

Scope of the course

- Mainly liquid flows, occasionally treating gas flows.
rarefied g.D. \leftarrow
- Characteristic flow dimension:
 $0(10\text{nm}) \leq L \leq 0(100\mu\text{m})$
- Major focus on theoretical understanding
not on the survey of new technologies

~~Concepts of micro flows~~

CASES at "L"

- (1) Macroscale description of fluid flow still valid
~~continuum~~
- $\left. \begin{array}{l} \text{mass conservation} \\ \text{momentum conservation} \\ \text{energy conservation} \end{array} \right\} \xrightarrow[\text{fluid}]{\text{Newtonian}} \text{Navier-Stokes eq.}$
- \Rightarrow most liquid flows
- $\left(\begin{array}{l} + \text{electromagnetic} \\ \text{terms} \\ + \text{surface tension} \\ \text{terms} \end{array} \right)$
- (2) Macroscale hypothesis breaks down
~~continuum~~
- \Rightarrow many gas flows

References: IJHMT vol. 46, 935-937 (2003)
 " " , 3941-3945 (2003)

* What are the micro-scale effects ?

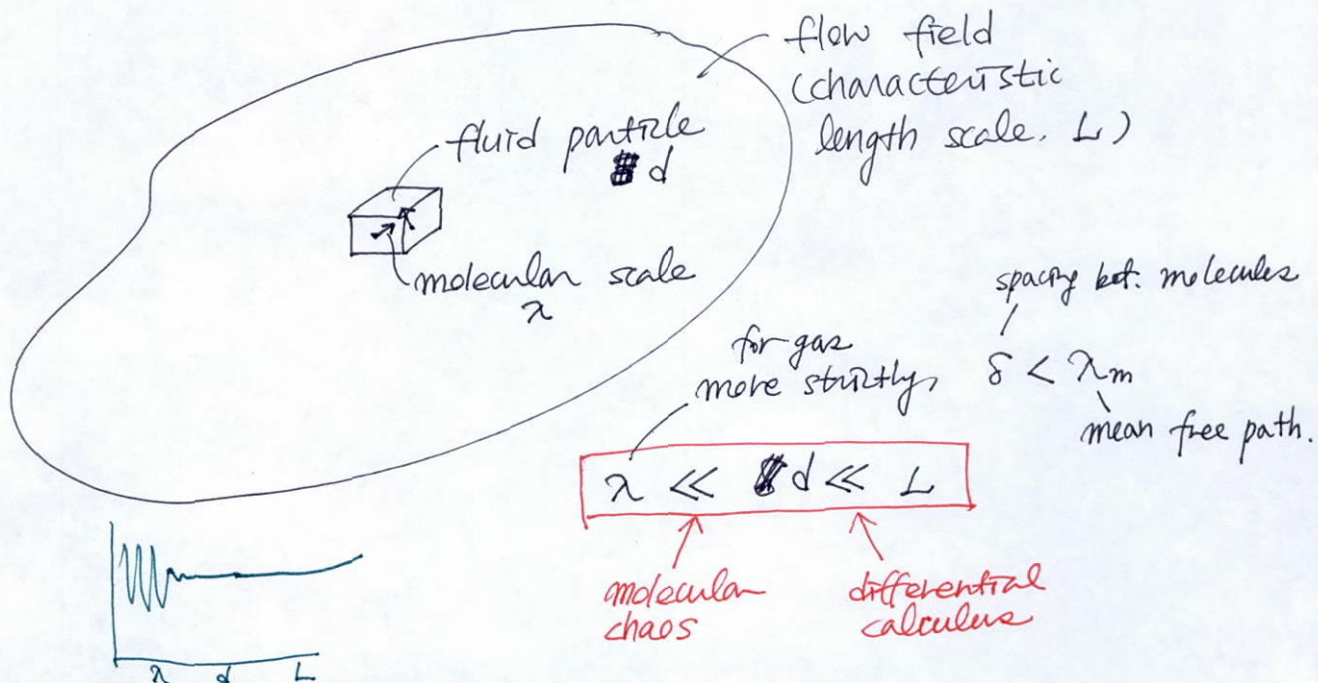
① Non-continuum behavior

② ~~Abnormal constitutive relations~~ (Non-equilibrium)

1. Non-continuum behavior.

Continuum hypothesis :

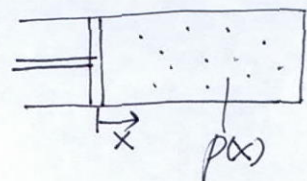
The spatial and temporal derivatives of all the macroscopic dependent variables (density, velocity, stress, heat flux) are defined as averages over elements exist in some reasonable sense.



