Naval Architecture & Ocean Engineering

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Engineering Mathematics 2

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Ordinary Differential Equations (2)

Separable Variables
Linear Equations
Exact Solution
Solutions by Substitution
Reduction of Order
Undetermined Coefficient
Variation of Parameters

Nonlinear - Reduction of Order
Nonlinear - Taylor Series
Nonlinear - A numerical Method

Separable Variables

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How to solve a D.E?

Separable Variables

How to solve a D.E?

Ex) Population dynamics

Separable Variables

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Ex) Population dynamics

$$\frac{dP(x)}{dx} = kP(x)$$

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Integration!!

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L.H.S:

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Definition 2.1 **Separable Equation**

A first order differential equation of the form

$$\frac{dy}{dx} = g(x)h(y)$$

is said to be separable or to have separable variables

Separable Variables

☑ **Example 1**

Solving a Separable DE

Separable Variables

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Solving a Separable DE

Solve

Separable Variables

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Solving a Separable DE

Solve $(1+x)dy - ydx = 0$

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Dividing by

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$$\frac{dy}{y} = \frac{dx}{(1+x)}$$

Separable Variables

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Separable Variables

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Dividing by $(1+x)y$

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$$\ln|y| = \ln|1+x| + c_1$$

Separable Variables

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$$\ln|y| = \ln|1+x| + c_1$$

$$y = e^{\ln|1+x|+c_1} = e^{\ln|1+x|} \cdot e^{c_1}$$

$$= |1+x|\, e^{c_1}$$

$$= \pm e^{c_1}(1+x)$$

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$$= \pm e^{c_1}(1+x)$$

$$y = c(1+x)$$

Separable Variables

☑ **Example 2**

Solution Curve

Solve the IVP

$$\frac{dy}{dx} = -\frac{x}{y}, \quad y(4) = 3$$

Separable Variables

☑ Example 2
Solution Curve

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$$ydy = -xdx$$

$$\int ydy = -\int xdx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c_1$$

Separable Variables

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$$x^2 + y^2 = c^2$$

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Substitute x=4 and y=-3 for satisfy initial condition $y(4) = 3$

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$$x^2 + y^2 = c^2$$

Substitute x=4 and y=-3 for satisfy initial condition $y(4) = 3$

$$4^2 + (-3)^2 = c^2$$

$$c^2 = 25$$

$$\therefore x^2 + y^2 = 25$$

Separable Variables

☑ Example 3
Losing a Solution

Solve $$\frac{dy}{dx} = y^2 - 4$$

Separable Variables

☑ Example 3
Losing a Solution

Solve $\dfrac{dy}{dx} = y^2 - 4$

$$\frac{dy}{y^2-4} = dx \quad (y \neq \pm 2)$$

$$\frac{1}{4}\left(\frac{1}{y-2} - \frac{1}{y+2}\right)dy = dx$$

$$\frac{1}{4}\int \left(\frac{1}{y-2} - \frac{1}{y+2}\right)dy = \int dx$$

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$$\frac{1}{4}(\ln|y-2| - \ln|y+2|) = x + c_1$$

$$\ln\left|\frac{y-2}{y+2}\right| = 4x + c_2$$

$$\frac{y-2}{y+2} = e^{4x+c_2} = ce^{4x}$$

$$y = 2\frac{1+ce^{4x}}{1-ce^{4x}} \quad (y \neq \pm 2)$$

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$y = \pm 2$ **is constant solution, but cannot be obtained from above solution**

Linear Equations

Linear Equations

Definition 2.2 **Linear Equation**

A first order differential equation of the form

$$a_1(x)\frac{dy}{dx}+a_0(x)y=g(x)$$

is said to be Linear equation in the dependant variable y.

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$$\frac{dy}{dx}+P(x)y=f(x)\xrightarrow{\text{solution}}y=y_c+y_p$$

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$$\frac{d}{dx}[y_c+y_p]+P(x)[y_c+y_p]$$
$$=\left[\frac{d}{dx}y_c+P(x)y_c\right]+\left[\frac{d}{dx}y_p+P(x)y_p\right]=f(x)$$

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☑ Homogeneous

$$f(x)=0$$

☑ Nonhomogeneous

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$$f(x)=0\xrightarrow[]{\text{solution}}y_c$$

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$$f(x)\neq 0$$

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☑ Nonhomogeneous

$$f(x)\neq 0\xrightarrow[y_p]{\text{solution}}$$

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☑ Nonhomogeneous

$$f(x)\neq 0\xrightarrow[y_p]{\text{solution}}\frac{dy_p}{dx}+P(x)y_p=f(x)$$

Linear Equations

Definition 2.2 **Linear Equation**

A first order differential equation of the form

$$a_1(x)\frac{dy}{dx}+a_0(x)y=g(x)$$

is said to be Linear equation in the dependant variable y.

☑ Standard form

$$\frac{dy}{dx}+P(x)y=f(x)\xrightarrow{\text{solution}}y=y_c+y_p$$

$$\frac{d}{dx}[y_c+y_p]+P(x)[y_c+y_p]$$
$$=\left[\frac{d}{dx}y_c+P(x)y_c\right]+\left[\frac{d}{dx}y_p+P(x)y_p\right]=f(x)$$

☑ Homogeneous

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Linear Equations

☑ **Variation of Parameters**

Linear Equations

☑ **Variation of Parameters**

- **Homogeneous solution**

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx \longrightarrow$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx \longrightarrow y_c = ce^{-\int P(x)dx}$$

$$= cy_1(x)$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx \longrightarrow y_c = ce^{-\int P(x)dx}$$

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Linear Equations

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Variation of Parameters

Linear Equations

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$\boxed{}$ Variation of Parameters

Linear Equations

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$$= \boxed{c}y_1(x)$$

- **Nonhomogeneous solution**

$$y_p = \boxed{u(x)}y_1(x)$$

Variation of Parameters

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx \longrightarrow y_c = ce^{-\int P(x)dx}$$

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- **Nonhomogeneous solution**

$$y_p = \boxed{u(x)}y_1(x) \qquad \underset{\text{Variation of Parameters}}{c \rightarrow u(x)}$$

Linear Equations

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- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Linear Equations

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Variation of Parameters

Find :

Linear Equations

☑ Variation of Parameters

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$$= \boxed{c}y_1(x)$$

- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Find : $u(x)$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx \longrightarrow y_c = ce^{-\int P(x)dx}$$

$$= \boxed{c}y_1(x)$$

- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Find : $\xrightarrow{u(x)}$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

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$$= \boxed{c}y_1(x)$$

- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Find : $\xrightarrow{u(x)}$ **Find :**

Linear Equations

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- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Find : $\xrightarrow{u(x)}$ **Find :** $y_p(x)$

Linear Equations

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- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Find : $\xrightarrow{u(x)}$ **Find :** $y_p(x)$

$$y_p(x) = u(x)y_1(x)$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

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Variation of Parameters

Find : $\xrightarrow{u(x)}$ **Find :** $y_p(x)$

$$y_p(x) = u(x)y_1(x)$$

substituting

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

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- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \to u(x)$

Variation of Parameters

Find : $\xrightarrow{u(x)}$ **Find :** $y_p(x)$

$$y_p(x) = u(x)y_1(x) \xrightarrow{\text{substituting}}$$

Linear Equations

☑ Variation of Parameters

- **Homogeneous solution**

$$\frac{dy_c}{dx} + P(x)y_c = 0 \Rightarrow \frac{dy_c}{y_c} = -P(x)dx \longrightarrow y_c = ce^{-\int P(x)dx}$$

$$= \boxed{c}y_1(x)$$

- **Nonhomogeneous solution**

Assumption: $y_p = \boxed{u(x)}y_1(x)$ $\quad c \rightarrow u(x)$

Variation of Parameters

Find : $\xrightarrow{u(x)}$ **Find :** $y_p(x)$

$$y_p(x) = u(x)y_1(x) \xrightarrow{\text{substituting}} \frac{dy_p}{dx} + P(x)y_p = f(x)$$

Linear Equations

☑ **Variation of Parameters**

Linear Equations

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$$\frac{dy_p}{dx} + P(x)y_p = f(x)$$

Linear Equations

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Linear Equations

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$$y_p = u(x)y_1(x)$$

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Linear Equations

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$$\frac{dy_p}{dx} + P(x)y_p = f(x)$$

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substituting

Linear Equations

☑ Variation of Parameters

$$\frac{dy_p}{dx} + P(x)y_p = f(x)$$

$y_p = u(x)y_1(x)$

substituting

L.H.S:

Linear Equations

☑ Variation of Parameters

$$\frac{dy_p}{dx} + P(x)y_p = f(x)$$

$y_p = u(x)y_1(x)$
substituting

L.H.S: $$\frac{dy_p}{dx} + P(x)y_p = \frac{d(uy_1)}{dx} + P(x)(uy_1)$$

$$= y_1\frac{du}{dx} + u\frac{dy_1}{dx} + P(x)uy_1$$

$$= y_1\frac{du}{dx} + u\left[\frac{dy_1}{dx} + P(x)y_1\right] = y_1\frac{du}{dx}$$

Linear Equations

☑ Variation of Parameters

$$\frac{dy_p}{dx} + P(x)y_p = f(x)$$

$y_p = u(x)y_1(x)$
substituting

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Linear Equations

☑ Variation of Parameters

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R.H.S: $f(x)$

Linear Equations

☑ Variation of Parameters

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R.H.S: $f(x)$

$$\therefore y_1\frac{du}{dx} = f(x)$$

Linear Equations

☑ **Variation of Parameters**

Linear Equations

☑ Variation of Parameters

$$y_1 \frac{du}{dx} = f(x)$$

Linear Equations

☑ Variation of Parameters

$$y_1 \frac{du}{dx} = f(x) \xrightarrow[\text{and integrating}]{\text{Separable variables}}$$

Linear Equations

☑ Variation of Parameters

$$y_1 \frac{du}{dx} = f(x) \xrightarrow[\text{and integrating}]{\text{Separable variables}} u = \int \frac{f(x)}{y_1(x)} dx$$

Linear Equations

☑ Variation of Parameters

$$y_1 \frac{du}{dx} = f(x) \xrightarrow[\text{and integrating}]{\text{Separable variables}} u = \int \frac{f(x)}{y_1(x)} dx$$

$$\therefore y_p = u(x) y_1(x) = \left(\int \frac{f(x)}{y_1(x)} dx \right) y_1(x)$$

Linear Equations

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Linear Equations

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Recall, $y_c = ce^{-\int P(x)dx} = cy_1(x)$

Linear Equations

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Linear Equations

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Linear Equations

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Recall, $y_c = c\boxed{e^{-\int P(x)dx}} = c\boxed{y_1(x)}$

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Linear Equations

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Now , we get

Linear Equations

☑ Variation of Parameters

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$$= \left(\int e^{\int P(x)dx} f(x) dx \right) e^{-\int P(x)dx}$$

Now , we get $\left[y_c = ce^{-\int P(x)dx} \right.$

Linear Equations

☑ Variation of Parameters

$$y_1 \frac{du}{dx} = f(x) \xrightarrow[\text{and integrating}]{\text{Separable variables}} u = \int \frac{f(x)}{y_1(x)} dx$$

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Recall, $y_c = c\boxed{e^{-\int P(x)dx}} = c\boxed{y_1(x)}$

$$= \left(\int e^{\int P(x)dx} f(x) dx \right) e^{-\int P(x)dx}$$

Now , we get

$$\begin{cases} y_c = ce^{-\int P(x)dx} \\ y_p = e^{-\int P(x)dx} \int e^{\int P(x)dx} f(x) dx \end{cases}$$

Linear Equations

☑ Variation of Parameters

$$y_1 \frac{du}{dx} = f(x) \xrightarrow[\text{and integrating}]{\text{Separable variables}} u = \int \frac{f(x)}{y_1(x)} dx$$

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Linear Equations

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Linear Equations

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Linear Equations

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$$= \left(\int e^{\int P(x)dx} f(x) dx \right) e^{-\int P(x)dx}$$

Now , we get

$$\begin{cases} y_c = ce^{-\int P(x)dx} \\ y_p = \underbrace{\boxed{e^{-\int P(x)dx}}}_{y_1(x)} \underbrace{\boxed{\int e^{\int P(x)dx} f(x)dx}}_{u(x)} \end{cases}$$

Linear Equations

Linear Equations

☑ **Variation of Parameters**

Linear Equations

☑ Variation of Parameters

$$\frac{dy}{dx} + P(x)y = f(x)$$

Linear Equations

☑ Variation of Parameters

$$\frac{dy}{dx} + P(x)y = f(x) \xrightarrow{\text{solution}} y = y_c + y_p$$

Linear Equations

☑ Variation of Parameters

$$\frac{dy}{dx} + P(x)y = f(x) \xrightarrow[y = y_c + y_p]{\text{solution}} \begin{cases} y_c = ce^{-\int P(x)dx} \\ y_p = e^{-\int P(x)dx} \int e^{\int P(x)dx} f(x)dx \end{cases}$$

Linear Equations

☑ Variation of Parameters

$$\frac{dy}{dx} + P(x)y = f(x) \xrightarrow[y = y_c + y_p]{\text{solution}} \begin{cases} y_c = ce^{-\int P(x)dx} \\ y_p = e^{-\int P(x)dx} \int e^{\int P(x)dx} f(x)dx \end{cases}$$

☑ Integrating factor

Linear Equations

☑ Variation of Parameters

$$\frac{dy}{dx} + P(x)y = f(x) \xrightarrow[y = y_c + y_p]{\text{solution}} \begin{cases} y_c = ce^{-\int P(x)dx} \\ y_p = e^{-\int P(x)dx} \int e^{\int P(x)dx} f(x)dx \end{cases}$$

☑ Integrating factor

$$y = ce^{-\int P(x)dx} + e^{-\int P(x)dx} \int e^{\int P(x)dx} f(x)dx$$

Multiplied by $\boxed{e^{\int P(x)dx}}$

$$e^{\int P(x)dx} y = c + \int e^{\int P(x)dx} f(x)dx$$

Linear Equations

☑ **Integrating factor**

Linear Equations

☑ **Integrating factor**

$$\frac{dy}{dx} + P(x)y = f(x)$$

Linear Equations

☑ Integrating factor

$$\frac{dy}{dx} + P(x)y = f(x)$$ **Linear Equation Standard Form**

Linear Equations

☑ Integrating factor

$$\frac{dy}{dx} + P(x)y = f(x)$$ **Linear Equation Standard Form**

Linear Equations

☑ Integrating factor

$$\frac{dy}{dx} + P(x)y = f(x)$$ **Linear Equation Standard Form**

$$y = ce^{-\int P(x)dx} + e^{-\int P(x)dx}\int e^{\int P(x)dx} f(x)dx$$

Linear Equations

☑ Integrating factor

$$\frac{dy}{dx} + P(x)y = f(x)$$ **Linear Equation Standard Form**

$$y = ce^{-\int P(x)dx} + e^{-\int P(x)dx}\int e^{\int P(x)dx} f(x)dx$$

Multiplied by

Linear Equations

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$$e^{\int P(x)dx} y = c + \int e^{\int P(x)dx} f(x)dx$$

Linear Equations

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$$e^{\int P(x)dx} y = c + \int e^{\int P(x)dx} f(x)dx$$

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Linear Equations

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$$\frac{d}{dx}\left[e^{\int P(x)dx} y\right] = e^{\int P(x)dx} f(x)$$

Linear Equations

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is differentiated

$$\frac{d}{dx}\left[e^{\int P(x)dx} y\right] = e^{\int P(x)dx} f(x)$$

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Solution!!!

$$e^{\int P(x)dx}\boxed{y} = c + \int e^{\int P(x)dx} f(x)dx$$

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$$\frac{d}{dx}\left[e^{\int P(x)dx} y\right] = e^{\int P(x)dx} f(x)$$

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Linear Equations

☑ Example 1
Solving a Linear DE

Solve

$$\frac{dy}{dx} - 3y = 6$$

Linear Equations

☑ Example 1

Solving a Linear DE

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$$\frac{dy}{dx} - 3y = 6$$

This equation is in the standard form

Linear Equations

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Solve

$$\frac{dy}{dx} - 3y = 6$$

This equation is in the standard form

$$\frac{dy}{dx} + p(x)y = f(x)$$

Linear Equations

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Solving a Linear DE

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$$\frac{dy}{dx} + p(x)y = f(x)$$

Integrating factor is

Linear Equations

☑ Example 1

Solving a Linear DE

Solve

$$\frac{dy}{dx} - 3y = 6$$

This equation is in the standard form

$$\frac{dy}{dx} + p(x)y = f(x)$$

Integrating factor is

$$e^{\int (-3)dx} = e^{-3x}$$

Linear Equations

☑ **Example 1**

Solving a Linear DE

Solve

$$\frac{dy}{dx} - 3y = 6$$

This equation is in the standard form

$$\frac{dy}{dx} + p(x)y = f(x)$$

Integrating factor is

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Multiplying integrating factor both side

Linear Equations

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Solving a Linear DE

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Multiplying integrating factor both side

$$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y = 6e^{-3x}$$

Linear Equations

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$$\frac{d}{dx}[e^{-3x}y] = 6e^{-3x}$$

Linear Equations

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Integrating both side

Linear Equations

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$$\frac{d}{dx}[e^{-3x}y] = 6e^{-3x}$$

Integrating both side

$$e^{-3x}y = -2e^{-3x} + c$$

Linear Equations

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Solving a Linear DE

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Multiplying integrating factor both side

$$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y = 6e^{-3x}$$

$$\frac{d}{dx}[e^{-3x}y] = 6e^{-3x}$$

Integrating both side

$$e^{-3x}y = -2e^{-3x} + c$$

$$y = -2 + ce^{3x}, \quad -\infty < x < \infty$$

Linear Equations

☑ Example 2

General Solution

Solve

$$x\frac{dy}{dx}-4y=x^6e^x$$

Linear Equations

☑ Example 2

General Solution

Solve

$$x\frac{dy}{dx}-4y=x^6e^x$$

Dividing by x gives the standard form

Linear Equations

☑ Example 2

General Solution

Solve

$$x\frac{dy}{dx}-4y=x^6e^x$$

Dividing by x gives the standard form

$$\frac{dy}{dx}-\frac{4}{x}y=x^5e^x$$

Linear Equations

☑ Example 2

General Solution

Solve

$$x\frac{dy}{dx}-4y=x^6e^x$$

Dividing by x gives the standard form

$$\frac{dy}{dx}-\frac{4}{x}y=x^5e^x$$

$$P(x)=-4/x \text{ and } f(x)=x^5e^x$$

Linear Equations

☑ Example 2

General Solution

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Dividing by x gives the standard form

