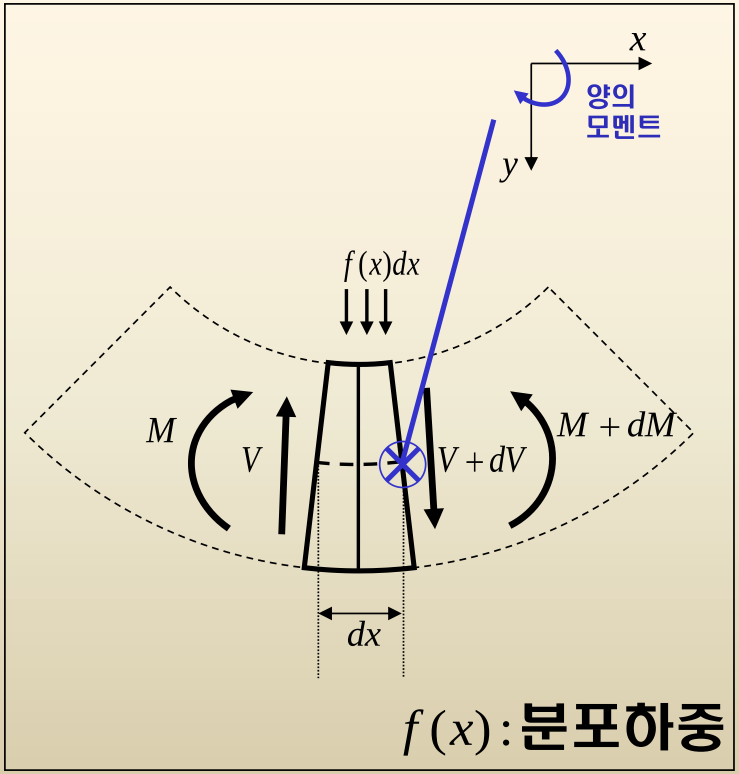
$$\frac{dV}{dx} = -f(x) \qquad \frac{dM}{dx} = V(x)$$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$: 탄성선의 미분방정식

$$\frac{d^3y}{dx^3} = -\frac{1}{EI}\cdot\frac{dM}{dx} = -\frac{1}{EI}\cdot V(x)$$

$$\frac{d^4y}{dx^4} = -\frac{1}{EI}\cdot\frac{dV}{dx} = \frac{1}{EI}\cdot f(x)$$

$$\boxed{\therefore EI\frac{d^4y}{dx^4} = f(x)}$$



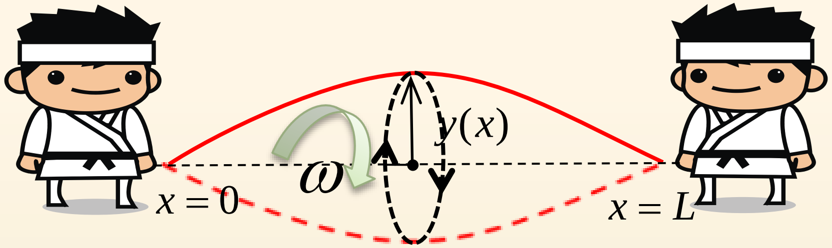
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ **Numerical Method**

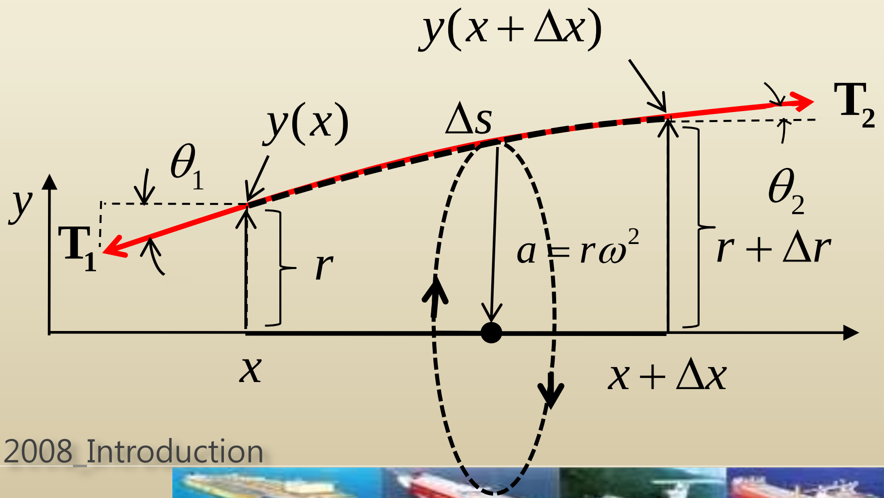
⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Rotating String



ρ : string density
ω : string angular velocity
T : magnitude of tension



$$\sum F_x = \mathbf{T}_1\cos\theta_1 - \mathbf{T}_2\cos\theta_2 = 0$$

$$\mathbf{T}_1\cos\theta_1 = \mathbf{T}_2\cos\theta_2 = T$$

$$\sum F_y = \mathbf{T}_2\sin\theta_2 - \mathbf{T}_1\sin\theta_1$$

$$= T\frac{\sin\theta_2}{\cos\theta_2} - T\frac{\sin\theta_1}{\cos\theta_1} \because \mathbf{T}_1\cos\theta_1 = \mathbf{T}_2\cos\theta_2 = T$$

$$= T\tan\theta_2 - T\tan\theta_1$$

$$= T[y'(x+\Delta x) - y'(x)]$$

$$\because \tan\theta_1 = \left.\frac{dy}{dx}\right|_x, \tan\theta_2 = \left.\frac{dy}{dx}\right|_{x+\Delta x}$$

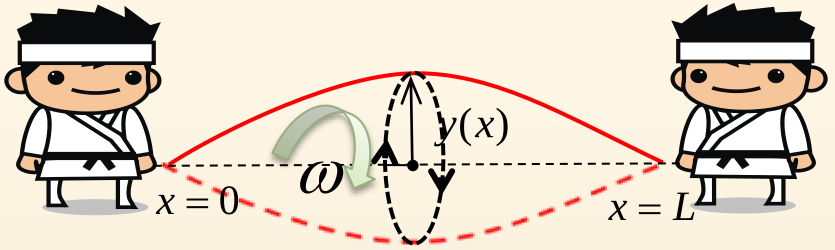
Nonlinearity

Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

Linearization ⇩

Linear Mathematical Model ⇨ Analytic Solution

Ex) Rotating String



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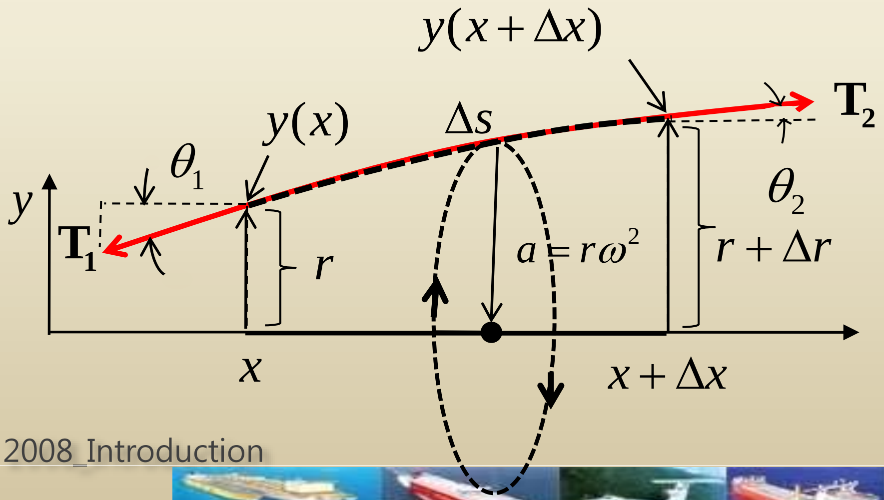


$$\sum F_y = T[y'(x+\Delta x) - y'(x)]$$

assum.: $\Delta x \ll 1$ linearization

Acceleration point in the direction opposite to the positive y direction

Mass: $m = \rho\Delta s \approx \rho\Delta x$

Centripetal acceleration: $a = \ominus r\omega^2$

When Δx is small,

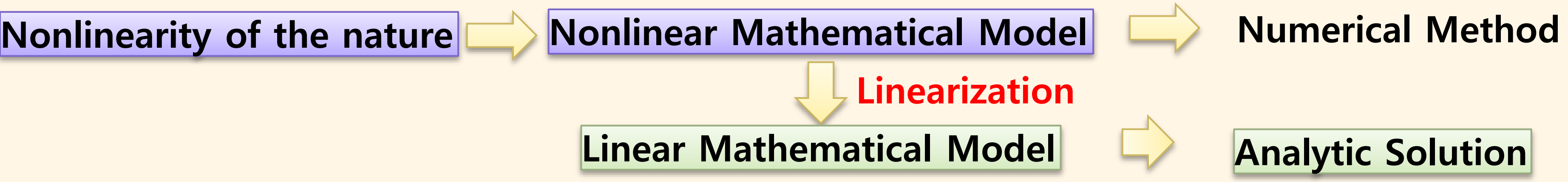
$$\boxed{r+\Delta r = r} + r'\Delta x + \cdots = \boxed{y} + y'\Delta x + \cdots$$

linearization

$$r+\Delta r \approx r = y\ , a = -r\omega^2 = -y\omega^2$$

$$\therefore \sum F_y = ma \approx -(\rho\Delta x)y\omega^2$$

Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

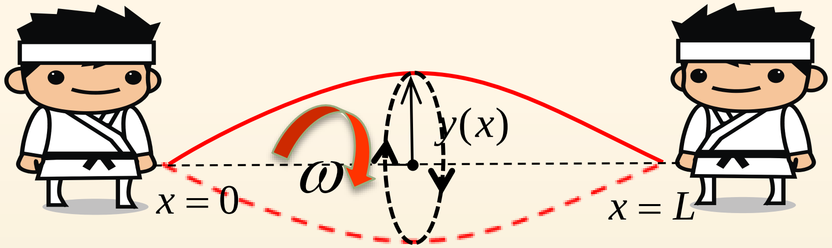
Numerical Method

Linearization

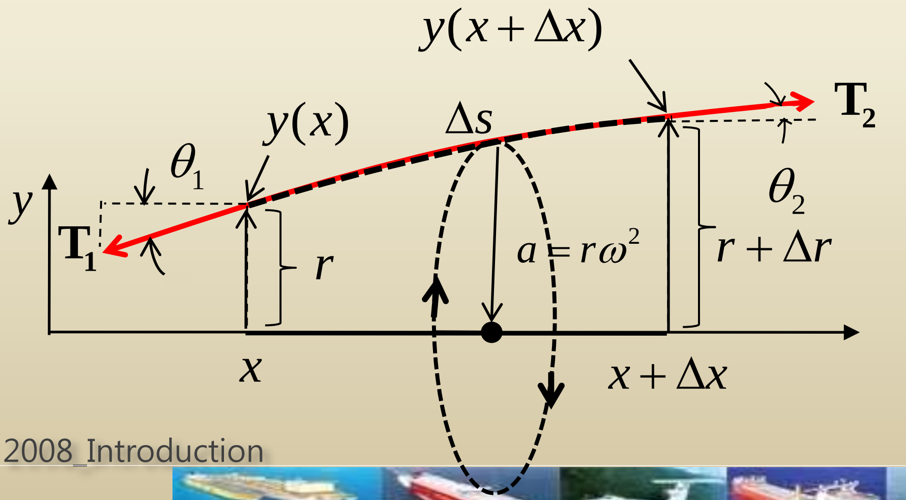
Linear Mathematical Model

Analytic Solution

Ex) Rotating String



ρ : string density
ω : string angular velocity
T : magnitude of tension



$$\sum F_y = T[y'(x+\Delta x) - y'(x)]$$

$$\sum F_y = ma \approx -(\rho\Delta x)y\omega^2$$

$$\therefore T[y'(x+\Delta x) - y'(x)] = -(\rho\Delta x)y\omega^2$$

$$T\frac{y'(x+\Delta x) - y'(x)}{\Delta x} + \rho\omega^2 y = 0$$

$$\frac{y'(x+\Delta x) - y'(x)}{\Delta x} \approx \frac{d^2 y}{dx^2}$$

$$\therefore T\frac{d^2 y}{dx^2} + \rho\omega^2 y = 0$$

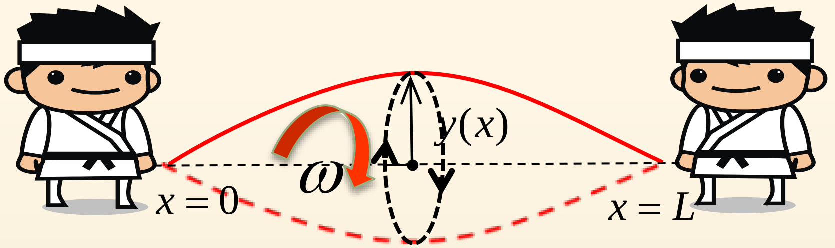
Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ Linearization

Linear Mathematical Model → Analytic Solution

Ex) Rotating String



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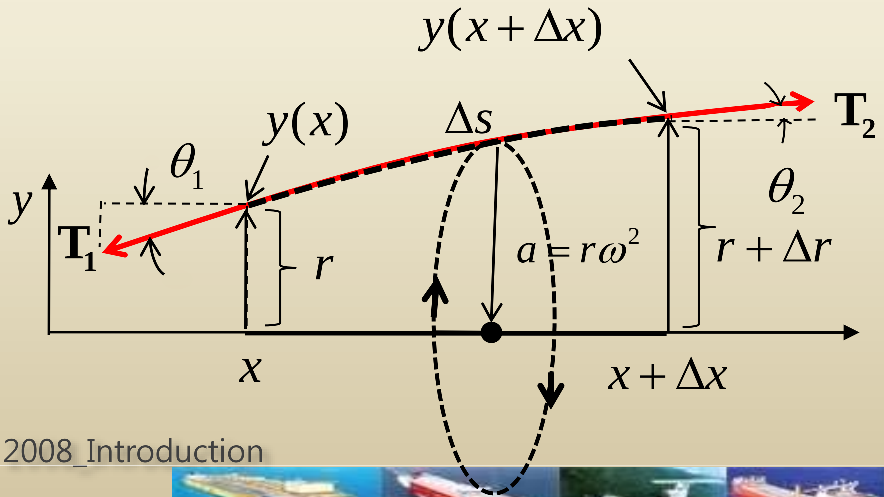

$$\sum F_y = T[y'(x+\Delta x) - y'(x)] \ , ma \approx -(\rho\Delta x)y\omega^2$$

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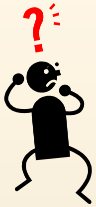


What if ..

$$\boxed{r+\Delta r = r + r'\Delta x} + \cdots = \boxed{y + y'\Delta x} + \cdots$$

$$then, a = -(r+\Delta r)\omega^2 = -(y+y'\Delta x)\omega^2$$

$$\therefore T[y'(x+\Delta x) - y'(x)] = -(\rho\Delta x)(y+y'\Delta x)\omega^2$$

$$\Rightarrow T\frac{d^2y}{dx^2} + \rho\omega^2\underline{\Delta x y'} + \rho\omega^2 y = 0$$

Not a form of $\frac{dy}{dx}$ or polynomial of x

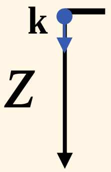
$\Delta x y' \approx 0 \because \Delta x \ll 1, \Delta s \approx \Delta x$ means y'(=dy/dx) is small too

$$\therefore T\frac{d^2y}{dx^2} + \rho\omega^2 y = 0$$

Spring/Mass Systems: Driven Motion

$$z = z(t)\ , \quad z'' = \frac{d^2 z}{dt^2}$$

①



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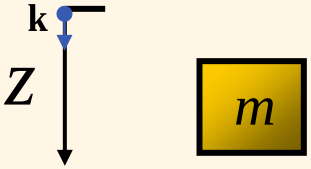

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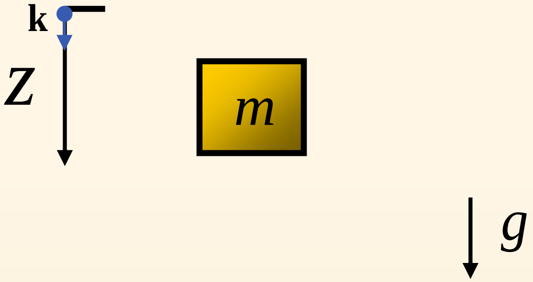

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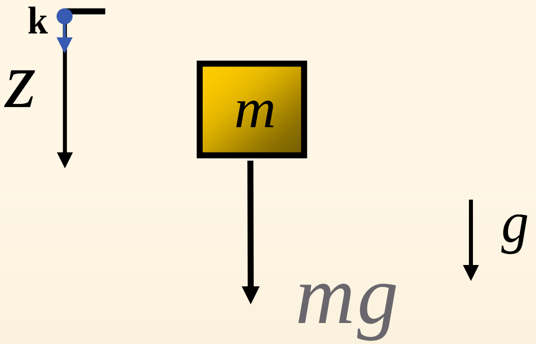

:Gravity force

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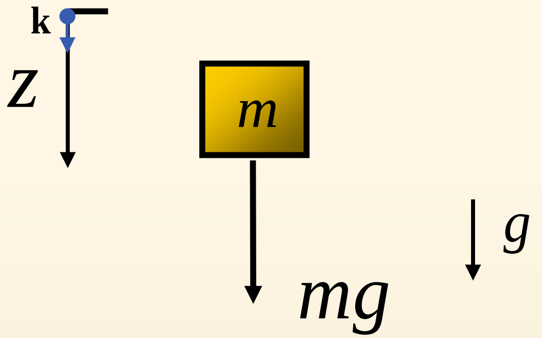

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By Newton's 2nd law,

$$m\mathbf{z}'' = \mathbf{F}$$

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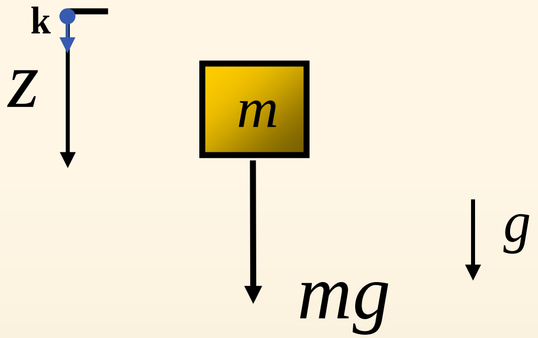

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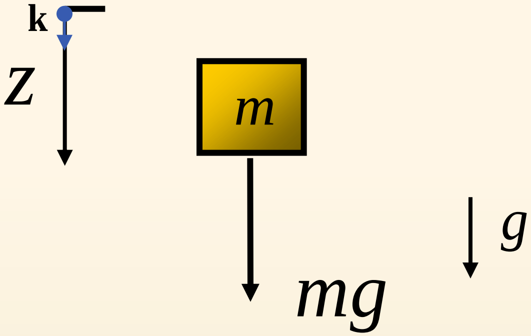

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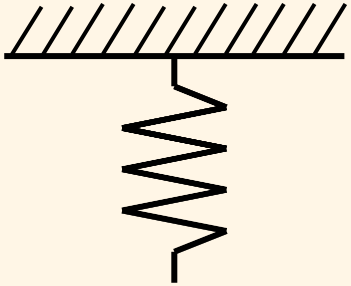

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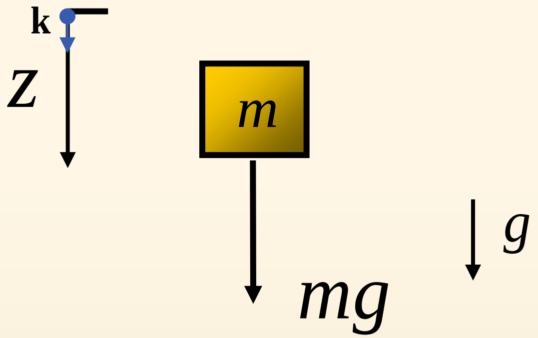

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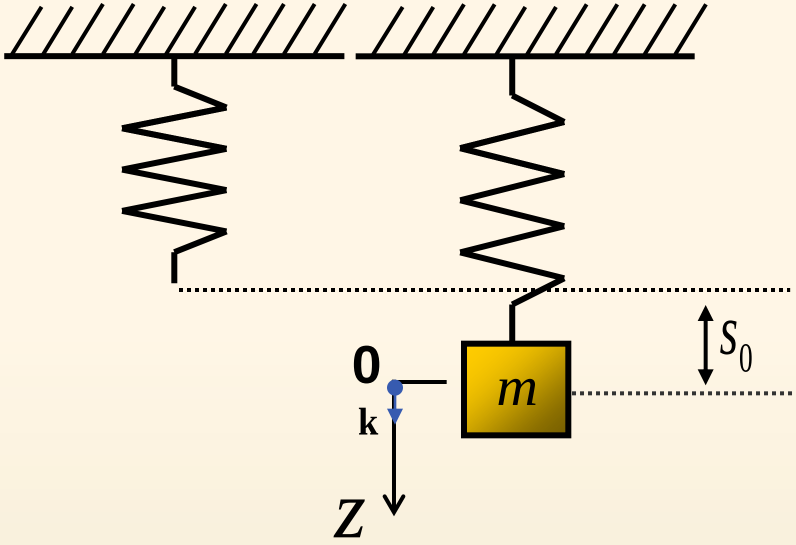

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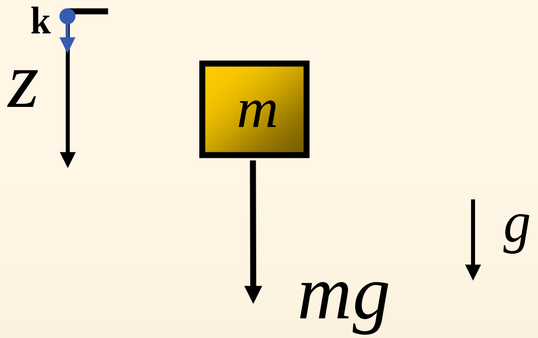

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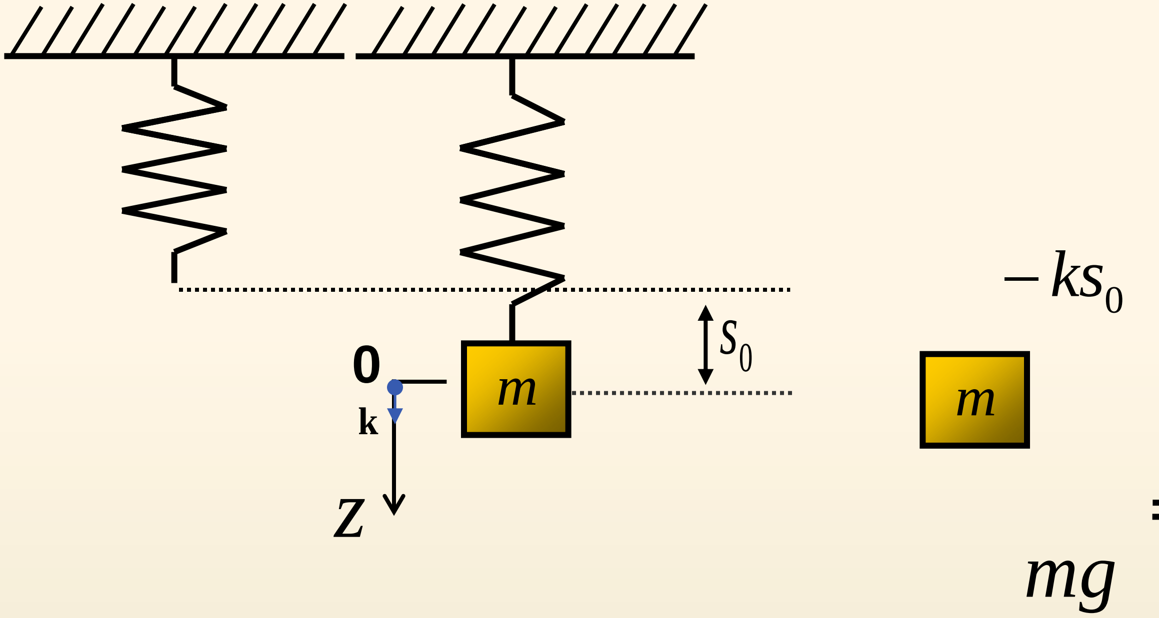

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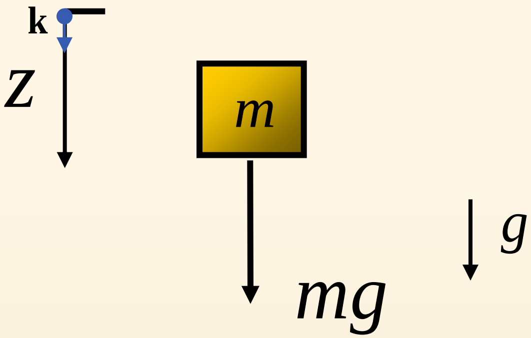

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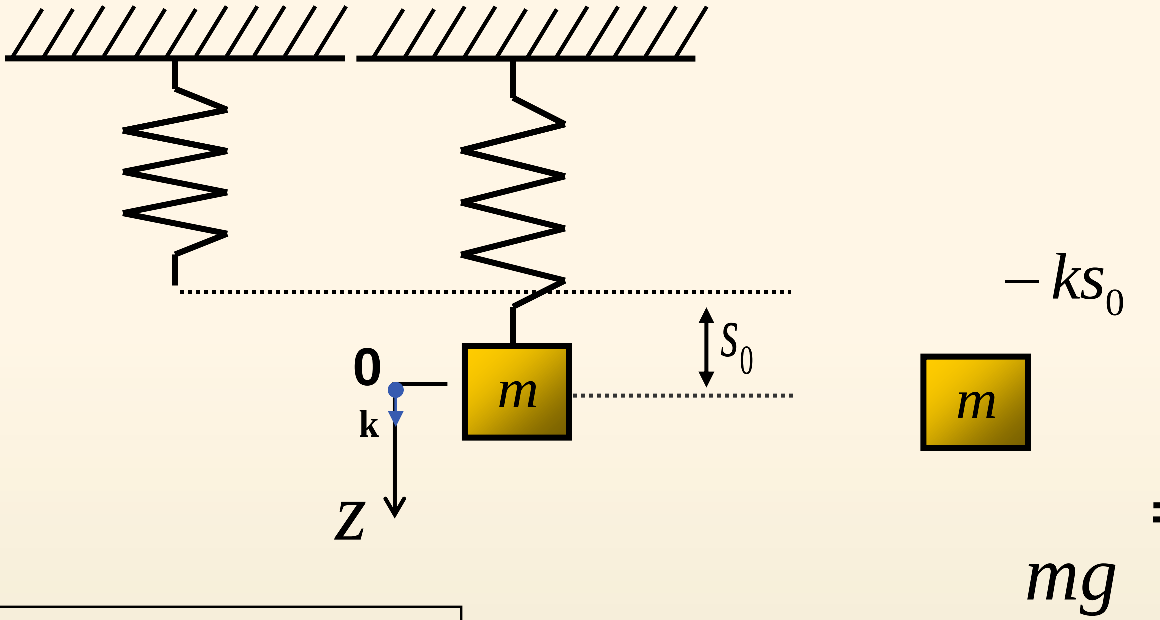

Nonlinearity of spring

$$\mathbf{F}(z) = -k\mathbf{z} - k_1\mathbf{z}^3$$

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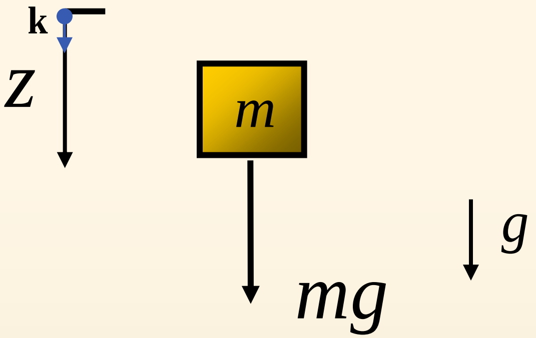

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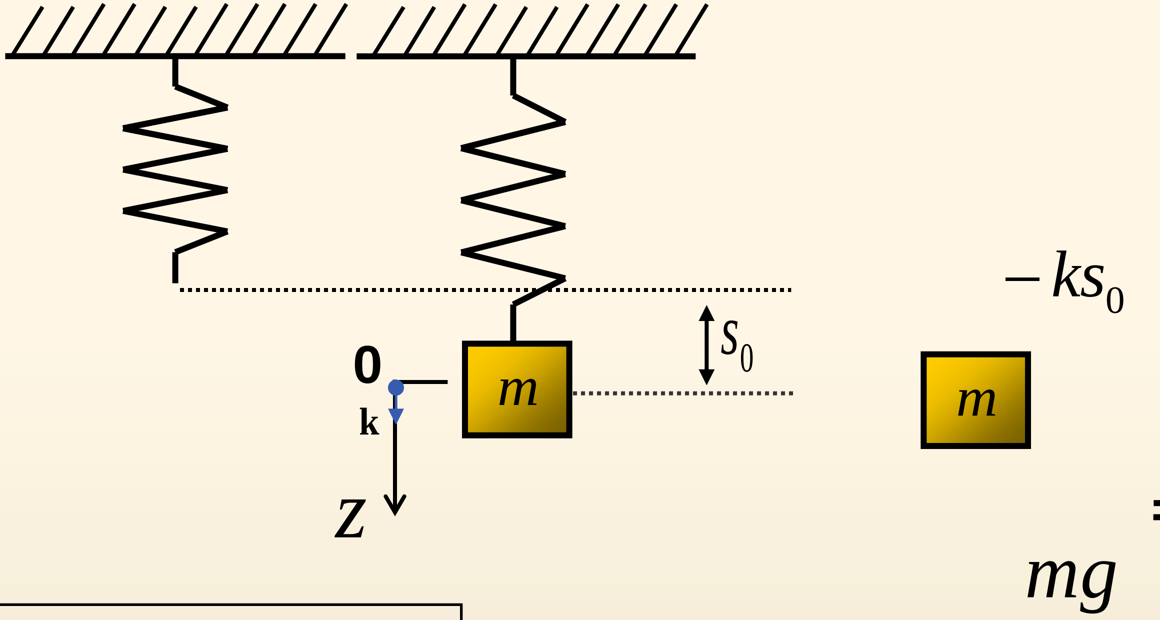

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linearize

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$$F \propto z$$
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k : spring constant

Spring/Mass Systems: Driven Motion

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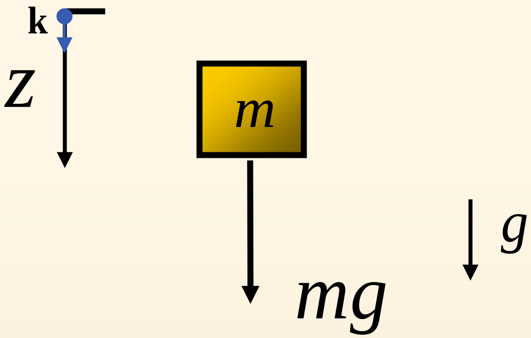

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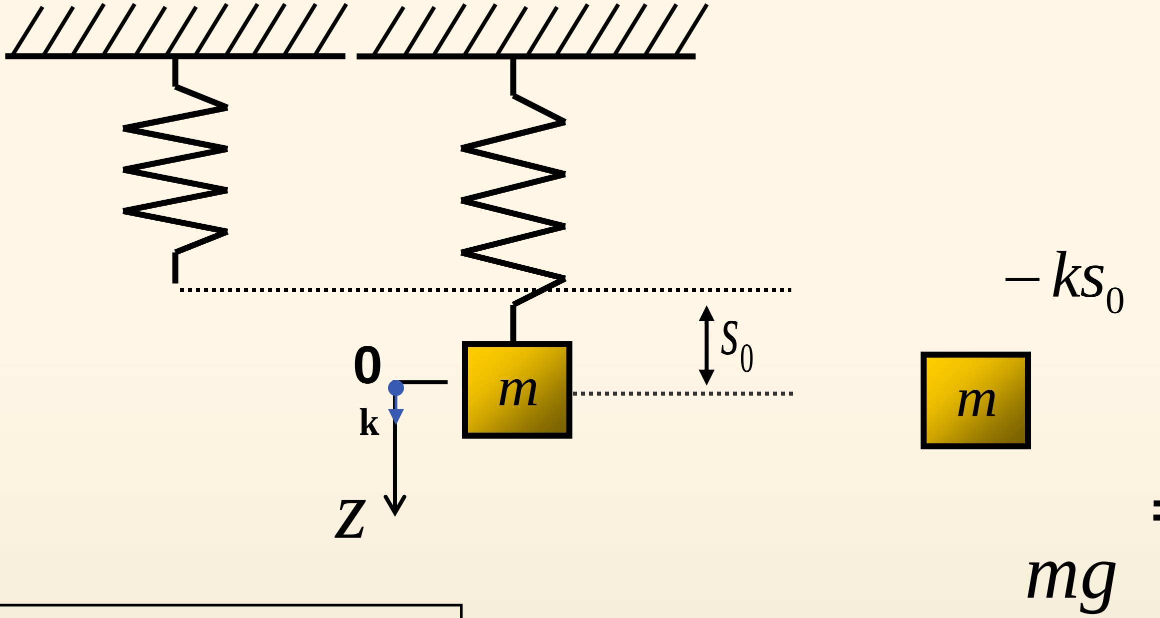

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○ **opposite to the direction of displacement**

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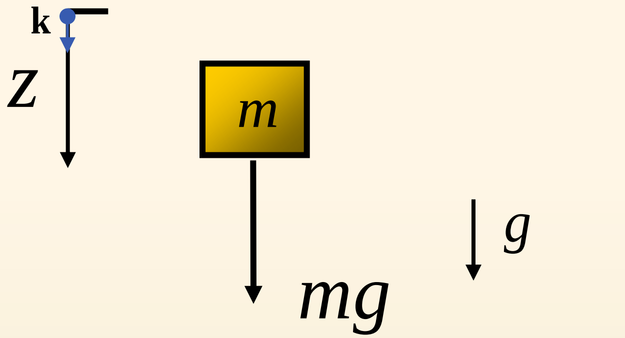

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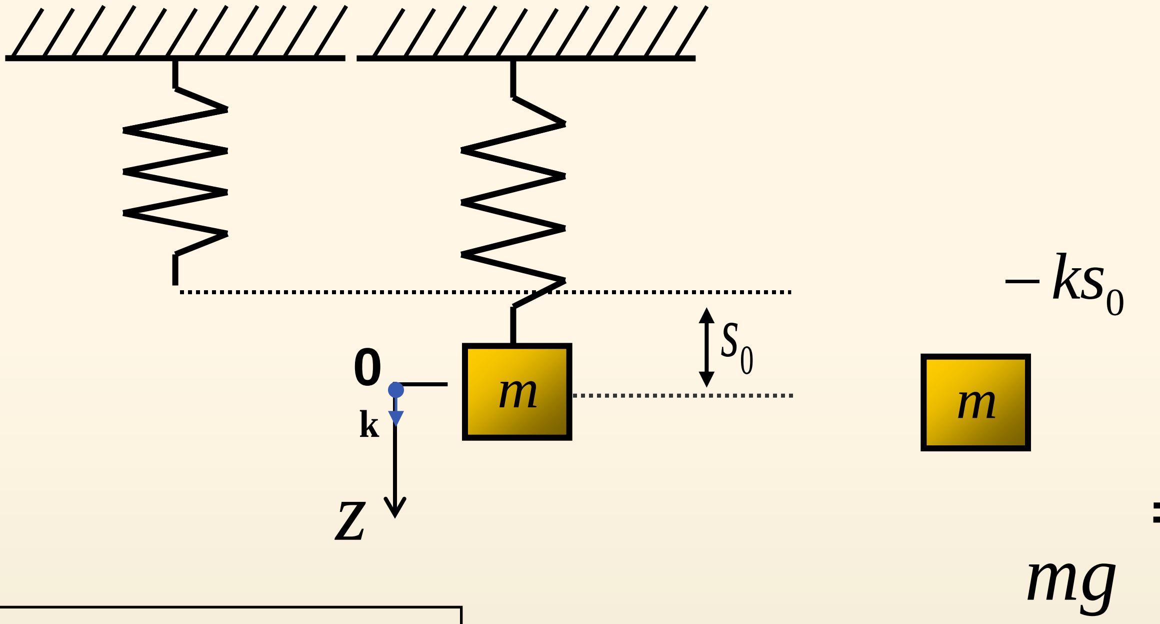

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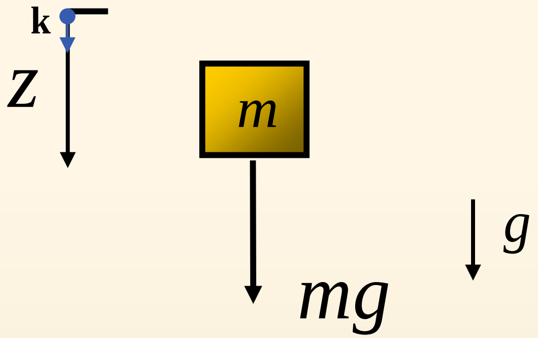

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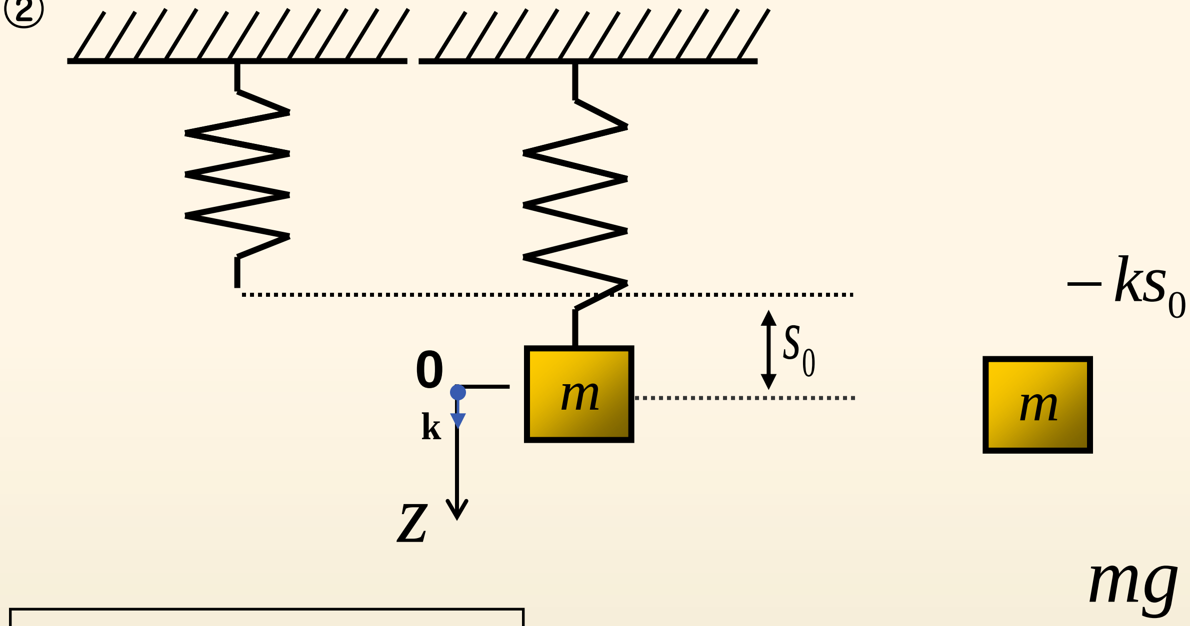

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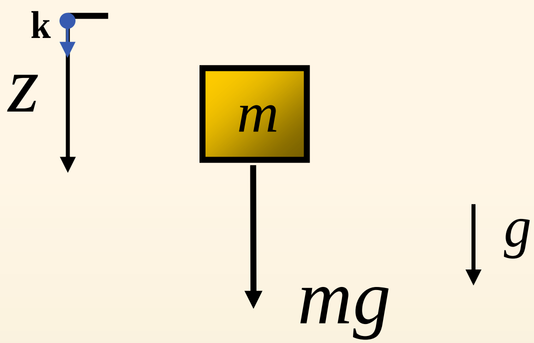

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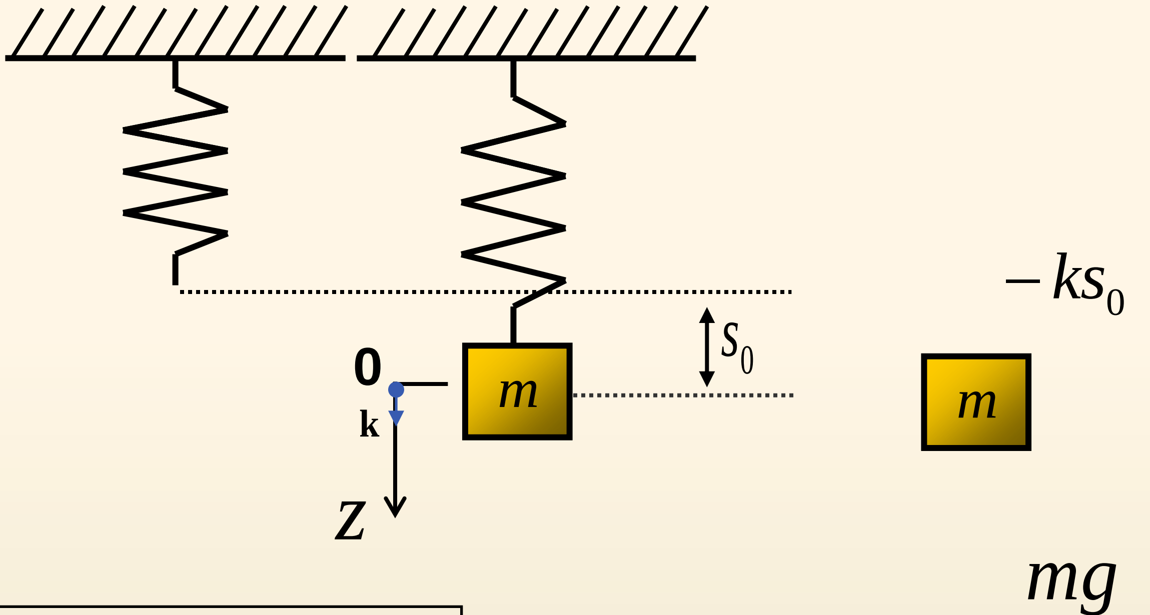

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$$= 0 \quad (\because z'' = 0)$$

: static equilibrium

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Spring/Mass Systems: Driven Motion

$z = z(t)\ , \quad z'' = \dfrac{d^2 z}{dt^2}$

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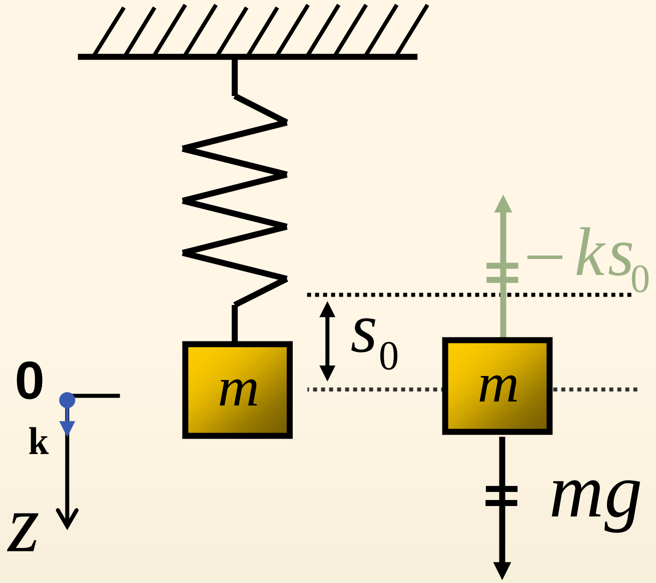

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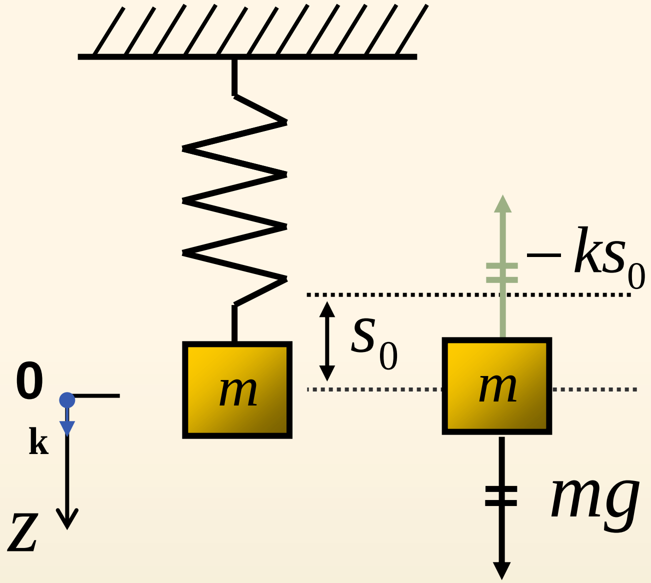

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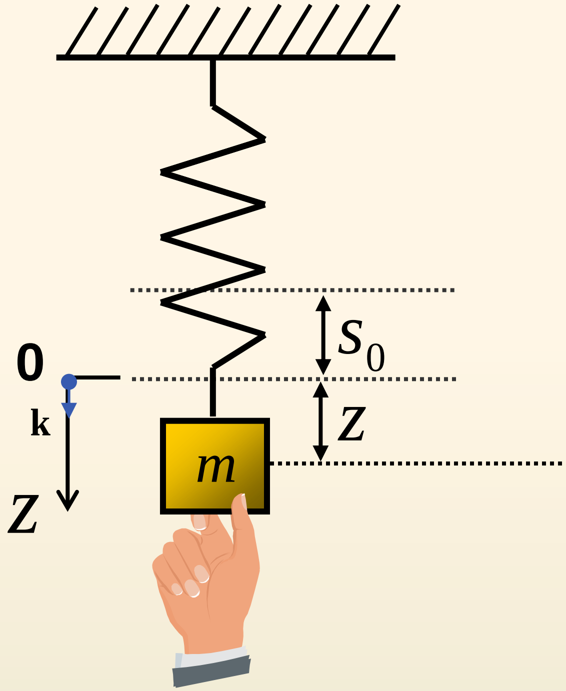

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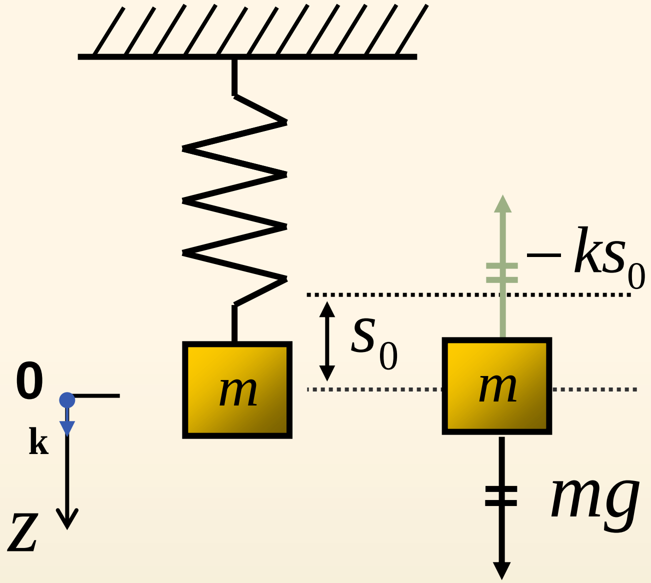