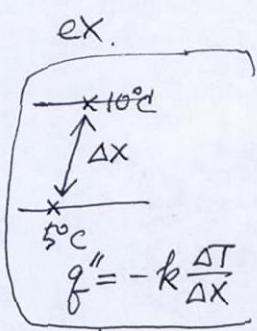
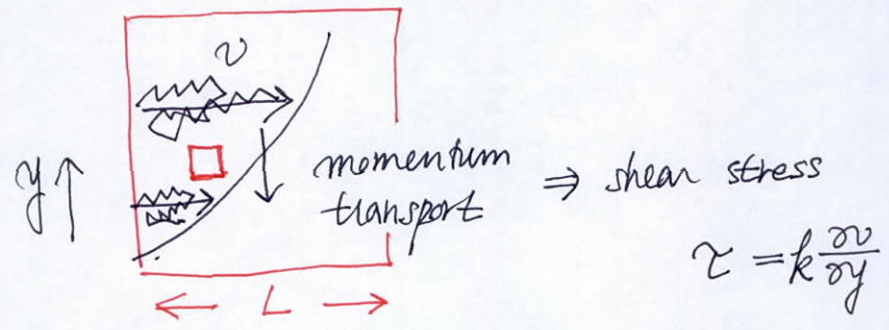
2. Non-thermodynamic equilibrium

- Thermodynamic equilibrium

$\tau_{\text{characteristic flow}} \gg \tau_{\text{molecular mixing}}$



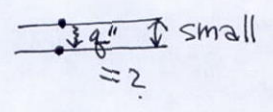
- process to equilibrium (transport phenomena)
 - diffusion of matter
 - conduction of heat
 - momentum transport (internal friction) : shearing



if $L \downarrow$ $\tau = k_1 \frac{dv}{dy} + k_2 \frac{d^2v}{dy^2} + \dots$

OR new form / new physics

what if

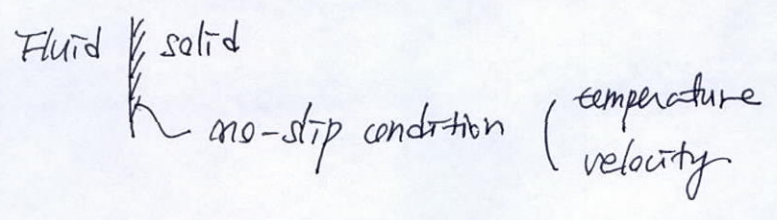


In macroscopic modeling
constitutive relations

- for matter : Fick's law
- heat : Fourier's law
- momentum : Newtonian fluid

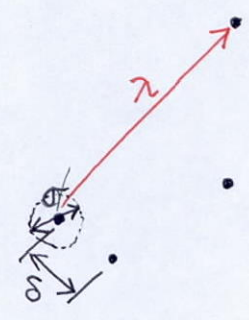
$$L \gg \lambda$$

- Boundary conditions (equilibrium with boundary)



Molecular scales for gases

- diameter of a molecule, σ
 $\sigma \approx 3-4 \times 10^{-10} \text{ m} \sim 0.1 \text{ nm}$
 1 \AA .



- average separation of molecules, δ

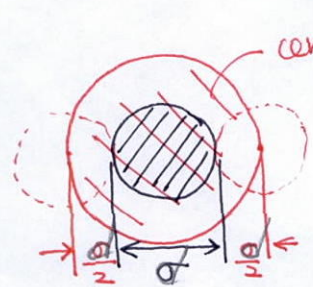
$T = 0^\circ\text{C}, P = 1 \text{ atm}$

Avogadro's law : $N = 2.69 \times 10^{19} / \text{cm}^3$

$\delta \approx 3.3 \times 10^{-9} \text{ m} = 3.3 \text{ nm} = 33 \text{ \AA} \sim 10 \sigma$
 $(= \sqrt[3]{10^{27}/N})$