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in standard form are continuous on (0,∞)

Linear Equations

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Integrating factor is

$$e^{-4\int(\frac{1}{x})dx}=e^{-4\ln x}=e^{\ln x^{-4}}=x^{-4}$$

Linear Equations

☑ Example 2

General Solution

Solve

$$x\frac{dy}{dx}-4y=x^6e^x$$

Dividing by x gives the standard form

$$\frac{dy}{dx}-\frac{4}{x}y=x^5e^x$$

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Multiplying integrating factor both side

Linear Equations

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Multiplying integrating factor both side

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Linear Equations

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General Solution

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Multiplying integrating factor both side

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$$\frac{d}{dx}[x^{-4}y]=xe^x$$

Linear Equations

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Linear Equations

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Multiplying integrating factor both side

$$x^{-4}\frac{dy}{dx}-4x^{-5}y=xe^x$$

$$\frac{d}{dx}[x^{-4}y]=xe^x$$

Integrating both side

$$x^{-4}y=xe^x-e^x+c$$

Linear Equations

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General Solution

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Multiplying integrating factor both side

$$x^{-4}\frac{dy}{dx}-4x^{-5}y=xe^x$$

$$\frac{d}{dx}[x^{-4}y]=xe^x$$

Integrating both side

$$x^{-4}y=xe^x-e^x+c$$

$$y=x^5e^x-x^4e^x+cx^4,\ 0<x<\infty$$

Linear Equations

☑ Example 3
General Solution

Find a general solution of

$$(x^2-9)\frac{dy}{dx}+xy=0$$

Linear Equations

☑ Example 3
General Solution

Find a general solution of

$$(x^2-9)\frac{dy}{dx}+xy=0$$

Write DE in standard form

Linear Equations

☑ Example 3
General Solution

Find a general solution of

$$(x^2-9)\frac{dy}{dx}+xy=0$$

Write DE in standard form

$$\frac{dy}{dx}+\frac{x}{x^2-9}y=0$$

Linear Equations

☑ Example 3
General Solution

Find a general solution of

$$(x^2-9)\frac{dy}{dx}+xy=0$$

Write DE in standard form

$$\frac{dy}{dx}+\frac{x}{x^2-9}y=0$$

$$P(x)=x/(x^2-9)$$

Linear Equations

☑ Example 3
General Solution

Find a general solution of

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Write DE in standard form

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is continuous on
(-∞,-3), (-3,3), (3,∞)

Linear Equations

☑ **Example 3**

General Solution

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Linear Equations

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Linear Equations

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Multiplying integrating factor both side

Linear Equations

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Multiplying integrating factor both side

$$\sqrt{x^2-9}\frac{dy}{dx}+\frac{x}{\sqrt{x^2-9}}y=0$$

Linear Equations

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General Solution

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Multiplying integrating factor both side

$$\sqrt{x^2-9}\frac{dy}{dx}+\frac{x}{\sqrt{x^2-9}}y=0$$

$$\frac{d}{dx}[\sqrt{x^2-9}y]=0$$

Linear Equations

☑ Example 3

General Solution

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Write DE in standard form

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$$e^{\int(\frac{x}{x^2-9})dx}=e^{\frac{1}{2}\int(\frac{2x}{x^2-9})dx}=e^{\frac{1}{2}\ln|x^2-9|}=\sqrt{x^2-9}$$

Multiplying integrating factor both side

$$\sqrt{x^2-9}\frac{dy}{dx}+\frac{x}{\sqrt{x^2-9}}y=0$$

$$\frac{d}{dx}[\sqrt{x^2-9}y]=0$$

Integrating both side

Linear Equations

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Write DE in standard form

$$\frac{dy}{dx}+\frac{x}{x^2-9}y=0$$

$$P(x)=x/(x^2-9)$$

is continuous on
(-∞,-3), (-3,3), (3,∞)

On (-∞,-3), (3,∞) integrating factor is

$$e^{\int(\frac{x}{x^2-9})dx}=e^{\frac{1}{2}\int(\frac{2x}{x^2-9})dx}=e^{\frac{1}{2}\ln|x^2-9|}=\sqrt{x^2-9}$$

Multiplying integrating factor both side

$$\sqrt{x^2-9}\frac{dy}{dx}+\frac{x}{\sqrt{x^2-9}}y=0$$

$$\frac{d}{dx}[\sqrt{x^2-9}y]=0$$

Integrating both side

$$\sqrt{x^2-9}y=c$$

$$y=\frac{c}{\sqrt{x^2-9}},\quad x>3\ or\ x<-3$$

Linear Equations

☑ Example 4

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = x, \; y(0) = 4$$

Linear Equations

☑ Example 4

Initial-Value Problem

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Integrating factor is

Linear Equations

☑ Example 4

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = x, \; y(0) = 4$$

Integrating factor is

$$e^{\int (1)dx} = e^{x}$$

Linear Equations

☑ Example 4

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = x,\ y(0) = 4$$

Integrating factor is

$$e^{\int (1)dx} = e^{x}$$

Multiplying integrating factor both side and rewrite

Linear Equations

☑ Example 4

Initial-Value Problem

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x}y] = xe^{x}$$

Linear Equations

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x}y] = xe^{x}$$

$$e^{x}y = xe^{x} - e^{x} + c$$

Linear Equations

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x}y] = xe^{x}$$

$$e^{x}y = xe^{x} - e^{x} + c$$

$$y = x - 1 + ce^{-x}$$

Linear Equations

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Substitute x=0 and y=4 for satisfy initial condition

Linear Equations

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Linear Equations

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^x y] = xe^x$$

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Substitute x=0 and y=4 for satisfy initial condition $y(0) = 4$

$$4 = 0 - 1 + c,\ \ c = 5$$

Linear Equations

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$$y = x - 1 + ce^{-x}$$

Substitute x=0 and y=4 for satisfy initial condition $y(0) = 4$

$$4 = 0 - 1 + c,\ \ c = 5$$

$$y = x - 1 + 5e^{-x},\ -\infty < x < \infty$$

Linear Equations

☑ Example 5

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

$$\text{where } f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

Linear Equations

☑ Example 5

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

$$\text{where } f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

Integrating factor is

Linear Equations

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$$e^{\int (1)dx} = e^{x}$$

(a) For 0≤x≤1

Linear Equations

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Multiplying integrating factor both side and rewrite

Linear Equations

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x} y] = e^{x}$$

Linear Equations

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^x y] = e^x$$

$$e^x y = e^x + c$$

$$y = 1 + ce^{-x}$$

Linear Equations

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Substitute x=0 and y=0 for satisfy initial condition $y(0) = 0$

$$0 = 1 + c,\ c = -1$$

Linear Equations

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Multiplying integrating factor both side and rewrite

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$$y = 1 + ce^{-x}$$

Substitute x=0 and y=0 for satisfy initial condition $y(0) = 0$

$$0 = 1 + c,\ c = -1$$

$$y = 1 - e^{-x}$$

Continued…

Linear Equations

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$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

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Linear Equations

☑ Example 5

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Linear Equations

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Linear Equations

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(b) For x>1

Linear Equations

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$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

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Integrating factor is

$$e^{\int (1)dx} = e^{x}$$

(b) For x>1 $f(x) = 0$

Linear Equations

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$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

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$$e^{\int (1)dx} = e^{x}$$

(b) For x>1 $f(x) = 0$

Multiplying integrating factor both side and rewrite

Linear Equations

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(b) For x>1 $f(x) = 0$

Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x} y] = 0$$

Linear Equations

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(b) For x>1 $f(x) = 0$

Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x} y] = 0$$

$$e^{x} y = c$$

$$y = ce^{-x}$$

Linear Equations

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$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

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Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x} y] = 0$$

$$e^{x} y = c$$

$$y = ce^{-x}$$

If y(x) is continuous at x=1,

Linear Equations

☑ Example 5

Initial-Value Problem

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$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

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Integrating factor is

$$e^{\int (1)dx} = e^{x}$$

(b) For x>1 $f(x) = 0$

Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^{x} y] = 0$$

$$e^{x} y = c$$

$$y = ce^{-x}$$

If y(x) is continuous at x=1,

$$\lim_{x \to 1^{+}} y(x) = y(1)$$

Linear Equations

☑ Example 5

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

$$\text{where } f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

Integrating factor is

$$e^{\int (1)dx} = e^x$$

(b) For x>1 $f(x) = 0$

Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^x y] = 0$$

$$e^x y = c$$

$$y = ce^{-x}$$

If y(x) is continuous at x=1,

$$\lim_{x \to 1^+} y(x) = y(1)$$

$$ce^{-1} = 1 - e^{-1},\ c = e - 1$$

Linear Equations

☑ Example 5

Initial-Value Problem

Solve the IVP

$$\frac{dy}{dx} + y = f(x),\ y(0) = 0$$

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Integrating factor is

$$e^{\int (1)dx} = e^x$$

(b) For x>1 $f(x) = 0$

Multiplying integrating factor both side and rewrite

$$\frac{d}{dx}[e^x y] = 0$$

$$e^x y = c$$

$$y = ce^{-x}$$

If y(x) is continuous at x=1,

$$\lim_{x \to 1^+} y(x) = y(1)$$

$$ce^{-1} = 1 - e^{-1},\ c = e - 1$$

$$y = \begin{cases} 1 - e^{-x}, & 0 \le x \le 1 \\ (e-1)e^{-x}, & x > 1 \end{cases}$$

Linear Equations

☑ Example 6

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x, \; y(0) = 1$$

Linear Equations

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The Error Function

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$$\frac{dy}{dx} - 2xy = x, \; y(0) = 1$$

The integrating factor is

Linear Equations

☑ Example 6

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x,\ y(0) = 1$$

The integrating factor is

$$e^{\int (-2x)dx} = e^{-x^2}$$

Linear Equations

☑ Example 6

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x,\ y(0) = 1$$

The integrating factor is

$$e^{\int (-2x)dx} = e^{-x^2}$$

Rearrange the equation

Linear Equations

☑ Example 6

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x,\ y(0) = 1$$

The integrating factor is

$$e^{\int (-2x)dx} = e^{-x^2}$$

Rearrange the equation

$$\frac{d}{dx}[e^{-x^2} y] = 2e^{-x^2}$$

Linear Equations

☑ Example 6

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x,\ y(0) = 1$$

The integrating factor is

$$e^{\int (-2x)dx} = e^{-x^2}$$

Rearrange the equation

$$\frac{d}{dx}[e^{-x^2} y] = 2e^{-x^2}$$

$$y = 2e^{x^2} \int_0^x e^{-t^2} dt + ce^{x^2}$$

Linear Equations

☑ **Example 6**

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x,\ y(0) = 1$$

The integrating factor is

$$e^{\int (-2x)dx} = e^{-x^2}$$

Rearrange the equation

$$\frac{d}{dx}[e^{-x^2} y] = 2e^{-x^2}$$

$$y = 2e^{x^2} \int_0^x e^{-t^2} dt + ce^{x^2}$$

$$y(0) = 1$$

Linear Equations

☑ **Example 6**

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The integrating factor is

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$y(0) = 1$ **gives c=1 , So**

Linear Equations

☑ Example 6

The Error Function

Solve the IVP

$$\frac{dy}{dx} - 2xy = x,\ y(0) = 1$$

The integrating factor is

$$e^{\int (-2x)dx} = e^{-x^2}$$

Rearrange the equation

$$\frac{d}{dx}[e^{-x^2} y] = 2e^{-x^2}$$

$$y = 2e^{x^2} \int_0^x e^{-t^2} dt + ce^{x^2}$$

$y(0) = 1$ **gives c=1 , So**

$$y = 2e^{x^2} \int_0^x e^{-t^2} dt + e^{x^2}$$

$$= e^{x^2}[1 + \sqrt{\pi} erf(x)]$$

$$(erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt)$$

Exact Equations

Exact Equations

$$z = f(x, y)$$

Exact Equations

$$z = f(x, y)$$

Total differential

Exact Equations

$$z = f(x, y)$$

Total differential

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

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If,

Exact Equations

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$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

If, $f(x, y) = const.$

Exact Equations

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Exact differential to the $f(x, y)$

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$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

If, $f(x, y) = const.$

Exact equation

$$\boxed{\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy} = 0$$

Exact differential to the $f(x, y)$

$$z = f(x, y) = xy$$

Exact Equations

$$z = f(x, y)$$

Total differential

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

If, $f(x, y) = const.$

Exact equation

$$\boxed{\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy} = 0$$

Exact differential to the $f(x, y)$

$$z = f(x, y) = xy$$

$$dz = ydx + xdy$$

Exact Equations

$$z = f(x, y)$$

Total differential

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Exact differential to the $f(x, y)$

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If, $xy = c$

Exact Equations

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If, $xy = c$ 기하학적 의미

$$ydx + xdy = 0$$

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If, $xy = c$ 기하학적 의미

$$ydx + xdy = 0$$

$$z = f(x, y) \quad \text{i.e. } f(x, y) = xy$$

Exact Equations

$$z = f(x, y)$$

Total differential

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

If, $f(x, y) = const.$

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If, $xy = c$ 기하학적 의미

$$ydx + xdy = 0$$

$$z = f(x, y) \quad \text{i.e. } f(x, y) = xy$$

$$xy = c \ , (c = \text{constant})$$

Exact Equations

$$z = f(x, y)$$

Total differential

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

If, $f(x, y) = const.$

Exact equation

$$\boxed{\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = 0}$$

Exact differential to the $f(x, y)$

$$z = f(x, y) = xy$$

$$dz = ydx + xdy$$

If, $xy = c$ 기하학적 의미

$$ydx + xdy = 0$$

$$z = f(x, y) \quad \text{i.e. } f(x, y) = xy$$

$$xy = c \ , (c = \text{constant})$$

$$\boxed{y = \frac{c}{x}} \qquad y \dashrightarrow y(x)$$

y is changing to dependent variable on x.

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y is changing to dependent variable on x.

Generally the solution is a curve.

$$f(x, y(x)) = xy = c$$

Exact Equations

Exact Equations

Definition 2.3

Exact Equation

$M(x,y)dx + N(x,y)dy$ **is an exact differential if it corresponds to the differential of some function** $f(x,y)$.

The first-order differential equation of the form

$$M(x,y)dx + N(x,y)dy = 0$$

is said to be exact equation if the expression on the left side is an exact differential .

Exact Equations

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Theorem 2.1 **Criterion for an Exact Differential**

Let $M(x,y)$ and $N(x,y)$ be continuous and have continuous first partial derivatives in a rectangular region R defined by $a < x < b, c < y < d$.

Then a necessary and sufficient condition that $M(x,y)dx + N(x,y)dy$ be an exact differential is. $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

Exact Equations

☑ **Proof of necessity**

Exact Equations

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$$\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$$

$$=M(x,y)dx+N(x,y)dy$$

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$$\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$$

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Exact Equations

☑ **Method of solution**

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Recognizing exact equation

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Exact Equations

☑ Example 1

Solving an Exact DE

Solve $2xydx+(x^2-1)dy=0$

Exact Equations

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Solving an Exact DE

Solve $2xydx + (x^2 - 1)dy = 0$

$$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$$

Exact Equations

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Solving an Exact DE

Solve $2xydx + (x^2 - 1)dy = 0$

$$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$$

Thus the equation is exact.
So there exists function such that

Exact Equations

☑ Example 1

Solving an Exact DE

Solve $2xydx+(x^2-1)dy=0$

$$\frac{\partial M}{\partial y}=2x=\frac{\partial N}{\partial x}$$

Thus the equation is exact.
So there exists function such that

$$\frac{\partial f}{\partial x}=2xy \text{ and } \frac{\partial f}{\partial y}=x^2-1 \quad \cdots(1)$$

Exact Equations

☑ Example 1

Solving an Exact DE

Solve $2xydx+(x^2-1)dy=0$

$$\frac{\partial M}{\partial y}=2x=\frac{\partial N}{\partial x}$$

Thus the equation is exact.
So there exists function such that

$$\frac{\partial f}{\partial x}=2xy \text{ and } \frac{\partial f}{\partial y}=x^2-1 \quad \cdots(1)$$

By integration first term,

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$$y=\frac{c}{(1-x^2)}$$

Exact Equations

☑ Example 2
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$$(e^{2y} - y\cos xy)dx$$
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$$(e^{2y} - y\cos xy)dx + (2xe^{2y} - x\cos xy + 2y)dy = 0$$

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Exact Equations

☑ **Example 3**

Initial-Value Problem

Solve IVP

$$\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}, \; y(0) = 2$$

Exact Equations

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$$\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}, \quad y(0) = 2$$

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$$(\cos x \sin x - xy^2)dx + y(1-x^2)dy = 0$$

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$$f(0,2) = \frac{1}{2}\cdot 4 \cdot 1 - \frac{1}{2}\cdot 1 + c = 0, \quad c = -\frac{3}{2}$$

Exact Equations

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$$\frac{\partial M}{\partial y}=-2xy=\frac{\partial N}{\partial x}$$

The equation is exact.

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$$y^2(1-x^2)-\cos^2 x=3$$

Exact Equations

☑ **Integrating factors**

Exact Equations

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$$M(x,y)dx + N(x,y)dy = 0$$

Exact Equations

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$$M(x,y)dx + N(x,y)dy = 0$$

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Exact Equations

☑ Integrating factors

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If we take $\mu = \mu(y)$

In a similar way

Exact Equations

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Exact Equations

☑ **Example 4**

Nonexact DE Made Exact

Solve

$$xydx + (2x^2 + 3y^2 - 20)dy = 0$$

Exact Equations

☑ **Example 4**

Nonexact DE Made Exact

Solve

$$xydx + (2x^2 + 3y^2 - 20)dy = 0$$

$$\frac{\partial M}{\partial y} = x \neq \frac{\partial N}{\partial x} = 4x$$

Exact Equations

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Nonexact DE Made Exact

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So we need integrating factor μ to make the equation exact.