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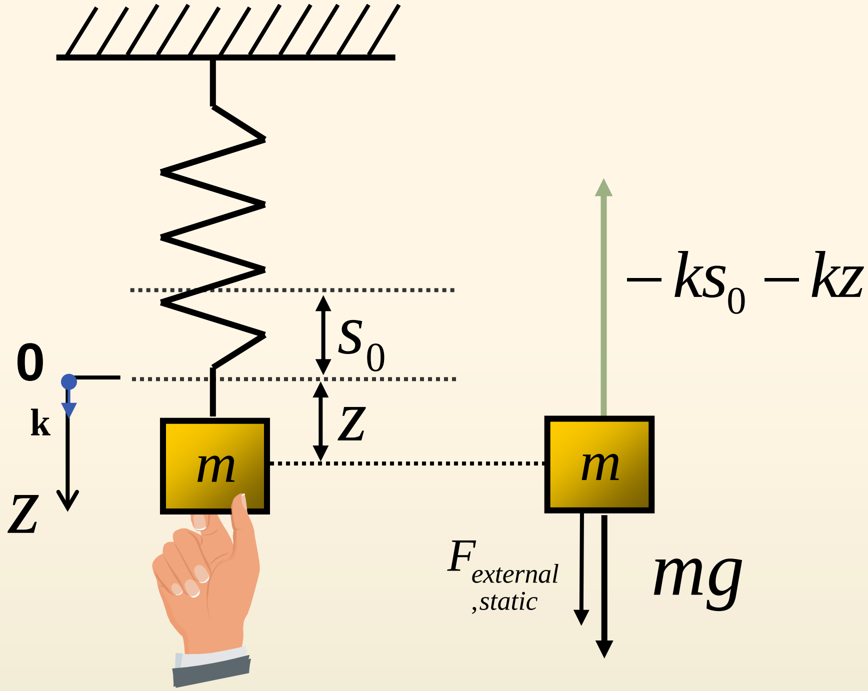

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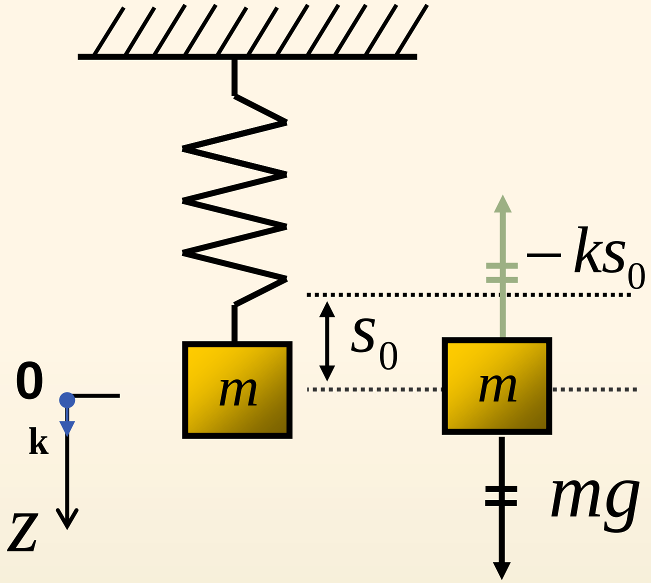

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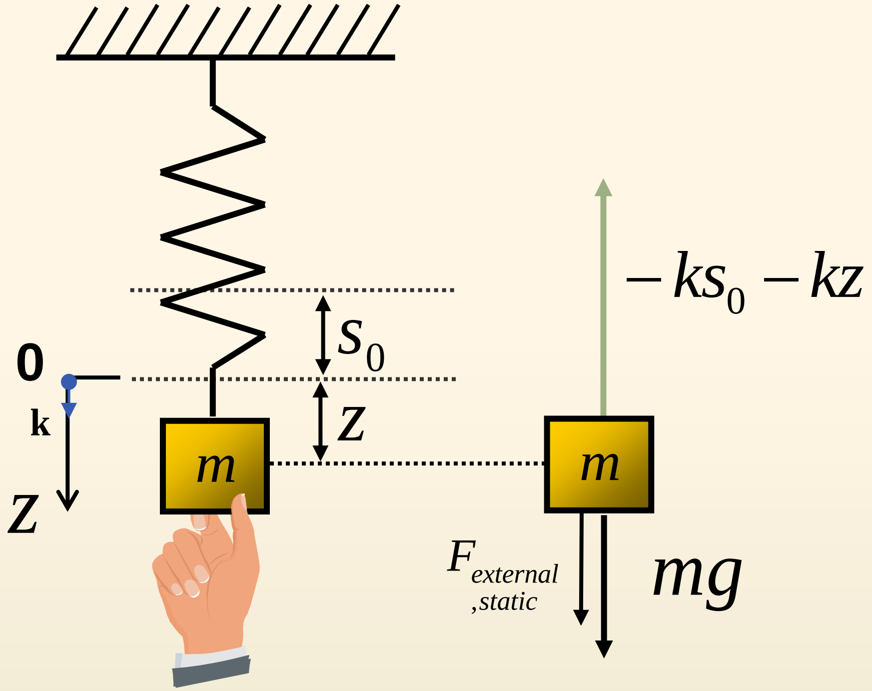

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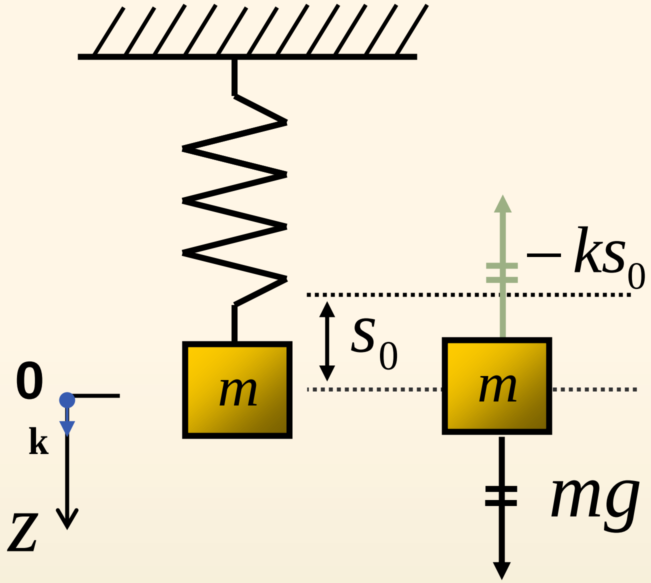

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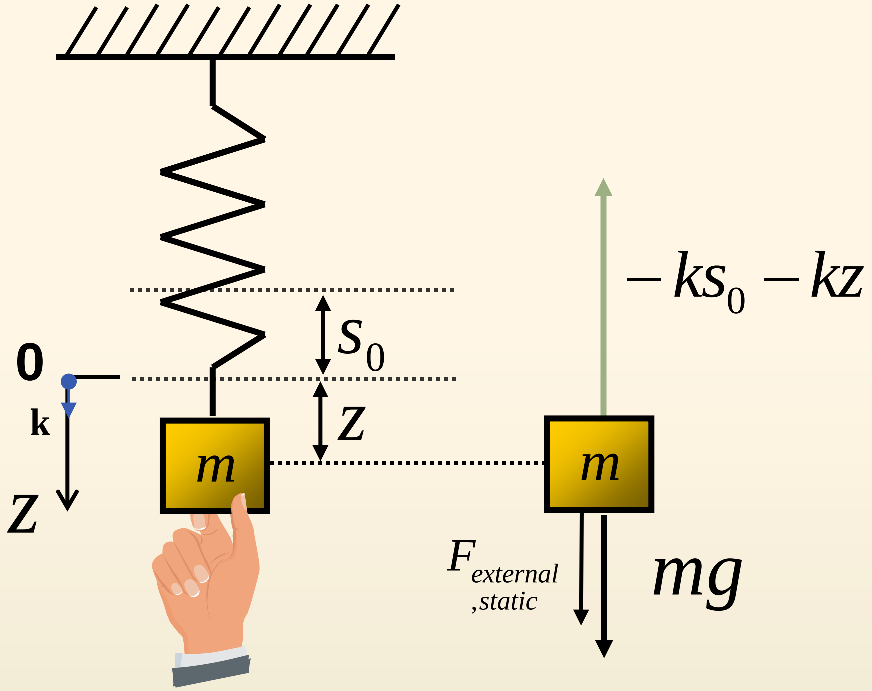

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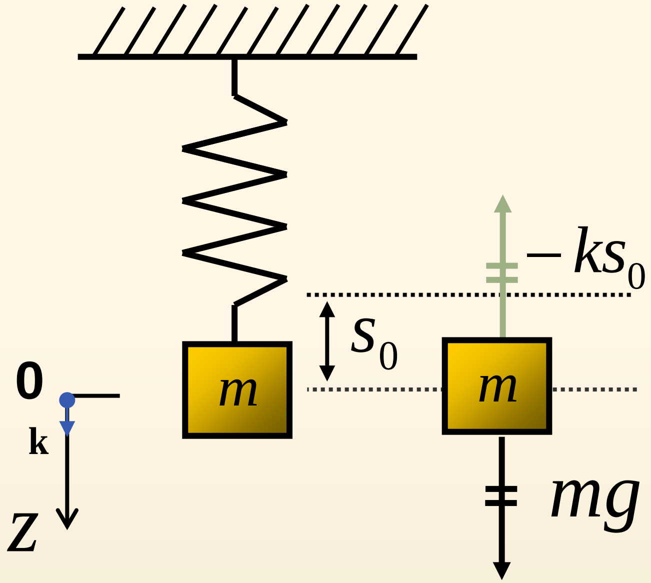

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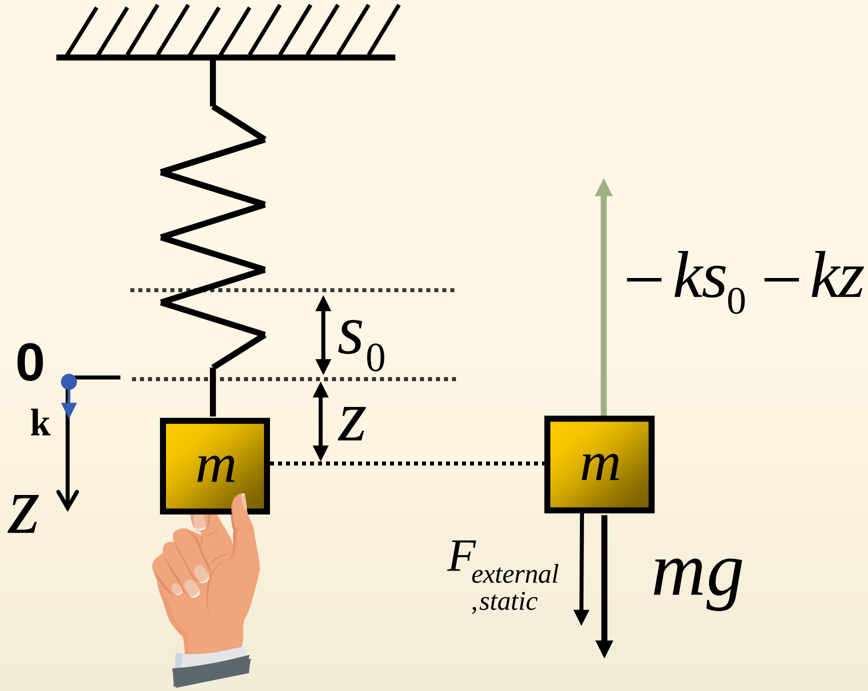

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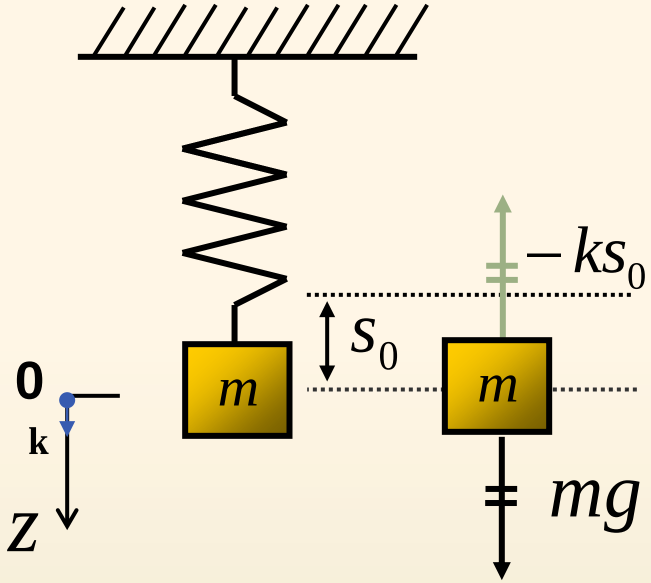

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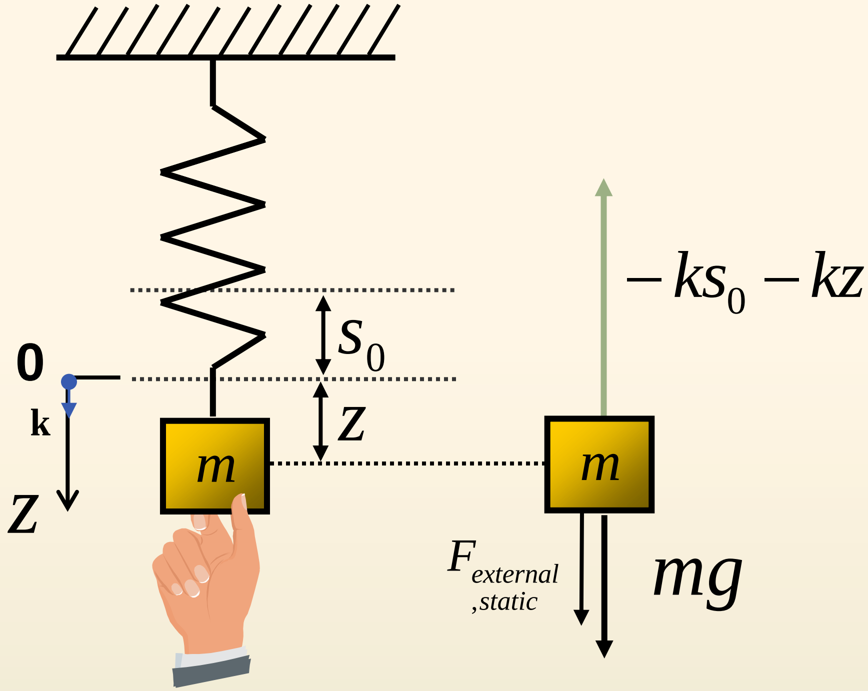

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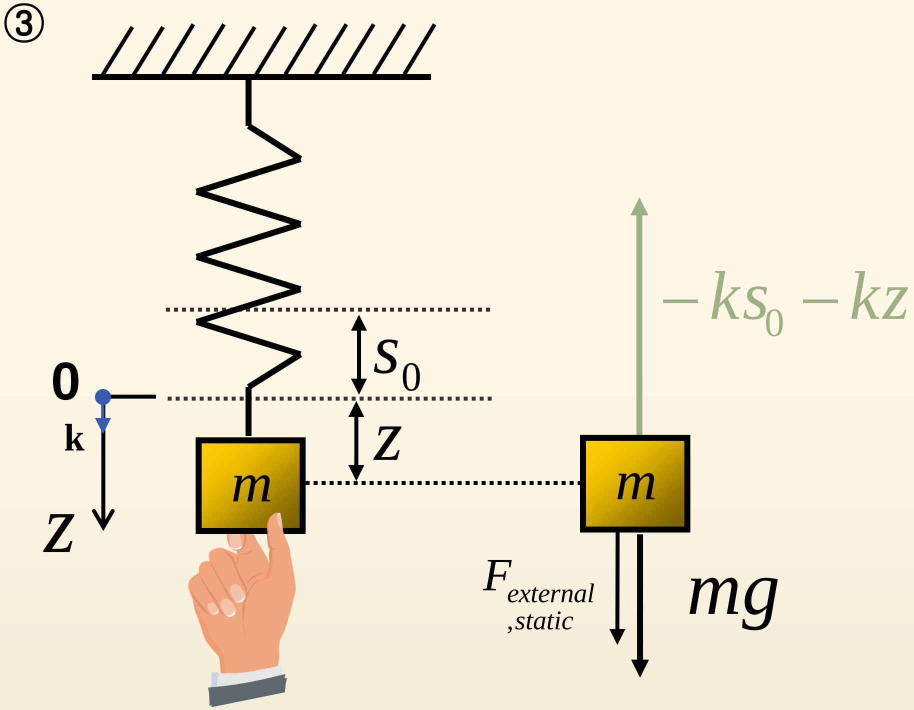


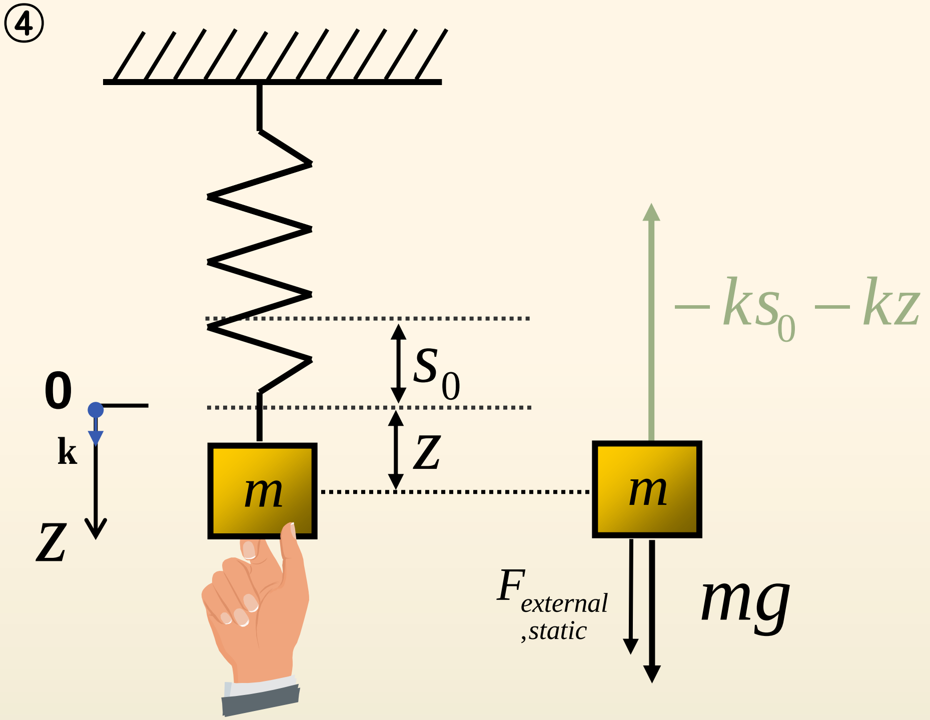
$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= 0 \quad (\because z'' = 0)$$

④



$$m\mathbf{z}'' = \mathbf{F}$$

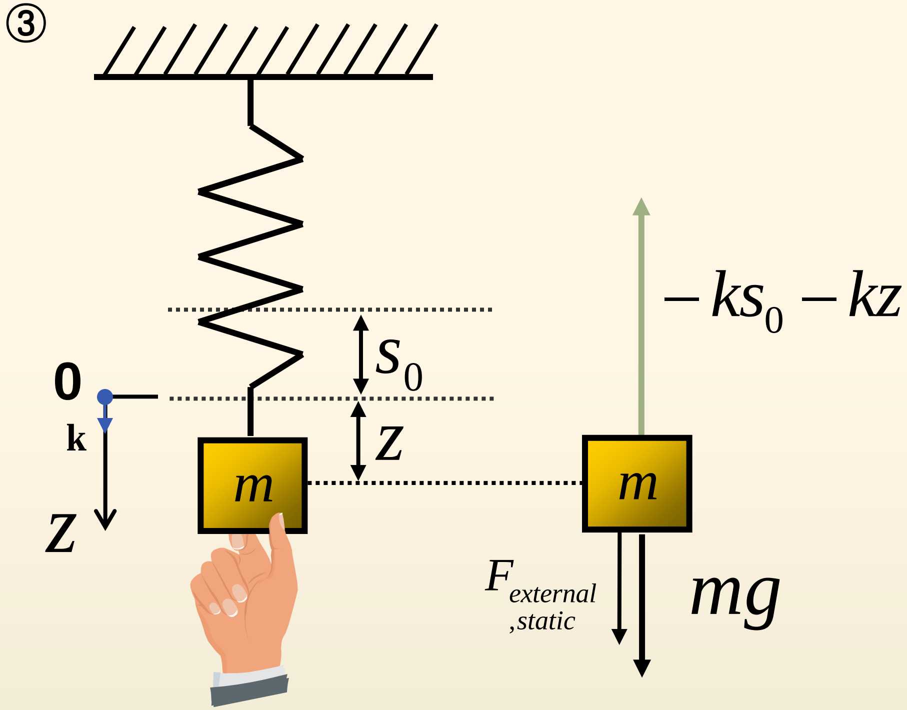
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

Spring/Mass Systems: Driven Motion

$z = z(t)\ , \quad z'' = \dfrac{d^2z}{dt^2}$

③



$$m\mathbf{z}'' = \mathbf{F}$$
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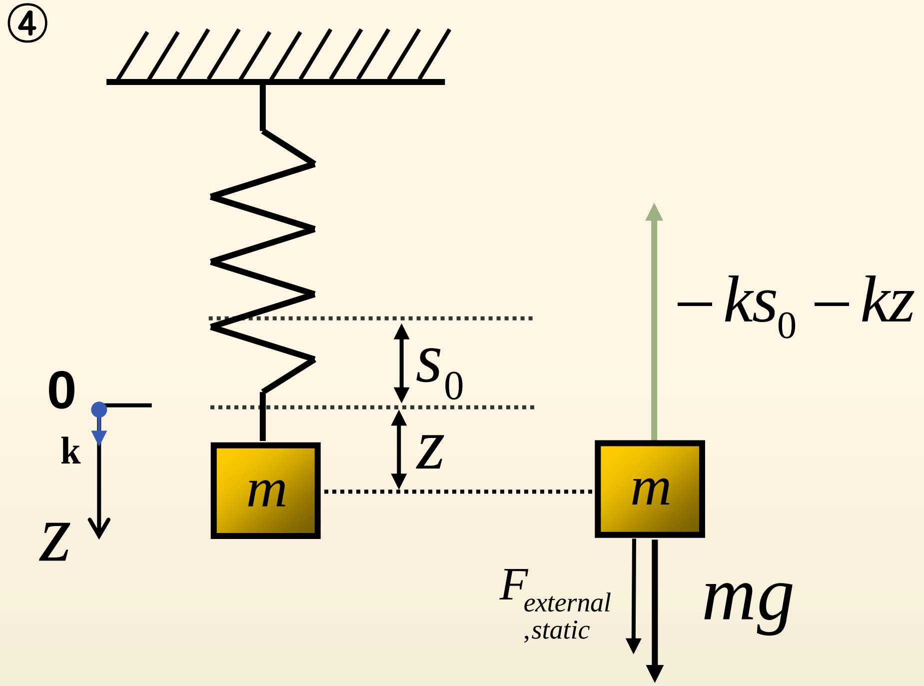
$$= 0 \quad (\because z'' = 0)$$

④

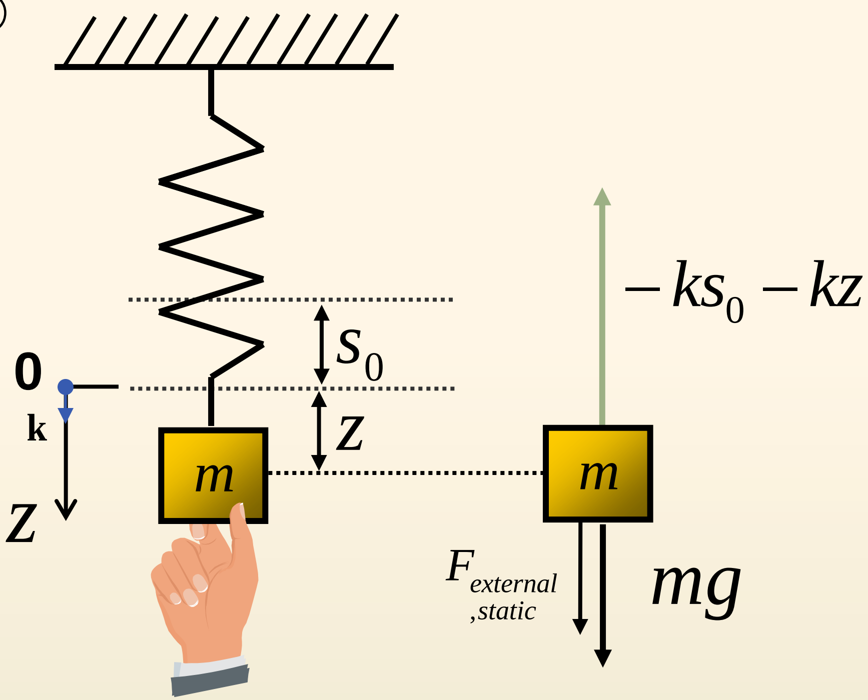


$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= -kz\mathbf{k} \ + \mathbf{F}_{external,static}$$

Spring/Mass Systems: Driven Motion

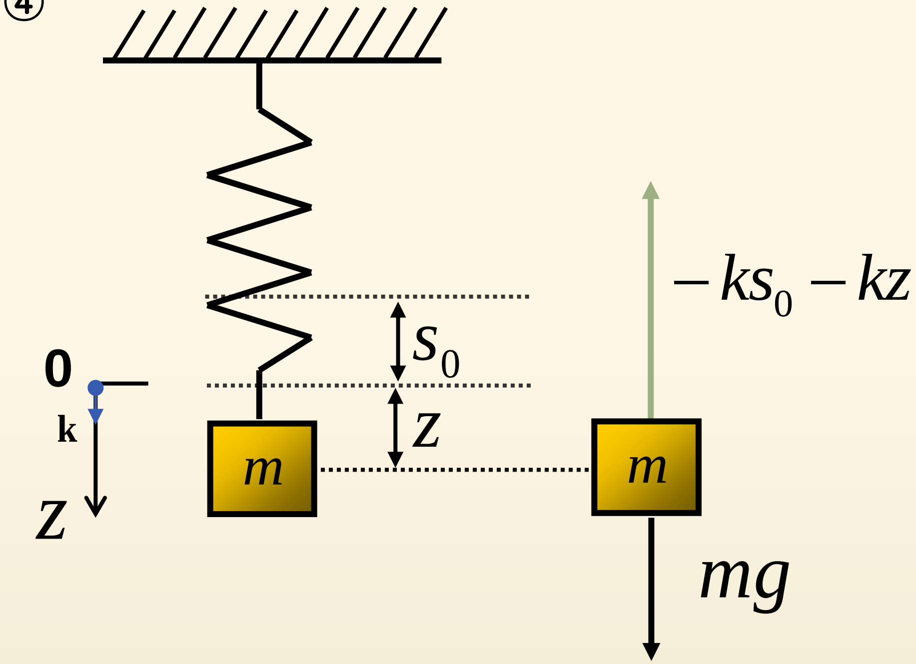
$z = z(t)\ , \quad z'' = \dfrac{d^2 z}{dt^2}$

③



$$\begin{aligned} m\mathbf{z}'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static} \\ &= -kz\mathbf{k} + \mathbf{F}_{external,static} \\ &= 0 \quad (\because z'' = 0) \end{aligned}$$

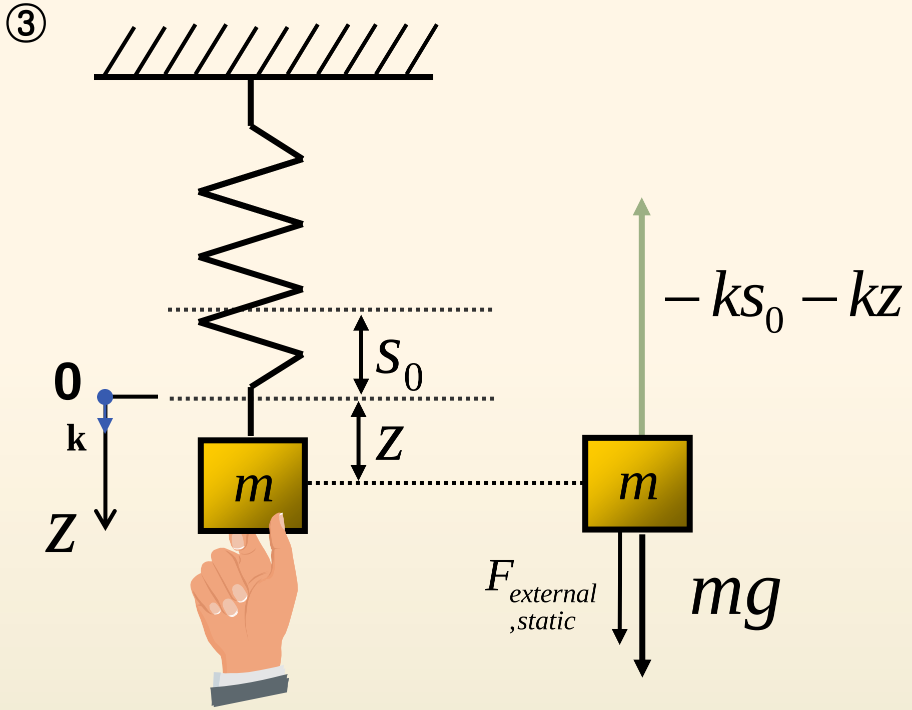
④



$$\begin{aligned} m\mathbf{z}'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \\ &= -kz\mathbf{k} \end{aligned}$$

Spring/Mass Systems: Driven Motion

$z = z(t)\ , \quad z'' = \dfrac{d^2 z}{dt^2}$



$$m\mathbf{z}'' = \mathbf{F}$$
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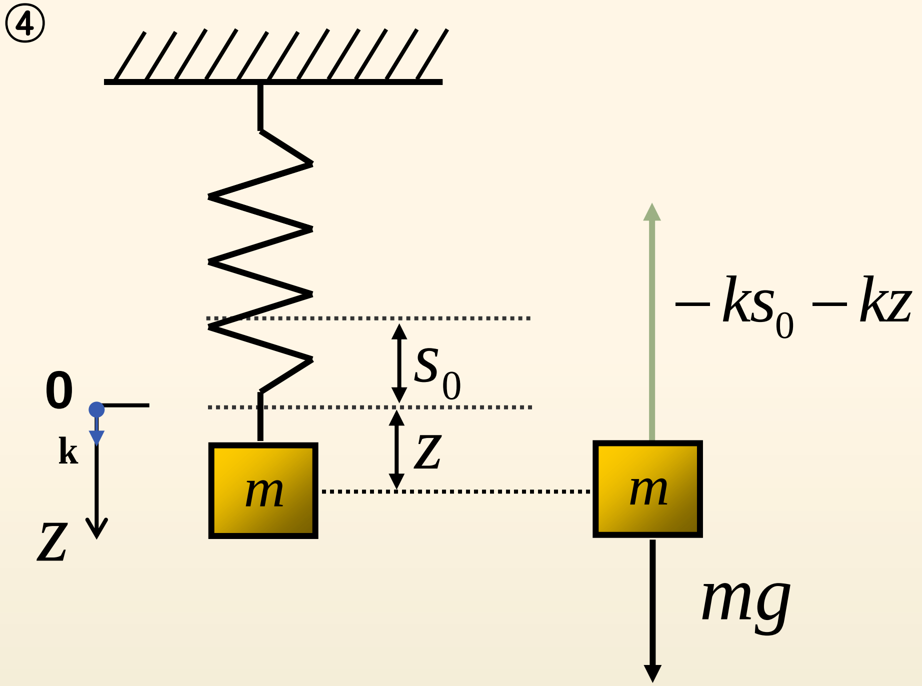


$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k}$$
$$= -kz\mathbf{k}$$

↑ Physical Phenomenon

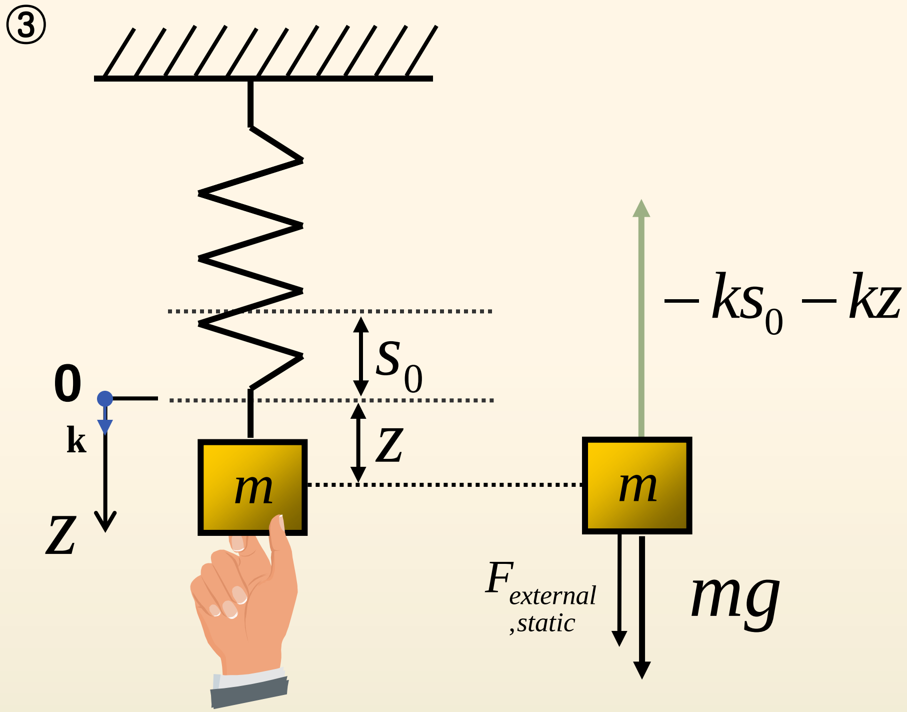
↓ Mathematical Equation

$$\boxed{m\mathbf{z}'' + k\mathbf{z} = 0}$$

Oscillation by the restoring force

Spring/Mass Systems: Driven Motion

$$z = z(t)\ , \qquad z'' = \frac{d^2 z}{dt^2}$$



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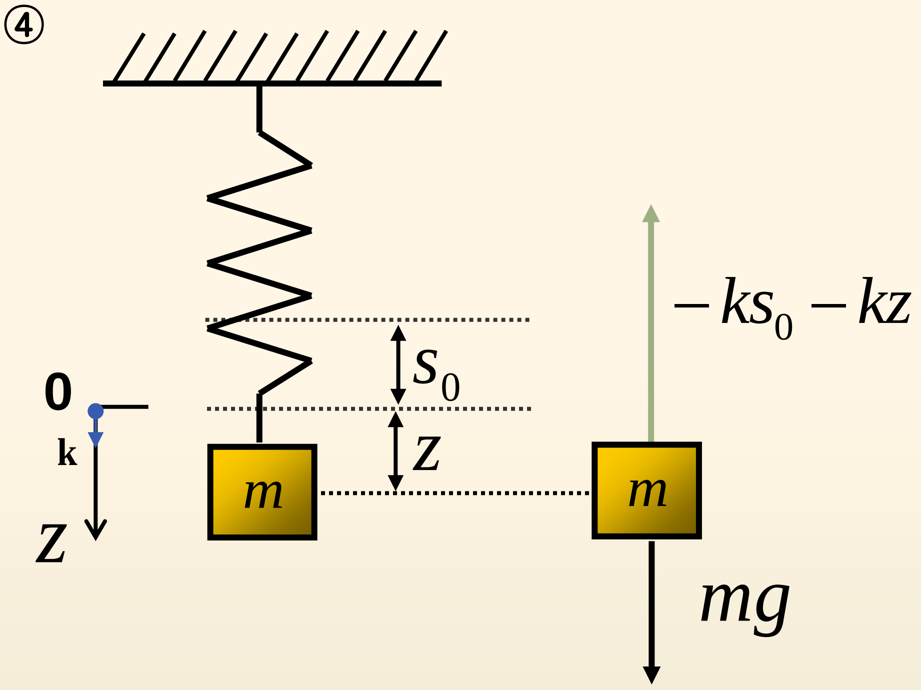


$$\begin{aligned} m\mathbf{z}'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \\ &= -kz\mathbf{k} \end{aligned}$$

restoring force

↑ Physical Phenomenon

↓ Mathematical Equation

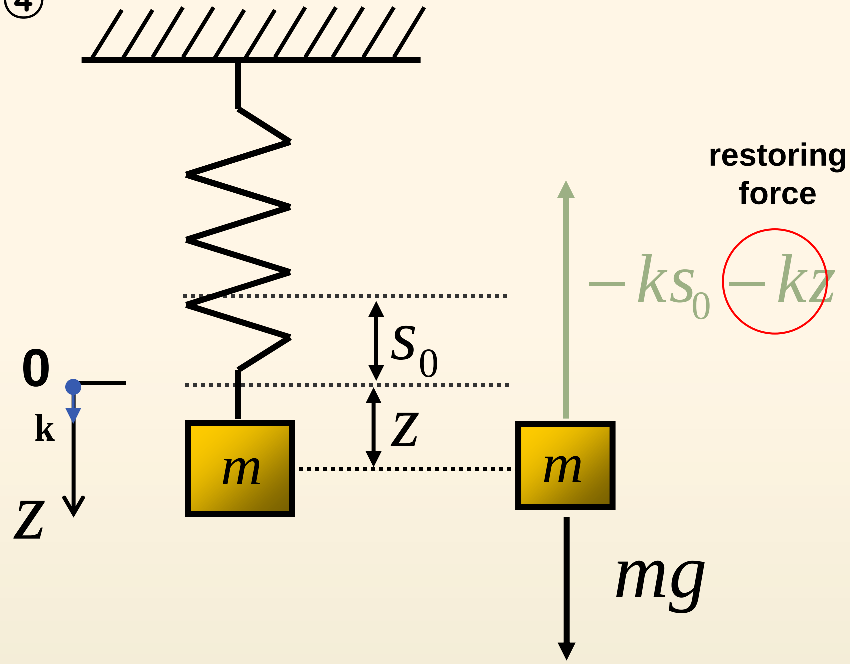
$$m\mathbf{z}'' + k\mathbf{z} = 0$$

Oscillation by the restoring force

Spring/Mass Systems: Driven Motion

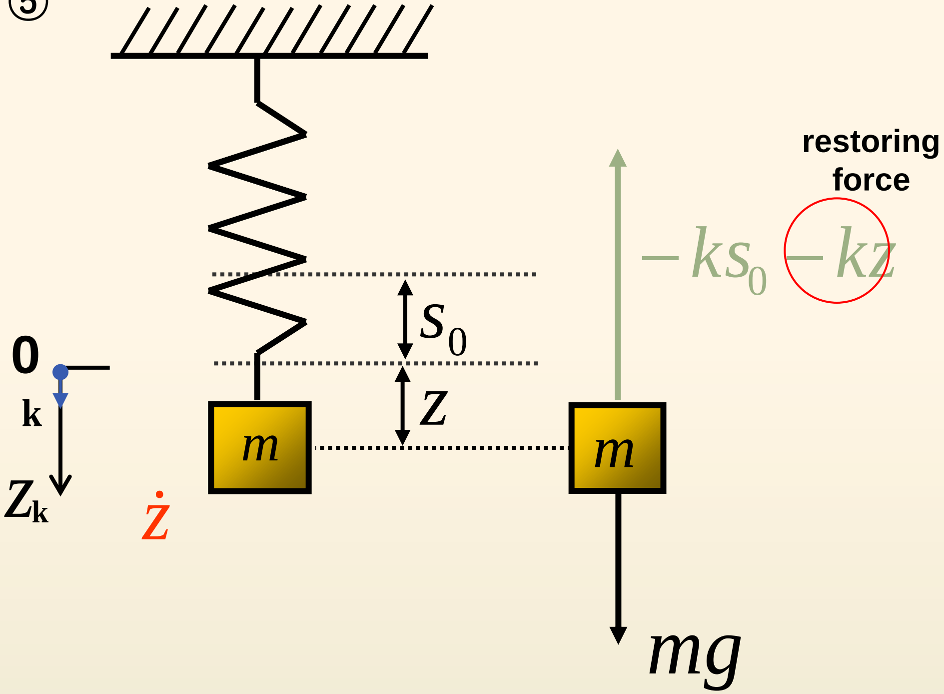
$$z = z(t)\ , \qquad z'' = \frac{d^2 z}{dt^2}$$

④



$$\begin{aligned} m\mathbf{z}'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \\ &= -kz\mathbf{k} \end{aligned}$$

⑤



$$\begin{aligned} m\mathbf{z}'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \\ &= -kz\mathbf{k} \end{aligned}$$

⇧ Physical Phenomenon

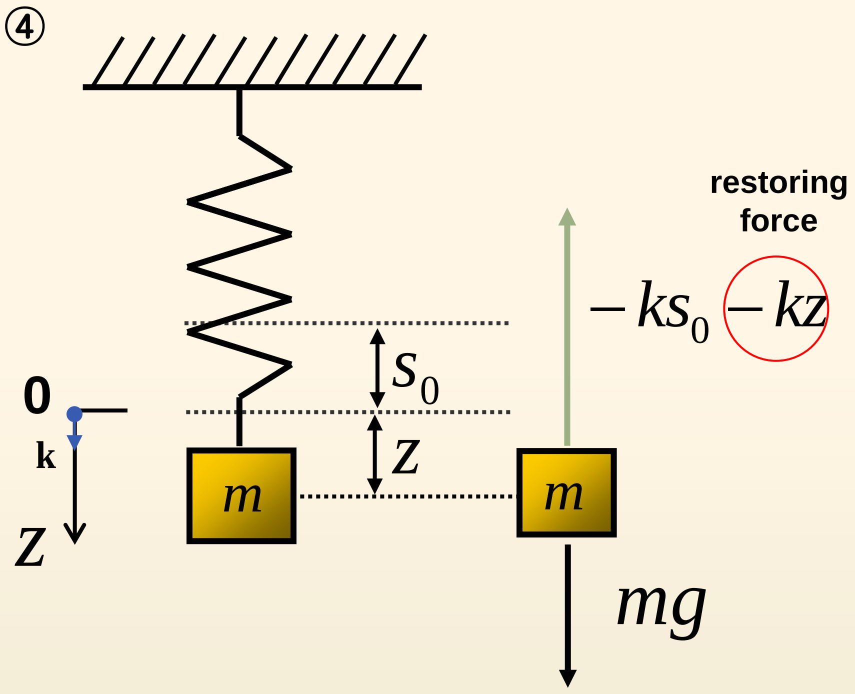
⇩ Mathematical Equation

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

oscillation by the restoring force

Spring/Mass Systems: Driven Motion

$$z = z(t)\ ,\quad z'' = \frac{d^2z}{dt^2}$$



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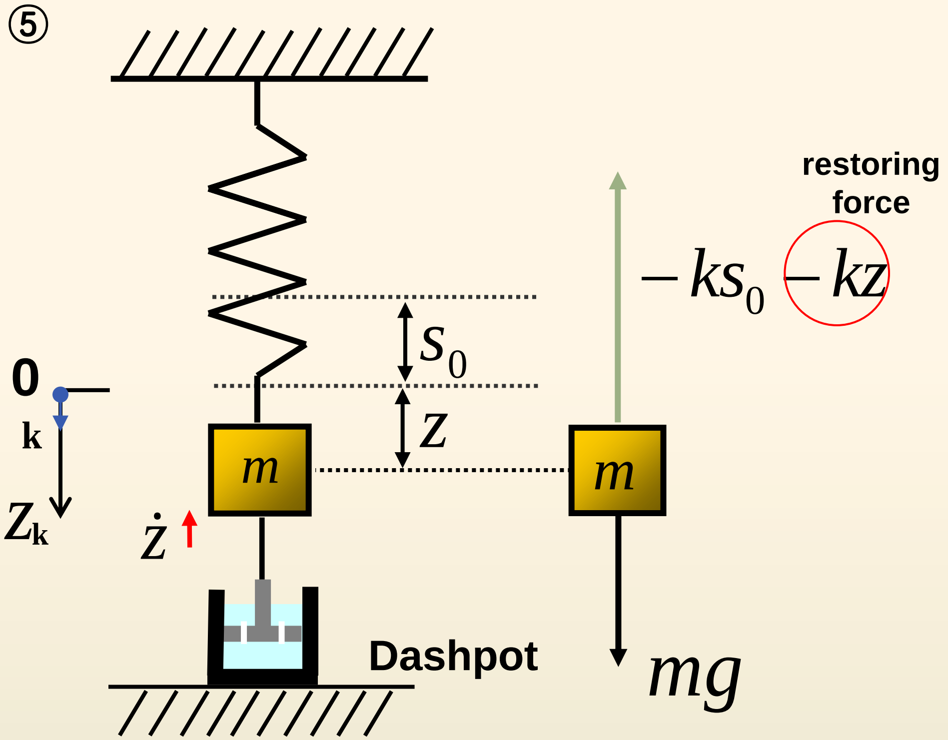

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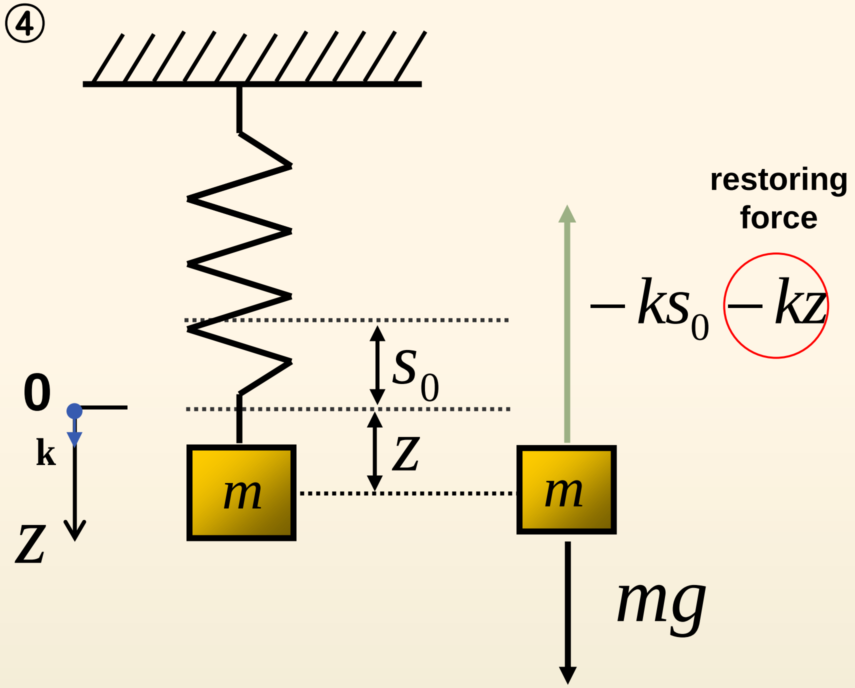
⇧ Physical Phenomenon
⇩ Mathematical Equation