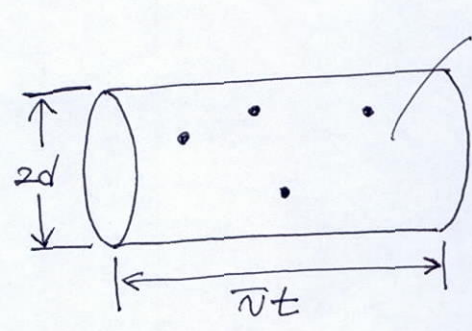
- Mean free path

\equiv average distance that a molecule travels between successive collisions.



center of neighboring molecule

effective collision area = πd^2



number density n
 (# molecules / vol)

$\lambda_{\text{app}} = \frac{\text{length of the path}}{\# \text{ of collisions}} = \frac{\bar{v}t}{\pi d^2 \bar{v}t n} = \frac{1}{\pi d^2 n}$

target molecules are at rest

\Rightarrow average relative velocity

$\bar{v}_{\text{rel}} = \sqrt{2} \bar{v}$

$\therefore \lambda = \frac{\bar{v}t}{\pi d^2 \sqrt{2} \bar{v}t n} = \frac{1}{\sqrt{2} \pi d^2 n}$

effective volume swept out in time t

Mean free path of ideal gas

$$n = \frac{\hat{n} N_A}{V} \quad [\hat{n}] = \text{mole}/\text{m}^3$$

$$= \frac{\hat{n} N_A}{\hat{n} RT/p} = \frac{N_A \cdot p}{RT}$$

$$\therefore \lambda = \frac{RT}{\sqrt{2} d^2 N_A p} \quad \left(\begin{array}{l} \downarrow \text{ as } p \uparrow, T = \text{const} \\ \uparrow \text{ as } T \uparrow, p = \text{const} \end{array} \right.$$

$$T = 0^\circ\text{C}, \quad p = 1 \text{ atm}$$

$$\lambda = 93 \text{ nm} \sim 300 \sigma \\ \sim 30 \delta$$

→ next page Kn

Molecular scale for liquids

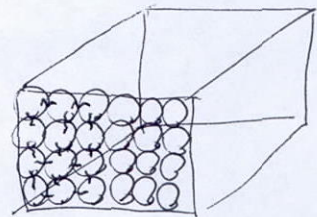
$$\frac{d^3}{\delta^3} \sim \frac{\hat{M}}{\rho N_A} \quad \hat{M}: \text{molecular weight (kg/kmol)}$$

$$\text{water: } \hat{M} = 18 \text{ kg/kmol}, \quad \rho = 10^3 \text{ kg/m}^3$$

$$N_A = 6.02252 \times 10^{23} / \text{mole}$$

$$\sigma \sim 3 \times 10^{-10} \text{ m} = 0.3 \text{ nm} = 3 \text{ \AA}.$$

(δ)



see handout. (1p)
MEMS hb.

Non-continuum flow regimes

L : characteristic length scale of boundaries.

$$Kn = \frac{\lambda}{L} \quad : \quad \text{Knudsen number}$$

$Kn > 10$: free molecule flow

$10 > Kn > 0.1$: transition

$0.1 > Kn > 0.01$: slip

$Kn < 0.01$: continuum \uparrow Lecture 1

Boundary conditions at solid / fluid interfaces

* No-slip B.C. : only a hypothesis

* Navier's slip B.C. (Gas-dynamic eq.)

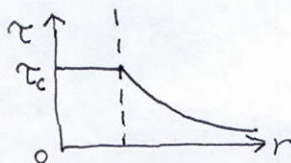
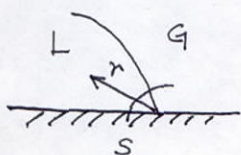
$$u_s = \frac{\beta \tau}{\mu}$$

u_s : slip velocity. τ : surface shear stress

β : slip length μ : viscosity

OR :
$$u_s = \left(\beta \frac{\partial u}{\partial y} \right)_{y=0}$$

* Yield stress condition (contact-line dynamics)



No-slip & Slip (liquids)