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Depend on x and y

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$$\frac{N_x - M_y}{M} = \frac{4x - x}{xy} = \frac{3}{y}$$

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The integrating factor is then

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Depend on x and y

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Depend only on y

The integrating factor is then

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Exact Equations

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Nonexact DE Made Exact

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$$f(x,y)=\frac{1}{2}x^2y^4+g(y)$$

Exact Equations

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After Multiplying integrating factor

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$$g(y) = \frac{1}{2}y^6 - 5y^4 + c$$

Exact Equations

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$$g(y) = \frac{1}{2}y^6 - 5y^4 + c$$

$$\therefore f(x,y) = \frac{1}{2}x^2y^4 + \frac{1}{2}y^6 - 5y^4 + c$$

$$= 0$$

Solutions by Substitutions

☑ **Substitution**

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y)$$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow$$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

$$\downarrow$$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

$\downarrow$

New D.E.

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx}$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx}$ $\longrightarrow$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx}$ $\xrightarrow{\text{find}}$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u)$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x, y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u) \longrightarrow$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x,y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u) \xrightarrow{\text{Find u}}$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x,y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u) \xrightarrow{\text{Find u}} u = \phi(x)$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x,y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u) \xrightarrow{\text{Find u}} u = \phi(x)$

Then, final solution is

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x,y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u) \xrightarrow{\text{Find u}} u = \phi(x)$

Then, final solution is

$$y = g(x,u) = g(x,\phi(x))$$

Solutions by Substitutions

☑ Substitution

$$\frac{dy}{dx} = f(x,y) \Rightarrow y = g(x,u)$$

$$\frac{dy}{dx} = \frac{\partial}{\partial x} g(x,u) + \frac{\partial}{\partial u} g(x,u) \frac{du}{dx}$$

New D.E.

Solving for $\frac{du}{dx} \xrightarrow{\text{find}} \frac{du}{dx} = F(x,u) \xrightarrow{\text{Find u}} u = \phi(x)$

Then, final solution is

$$y = g(x,u) = g(x,\phi(x))$$

이것을 만족하는 $\phi(x)$ 를 구하는 것

Solutions by Substitutions

☑ **Homogeneous**

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

If

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

If $f(tx, ty) = t^{\alpha} f(x, y)$

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

If $f(tx, ty) = t^{\alpha} f(x, y)$

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

If $f(tx,ty) = t^{\alpha} f(x,y)$

$$f(x,y) = x^3 + y^3$$

$$f(tx,ty) = (tx)^3 + (ty)^3 = t^3(x^3 + y^3)$$

$$\therefore\ f(tx,ty) = t^3 f(x,y)$$

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

If $f(tx,ty) = t^{\alpha} f(x,y)$

$$f(x,y) = x^3 + y^3$$

$$f(tx,ty) = (tx)^3 + (ty)^3 = t^3(x^3 + y^3)$$

$$\therefore\ f(tx,ty) = t^3 f(x,y)$$

Vs.

Solutions by Substitutions

☑ Homogeneous

f is homogeneous function of degree α

If $f(tx,ty) = t^{\alpha} f(x,y)$

$$f(x,y) = x^3 + y^3$$

$$f(tx,ty) = (tx)^3 + (ty)^3 = t^3(x^3 + y^3)$$

$$\therefore\ f(tx,ty) = t^3 f(x,y)$$

Vs. $f(x,y) = x^3 + y^3 + 1$

Solutions by Substitutions

Solutions by Substitutions

☑ **Homogeneous**

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y) \ , N(tx,ty) = t^{\alpha} N(x,y)$$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y) \ , N(tx,ty) = t^{\alpha} N(x,y)$$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y) \ , N(tx,ty) = t^{\alpha} N(x,y)$$

○ **Same degree**

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ **Same degree**

☑ Substitution

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y) \ , N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y) \ , N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots (1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y) \ , N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

or

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

or $M(1,u)dx + N(1,u)dy = 0 \cdots(2)$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

or $M(1,u)dx + N(1,u)dy = 0 \cdots(2)$

from

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

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from $y = ux$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

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from $y = ux \longrightarrow$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

or $M(1,u)dx + N(1,u)dy = 0 \cdots(2)$

from $y = ux \longrightarrow dy = udx + xdu$

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

or $M(1,u)dx + N(1,u)dy = 0 \cdots(2)$

from $y = ux \longrightarrow dy = udx + xdu$

So (2) is

Solutions by Substitutions

☑ Homogeneous

$$M(x,y)dx + N(x,y)dy = 0 \quad \cdots(1)$$

Equation is homogeneous if

$$M(tx,ty) = t^{\alpha} M(x,y)\ ,\ N(tx,ty) = t^{\alpha} N(x,y)$$

○ Same degree

☑ Substitution

Then, We can rewrite (1) by substituting $y = ux$

$$x^{\alpha} M(1,u)dx + x^{\alpha} N(1,u)dy = 0$$

or $M(1,u)dx + N(1,u)dy = 0 \cdots(2)$

from $y = ux \longrightarrow dy = udx + xdu$

So (2) is

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

Solutions by Substitutions

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

If we take

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

If we take $x = vy$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

If we take $x = vy$

In a similar way

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

If we take $x = vy$

In a similar way

$$\frac{M(v,1)}{(vM(v,1) + N(v,1))}dv = -\frac{dy}{y}$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

If we take $x = vy$

In a similar way

$$\frac{M(v,1)}{(vM(v,1) + N(v,1))}dv = -\frac{dy}{y}$$

$$v = \psi(x)$$

Solutions by Substitutions

$$M(1,u)dx + N(1,u)[udx + xdu] = 0$$

$$\downarrow$$

$$(M(1,u) + uN(1,u))dx = -xN(1,u)du$$

$$\downarrow$$

Separable variables

$$\frac{N(1,u)}{(M(1,u) + uN(1,u))}du = -\frac{dx}{x}$$

Solving it $u = \phi(x)$

Then, final solution is

$$y = ux = \phi(x)x$$

If we take $x = vy$

In a similar way

$$\frac{M(v,1)}{(vM(v,1) + N(v,1))}dv = -\frac{dy}{y}$$

$$v = \psi(x)$$

$$y = \frac{v}{x} = \frac{\psi(x)}{x}$$

Solutions by Substitutions

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

☑ **Bernoulli's Equation**

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n: real\ \ number$$

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n: real\ \ number$$

Solution : try

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n: real\ \ number$$

Solution : try $u = y^{1-n}$

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

☑ **Bernoulli's Equation**

$$if \;\; n = 0, n = 1$$

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n : real \;\; number$$

Solution : try $u = y^{1-n}$

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

$$if \;\; n = 0, n = 1$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n : real \;\; number$$

Solution : try $u = y^{1-n}$

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

$$if \;\; n = 0, n = 1$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n : real \;\; number$$

Solution : try $u = y^{1-n}$

☑ **Reduction to Separation of Variables**

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

$$if \;\; n = 0, n = 1$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n : real \;\; number$$

Solution : try $u = y^{1-n}$

☑ **Reduction to Separation of Variables**

$$\frac{dy}{dx} = f(Ax + By + C), \; where \;\; B \neq 0$$

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

$$if \;\; n = 0, n = 1$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n : real \;\; number$$

Solution : try $u = y^{1-n}$

☑ **Reduction to Separation of Variables**

$$\frac{dy}{dx} = f(Ax + By + C), \; where \;\; B \neq 0$$

Solution : try

Solutions by Substitutions

☑ **First-order linear D.E, standard form**

$$\frac{dy}{dx} + P(x)y = f(x)$$

$$if \;\; n = 0, n = 1$$

☑ **Bernoulli's Equation**

$$\frac{dy}{dx} + P(x)y = f(x)y^n, n: real \;\; number$$

Solution : try $u = y^{1-n}$

☑ **Reduction to Separation of Variables**

$$\frac{dy}{dx} = f(Ax + By + C), \; where \;\; B \neq 0$$

Solution : try $u = Ax + By + C$

Solutions by Substitutions

☑ Example 1

Solving a Homogeneous DE

Solve

$$(x^2 + y^2)dx + (x^2 - xy)dy = 0$$

Solutions by Substitutions

☑ Example 1

Solving a Homogeneous DE

Solve

$$(x^2+y^2)dx+(x^2-xy)dy=0$$

$$M(tx,ty)=t^2(x^2+y^2)$$

$$:2\text{-degree homogenous}$$

$$N(tx,ty)=t^2(x^2-xy)$$

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Solutions by Substitutions

☑ Example 1

Solving a Homogeneous DE

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Solutions by Substitutions

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$$y=\frac{1}{(-x^2+cx)}$$

Solutions by Substitutions

☑ **Example 3**

An Initial-Value Problem

Solve IVP

$$\frac{dy}{dx} = (-2x + y)^2 - 7,\ y(0) = 0$$

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$$\frac{dy}{dx} = (-2x + y)^2 - 7,\ y(0) = 0$$

Let $u = -2x + y$

then, $du/dx = -2 + dy/dx$

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$$\frac{1}{u^2 - 9} du = dx$$

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$$\frac{u-3}{u+3} = c \cdot e^{6x}$$

$$u - 3 = ce^{6x}u + 3ce^{6x}$$

$$u = 3\frac{1+ce^{6x}}{1-ce^{6x}},\quad y = 2 + 3\frac{1+ce^{6x}}{1-ce^{6x}}$$

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$$0 = 0 + 3\frac{1+c}{1-c}$$

$$c = -1$$

$$\therefore y = 2 + 3\frac{1-e^{6x}}{1+e^{6x}}$$

Reduction of Order

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☑ **For what?**

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- **When have only a single solution**

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$$y'' + P(x)y' + Q(x)y = 0$$ **Standard form**

$y_1(x)$ **: known**

$$y_2(x) = u(x)y_1(x),$$

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L.H.S:

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$y_1(x)$: known

$$\therefore\ y_1'' + P(x)y_1' + Q(x)y_1 = 0$$

Put them into the standard form

$$y'' + P(x)y' + Q(x)y = 0$$

L.H.S:

$$[u''(x)y_1(x) + 2u'(x)y_1'(x) + u(x)y_1''(x)] + P(x)[u'(x)y_1(x) + u(x)y_1'(x)] + Q(x)[u(x)y_1(x)]$$

$$= \boxed{u(x)[y_1''(x) + P(x)y_1'(x) + Q(x)y_1(x)]} + u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

$$= u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

Reduction of Order

$$y(x) = u(x)y_1(x)$$

$$y' = u'(x)y_1(x) + u(x)y_1'(x)$$

$$y'' = u''(x)y_1(x) + 2u'(x)y_1(x) + u(x)y_1''(x)$$

$y_1(x)$: known

$$\therefore\ y_1'' + P(x)y_1' + Q(x)y_1 = 0$$

Put them into the standard form

$$y'' + P(x)y' + Q(x)y = 0$$

L.H.S:

$$[u''(x)y_1(x) + 2u'(x)y_1'(x) + u(x)y_1''(x)] + P(x)[u'(x)y_1(x) + u(x)y_1'(x)] + Q(x)[u(x)y_1(x)]$$

$$= \boxed{u(x)[y_1''(x) + P(x)y_1'(x) + Q(x)y_1(x)]} + u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

$$= u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

R.H.S:

Reduction of Order

$$y(x) = u(x)y_1(x)$$

$$y' = u'(x)y_1(x) + u(x)y_1'(x)$$

$$y'' = u''(x)y_1(x) + 2u'(x)y_1(x) + u(x)y_1''(x)$$

$y_1(x)$: known

$$\therefore\ y_1'' + P(x)y_1' + Q(x)y_1 = 0$$

Put them into the standard form

$$y'' + P(x)y' + Q(x)y = 0$$

L.H.S:

$$[u''(x)y_1(x) + 2u'(x)y_1'(x) + u(x)y_1''(x)] + P(x)[u'(x)y_1(x) + u(x)y_1'(x)] + Q(x)[u(x)y_1(x)]$$

$$= \boxed{u(x)[y_1''(x) + P(x)y_1'(x) + Q(x)y_1(x)]} + u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

$$= u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

R.H.S: 0

Reduction of Order

$$y(x) = u(x)y_1(x)$$

$$y' = u'(x)y_1(x) + u(x)y_1'(x)$$

$$y'' = u''(x)y_1(x) + 2u'(x)y_1(x) + u(x)y_1''(x)$$

$y_1(x)$: known

$$\therefore y_1'' + P(x)y_1' + Q(x)y_1 = 0$$

Put them into the standard form

$$y'' + P(x)y' + Q(x)y = 0$$

L.H.S:

$$[u''(x)y_1(x) + 2u'(x)y_1'(x) + u(x)y_1''(x)] + P(x)[u'(x)y_1(x) + u(x)y_1'(x)] + Q(x)[u(x)y_1(x)]$$

$$= \boxed{u(x)[y_1''(x) + P(x)y_1'(x) + Q(x)y_1(x)]} + u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

$$= u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

R.H.S: 0

$$\therefore u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x) = 0$$

Reduction of Order

$$y(x) = u(x)y_1(x)$$

$$y' = u'(x)y_1(x) + u(x)y_1'(x)$$

$$y'' = u''(x)y_1(x) + 2u'(x)y_1(x) + u(x)y_1''(x)$$

$y_1(x)$: known

$$\therefore y_1'' + P(x)y_1' + Q(x)y_1 = 0$$

Put them into the standard form

$$y'' + P(x)y' + Q(x)y = 0$$

L.H.S:

$$[u''(x)y_1(x) + 2u'(x)y_1'(x) + u(x)y_1''(x)] + P(x)[u'(x)y_1(x) + u(x)y_1'(x)] + Q(x)[u(x)y_1(x)]$$

$$= \boxed{u(x)[y_1''(x) + P(x)y_1'(x) + Q(x)y_1(x)]} + u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

$$= u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x)$$

R.H.S: 0

$$\therefore u'(x)[2y_1'(x) + P(x)y_1(x)] + u''(x)y_1(x) = 0$$

From this find $u(x)$ **!**

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then**

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ then $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

divided by $w(x)y_1(x)$ ..... $[2\dfrac{y_1'(x)}{y_1(x)}+P(x)]+\dfrac{w'(x)}{w(x)}=0$

$$2\frac{1}{y_1(x)}\frac{dy_1(x)}{dx}+P(x)+\frac{1}{w(x)}\frac{dw(x)}{dx}=0$$

multiplied by dx ..... $2\dfrac{dy_1}{y_1(x)}+P(x)dx+\dfrac{dw}{w(x)}=0$

$$2\ln|y_1(x)|+\int P(x)dx+\ln|w(x)|=c$$

$$\ln|y^2{}_1(x)w(x)|=-\int P(x)dx+c$$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c}$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

$$w=u'(x)$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then**

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then** $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then** $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c}$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then** $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then** $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

$$w(x)=\frac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then** $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

$$w(x)=\frac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$$

since

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ then $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

$$w(x)=\frac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$$

since $w(x)=u'(x)$ ..... $u'(x)=\dfrac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$

$$u(x)=c_1\int\frac{e^{-\int P(x)dx}}{y^2{}_1(x)}dx+c_2$$

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ **then** $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

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since $w(x)=u'(x)$ ..... $u'(x)=\dfrac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$

$$u(x)=c_1\int\frac{e^{-\int P(x)dx}}{y^2{}_1(x)}dx+c_2$$

by choosing

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ then $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

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since $w(x)=u'(x)$ ..... $u'(x)=\dfrac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$

$$u(x)=c_1\int\frac{e^{-\int P(x)dx}}{y^2{}_1(x)}dx+c_2$$

by choosing $c_1=1, c_2=0$ .....

Reduction of Order

$$u'(x)[2y_1'(x)+P(x)y_1(x)]+u''(x)y_1(x)=0$$

Let $w=u'(x)$ then $w(x)[2y_1'(x)+P(x)y_1(x)]+w'(x)y_1(x)=0$

$$y^2{}_1(x)w(x)=e^{-\int P(x)dx+c} \qquad c_1=e^c\neq 0$$

$$w(x)=\frac{c_1}{y^2{}_1(x)}e^{-\int P(x)dx}$$

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$$u(x)=c_1\int\frac{e^{-\int P(x)dx}}{y^2{}_1(x)}dx+c_2$$

by choosing $c_1=1, c_2=0$ ..... $y_2=u(x)y_1(x)=y_1(x)\int\dfrac{e^{-\int P(x)dx}}{y^2{}_1(x)}dx$

Reduction of Order

☑ **General solution**

Reduction of Order

☑ **General solution**

$$y'' + P(x)y' + Q(x)y = 0$$

$$y(x) = ay_1(x) + by_2(x) = ay_1(x) + bu(x)y_1(x)$$

Reduction of Order

☑ **General solution**

$$y'' + P(x)y' + Q(x)y = 0$$

$$y(x) = \underline{ay_1(x) + by_2(x)} = ay_1(x) + bu(x)y_1(x)$$

Reduction of Order

☑ **General solution**

$$y'' + P(x)y' + Q(x)y = 0$$

$$y(x) = \underline{ay_1(x) + by_2(x)} = ay_1(x) + bu(x)y_1(x)$$

$$u(x) = c_1 \int \frac{e^{-\int P(x)dx}}{y^2{}_1(x)} dx + c_2$$

Reduction of Order

☑ General solution

$$y'' + P(x)y' + Q(x)y = 0$$

$$y(x) = \underline{ay_1(x) + by_2(x)} = ay_1(x) + bu(x)y_1(x)$$

$$u(x) = c_1 \int \frac{e^{-\int P(x)dx}}{y^2{}_1(x)} dx + c_2$$

$$y(x) = ay_1(x) + b\left(c_1 \int \frac{e^{-\int P(x)dx}}{y^2{}_1(x)} dx + c_2 \right) y_1(x)$$

$$= (a + bc_2)y_1(x) + bc_1 \left(\int \frac{e^{-\int P(x)dx}}{y^2{}_1(x)} dx \right) y_1(x)$$

$$= a'y_1(x) + b'y_2(x)$$

Reduction of Order

☑ General solution

$$y'' + P(x)y' + Q(x)y = 0$$

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$$= \underline{a'y_1(x) + b'y_2(x)}$$

Reduction of Order

Recall reduction of order

Reduction of Order

Recall reduction of order

$$\boxed{m\lambda^2 + c\lambda + k = 0}$$

Reduction of Order

Recall reduction of order

$$\boxed{m\lambda^2 + c\lambda + k = 0} \quad \text{in case, } c^2 - 4mk = 0$$

Reduction of Order

Recall reduction of order

$$\boxed{m\lambda^2 + c\lambda + k = 0} \quad \text{in case, } c^2 - 4mk = 0$$

$$\lambda_1 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} = -\frac{c}{2m}$$