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

oscillation by the restoring force

Spring/Mass Systems: Driven Motion

$z = z(t)\ ,\quad z'' = \dfrac{d^2 z}{dt^2}$



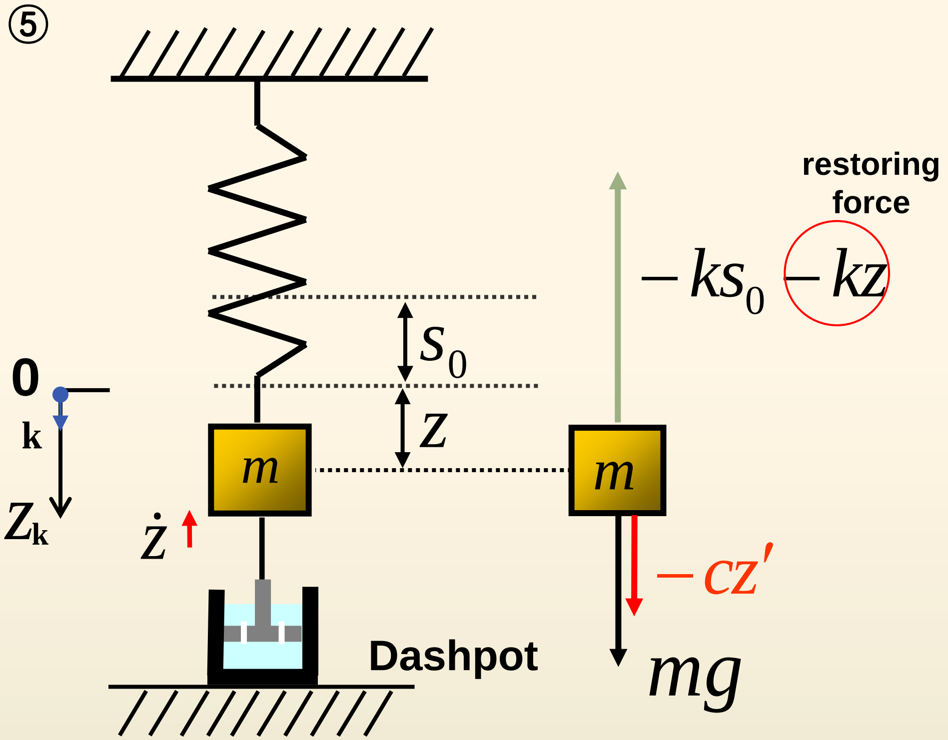
$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k}$$
$$= -kz\mathbf{k}$$

Physical Phenomenon

Mathematical Equation

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

oscillation by the restoring force



$$m\mathbf{z}'' = \mathbf{F}$$
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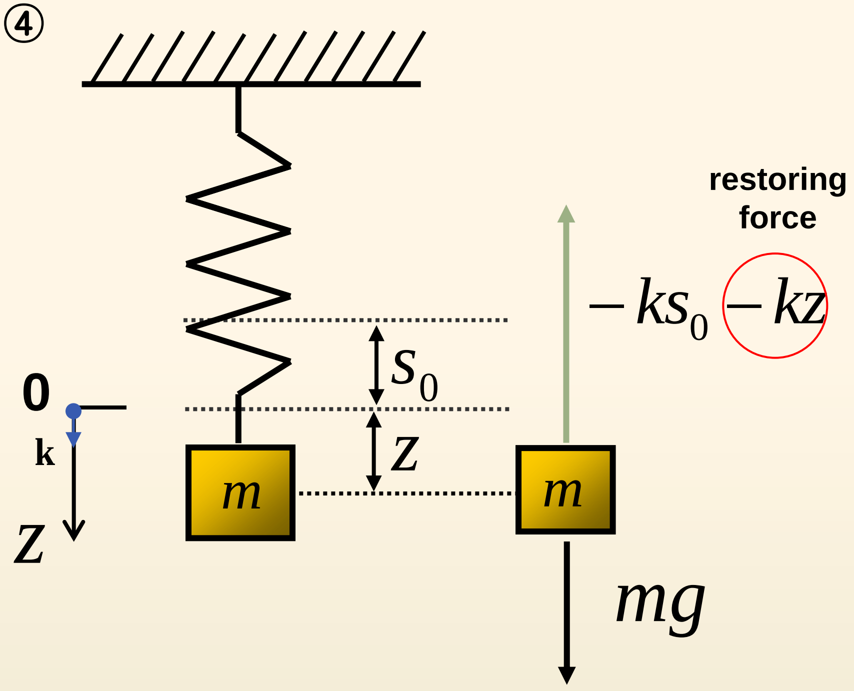
Spring/Mass Systems: Driven Motion

$$z = z(t)\ , \qquad z'' = \frac{d^2 z}{dt^2}$$



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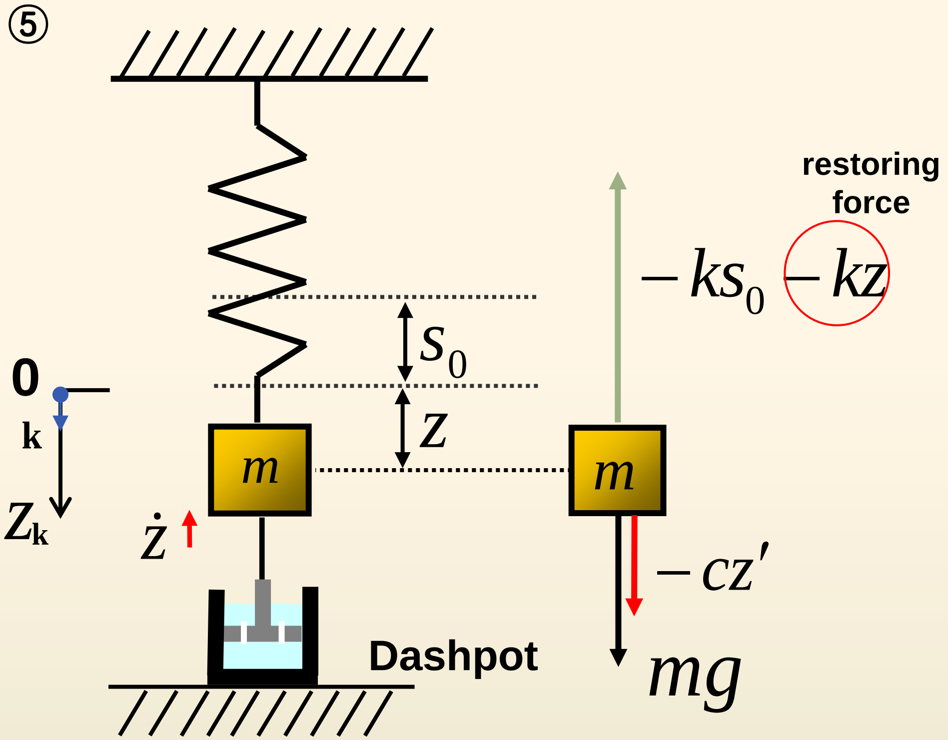


$$\begin{aligned} m\mathbf{z}'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} \\ &= -kz\mathbf{k} \end{aligned}$$

⇧ Physical Phenomenon

⇩ Mathematical Equation

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

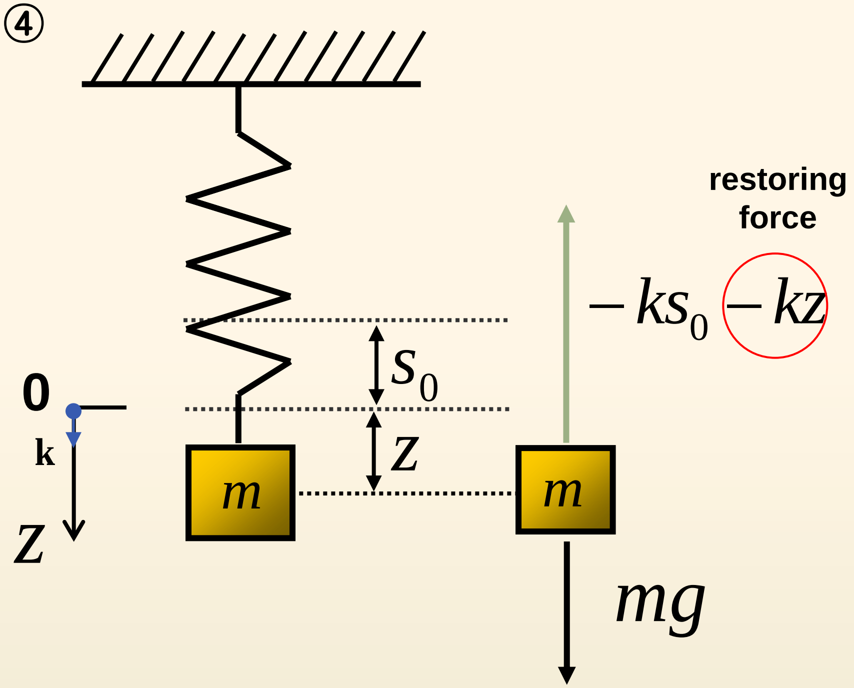
oscillation by the restoring force

2008

Spring/Mass Systems: Driven Motion

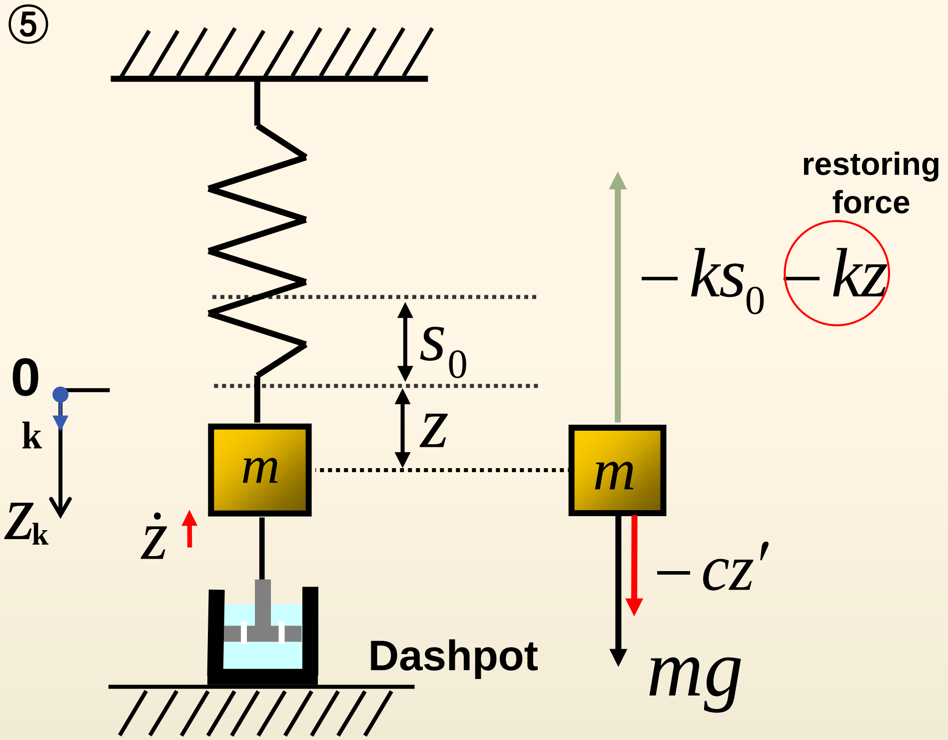
$$z = z(t)\ , \qquad z'' = \frac{d^2 z}{dt^2}$$

④



$$
\begin{aligned}
m\mathbf{z}'' &= \mathbf{F} \\
&= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \\
&= -kz\mathbf{k}
\end{aligned}
$$

⑤



$$
\begin{aligned}
m\mathbf{z}'' &= \mathbf{F} \\
&= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} \\
&= -kz\mathbf{k} - cz'\mathbf{k}
\end{aligned}
$$

Physical Phenomenon

Mathematical Equation

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

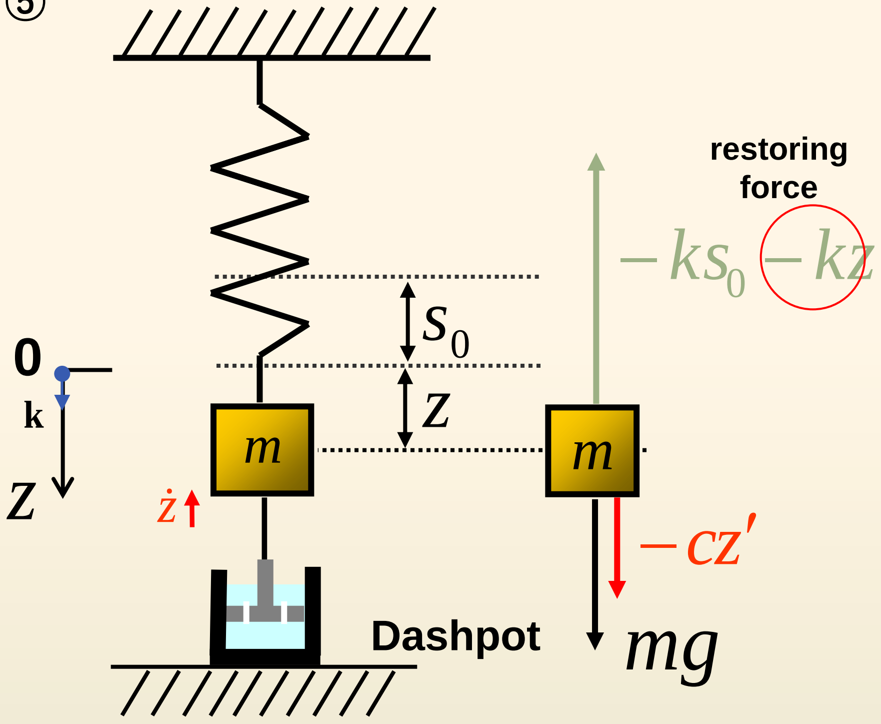
oscillation by the restoring force

$$m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = 0$$

Spring/Mass Systems: Driven Motion

$z = z(t)$, $z'' = \dfrac{d^2 z}{dt^2}$

⑤



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \quad -cz'\mathbf{k}$$
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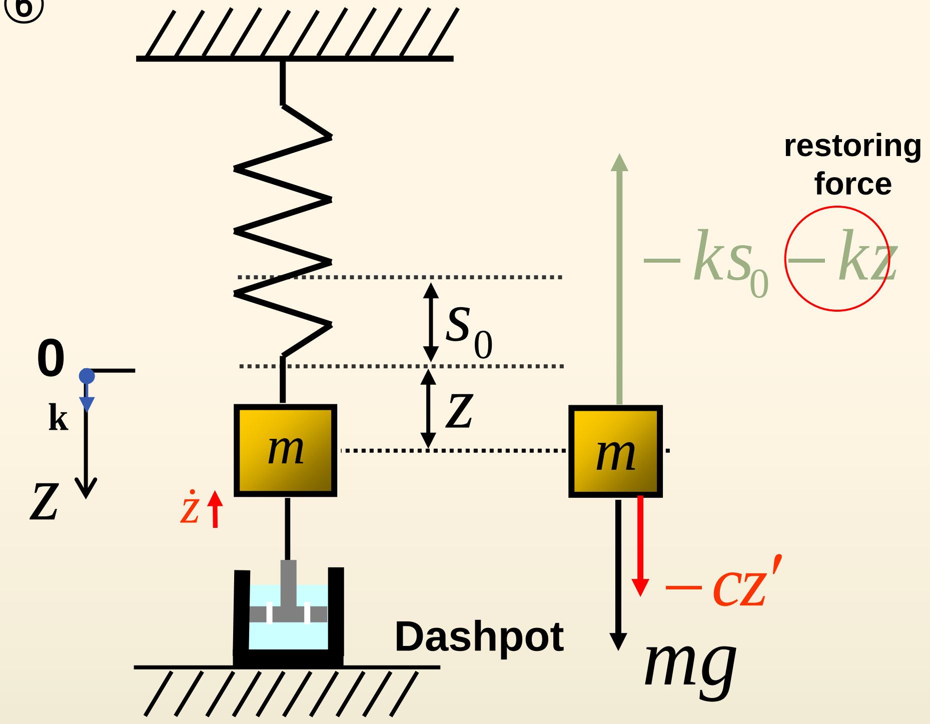
⑥



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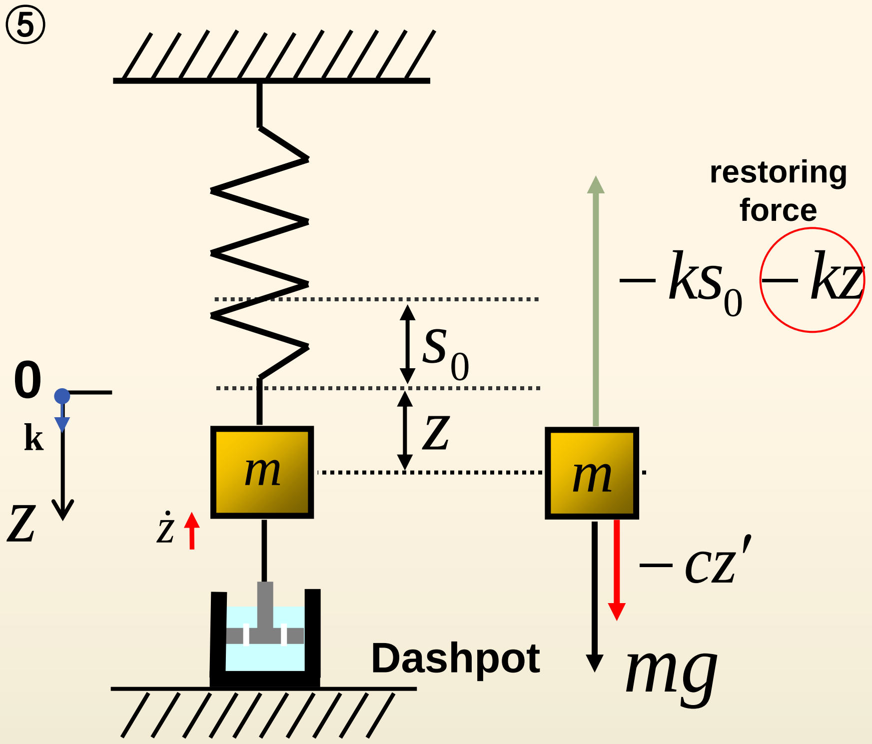
⇧ Physical Phenomenon

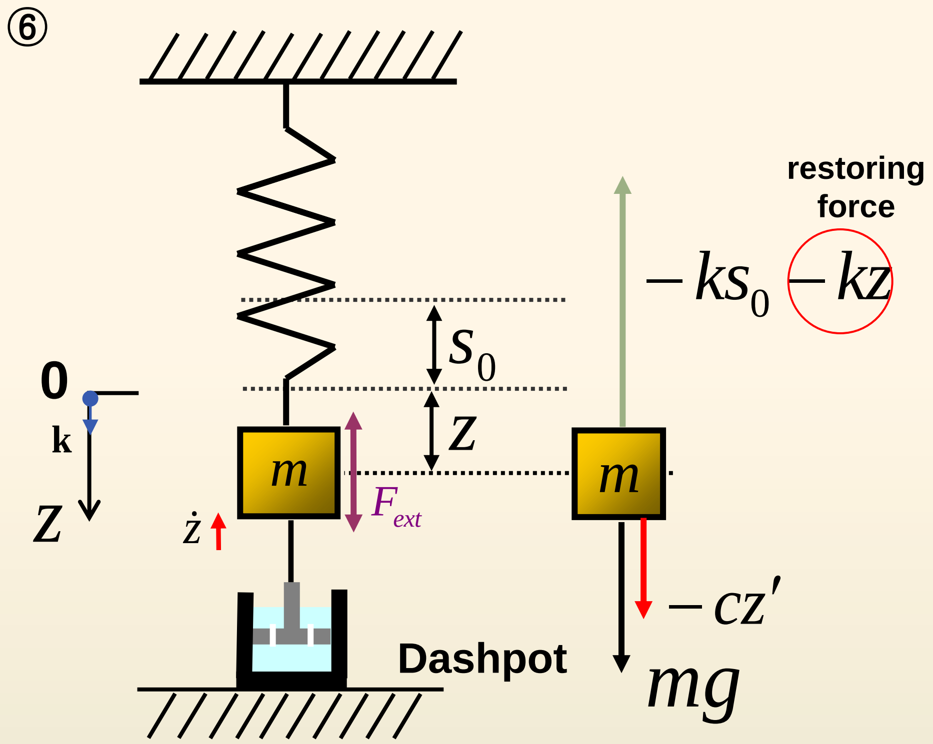
⇩ Mathematical Equation

$$m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = 0$$

Spring/Mass Systems: Driven Motion

$$z = z(t)\,, \qquad z'' = \frac{d^2 z}{dt^2}$$





$$\mathbf{m}\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \quad - cz'\mathbf{k}$$
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$$\mathbf{m}\mathbf{z}'' = \mathbf{F}$$
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Physical Phenomenon

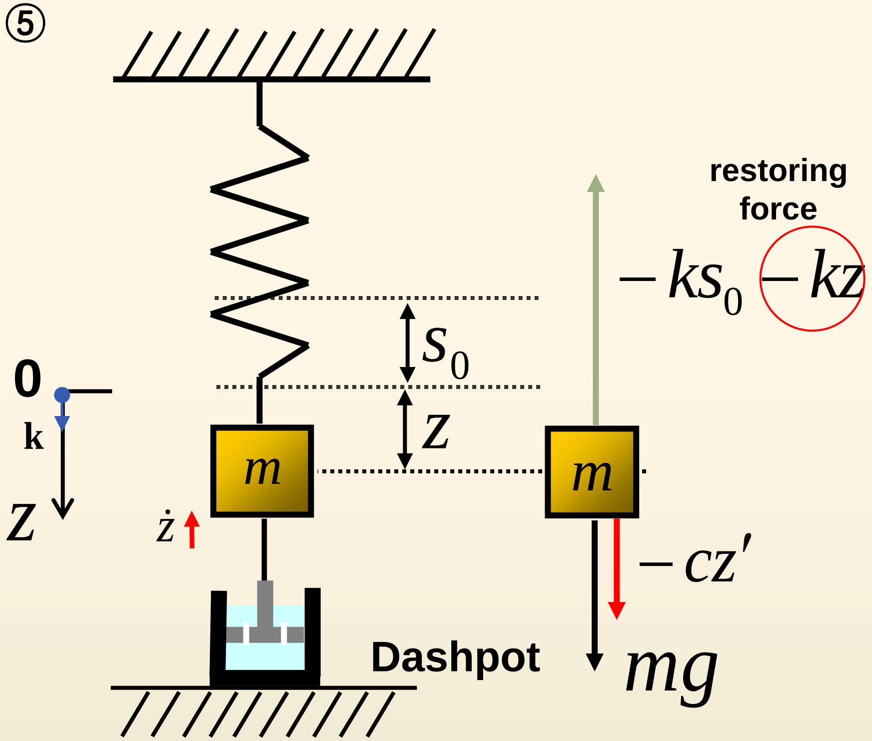
Mathematical Equation

$$\mathbf{m}\mathbf{z}'' + \mathbf{c}\mathbf{z}' + \mathbf{k}\mathbf{z} = 0$$

Spring/Mass Systems: Driven Motion

$z = z(t)$, $z'' = \dfrac{d^2 z}{dt^2}$

⑤



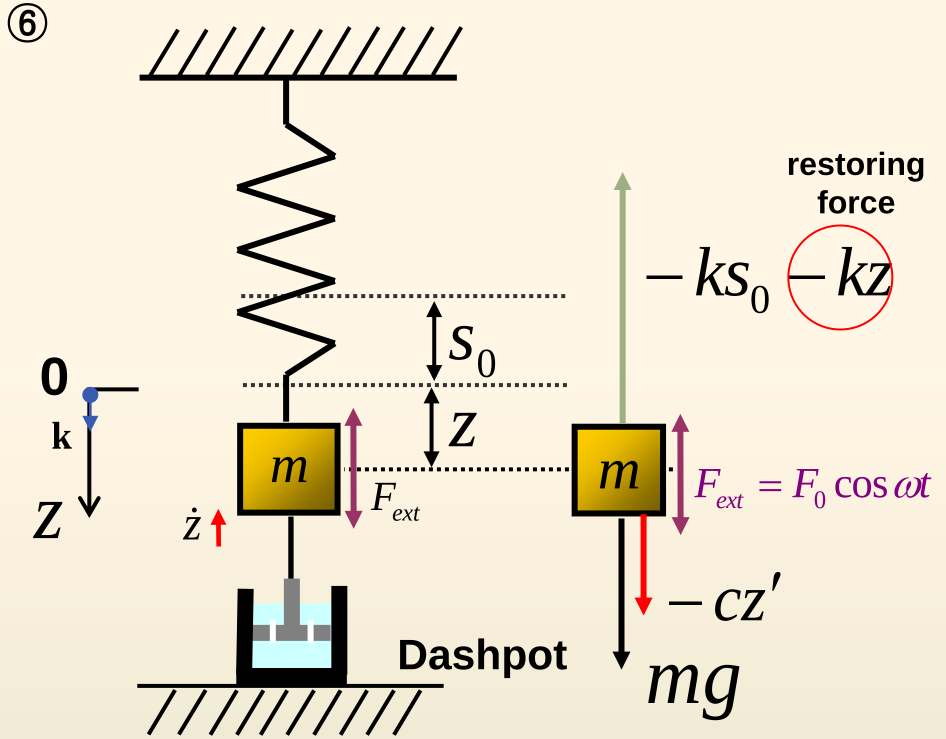
$$\mathbf{m}z'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \quad -cz'\mathbf{k}$$
$$= -kz\mathbf{k} \quad -cz'\mathbf{k}$$

Physical Phenomenon

Mathematical Equation

$$\mathbf{m}z'' + cz' + kz = 0$$

⑥



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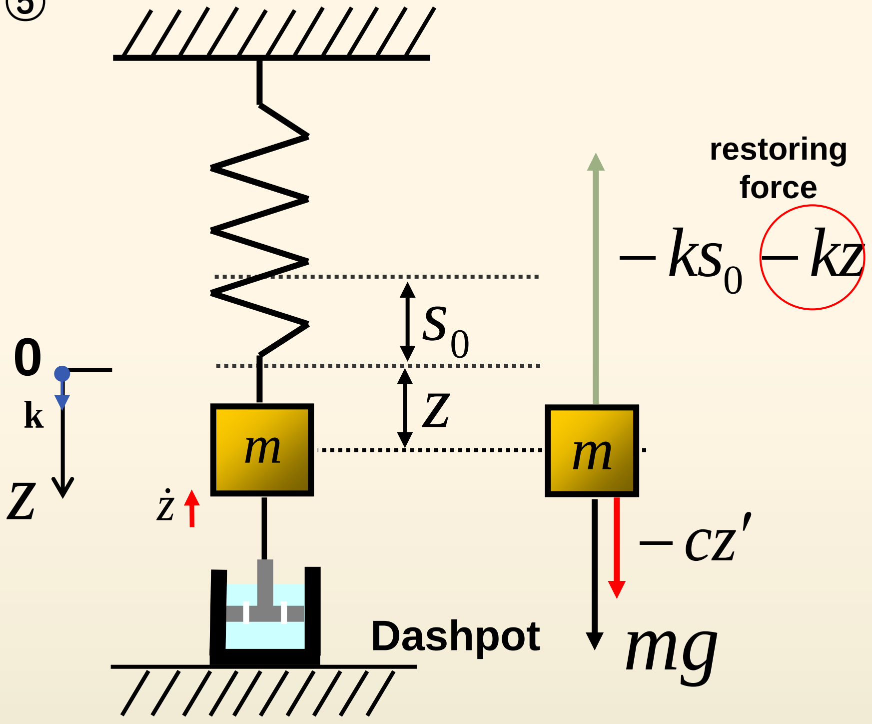
Spring/Mass Systems: Driven Motion

$z = z(t)\ ,\qquad z'' = \dfrac{d^2z}{dt^2}$

⑤



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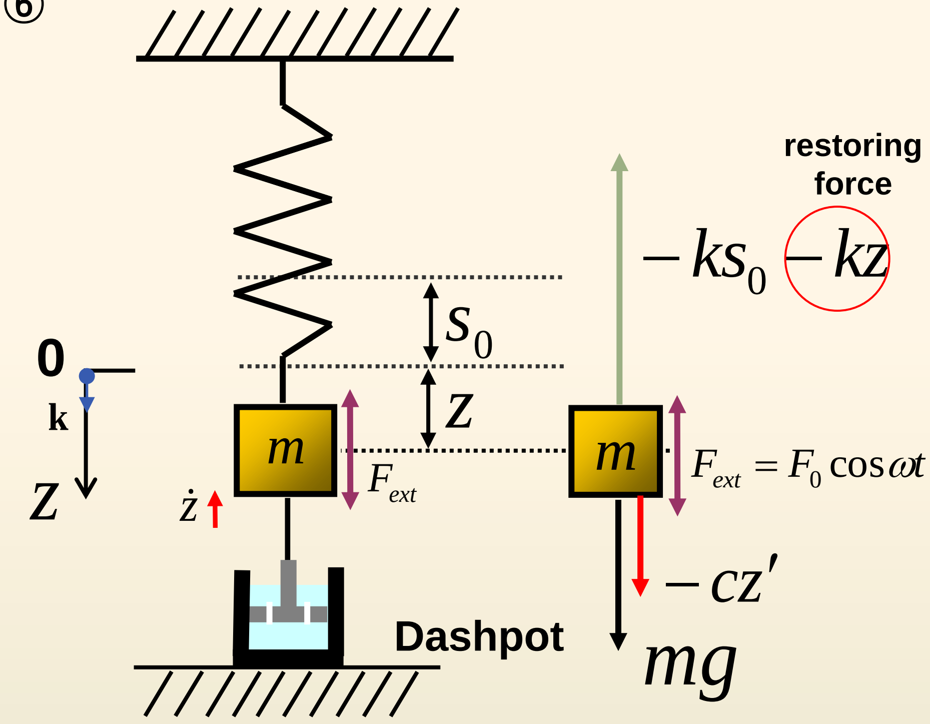
⑥



$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0}\cos\omega t$$

$$= -kz\mathbf{k} - cz'\mathbf{k}$$

⇧ Physical Phenomenon

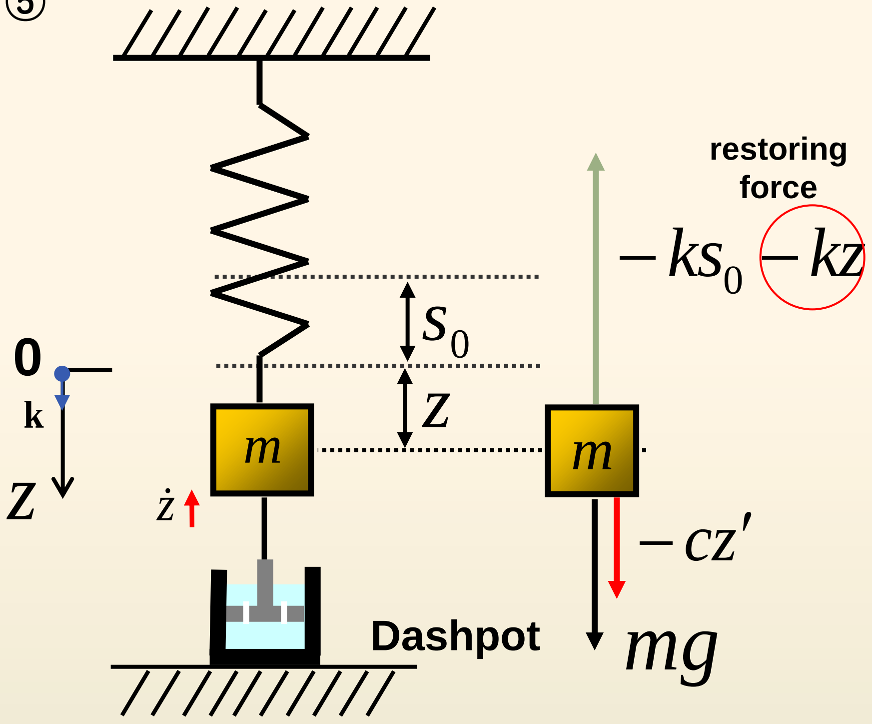
⇩ Mathematical Equation

$$m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = 0$$

Spring/Mass Systems: Driven Motion

$z = z(t)\,,\quad z'' = \dfrac{d^2z}{dt^2}$

⑤



$$m\mathbf{z}'' = \mathbf{F}$$

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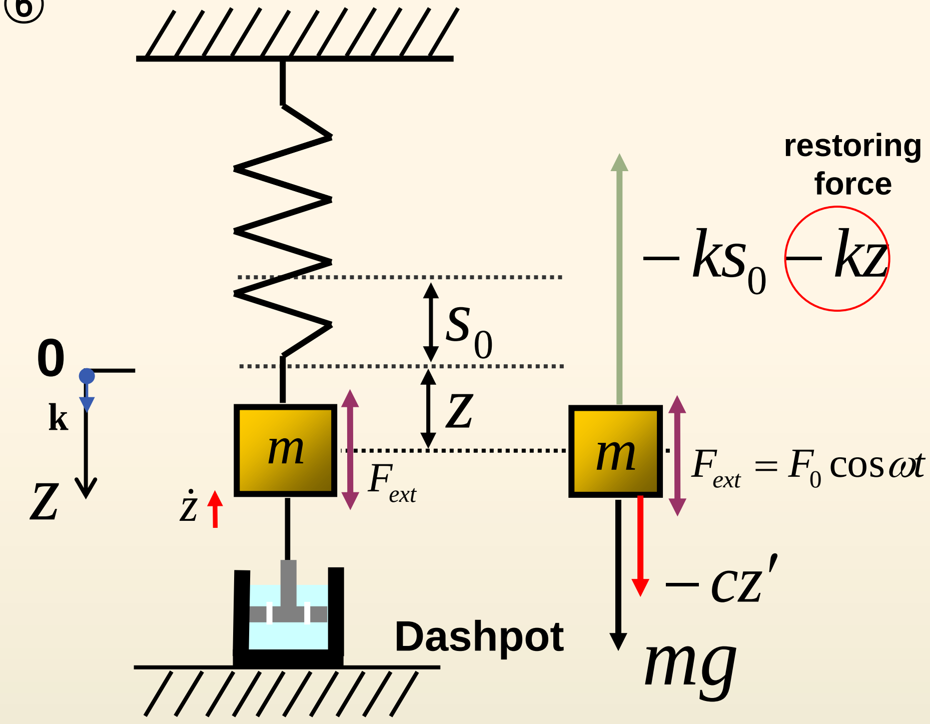
Physical Phenomenon

Mathematical Equation

$$m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = 0$$

⑥



$$m\mathbf{z}'' = \mathbf{F}$$

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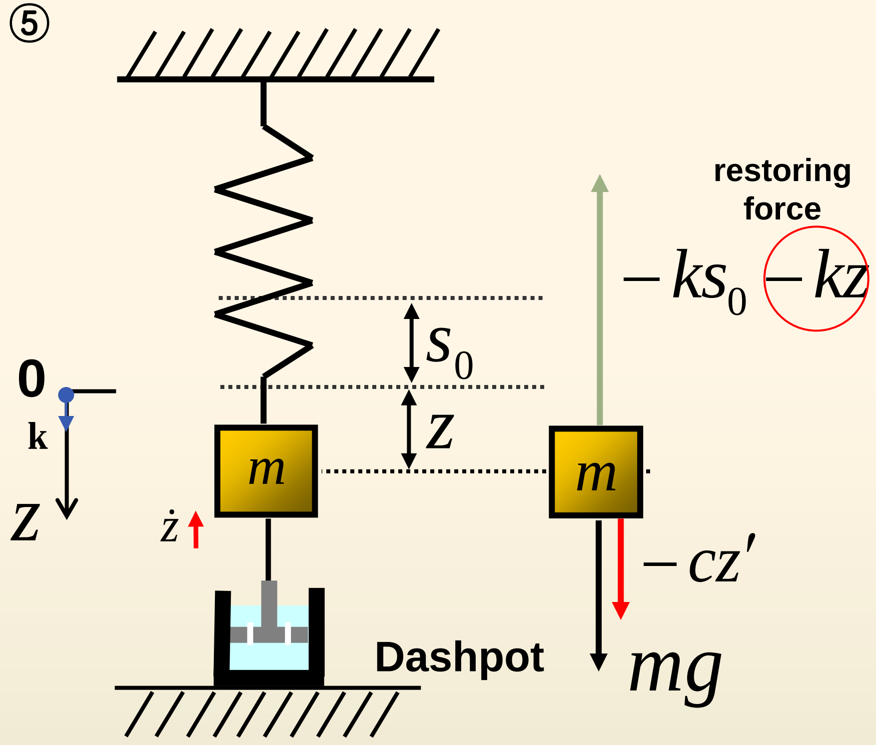
Spring/Mass Systems: Driven Motion

$$z = z(t)\,, \qquad z'' = \frac{d^2 z}{dt^2}$$

⑤



$$\mathbf{mz''} = \mathbf{F}$$

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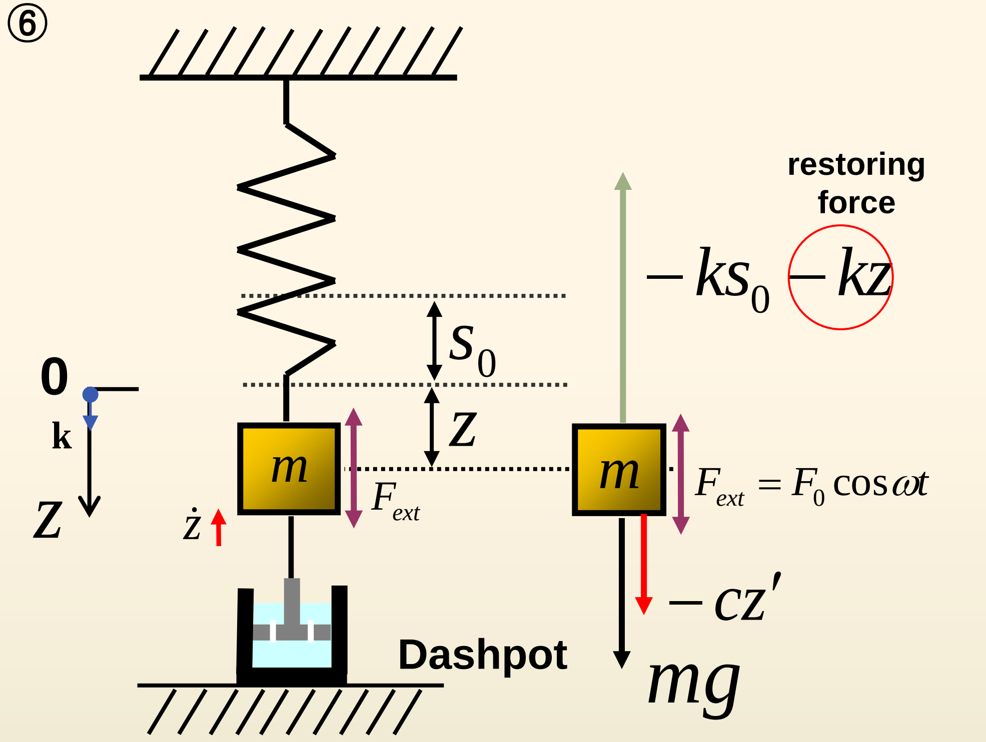
$$= -kz\mathbf{k} \quad - cz'\mathbf{k}$$

Physical Phenomenon

Mathematical Equation

$$\mathbf{mz''} + \mathbf{cz'} + \mathbf{kz} = 0$$

⑥



$$\mathbf{mz''} = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0}\cos\omega t$$

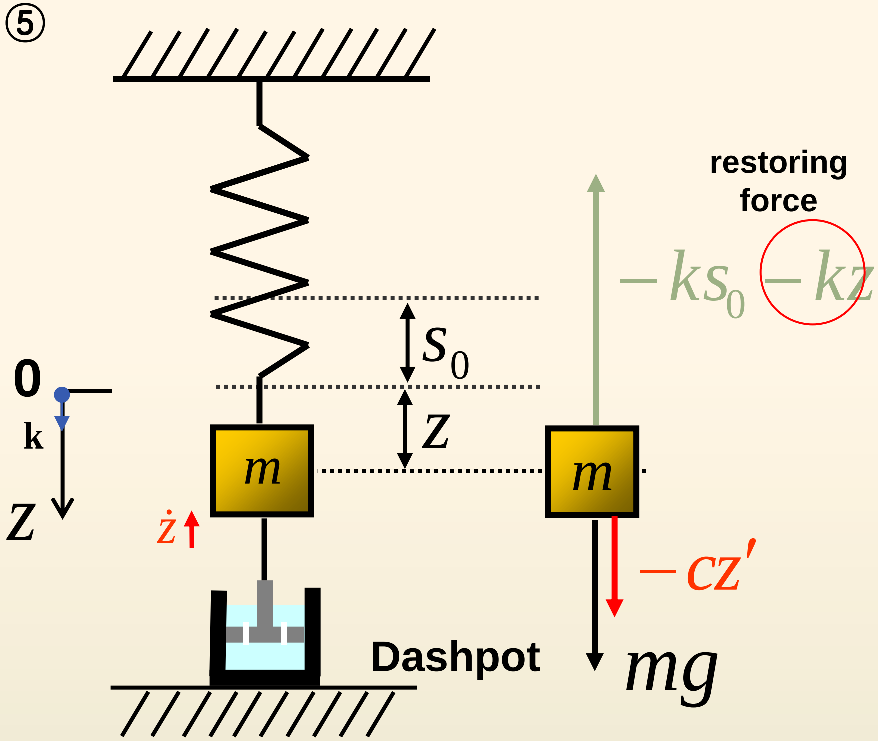
$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0}\cos\omega t$$

$$\mathbf{mz''} + \mathbf{cz'} + \mathbf{kz} = \mathbf{F_0}\cos\omega t$$

Spring/Mass Systems: Driven Motion

$z = z(t)\ ,\quad z'' = \dfrac{d^2 z}{dt^2}$

⑤



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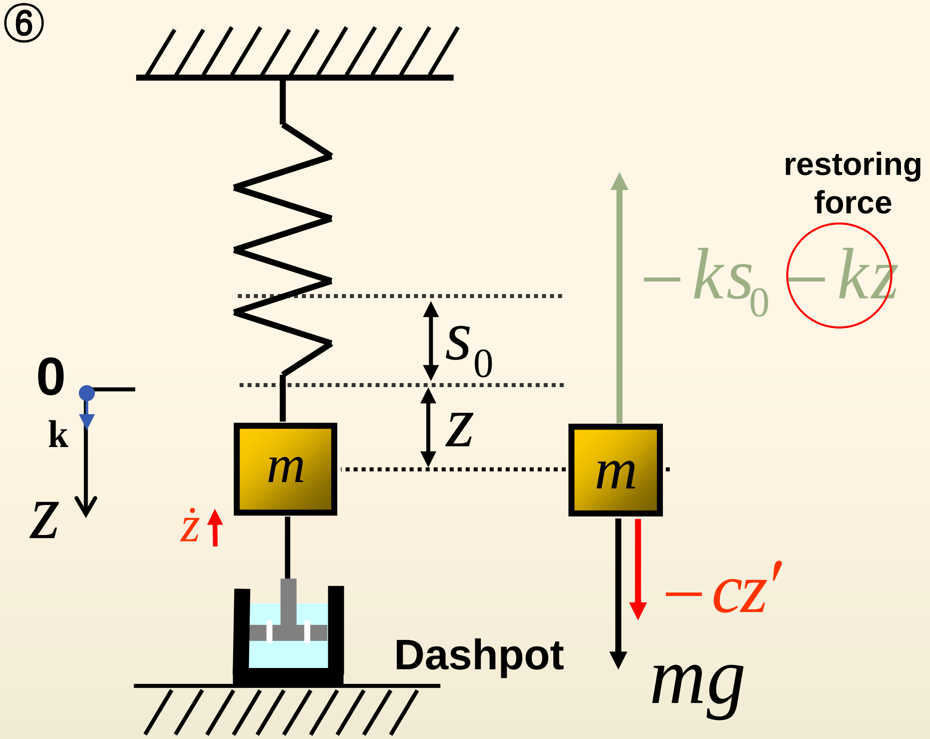
Physical Phenomenon

Mathematical Equation

$$\mathbf{mz''} + \mathbf{cz'} + \mathbf{kz} = 0$$

⑥



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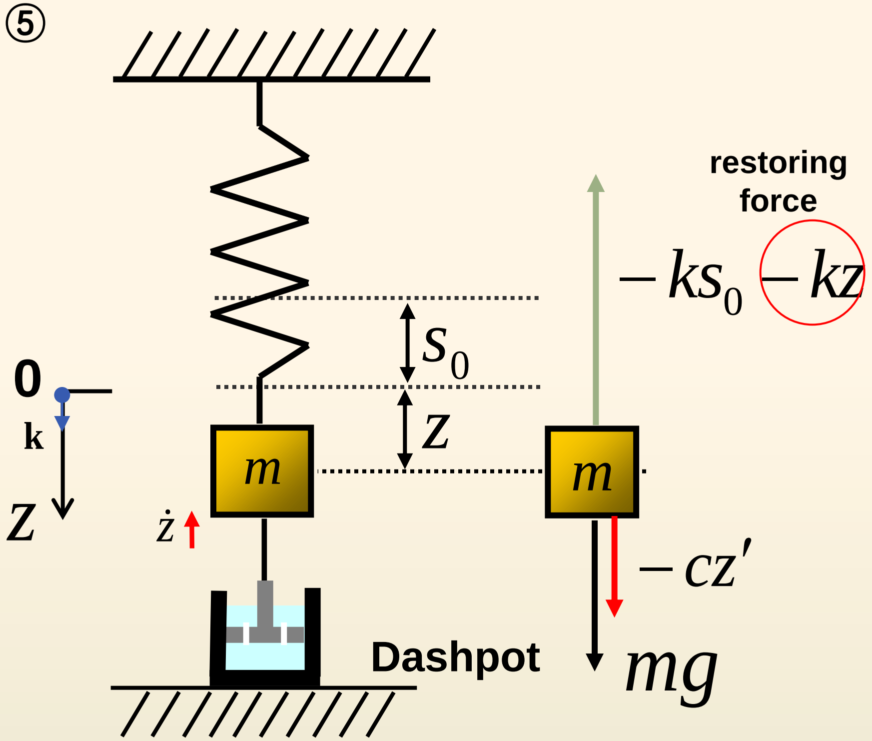
Spring/Mass Systems: Driven Motion

$z = z(t)\,,\quad z'' = \dfrac{d^2 z}{dt^2}$



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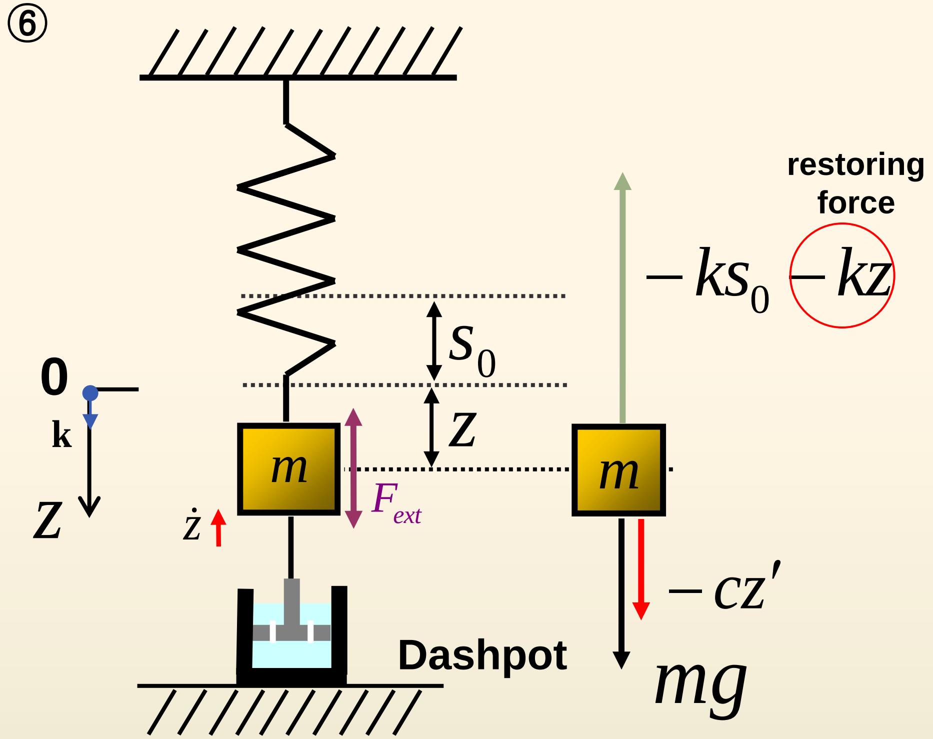
Physical Phenomenon

