$t > 2 \qquad y = \int_0^t \sim dt = \int_1^2 g(t-\tau)\, r(\tau)\, d\tau$

$= e^{-(t-2)} - e^{-(t-1)} - \frac{1}{2}\left[e^{-2(t-2)} - e^{-2(t-1)} \right]$

* Integral Eq.

Ex. ~~5~~ 6. $\quad y(t) = t + \int_0^t y(\tau) \sin(t-\tau)\, d\tau$

$y = t + (y * \sin t)$

$Y = \frac{1}{s^2} + Y \cdot \frac{1}{s^2+1}$

$Y\left(1 - \frac{1}{s^2+1}\right) = \frac{1}{s^2}$

$Y = \frac{s^2+1}{s^4} = \frac{1}{s^2} + \frac{1}{s^4}$

$= \frac{1}{s^2} + \frac{3!}{s^4} \cdot \frac{1}{3!}$

$y = \mathcal{L}^{-1}(Y) = t + \frac{1}{6} t^3$ $\rightarrow$ p.153

PS 5.5 # 7, 17, 25, 31

p.153

§ ~~5.6.~~ Partial Fractions

Case 1. unrepeated factor $(s-a)$

Ex. 1. $\quad y'' + y' - 6y = 1. \qquad y(0) = 0. \quad y'(0) = 1$

$Y(s) = \frac{s+1}{s(s-2)(s+3)} = \frac{A_1}{s} + \frac{A_2}{s-2} + \frac{A_3}{s+3}$

$A_1 = -\frac{1}{6}, \quad A_2 = \frac{3}{10}, \quad A_3 = -\frac{2}{15}$

Case 2. Repeated factor $(s-a)^m$

Ex. 2 $$Y(s) = \frac{s^3-4s^2+4}{s^2(s-2)(s-1)} = \underbrace{\frac{A_2}{s^2} + \frac{A_1}{s}} + \frac{B}{s-2} + \frac{C}{s-1}$$

$$s^3-4s^2+4 = A_2(s^2-3s+2) + A_1(s^3-3s^2+2s) + B(s^3-s^2) + C(s^3-2s^2)$$

$$= (A_1+B+C)s^3 + (A_2-3A_1-B-2C)s^2$$

$$+ (-3A_2+2A_1)s + 2A_2$$

$2A_2 = 4$, $\quad A_2 = 2$

$2A_1 = 3A_2$. $\quad A_1 = 3$

$2-9-B-2C = -4$.

$\quad B+2C = -3$

$3+B+C = 1$

$\quad B+C = -2$

$\quad C = -1$

$\quad B = -1$. $\qquad y = 2t+3-e^{2t}-e^{t}$

Case 3. Unrepeated Complex factors $(s-a)(s-\bar{a})$

Ex. 3. $$\frac{20}{(s^2+4)(s^2+2s+2)} = \frac{As+B}{s^2+4} + \frac{Ms+N}{s^2+2s+2}$$

~sin2t

$$= \frac{-2s-2}{s^2+4} + \underbrace{\frac{2s+6}{s^2+2s+2}}_{\parallel}$$

~cos2t

$$\frac{2(s+1)+4}{(s+1)^2+1}$$ → ~$e^{-t}\sin t$

~$e^{-t}\cos t$

Case 4 Repeated Complex Factors

$$\frac{f(s)}{[(s-a)(s-\bar{a})]^2} = \frac{As+B}{[(s-a)(s-\bar{a})]^2} + \frac{Ms+N}{(s-a)(s-\bar{a})}$$ → p.156

PS 5.6. # 1, 7, 11

→ p.156

~~5~~ 6.7. Systems of ~~Differential Eq~~ ODEs

$$\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + g_1(t) \\ y_2' = a_{21}y_1 + a_{22}y_2 + g_2(t) \end{cases}$$