Reduction of Order

Recall reduction of order

$$\boxed{m\lambda^2 + c\lambda + k = 0} \quad \text{in case, } c^2 - 4mk = 0$$

$$\lambda_1 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} = -\frac{c}{2m} \quad \Rightarrow \quad z_1 = e^{\lambda_1 t}$$

Reduction of Order

Recall reduction of order

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$$z_2 = u(t)z_1(t) = z_1(t)\int \frac{e^{-\int P(t)dt}}{z_1^2(t)}dt$$

Reduction of Order

Recall reduction of order

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$$\boxed{\begin{aligned} &\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0 \\ &\lambda^2 + P(t)\lambda + G(t) = 0 \end{aligned}}$$

Reduction of Order

Recall reduction of order

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$$\boxed{\begin{aligned} &\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0 \\ &\lambda^2 + P(t)\lambda + G(t) = 0 \end{aligned}} \quad \Rightarrow \quad P(t) = \frac{c}{m} = -2\lambda_1$$

Reduction of Order

Recall reduction of order

$$P(t) = \frac{c}{m} = -2\lambda_1 \qquad z_1 = e^{\lambda_1 t}$$

Reduction of Order

Recall reduction of order

$$P(t) = \frac{c}{m} = -2\lambda_1 \qquad z_1 = e^{\lambda_1 t}$$

$$\Rightarrow \quad z_2 = z_1(t)\int \frac{e^{-\int P(t)dt}}{z_1^2(t)}dt$$

$$= e^{\lambda_1 t}\int \frac{e^{-\int(-2\lambda_1)dt}}{e^{2\lambda_1 t}}dt = e^{\lambda_1 t}\int \frac{e^{2\lambda_1 t}}{e^{2\lambda_1 t}}dt = e^{\lambda_1 t}\int dt$$

$$= te^{\lambda_1 t}$$

Undetermined Coefficients

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c
  - **General Solution of associated homogeneous differential equation**

Undetermined Coefficients

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$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c
  - **General Solution of associated homogeneous differential equation**

$$a_n(x) y^n + a_{n-1}(x) y^{n-1} + \cdots + a_1(x) y' + a_0(x) y = 0$$

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

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$$a_n(x) y^n + a_{n-1}(x) y^{n-1} + \cdots + a_1(x) y' + a_0(x) y = 0$$

- **Find any particular solution** y_p

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c
  - **General Solution of associated homogeneous differential equation**

$$a_n(x) y^n + a_{n-1}(x) y^{n-1} + \cdots + a_1(x) y' + a_0(x) y = 0$$

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Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c
  - **General Solution of associated homogeneous differential equation**

$$a_n(x) y^n + a_{n-1}(x) y^{n-1} + \cdots + a_1(x) y' + a_0(x) y = 0$$

- **Find any particular solution** y_p
  - **Method of undetermined coefficients**

    y_p **motivated by the kinds of functions that make up the input function** $g(x)$

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c
  - **General Solution of associated homogeneous differential equation**

$$a_n(x) y^n + a_{n-1}(x) y^{n-1} + \cdots + a_1(x) y' + a_0(x) y = 0$$

- **Find any particular solution** y_p
  - **Method of undetermined coefficients**

    y_p **motivated by the kinds of functions that make up the input function** $g(x)$

    – **applicable when** $g(x)$ **is a constant, a polynomial function, exponential function, sine or cosine function or finite sums and products of these functions.**

Undetermined Coefficients

☑ **Two things to solve a nonhomogeneous differential equation**

$$a_n y^n + a_{n-1} y^{n-1} + \cdots + a_1 y' + a_0 y = g(x),\ a_k = const, k = 1,2,..,n,\ \ g(x) \neq 0$$

- **Find the complementary function** y_c
  - **General Solution of associated homogeneous differential equation**

$$a_n(x) y^n + a_{n-1}(x) y^{n-1} + \cdots + a_1(x) y' + a_0(x) y = 0$$

- **Find any particular solution** y_p
  - **Method of undetermined coefficients**

    y_p **motivated by the kinds of functions that make up the input function** $g(x)$

    – **applicable when** $g(x)$ **is a constant, a polynomial function, exponential function, sine or cosine function or finite sums and products of these functions.**

$what\ \ about\ g(x) = \tan x?$

Undetermined Coefficients

☑ **Method of undetermined coefficients**

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **By Superposition**

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **By Superposition**
  - **Step 1 : find the solution of the associated homogeneous equation**

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **By Superposition**
  - **Step 1 : find the solution of the associated homogeneous equation**
  - **Step 2 : superposition of nonhomogeneous equations**

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **By Superposition**
  - **Step 1 : find the solution of the associated homogeneous equation**
  - **Step 2 : superposition of nonhomogeneous equations**
  - **Step 3 : general solution**

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 1

solve homogeneous equation

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 1

solve homogeneous equation

$$y'' + 4y' - 2y = 0$$

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 1

solve homogeneous equation

$$y'' + 4y' - 2y = 0$$

Substitute

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 1

solve homogeneous equation

$$y'' + 4y' - 2y = 0$$

Substitute $y = e^{mx}$

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 1

solve homogeneous equation

$$y'' + 4y' - 2y = 0$$

Substitute $y = e^{mx}$

$$m^2 + 4m - 2 = 0$$

$$m_1 = -2 - \sqrt{6},\ \ m_2 = -2 + \sqrt{6}$$

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$$y_c = c_1 e^{-(2+\sqrt{6})x} + c_2 e^{(-2+\sqrt{6})x}$$

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Assume Particular solution

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Assume Particular solution

$$y_P = Ax^2 + Bx + C$$

Undetermined Coefficients

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 # Step 2

Assume Particular solution

$$y_P = Ax^2 + Bx + C$$

Then, $y_P' = 2Ax + B$

Undetermined Coefficients

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Assume Particular solution

$$y_P = Ax^2 + Bx + C$$

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 # Step 2

Assume Particular solution

$$y_P = Ax^2 + Bx + C$$

Then, $y_P' = 2Ax + B \qquad y_P'' = 2A$

$$y_P'' + 4y_p' - 2y_p$$

$$= 2A + 8Ax + 4B - 2Ax^2 - 2Bx - 2C$$

$$= -2Ax^2 + (8A - 2B)x + 2A + 4B - 2C$$

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

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$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

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☑ **Example 1**

General Solution Using Undetermined Coefficients

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 # Step 2

Assume Particular solution

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

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Assume Particular solution

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Assume Particular solution

$$y_P = Ax^2 + Bx + C$$

$$\boxed{-2A}x^2 + \boxed{(8A - 2B)}x + 2A + 4B - 2C$$

$$= \boxed{2}x^2 \boxed{-3}x + 6$$

Undetermined Coefficients

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Assume Particular solution

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Assume Particular solution

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$$= \boxed{2}x^2 \boxed{-3}x + \boxed{6}$$

$$2A = 2, \quad 8A - 2B = -3,$$

$$2A + 4B - 2C = 6$$

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

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$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 2

Assume Particular solution

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$$\boxed{-2A}x^2 + \boxed{(8A - 2B)}x + \boxed{2A + 4B - 2C}$$

$$= \boxed{2}x^2 \boxed{-3}x + \boxed{6}$$

$$2A = 2, \quad 8A - 2B = -3,$$

$$2A + 4B - 2C = 6$$

$$A = 1, \quad B = \frac{11}{2}, \quad C = 9$$

Undetermined Coefficients

☑ **Example 1**

General Solution Using Undetermined Coefficients

Solve

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

 # Step 2

Assume Particular solution

$$y_P = Ax^2 + Bx + C$$

$$\boxed{-2A}x^2 + \boxed{(8A - 2B)}x + \boxed{2A + 4B - 2C}$$

$$= \boxed{2}x^2 \boxed{-3}x + \boxed{6}$$

$$2A = 2, \quad 8A - 2B = -3,$$

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$$A = 1, \quad B = \frac{11}{2}, \quad C = 9$$

$$y_P = x^2 + \frac{11}{2}x + 9$$

Undetermined Coefficients

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Assume Particular solution

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$$= 2x^2 - 3x + 6$$

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$$y_P = x^2 + \frac{11}{2}x + 9$$

 # Step 3

General solution

Undetermined Coefficients

☑ **Example 1**
General Solution Using Undetermined Coefficients

Solve

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 # Step 2
Assume Particular solution

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$$-2Ax^2 + (8A - 2B)x + 2A + 4B - 2C$$
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$$2A = 2, \quad 8A - 2B = -3,$$
$$2A + 4B - 2C = 6$$

$$A = 1, \quad B = \frac{11}{2}, \quad C = 9$$

$$y_P = x^2 + \frac{11}{2}x + 9$$

 # Step 3
General solution

$$y = y_c + y_p$$

$$= c_1 e^{-(2+\sqrt{6})x} + c_2 e^{(-2+\sqrt{6})x} + x^2 + \frac{11}{2}x + 9$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Assume Particular solution

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Assume Particular solution

$$y_P = A\cos 3x + B\sin 3x$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Assume Particular solution

$$y_P = A\cos 3x + B\sin 3x$$

Then,

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Assume Particular solution

$$y_P = A\cos 3x + B\sin 3x$$

Then,

$$y_P' = -3A\sin 3x + 3B\cos 3x$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Assume Particular solution

$$y_P = A\cos 3x + B\sin 3x$$

Then,

$$y_P' = -3A\sin 3x + 3B\cos 3x$$

$$y_P'' = -9A\cos 3x - 9B\sin 3x$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

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Then,

$$y_P' = -3A\sin 3x + 3B\cos 3x$$

$$y_P'' = -9A\cos 3x - 9B\sin 3x$$

$$y_p'' - y_p' + y_p$$

$$= -9Ac - 9Bs + 3As - 3Bc + Ac + Bs$$

$$= (3A - 8B)\sin 3x$$

$$-(8A + 3B)\cos 3x$$

$$= 2\sin 3x + 0 \cdot \cos 3x$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

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$$y_p'' - y_p' + y_p$$

$$= -9Ac - 9Bs + 3As - 3Bc + Ac + Bs$$

$$= \boxed{(3A - 8B)}\sin 3x$$

$$- \boxed{(8A + 3B)}\cos 3x$$

$$= \boxed{2}\sin 3x + \boxed{0}\cdot\cos 3x$$

Undetermined Coefficients

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Particular Solution Using Undetermined Coefficients

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$$= \boxed{2}\sin 3x + \boxed{0}\cdot\cos 3x$$

$$3A - 8B = 2$$

$$8A + 3B = 0$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

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$$3A - 8B = 2$$

$$8A + 3B = 0$$

$$A = \frac{6}{73}, \quad B = -\frac{16}{73}$$

Undetermined Coefficients

☑ **Example 2**

Particular Solution Using Undetermined Coefficients

Find particular solution of

$$y'' - y' + y = 2\sin 3x$$

Assume Particular solution

$$y_P = A\cos 3x + B\sin 3x$$

Then,

$$y_P' = -3A\sin 3x + 3B\cos 3x$$

$$y_P'' = -9A\cos 3x - 9B\sin 3x$$

$$y_p'' - y_p' + y_p$$

$$= -9Ac - 9Bs + 3As - 3Bc + Ac + Bs$$

$$= \boxed{(3A - 8B)}\sin 3x$$

$$- \boxed{(8A + 3B)}\cos 3x$$

$$= \boxed{2}\sin 3x + \boxed{0}\cdot\cos 3x$$

$$3A - 8B = 2$$

$$8A + 3B = 0$$

$$A = \frac{6}{73}, \quad B = -\frac{16}{73}$$

$$\therefore y_P = \frac{6}{73}\cos 3x - \frac{16}{73}\sin 3x$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 1

solve homogeneous equation

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 1

solve homogeneous equation

$$y'' - 2y' - 3y = 0$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 1

solve homogeneous equation

$$y'' - 2y' - 3y = 0$$

Substitute

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

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 # Step 1

solve homogeneous equation

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Substitute $y = e^{mx}$

Undetermined Coefficients

☑ **Example 3**
Forming y_p by Superposition

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$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 1
solve homogeneous equation

$$y'' - 2y' - 3y = 0$$

Substitute $y = e^{mx}$

$$m^2 - 2m - 3 = (m-3)(m+1) = 0$$

$$m_1 = -1,\ \ m_2 = 3$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

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 # Step 2

Assume that

Undetermined Coefficients

☑ **Example 3**
Forming y_p by Superposition

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 # Step 1
solve homogeneous equation

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$$m_1 = -1, \ m_2 = 3$$

$$y_c = c_1 e^{-x} + c_2 e^{3x}$$

 # Step 2
Assume that

$$g(x) = g_1(x) + g_2(x)$$

$$= polynomial + exponetial$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

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$$y'' - 2y' - 3y = 0$$

Substitute $y = e^{mx}$

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Correspondingly,

Undetermined Coefficients

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Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 1

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Correspondingly, $y_p = y_{p_1} + y_{p_2}$

Undetermined Coefficients

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$$g(x) = g_1(x) + g_2(x)$$

$$= polynomial + exponetial$$

Correspondingly, $y_p = y_{p_1} + y_{p_2}$

where, $y_{p_1} = Ax + B$

Undetermined Coefficients

☑ **Example 3**
Forming y_p by Superposition

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$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

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 # Step 2
Assume that

$$g(x) = g_1(x) + g_2(x)$$

$$= polynomial + exponetial$$

Correspondingly, $y_p = y_{p_1} + y_{p_2}$

where, $y_{p_1} = Ax + B$

$$y_{P_2} = Cxe^{2x} + Ee^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 2

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

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 # Step 2

$$y_p = y_{p_1} + y_{p_2}$$

$$= Ax + B + Cxe^{2x} + Ee^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 2

$$y_p = y_{p_1} + y_{p_2}$$

$$= Ax + B + Cxe^{2x} + Ee^{2x}$$

$$y_p' = A + Ce^{2x} + 2Cxe^{2x} + 2Ee^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 2

$$y_p = y_{p_1} + y_{p_2}$$

$$= Ax + B + Cxe^{2x} + Ee^{2x}$$

$$y_p' = A + Ce^{2x} + 2Cxe^{2x} + 2Ee^{2x}$$

$$y_p'' = 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x} + 4Ee^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 2

$$y_p = y_{p_1} + y_{p_2}$$
$$= Ax + B + Cxe^{2x} + Ee^{2x}$$

$$y_p' = A + Ce^{2x} + 2Cxe^{2x} + 2Ee^{2x}$$

$$y_p'' = 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x} + 4Ee^{2x}$$

$$y_p'' - 2y_p' - 3y_p$$
$$= 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x} + 4Ee^{2x}$$
$$-2A - 2Ce^{2x} - 4Cxe^{2x} - 4Ee^{2x}$$
$$-3Ax - 3B - 3Cxe^{2x} - 3Ee^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 2

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$$y_p'' - 2y_p' - 3y_p$$
$$= 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x} + 4Ee^{2x}$$
$$- 2A - 2Ce^{2x} - 4Cxe^{2x} - 4Ee^{2x}$$
$$- 3Ax - 3B - 3Cxe^{2x} - 3Ee^{2x}$$
$$= -3Ax - (3B + 2A)$$
$$+ (2C - 3E)e^{2x} - 3Cxe^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

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 # Step 2

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$$= -3Ax - (3B + 2A)$$
$$+ (2C - 3E)e^{2x} - 3Cxe^{2x}$$
$$= 4x - 5 + 0 + 6xe^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

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$$y_p'' - 2y_p' - 3y_p$$

$$= 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x} + 4Ee^{2x}$$

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$$-3Ax - 3B - 3Cxe^{2x} - 3Ee^{2x}$$

$$= -3Ax - (3B + 2A)$$

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Undetermined Coefficients

☑ **Example 3**
Forming y_p by Superposition

Solve

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 # Step 2

$$y_p = y_{p_1} + y_{p_2}$$
$$= Ax + B + Cxe^{2x} + Ee^{2x}$$

$$y_p'' - 2y_p' - 3y_p$$
$$= -3Ax - (3B + 2A)$$
$$+ (2C - 3E)e^{2x} - 3Cxe^{2x}$$
$$= 4x - 5 + 0 + 6xe^{2x}$$

$$-3A = 4, \quad 3B + 2A = 5,$$
$$-3C = 6, \quad 2C - 3E = 0$$

$$A = -\frac{4}{3}, \; B = \frac{23}{9}, \; C = -2, \; E = -\frac{4}{3}$$

$$\therefore y_p = -\frac{4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 3

General solution

$$y_c = c_1 e^{-x} + c_2 e^{3x}$$

$$y_p = -\frac{4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

 # Step 3

General solution

$$y_c = c_1 e^{-x} + c_2 e^{3x}$$

$$y_p = -\frac{4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}$$

$$y = y_c + y_p$$

$$= c_1 e^{-x} + c_2 e^{3x} - \frac{4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}$$

Undetermined Coefficients

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Forming y_p by Superposition

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※ We can solve this example by solving two simpler problem

Undetermined Coefficients

☑ **Example 3**

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$$\begin{cases} y'' - 2y' - 3y = 4x - 5 \\ y'' - 2y' - 3y = 6xe^{2x} \end{cases}$$

Undetermined Coefficients

☑ **Example 3**

Forming y_p by Superposition

Solve

$$y'' - 2y' - 3y = \boxed{4x - 5} + 6xe^{2x}$$

 # Step 3

General solution

$$y_c = c_1 e^{-x} + c_2 e^{3x}$$

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Undetermined Coefficients

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$$\begin{cases} y'' - 2y' - 3y = \boxed{4x - 5} \\ y'' - 2y' - 3y = \boxed{6xe^{2x}} \end{cases}$$

Undetermined Coefficients

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$$\begin{cases} y'' - 2y' - 3y = \boxed{4x - 5} \\ y'' - 2y' - 3y = \boxed{6xe^{2x}} \end{cases}$$

$$y_{p_1} = -\frac{4}{3}x + \frac{23}{9},$$

Undetermined Coefficients

☑ Example 3

Forming y_p by Superposition

Solve

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$$\begin{cases} y'' - 2y' - 3y = \boxed{4x - 5} \\ y'' - 2y' - 3y = \boxed{6xe^{2x}} \end{cases}$$

$$y_{p_1} = -\frac{4}{3}x + \frac{23}{9}, \quad y_{p_2} = -2xe^{2x} - \frac{4}{3}e^{2x}$$