Mathematical Equation

$$\mathbf{mz}'' + \mathbf{cz}' + \mathbf{kz} = 0$$



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Spring/Mass Systems: Driven Motion

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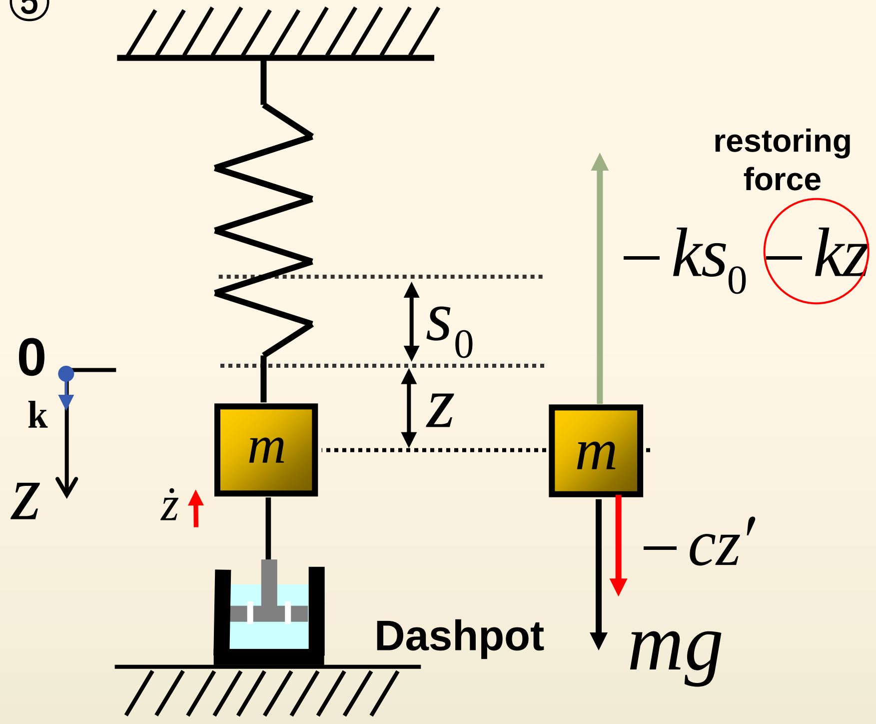
⑤



$$m\mathbf{z}'' = \mathbf{F}$$

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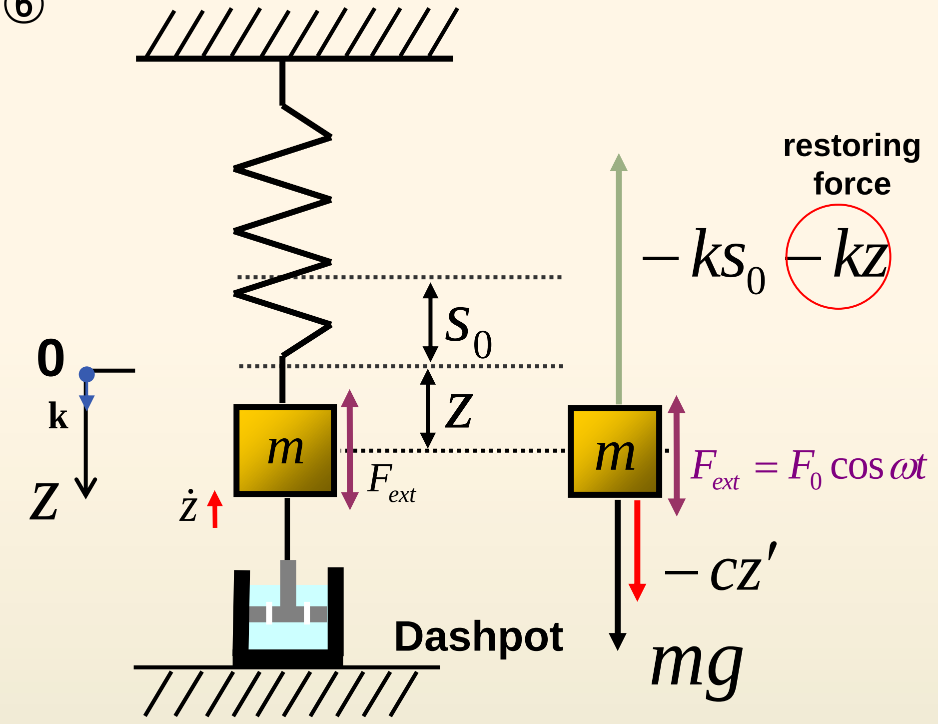
Physical Phenomenon

Mathematical Equation

$$m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = 0$$

⑥



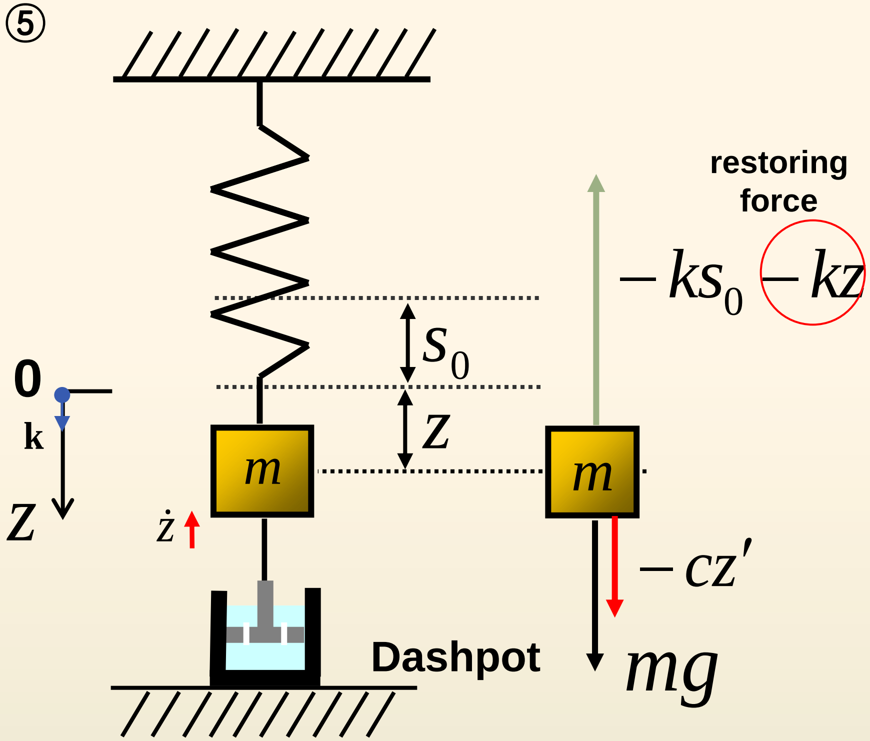
$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$

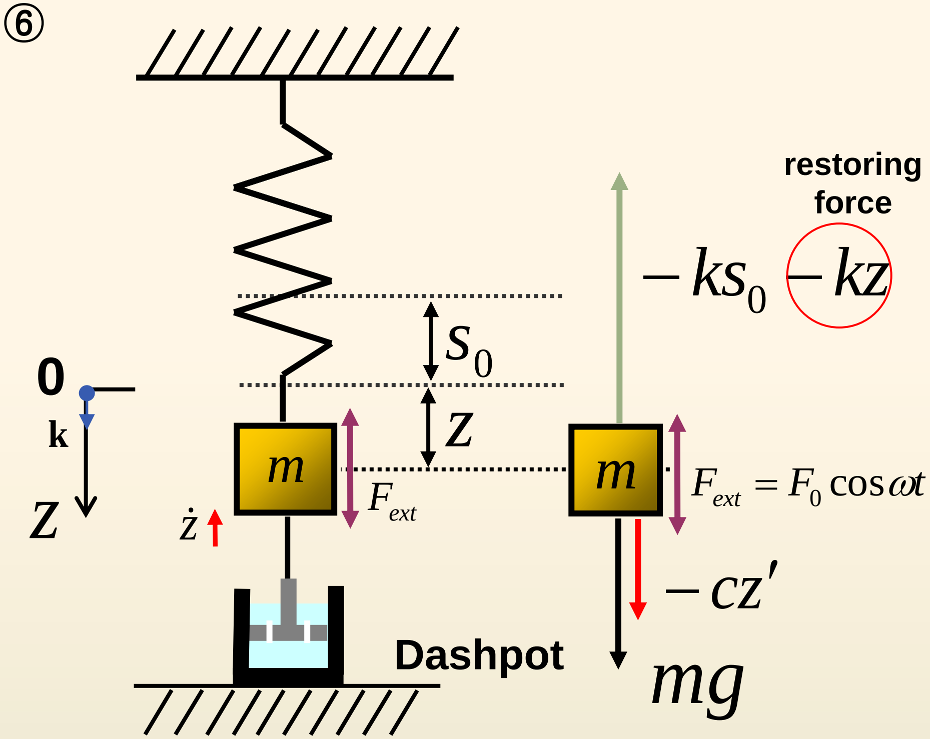
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

Spring/Mass Systems: Driven Motion

$z = z(t)\ ,\quad z'' = \frac{d^2 z}{dt^2}$



$$\mathbf{m}z'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$



$$\mathbf{m}z'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0}\cos \omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

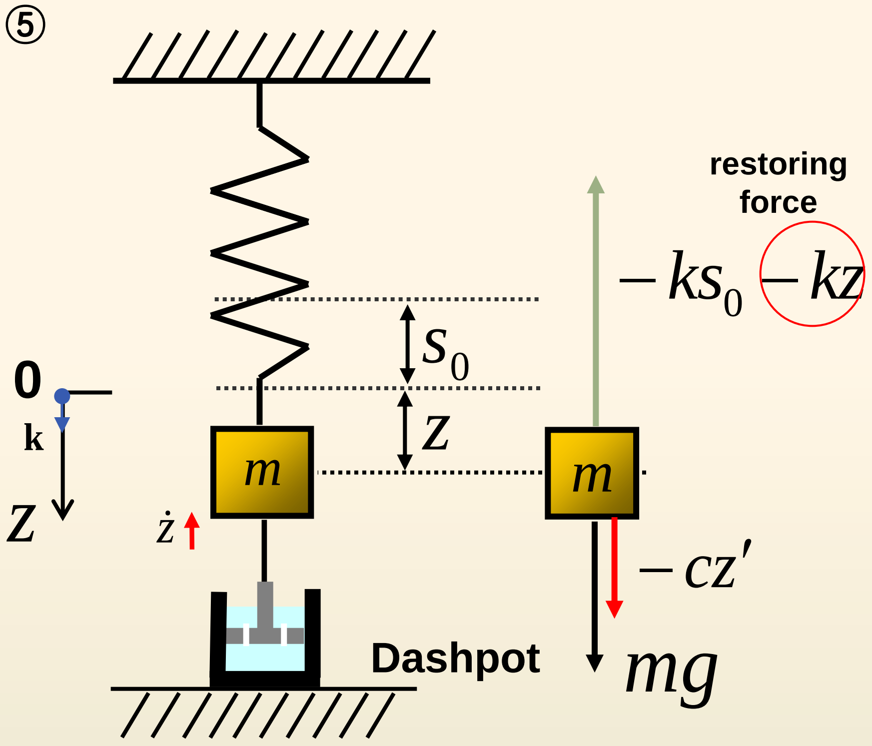
⇧ Physical Phenomenon

⇩ Mathematical Equation

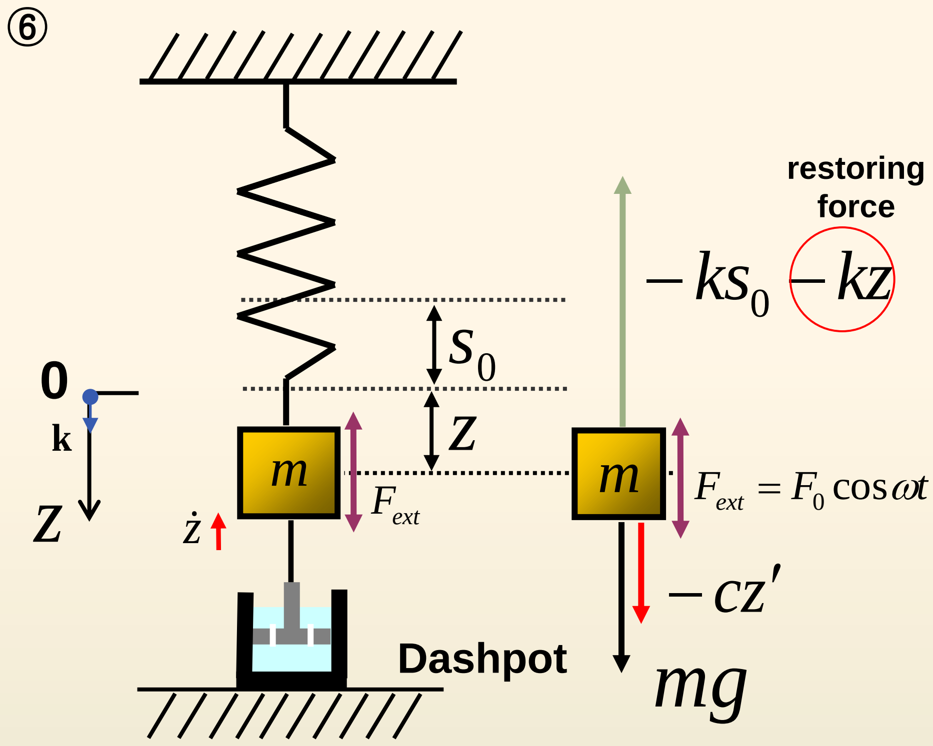
$$\mathbf{m}z'' + cz' + kz = 0$$

Spring/Mass Systems: Driven Motion

$z = z(t)\ , \quad z'' = \dfrac{d^2 z}{dt^2}$



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0} \cos \omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0} \cos \omega t$$

Physical Phenomenon

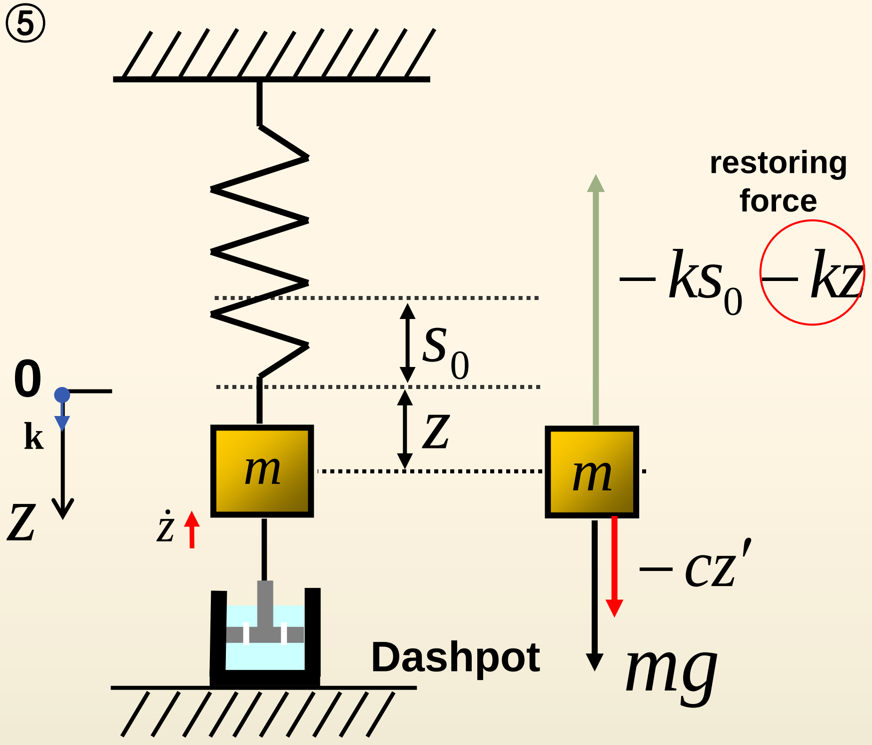
Mathematical Equation

$$m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = 0$$

Spring/Mass Systems: Driven Motion

$$z = z(t)\ ,\qquad z'' = \frac{d^2 z}{dt^2}$$

⑤

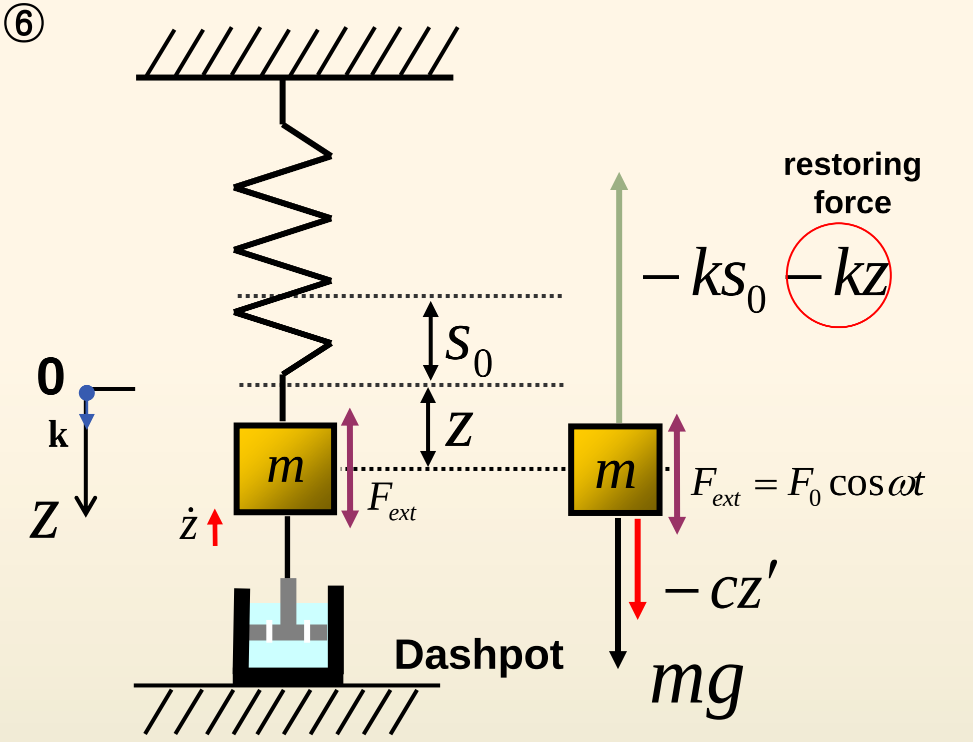


$$\mathbf{mz}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$

$$= -kz\mathbf{k} - cz'\mathbf{k}$$

⑥



$$\mathbf{mz}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0} \cos \omega t$$

$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0} \cos \omega t$$

Physical Phenomenon

Mathematical Equation

$$\mathbf{mz}'' + \mathbf{cz}' + \mathbf{kz} = 0$$

$$\mathbf{mz}'' + \mathbf{cz}' + \mathbf{kz} = \mathbf{F}_0 \cos \omega t$$

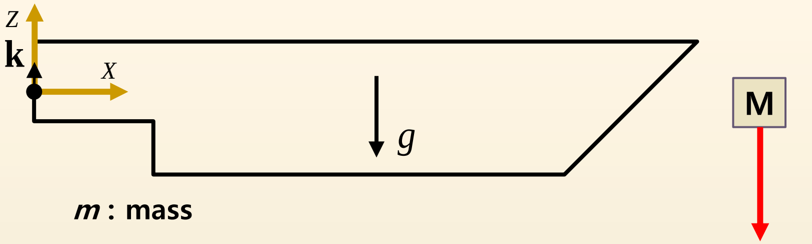
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 1

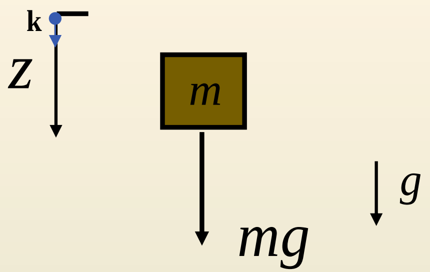


m : mass

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

✓ Mass-Spring-Damper system

①



By Newton's 2nd law,

$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k}$$

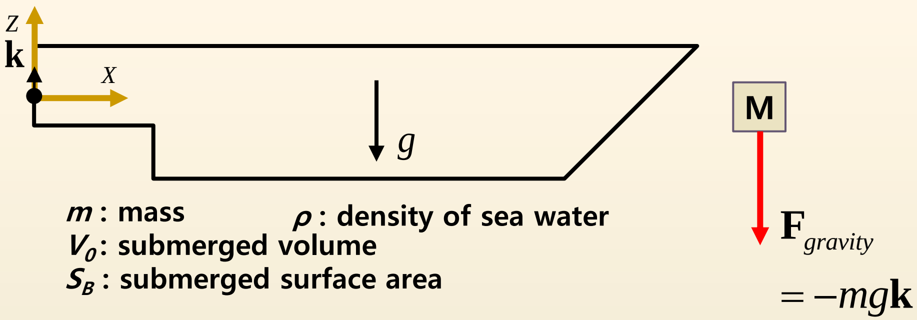
Nonlinearity

Nonlinearity of the nature ⇒ **Nonlinear Mathematical Model** ⇒ Numerical Method

⇓ Linearization

Linear Mathematical Model ⇒ **Analytic Solution**

Ex) Heave Motion of a Ship – step 2



m : mass

ρ : density of sea water

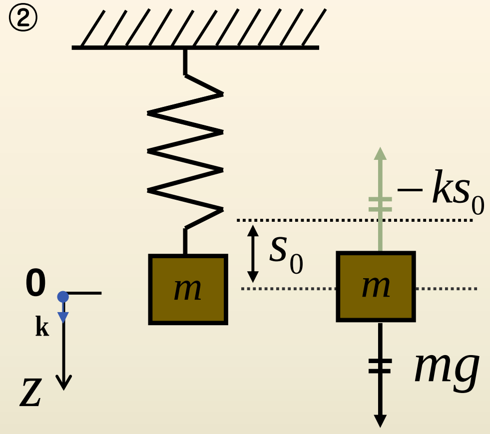
V_0: submerged volume

S_B : submerged surface area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

✓ Mass-Spring-Damper system

②



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \quad (\because z'' = 0)$$

: static equilibrium

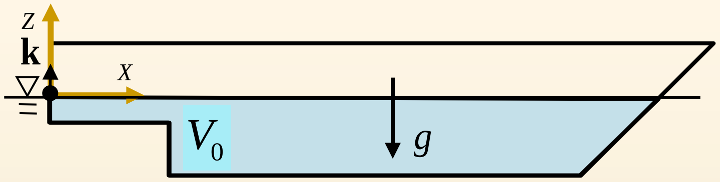
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

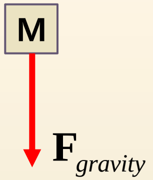
↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 2



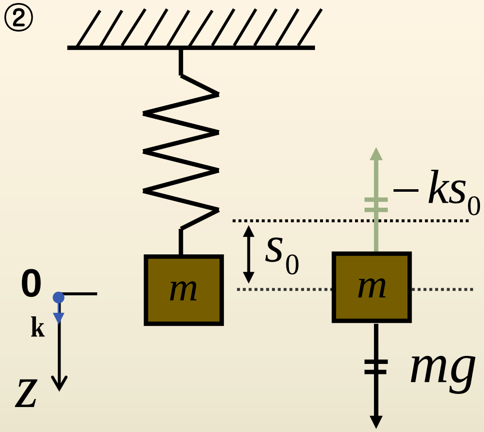
m : mass
ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area



$$= -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \quad (\because z'' = 0)$$

: static equilibrium

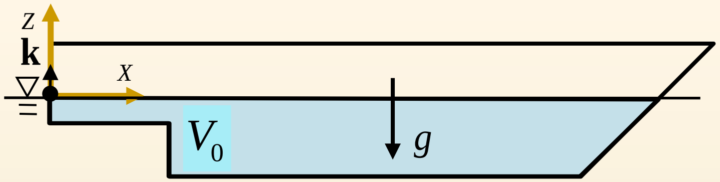
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

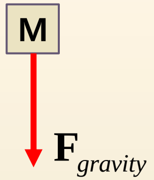
↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 2

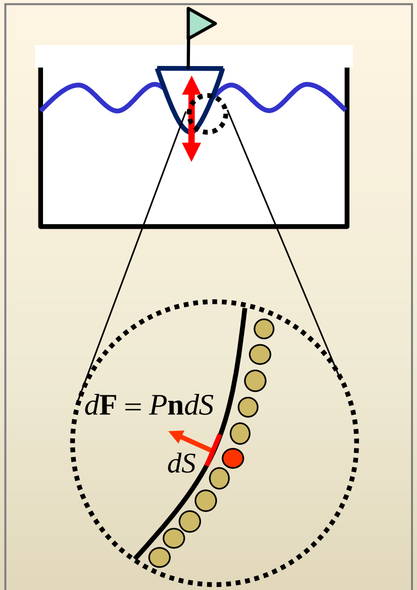


m : mass
ρ : density of sea water
V₀: submerged volume
S_B : submerged surface area



$$= -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

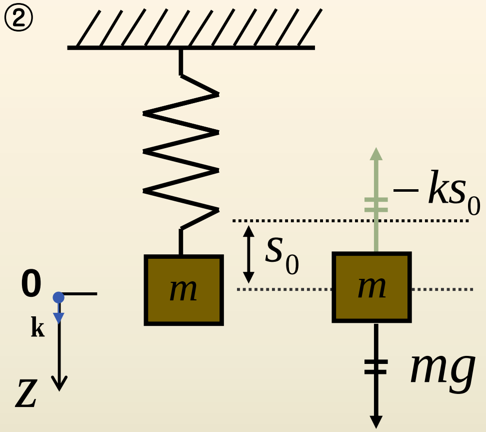


dS : infinitesimal submerged surface area

$d\mathbf{F}$: force exerted by the infinitesimal fluid element on dS

$\mathbf{n}$: normal vector of dS

✓ **Mass-Spring-Damper system**



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \quad (\because z'' = 0)$$

: static equilibrium

Nonlinearity

Nonlinearity of the nature ⇒ **Nonlinear Mathematical Model** ⇒ Numerical Method

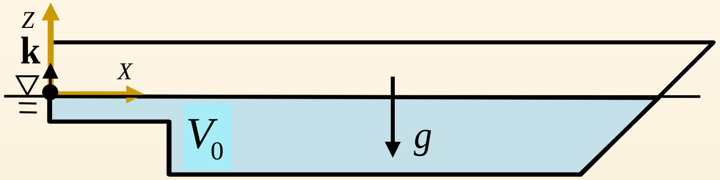
⇓ Linearization

Linear Mathematical Model ⇒ **Analytic Solution**

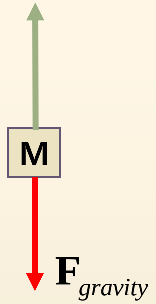
Ex) Heave Motion of a Ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

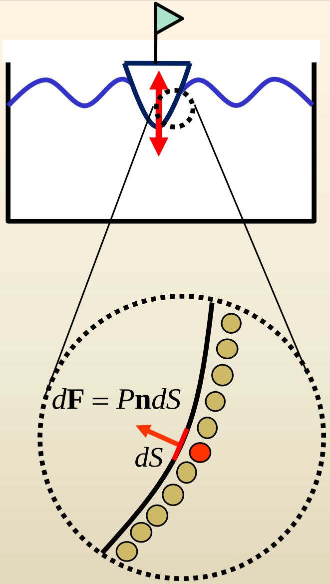


m : mass *ρ* : density of sea water
V_0: submerged volume
S_B : submerged surface area



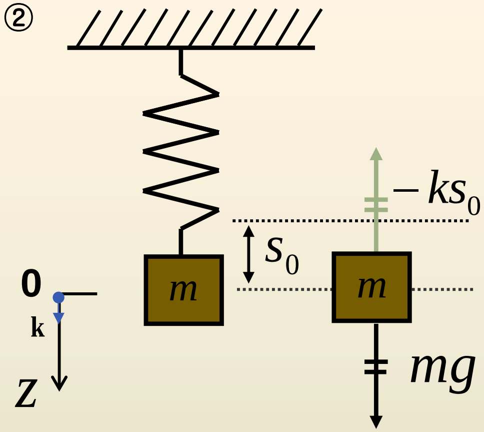
$= -mg\mathbf{k}$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$



dS : infinitesimal submerged surface area
$d\mathbf{F}$: force exerted by the infinitesimal fluid element on dS
$\mathbf{n}$: normal vector of dS

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \quad (\because z'' = 0)$$

: static equilibrium

Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

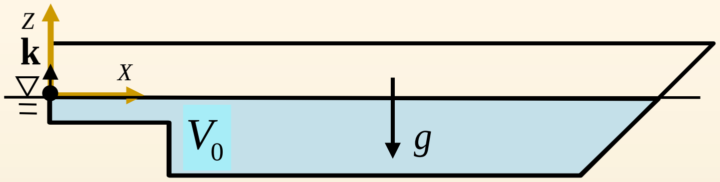
↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

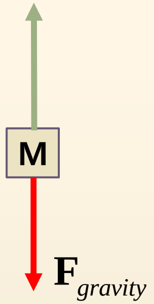


m : mass

ρ : density of sea water

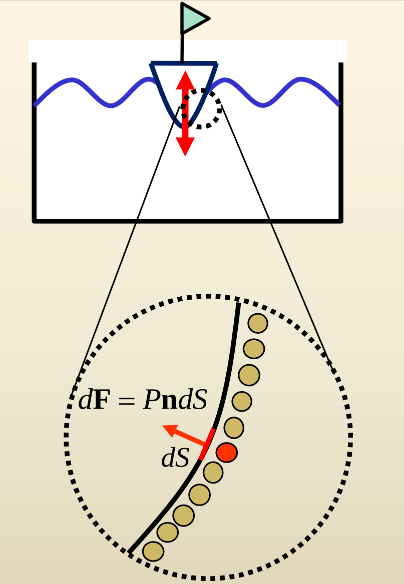
V_0: submerged volume

S_B : submerged surface area



$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F} = \mathbf{F}_{gravity} + \mathbf{F}_{static} = -mg\mathbf{k}$$



dS : infinitesimal submerged surface area

$d\mathbf{F}$: force exerted by the infinitesimal fluid element on dS

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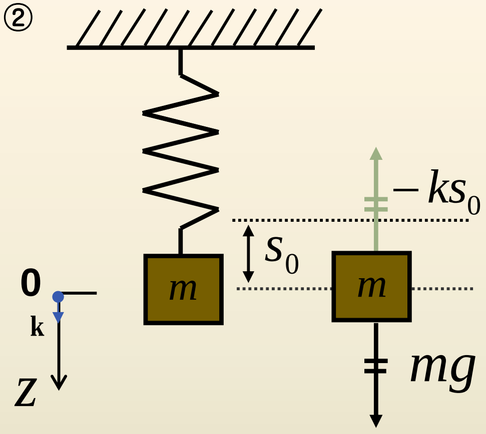
✓ Mass-Spring-Damper system



$$mz'' = \mathbf{F} = mg\mathbf{k} - ks_0\mathbf{k} = 0 \quad (\because z'' = 0)$$

: static equilibrium

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

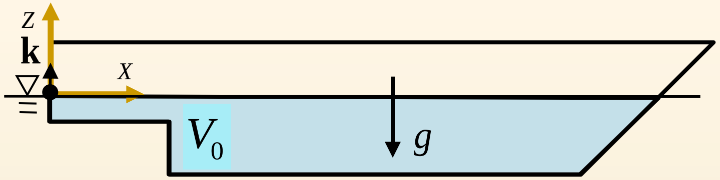
⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

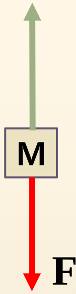


m : mass

ρ : density of sea water

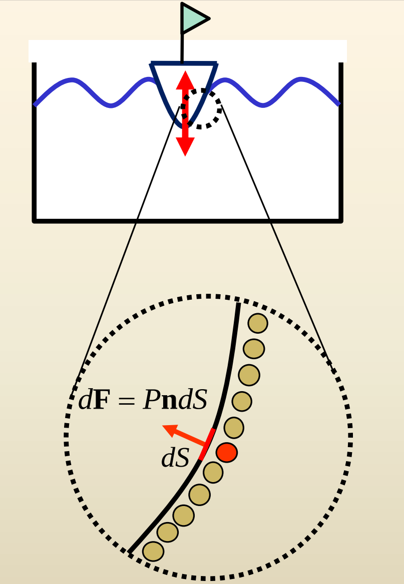
V_0: submerged volume

S_B : submerged surface area



$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k}$$



dS : infinitesimal submerged surface area

$d\mathbf{F}$: force exerted by the infinitesimal fluid element on dS

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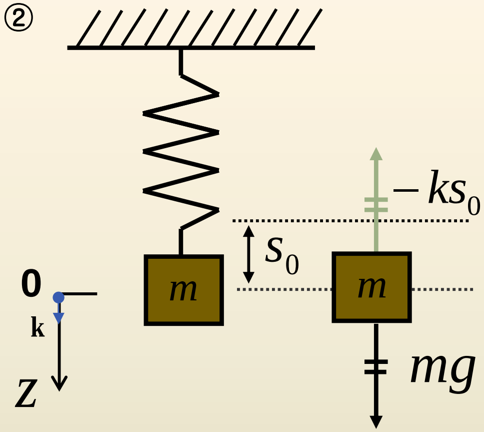
✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \quad (\because z'' = 0)$$

: static equilibrium

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

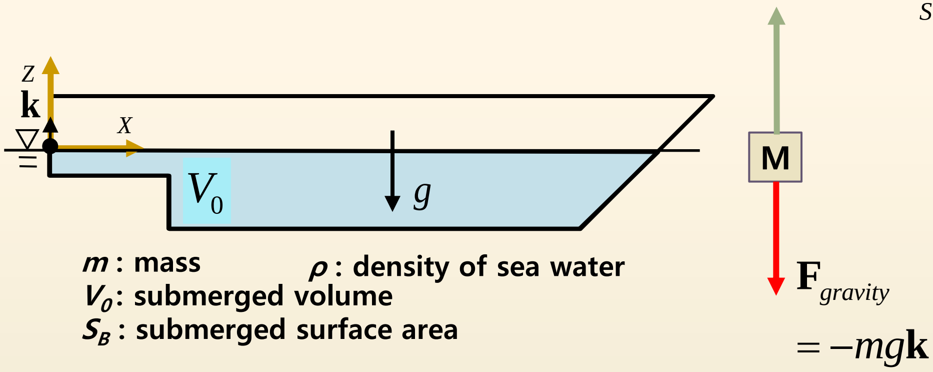
⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

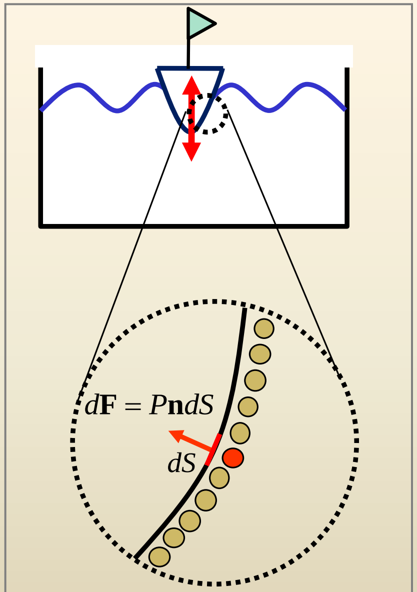
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$



m : mass
ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k}$$
$$= 0 \ (\because \ddot{z} = 0)$$

: static equilibrium



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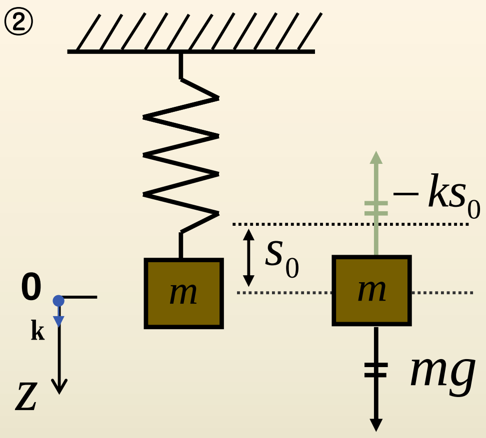
$d\mathbf{F}$: force exerted by the infinitesimal fluid element on dS
$\mathbf{n}$: normal vector of dS

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \ (\because z'' = 0)$$

: static equilibrium

Nonlinearity

Nonlinearity of the nature ⇒ **Nonlinear Mathematical Model** ⇒ Numerical Method

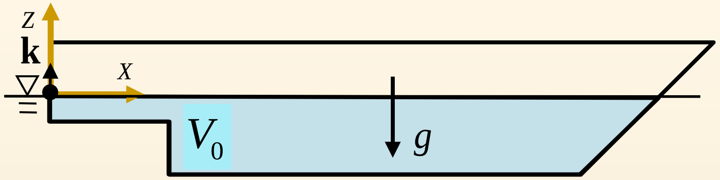
⇓ **Linearization**

Linear Mathematical Model ⇒ **Analytic Solution**

Ex) Heave Motion of a Ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

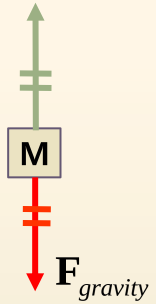


m : mass

ρ : density of sea water

V₀: submerged volume

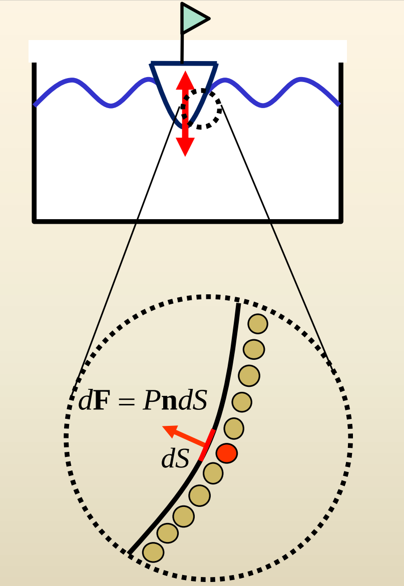
S_B : submerged surface area



$= -mg\mathbf{k}$

$$
\begin{aligned}
m\ddot{\mathbf{z}} &= \mathbf{F} \\
&= \mathbf{F}_{gravity} + \mathbf{F}_{static} \\
&= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \\
&= 0 \ (\because \ddot{z} = 0)
\end{aligned}
$$

: static equilibrium



dS : infinitesimal submerged surface area

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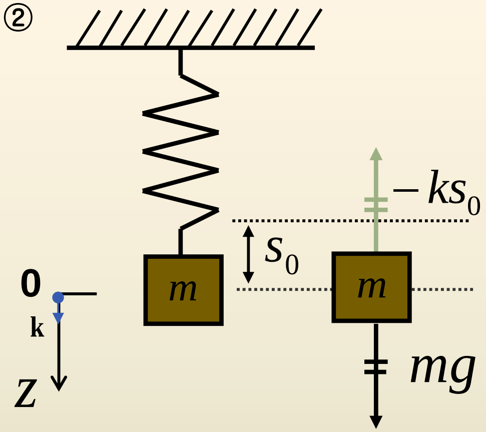
✓ Mass-Spring-Damper system



$$
\begin{aligned}
m\mathbf{z}'' &= \mathbf{F} \\
&= mg\mathbf{k} - ks_0\mathbf{k} \\
&= 0 \ \ (\because z'' = 0)
\end{aligned}
$$

: static equilibrium

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

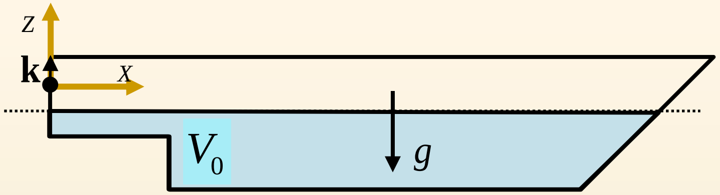
⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$



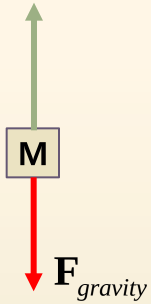
m : mass

ρ : density of sea water

V_0: submerged volume

S_B : submerged surface area

Awp : waterplane area



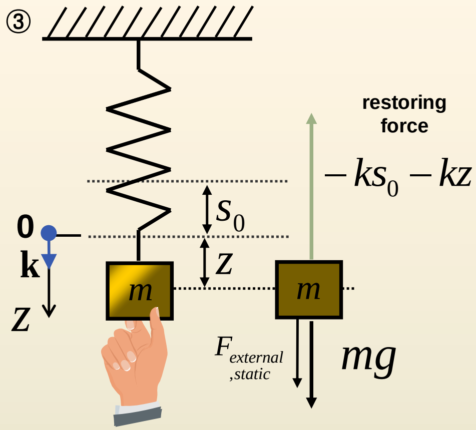
$$= -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$

$$= -mg\mathbf{k}$$

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= 0 \quad (\because z'' = 0)$$

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

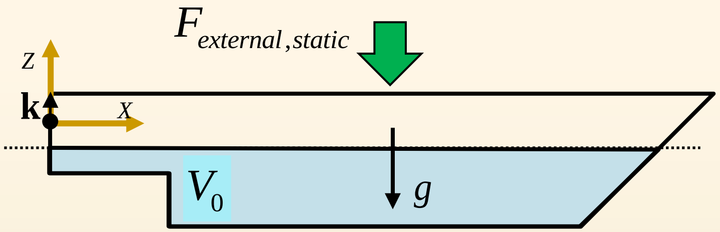
⇩ Linearization

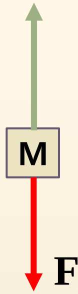
Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3

$$\mathbf{F}_{static}=\iint_{S_B} P_{static}\mathbf{n}dS$$

✓ Archimedes' Principle $\mathbf{F}_{static}=\rho g V_0 \mathbf{k}$





$$\mathbf{F}_{gravity}$$
$$=-mg\mathbf{k}$$

m : mass

ρ : density of sea water

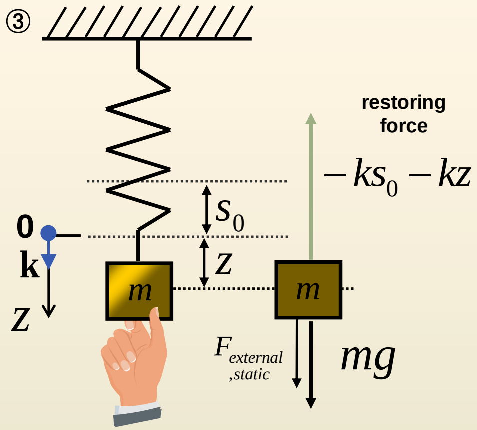
V_0: submerged volume

S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}}=\mathbf{F}$$
$$=\mathbf{F}_{gravity}+\mathbf{F}_{static}$$
$$=-mg\mathbf{k}$$

✓ Mass-Spring-Damper system



$$mz''=\mathbf{F}$$
$$=mg\mathbf{k}-ks_0\mathbf{k}-kz\mathbf{k}+\mathbf{F}_{external,static}$$
$$=-kz\mathbf{k}+\mathbf{F}_{external,static}$$
$$=0 \quad (\because z''=0)$$

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

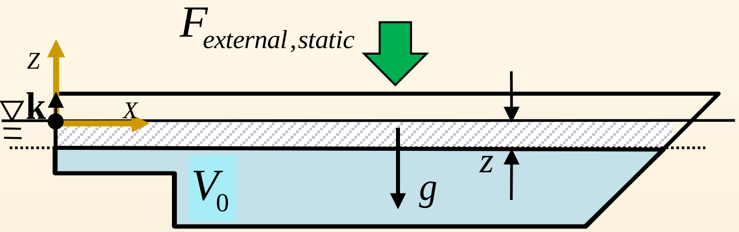
⇩ **Linearization**

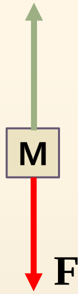
Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3

$$\mathbf{F}_{static} = \iint_{S_B} P_{static} \mathbf{n} dS$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$





$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

m : mass

ρ : density of sea water

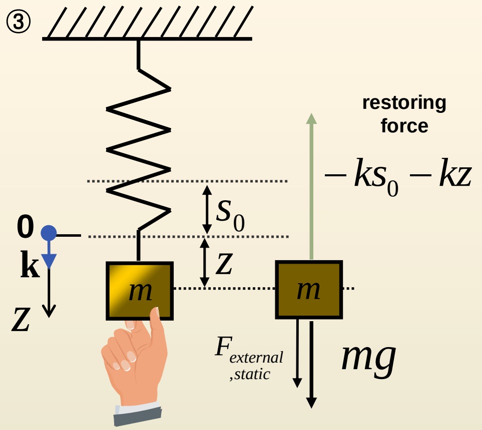
V₀: submerged volume

S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k}$$

✓ Mass-Spring-Damper system



$$mz'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
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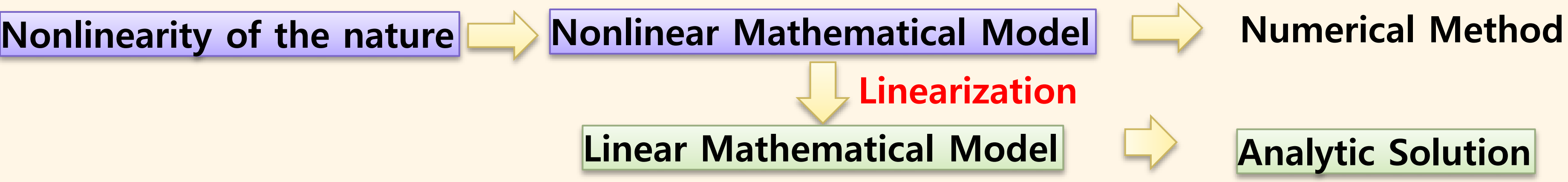
Nonlinearity

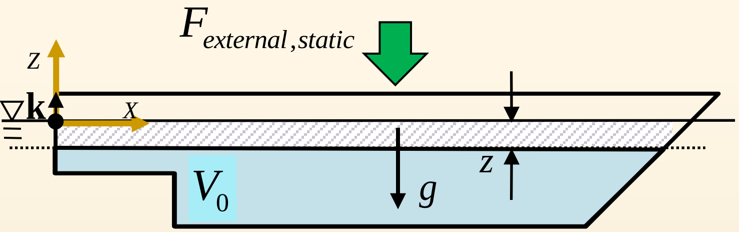


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



- ***m*** : mass
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$$m\ddot{\mathbf{z}} = \mathbf{F}$$

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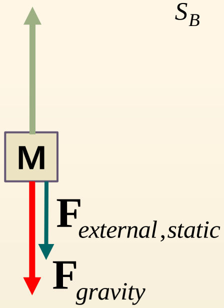
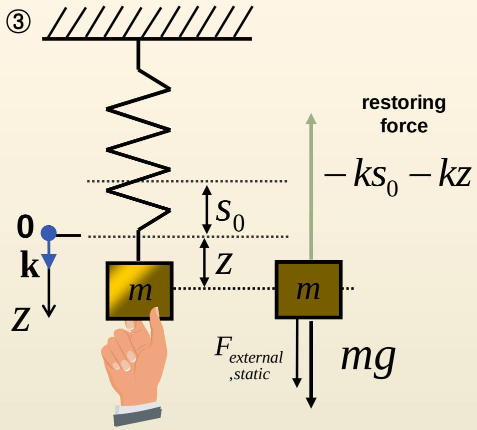


$$= -mg\mathbf{k}$$

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system



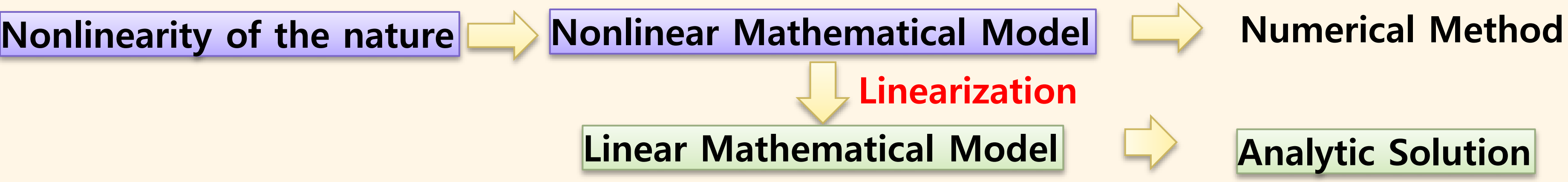
$$mz'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= 0 \quad (\because z'' = 0)$$