* No-slip B.C. : macroscopic scale
microscale flow with hydrophilic wall

* Slip B.C. : microscale flow with hydrophobic wall
high shear rate

Ref.) . Trethewey & Menhart, "Apparent fluid slip
at hydrophobic microchannel walls," Phys. Fluids 14,
L9 (2002)

• Choi et al. Phys. Fluids 15. 2897 (2003)

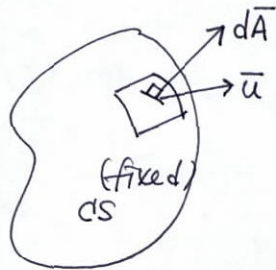
• Arkilic, Schmidt, Breuer, J. MEMS. 6. 167. (1997)

- Generating mechanism : nanobubbles ?

Basic conservation equations

(quick, informal)

§1. Mass conservation



$$\int_S \rho \bar{u} \cdot d\bar{A} = - \int_V \frac{\partial \rho}{\partial t} dV$$

Gauss's theorem, $\int_S \rho \bar{u} \cdot d\bar{A} = \int_V \nabla \cdot \rho \bar{u} dV$

$$\therefore \int_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \bar{u} \right) dV = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \bar{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \bar{u} + (\bar{u} \cdot \nabla) \rho = 0$$

$$\boxed{\frac{D\rho}{Dt} + \rho \nabla \cdot \bar{u} = 0}$$

for incompressible flow, $\rho = \text{const}$ ($\frac{D\rho}{Dt} = 0$)

$$\boxed{\nabla \cdot \bar{u} = 0}$$

§2. Momentum conservation

Newton's second law applied to a fluid particle

$$\int \rho \frac{D\bar{u}}{Dt} = \bar{F}_{\text{body}} + \bar{F}_{\text{surface}}$$

(gravitational
electrical
electromagnetic)

for now $\bar{f}_b = \rho \bar{g}$.

$$\bar{f}_s = \nabla \cdot \bar{T} = \frac{\partial \tau_{ij}}{\partial x_j}$$

} dyadic

$$\rho \frac{D\bar{u}}{Dt} = \nabla \cdot \bar{T} + \rho \bar{g}$$

Newtonian fluid: $\tau_{ij} = -p \delta_{ij} + 2\mu \left(\epsilon_{ij} - \frac{1}{3} \epsilon_{kk} \delta_{ij} \right)$

where $\epsilon_{kk} = \frac{\partial u_k}{\partial x_k} = \nabla \cdot \bar{u}$
 $\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$

In Cartesian tensor notation,

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left\{ \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right\} + \rho g_i$$

Navier-Stokes eq.

⊙ for const.-density flows $\epsilon_{kk} = 0$.

$$\tau_{xx} = -p + 2\mu \frac{\partial u}{\partial x}$$

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

⋮

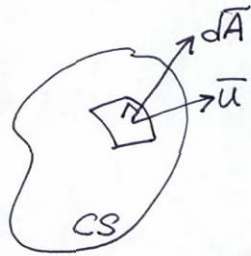
⊖ $\mu = \text{const}$

then we get

$$\rho \frac{D\bar{u}}{Dt} = -\nabla p + \mu \nabla^2 \bar{u} + \rho \bar{g}$$

N-S. eq for const. density & const. ~~visc~~ flows

§ 3. Energy conservation



$$\int_S (\boldsymbol{\pi} \cdot \vec{u}) \cdot d\vec{A} - \int_S \vec{q} \cdot d\vec{A} + \int_V \rho \vec{g} \cdot \vec{u} dV$$

$$= \int_V \rho \left[\frac{D}{Dt} \left(\frac{1}{2} |\vec{u}|^2 + e \right) \right] dV$$

Gauss theorem.

$$\rho \frac{D}{Dt} \left(e + \frac{1}{2} |\vec{u}|^2 \right) = -\nabla \cdot \vec{q} + \rho \vec{g} \cdot \vec{u} + \nabla \cdot (\boldsymbol{\pi} \cdot \vec{u})$$

OR

$$\rho \frac{D}{Dt} \left(e + \frac{1}{2} u_i u_i \right) = -\frac{\partial q_i}{\partial x_i} + \rho g_i u_i + \frac{\partial}{\partial x_j} \tau_{ij} u_i$$

for Newtonian.