Undetermined Coefficients

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$$\begin{cases} y'' - 2y' - 3y = \boxed{4x - 5} \\ y'' - 2y' - 3y = \boxed{6xe^{2x}} \end{cases}$$

$$y_{p_1} = -\frac{4}{3}x + \frac{23}{9}, \quad y_{p_2} = -2xe^{2x} - \frac{4}{3}e^{2x}$$

Are particular solution of each DE

Undetermined Coefficients

☑ **Example 4**

Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

Undetermined Coefficients

☑ **Example 4**

Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

Undetermined Coefficients

☑ **Example 4**

Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

$$y_p = Ae^x$$

Undetermined Coefficients

☑ **Example 4**

Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

$$y_p = Ae^x$$

$$y_p'' - 5y_p' + 4y_p$$

$$= (A - 5A + 4A)e^x = 0 \cdot e^x = 0$$

Undetermined Coefficients

☑ **Example 4**
Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

$$y_p = Ae^x$$

$$y_p'' - 5y_p' + 4y_p$$

$$= (A - 5A + 4A)e^x = 0 \cdot e^x = 0$$

$$0 = 8e^x \ ??$$

Undetermined Coefficients

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Undetermined Coefficients

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Let assume particular solution as

Undetermined Coefficients

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Let assume particular solution as

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then,

Undetermined Coefficients

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$$y'' - 5y' + 4y = 8e^x$$

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$$y_p = Ae^x$$

$$y_p'' - 5y_p' + 4y_p$$

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$$0 = 8e^x \ ??$$

Let assume particular solution as

$$y_p = Axe^x$$

then,

$$y_p' = Ae^x + Axe^x$$

Undetermined Coefficients

☑ Example 4
Glitch in the Method

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$$y'' - 5y' + 4y = 8e^x$$

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$$y_p'' - 5y_p' + 4y_p$$

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Let assume particular solution as

$$y_p = Axe^x$$

then,

$$y_p' = Ae^x + Axe^x$$

$$y_p'' = 2Ae^x + Axe^x$$

Undetermined Coefficients

☑ Example 4
Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

$$y_p = Ae^x$$

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$$0 = 8e^x \ ??$$

Let assume particular solution as

$$y_p = Axe^x$$

then,

$$y_p' = Ae^x + Axe^x$$

$$y_p'' = 2Ae^x + Axe^x$$

$$y_p'' - 5y_p' + 4y_p$$

$$= (2A + Ax - 5A - 5Ax + 4Ax)e^x$$

$$= -3Ae^x = 8e^x$$

Undetermined Coefficients

☑ Example 4
Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

$$y_p = Ae^x$$

$$y_p'' - 5y_p' + 4y_p$$

$$= (A - 5A + 4A)e^x = 0 \cdot e^x = 0$$

$$0 = 8e^x \ ??$$

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Undetermined Coefficients

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$$y_p'' - 5y_p' + 4y_p$$

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$$0 = 8e^x \ ??$$

Let assume particular solution as

$$y_p = Axe^x$$

then,

$$y_p' = Ae^x + Axe^x$$

$$y_p'' = 2Ae^x + Axe^x$$

$$y_p'' - 5y_p' + 4y_p$$

$$= (2A + Ax - 5A - 5Ax + 4Ax)e^x$$

$$= -3Ae^x = 8e^x$$

$$A = -\frac{8}{3}$$

Undetermined Coefficients

☑ Example 4
Glitch in the Method

Find a particular solution of

$$y'' - 5y' + 4y = 8e^x$$

If we assume particular solution

$$y_p = Ae^x$$

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$$= (A - 5A + 4A)e^x = 0 \cdot e^x = 0$$

$$0 = 8e^x \ ??$$

Let assume particular solution as

$$y_p = Axe^x$$

then,

$$y_p' = Ae^x + Axe^x$$

$$y_p'' = 2Ae^x + Axe^x$$

$$y_p'' - 5y_p' + 4y_p$$

$$= (2A + Ax - 5A - 5Ax + 4Ax)e^x$$

$$= -3Ae^x = 8e^x$$

$$A = -\frac{8}{3} \qquad y_p = -\frac{8}{3}xe^x$$

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **Glitch in the method**

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **Glitch in the method**
  - **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

Undetermined Coefficients

☑ Method of undetermined coefficients

- ■ Glitch in the method
  - Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation
  - Case II : A function in the assumed particular solution is also a solution of the associated homogeneous differential equation

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **Glitch in the method**
  - **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$ **Form of** y_p

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	**Form of** y_p
$1(any\ const.)$	A

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
$1(any\ const.)$	A
$5x+7$	$Ax+B$

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
$1(any\ const.)$	A
$5x+7$	$Ax+B$
$3x^2-2$	Ax^2+Bx+C

Undetermined Coefficients

☑ Method of undetermined coefficients

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- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
$1(any\ const.)$	A
$5x+7$	$Ax+B$
$3x^2-2$	Ax^2+Bx+C
x^3-x+1	Ax^3+Bx^2+Cx+E

Undetermined Coefficients

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- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
$1(any\ const.)$	A
$5x+7$	$Ax+B$
$3x^2-2$	Ax^2+Bx+C
x^3-x+1	Ax^3+Bx^2+Cx+E
$\sin 4x$	$A\cos 4x+B\sin 4x$

Undetermined Coefficients

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$5x+7$	$Ax+B$
$3x^2-2$	Ax^2+Bx+C
x^3-x+1	Ax^3+Bx^2+Cx+E
$\sin 4x$	$A\cos 4x+B\sin 4x$
$\cos 4x$	$A\cos 4x+B\sin 4x$

Undetermined Coefficients

☑ **Method of undetermined coefficients**

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Undetermined Coefficients

☑ Method of undetermined coefficients

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Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
e^{5x}	Ae^{5x}

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
e^{5x}	Ae^{5x}
$(9x-2)e^{5x}$	$(Ax+B)e^{5x}$

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case I : No function in the assumed particular solution is a solution of the associated homogeneous differential equation**

$g(x)$	Form of y_p
e^{5x}	Ae^{5x}
$(9x-2)e^{5x}$	$(Ax+B)e^{5x}$
x^2e^{5x}	$(Ax^2+Bx+C)e^{5x}$

Undetermined Coefficients

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e^{5x}	Ae^{5x}
$(9x-2)e^{5x}$	$(Ax+B)e^{5x}$
x^2e^{5x}	$(Ax^2+Bx+C)e^{5x}$
$e^{3x}\sin 4x$	$Ae^{3x}\cos 4x + Be^{3x}\sin 4x$

Undetermined Coefficients

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$(9x-2)e^{5x}$	$(Ax+B)e^{5x}$
x^2e^{5x}	$(Ax^2+Bx+C)e^{5x}$
$e^{3x}\sin 4x$	$Ae^{3x}\cos 4x+Be^{3x}\sin 4x$
$5x^2\sin 4x$	$(Ax^2+Bx+C)\cos 4x+(Ex^2+Fx+G)\sin 4x$

Undetermined Coefficients

☑ Method of undetermined coefficients

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$e^{3x}\sin 4x$	$Ae^{3x}\cos 4x+Be^{3x}\sin 4x$
$5x^2\sin 4x$	$(Ax^2+Bx+C)\cos 4x+(Ex^2+Fx+G)\sin 4x$
$xe^{3x}\cos 4x$	$(Ax+B)e^{3x}\cos 4x+(Cx+E)e^{3x}\sin 4x$

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **Glitch in the method**

Undetermined Coefficients

☑ **Method of undetermined coefficients**

- **Glitch in the method**
  - **Case II : A function in the assumed particular solution is also a solution of the associated homogeneous differential equation**

Undetermined Coefficients

☑ Method of undetermined coefficients

■ Glitch in the method

- **Case II : A function in the assumed particular solution is also a solution of the associated homogeneous differential equation**

Multiplication Rule : if any y_p contains terms that duplicate terms in y_c then that y_{pi} must multiplied by x^n , where *n* is the smallest positive integer that eliminates that duplication.

Undetermined Coefficients

☑ Example 5

Forms of Particular Solutions-Case I

Determine the form of a particular solution of

(a) $y'' - 8y' + 25y = 5x^3 e^{-x} - 7e^{-x}$

(b) $y'' + 4y = x\cos x$

Undetermined Coefficients

☑ Example 5

Forms of Particular Solutions-Case I

Determine the form of a particular solution of

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(a)

Undetermined Coefficients

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(a) $y'' - 8y' + 25y = 5x^3 e^{-x} - 7e^{-x}$

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(a) $g(x) = (5x^3 - 7)e^{-x}$

Undetermined Coefficients

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Undetermined Coefficients

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$$y_p = (Ax^3 + Bx^2 + Cx + E)e^{-x}$$

Undetermined Coefficients

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It is not duplicated with y_c

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Undetermined Coefficients

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Undetermined Coefficients

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It is not duplicated with y_c

$$y_c = c_1 \cos 2x + c_2 \sin 2x$$

Undetermined Coefficients

☑ **Example 6**

Forming y_p by Superposition-Case I

Determine the form of a particular solution of

$$y'' - 9y' + 14y$$
$$= 3x^2 - 5\sin 2x + 7xe^{6x}$$

Undetermined Coefficients

☑ **Example 6**

Forming y_p by Superposition-Case I

Determine the form of a particular solution of

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 # Corresponding to $3x^2$ **we assume**

Undetermined Coefficients

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Undetermined Coefficients

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Determine the form of a particular solution of

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 # Corresponding to $3x^2$ **we assume**

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Undetermined Coefficients

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Undetermined Coefficients

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 # Corresponding to $7xe^{6x}$ **we assume**

$$y_{p_3} = (Gx + H)e^{6x}$$

Undetermined Coefficients

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 # Corresponding to $7xe^{6x}$ **we assume**

$$y_{p_3} = (Gx + H)e^{6x}$$

$$y = y_{p_1} + y_{p_2} + y_{p_3}$$
$$= Ax^2 + Bx + C$$
$$+ E\cos 2x + F\sin 2x$$
$$+ (Gx + H)e^{6x}$$

Undetermined Coefficients

☑ Example 7
Particular Solution-Case II

Find a particular solution of

$$y'' - 2y' + y = e^x$$

Undetermined Coefficients

☑ Example 7
Particular Solution-Case II

Find a particular solution of

$$y'' - 2y' + y = e^x$$

Complementary function is

Undetermined Coefficients

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Find a particular solution of

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Complementary function is

$$y_c = c_1 e^x + c_2 x e^x$$

Undetermined Coefficients

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Undetermined Coefficients

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Undetermined Coefficients

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Undetermined Coefficients

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Undetermined Coefficients

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Undetermined Coefficients

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$$y_p = Ax^2 e^x$$

it is ***not*** associate with y_c

Undetermined Coefficients

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it is ***not*** associate with y_c

$$y_p' = 2Axe^x + Ax^2 e^x$$

$$y_p'' = 2Ae^x + 4Axe^x + Ax^2 e^x$$

Undetermined Coefficients

☑ Example 7
Particular Solution-Case II

Find a particular solution of

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Undetermined Coefficients

☑ **Example 7**

Particular Solution-Case II

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$$y_p = Ax^2 e^x$$

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Undetermined Coefficients

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$$= 2Ae^x + 4Axe^x + Ax^2e^x$$

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$$= 2Ae^x = e^x$$

$$2A = 1,\ A = \frac{1}{2}$$

Undetermined Coefficients

☑ **Example 7**

Particular Solution-Case II

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$$y_p' = 2Axe^x + Ax^2e^x$$

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$$y_p'' - 2y_p' + y_p$$

$$= 2Ae^x + 4Axe^x + Ax^2e^x$$

$$- 4Axe^x - 2Ax^2e^x + Ax^2e^x$$

$$= 2Ae^x = e^x$$

$$2A = 1,\ A = \frac{1}{2}$$

$$\therefore y_p = \frac{1}{2}x^2e^x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Undetermined Coefficients

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An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

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Complementary function is

Undetermined Coefficients

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Let's assume particular solution as

Undetermined Coefficients

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Let's assume particular solution as

$$y_p = y_{p_1} + y_{p_2}$$

$$= polynomial + trigonometric$$

Undetermined Coefficients

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$$y_{p_1} = Ax + B$$

Undetermined Coefficients

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$$y(\pi) = 0, \quad y'(\pi) = 2$$

Complementary function is

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Let's assume particular solution as

$$y_p = y_{p_1} + y_{p_2}$$
$$= polynomial + trigonometric$$

$$y_{p_1} = Ax + B$$

$$y_{p_2} = C\cos x + E\sin x$$

Undetermined Coefficients

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$$y_{p_2}$$

Undetermined Coefficients

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y_{p_2} **is associated with** y_c **so**

Undetermined Coefficients

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y_{p_2} **is associated with** y_c **so**

$$\text{Let } y_{p_2} = Cx\cos x + Ex\sin x$$

Undetermined Coefficients

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Let's assume particular solution as

$$y_p = y_{p_1} + y_{p_2}$$

$$= polynomial + trigonometric$$

$$y_{p_1} = Ax + B$$

$$y_{p_2} = C\cos x + E\sin x$$

y_{p_2} **is associated with y_c so**

$$\text{Let } y_{p_2} = Cx\cos x + Ex\sin x$$

$$\therefore y_p = Ax + B + Cx\cos x + Ex\sin x$$

Undetermined Coefficients

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An Initial-Value Problem

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Undetermined Coefficients

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$$y_p = Ax + B$$

$$+ Cx\cos x + Ex\sin x$$

Undetermined Coefficients

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An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Assume particular solution as

$$y_p = Ax + B$$

$$+ Cx\cos x + Ex\sin x$$

$$y_p' = A + C\cos x - Cx\sin x$$

$$+ E\sin x + Ex\cos x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Assume particular solution as

$$y_p = Ax + B$$

$$+ Cx\cos x + Ex\sin x$$

$$y_p' = A + C\cos x - Cx\sin x$$

$$+ E\sin x + Ex\cos x$$

$$y_p'' = -2C\sin x - Cx\cos x$$

$$+ 2E\cos x - Ex\sin x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Assume particular solution as

$$y_p = Ax + B$$

$$+ Cx\cos x + Ex\sin x$$

$$y_p' = A + C\cos x - Cx\sin x$$

$$+ E\sin x + Ex\cos x$$

$$y_p'' = -2C\sin x - Cx\cos x$$

$$+ 2E\cos x - Ex\sin x$$

$$y'' + y = Ax + B$$

$$- 2C\sin x + 2E\cos x$$

$$= 4x + 10\sin x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Assume particular solution as

$$y_p = Ax + B$$

$$+ Cx\cos x + Ex\sin x$$

$$y_p' = A + C\cos x - Cx\sin x$$

$$+ E\sin x + Ex\cos x$$

$$y_p'' = -2C\sin x - Cx\cos x$$

$$+ 2E\cos x - Ex\sin x$$

$$y'' + y = Ax + B$$

$$-2C\sin x + 2E\cos x$$

$$= 4x + 10\sin x$$

$$A = 4,\ B = 0,\ C = -5,\ E = 0$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Assume particular solution as

$$y_p = Ax + B + Cx\cos x + Ex\sin x$$

$$y_p' = A + C\cos x - Cx\sin x + E\sin x + Ex\cos x$$

$$y_p'' = -2C\sin x - Cx\cos x + 2E\cos x - Ex\sin x$$

$$y'' + y = Ax + B - 2C\sin x + 2E\cos x = 4x + 10\sin x$$

$$A = 4,\ B = 0,\ C = -5,\ E = 0$$

$$\therefore y_p = 4x - 5x\cos x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

General solution is

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

General solution is

$$y = c_1 \cos x + c_2 \sin x + 4x - 5x\cos x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

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$$y'' + y = 4x + 10\sin x,$$

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To satisfy initial condition

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

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Undetermined Coefficients

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An Initial-Value Problem

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$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

General solution is

$$y = c_1 \cos x + c_2 \sin x + 4x - 5x\cos x$$

To satisfy initial condition

$$y(\pi) = 0, \quad y'(\pi) = 2$$

$$0 = c_1 \cos\pi + c_2 \sin\pi + 4\pi - 5\pi\cos\pi$$

$$= c_1 - \pi$$

$$c_1 = \pi$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

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$$= c_1 - \pi$$

$$c_1 = \pi$$

$$y' = -c_1 \sin x + c_2 \cos x + 4 - 5\cos x + 5x\sin x$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

General solution is

$$y = c_1 \cos x + c_2 \sin x + 4x - 5x\cos x$$

To satisfy initial condition

$$y(\pi) = 0, \quad y'(\pi) = 2$$

$$0 = c_1 \cos\pi + c_2 \sin\pi + 4\pi - 5\pi\cos\pi$$

$$= c_1 - \pi$$

$$c_1 = \pi$$

$$y' = -c_1 \sin x + c_2 \cos x + 4 - 5\cos x + 5x\sin x$$

$$2 = -c_1 \sin\pi + c_2 \cos\pi + 4 - 5\cos\pi + 5\pi\sin\pi$$

$$= c_2 - 1$$

$$c_2 = 3$$

Undetermined Coefficients

☑ **Example 8**

An Initial-Value Problem

Solve IVP

$$y'' + y = 4x + 10\sin x,$$

$$y(\pi) = 0, \quad y'(\pi) = 2$$

General solution is

$$y = c_1 \cos x + c_2 \sin x + 4x - 5x\cos x$$

To satisfy initial condition

$$y(\pi) = 0, \quad y'(\pi) = 2$$

$$0 = c_1 \cos\pi + c_2 \sin\pi + 4\pi - 5\pi\cos\pi$$

$$= c_1 - \pi$$

$$c_1 = \pi$$

$$y' = -c_1 \sin x + c_2 \cos x + 4 - 5\cos x + 5x\sin x$$

$$2 = -c_1 \sin\pi + c_2 \cos\pi + 4 - 5\cos\pi + 5\pi\sin\pi$$

$$= c_2 - 1$$

$$c_2 = 3$$

$$\therefore y = \pi\cos x + 3\sin x + 4x - 5x\cos x$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

particular solution assumed as

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

particular solution assumed as

$$y_p = \underbrace{Ax^2 + Bx + C}_{y_{p1}} + \underbrace{Ee^{3x}}_{y_{p2}}$$

Undetermined Coefficients

☑ **Example 9**
Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

particular solution assumed as

$$y_p = \underbrace{Ax^2 + Bx + C}_{y_{p1}} + \underbrace{Ee^{3x}}_{y_{p2}}$$

$$y_c = c_1 e^{3x} + c_2 x e^{3x}$$

Undetermined Coefficients

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$$y_c = c_1 e^{3x} + c_2 x e^{3x}$$

y_{p_2} **is associated with y_c, So multiply x^2 (because multiple of x also associated with y_c) in that solution**