Nonlinearity


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 3

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

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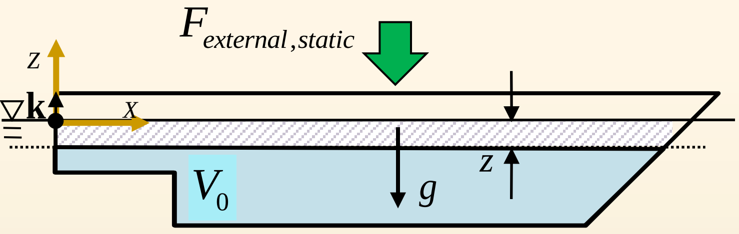

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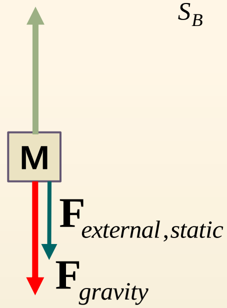
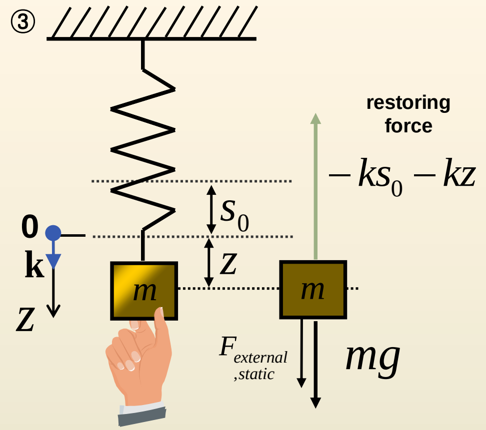


$= -mg\mathbf{k}$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k}$$

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
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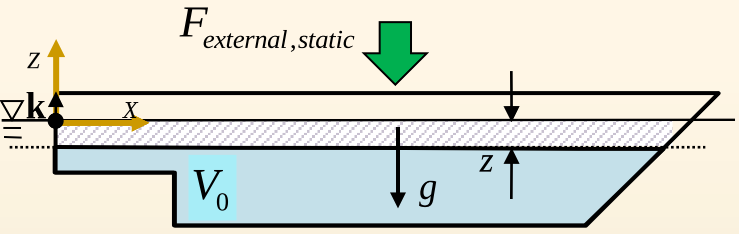
Nonlinearity

Nonlinearity of the nature ⇒ Nonlinear Mathematical Model ⇒ Numerical Method

⇓ Linearization

Linear Mathematical Model ⇒ Analytic Solution

Ex) Heave Motion of a Ship – step 3



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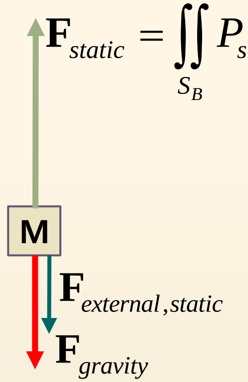
$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k}$$

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

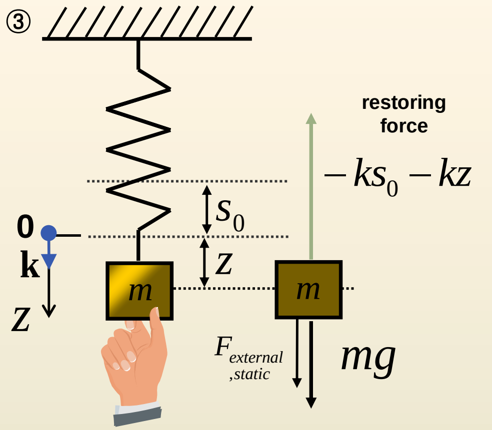


$$= -mg\mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system

③



$$m\mathbf{z}'' = \mathbf{F}$$
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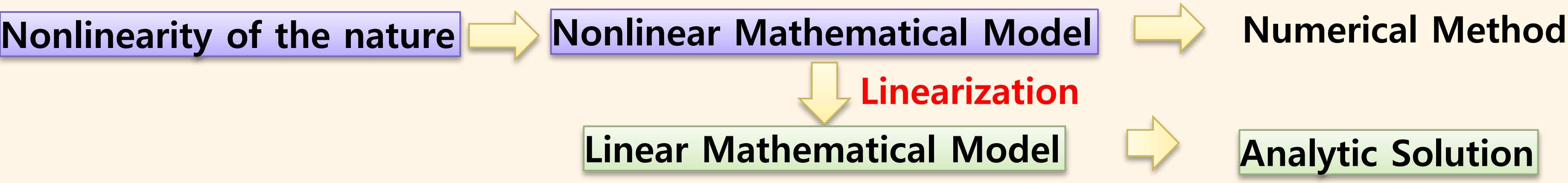
Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

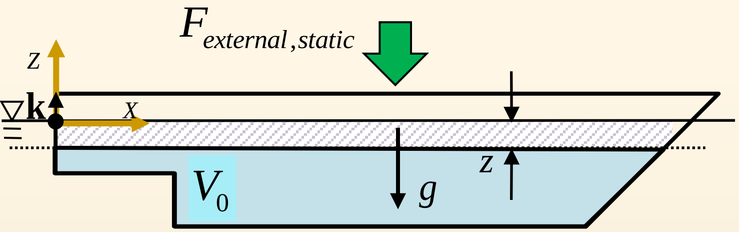
Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

Ex) Heave Motion of a Ship – step 3



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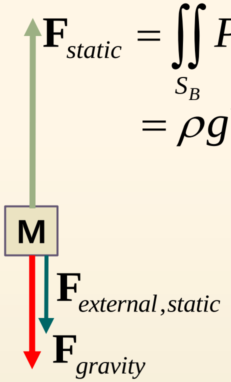
$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k}$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

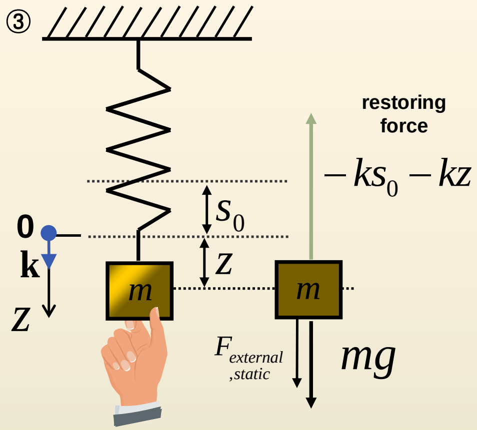
$$\mathbf{F}_{external,static}$$

$$\mathbf{F}_{gravity}$$

$$= -mg\mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system



$$mz'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

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$$= 0 \quad (\because z'' = 0)$$

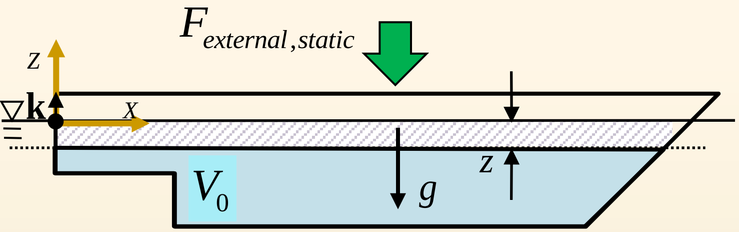
Nonlinearity

Nonlinearity of the nature ⇒ **Nonlinear Mathematical Model** ⇒ Numerical Method

⇓ Linearization

Linear Mathematical Model ⇒ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



m : mass

ρ : density of sea water

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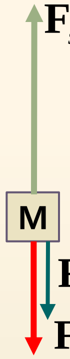
S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k}$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0\mathbf{k} + \mathbf{F}_{addtional\,bouyancy}$$

additional buoyancy caused by additional displacement z

$\mathbf{F}_{external,static}$

$\mathbf{F}_{gravity}$

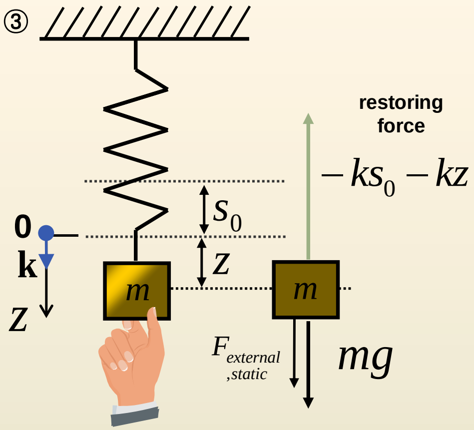
$$= -mg\mathbf{k}$$

if, z is small

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

✓ Mass-Spring-Damper system

③



$$mz'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= 0 \quad (\because z'' = 0)$$

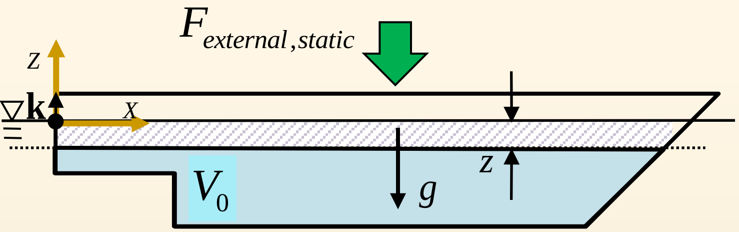
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



m **: mass** ***ρ*** **: density of sea water**

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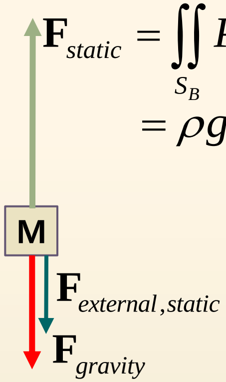
Awp **: waterplane area**

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k}$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static} \mathbf{n} dS$$

$$= \rho g V_0 \mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$

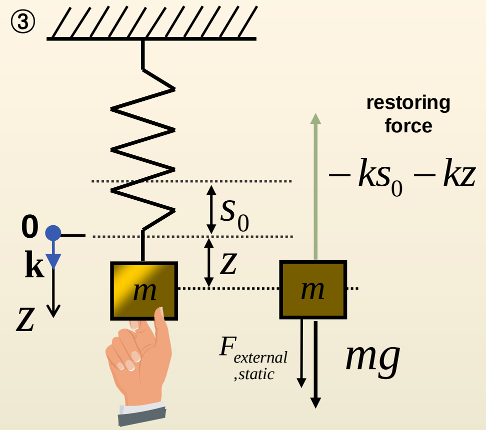
$$= -\rho g A_{WP} \mathbf{z}$$

$$= -k\mathbf{z}$$

$$, k = \rho g A_{WP}$$

✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= 0 \quad (\because z'' = 0)$$

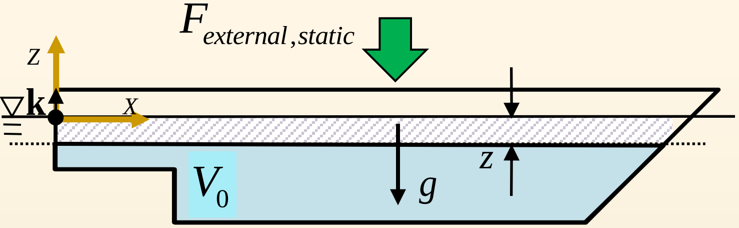
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



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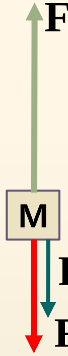


$$\mathbf{F}_{static}=\iint_{S_B}P_{static}\mathbf{n}dS$$
$$=\rho gV_0\mathbf{k}+\underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

$\mathbf{F}_{external,static}$

$\mathbf{F}_{gravity}$

$$=-mg\mathbf{k}$$

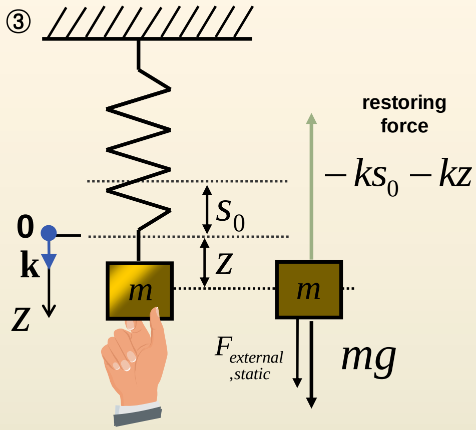
if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$
$$=-\rho gA_{WP}\mathbf{z}$$
$$=-k\mathbf{z}$$
$$,k=\rho gA_{WP}$$

$$m\ddot{\mathbf{z}}=\mathbf{F}$$
$$=\mathbf{F}_{gravity}+\mathbf{F}_{static}+\mathbf{F}_{external,static}$$
$$=-mg\mathbf{k}+\rho gV_0\mathbf{k}\ \ -\rho gA_{wp}\mathbf{z}$$

✓ Archimedes' Principle $\mathbf{F}_{static}=\rho gV_0\mathbf{k}$

✓ Mass-Spring-Damper system



$$mz''=\mathbf{F}$$
$$=mg\mathbf{k}-ks_0\mathbf{k}-kz\mathbf{k}+\mathbf{F}_{external,static}$$
$$=-kz\mathbf{k}+\mathbf{F}_{external,static}$$
$$=0\quad(\because z''=0)$$

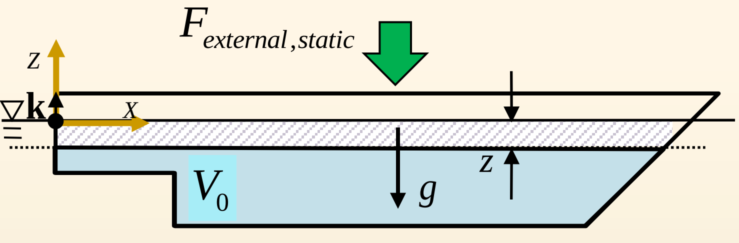
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



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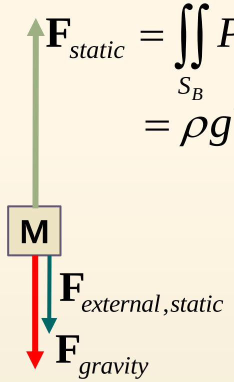


$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0\mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

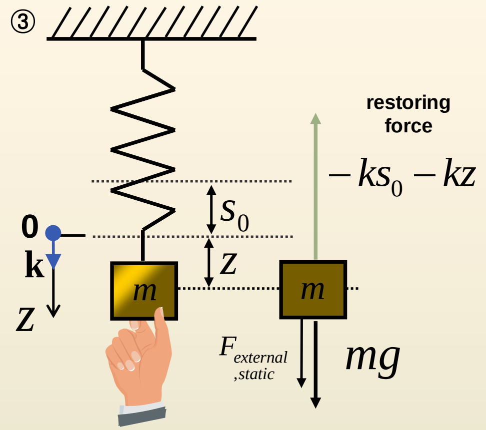
if, z is small

$$\mathbf{F}_{addtional\,bouyancy} = -\rho g A_{WP}\mathbf{z} = -k\mathbf{z} \quad , k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F} = \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static} = -mg\mathbf{k} + \rho g V_0\mathbf{k} \ -\rho g A_{wp}\mathbf{z} \ +\mathbf{F}_{external,static}$$

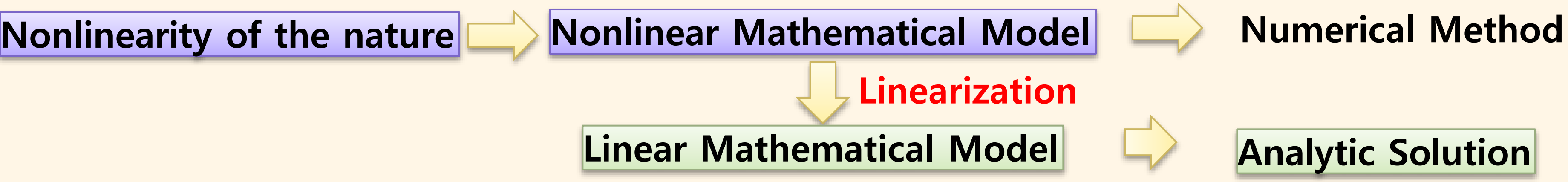
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

✓ Mass-Spring-Damper system



$$m z'' = \mathbf{F} = mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static} = -kz\mathbf{k} + \mathbf{F}_{external,static} = 0 \quad (\because z'' = 0)$$

Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

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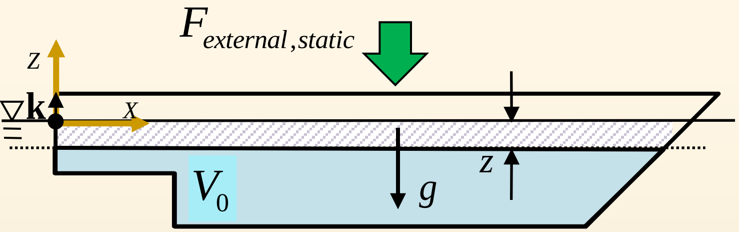
Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



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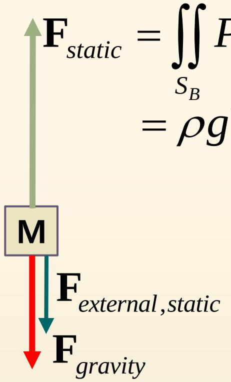
$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \ - \rho g A_{wp}\mathbf{z} \ + \mathbf{F}_{external,static}$$

$$= -\rho g A_{wp}\mathbf{z} \ + \mathbf{F}_{external,static}$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} + \underline{\mathbf{F}_{addtional\ bouyancy}}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\ bouyancy}$$

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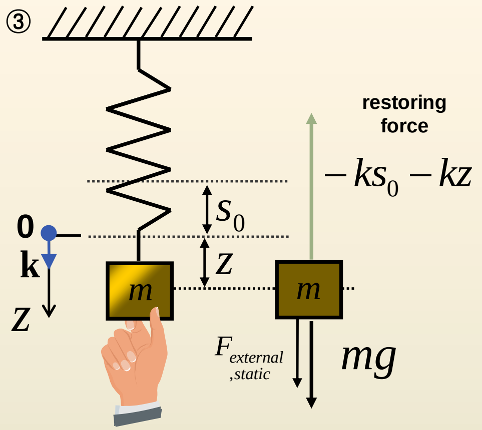
✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**

③



$$m\mathbf{z}'' = \mathbf{F}$$

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Nonlinearity

Nonlinearity of the nature ⇒ **Nonlinear Mathematical Model** ⇒ Numerical Method

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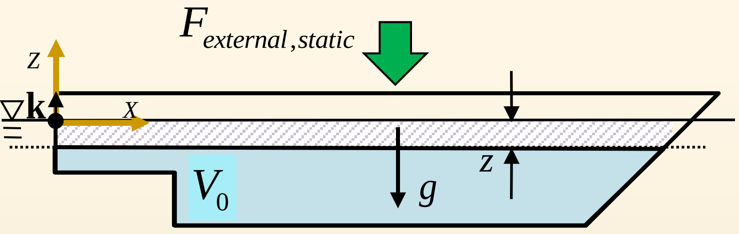
Linear Mathematical Model ⇒ **Analytic Solution**

Ex) Heave Motion of a Ship – step 3



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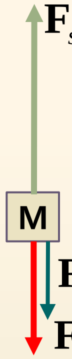
ρ : density of sea water

V₀: submerged volume

S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -k\mathbf{z} + \mathbf{F}_{external,static}$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

$$= -mg\mathbf{k}$$

additional buoyancy caused by additional displacement z

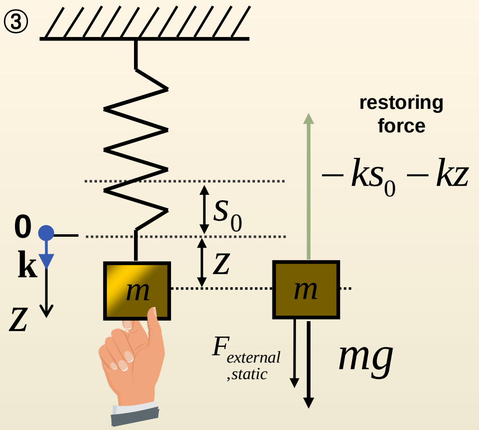
if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$
$$= -\rho g A_{WP}\mathbf{z}$$
$$= -k\mathbf{z}$$
$$, k = \rho g A_{WP}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

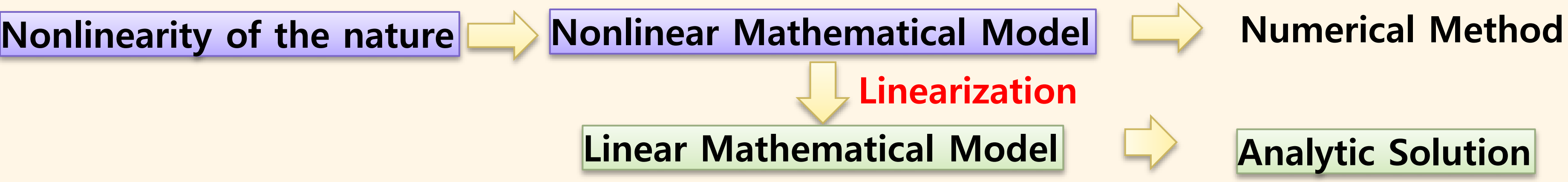
✓ **Mass-Spring-Damper system**

③



$$mz'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= 0 \quad (\because z'' = 0)$$

Nonlinearity

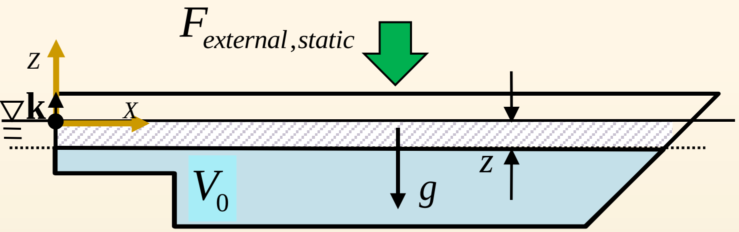


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ **Linearization**

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 3



m : mass

ρ : density of sea water

V₀: submerged volume

S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

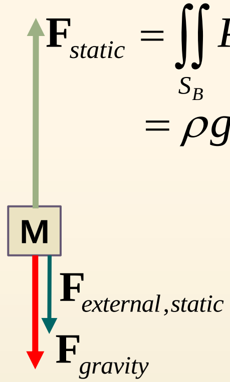
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \; -\rho g A_{wp}\mathbf{z} \; + \mathbf{F}_{external,static}$$

$$= -\rho g A_{wp}\mathbf{z} \; + \mathbf{F}_{external,static}$$

$$= -k\mathbf{z} \; + \mathbf{F}_{external,static}$$

$$= 0 \; (\because \ddot{z} = 0)$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\,bouyancy}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$

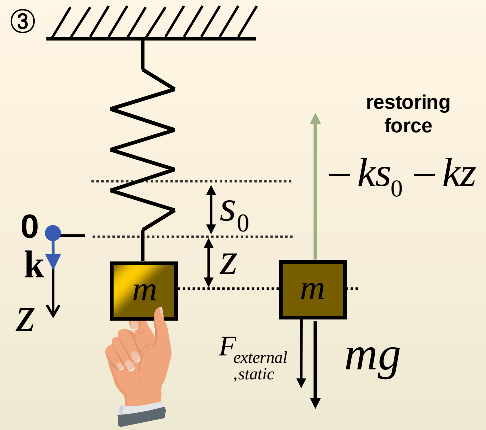
$$= -\rho g A_{WP}\mathbf{z}$$

$$= -k\mathbf{z}$$

$$, k = \rho g A_{WP}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system



$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$= 0 \quad (\because z'' = 0)$$

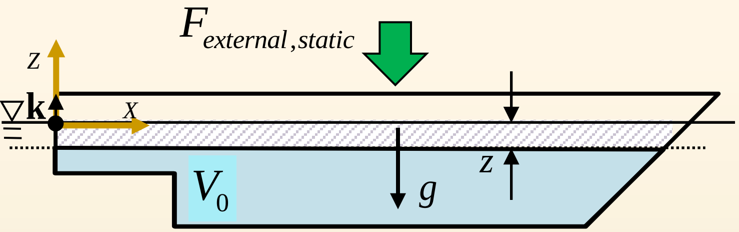
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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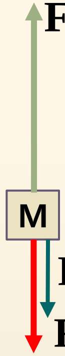

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0\mathbf{k} + \mathbf{F}_{addtional\,bouyancy}$$

additional buoyancy caused by additional displacement z

$\mathbf{F}_{external,static}$

$$\mathbf{F}_{gravity}$$

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$
$$= -\rho g A_{WP}\mathbf{z}$$
$$= -k\mathbf{z}$$
$$, k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
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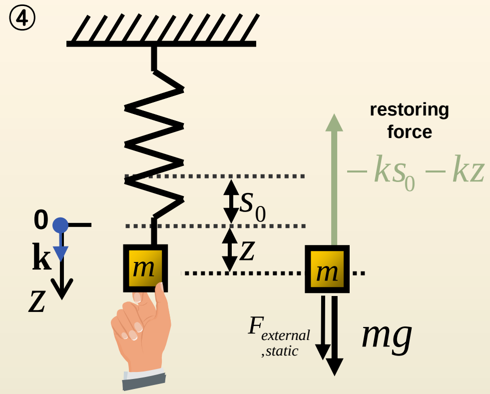
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

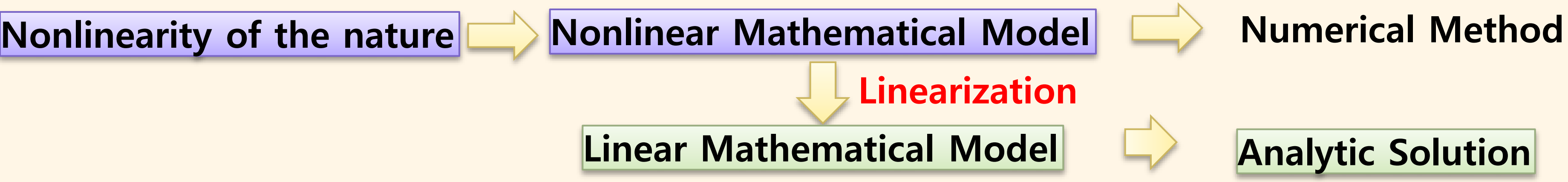
✓ **Mass-Spring-Damper system**

④



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

Nonlinearity

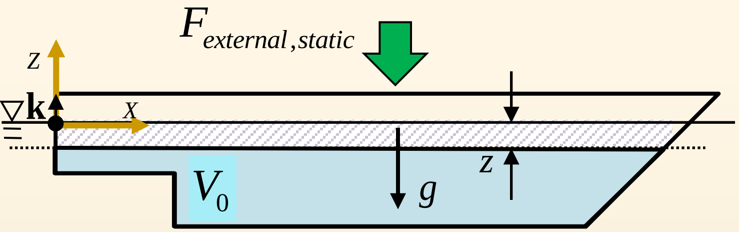


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 4



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$$m\ddot{\mathbf{z}} = \mathbf{F}$$
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$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -k\mathbf{z} + \mathbf{F}_{external,static} \quad , k = \rho g A_{WP}$$
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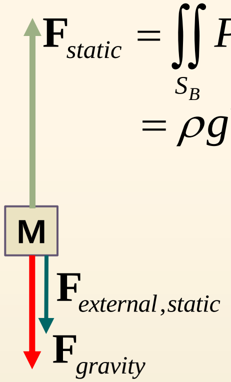


$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0\mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

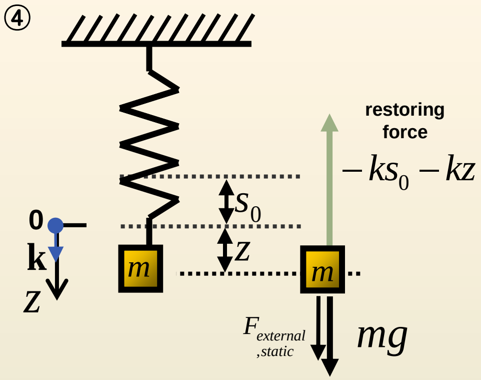
if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$
$$= -\rho g A_{WP}\mathbf{z}$$
$$= -k\mathbf{z}$$
$$, k = \rho g A_{WP}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

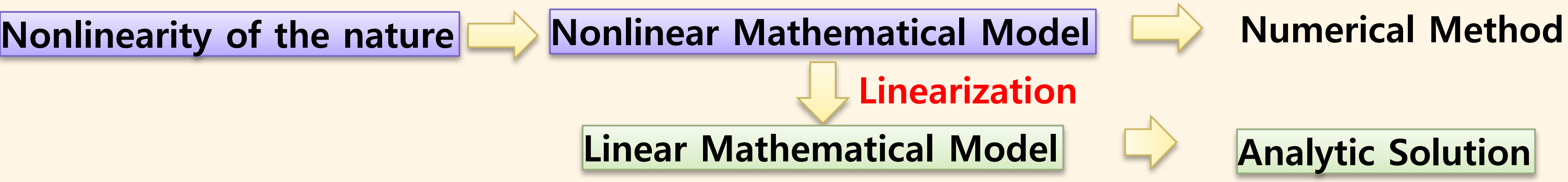
✓ Mass-Spring-Damper system

④



$$\boxed{\begin{aligned} mz'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static} \\ &= -kz\mathbf{k} \quad + \mathbf{F}_{external,static} \end{aligned}}$$

Nonlinearity

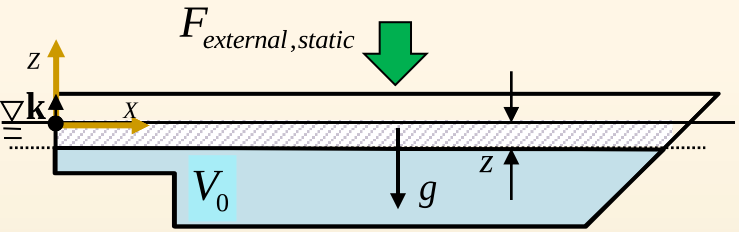


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

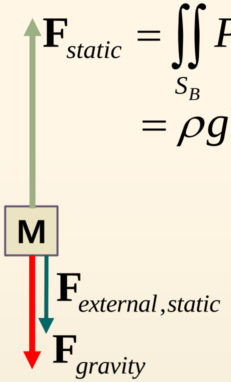
↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



m : mass
ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
A_{wp} : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy} = -\rho g A_{WP}\mathbf{z} = -k\mathbf{z} \quad , k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \; -\rho g A_{wp}\mathbf{z} \; + \mathbf{F}_{external,static}$$

$$= -\rho g A_{wp}\mathbf{z} \; + \mathbf{F}_{external,static}$$

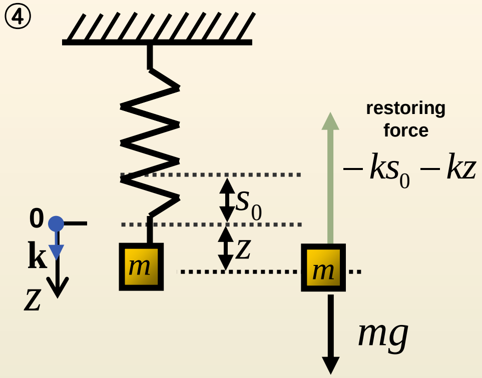
$$= -k\mathbf{z} \; + \mathbf{F}_{external,static} \; , k = \rho g A_{WP}$$

$$= 0 \; (\because \ddot{z} = 0)$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

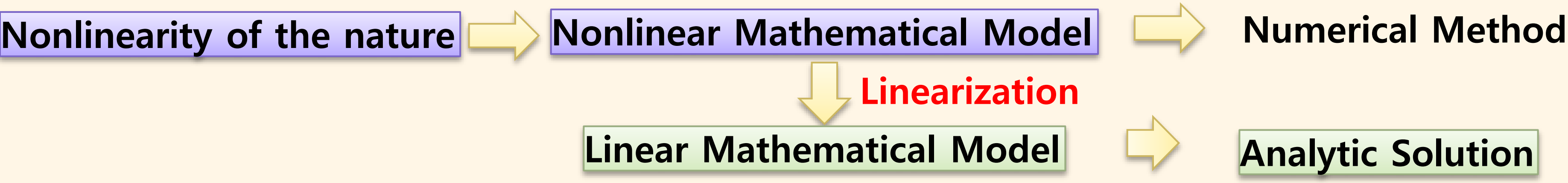
✓ Mass-Spring-Damper system

④



$$\boxed{\begin{aligned} mz'' &= \mathbf{F} \\ &= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \; + \mathbf{F}_{external,static} \\ &= -kz\mathbf{k} \quad + \mathbf{F}_{external,static} \end{aligned}}$$

Nonlinearity

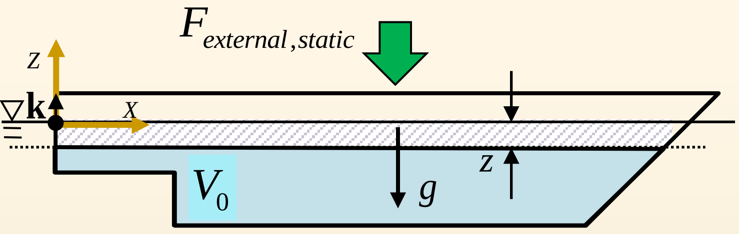


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



m : mass　　***ρ*** : density of sea water

V₀: submerged volume

S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

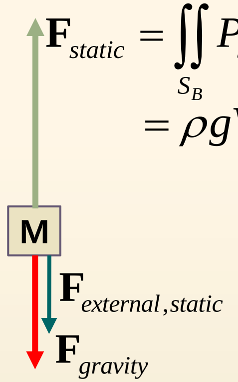
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k} + \rho g V_0\mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$

$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$

$$= -k\mathbf{z} + \mathbf{F}_{external,static} \; , k = \rho g A_{WP}$$

$$= 0 \; (\because \ddot{z} = 0)$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0\mathbf{k} + \underline{\mathbf{F}_{addtional\,bouyancy}}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$

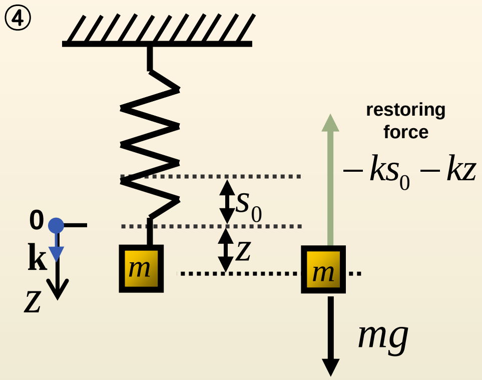
$$= -\rho g A_{WP}\mathbf{z}$$

$$= -k\mathbf{z}$$

$$, k = \rho g A_{WP}$$

✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

✓ **Mass-Spring-Damper system**

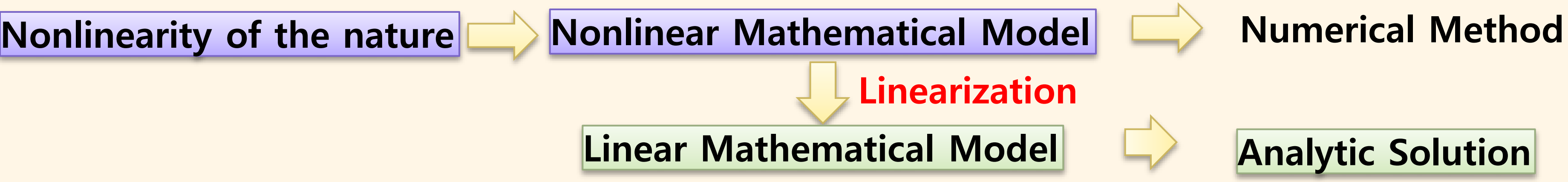


$$mz'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k}$$

$$= -kz\mathbf{k}$$

Nonlinearity

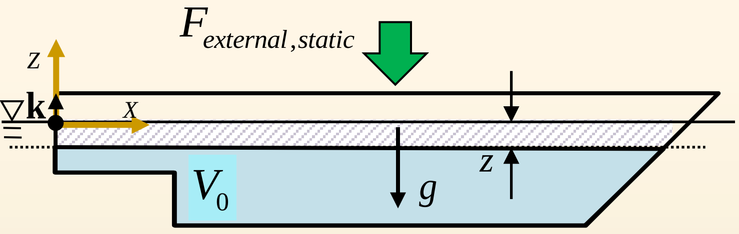


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ **Linearization**

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



m : mass ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
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$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
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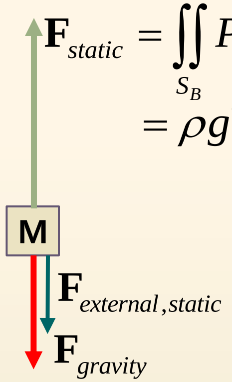


$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\ bouyancy}$$

additional buoyancy caused by additional displacement z

$$= -mg\mathbf{k}$$

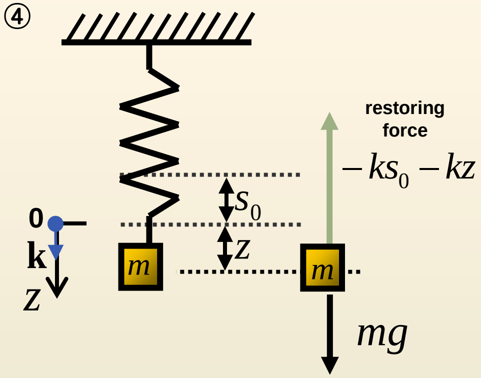
if, z is small

$$\mathbf{F}_{addtional\ bouyancy}$$
$$= -\rho g A_{WP}\mathbf{z}$$
$$= -k\mathbf{z}$$
$$, k = \rho g A_{WP}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**

④



$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k}$$
$$= -kz\mathbf{k}$$

$$\boxed{m\mathbf{z}'' + k\mathbf{z} = 0}$$ Oscillation by the restoring force

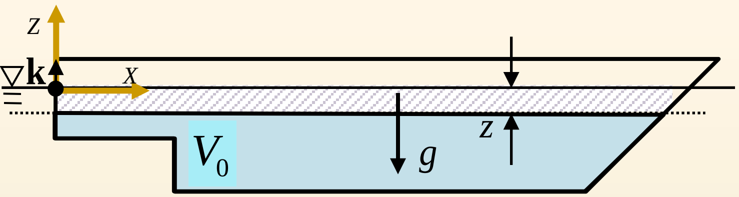
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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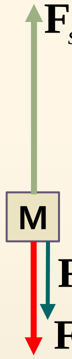

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} + \underline{\mathbf{F}_{addtional\ bouyancy}}$$

additional buoyancy caused by additional displacement z

$$\mathbf{F}_{external,static}$$
$$\mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\ bouyancy}$$
$$= -\rho g A_{WP}\mathbf{z}$$
$$= -k\mathbf{z}$$
$$, k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k} + \rho g V_0\mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -k\mathbf{z} + \mathbf{F}_{external,static} \quad , k = \rho g A_{WP}$$
$$= 0 \ (\because \ddot{z} = 0)$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

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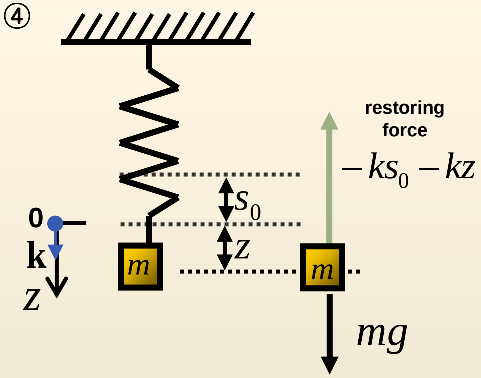
④



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Oscillation by the restoring force

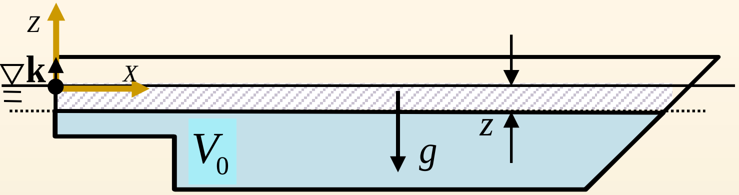
Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ **Linearization**

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 4



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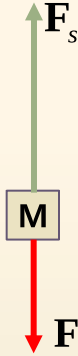

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additional buoyancy caused by additional displacement z

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$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$

$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$

$$= -k\mathbf{z} + \mathbf{F}_{external,static} \quad , k = \rho g A_{WP}$$

$$= 0 \ (\because \ddot{z} = 0)$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$

$$= -\rho g A_{WP}\mathbf{z}$$

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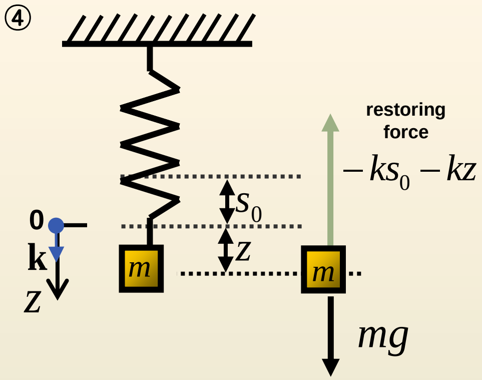

$$mz'' = \mathbf{F}$$

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$$\boxed{mz'' + k\mathbf{z} = 0}$$ Oscillation by the restoring force

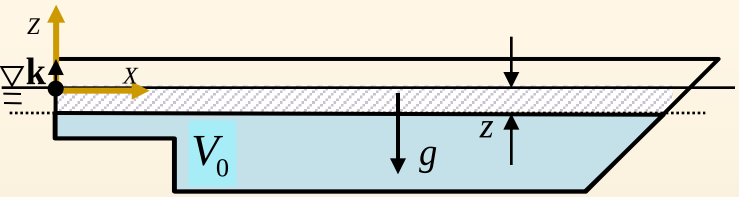
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

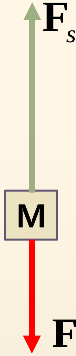
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \;\; -\rho g A_{wp}\mathbf{z}$$

$$= -\rho g A_{wp}\mathbf{z}$$

$$= -k\mathbf{z} \qquad , k = \rho g A_{WP}$$

$$= 0 \; (\because \ddot{z} = 0)$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\,bouyancy}$$

additional buoyancy caused by additional displacement z

$$\mathbf{F}_{gravity}$$

$$= -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy}$$

$$= -\rho g A_{WP}\mathbf{z}$$

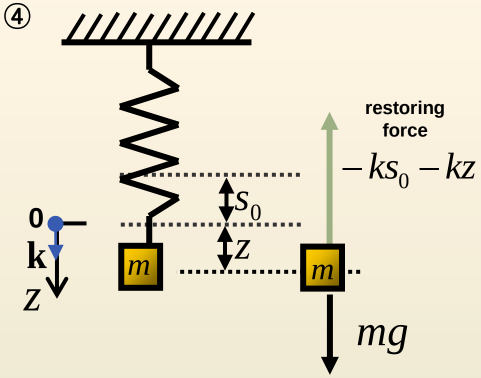
$$= -k\mathbf{z}$$

$$, k = \rho g A_{WP}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system

④



$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k}$$

$$= -kz\mathbf{k}$$

$$\boxed{m\mathbf{z}'' + k\mathbf{z} = 0}$$ Oscillation by the restoring force

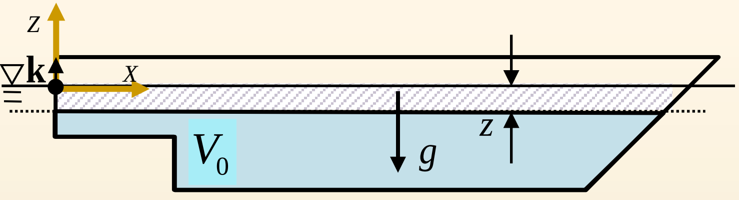
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



m : mass
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V₀: submerged volume
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Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
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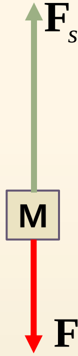

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$$= -\rho g A_{WP}\mathbf{z}$$
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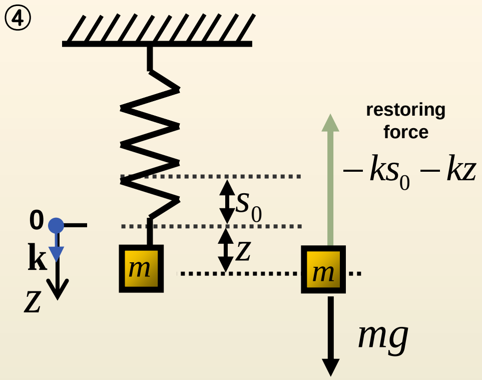

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$$m\mathbf{z}'' + k\mathbf{z} = 0$$ Oscillation by the restoring force

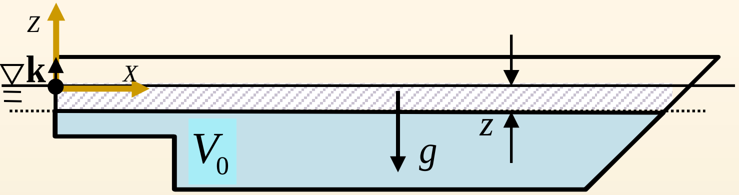
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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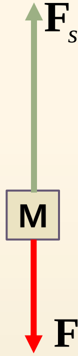

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additional buoyancy caused by additional displacement z

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if, z is small

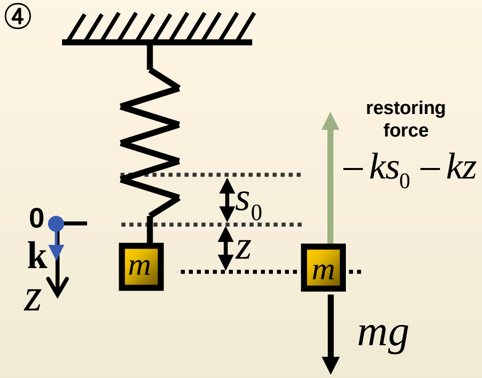
$$\mathbf{F}_{addtional\ bouyancy} = -\rho g A_{WP}\mathbf{z} = -k\mathbf{z} \quad , k = \rho g A_{WP}$$

Linearized Restoring Force

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system

④



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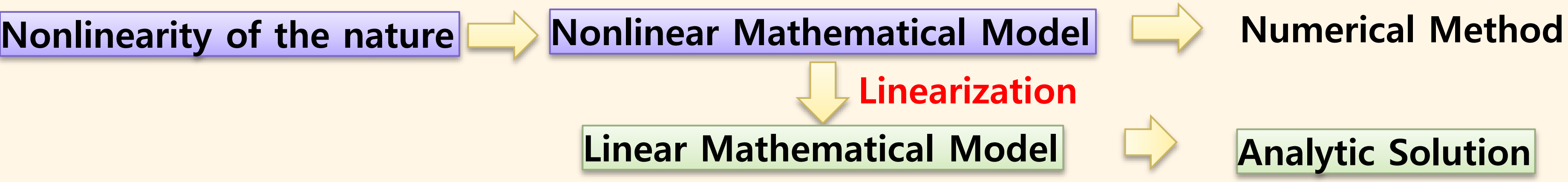
Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

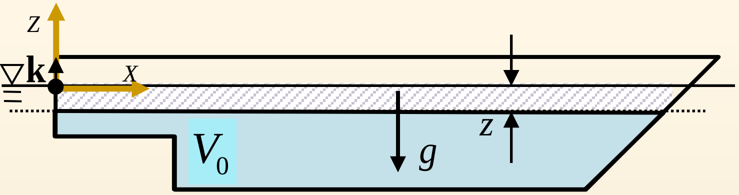
Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

Ex) Heave Motion of a Ship – step 4



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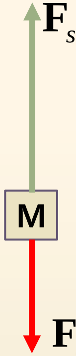

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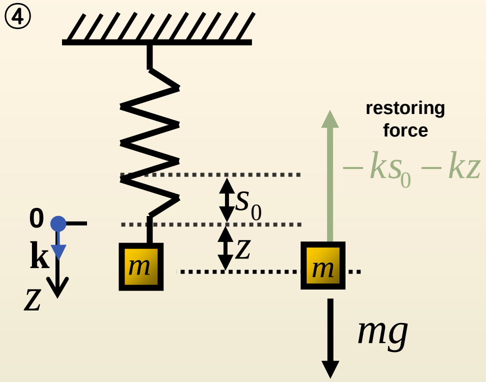

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$$\boxed{m\mathbf{z}'' + k\mathbf{z} = 0}$$ Oscillation by the restoring force

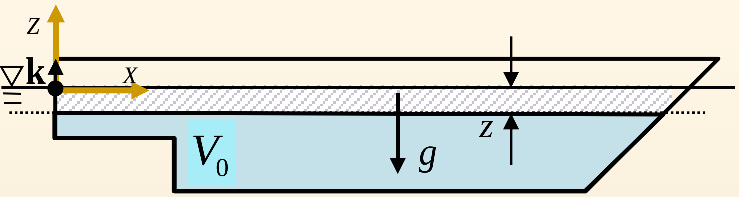
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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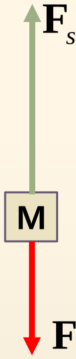
Ship will oscillate forever?



