Part A. Ordinary Differential Equations (ODE)

$\updownarrow$

Partial " " (PDE)

ODE : $f(x)$ $\qquad \frac{df}{dx} = c$

PDE : $f(x, y, z, t)$ $\qquad \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} + \frac{\partial f}{\partial t} = 0.$

Chap. 1. First-order Ordinary Differential Equations.

1.1. Basic Concepts.

① $y' = \cos x$ (1st)

② $y' + y = 0$ (1st)

③ $xy' + y^2 = 0$ (1st)

④ $y'' + y = 0$ (2nd)

General form of 1st-order DE

$$F(x, y, y') = 0$$

OR $\quad y' = f(x, y)$

Example of problem solving

~~Problem~~: Ex.5

radioactive substance ${}_{88}Ra^{226}$

$m_0 = \cancel{1}\,0.5\,g$

Find : m. at $t (= 10^3\,yr.)$

(1) Modeling :

* based on understanding of physics

(decomposing "rate") $\propto$ (present amount) $\leftarrow$ fluid, heat, particle physics, EM. ..., biophysics

theoretician (experimentalist)

mass $= y$

$$\frac{dy}{dt} = ky \qquad (k<0). \qquad y(0) = 0.5$$

"experimentalist"

$$\boxed{k = -1.4\times 10^{-11}\ \text{sec}^{-1}}$$

(2) Solving :

$y(t) = ce^{kt}$: General solution

~~(3) Determination of a~~ particular solution

* condition given in the problem: $y(t=0) = \cancel{2}\,0.5$

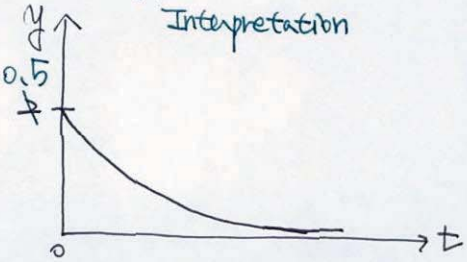
Initial condition

$y(0) = c = \cancel{2}\,0.5$

$\therefore \boxed{y(t) = \overset{0.5}{\cancel{2}}e^{kt}}$

Particular solution to the DE.

(3) Interpretation



~~(4)~~ checking :

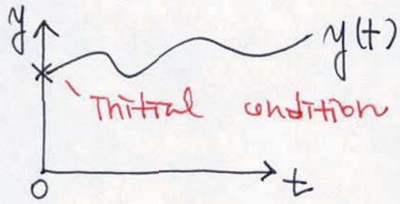
(i) DE ? $\frac{d}{dt}(\overset{y}{\frac{1}{2}e^{kt}}) = \frac{1}{2}ke^{kt} = k(\overset{y}{\frac{1}{2}e^{kt}})$

(ii) IC ? $y(0) = \frac{1}{2}e^{k(0)} = \frac{1}{2}$.

~~In 10^3 year?~~

$\cancel{y(t = 365\times 24\times 3600 \times 1000) = 2e^{(-1.4\times10^{-11})(3.2\times10^{10})} = 1.3g}$ //

Foregoing example



$$\boxed{\frac{dy}{dt} = f(t, y) \quad . \quad y(t_0) = y_0}$$

$\Rightarrow$ INITIAL VALUE PROBLEM (IVP)

More problems

1. $y' = x^2$

$\frac{dy}{dx} = x^2$. $\quad dy = x^2 dx \quad \int dy = \int x^2 dx$

$\therefore y = \frac{1}{3}x^3 + c \quad$: G.S.

2. $y' = y \tan x \quad . \quad y(0) = \frac{\pi}{2} \quad$: IVP

$\frac{dy}{dx} = y \tan x. \quad \int \frac{dy}{y} = \int \tan x \, dx$

$\ln y = -\ln \cos x = \ln \sec x + c^*$

$y = e^{\ln \sec x} e^{c^*} = c \sec x \quad$: G.S.

$y(0) = c = \frac{\pi}{2}$

$\therefore y = \frac{\pi}{2} \sec x \quad$: P.S. ✓

3. Practice : problem 24 in p.9

X Problem 24. (optional)

air pressure : y

height : x

· physical info. (governing eq.) : $\frac{dy}{dx} = ky$

· condition : ① $y(x=18{,}000\,ft.) = \frac{1}{2}y(x=0)$. ② $y(x=0) = 1\,atm$

sol.) $y = ce^{kx}$

①: $y(x=18000) = ce^{18000k} = \frac{1}{2}c$ ②: $c = 1$

$$\exp(18{,}000k) = \frac{1}{2}$$

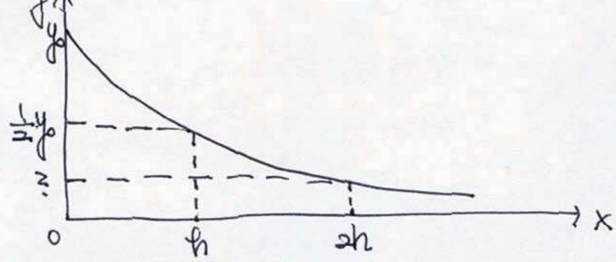
$$k = \frac{1}{18000}\ln\frac{1}{2}$$

$$y(x=35000\,ft.) = c\exp\left(\frac{35000}{18000}\ln\frac{1}{2}\right) = c\exp(-1.35)$$