Undetermined Coefficients

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$$y_p = Ax^2 + Bx + C + Ex^2 e^{3x}$$

Undetermined Coefficients

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Using the Multiplication Rule

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particular solution assumed as

$$y_p = \underbrace{Ax^2 + Bx + C}_{y_{p1}} + \underbrace{Ee^{3x}}_{y_{p2}}$$

$$y_c = c_1 e^{3x} + c_2 x e^{3x}$$

y_{p_2} **is associated with y_c, So multiply x^2 (because multiple of x also associated with y_c) in that solution**

$$y_p = Ax^2 + Bx + C + Ex^2 e^{3x}$$

$$y_p' = 2Ax + B + 2Exe^{3x} + 3Ex^2 e^{3x}$$

Undetermined Coefficients

☑ **Example 9**
Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

particular solution assumed as

$$y_p = \underbrace{Ax^2 + Bx + C}_{y_{p1}} + \underbrace{Ee^{3x}}_{y_{p2}}$$

$$y_c = c_1 e^{3x} + c_2 x e^{3x}$$

y_{p_2} **is associated with y_c, So multiply x^2 (because multiple of x also associated with y_c) in that solution**

$$y_p = Ax^2 + Bx + C + Ex^2 e^{3x}$$

$$y_p' = 2Ax + B + 2Exe^{3x} + 3Ex^2 e^{3x}$$

$$y_p'' = 2A + 2Ee^{3x} + 12Exe^{3x} + 9Ex^2 e^{3x}$$

Undetermined Coefficients

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Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

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$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

Undetermined Coefficients

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$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

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Undetermined Coefficients

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Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

$$y_p' = 2Ax + B + 2Exe^{3x} + 3Ex^2e^{3x}$$

$$y_p'' = 2A + 2Ee^{3x}$$

$$+12Exe^{3x} + 9Ex^2e^{3x}$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

$$y_p' = 2Ax + B + 2Exe^{3x} + 3Ex^2e^{3x}$$

$$y_p'' = 2A + 2Ee^{3x} + 12Exe^{3x} + 9Ex^2e^{3x}$$

$$y_p'' - 6y_p' + 9y_p$$

$$= 9Ax^2 + (-12A + 9B)x + 2A - 6B + 9C + 2Ee^{3x}$$

$$= 6x^2 + 2 - 12e^{3x}$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

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$$+12Exe^{3x} + 9Ex^2e^{3x}$$

$$y_p'' - 6y_p' + 9y_p$$

$$= 9Ax^2 + (-12A + 9B)x + 2A - 6B$$

$$+9C + 2Ee^{3x}$$

$$= 6x^2 + 2 - 12e^{3x}$$

$$A = \frac{2}{3},\ B = \frac{8}{9},\ C = \frac{2}{3},\ E = -6$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

$$y_p' = 2Ax + B + 2Exe^{3x} + 3Ex^2e^{3x}$$

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$$= 9Ax^2 + (-12A + 9B)x + 2A - 6B$$

$$+9C + 2Ee^{3x}$$

$$= 6x^2 + 2 - 12e^{3x}$$

$$A = \frac{2}{3},\ B = \frac{8}{9},\ C = \frac{2}{3},\ E = -6$$

$$\therefore y_p = \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2e^{3x}$$

Undetermined Coefficients

☑ **Example 9**

Using the Multiplication Rule

Solve

$$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$$

$$y_p = Ax^2 + Bx + C + Ex^2e^{3x}$$

$$y_p' = 2Ax + B + 2Exe^{3x} + 3Ex^2e^{3x}$$

$$y_p'' = 2A + 2Ee^{3x} + 12Exe^{3x} + 9Ex^2e^{3x}$$

$$y_p'' - 6y_p' + 9y_p$$

$$= 9Ax^2 + (-12A + 9B)x + 2A - 6B + 9C + 2Ee^{3x}$$

$$= 6x^2 + 2 - 12e^{3x}$$

$$A = \frac{2}{3},\ B = \frac{8}{9},\ C = \frac{2}{3},\ E = -6$$

$$\therefore y_p = \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2e^{3x}$$

$$y = y_c + y_p$$

$$= c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2e^{3x}$$

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

*Dennis G. Zill, Michael R. Cullen, Advanced Engineering Mathematics 3rd Edition, Johns and Bartlett, 2006, p130

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

Substitute

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Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

Substitute $y = e^{mx}$

$$m^3 + m^2 = m^2(m+1) = 0$$

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Undetermined Coefficients

☑ Example 10

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

Substitute $y = e^{mx}$

$$m^3 + m^2 = m^2(m+1) = 0$$

$$m_1 = m_2 = 0 \ and \ m_3 = -1$$

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Undetermined Coefficients

☑ Example 10

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$$y_c = c_1 + c_2 x + c_3 e^{-x}$$

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Undetermined Coefficients

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$$y_c = c_1 + c_2 x + c_3 e^{-x}$$

Based on entry 10 in table 3.1*

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Undetermined Coefficients

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Based on entry 10 in table 3.1*

$$y_p = Ae^x \cos x + Be^x \sin x$$

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Undetermined Coefficients

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$$y''' + y'' = e^x \cos x$$

Substitute $y = e^{mx}$

$$m^3 + m^2 = m^2(m+1) = 0$$

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$$y_c = c_1 + c_2 x + c_3 e^{-x}$$

Based on entry 10 in table 3.1*

$$y_p = Ae^x \cos x + Be^x \sin x$$

It is not duplicate with y_c

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Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

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$$y_c = c_1 + c_2 x + c_3 e^{-x}$$

Based on entry 10 in table 3.1*

$$y_p = Ae^x \cos x + Be^x \sin x$$

It is not duplicate with y_c

$$y_p' = (A+B)e^x \cos x + (B-A)e^x \sin x$$

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Undetermined Coefficients

☑ Example 10

Third-Order DE- Case I

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$$y''' + y'' = e^x \cos x$$

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Based on entry 10 in table 3.1*

$$y_p = Ae^x \cos x + Be^x \sin x$$

It is not duplicate with y_c

$$y_p' = (A+B)e^x \cos x + (B-A)e^x \sin x$$

$$y_p'' = (2B)e^x \cos x - 2Ae^x \sin x$$

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Solve

$$y''' + y'' = e^x \cos x$$

Substitute $y = e^{mx}$

$$m^3 + m^2 = m^2(m+1) = 0$$

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$$y_c = c_1 + c_2 x + c_3 e^{-x}$$

Based on entry 10 in table 3.1*

$$y_p = Ae^x \cos x + Be^x \sin x$$

It is not duplicate with y_c

$$y_p' = (A+B)e^x \cos x + (B-A)e^x \sin x$$

$$y_p'' = (2B)e^x \cos x - 2Ae^x \sin x$$

$$y_p''' = 2(-A+B)e^x \cos x - 2(A+B)e^x \sin x$$

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Undetermined Coefficients

☑ Example 10

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

Substitute $y = e^{mx}$

$$m^3 + m^2 = m^2(m+1) = 0$$

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$$y_c = c_1 + c_2 x + c_3 e^{-x}$$

Based on entry 10 in table 3.1*

$$y_p = Ae^x \cos x + Be^x \sin x$$

It is not duplicate with y_c

$$y_p' = (A+B)e^x \cos x + (B-A)e^x \sin x$$

$$y_p'' = (2B)e^x \cos x - 2Ae^x \sin x$$

$$y_p''' = 2(-A+B)e^x \cos x - 2(A+B)e^x \sin x$$

$$y_p''' + y_p''$$

$$= (-2A+3B)e^x \cos x - (4A+2B)e^x \sin x$$

$$= e^x \cos x$$

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Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

$$y_p = Ae^x \cos x + Be^x \sin x$$

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$$y_p''' + y_p''$$

$$= (-2A + 3B)e^x \cos x$$

$$-(4A + 2B)e^x \sin x$$

$$= e^x \cos x$$

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

$$y_p = Ae^x \cos x + Be^x \sin x$$

$$y_p''' + y_p''$$

$$= (-2A + 3B)e^x \cos x$$

$$-(4A + 2B)e^x \sin x$$

$$= e^x \cos x$$

$$\begin{cases} -2A + 3B = 1 \\ 4A + 2B = 0 \end{cases}$$

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

$$y_p = Ae^x \cos x + Be^x \sin x$$

$$y_p''' + y_p''$$

$$= (-2A + 3B)e^x \cos x$$

$$-(4A + 2B)e^x \sin x$$

$$= e^x \cos x$$

$$\begin{cases} -2A + 3B = 1 \\ 4A + 2B = 0 \end{cases}$$

$$A = -\frac{1}{10}, \quad B = \frac{1}{5}$$

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

$$y_p = Ae^x \cos x + Be^x \sin x$$

$$y_p''' + y_p''$$

$$= (-2A + 3B)e^x \cos x$$

$$-(4A + 2B)e^x \sin x$$

$$= e^x \cos x$$

$$\begin{cases} -2A + 3B = 1 \\ 4A + 2B = 0 \end{cases}$$

$$A = -\frac{1}{10}, \quad B = \frac{1}{5}$$

$$\therefore y_p = -\frac{1}{10} e^x \cos x + \frac{1}{5} e^x \sin x$$

Undetermined Coefficients

☑ **Example 10**

Third-Order DE- Case I

Solve

$$y''' + y'' = e^x \cos x$$

$$y_p = Ae^x \cos x + Be^x \sin x$$

$$y_p''' + y_p''$$

$$= (-2A + 3B)e^x \cos x$$

$$-(4A + 2B)e^x \sin x$$

$$= e^x \cos x$$

$$\begin{cases} -2A + 3B = 1 \\ 4A + 2B = 0 \end{cases}$$

$$A = -\frac{1}{10}, \quad B = \frac{1}{5}$$

$$\therefore y_p = -\frac{1}{10} e^x \cos x + \frac{1}{5} e^x \sin x$$

$$y = y_c + y_p = c_1 + c_2 x + c_3 e^{-x}$$

$$-\frac{1}{10} e^x \cos x + \frac{1}{5} e^x \sin x$$

Undetermined Coefficients

☑ **Example 11**

Fourth-Order DE- Case II

Determine the form of particular solution

$$y^{(4)} + y''' = 1 - x^2 e^{-x}$$

Undetermined Coefficients

☑ **Example 11**

Fourth-Order DE- Case II

Determine the form of particular solution

$$y^{(4)} + y''' = 1 - x^2 e^{-x}$$

Substitute $y = e^{mx}$

$$m^4 + m^3 = m^3(m+1)$$

$$m_1 = m_2 = m_3 = 0 \ and \ m_4 = -1$$

$$y_c = c_1 + c_2 x + c_3 x^2 + c_4 e^{-x}$$

Undetermined Coefficients

☑ **Example 11**

Fourth-Order DE- Case II

Determine the form of particular solution

$$y^{(4)} + y''' = 1 - x^2 e^{-x}$$

Substitute $y = e^{mx}$

$$m^4 + m^3 = m^3(m+1)$$

$$m_1 = m_2 = m_3 = 0 \ and \ m_4 = -1$$

$$y_c = c_1 + c_2 x + c_3 x^2 + c_4 e^{-x}$$

$$y_p = \underbrace{A}_{y_{p1}} + \underbrace{Bx^2 e^{-x} + Cxe^{-x} + Ee^{-x}}_{y_{p2}}$$

Undetermined Coefficients

☑ **Example 11**

Fourth-Order DE- Case II

Determine the form of particular solution

$$y^{(4)} + y''' = 1 - x^2 e^{-x}$$

Substitute $y = e^{mx}$

$$m^4 + m^3 = m^3(m+1)$$

$$m_1 = m_2 = m_3 = 0 \ and \ m_4 = -1$$

$$y_c = c_1 + c_2 x + c_3 x^2 + c_4 e^{-x}$$

$$y_p = \underbrace{A}_{y_{p1}} + \underbrace{Bx^2 e^{-x} + Cxe^{-x} + Ee^{-x}}_{y_{p2}}$$

We see that the duplications between y_c **and** y_p **are eliminated when** y_{p1} **is multiplied by** x^3 **and** y_{p2} **is multiplied by** x **.**

$$y_p = Ax^3 + Bx^3 e^{-x} + Cx^2 e^{-x} + Exe^{-x}$$

Variation of Parameters

C.f.)Variation of parameters of first-order differential equation

C.f.)Reduction of order second order homogeneous

☑ Method of Variation of Parameters

Variation of Parameters

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☑ Method of Variation of Parameters

- Has a distinct advantage in that it *always* yields a particular solution y_p provided the associated homogeneous equation can be solved.

Variation of Parameters

C.f.)Variation of parameters of first-order differential equation

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☑ Method of Variation of Parameters

- Has a distinct advantage in that it *always* yields a particular solution y_p provided the associated homogeneous equation can be solved.

- In addition, unlike undetermined coefficients, is *not* limited to cases where the input function is a combination of four types of functions, nor is it limited to differential equations with constant coefficients.

Variation of Parameters

☑ **A linear second-order differential equation in standard form**

Variation of Parameters

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$$y'' + P(x)y' + Q(x) = f(x)$$

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$$y_p'' + P(x)y_p' + Q(x) = f(x)$$

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$$y_p'' + P(x)y_p' + Q(x) = f(x)$$ **Find particular solution**

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Substitution

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Substitution

$$y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$

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$$y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$

Where $y_1(x)$ **and** $y_2(x)$ **form a fundamental set of solutions on I of the associated homogeneous form**

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Substitution

$$y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$

Where $y_1(x)$ **and** $y_2(x)$ **form a fundamental set of solutions on I of the associated homogeneous form**

$$y_p' = u_1'(x)y_1(x) + u_1(x)y_1'(x) + u_2'(x)y_2(x) + u_2(x)y_2'(x)$$

$$y_p'' = u_1''(x)y_1(x) + 2u_1'(x)y_1(x) + u_1(x)y_1''(x)$$
$$+ u_2''(x)y_2(x) + 2u_2'(x)y_2(x) + u_2(x)y_2''(x)$$

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L.H.S.:

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L.H.S.:

$$[u_1''(x)y_1(x) + 2u_1'(x)y_1(x) + u_1(x)y_1''(x) + u_2''(x)y_2(x) + 2u_2'(x)y_2(x) + u_2(x)y_2''(x)] +$$
$$P(x)[u_1'(x)y_1(x) + u_1(x)y_1'(x) + u_2'(x)y_2(x) + u_2(x)y_2'(x)] +$$
$$Q(x)$$

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$$P(x)[u_1'(x)y_1(x) + u_1(x)y_1'(x) + u_2'(x)y_2(x) + u_2(x)y_2'(x)] +$$
$$Q(x)$$

$$= u_1(x)[y_1''(x) + P(x)y_1'(x) + Q(x)y_1(x)] + u_1'(x)[2y_1'(x) + P(x)y_1(x)] + u_1''(x)y_1(x) +$$
$$u_2(x)[y_2''(x) + P(x)y_2'(x) + Q(x)y_2(x)] + u_2'(x)[2y_2'(x) + P(x)y_2(x)] + u_2''(x)y_2(x)$$

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$$[u_1''(x)y_1(x) + 2u_1'(x)y_1(x) + u_1(x)y_1''(x) + u_2''(x)y_2(x) + 2u_2'(x)y_2(x) + u_2(x)y_2''(x)] +$$
$$P(x)[u_1'(x)y_1(x) + u_1(x)y_1'(x) + u_2'(x)y_2(x) + u_2(x)y_2'(x)] +$$
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$$u_2(x)[y_2''(x) + P(x)y_2'(x) + Q(x)y_2(x)] + u_2'(x)[2y_2'(x) + P(x)y_2(x)] + u_2''(x)y_2(x)$$

$$= u_1'(x)[2y_1'(x) + P(x)y_1(x)] + y_1(x)u_1''(x) +$$
$$u_2'(x)[2y_2'(x) + P(x)y_2(x)] + y_2(x)u_2''(x)$$

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$$= y_1'(x)u_1'(x) + y_1'(x)u_1'(x) + P(x)y_1(x)u_1'(x) + y_1(x)u_1''(x) + y_2'(x)u_2'(x) + y_2'(x)u_2'(x) + P(x)y_2(x)u_2'(x) + y_2(x)u_2''(x)$$

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$$= y_1'(x)u_1'(x) + y_1'(x)u_1'(x) + P(x)y_1(x)u_1'(x) + y_1(x)u_1''(x) + y_2'(x)u_2'(x) + y_2'(x)u_2'(x) + P(x)y_2(x)u_2'(x) + y_2(x)u_2''(x)$$

$$= \frac{d}{dx}[y_1(x)u_1'(x)] + y_1'(x)u_1'(x) + P(x)u_1'(x)y_1(x) + \frac{d}{dx}[y_2(x)u_2'(x)] + y_2'(x)u_2'(x) + P(x)u_2'(x)y_2(x)$$

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$$= \boxed{y_1'(x)u_1'(x)} + y_1'(x)u_1'(x) + P(x)y_1(x)u_1'(x) + \boxed{y_1(x)u_1''(x)} +$$
$$y_2'(x)u_2'(x) + y_2'(x)u_2'(x) + P(x)y_2(x)u_2'(x) + y_2(x)u_2''(x)$$

$$= \frac{d}{dx}[y_1(x)u_1'(x)] + y_1'(x)u_1'(x) + P(x)u_1'(x)y_1(x) +$$
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$$y_2'(x)u_2'(x) + y_2'(x)u_2'(x) + P(x)y_2(x)u_2'(x) + y_2(x)u_2''(x)$$

$$= \boxed{\frac{d}{dx}[y_1(x)u_1'(x)]} + y_1'(x)u_1'(x) + P(x)u_1'(x)y_1(x) +$$
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$$= \frac{d}{dx}[y_1(x)u_1'(x)] + u_1'(x)y_1'(x) + P(x)u_1'(x)y_1(x) +$$

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R.H.S.: $f(x)$

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R.H.S.: $f(x)$

$$\therefore \frac{d}{dx}[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + P(x)[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + y_1'(x)u_1'(x) + y_2'(x)u_2'(x) = f(x)$$

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$$y_p'' + P(x)y_p' + Q(x) = f(x) \qquad y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$

L.H.S.:

$$\frac{d}{dx}[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + P(x)[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + y_1'(x)u_1'(x) + y_2'(x)u_2'(x)$$