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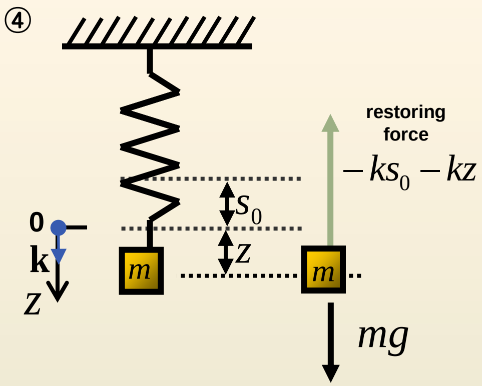
✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**



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$$m\mathbf{z}'' + k\mathbf{z} = 0$$

Oscillation by the restoring force

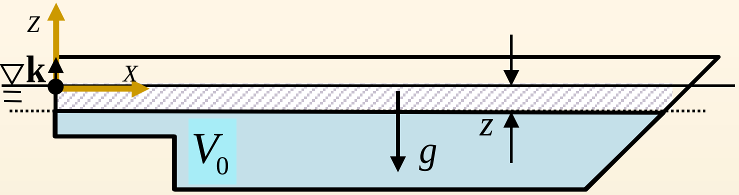
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ **Linearization**

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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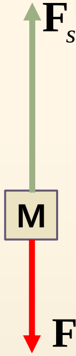

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$$m\ddot{\mathbf{z}} = \mathbf{F}$$
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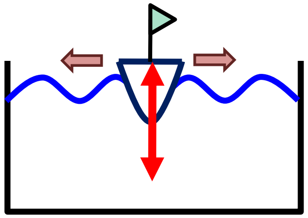
Ship will oscillate forever?

Energy is dissipated by radiation wave →

정수 중 선박의 강제 운동에 의해 발생한 힘

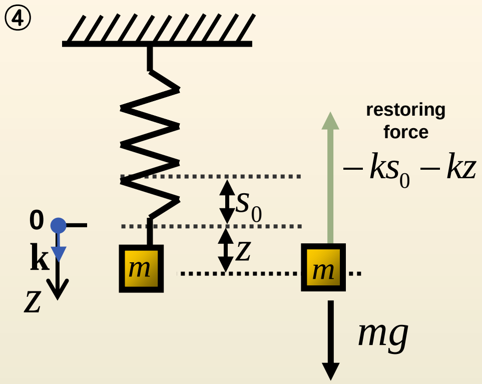


Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**



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$$\boxed{m\mathbf{z}'' + k\mathbf{z} = 0}$$ Oscillation by the restoring force

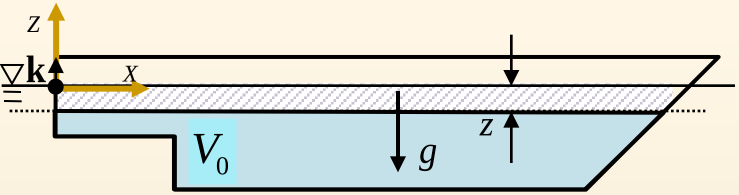
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 4



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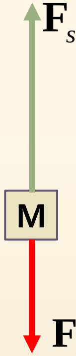

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0\mathbf{k} - \rho g A_{WP}\mathbf{z} = \rho g V_0\mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0\mathbf{k} - \rho g A_{wp}\mathbf{z}$$
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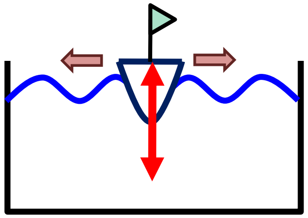
$$= -k\mathbf{z}$$

Ship will oscillate forever?

Energy is dissipated by radiation wave ⇨

정수 중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$

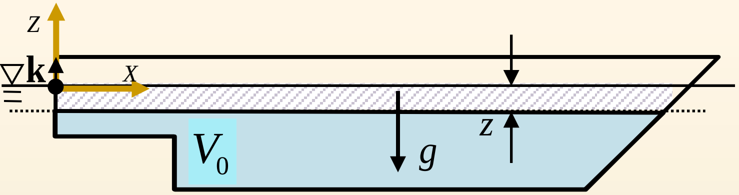
Nonlinearity

Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ Analytic Solution

Ex) Heave Motion of a Ship – step 5



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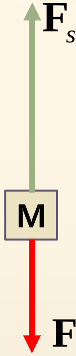


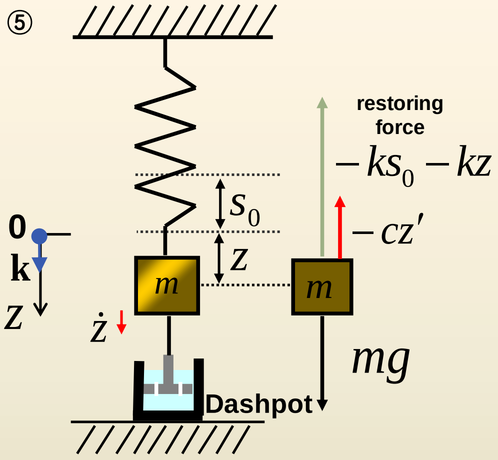
$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0\mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0\mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0\mathbf{k} - \rho g A_{wp}\mathbf{z}$$
$$= -\rho g A_{wp}\mathbf{z}$$
$$= -k\mathbf{z}$$

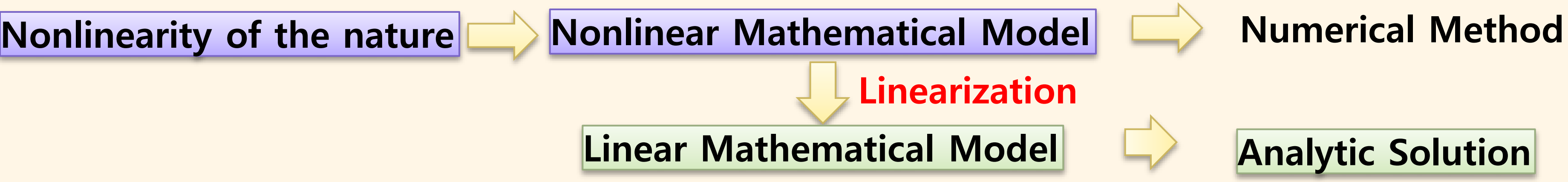
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0\mathbf{k}$

✓ Mass-Spring-Damper system



$$mz'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

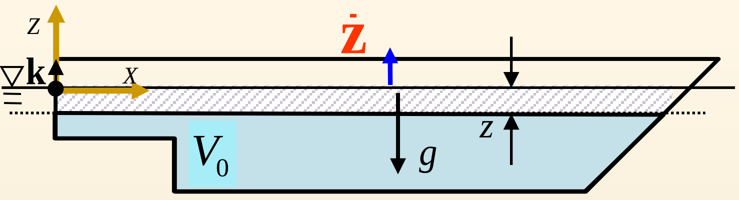
Nonlinearity


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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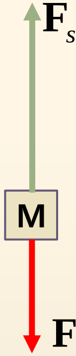


$$\mathbf{F}_{static} = \iint_{S_B} P_{static} \mathbf{n} dS$$

$$= \rho g V_0 \mathbf{k} - \rho g A_{WP} \mathbf{z}$$

$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

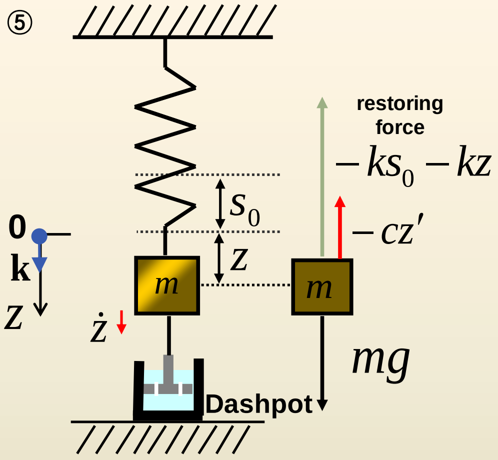
$$\mathbf{F}_{gravity}$$

$$= -mg\mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system

⑤



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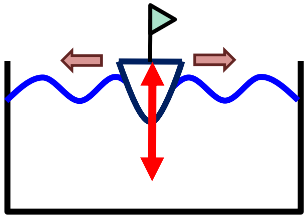
정수 중 선박의 강제
운동에 의해 발생한 힘



Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation} \mathbf{n} dS$$

$$= -c\dot{\mathbf{z}}$$

opposite to velocity

2008_Introduction

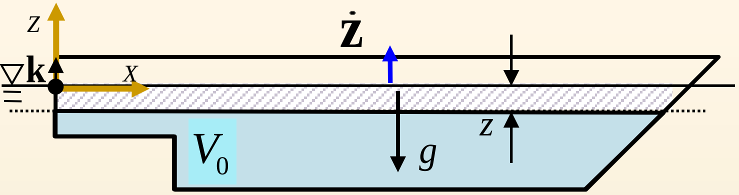
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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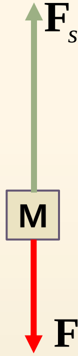

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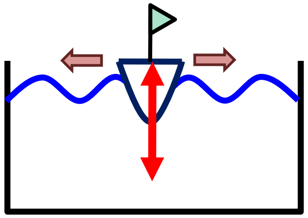
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

정수 중 선박의 강제
운동에 의해 발생한 힘



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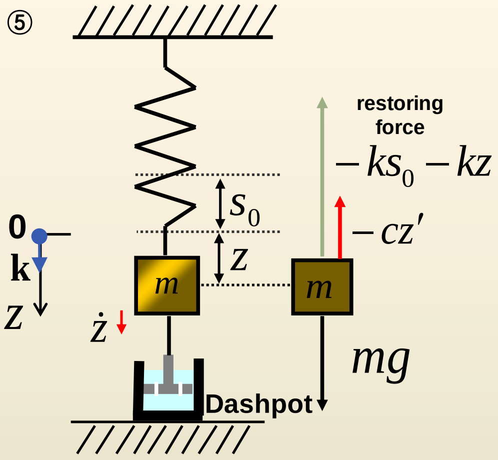
opposite to velocity

c : damping coefficient

✓ Mass-Spring-Damper system



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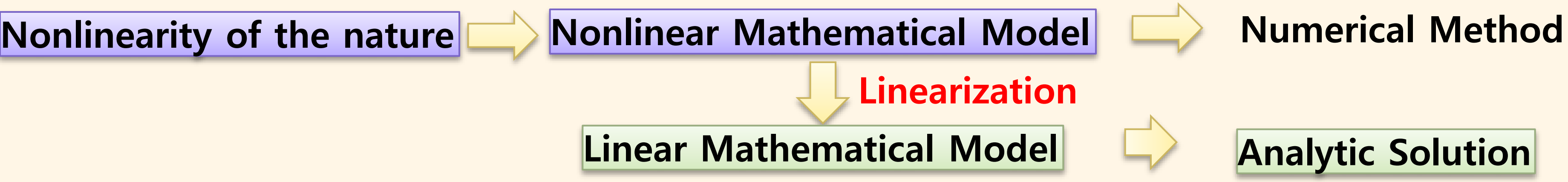
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

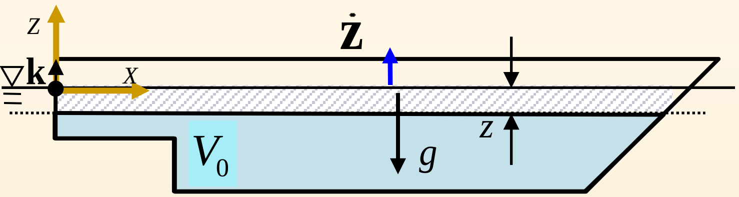
Nonlinearity


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



m : mass

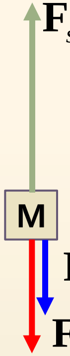
ρ : density of sea water

V₀: submerged volume

S_B : submerged surface area

Awp : waterplane area

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z}$$
$$= -\rho g A_{wp}\mathbf{z}$$
$$= -k\mathbf{z}$$

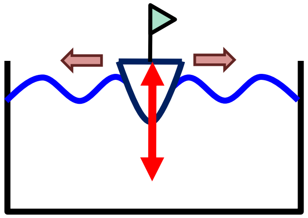


$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}}$$

$$\mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

정수 중 선박의 강제
운동에 의해 발생한 힘



Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$
$$= -c\dot{\mathbf{z}}$$

opposite to velocity

c : damping coefficient

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

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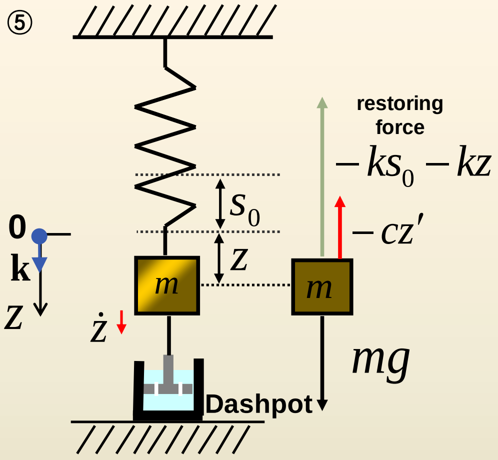

$$m\mathbf{z}'' = \mathbf{F}$$
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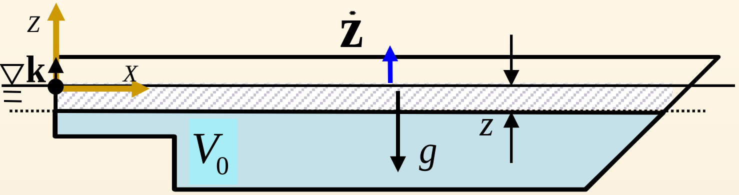
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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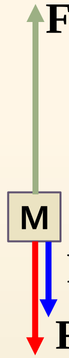

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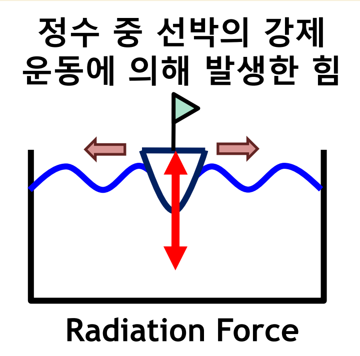

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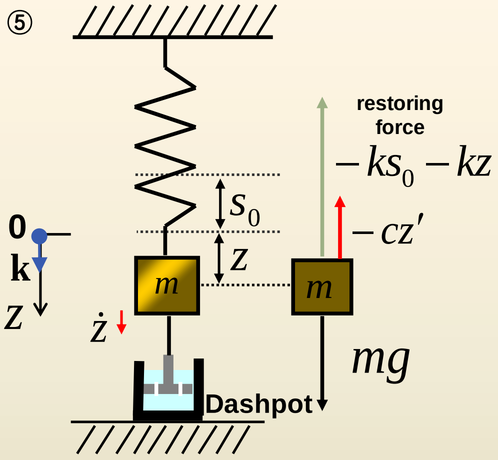

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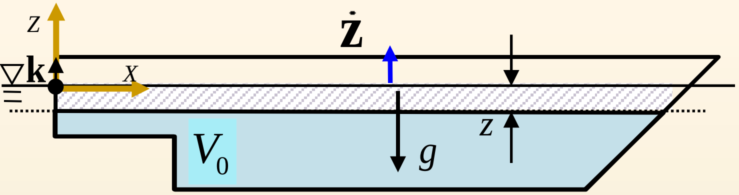
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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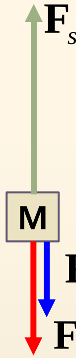

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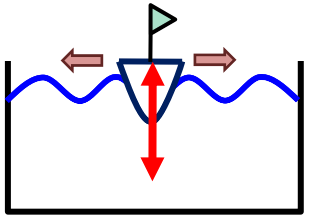
정수 중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

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opposite to velocity

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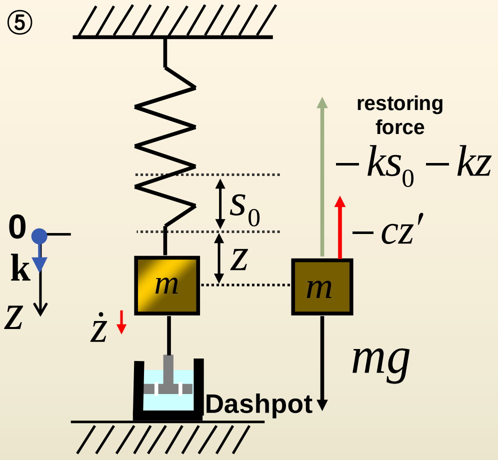

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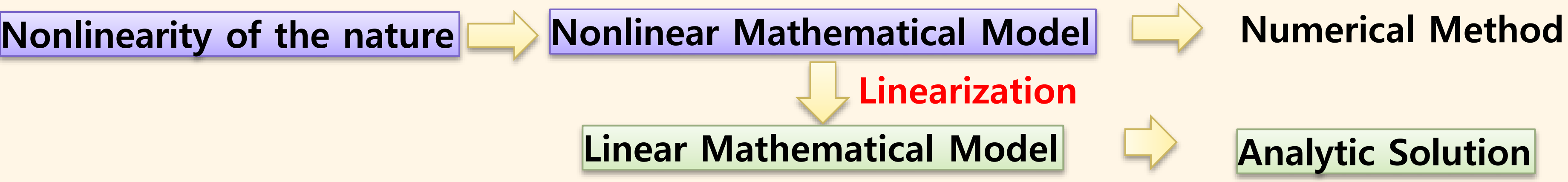
Nonlinearity

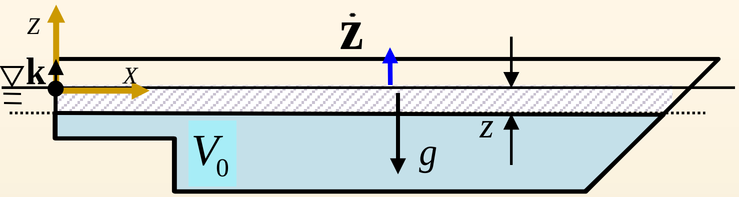


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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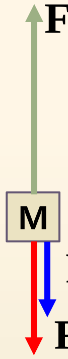

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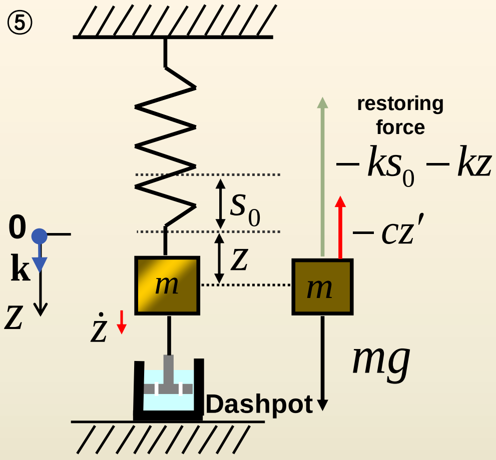
⑤



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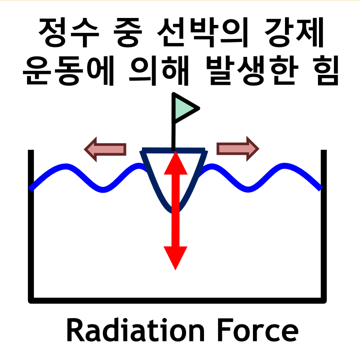

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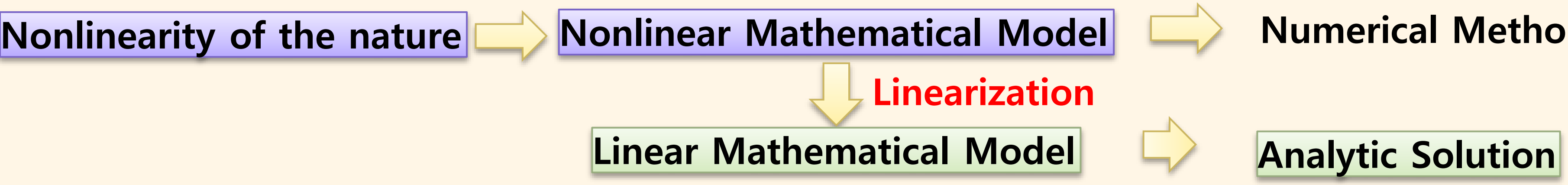
Nonlinearity

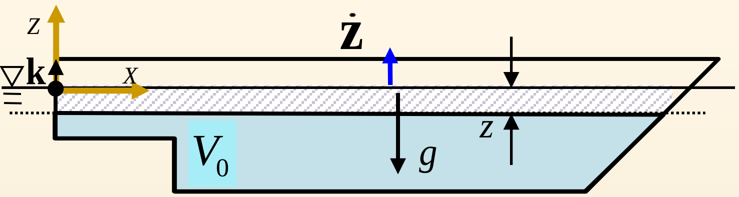


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 5



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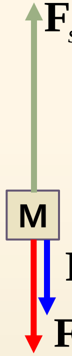

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✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

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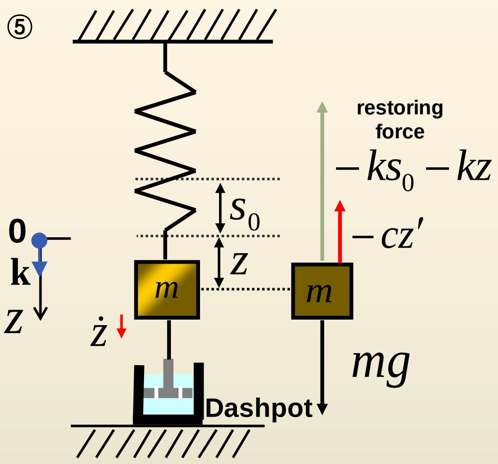

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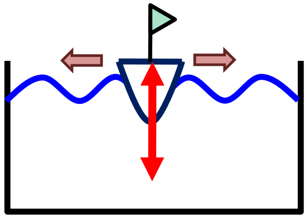
정수 중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

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opposite to velocity

c : damping coefficient

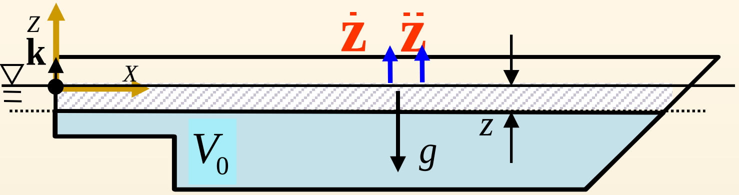
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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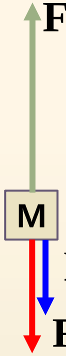

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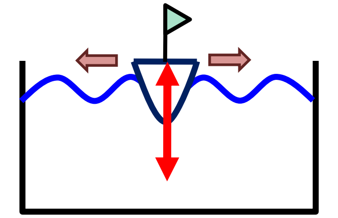

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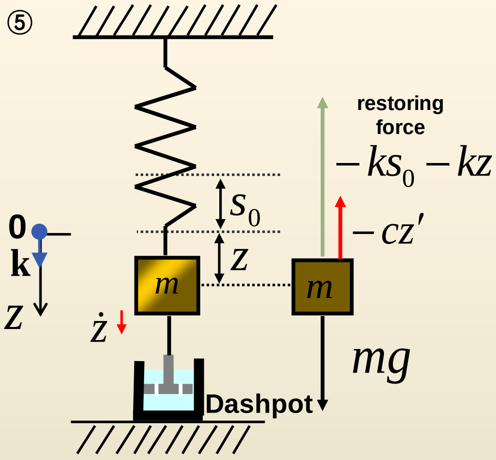

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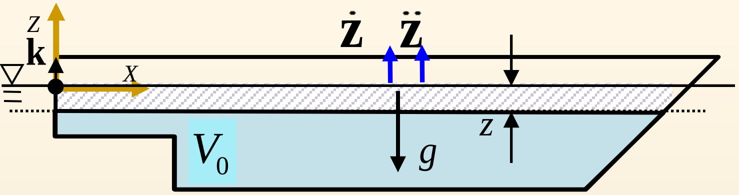
Ex) Heave Motion of a Ship – step 5



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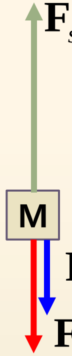

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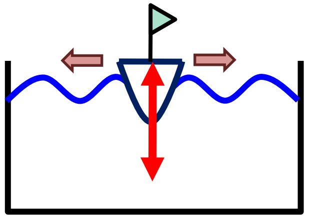

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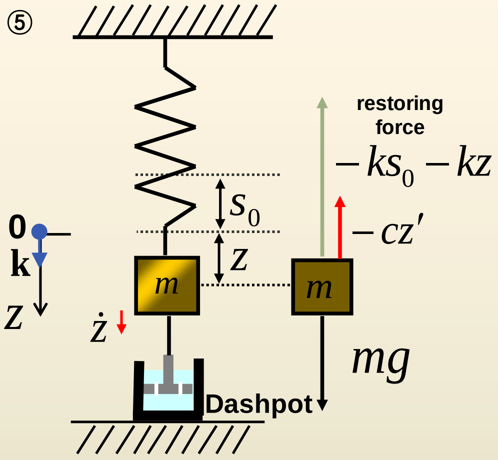

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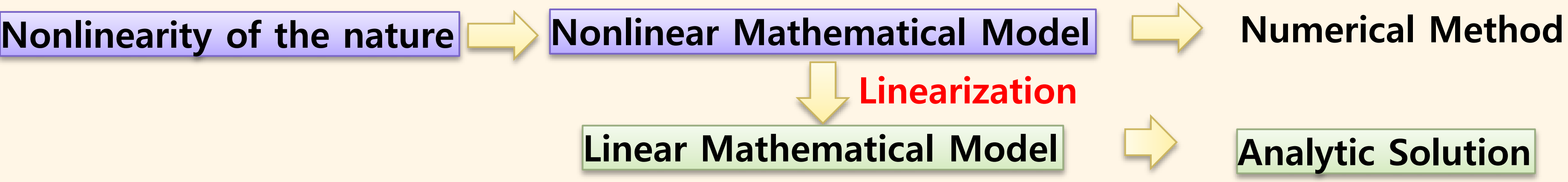
Nonlinearity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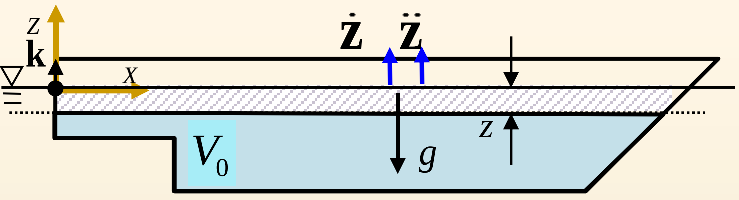

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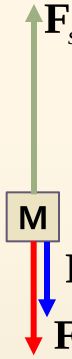

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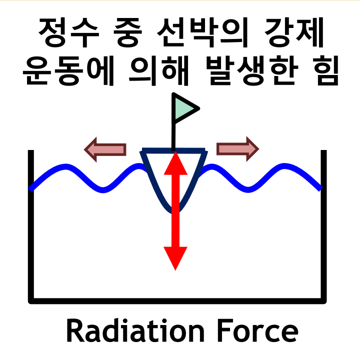

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opposite to velocity

opposite to acceleration

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m_a : added mass

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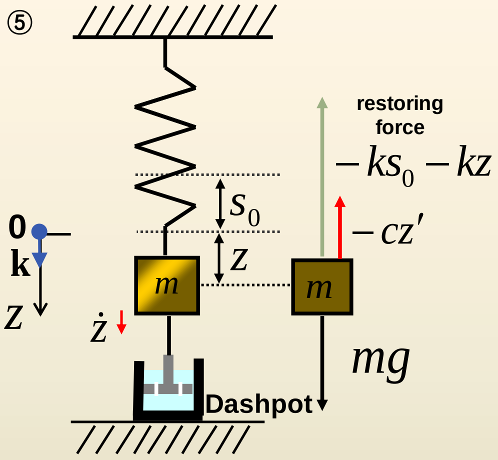

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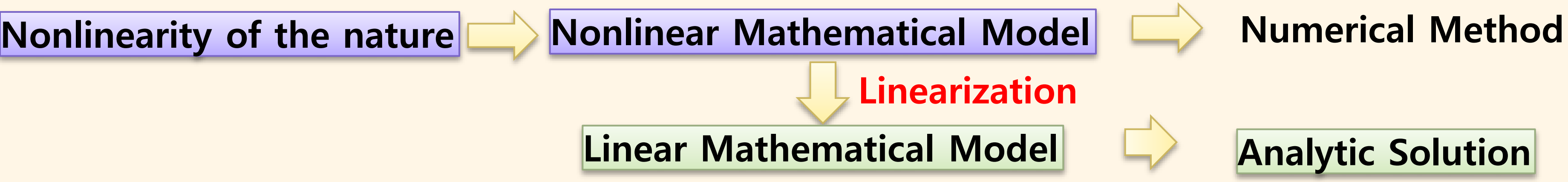
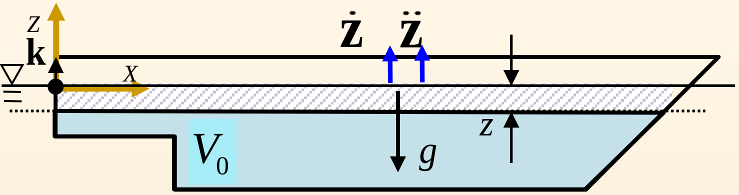
Nonlinearity


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

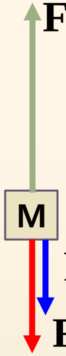
Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



m : mass
ρ : density of sea water
V₀: submerged volume
S_B : submerged surface area
Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

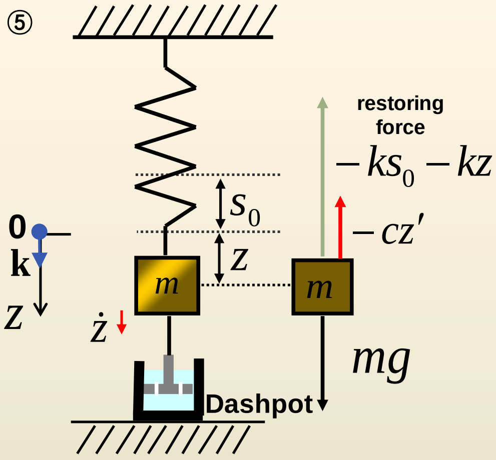
$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \; -\rho g A_{wp}\mathbf{z} \; -c\dot{\mathbf{z}}$$
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}}$$
$$= -k\mathbf{z} \; -c\dot{\mathbf{z}}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system



$$mz'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \; -cz'\mathbf{k}$$
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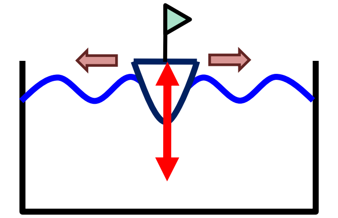


정수 중 선박의 강제
운동에 의해 발생한 힘

Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$
$$= -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

opposite to velocity

opposite to acceleration

c : damping coefficient
m_a : added mass

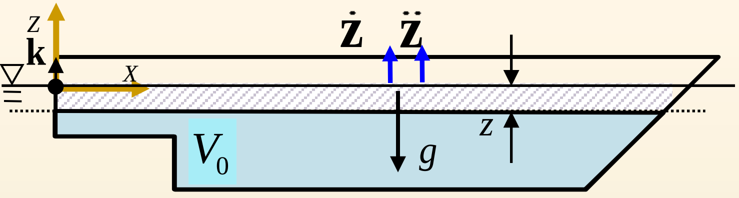
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ **Numerical Method**

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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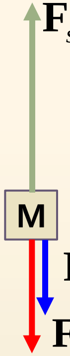

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$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$
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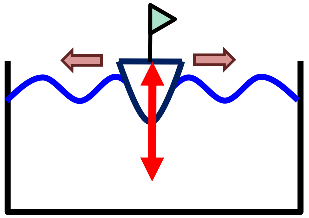
정수 중 선박의 강제
운동에 의해 발생한 힘



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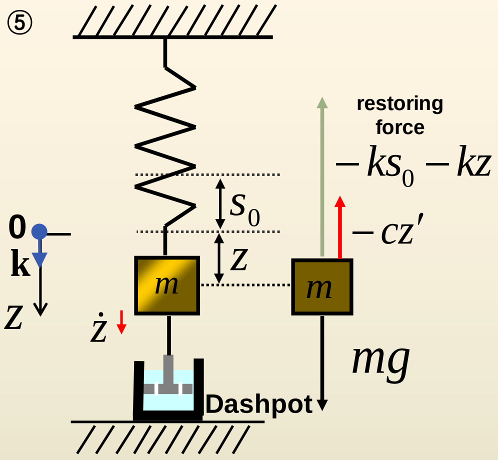

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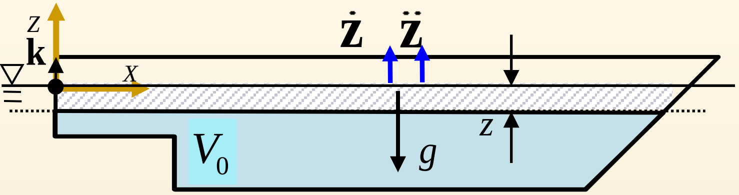
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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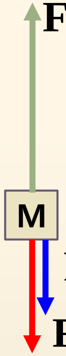

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정수 중 선박의 강제
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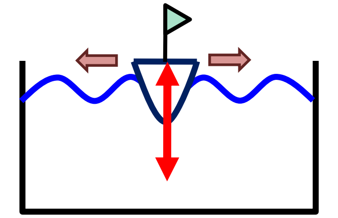

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opposite to velocity

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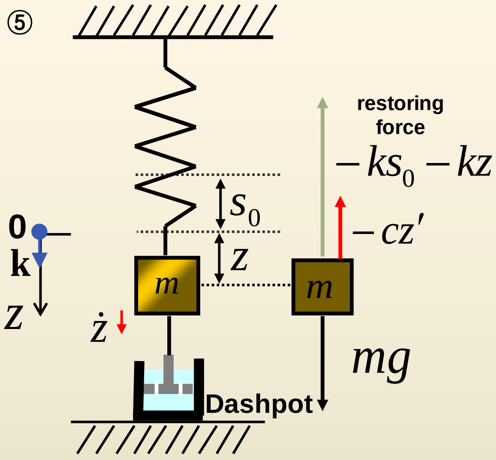

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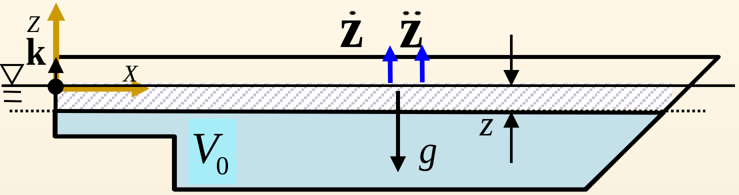
Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Heave Motion of a Ship – step 5



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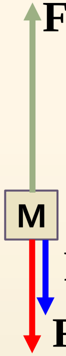

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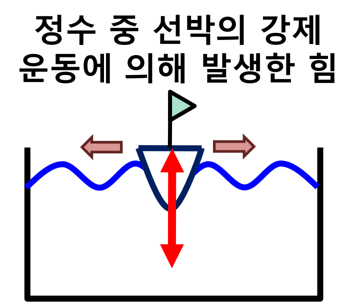

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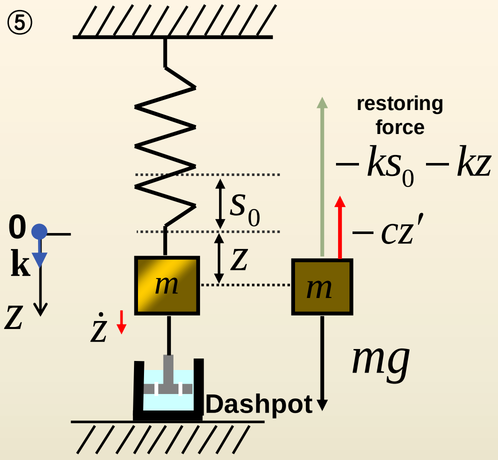

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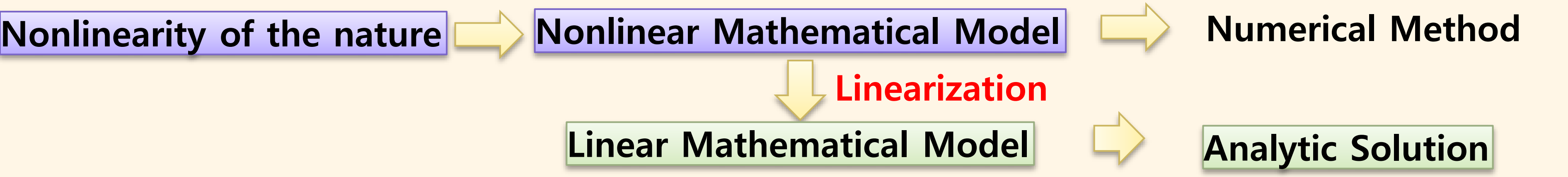
Nonlinearity

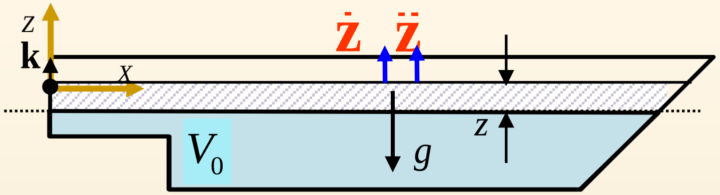


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 6



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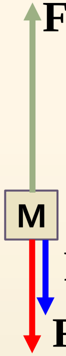

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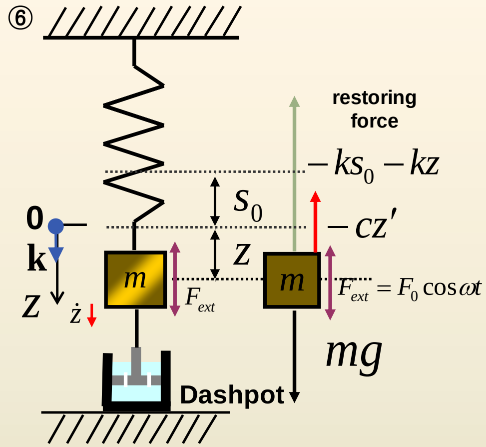

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$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$
$$= -mg\mathbf{k} + \rho g V_0\mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
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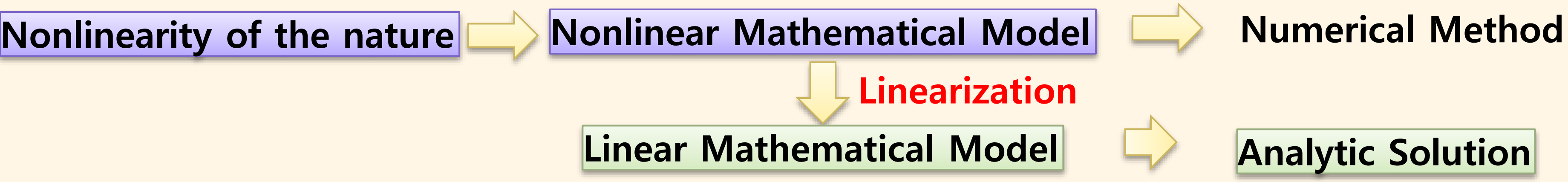
mₐ : added mass

Nonlinearity

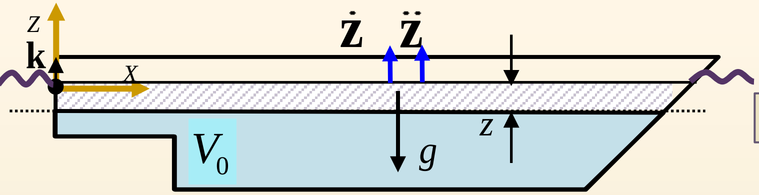


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 6



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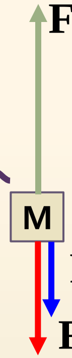

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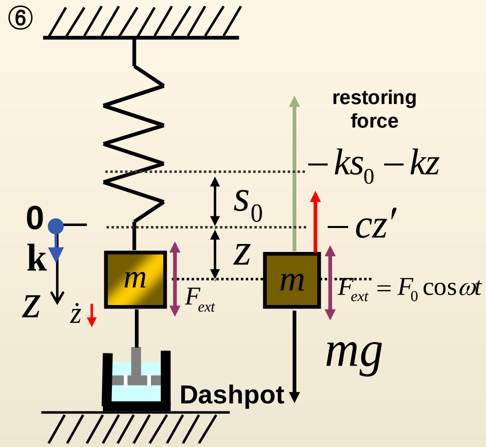

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$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} \; - cz'\mathbf{k} + \mathbf{F_0} \cos \omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k} \; + \mathbf{F_0} \cos \omega t$$

c : damping coefficient

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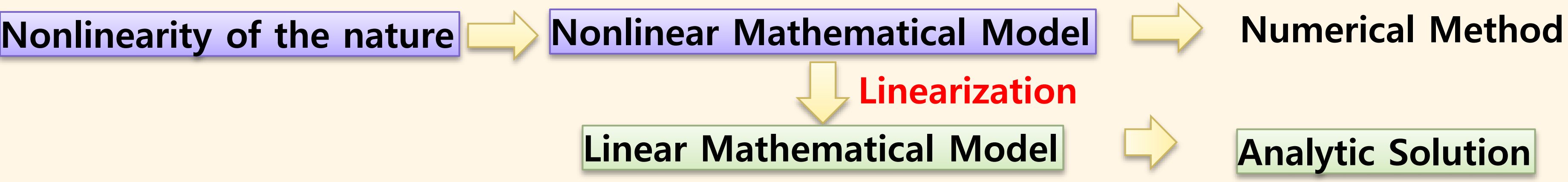
Nonlinearity

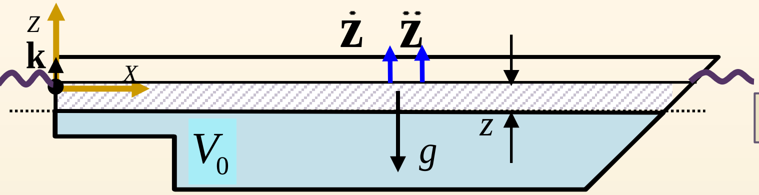


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → Analytic Solution

Ex) Heave Motion of a Ship – step 6



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$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} \ -\rho g A_{wp}\mathbf{z} \ -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

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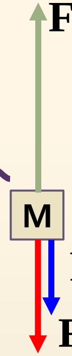

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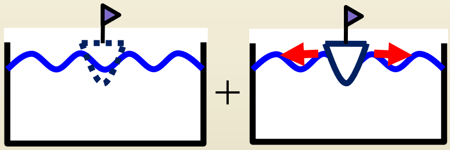
Wave force



Froude-Kriloff Force   Diffraction Force

$$\mathbf{F}_{wave\ exciting}$$

$$= \iint_{S_B} P_{wave\ exciting}\mathbf{n}dS$$

$$(= \mathbf{F}_{exciting})$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

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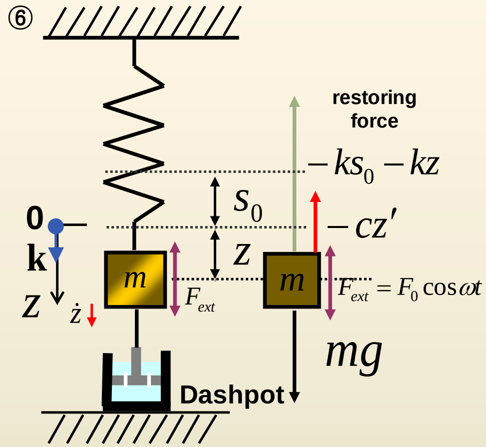
⑥



$$m\mathbf{z}'' = \mathbf{F}$$

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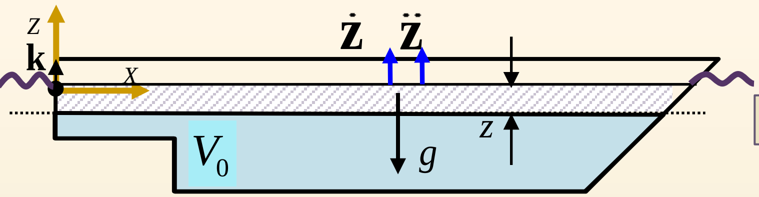
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



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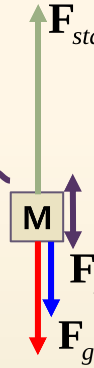

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$$\mathbf{F}_{exciting}$$

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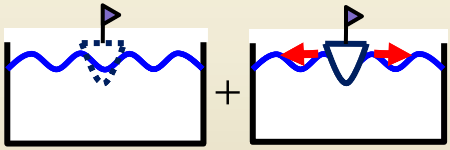
Wave force



Froude-Kriloff Force + Diffraction Force

$$\mathbf{F}_{wave\ exciting}$$
$$= \iint_{S_B} P_{wave\ exciting}\mathbf{n}dS$$
$$\left(= \mathbf{F}_{exciting}\right)$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

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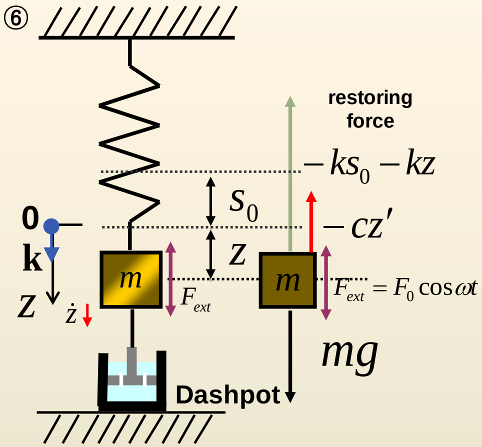
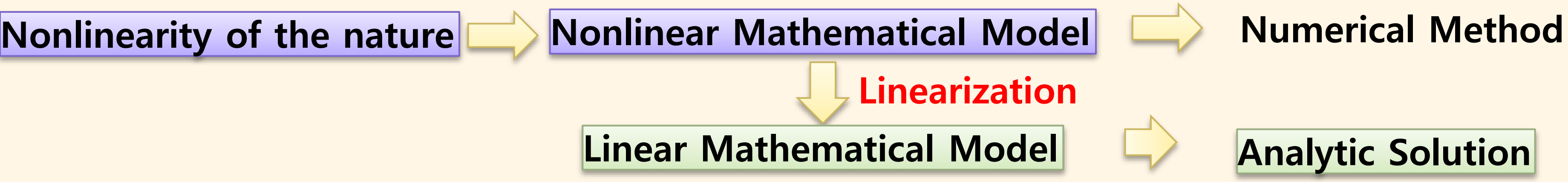


$$mz'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F_0}\cos\omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k} \; + \mathbf{F_0}\cos\omega t$$