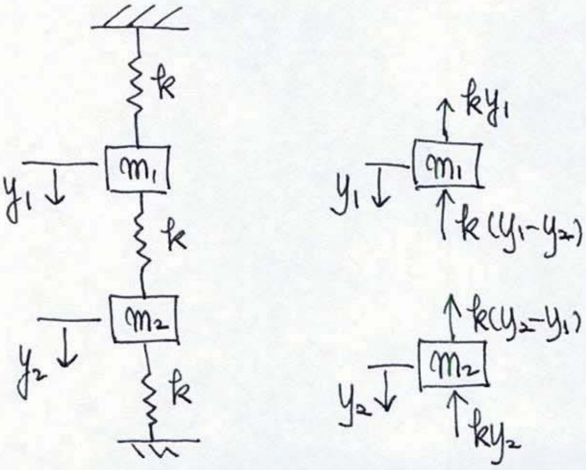
L.T.

$$\begin{cases} sY_1 - y_1(0) = a_{11}Y_1 + a_{12}Y_2 + G_1 \\ sY_2 - y_2(0) = a_{21}Y_1 + a_{22}Y_2 + G_2 \end{cases}$$

$$\left.\begin{array}{l} (a_{11}-s)Y_1 + a_{12}Y_2 = -y_1(0) - G_1(s) \\ a_{21}Y_1 + (a_{22}-s)Y_2 = -y_2(0) - G_2(s) \end{array}\right\} \text{algebraic eq.}$$

Ex. 1 / Ex. 2.

Ex. 3. Model of two masses on springs



$\downarrow\ m_1y_1'' = -ky_1 - k(y_1-y_2)$

$\cdot\ m_1y_1'' = -2ky_1 + ky_2$

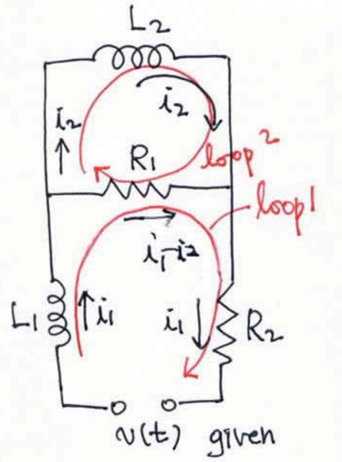
$\downarrow\ m_2y_2'' = -k(y_2-y_1) - ky_2$

$\cdot\ m_2y_2'' = ky_1 - 2ky_2$

$$\begin{cases} m_1y_1'' = -2ky_1 + ky_2 \\ m_2y_2'' = ky_1 - 2ky_2 \end{cases}$$

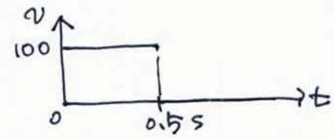
PS. 5.7. # 5, 13,

Ex. 2. Electrical network



$i_1(t)$, $i_2(t)$ = ?

$i(0) = i'(0) = 0$.



$v(t) = 100\,[u(0) - u(t-0.5)]$

Kirchhoff's voltage law

$$L_1 i_1' + R_1(i_1 - i_2) + i_1 R_2 = v(t) \qquad : \text{loop 1}$$

$$L_2 i_2' + R_1(i_2 - i_1) = 0. \qquad : \text{loop 2}$$

6.8

Def. $\mathcal{L}(f) = \int_0^\infty e^{-st} f(t)\, dt$. Linear: $\mathcal{L}(af+bg) = a\mathcal{L}(f) + b\mathcal{L}(g)$

Shifting $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

$e^{-as} F(s) = \mathcal{L}\{f(t-a)\, u(t-a)\}$

Diff. $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$

$\frac{d}{ds} F(s) = -\mathcal{L}\{t f(t)\}$

Integ. $\mathcal{L}\{\int_0^t f(\tau)\, d\tau\} = \frac{1}{s}\mathcal{L}(f)$

$\int_s^\infty F(\tilde{s})\, d\tilde{s} = \int_s^\infty \mathcal{L}(f)\, d\tilde{s} = \mathcal{L}\{\frac{f(t)}{t}\}$

Convolution. $H = F \cdot G$

$\mathcal{L}(h) = \mathcal{L}(f)\, \mathcal{L}(g)$. $h = f * g$

$\mathcal{L}(f*g) = \mathcal{L}(f)\, \mathcal{L}(g)$

$f*g = \int_0^t f(t-\tau)\, g(\tau)\, d\tau = g*f$

Periodic fn. $f(t+p) = f(t)$

$\mathcal{L}(f) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)\, dt$

6.9. Table of LT's