R.H.S.: $f(x)$

$$\therefore \frac{d}{dx}[\underbrace{y_1(x)u_1'(x) + y_2(x)u_2'(x)}_{0}] + P(x)[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + y_1'(x)u_1'(x) + y_2'(x)u_2'(x) = f(x)$$

Variation of Parameters

☑ **A linear second-order differential equation in standard form**

$$y_p'' + P(x)y_p' + Q(x) = f(x) \qquad y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$

L.H.S.:

$$\frac{d}{dx}[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + P(x)[y_1(x)u_1'(x) + y_2(x)u_2'(x)] + y_1'(x)u_1'(x) + y_2'(x)u_2'(x)$$

R.H.S.: $f(x)$

Further assumption

$$\therefore \frac{d}{dx}[\underbrace{y_1(x)u_1'(x) + y_2(x)u_2'(x)}_{\to 0}] + P(x)[\underbrace{y_1(x)u_1'(x) + y_2(x)u_2'(x)}_{\to 0}] + y_1'(x)u_1'(x) + y_2'(x)u_2'(x) = f(x)$$

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Then,

$$y_1(x)u_1'(x) + y_2(x)u_2'(x) = 0$$
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Variation of Parameters

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Variation of Parameters

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Variation of Parameters

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Then,

$$\begin{aligned} y_1(x)u_1'(x) + y_2(x)u_2'(x) &= 0 \\ y_1'(x)u_1'(x) + y_2'(x)u_2'(x) &= f(x) \end{aligned} \Rightarrow \begin{bmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{bmatrix} \begin{bmatrix} u_1'(x) \\ u_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ f(x) \end{bmatrix}$$

$$\therefore \begin{bmatrix} u_1'(x) \\ u_2'(x) \end{bmatrix} = \begin{bmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ f(x) \end{bmatrix}$$

Variation of Parameters

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(Cramer's Rule)

$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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Variation of Parameters

☑ **A linear second-order differential equation in standard form : Summary**

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$$y'' + P(x)y' + Q(x) = f(x)$$

1) From associated homogeneous equation, fundamental set

Variation of Parameters

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1) From associated homogeneous equation, fundamental set $\longrightarrow$ $y_c = c_1 y_1(x) + c_2 y_2(x)$

Variation of Parameters

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$$y'' + P(x)y' + Q(x) = f(x)$$

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2) substitution

Variation of Parameters

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$$y_p'' + P(x) y_p' + Q(x) = f(x)$$

$$\Downarrow$$

$$y_1(x) u_1'(x) + y_2(x) u_2'(x) = 0$$

Variation of Parameters

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$$\Downarrow$$

$$y_1(x)u_1'(x) + y_2(x)u_2'(x) = 0$$
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Variation of Parameters

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Variation of Parameters

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Variation of Parameters

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5) General Solution

Variation of Parameters

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$$y'' + P(x)y' + Q(x) = f(x)$$

$$u_1'(x) = \frac{W_1}{W} = -\frac{y_2(x)f(x)}{W}$$

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4) Find $u_1(x), u_2(x)$ $\longrightarrow$

$$u_1(x) = -\int \frac{y_2(x)f(x)}{W}dx$$

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5) General Solution $\longrightarrow$

$$y = c_1y_1 + c_2y_2 + u_1y_1 + u_2y_2$$

Variation of Parameters

☑ **A linear third-order differential equation in standard form:**

Variation of Parameters

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$$y = u_1(x)y_1(x) + u_2(x)y_2(x) + u_3(x)y_3(x)$$

Variation of Parameters

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$$y = u_1(x)y_1(x) + u_2(x)y_2(x) + u_3(x)y_3(x)$$

$$u_1'(x) = \frac{W_1}{W}, u_2'(x) = \frac{W_2}{W}, u_3'(x) = \frac{W_3}{W}$$

Variation of Parameters

☑ **A linear third-order differential equation in standard form:**

$$y = u_1(x)y_1(x) + u_2(x)y_2(x) + u_3(x)y_3(x)$$

Where,

$$u_1'(x) = \frac{W_1}{W}, u_2'(x) = \frac{W_2}{W}, u_3'(x) = \frac{W_3}{W}$$

Variation of Parameters

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$$y = u_1(x)y_1(x) + u_2(x)y_2(x) + u_3(x)y_3(x)$$

$$u_1'(x) = \frac{W_1}{W}, u_2'(x) = \frac{W_2}{W}, u_3'(x) = \frac{W_3}{W}$$

Where,

$$W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ f(x) & y_2'' & y_3'' \end{vmatrix}, W_1 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & f(x) & y_3'' \end{vmatrix}, W_1 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & f(x) \end{vmatrix},$$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

[Cramer's Rule]

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$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \; x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \; x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$a_{11}x_1 + a_{12}x_2 = b_1 \cdots (1)$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \cdots (2)$$

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$a_{11}x_1 + a_{12}x_2 = b_1 \cdots (1)$$
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Multiplying (1) by a_{22}

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \; x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$a_{11}x_1 + a_{12}x_2 = b_1 \cdots (1)$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \cdots (2)$$

Multiplying (1) by a_{22}

$$a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = a_{22}b_1 \cdots (3)$$

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \; x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$a_{11}x_1 + a_{12}x_2 = b_1 \cdots (1)$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \cdots (2)$$

Multiplying (1) by a_{22}

$$a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = a_{22}b_1 \cdots (3)$$

Multiplying (2) by a_{12}

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \; x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$a_{11}x_1 + a_{12}x_2 = b_1 \cdots (1)$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \cdots (2)$$

Multiplying (1) by a_{22}

$$a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = a_{22}b_1 \cdots (3)$$

Multiplying (2) by a_{12}

$$a_{12}a_{21}x_1 + a_{12}a_{22}x_2 = a_{12}b_2 \cdots (4)$$

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$\Rightarrow \quad x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$a_{11}x_1 + a_{12}x_2 = b_1 \cdots (1)$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \cdots (2)$$

Multiplying (1) by a_{22}

$$a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = a_{22}b_1 \cdots (3)$$

Multiplying (2) by a_{12}

$$a_{12}a_{21}x_1 + a_{12}a_{22}x_2 = a_{12}b_2 \cdots (4)$$

(3)-(4):

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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Multiplying (1) by a_{22}

$$a_{11}a_{22}x_1 + a_{12}a_{22}x_2 = a_{22}b_1 \cdots (3)$$

Multiplying (2) by a_{12}

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$$(a_{11}a_{22} - a_{12}a_{21})x_1 = a_{22}b_1 - a_{12}b_2$$

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(3)-(4):

$$(a_{11}a_{22} - a_{12}a_{21})x_1 = a_{22}b_1 - a_{12}b_2$$

$$\therefore x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

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$$a_{12}a_{21}x_1 + a_{12}a_{22}x_2 = a_{12}b_2 \cdots (4)$$

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$$(a_{11}a_{22} - a_{12}a_{21})x_1 = a_{22}b_1 - a_{12}b_2$$

$$\therefore x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

Similarly for

[Cramer's Rule]

$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$a_{12}a_{21}x_1 + a_{12}a_{22}x_2 = a_{12}b_2 \cdots (4)$$

(3)-(4):

$$(a_{11}a_{22} - a_{12}a_{21})x_1 = a_{22}b_1 - a_{12}b_2$$

$$\therefore x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

Similarly for x_2

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$$a_{11}x_1 + a_{12}x_2 = b_1$$
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$$a_{12}a_{21}x_1 + a_{12}a_{22}x_2 = a_{12}b_2 \cdots (4)$$

(3)-(4):

$$(a_{11}a_{22} - a_{12}a_{21})x_1 = a_{22}b_1 - a_{12}b_2$$

$$\therefore x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

Similarly for x_2

$$\therefore x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Variation of Parameter

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General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Substitute

Variation of Parameter

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General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Substitute $y = e^{mx}$

Variation of Parameter

☑ Example 1

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Substitute $y = e^{mx}$

$$m^2 - 4m + 4 = (m-2)^2$$

Variation of Parameter

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$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

$$W(e^{2x}, xe^{2x}) = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$$= e^{4x}$$

Variation of Parameter

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Solve

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Substitute $y = e^{mx}$

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$$= e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & xe^{2x} \\ (x+1)e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$$= -(x+1)xe^{4x}$$

Variation of Parameter

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Substitute $y = e^{mx}$

$$m^2 - 4m + 4 = (m-2)^2$$

$$m_1 = m_2 = 2$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

$$W(e^{2x}, xe^{2x}) = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$$= e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & xe^{2x} \\ (x+1)e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$$= -(x+1)xe^{4x}$$

$$W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & (x+1)e^{2x} \end{vmatrix}$$

$$= (x+1)e^{4x}$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Variation of Parameter

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General Solution Using Variation of Parameter

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$$y'' - 4y' + 4y = (x+1)e^{2x}$$

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Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Assume

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Assume

$$y_p = u_1(x) y_1(x) + u_2(x) y_2(x)$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Assume

$$y_p = u_1(x) y_1(x) + u_2(x) y_2(x)$$

$$u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}} = -x^2 - x$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

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$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

$$u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}} = -x^2 - x$$

$$u_2' = \frac{(x+1)e^{4x}}{e^{4x}} = x+1$$

Variation of Parameter

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$$u_2' = \frac{(x+1)e^{4x}}{e^{4x}} = x+1$$

$$u_1 = -\frac{1}{3}x^3 - \frac{1}{2}x^2$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Assume

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

$$u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}} = -x^2 - x$$

$$u_2' = \frac{(x+1)e^{4x}}{e^{4x}} = x+1$$

$$u_1 = -\frac{1}{3}x^3 - \frac{1}{2}x^2$$

$$u_2 = \frac{1}{2}x^2 + x$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Assume

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

$$u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}} = -x^2 - x$$

$$u_2' = \frac{(x+1)e^{4x}}{e^{4x}} = x+1$$

$$u_1 = -\frac{1}{3}x^3 - \frac{1}{2}x^2$$

$$u_2 = \frac{1}{2}x^2 + x$$

$$y_p = (-\frac{1}{3}x^2 - \frac{1}{2}x^2)e^{2x} + (\frac{1}{2}x^2 + x)xe^{2x}$$

$$= (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^{2x}$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Complementary function is

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Complementary function is

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Complementary function is

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Particular solution is

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Complementary function is

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Particular solution is

$$y_p = (-\frac{1}{3}x^2 - \frac{1}{2}x^2)e^{2x} + (\frac{1}{2}x^2 + x)xe^{2x}$$

$$= (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^{2x}$$

Variation of Parameter

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General Solution Using Variation of Parameter

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$$y'' - 4y' + 4y = (x+1)e^{2x}$$

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$$= (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^{2x}$$

then general Solution is

Variation of Parameter

☑ **Example 1**

General Solution Using Variation of Parameter

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Complementary function is

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

Particular solution is

$$y_p = (-\frac{1}{3}x^2 - \frac{1}{2}x^2)e^{2x} + (\frac{1}{2}x^2 + x)xe^{2x}$$

$$= (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^{2x}$$

then general Solution is

$$y = y_c + y_p$$

$$= c_1 e^{2x} + c_2 x e^{2x} (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^{2x}$$

Variation of Parameter

☑ **Example 2**

General Solution Using Variation of Parameter

Solve

$$y'' + 9y = \frac{1}{4}\csc 3x$$

Variation of Parameter

☑ **Example 2**

General Solution Using Variation of Parameter

Solve

$$y'' + 9y = \frac{1}{4}\csc 3x$$

$$m^2 + 9 = 0$$

$$m_1 = 3i, m_2 = -3i$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x$$

Assume

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

$$W(\cos 3x, \sin 3x)$$

$$= \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 9$$

$$W_1 = \begin{vmatrix} 0 & \sin 3x \\ \frac{1}{4}\csc 3x & 3\cos 3x \end{vmatrix} = -1$$

$$W_2 = \begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \frac{1}{4}\csc 3x \end{vmatrix} = \frac{1}{4}\frac{\cos 3x}{\sin 3x}$$

Variation of Parameter

☑ **Example 2**

General Solution Using Variation of Parameter

Solve

$$y'' + 9y = \frac{1}{4}\csc 3x$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x$$

Assume

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

Variation of Parameter

☑ **Example 2**

General Solution Using Variation of Parameter

Solve

$$y'' + 9y = \frac{1}{4}\csc 3x$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x$$

Assume

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

$$u_1' = \frac{-1/4}{3} = -\frac{1}{12}$$

Variation of Parameter

☑ **Example 2**

General Solution Using Variation of Parameter

Solve

$$y'' + 9y = \frac{1}{4}\csc 3x$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x$$

Assume

$$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$$

$$u_1' = \frac{-1/4}{3} = -\frac{1}{12}$$

$$u_2' = \frac{1}{12}\frac{\cos 3x}{\sin 3x}$$

Variation of Parameter

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Variation of Parameter

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$$y_p = -\frac{1}{12}x\cos 3x + \frac{1}{36}(\sin 3x)\ln|\sin 3x|$$

Variation of Parameter

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$$y'' + 9y = \frac{1}{4}\csc 3x$$

Variation of Parameter

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Complementary function is

Variation of Parameter

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Variation of Parameter

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then general Solution is

$$y = y_c + y_p$$

$$= c_1 e^{2x} + c_2 x e^{2x}\left(\frac{1}{6}x^3 + \frac{1}{2}x^2\right)e^{2x}$$

$$-\frac{1}{12} x \cos 3x + \frac{1}{36}(\sin 3x)\ln|\sin 3x|$$

Variation of Parameter

☑ **Example 3**

General Solution Using Variation of Parameter

Solve

$$y'' - y = \frac{1}{x}$$

Variation of Parameter

☑ **Example 3**

General Solution Using Variation of Parameter

Solve

$$y'' - y = \frac{1}{x}$$

$$m^2 - 1 = 0$$

$$m_1 = 1, m_2 = -1$$

Variation of Parameter

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$$y'' - y = \frac{1}{x}$$

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Variation of Parameter

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$$y_p = u_1(x) y_1(x) + u_2(x) y_2(x)$$

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$$y_p = u_1(x) y_1(x) + u_2(x) y_2(x)$$

$$W(e^x, e^{-x})$$

$$= \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -2$$

Variation of Parameter

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$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{1}{x} & -e^{-x} \end{vmatrix} = -\frac{e^{-x}}{x} = -\frac{1}{xe^x}$$

Variation of Parameter

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$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{1}{x} \end{vmatrix} = \frac{e^x}{x}$$

Variation of Parameter

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$$u_1 = \frac{1}{2}\int \frac{e^t}{t} dt$$

Variation of Parameter

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$$y_p = \frac{1}{2} e^x \int \frac{e^t}{t} dt - \frac{1}{2} e^{-x} \int \frac{e^{-t}}{t} dt$$

Variation of Parameter

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General Solution Using Variation of Parameter

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$$y'' - y = \frac{1}{x}$$

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Variation of Parameter

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$$y_c = c_1 e^{x} + c_2 e^{-x}$$

Particular solution is

Variation of Parameter

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General Solution Using Variation of Parameter

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then general Solution is

Variation of Parameter

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$$y_c = c_1 e^x + c_2 e^{-x}$$

Particular solution is

$$y_p = \frac{1}{2} e^x \int \frac{e^t}{t} dt - \frac{1}{2} e^{-x} \int \frac{e^{-t}}{t} dt$$

then general Solution is

$$\begin{aligned} y &= y_c + y_p \\ &= c_1 e^x + c_2 e^{-x} \\ &\quad - \frac{1}{2} e^x \int \frac{e^t}{t} dt - \frac{1}{2} e^{-x} \int \frac{e^{-t}}{t} dt \end{aligned}$$

Nonlinear Equations

Nonlinear Equations

☑ **Some Differences**

Nonlinear Equations

☑ **Some Differences**

- **Nonlinear Equation *don't posses Superposability***

Nonlinear Equations

☑ Some Differences

- **Nonlinear Equation *don't posses Superposability***

  Even though y_1 and y_2 **is a solution of nonlinear DE,**

  $y = c_1 y_1 + c_2 y_2$ **Is not a solution of nonlinear DEc**

Nonlinear Equations

☑ Some Differences

- **Nonlinear Equation *don't posses Superposability***

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- **Higher order nonlinear ODE is *difficult to solve analytically***

Nonlinear Equations

☑ Some Differences

- **Nonlinear Equation *don't posses Superposability***

  Even though y_1 and y_2 **is a solution of nonlinear DE,**

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- **Higher order nonlinear ODE is *difficult to solve analytically***

- ***Special* kind of nonlinear 2nd order ODE can be solved analytically by following method**

Nonlinear Equations

☑ **Reduction of Order**

Nonlinear Equations

☑ **Reduction of Order**

$$F(x, y', y'') = 0$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

Dependent variable y is missing

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

Dependent variable y is missing

$$ex) \quad y'' = 2x(y')^2$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0 \qquad\qquad F(y, y', y'') = 0$$

Dependent variable y is missing

$$ex)\ \ y'' = 2x(y')^2$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

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$$ex)\ \ y'' = 2x(y')^2$$

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independent variable x is missing