Nonlinearity

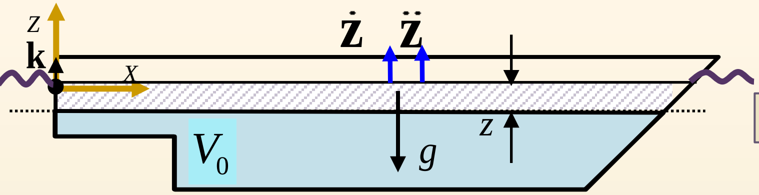


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

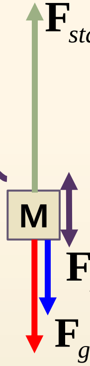
Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



m : mass
ρ : density of sea water
V₀: submerged volume
S_B : submerged surface area
Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{exciting}$$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity}$$
$$= -mg\mathbf{k}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation} + \mathbf{F}_{exciting}$$
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$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
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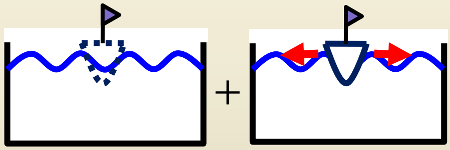
c : damping coefficient
m_a : added mass

Wave force



Froude-Kriloff Force　　Diffraction Force

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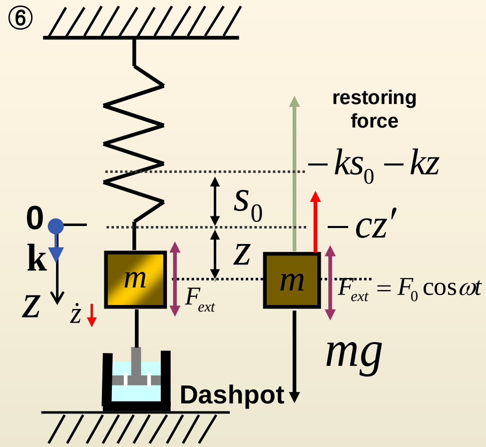

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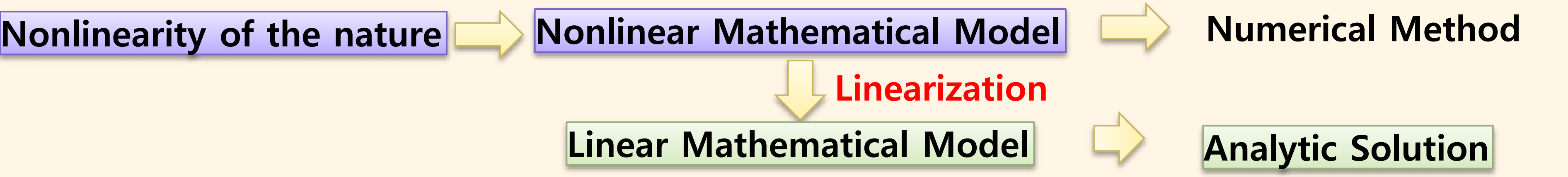
Nonlinearity

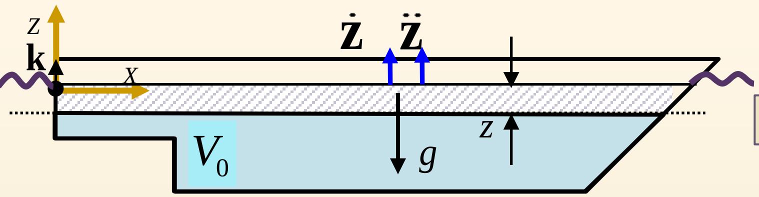


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



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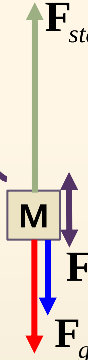

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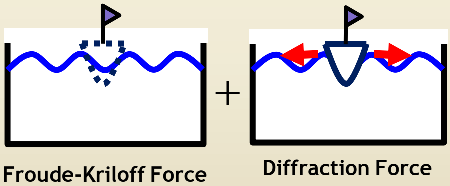
Wave force



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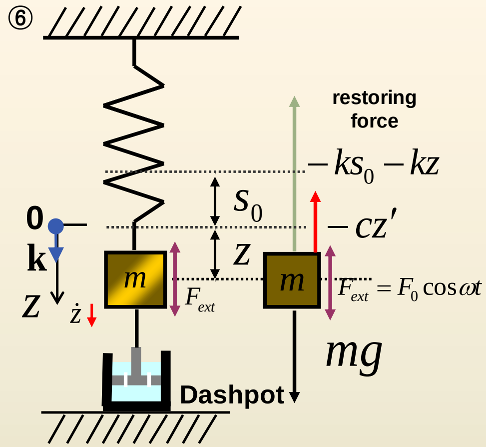

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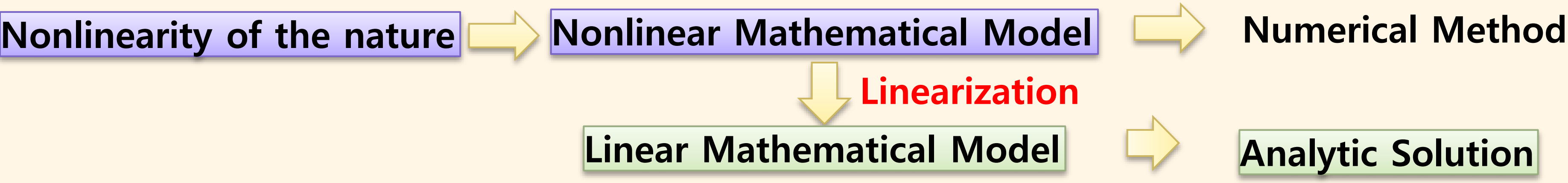
Nonlinearity

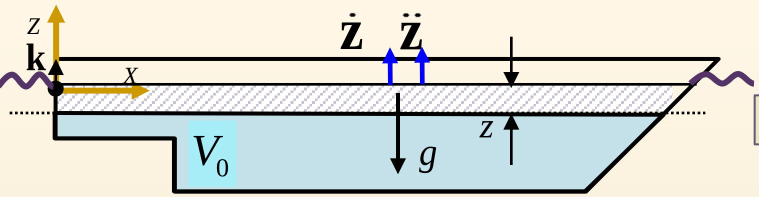


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



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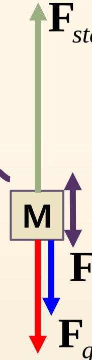

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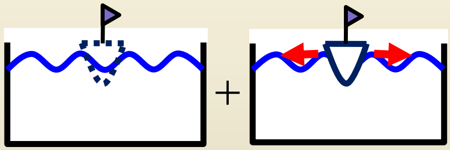
Wave force



Froude-Kriloff Force

Diffraction Force

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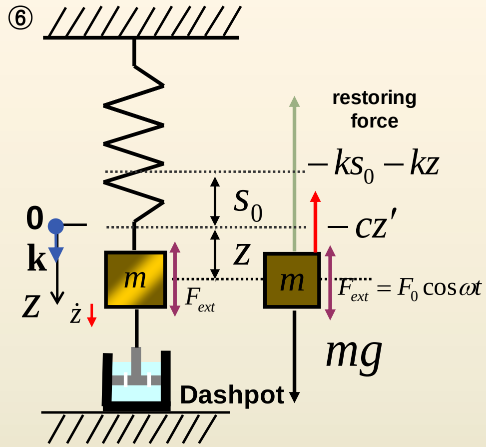

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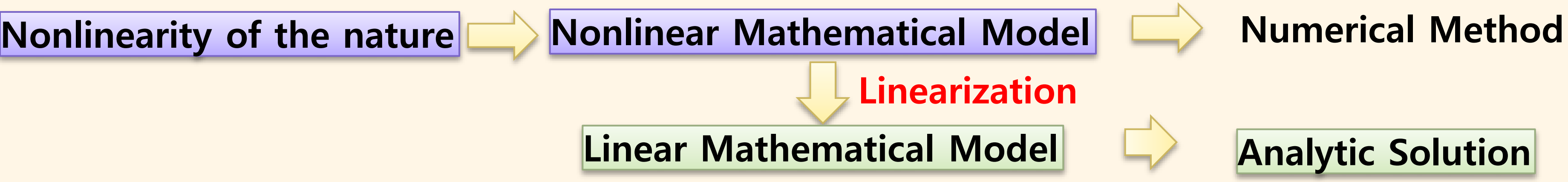
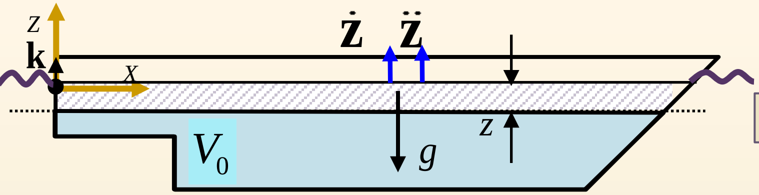
Nonlinearity


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



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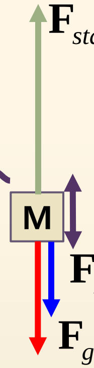

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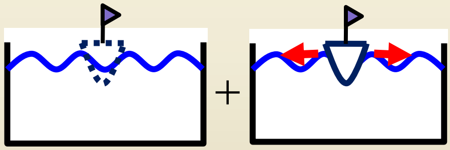
Wave force



Froude-Kriloff Force **Diffraction Force**

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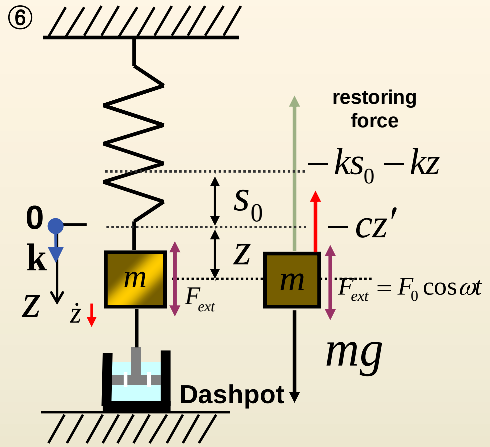

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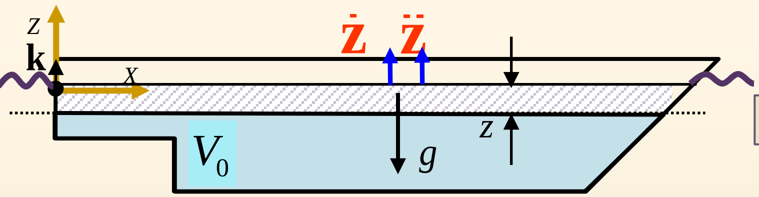
Nonlinearity

Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ **Linearization**

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



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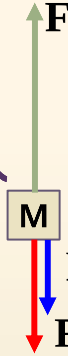

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✓ **Archimedes' Principle** $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

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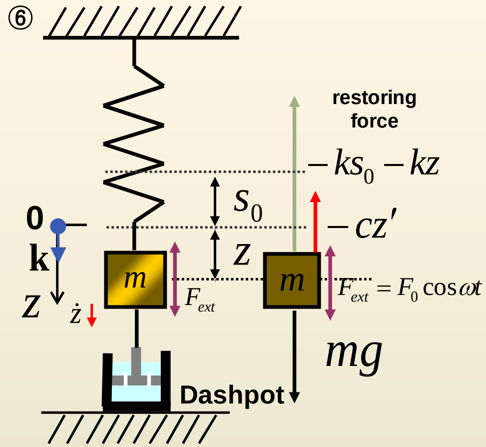

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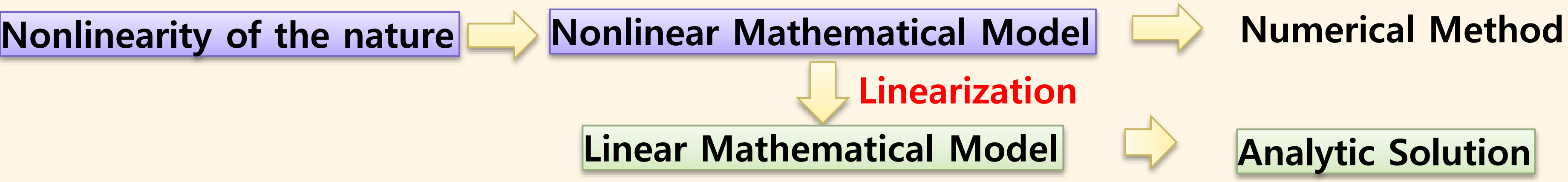
Nonlinearity

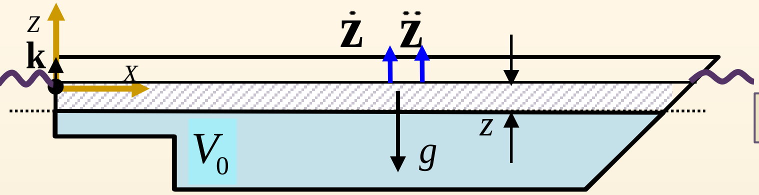


Nonlinearity of the nature → **Nonlinear Mathematical Model** → **Numerical Method**

↓ **Linearization**

Linear Mathematical Model → **Analytic Solution**

Ex) Heave Motion of a Ship – step 6



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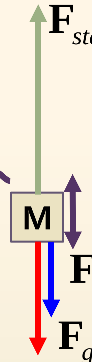

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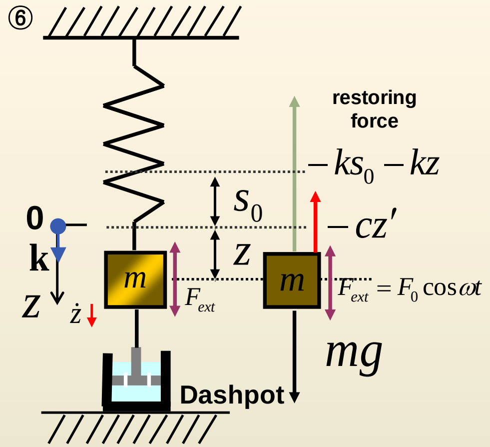

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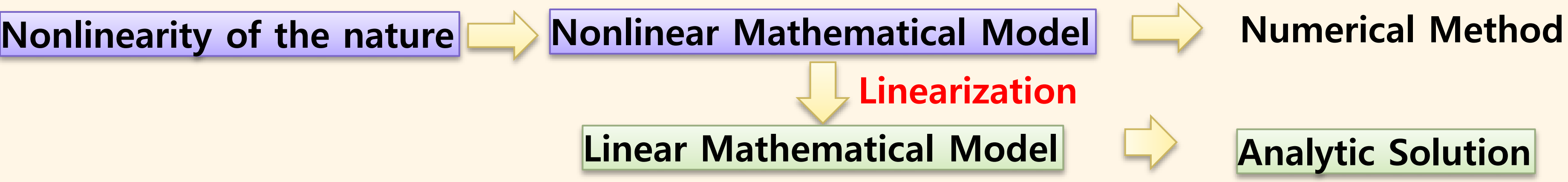
Nonlinearity



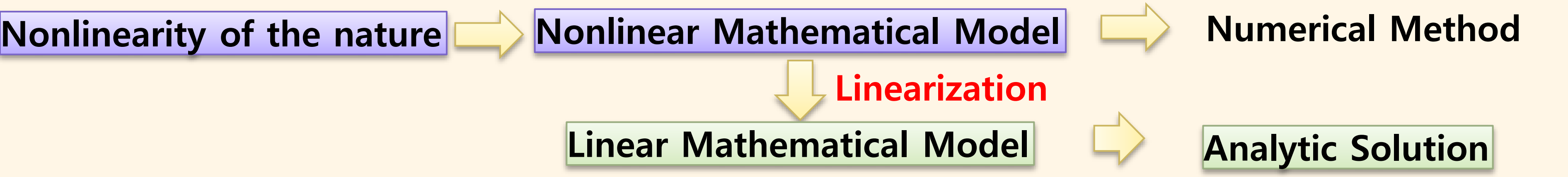
Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Roll Motion of a Ship

Nonlinearity



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Ex) Roll Motion of a Ship

[Dynamics for roll motion]

Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

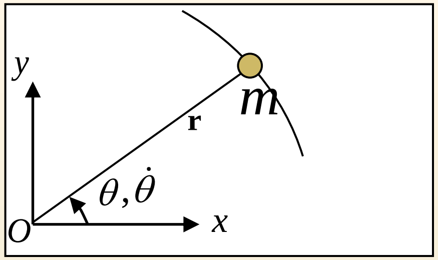
Ex) Roll Motion of a Ship

[Dynamics for roll motion]

Angular momentum defined : $L = I\dot{\theta}$

$$where, I = m\mathbf{r}^2$$:moment of inertia

$$\theta = \theta(t)$$



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Linear Mathematical Model → **Analytic Solution**

Ex) Roll Motion of a Ship

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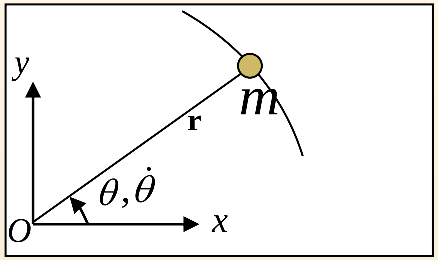
Rate of change of Angular momentum :

$$\frac{dL}{dt} = I\frac{d(\dot{\theta})}{dt} = I\ddot{\theta} \quad (I : \text{constant})$$



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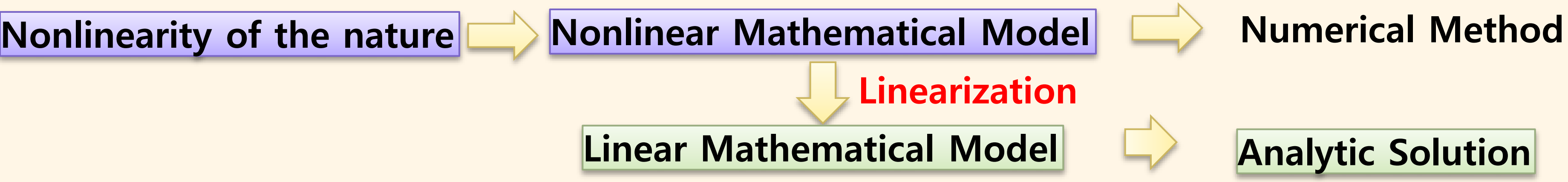

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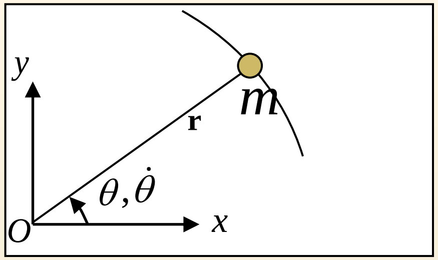

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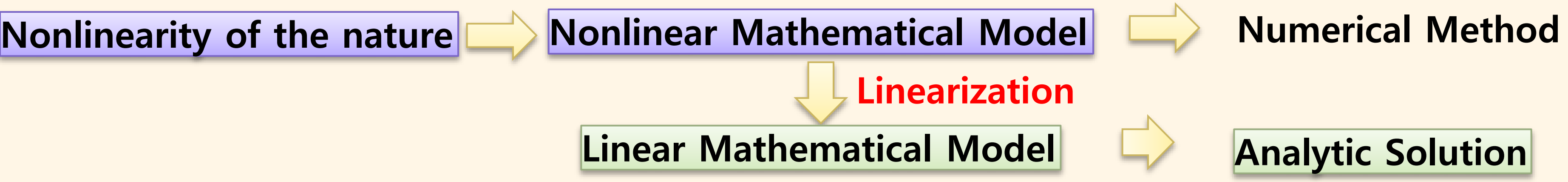
Euler's Equation: $$\frac{dL}{dt} = I\ddot{\theta} = \tau_{net}$$

Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

Linearization

Linear Mathematical Model

Analytic Solution

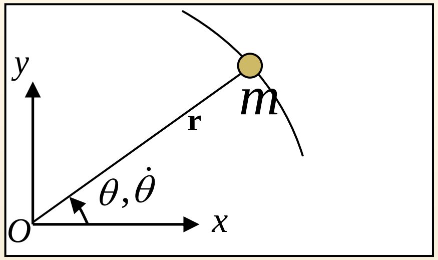
Ex) Roll Motion of a Ship

[Dynamics for roll motion]

Angular momentum defined : $L = I\dot{\theta}$

$where, I = m\mathbf{r}^2$:moment of inertia

$\theta = \theta(t)$



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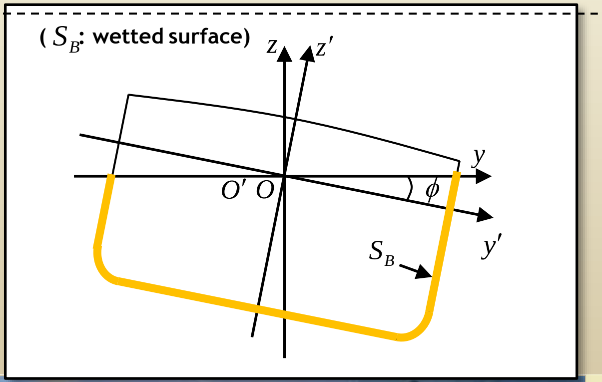

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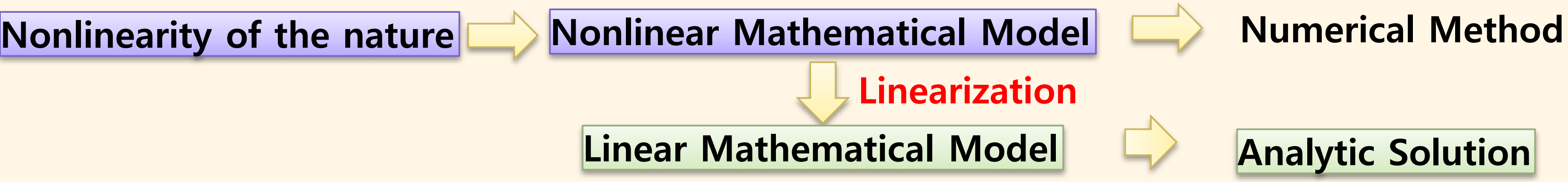

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → Analytic Solution

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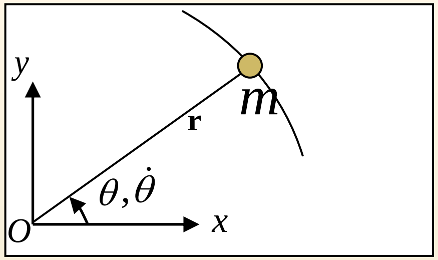

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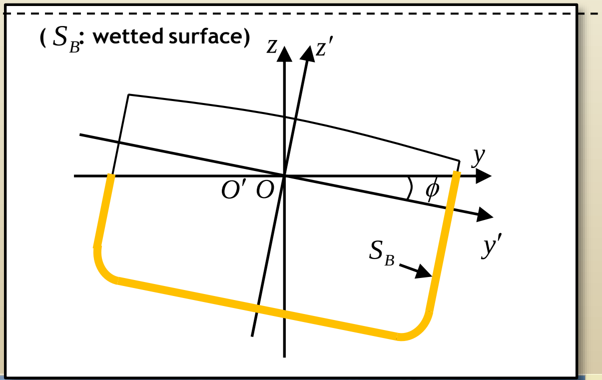
Euler's Equation for roll motion:

$$I\phi'' = \sum M$$



Nonlinearity

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Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

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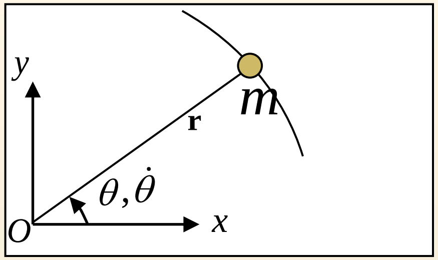

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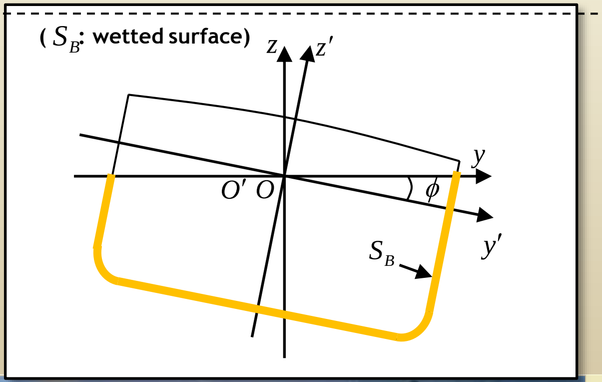

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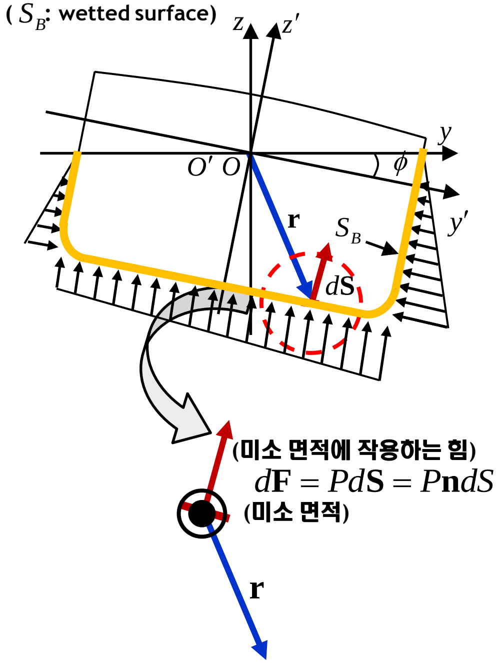
Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Roll Motion of a Ship



$$d\mathbf{F} = Pd\mathbf{S} = P\mathbf{n}dS$$

- Hydrostatic Moment : (모멘트)=(거리) X (힘)
  - ✓ 미소 면적에 작용하는 모멘트 :

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  - ✓ Total moment :

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r} \times \mathbf{n})dS$$

Nonlinearity

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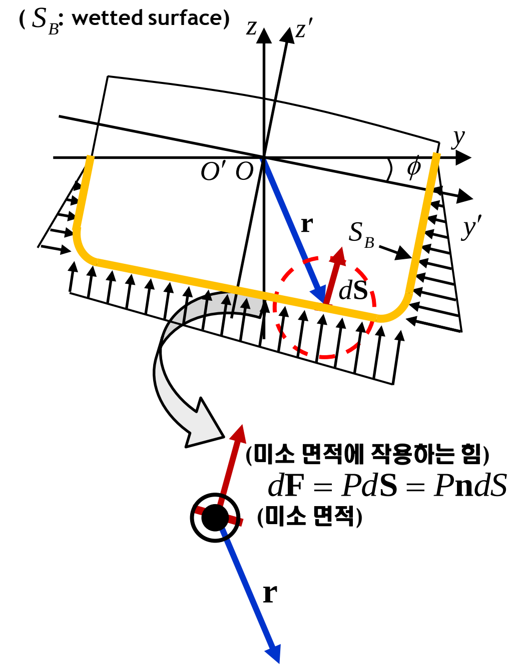
⇓ Linearization

Linear Mathematical Model ⇒ **Analytic Solution**

Ex) Roll Motion of a Ship

$$I\phi'' = \sum M$$



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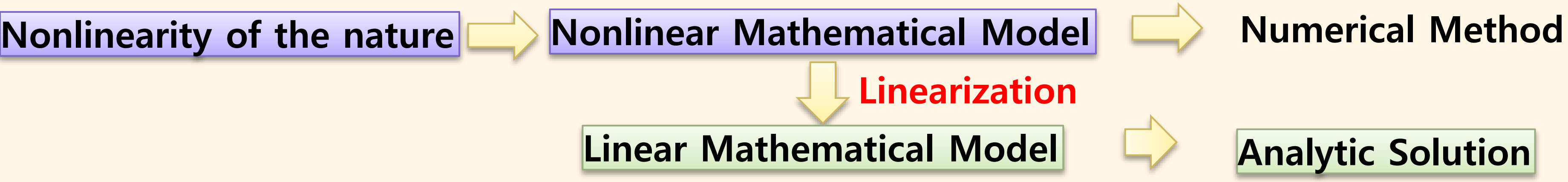

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Roll Motion of a Ship

$$I\phi'' = \sum M$$

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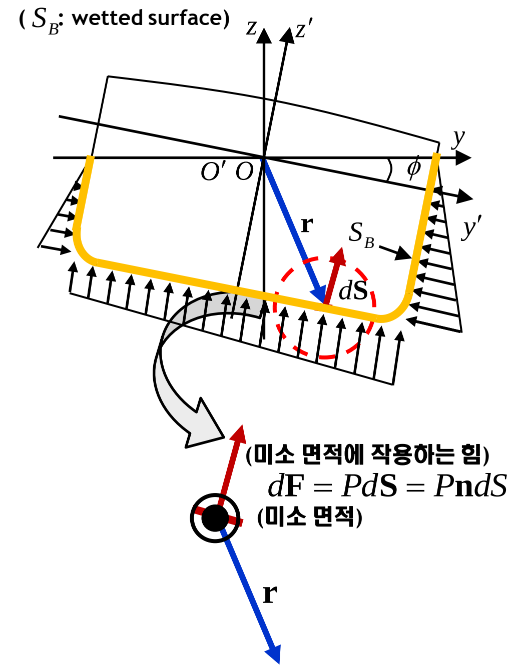

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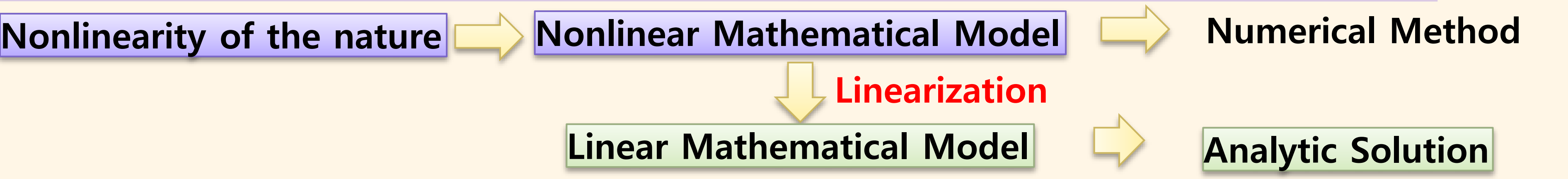
Nonlinearity


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

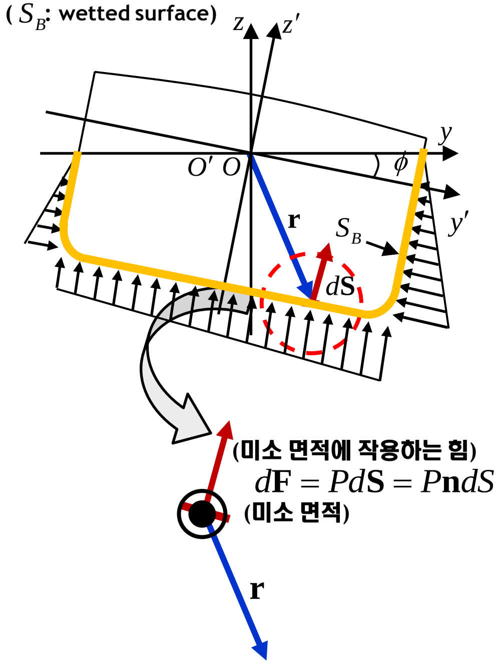
Linear Mathematical Model → **Analytic Solution**

Ex) Roll Motion of a Ship

$$I\phi'' = \sum M$$

$$\begin{aligned} I\phi'' &= M_{body} + M_{surface} \\ &= M_{body} + \iint P_{static}(\mathbf{r}\times\mathbf{n})ds + \iint P_{dynamic}(\mathbf{r}\times\mathbf{n})ds + M_{external} \\ &= M_{gravity} + M_{buoyancy} + M_{damping} + M_{added} + M_{external} \end{aligned}$$

(S_B: wetted surface)



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Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ **Numerical Method**

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Linear Mathematical Model ⇨ **Analytic Solution**

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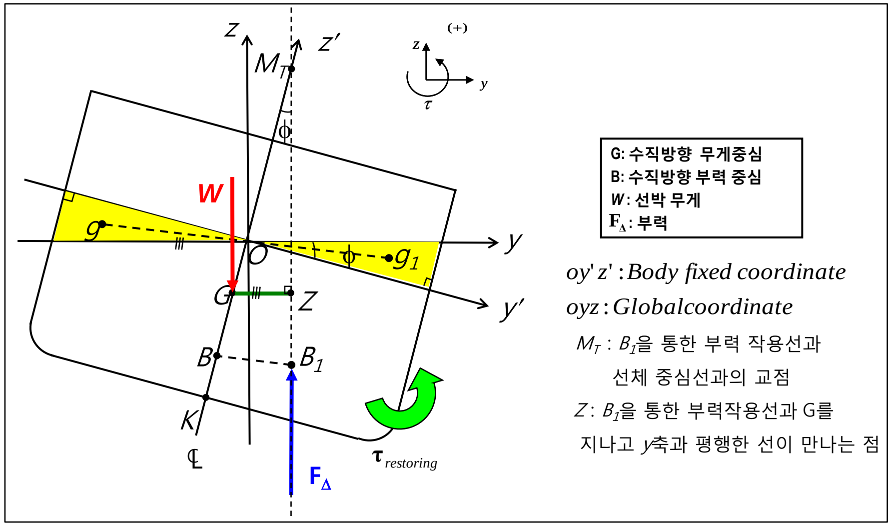

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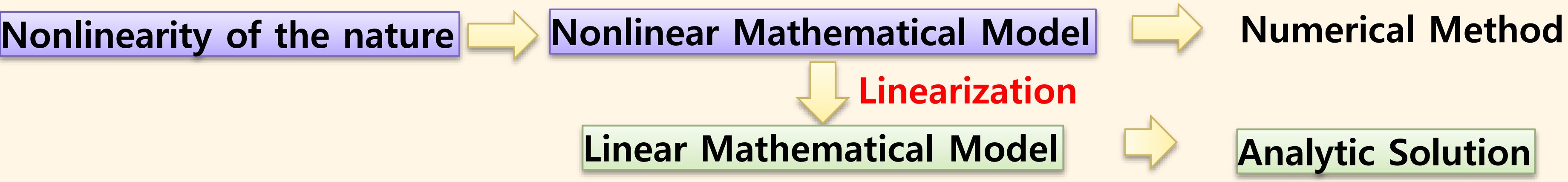
Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

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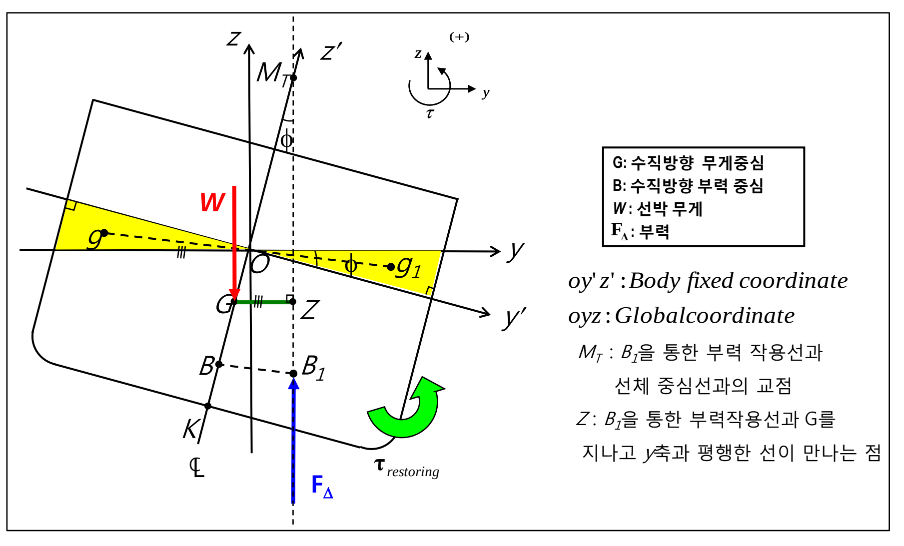

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Nonlinearity

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Roll Motion of a Ship

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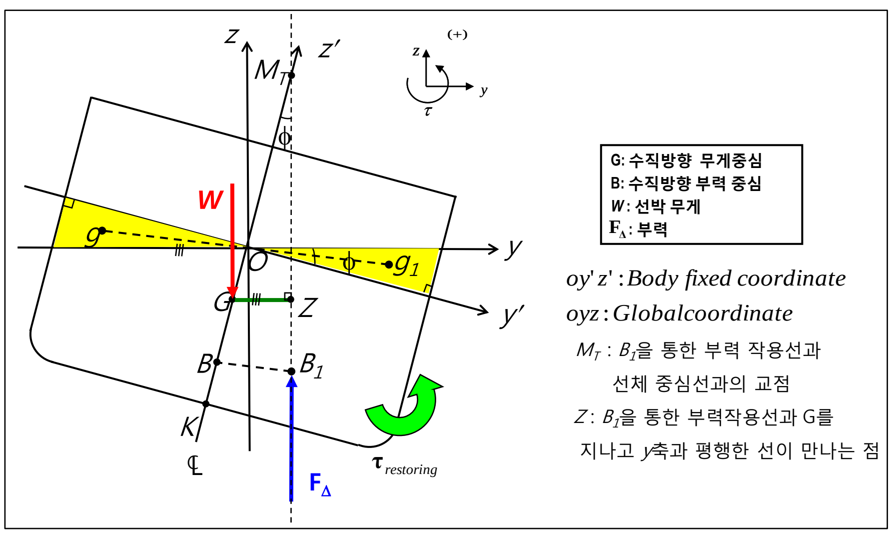

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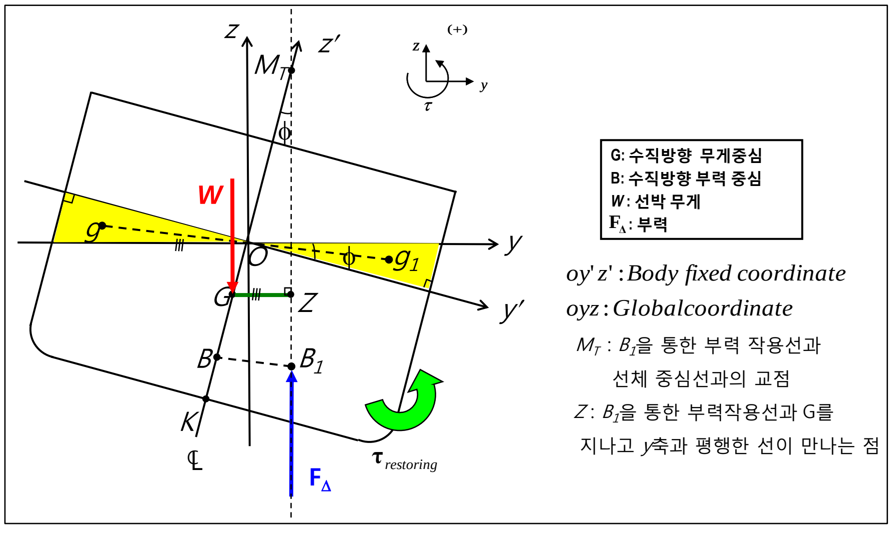

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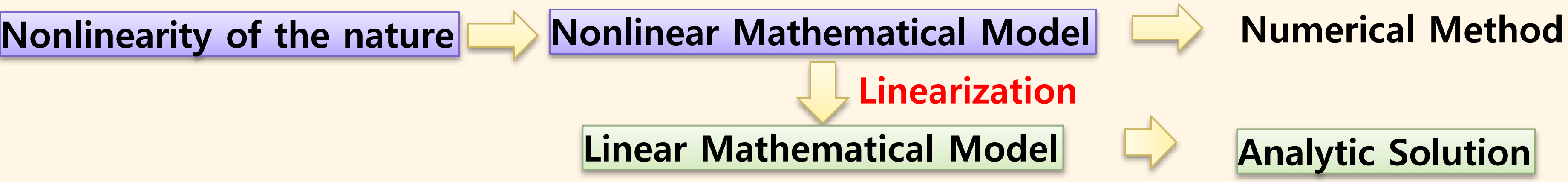

Nonlinearity of the nature → **Nonlinear Mathematical Model** → **Numerical Method**

Linearization

Linear Mathematical Model → **Analytic Solution**

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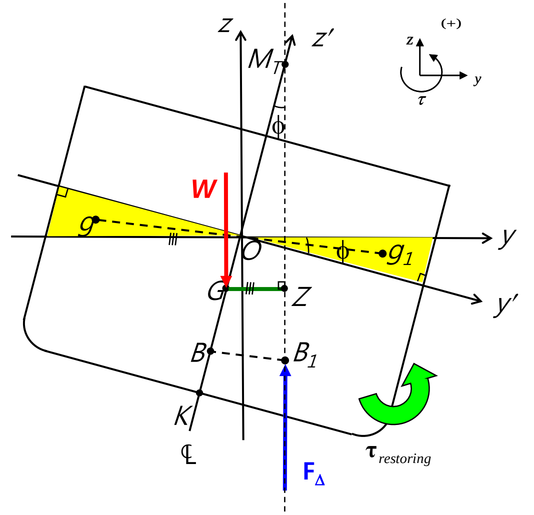

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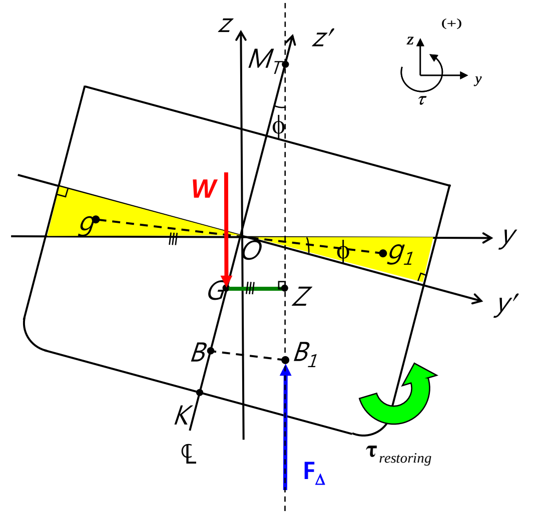

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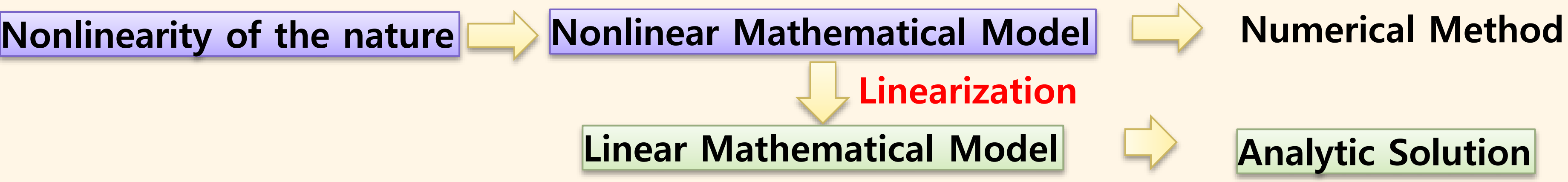

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↓ **Linearization**

Linear Mathematical Model → **Analytic Solution**

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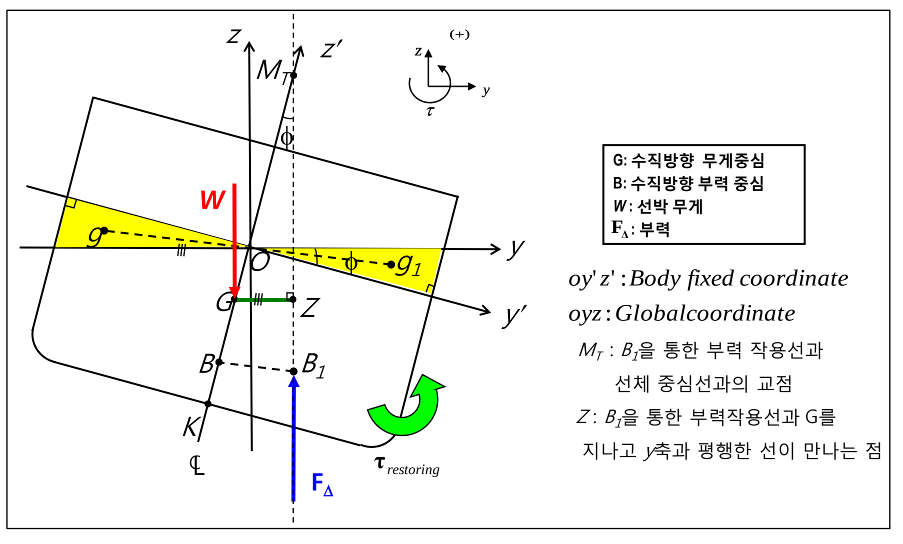
5) $M_{external}$ **: external moment**



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$$\therefore I\phi'' = (-W)\times 0 + (\Delta\overline{GZ}) + (-b\phi') + (-I_{add}\phi'') + M_{ext}$$

Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

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Ex) Roll Motion of a Ship

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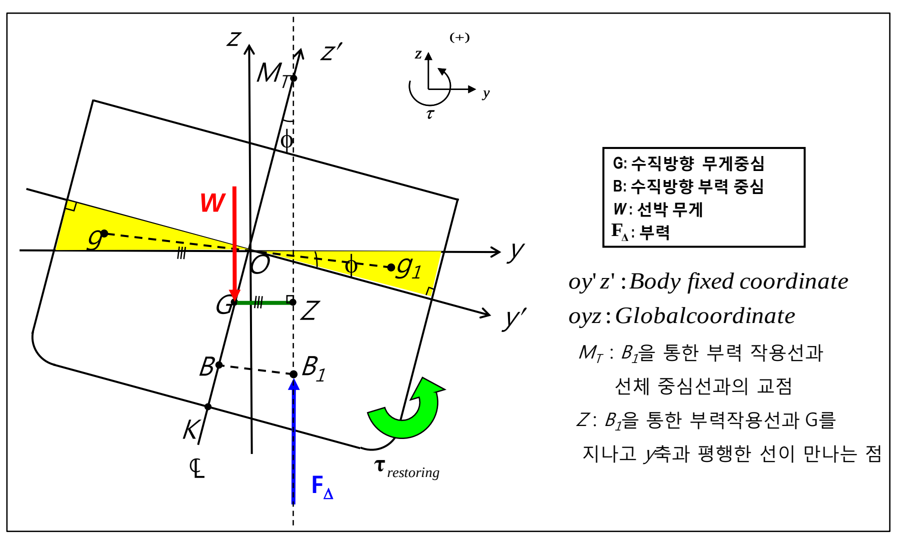
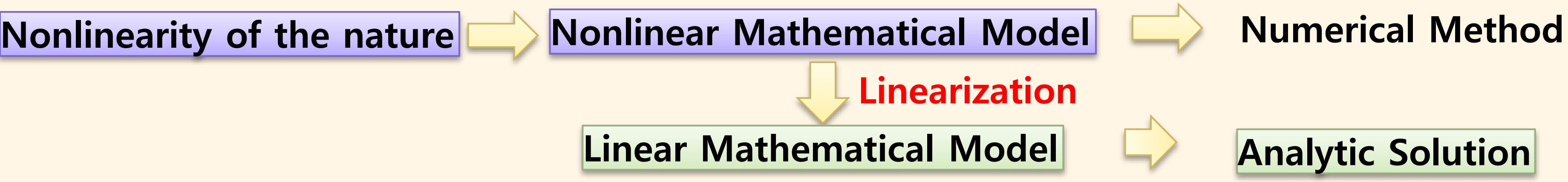
✓ Archimedes' Principle

$$W = \Delta$$



Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) Roll Motion of a Ship

$$I\phi'' = (-W)\times 0 + (\Delta\overline{GZ}) + (-b\phi') + (-I_{add}\phi'') + M_{ext}$$

$$\therefore (I + I_{add})\phi'' + b\phi' - \Delta\overline{GZ} = M_{external}$$

✓ Archimedes' Principle

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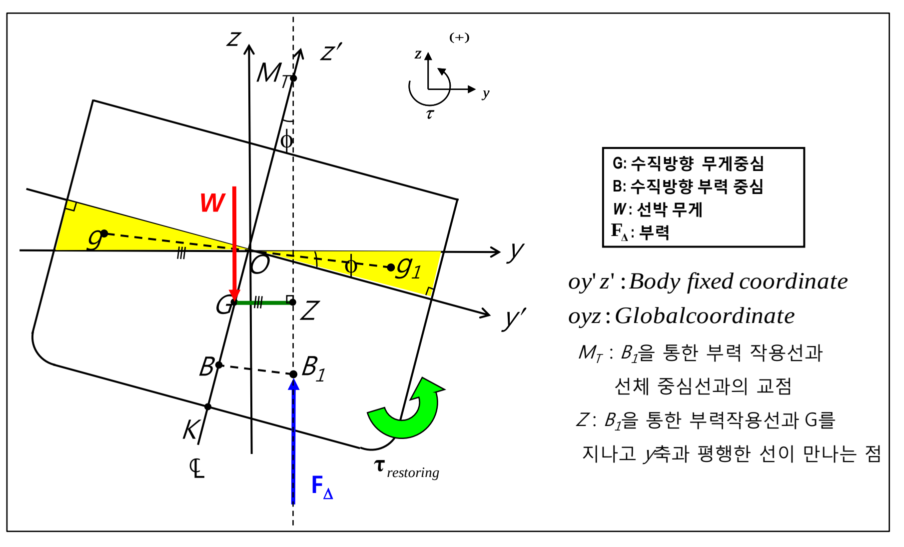

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Linear Mathematical Model ⇨ **Analytic Solution**

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✓ Archimedes' Principle

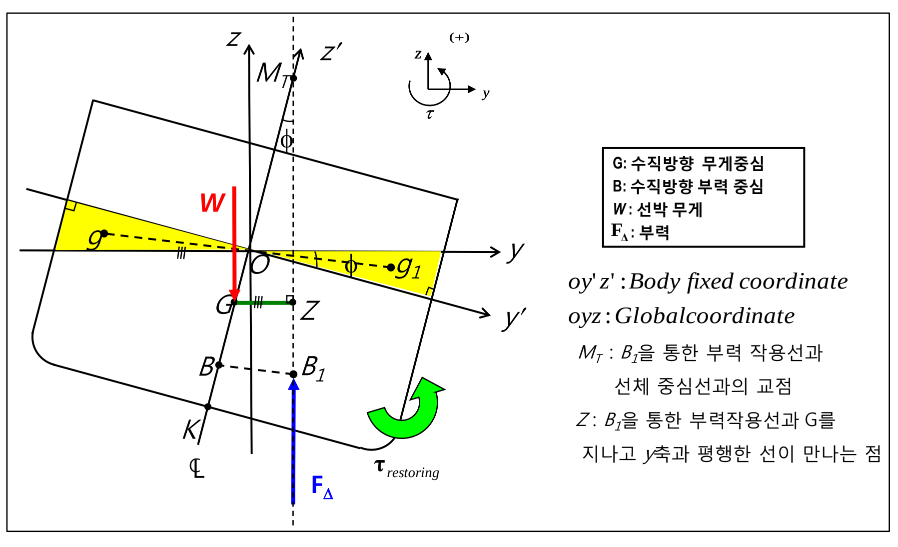
$$W = \Delta$$

$$I\phi'' = (-W)\times 0 + (\Delta\overline{GZ}) + (-b\phi') + (-I_{add}\phi'') + M_{ext}$$

$$\therefore (I + I_{add})\phi'' + b\phi' - \Delta\overline{GZ} = M_{external}$$

Consider restoring moment for the point G

$$\tau_{restoring} = \Delta\underline{\overline{GZ}} \qquad , \boldsymbol{\Delta = -W}$$



G: 수직방향 무게중심
B: 수직방향 부력 중심
W: 선박 무게
$\mathbf{F}_\Delta$: 부력

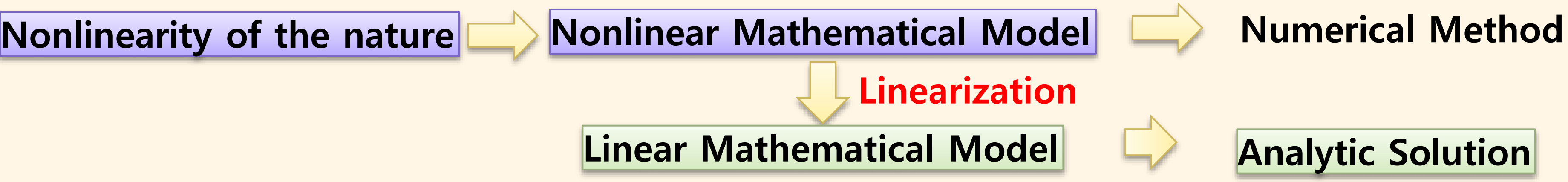
$oy'z'$: *Body fixed coordinate*

oyz : *Globalcoordinate*

M_T : B_1을 통한 부력 작용선과 선체 중심선과의 교점

Z : B_1을 통한 부력작용선과 G를 지나고 y축과 평행한 선이 만나는 점

Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Roll Motion of a Ship

$$I\phi'' = (-W)\times 0 + (\Delta\overline{GZ}) + (-b\phi') + (-I_{add}\phi'') + M_{ext}$$

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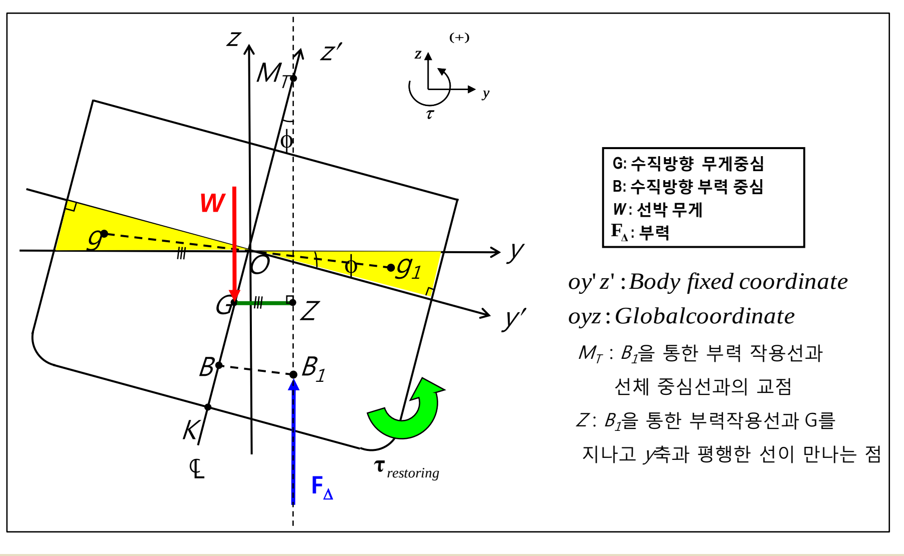

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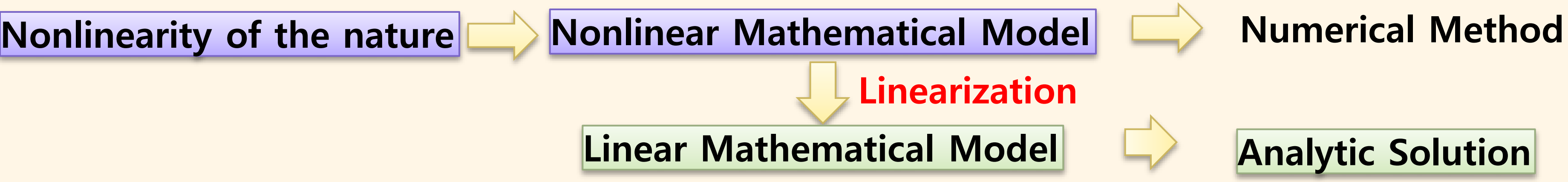
Assum. M_T **doesn't change for small** ϕ $(<10^\circ)$

$\overline{GM}_T$: **Metercenter Height**

$$\underline{\overline{GZ} \approx -\overline{GM}_T \cdot \sin\phi}$$

Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

Linearization

Linear Mathematical Model

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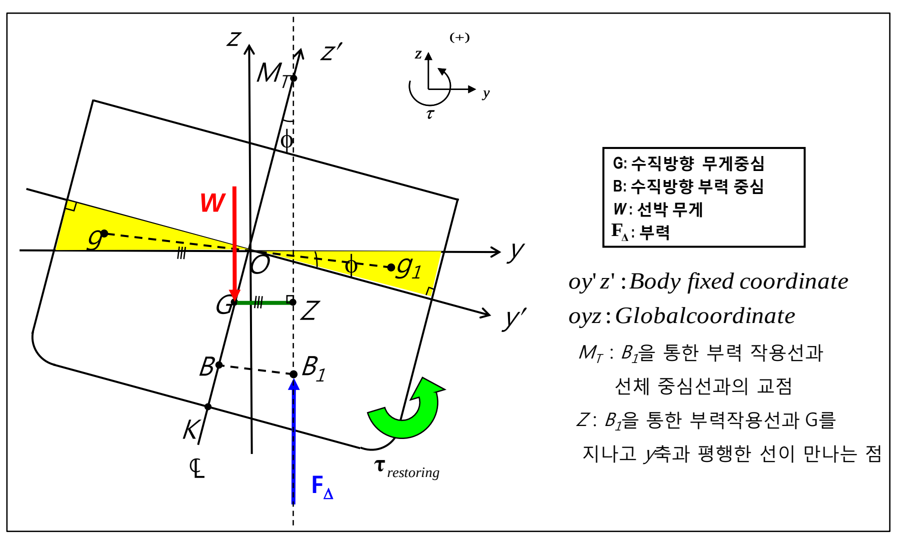

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$$\underline{\overline{GZ} \approx -\overline{GM}_T \cdot \sin\phi}$$

$$\therefore (I + I_{add})\phi'' + b\phi' + \Delta\overline{GM}_T \sin\phi = M_{external}$$

Nonlinearity

$$\boxed{\sin\theta = \theta} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Nonlinearity of the nature ⇨ **Nonlinear Mathematical Model** ⇨ **Numerical Method**

⇩ **Linearization**

Linear Mathematical Model ⇨ **Analytic Solution**

Ex) Roll Motion of a Ship

$$\therefore (I + I_{add})\phi'' + b\phi' + \Delta\overline{GM_T}\sin\phi = M_{external}$$

✓ **Archimedes' Principle**

$$W = \Delta$$

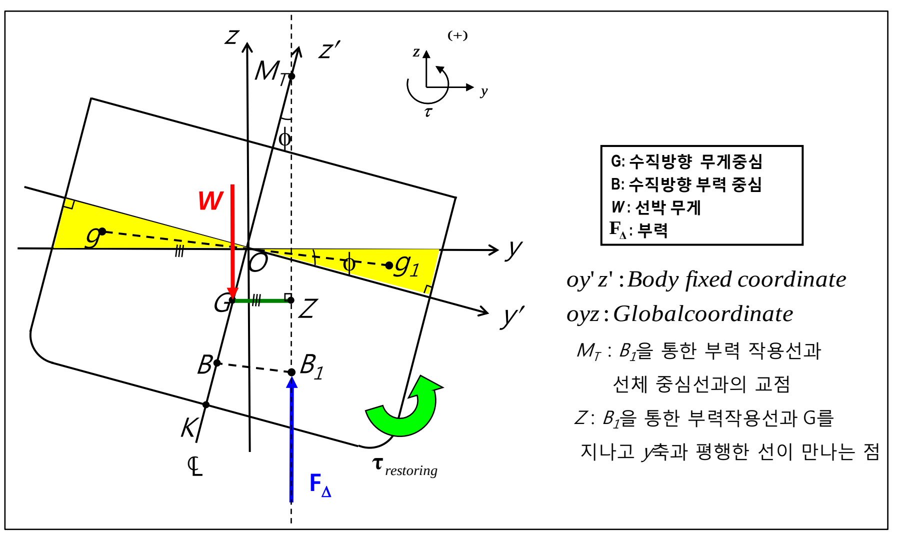
For small ϕ $\boxed{\sin\phi \cong \phi}$

$$(I + I_{add})\phi'' + b\phi' + \Delta\overline{GM_T}\boxed{\phi} = M_{external}$$

정적 평형상태 서는 가속도와 속도 성분이 사라지므로,

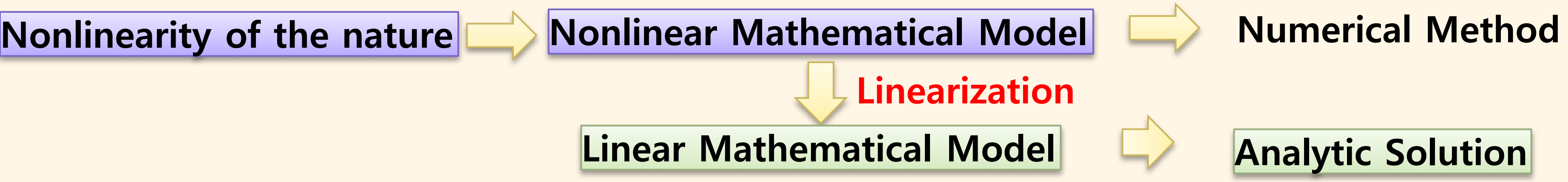
$$\Delta\overline{GM_T}\phi = M_{external}$$

양변의 모멘트가 같아지는 ϕ 까지 기울어 진다.



Nonlinearity

$$\boxed{\sin\theta = \theta} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$



Nonlinearity of the nature → **Nonlinear Mathematical Model** → **Numerical Method**

Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) Roll Motion of a Ship

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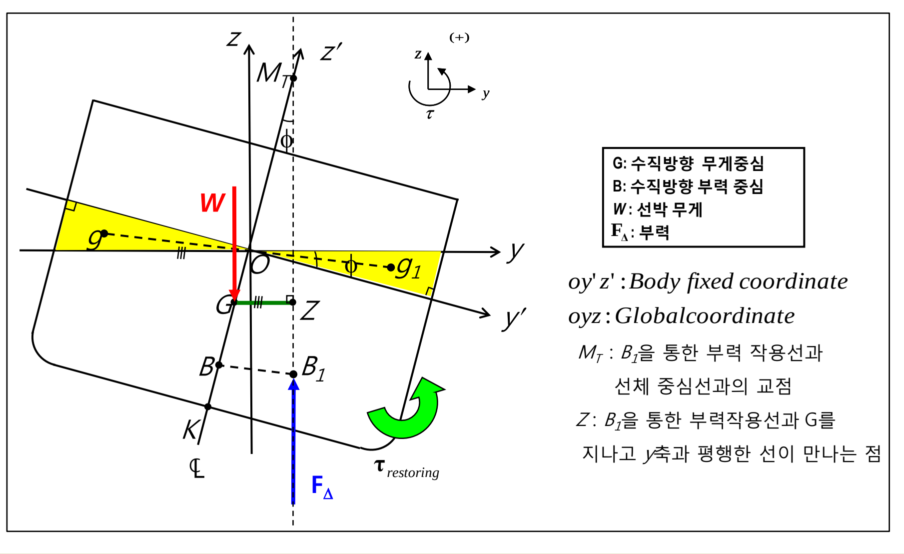


$$\therefore (I + I_{add})\phi'' + b\phi' + \Delta\overline{GM}_T \phi = M_{external} \longleftarrow$$

Second order Linear Ordinary Differential Equation

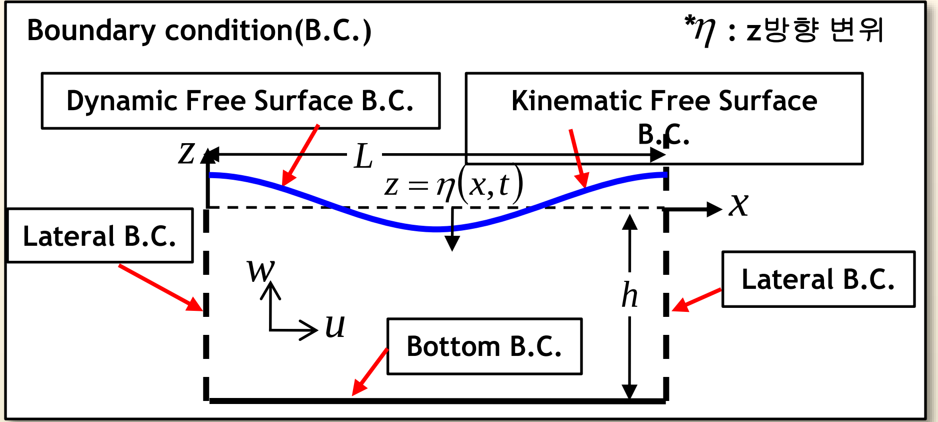
Nonlinearity

Nonlinearity of the nature ⇒ Nonlinear Mathematical Model ⇒ Numerical Method

⇓ Linearization

Linear Mathematical Model ⇒ Analytic Solution

Ex) 해양파 Free surface Boundary Condition



<Summary of the 2-D periodic water wave boundary condition>

① Kinematic Free Surface B.C.(KFSBC)

$$\frac{\partial\Phi}{\partial z}-\frac{\partial\eta}{\partial t}-\frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x}=0 \quad (\text{on } \mathrm{z}=\eta)$$

② Bottom B.C. (BBC)

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=-h}=0$$

③ Dynamic Free Surface B.C. (DFSBC)

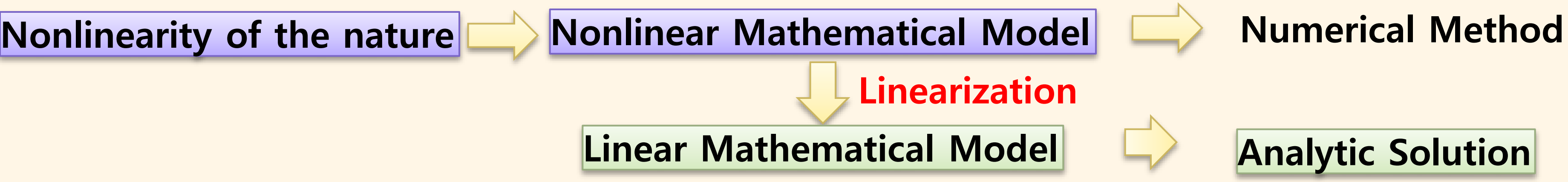
$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left|\nabla\Phi\right|^2+g\eta=0 \quad (\text{on } \mathrm{z}=\eta)$$

④ Lateral B.C.

$$\Phi(x,z,t)=\Phi(x,z,t+T)$$

$$\Phi(x,z,t)=\Phi(x+L,z,t)$$

Nonlinearity



Nonlinearity of the nature

Nonlinear Mathematical Model

Numerical Method

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Ex) 해양파 Free surface Boundary Condition

① Kinematic Free Surface B.C.(KFSBC)

$$\frac{\partial\Phi}{\partial z}-\frac{\partial\eta}{\partial t}-\frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x}=0 \quad (\text{on } z=\eta)$$

Tayler series로 전개하면,

(High Order Term)

$$\left(\frac{\partial\Phi}{\partial z}-\frac{\partial\eta}{\partial t}-\frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x}\right)_{z=\eta}=\left(\frac{\partial\Phi}{\partial z}-\frac{\partial\eta}{\partial t}-\cancel{\frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x}}\right)_{z=0}+\eta\frac{\partial}{\partial z}\left(\cancel{\frac{\partial\Phi}{\partial z}}-\cancel{\frac{\partial\eta}{\partial t}}-\cancel{\frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x}}\right)_{z=0}+\cancel{H.O.T}=0$$

여기서 파장에 비해 **파고가 작다**고 가정했으므로, $\eta<<1$

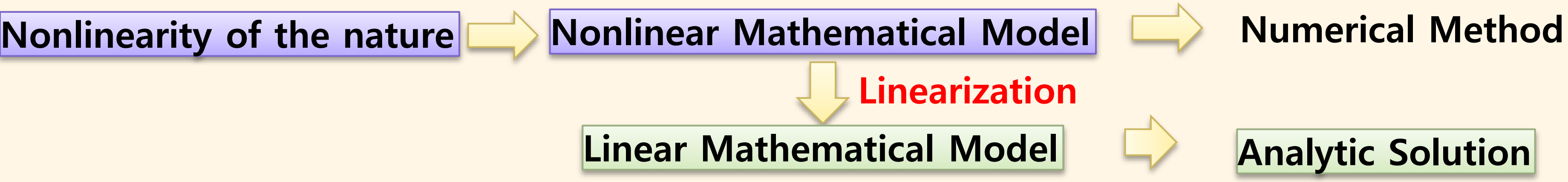
$$u\big|_{z=0}=\frac{\partial\Phi}{\partial x}\bigg|_{z=0}<<1\ ,\ w\big|_{z=0}=\frac{\partial\Phi}{\partial z}\bigg|_{z=0}<<1$$

작은 텀이 두 개 이상 곱해진 경우를 무시하면,

$$\left(\frac{\partial\Phi}{\partial z}-\frac{\partial\eta}{\partial t}\right)_{z=0}=0$$

=> Linearized Kinematic Free Surface B.C.(KFSBC)

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → Analytic Solution

Ex) 해양파 Free surface Boundary Condition

③ Dynamic Free Surface B.C. (DFSBC)

$$\frac{\partial \Phi}{\partial t}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+g\eta=0 \quad (\text{on } z=\eta)$$

Tayler series로 전개하면,

(High Order Term)

$$\left(\frac{\partial \Phi}{\partial t}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+g\eta\right)_{z=\eta}=\left(\frac{\partial \Phi}{\partial t}+\cancel{\frac{1}{2}\left|\nabla \Phi\right|^{2}}+g\eta\right)_{z=0}+\eta\frac{\partial}{\partial z}\left(\cancel{\frac{\partial \Phi}{\partial t}}+\cancel{\frac{1}{2}\left|\nabla \Phi\right|^{2}}+\cancel{g\eta}\right)_{z=0}+\cancel{H.O.T}=0$$

여기서 파장에 비해 **파고가 작다**고 가정했으므로, $\eta << 1$

$$u\big|_{z=0}=\left.\frac{\partial \Phi}{\partial x}\right|_{z=0}<<1 \ , \ w\big|_{z=0}=\left.\frac{\partial \Phi}{\partial z}\right|_{z=0}<<1$$

작은 텀이 두 개 이상 곱해진 경우를 무시하면,

$$\left(\frac{\partial \Phi}{\partial t}+g\eta\right)_{z=0}=0 \Rightarrow \eta=-\frac{1}{g}\frac{\partial \Phi}{\partial t} \quad (\text{on } z=0)$$

=> Linearized Dynamic Free Surface B.C.(DFSBC)

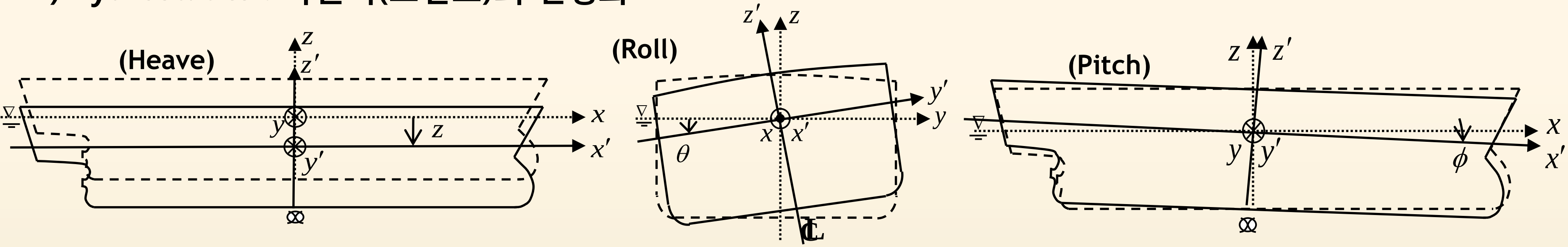
Nonlinearity

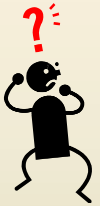
Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

⇩ Linearization

Linear Mathematical Model ⇨ Analytic Solution

Ex) hydrostatics : 복원력(모멘트)의 선형화





초기 자세(z ,θ ,ϕ)에서

복원력($\mathbf{F}$), 횡 방향 복원 모멘트($\mathbf{M}_T$), 종 방향 복원 모멘트($\mathbf{M}_L$)를 알고 있을 때,

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Nonlinearity

Nonlinearity of the nature ⇨ Nonlinear Mathematical Model ⇨ Numerical Method

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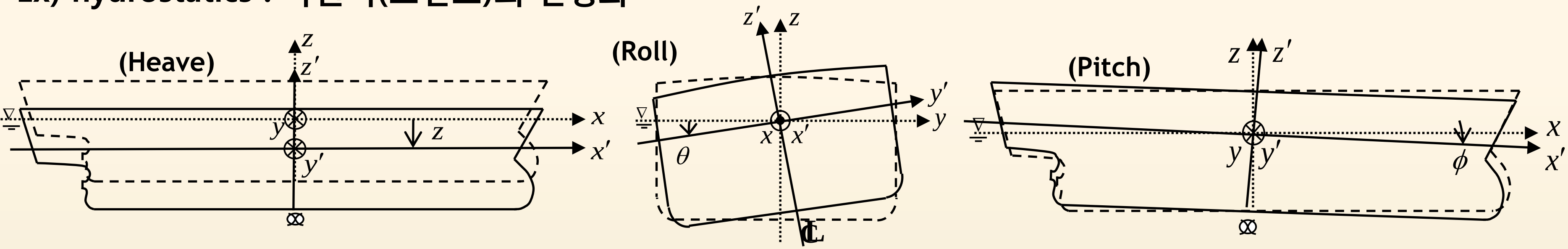

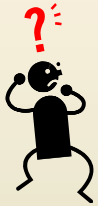

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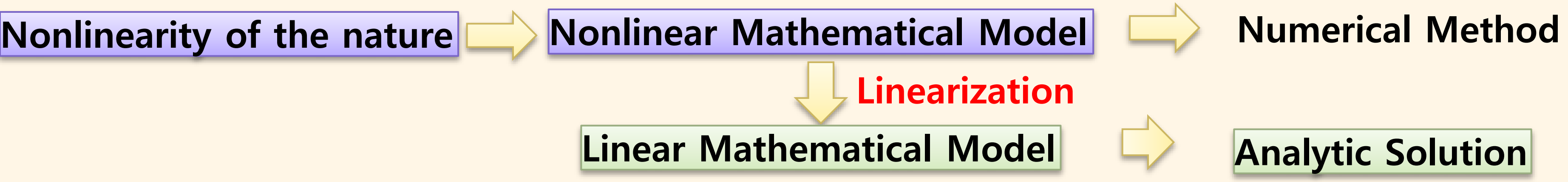
$$\mathbf{F}(z,\theta,\phi)$$

$$\mathbf{M}_T(z,\theta,\phi)$$

$$\mathbf{M}_L(z,\theta,\phi)$$

Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

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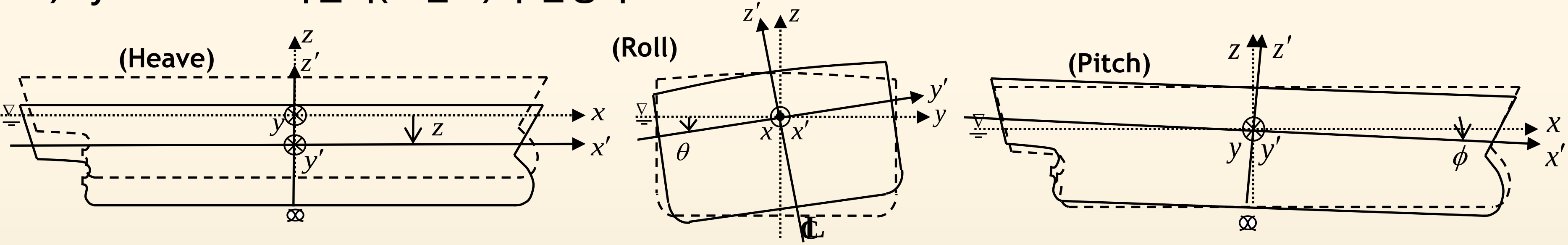

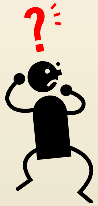

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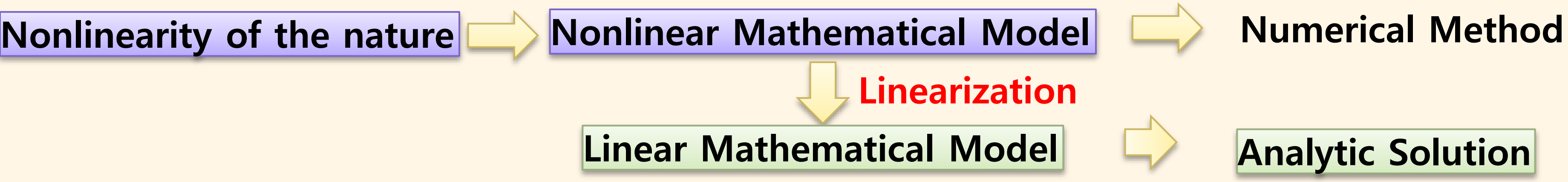
$$\begin{matrix} \mathbf{F}(z,\theta,\phi) & & \mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \\ \mathbf{M}_T(z,\theta,\phi) & \xrightarrow{?} & \mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \\ \mathbf{M}_L(z,\theta,\phi) & & \mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \end{matrix}$$

Nonlinearity



Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ **Linearization**

Linear Mathematical Model → Analytic Solution

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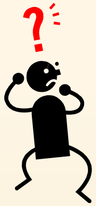

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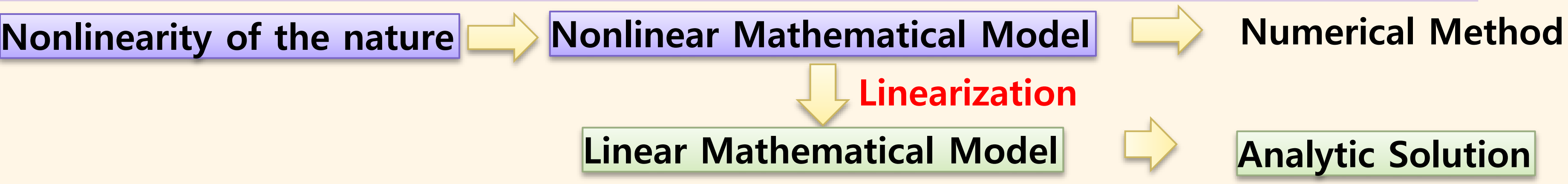
$$\begin{array}{lcl}
\mathbf{F}(z,\theta,\phi) & & \mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \\
\mathbf{M}_T(z,\theta,\phi) & \xrightarrow{?} & \mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \\
\mathbf{M}_L(z,\theta,\phi) & & \mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)
\end{array}$$

Nonlinearity



Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

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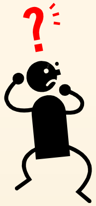

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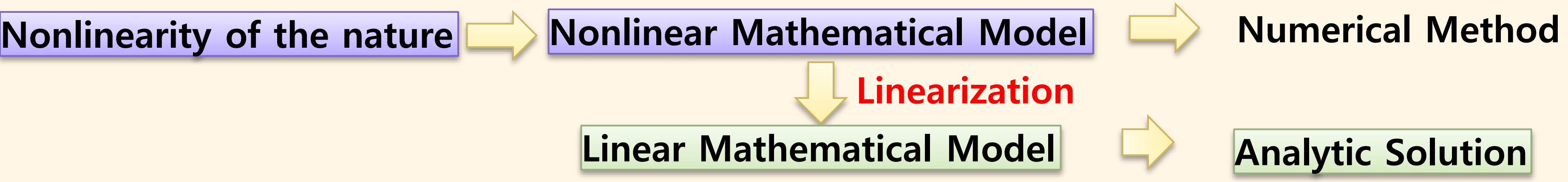
Taylor series expansion

$$\mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)=\mathbf{F}(z,\theta,\phi)+\frac{\partial\mathbf{F}}{\partial z}\Delta z+\frac{\partial\mathbf{F}}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{F}}{\partial\phi}\Delta\phi+\cdots$$

$$\mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)=\mathbf{M}_T(z,\theta,\phi)+\frac{\partial\mathbf{M}_T}{\partial z}\Delta z+\frac{\partial\mathbf{M}_T}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_T}{\partial\phi}\Delta\phi+\cdots$$

$$\mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)=\mathbf{M}_L(z,\theta,\phi)+\frac{\partial\mathbf{M}_L}{\partial z}\Delta z+\frac{\partial\mathbf{M}_L}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_L}{\partial\phi}\Delta\phi+\cdots$$

Nonlinearity

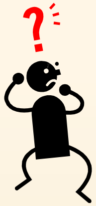


Nonlinearity of the nature → **Nonlinear Mathematical Model** → Numerical Method

↓ Linearization

Linear Mathematical Model → **Analytic Solution**

Ex) hydrostatics : 복원력(모멘트)의 선형화



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$$\begin{array}{ccc} \mathbf{F}(z,\theta,\phi) & & \mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \\ \mathbf{M}_T(z,\theta,\phi) & \xrightarrow{?} & \mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \\ \mathbf{M}_L(z,\theta,\phi) & & \mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi) \end{array}$$

Taylor series expansion

$$\mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)=\mathbf{F}(z,\theta,\phi)+\frac{\partial\mathbf{F}}{\partial z}\Delta z+\frac{\partial\mathbf{F}}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{F}}{\partial\phi}\Delta\phi+\cdots$$

선형화

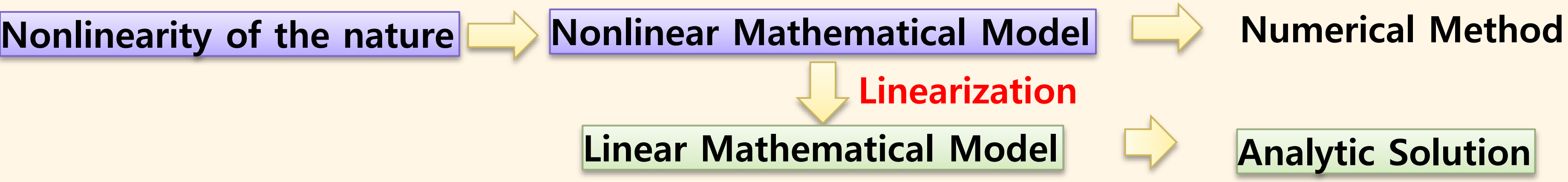
$$\mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)=\mathbf{M}_T(z,\theta,\phi)+\frac{\partial\mathbf{M}_T}{\partial z}\Delta z+\frac{\partial\mathbf{M}_T}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_T}{\partial\phi}\Delta\phi+\cdots$$

선형화

$$\mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)=\mathbf{M}_L(z,\theta,\phi)+\frac{\partial\mathbf{M}_L}{\partial z}\Delta z+\frac{\partial\mathbf{M}_L}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_L}{\partial\phi}\Delta\phi+\cdots$$

선형화

Nonlinearity

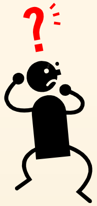


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) hydrostatics : 복원력(모멘트)의 선형화



초기 자세(z ,θ ,ϕ)에서

복원력($\mathbf{F}$), 횡 방향 복원 모멘트($\mathbf{M}_T$), 종 방향 복원 모멘트($\mathbf{M}_L$)를 알고 있을 때,

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$$\mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)-\mathbf{F}(z,\theta,\phi)=\frac{\partial\mathbf{F}}{\partial z}\Delta z+\frac{\partial\mathbf{F}}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{F}}{\partial\phi}\Delta\phi$$

$$\mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)-\mathbf{M}_T(z,\theta,\phi)=\frac{\partial\mathbf{M}_T}{\partial z}\Delta z+\frac{\partial\mathbf{M}_T}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_T}{\partial\phi}\Delta\phi$$

$$\mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)-\mathbf{M}_L(z,\theta,\phi)=\frac{\partial\mathbf{M}_L}{\partial z}\Delta z+\frac{\partial\mathbf{M}_L}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_L}{\partial\phi}\Delta\phi$$

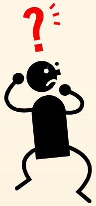
Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Nonlinear Mathematical Model ↓ Linearization

Linear Mathematical Model → Analytic Solution

Ex) hydrostatics : 복원력(모멘트)의 선형화



초기 자세(z ,θ ,ϕ)에서

복원력($\mathbf{F}$), 횡 방향 복원 모멘트($\mathbf{M}_T$), 종 방향 복원 모멘트($\mathbf{M}_L$)를 알고 있을 때,

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$$\mathbf{F}(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)-\mathbf{F}(z,\theta,\phi)=\frac{\partial \mathbf{F}}{\partial z}\Delta z+\frac{\partial \mathbf{F}}{\partial \theta}\Delta\theta+\frac{\partial \mathbf{F}}{\partial \phi}\Delta\phi$$

$$\mathbf{M}_T(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)-\mathbf{M}_T(z,\theta,\phi)=\frac{\partial \mathbf{M}_T}{\partial z}\Delta z+\frac{\partial \mathbf{M}_T}{\partial \theta}\Delta\theta+\frac{\partial \mathbf{M}_T}{\partial \phi}\Delta\phi$$

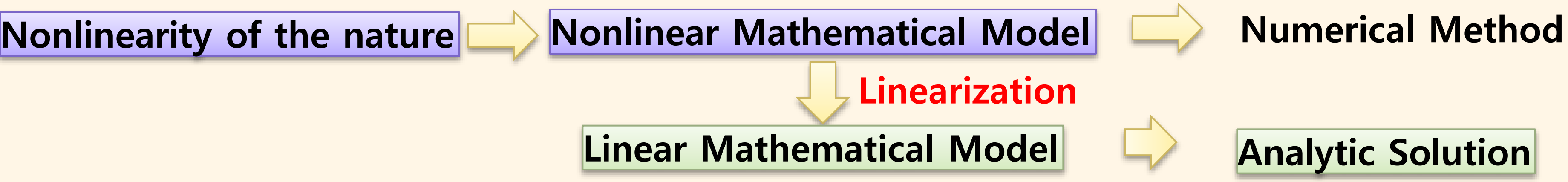
$$\mathbf{M}_L(z+\Delta z,\theta+\Delta\theta,\phi+\Delta\phi)-\mathbf{M}_L(z,\theta,\phi)=\frac{\partial \mathbf{M}_L}{\partial z}\Delta z+\frac{\partial \mathbf{M}_L}{\partial \theta}\Delta\theta+\frac{\partial \mathbf{M}_L}{\partial \phi}\Delta\phi$$

$$\Delta\mathbf{F}=\frac{\partial \mathbf{F}}{\partial z}\Delta z+\frac{\partial \mathbf{F}}{\partial \theta}\Delta\theta+\frac{\partial \mathbf{F}}{\partial \phi}\Delta\phi$$

$$\Delta\mathbf{M}_T=\frac{\partial \mathbf{M}_T}{\partial z}\Delta z+\frac{\partial \mathbf{M}_T}{\partial \theta}\Delta\theta+\frac{\partial \mathbf{M}_T}{\partial \phi}\Delta\phi$$

$$\Delta\mathbf{M}_L=\frac{\partial \mathbf{M}_L}{\partial z}\Delta z+\frac{\partial \mathbf{M}_L}{\partial \theta}\Delta\theta+\frac{\partial \mathbf{M}_L}{\partial \phi}\Delta\phi$$

Nonlinearity

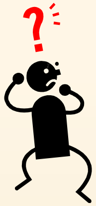


Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

Linearization

Linear Mathematical Model → Analytic Solution

Ex) hydrostatics : 복원력(모멘트)의 선형화



초기 자세(z , θ , ϕ)에서

복원력($\mathbf{F}$), 횡 방향 복원 모멘트($\mathbf{M}_T$), 종 방향 복원 모멘트($\mathbf{M}_L$)를 알고 있을 때,

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$$\Delta\mathbf{F} = \frac{\partial\mathbf{F}}{\partial z}\Delta z + \frac{\partial\mathbf{F}}{\partial\theta}\Delta\theta + \frac{\partial\mathbf{F}}{\partial\phi}\Delta\phi$$

$$\Delta\mathbf{M}_T = \frac{\partial\mathbf{M}_T}{\partial z}\Delta z + \frac{\partial\mathbf{M}_T}{\partial\theta}\Delta\theta + \frac{\partial\mathbf{M}_T}{\partial\phi}\Delta\phi$$

$$\Delta\mathbf{M}_L = \frac{\partial\mathbf{M}_L}{\partial z}\Delta z + \frac{\partial\mathbf{M}_L}{\partial\theta}\Delta\theta + \frac{\partial\mathbf{M}_L}{\partial\phi}\Delta\phi$$

Nonlinearity

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ **Linearization**

Linear Mathematical Model → Analytic Solution

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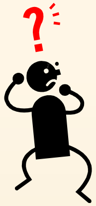

초기 자세(z ,θ ,ϕ)에서

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✓ Matrix로 표현 $\mathbf{b}=\mathbf{A}\mathbf{x}$

$$\Delta\mathbf{F}=\frac{\partial\mathbf{F}}{\partial z}\Delta z+\frac{\partial\mathbf{F}}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{F}}{\partial\phi}\Delta\phi$$

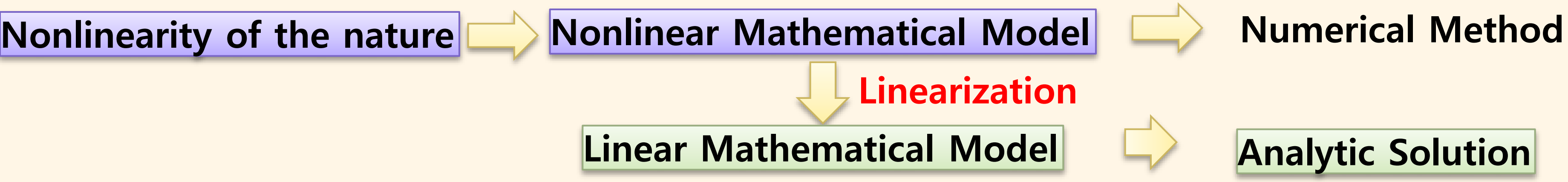
$$\Delta\mathbf{M}_T=\frac{\partial\mathbf{M}_T}{\partial z}\Delta z+\frac{\partial\mathbf{M}_T}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_T}{\partial\phi}\Delta\phi$$

$$\Delta\mathbf{M}_L=\frac{\partial\mathbf{M}_L}{\partial z}\Delta z+\frac{\partial\mathbf{M}_L}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_L}{\partial\phi}\Delta\phi$$

변수 3개, 식 3개 ⇨

$$\begin{pmatrix}\Delta\mathbf{F}\\ \Delta\mathbf{M}_T\\ \Delta\mathbf{M}_L\end{pmatrix}=\begin{pmatrix}\frac{\partial\mathbf{F}}{\partial z} & \frac{\partial\mathbf{F}}{\partial\theta} & \frac{\partial\mathbf{F}}{\partial\phi}\\ \frac{\partial\mathbf{M}_T}{\partial z} & \frac{\partial\mathbf{M}_T}{\partial\theta} & \frac{\partial\mathbf{M}_T}{\partial\phi}\\ \frac{\partial\mathbf{M}_L}{\partial z} & \frac{\partial\mathbf{M}_L}{\partial\theta} & \frac{\partial\mathbf{M}_L}{\partial\phi}\end{pmatrix}\begin{pmatrix}\Delta z\\ \Delta\theta\\ \Delta\phi\end{pmatrix}$$

Nonlinearity



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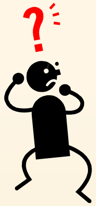

초기 자세(z ,θ ,ϕ)에서

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$$\Delta\mathbf{F}=\frac{\partial\mathbf{F}}{\partial z}\Delta z+\frac{\partial\mathbf{F}}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{F}}{\partial\phi}\Delta\phi$$

$$\Delta\mathbf{M}_T=\frac{\partial\mathbf{M}_T}{\partial z}\Delta z+\frac{\partial\mathbf{M}_T}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_T}{\partial\phi}\Delta\phi$$

$$\Delta\mathbf{M}_L=\frac{\partial\mathbf{M}_L}{\partial z}\Delta z+\frac{\partial\mathbf{M}_L}{\partial\theta}\Delta\theta+\frac{\partial\mathbf{M}_L}{\partial\phi}\Delta\phi$$

1. 자세의 변화량이 주어져 있을 때, 힘(모멘트)의 변화량을 구하는 경우

$$\mathbf{b}=\mathbf{A}\mathbf{x}$$

$$\overset{\text{Find}}{\begin{pmatrix}\Delta\mathbf{F}\\ \Delta\mathbf{M}_T\\ \Delta\mathbf{M}_L\end{pmatrix}}=\overset{\text{Given}}{\begin{pmatrix}\frac{\partial\mathbf{F}}{\partial z}&\frac{\partial\mathbf{F}}{\partial\theta}&\frac{\partial\mathbf{F}}{\partial\phi}\\ \frac{\partial\mathbf{M}_T}{\partial z}&\frac{\partial\mathbf{M}_T}{\partial\theta}&\frac{\partial\mathbf{M}_T}{\partial\phi}\\ \frac{\partial\mathbf{M}_L}{\partial z}&\frac{\partial\mathbf{M}_L}{\partial\theta}&\frac{\partial\mathbf{M}_L}{\partial\phi}\end{pmatrix}}\overset{\text{Given}}{\begin{pmatrix}\Delta z\\ \Delta\theta\\ \Delta\phi\end{pmatrix}}$$

※$\mathbf{A}$ 가 선형화되어 있기 때문에 반복 계산 (iteration)을 해야 함

2. 힘(모멘트)의 변화량이 주어져 있을 때, 자세의 변화량을 구하는 경우

$$\mathbf{x}=\mathbf{A}^{-1}\mathbf{x}$$

$$\overset{\text{Given}}{\begin{pmatrix}\Delta\mathbf{F}\\ \Delta\mathbf{M}_T\\ \Delta\mathbf{M}_L\end{pmatrix}}=\overset{\text{Given}}{\begin{pmatrix}\frac{\partial\mathbf{F}}{\partial z}&\frac{\partial\mathbf{F}}{\partial\theta}&\frac{\partial\mathbf{F}}{\partial\phi}\\ \frac{\partial\mathbf{M}_T}{\partial z}&\frac{\partial\mathbf{M}_T}{\partial\theta}&\frac{\partial\mathbf{M}_T}{\partial\phi}\\ \frac{\partial\mathbf{M}_L}{\partial z}&\frac{\partial\mathbf{M}_L}{\partial\theta}&\frac{\partial\mathbf{M}_L}{\partial\phi}\end{pmatrix}}\overset{\text{Find}}{\begin{pmatrix}\Delta z\\ \Delta\theta\\ \Delta\phi\end{pmatrix}}$$