$$= 0.259\,c$$

$$\therefore\ y = 0.259\ atm$$

graphically

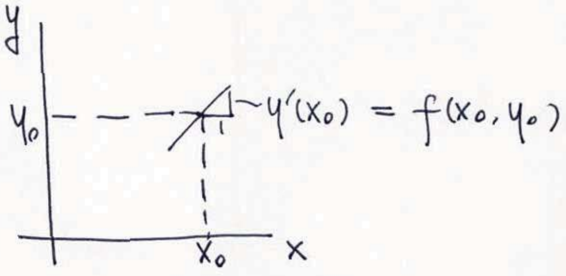


$$y = ce^{k(2h)} = ce^{k(h+h)} = c\,e^{kh}\cdot e^{kh}$$

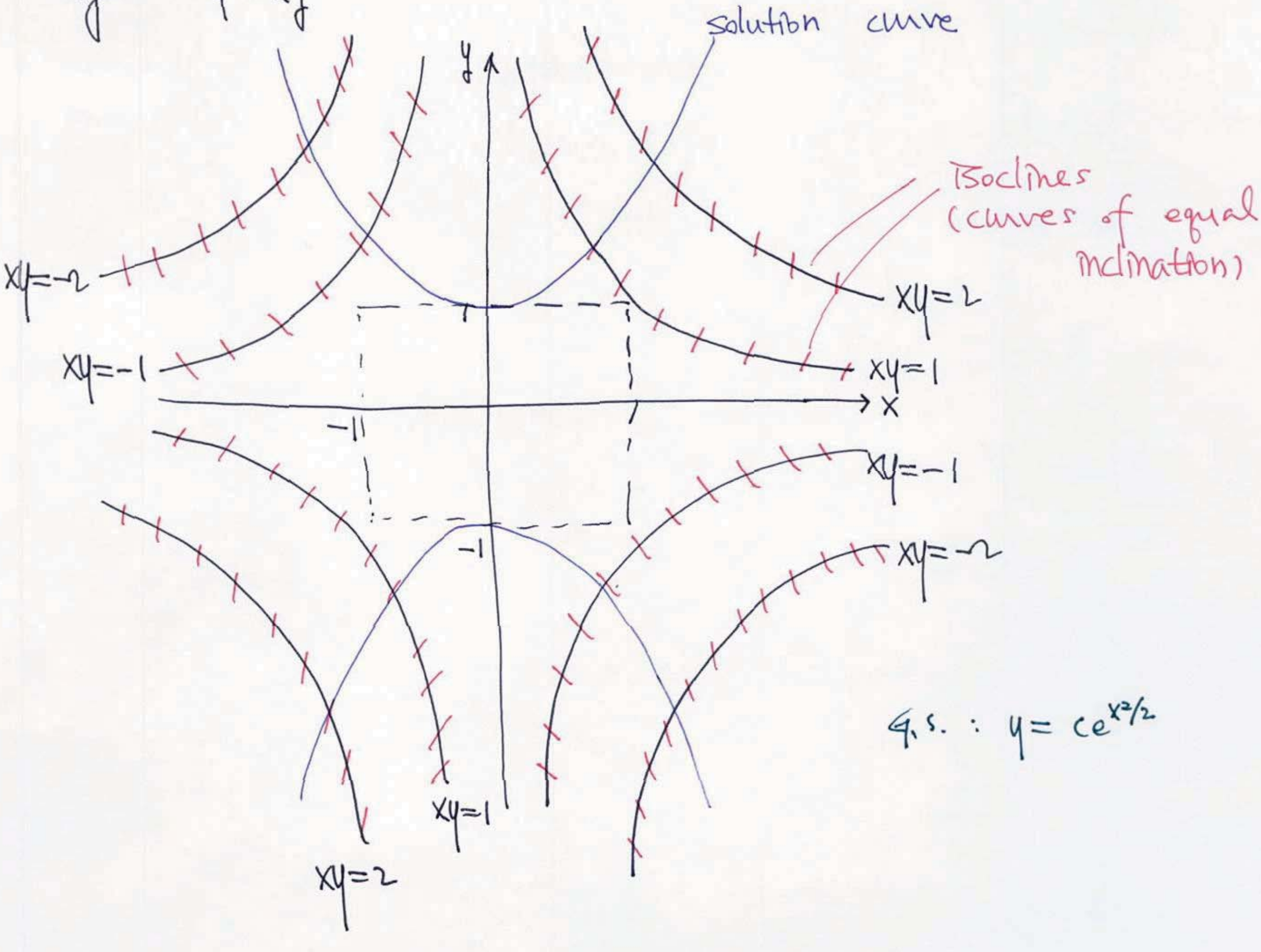
$$= y_0\cdot \underbrace{e^{kh}}_{=\frac{1}{2}}\cdot\underbrace{e^{kh}}_{=\frac{1}{2}} = \frac{1}{4}y_0$$: exponential decay.

1.2. Geometric meaning of $y' = f(x, y)$

- $y' = f(x, y)$: slope of $y(x)$



e.g. $y' = xy$



G.S. : $y = ce^{x^2/2}$

- < Direction field >

* useful for complicated solutions