Fourier's law ($\vec{q} = -k \nabla T$)

$$\rho \frac{De}{Dt} = -\rho \frac{\partial u_i}{\partial x_i} + \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right) + \Phi$$

where Φ is the dissipation function

$$\Phi = 2\mu \left(\epsilon_{ij} \epsilon_{ij} - \frac{1}{3} \epsilon_{ii}^2 \right) > 0$$

for incompressible flows

$$\rho \frac{De}{Dt} = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right) + \Phi$$

- $de \approx c_p dT$. • incompressible $\Rightarrow c_p \approx c_v = c$.
- $k = \text{const.}$

$$\Rightarrow \rho c \frac{DT}{Dt} = k \nabla^2 T + \Phi$$

for $\frac{|\Phi|}{|k\nabla^2 T|} \ll 1,$

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

$$\alpha = \frac{k}{\rho c} : \text{thermal diffusivity}$$

§4. Species conservation

← for multicomponent fluid

continuity eq for i -th species

$$\int_S \rho_i \bar{u}_i \cdot d\bar{A} = - \int_V \frac{\partial \rho_i}{\partial t} dV + \int_V r_i dV$$

} species production
by chemical reaction
[kg/m³.s]

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \bar{u}_i = r_i$$

Introducing mass average velocity: \bar{u}

$$\rho \bar{u} = \sum \rho_i \bar{u}_i$$

- mass flux of i -th species w.r.t. \bar{u}

$$\bar{J}_i = \rho_i (\bar{u}_i - \bar{u})$$

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \bar{u} = - \nabla \cdot \bar{J}_i + r_i$$

mass diffusivity
[m²/s]

for binary system,

$$\bar{J}_i = -\rho D_{12} \nabla w_i$$

$$= -D_{12} \nabla \rho_i$$

$$w_i = \frac{\rho_i}{\rho}$$

$$\frac{\partial p_i}{\partial t} + \nabla \cdot \rho \bar{u} = \nabla \cdot (\underbrace{\rho D_{12}}_{D_{12}} \nabla w_i) + r_i$$

for dilute liquid solutions, $\nabla \cdot \bar{u} = 0$

$$\frac{\partial p_i}{\partial t} + \bar{u} \cdot \nabla p_i = D_{12} \nabla^2 p_i + r_i$$

dividing by M_i (molar mass)

$$c_i = \frac{p_i}{M_i} = \frac{\text{no. of moles of species } i}{\text{volume of solution}} \left[\frac{\text{mol}}{\text{m}^3} \right]$$

$$\frac{\partial c_i}{\partial t} + \bar{u} \cdot \nabla c_i = D_{12} \nabla^2 c_i + R_i$$

→ convective diffusion eq

$$\boxed{\frac{Dc}{Dt} = D \nabla^2 c}$$

compare with energy eq

$$\frac{DT}{Dt} = \alpha \nabla^2 T$$

Nondimensional parameters

$$\text{Momentum: } Re = \frac{|\rho \frac{D\bar{u}}{Dt}|}{|\mu \nabla^2 \bar{u}|} = \frac{\rho U^2 / L}{\mu U / L^2} = \frac{UL}{\nu}$$

= $\frac{\text{momentum transported by convection}}{\text{viscous diffusion}}$

$$\text{Energy: } Pe_T = \frac{|\frac{DT}{Dt}|}{|\alpha \nabla^2 T|} = \frac{UT/L}{\alpha T/L^2} = \frac{UL}{\alpha} = \frac{\text{heat transp. conv.}}{\text{cond.}}$$

thermal Peclet number

$$\text{Species: } Pe_D = \frac{|\frac{Dc}{Dt}|}{|D \nabla^2 c|} = \frac{UL}{D} = \frac{\text{mass transp. conv.}}{\text{diff.}}$$

diffusion Peclet number

* Writing Pe in terms of Re and fluid properties

$$Pe_T = \frac{UL}{\nu} \cdot \frac{\nu}{\alpha} = Re \cdot Pr \quad (Pr = \frac{\nu}{\alpha})$$

Prandtl

$$Pe_D = \frac{UL}{\nu} \frac{\nu}{D} = Re \cdot Sc \quad (Sc = \frac{\nu}{D})$$

Schmidt

· Lewis number = $Le = \frac{\alpha}{D}$