Nonlinear Equations

☑ Reduction of Order

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$$ex)\ \ y'' = 2x(y')^2$$

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independent variable x is missing

$$ex)\ \ yy'' = (y')^2$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

Dependent variable y is missing

$$ex)\ \ y'' = 2x(y')^2$$

$$F(y, y', y'') = 0$$

independent variable x is missing

$$ex)\ \ yy'' = (y')^2$$

When substitute

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

Dependent variable y is missing

$$ex)\ \ y'' = 2x(y')^2$$

$$F(y, y', y'') = 0$$

independent variable x is missing

$$ex)\ \ yy'' = (y')^2$$

When substitute $u = y'$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

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$$ex)\ \ y'' = 2x(y')^2$$

$$F(y, y', y'') = 0$$

independent variable x is missing

$$ex)\ \ yy'' = (y')^2$$

When substitute $u = y'$

$$F(x, u, u') = 0$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

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$$ex)\ \ y'' = 2x(y')^2$$

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$$ex)\ \ yy'' = (y')^2$$

When substitute $u = y'$

$$F(x, u, u') = 0$$

$$u' = 2xu^2$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

Dependent variable y is missing

$$ex)\ \ y'' = 2x(y')^2$$

$$F(y, y', y'') = 0$$

independent variable x is missing

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independent variable x is missing

$$ex)\ \ yy'' = (y')^2$$

When substitute $u = y'$

$$F(x, u, u') = 0$$

$$u' = 2xu^2$$

$$F(y, u, u') = 0$$

$$yu' = (u)^2$$

Nonlinear Equations

☑ Reduction of Order

$$F(x, y', y'') = 0$$

Dependent variable y is missing

$$ex)\ \ y'' = 2x(y')^2$$

$$F(y, y', y'') = 0$$

independent variable x is missing

$$ex)\ \ yy'' = (y')^2$$

When substitute $u = y'$

$$F(x, u, u') = 0$$

$$u' = 2xu^2$$

$$F(y, u, u') = 0$$

$$yu' = (u)^2$$

Changed as 1st order DE and can be Solved by separable variables

Nonlinear Equations

☑ **Example 1**

Dependent Variable y is missing

Solve

$$y'' = 2x(y')^2$$

Nonlinear Equations

☑ **Example 1**

Dependent Variable y is missing

Solve

$$y'' = 2x(y')^2$$

Let

Nonlinear Equations

☑ **Example 1**

Dependent Variable y is missing

Solve

$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

Nonlinear Equations

☑ **Example 1**

Dependent Variable y is missing

Solve

$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

$$du/dx = 2xu^2$$

Nonlinear Equations

☑ **Example 1**

Dependent Variable y is missing

Solve

$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

$$du/dx = 2xu^2$$

$$\frac{1}{u^2}du = 2xdx$$

Nonlinear Equations

☑ **Example 1**

Dependent Variable y is missing

Solve

$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

$$du/dx = 2xu^2$$

$$\frac{1}{u^2}du = 2xdx$$

$$-\frac{1}{u} = x^2 + c$$

Nonlinear Equations

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Dependent Variable y is missing

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$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

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For convenience, let

Nonlinear Equations

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Let $y' = u$ then, $du/dx = y''$

$$du/dx = 2xu^2$$

$$\frac{1}{u^2}du = 2xdx$$

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For convenience, let

$$-\frac{1}{u} = x^2 + c_1^{\ 2}$$

Nonlinear Equations

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$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

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$$\frac{1}{u^2}du = 2xdx$$

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For convenience, let

$$-\frac{1}{u} = x^2 + c_1^{\ 2}$$

$$\frac{dy}{dx} = -\frac{1}{x^2 + c_1^{\ 2}}$$

Nonlinear Equations

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$$y'' = 2x(y')^2$$

Let $y' = u$ then, $du/dx = y''$

$$du/dx = 2xu^2$$

$$\frac{1}{u^2}du = 2xdx$$

$$-\frac{1}{u} = x^2 + c$$

For convenience, let

$$-\frac{1}{u} = x^2 + c_1^{\ 2}$$

$$\frac{dy}{dx} = -\frac{1}{x^2 + c_1^{\ 2}}$$

$$y = -\int \frac{1}{x^2 + c_1^{\ 2}}dx$$

$$= -\frac{1}{c_1}\tan^{-1}\frac{x}{c_1} + c_2$$

Nonlinear Equations

☑ **Example 2**

Independent Variable x is missing

Solve

$$yy'' = (y')^2$$

Let

Nonlinear Equations

☑ **Example 2**

Independent Variable x is missing

Solve

$$yy'' = (y')^2$$

Let $y' = u$ then, $du/dx = y''$

Nonlinear Equations

☑ **Example 2**

Independent Variable x is missing

Solve

$$yy'' = (y')^2$$

Let $y' = u$ then, $du/dx = y''$

$$y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = \frac{du}{dy}y' = \frac{du}{dy}u$$

Nonlinear Equations

☑ **Example 2**

Independent Variable x is missing

Solve

$$yy'' = (y')^2$$

Let $y' = u$ then, $du/dx = y''$

$$y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = \frac{du}{dy}y' = \frac{du}{dy}u$$

$$yu\frac{du}{dy} = u^2$$

$$\frac{1}{u}du = \frac{1}{y}dy$$

Nonlinear Equations

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$$u = c_2 y$$

$$\frac{dy}{y} = c_2 dx$$

$$\ln|y| = c_2 x + c_3$$

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$$u = c_2 y$$

$$\frac{dy}{y} = c_2 dx$$

$$\ln|y| = c_2 x + c_3$$

$$y = c_4 e^{c_2 x}$$

Nonlinear Equations

☑ **Use of Taylor Series**

Nonlinear Equations

☑ Use of Taylor Series

$$y'' = F(x, y, y')$$

Nonlinear Equations

☑ Use of Taylor Series

$$y'' = F(x, y, y') \qquad y(x_0) = y_0, \quad y'(x_0) = y_1$$

Nonlinear Equations

☑ Use of Taylor Series

$$y'' = F(x, y, y') \qquad y(x_0) = y_0, \quad y'(x_0) = y_1$$

Taylor Expansion centered at x_0

Nonlinear Equations

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Taylor Expansion centered at x_0

$$\boxed{y(x) = y(x_0) + \frac{y'(x_0)}{1!}x + \frac{y''(x_0)}{2!}x^2 + \frac{y'''(x_0)}{3!}x^3 + \frac{y^{(4)}(x_0)}{4!}x^4 + \cdots}$$

Nonlinear Equations

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After substitute

Nonlinear Equations

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$$y'' = F(x, y, y') \qquad y(x_0) = y_0, \quad y'(x_0) = y_1$$

Taylor Expansion centered at x_0

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After substitute $y(x_0) = y_0, \quad y'(x_0) = y_1$

Nonlinear Equations

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After substitute $y(x_0) = y_0, \quad y'(x_0) = y_1$ **We could get**

Nonlinear Equations

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After substitute $y(x_0) = y_0, \quad y'(x_0) = y_1$ **We could get** $y''(x_0)$

Nonlinear Equations

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Higher derivatives are calculated by successive derivatives of DE

Nonlinear Equations

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Higher derivatives are calculated by successive derivatives of DE

$$y'''(x) = \frac{d}{dx}(y'') = \frac{d}{dx}(F(x, y, y'))$$

Nonlinear Equations

☑ Use of Taylor Series

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$$\cdots$$

$\Longrightarrow$ **Substitute**

Nonlinear Equations

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$$\cdots$$

$\Longrightarrow$ **Substitute** $x = x_0$

Nonlinear Equations

☑ Use of Taylor Series

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$$\cdots$$

$$\Longrightarrow \quad y'''(x_0)$$

Substitute

$$x = x_0$$

Nonlinear Equations

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$$\cdots$$

$$\xrightarrow[x = x_0]{\text{Substitute}}$$

$$y'''(x_0)$$

$$y^{(4)}(x_0)$$

Nonlinear Equations

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$$\cdots$$

$$\Longrightarrow$$

Substitute

$$x = x_0$$

$$y'''(x_0)$$

$$y^{(4)}(x_0)$$

$$\cdots$$

Nonlinear Equations

☑ **Example 3**

Taylor Series Solution of an IVP

Solve

$$y'' = x + y - y^2$$

$$y(0) = -1, \quad y'(0) = 1$$

Assume that solution exist

Nonlinear Equations

☑ **Example 3**

Taylor Series Solution of an IVP

Solve

$$y'' = x + y - y^2$$

$$y(0) = -1, \quad y'(0) = 1$$

Assume that solution exist

Taylor series expansion centered at 0 :

Nonlinear Equations

☑ Example 3

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$$y'''(x) = \frac{d}{dx}(x + y - y^2) = 1 + y' - 2yy'$$

Nonlinear Equations

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$$y^{(4)}(x) = \frac{d}{dx}(1 + y' - 2yy')$$

$$= y'' - 2(y')^2 - 2yy''$$

Nonlinear Equations

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$$= y'' - 2(y')^2 - 2yy''$$

$$y^{(5)}(x) = \frac{d}{dx}(y'' - 2(y')^2 - 2yy'')$$

$$= y^{(4)} - 6y'y'' - 2yy'''$$

Nonlinear Equations

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$$y(0) = -1, \; y'(0) = 1, \; y'''(0) = 4$$

Nonlinear Equations

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$$y^{(5)}(x) = \frac{d}{dx}(y'' - 2(y')^2 - 2yy'')$$

$$= y^{(4)} - 6y'y'' - 2yy'''$$

$$y(0) = -1, \; y'(0) = 1, \; y'''(0) = 4$$

$$y''(0) = -2, \; y^{(4)}(0) = -8$$

Nonlinear Equations

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$$y(0) = -1, \; y'(0) = 1, \; y'''(0) = 4$$

$$y''(0) = -2, \; y^{(4)}(0) = -8$$

$$y^{(5)}(0) = 24$$

Nonlinear Equations

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$$y(0) = -1, \; y'(0) = 1, \; y'''(0) = 4$$

$$y''(0) = -2, \; y^{(4)}(0) = -8$$

$$y^{(5)}(0) = 24$$

$$\therefore y(x) = -1 + x - x^2 + \frac{2}{3}x^3 - \frac{1}{3}x^4 + \frac{1}{5}x^5 \cdots$$

Nonlinear Equations

☑ Example 3

Taylor Series Solution of an IVP

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$$y(0) = -1, \; y'(0) = 1, \; y'''(0) = 4$$

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$$y^{(5)}(0) = 24$$

$$\therefore \boxed{y(x) = -1 + x - x^2 + \frac{2}{3}x^3 - \frac{1}{3}x^4 + \frac{1}{5}x^5 \cdots}$$

Nonlinear Equations

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$$= y^{(4)} - 6y'y'' - 2yy'''$$

$$y(0) = -1, \; y'(0) = 1, \; y'''(0) = 4$$

$$y''(0) = -2, \; y^{(4)}(0) = -8$$

$$y^{(5)}(0) = 24 \quad \textbf{(Near 0 !)}$$

$$\therefore \boxed{y(x) = -1 + x - x^2 + \frac{2}{3}x^3 - \frac{1}{3}x^4 + \frac{1}{5}x^5 \cdots}$$

A Numerical Method

☑ **approximating the solution**

A Numerical Method

☑ **approximating the solution**

$$y' = 0.1\sqrt{y} + 0.4x^2, y(2) = 4$$

A Numerical Method

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Nonlinear D.E.

A Numerical Method

☑ **approximating the solution**

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. $\longrightarrow$

A Numerical Method

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$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. $\longrightarrow$ **Analytically**

A Numerical Method

☑ **approximating the solution**

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. ——— $\boxed{\text{X}}$ → **Analytically**

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Nonlinear D.E. → ☒ → **Analytically**

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Nonlinear D.E. → ☒ → **Analytically**

↓

Qualitative

A Numerical Method

☑ **approximating the solution**

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → **Analytically**

↓

Qualitative

Direction Field

A Numerical Method

☑ **approximating the solution**

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → **Analytically**

↓ **Qualitative**

Direction Field

A Numerical Method

☑ approximating the solution

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Nonlinear D.E. → ☒ → Analytically

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Nonlinear D.E. → ☒ → Analytically

Qualitative

Direction Field

Numerically

Ex) Euler's Method

A Numerical Method

☑ approximating the solution

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → Analytically

Qualitative

Direction Field

Numerically

Ex) Euler's Method

$$y' = f(x, y),\ y(x_0) = y_0$$

A Numerical Method

☑ approximating the solution

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → Analytically

Qualitative

Direction Field

Numerically

Ex) Euler's Method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

A Numerical Method

☑ approximating the solution

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → Analytically

Qualitative

Direction Field

Numerically

Ex) Euler's Method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y(x_{n+1}) \approx y_{n+1} = L(x_{n+1})$$
$$= f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

A Numerical Method

☑ approximating the solution

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → Analytically

Qualitative

Direction Field

Numerically

Ex) Euler's Method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y(x_{n+1}) \approx y_{n+1} = L(x_{n+1})$$
$$= f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

A Numerical Method

☑ approximating the solution

$$y' = 0.1\sqrt{y} + 0.4x^2,\ y(2) = 4$$

Nonlinear D.E. → ☒ → Analytically

Qualitative

Direction Field

Numerically

Ex) Euler's Method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y(x_{n+1}) \approx y_{n+1} = L(x_{n+1})$$
$$= f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

Equation of the tangent line at x

A Numerical Method

A Numerical Method

☑ **Euler's method**

A Numerical Method

☑ **Euler's method**

$$y' = f(x, y),\ y(x_0) = y_0$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$y(x_0) = y_0$$

$$x_1 = x_0 + h$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

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$$y(x_1) \approx y_1 = L(x_1) = f(x_0, y_0)(x_1 - x_0) + y_0 = y_0 + hf(x_0, y_0)$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$x_1 = x_0 + h$$

$$y(x_0) = \boxed{y_0}$$

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A Numerical Method

☑ Euler's method

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$$x_2 = x_1 + h$$

$$y(x_2) \approx y_2 = L(x_2) = f(x_1, y_1)(x_2 - x_1) + y_1 = y_1 + hf(x_1, y_1)$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$x_1 = x_0 + h$$

$$x_2 = x_1 + h$$

$$y(x_0) = \boxed{y_0}$$

$$y(x_1) \approx \boxed{y_1} = L(x_1) = f(x_0, y_0)(x_1 - x_0) + y_0 = \boxed{y_0} + h\boxed{f(x_0, y_0)}$$

$$y(x_2) \approx y_2 = L(x_2) = f(x_1, y_1)(x_2 - x_1) + y_1 = \boxed{y_1} + h\boxed{f(x_1, y_1)}$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

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$$x_2 = x_1 + h$$

$$y(x_2) \approx y_2 = L(x_2) = f(x_1, y_1)(x_2 - x_1) + y_1 = \boxed{y_1} + h\boxed{f(x_1, y_1)}$$

$$x_3 = x_2 + h$$

$$y(x_3) \approx y_3 = L(x_3) = f(x_2, y_2)(x_3 - x_2) + y_2 = y_2 + hf(x_2, y_2)$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

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$$y(x_3) \approx y_3 = L(x_3) = f(x_2, y_2)(x_3 - x_2) + y_2 = \boxed{y_2} + h\boxed{f(x_2, y_2)}$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$x_1 = x_0 + h$$

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$$y(x_3) \approx y_3 = L(x_3) = f(x_2, y_2)(x_3 - x_2) + y_2 = y_2 + hf(x_2, y_2)$$

$$\vdots$$

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$x_1 = x_0 + h$$

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$$y(x_0) = y_0$$

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$$y(x_3) \approx y_3 = L(x_3) = f(x_2, y_2)(x_3 - x_2) + y_2 = y_2 + hf(x_2, y_2)$$

$$\vdots$$

Exact solution

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$x_1 = x_0 + h$$

$$x_2 = x_1 + h$$

$$x_3 = x_2 + h$$

$$y(x_0) = y_0$$

$$y(x_1) \approx y_1 = L(x_1) = f(x_0, y_0)(x_1 - x_0) + y_0 = y_0 + hf(x_0, y_0)$$

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$$y(x_3) \approx y_3 = L(x_3) = f(x_2, y_2)(x_3 - x_2) + y_2 = y_2 + hf(x_2, y_2)$$

$$\vdots$$

Exact solution **Approximation**

A Numerical Method

☑ Euler's method

$$y' = f(x, y),\ y(x_0) = y_0$$

Linearization of the unknown solution

$$y_{n+1} = L(x_{n+1}) = f(x_n, y_n)(x_{n+1} - x_n) + y_n$$

$$x_1 = x_0 + h$$

$$x_2 = x_1 + h$$

$$x_3 = x_2 + h$$

$$y(x_0) = y_0$$

$$y(x_1) \approx y_1 = L(x_1) = f(x_0, y_0)(x_1 - x_0) + y_0 = y_0 + hf(x_0, y_0)$$

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$$y(x_3) \approx y_3 = L(x_3) = f(x_2, y_2)(x_3 - x_2) + y_2 = y_2 + hf(x_2, y_2)$$

$$\vdots$$

Exact solution **Approximation**

Slope given at each point

A Numerical Method

☑ **Example 1**

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

A Numerical Method

☑ Example 1

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.1\sqrt{y_n} + 0.4x_n{}^2)$$

For h=0.1

n	x	y	f(x,y)
0	2	4	1.8
1	2.1	4.18	1.96845
2	2.2	4.376845	2.145209
3	2.3	4.591366	2.330275
4	2.4	4.824393	2.523645
5	2.5	5.076758	2.725317

$$y(2.5) \approx 5.076758$$

A Numerical Method

☑ **Example 1**

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.1\sqrt{y_n} + 0.4{x_n}^2)$$

A Numerical Method

☑ Example 1

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.1\sqrt{y_n} + 0.4x_n^{\ 2})$$

For h=0.05

n	x	y	f(x,y)
0	2	4	1.8
1	2.05	4.09	1.883237
2	2.1	4.184162	1.968552
3	2.15	4.282589	2.055944
4	2.2	4.385387	2.145413
5	2.25	4.492657	2.236959
6	2.3	4.604505	2.330581
7	2.35	4.721034	2.426279
8	2.4	4.842348	2.524053
9	2.45	4.968551	2.623902
10	2.5	5.099746	2.725826

$$y(2.5) \approx 5.099746$$

A Numerical Method

☑ **Example 1**

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.1\sqrt{y_n} + 0.4{x_n}^2)$$

A Numerical Method

☑ Example 1

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

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A Numerical Method

☑ Example 1

Euler's Method

Solve IVP

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.1\sqrt{y_n} + 0.4x_n^{\ 2})$$

For h=0.1 $y(2.5) \approx 5.076758$

For h=0.05 $y(2.5) \approx 5.099746$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

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A Numerical Method

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$$y = e^{0.1(x^2-1)}$$

For h=0.1

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

For h=0.1 $y(1.5) \approx 1.125878$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

For h=0.1 $y(1.5) \approx 1.125878$

For h=0.05

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

For h=0.1 $y(1.5) \approx 1.125878$

For h=0.05 $y(1.5) \approx 1.129467$

A Numerical Method

☑ Example 2

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

For h=0.1 $y(1.5) \approx 1.125878$

For h=0.05 $y(1.5) \approx 1.129467$

Real Solution

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

For h=0.1 $y(1.5) \approx 1.125878$

For h=0.05 $y(1.5) \approx 1.129467$

Real Solution $y(1.5) \approx 1.133148$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

<Percentage Relative error>

A Numerical Method

☑ **Example 2**

Comparison of Approximate and Exact Values

Solve IVP

$$y' = 0.2xy, \quad y(1) = 1$$

Use Euler's Method using h=0.1 and h=0.05

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(0.2x_n y_n)$$

$$y = e^{0.1(x^2-1)}$$

<Percentage Relative error>