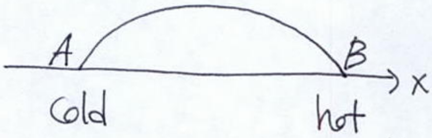
Motion of droplets induced by thermal gradient.

- Driving force



$$\frac{dT}{dx} > 0.$$

$$\frac{dS}{dx} > 0. \quad \frac{d\gamma}{dx} < 0.$$

$$f_d = -\frac{dU}{dX}$$

$$= +(\gamma_{SG} - \gamma_{SL})_B - (\gamma_{SG} - \gamma_{SL})_A$$

$$= +\gamma_B \cos\theta_{e,B} - \gamma_A \cos\theta_{e,A}$$

$$= (S+\gamma)_B - (S+\gamma)_A$$

$$= \frac{d}{dx}(S+\gamma) \cdot l$$

$$= (S' + \gamma')\, l$$

- Resisting force

thin drop ($h \ll L$): $\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$

B.C. $\begin{cases} y=0: & u=0 \\ y=h: & \mu \frac{\partial u}{\partial y} = \frac{d\gamma}{dx} \end{cases}$

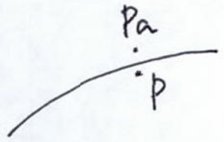
$$\mu \frac{\partial u}{\partial y} = \frac{dp}{dx} y + c_1$$

$$\mu u = \frac{1}{2}\frac{dp}{dx} y^2 + c_1 y + \cancel{c_2}^{\,0}$$

- $\mu \frac{\partial u}{\partial y}\Big|_{y=h} = \frac{d\gamma}{dx} = \frac{dp}{dx} h + c_1$

$$c_1 = -\frac{dp}{dx} h + \frac{d\gamma}{dx}$$

$$\mu u = \frac{1}{2}\frac{dp}{dx}y^2 + \left(-\frac{dp}{dx}h + \frac{d\gamma}{dx}\right)y$$



$$p = \rho g(h-y) + p_a + \gamma \cdot \kappa$$

$$\kappa \approx -\frac{\partial^2 h}{\partial x^2}$$

$$\cdot\ \frac{dp}{dx} = \rho g h' - \gamma h'''$$

$$\cdot\ \frac{d\gamma}{dx} = \frac{d\gamma}{dT}\frac{dT}{dx}$$

$$Q = hV = \int_0^h u\,dy$$

$$= \int_0^h \frac{1}{\mu}\left[\frac{1}{2}\frac{dp}{dx}y^2 + \left(-\frac{dp}{dx}h + \frac{d\gamma}{dx}\right)y\right]dy$$

$$= \frac{1}{\mu}\left(\frac{1}{6}\frac{dp}{dx}h^3 - \frac{dp}{dx}h^3/2 + \frac{d\gamma}{dx}\frac{h^2}{2}\right)$$

$$hV = \frac{1}{\mu}\left(-\frac{1}{3}h^3\frac{dp}{dx} + \frac{d\gamma}{dx}\frac{h^2}{2}\right)$$

$$-\frac{1}{3}h^3\frac{dp}{dx} + \frac{d\gamma}{dx}\frac{h^2}{2} = \mu hV$$

$$-\frac{1}{3}h^3\frac{dp}{dx} = \mu hV - \frac{d\gamma}{dx}\frac{h^2}{2}$$

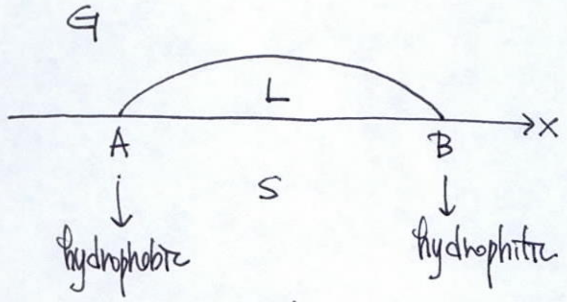
$$\frac{dp}{dx} = \frac{3}{h^3}\left(\frac{d\gamma}{dx}\frac{h^2}{2} - \mu hV\right)$$

$$\frac{dp}{dx} = \frac{3}{h^2}\left(\frac{h}{2}\frac{d\gamma}{dx} - \mu V\right)$$

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = -\frac{dp}{dx}h + \frac{d\gamma}{dx} = -\frac{3}{h}\left(\frac{h}{2}\frac{d\gamma}{dx} - \mu V\right) + \frac{d\gamma}{dx}$$

Motion of droplets induced by chemical gradient.

from F. Brochard, Langmuir. 5, 432-438 (1989)



$\vec{V} = ?$

(1) Driving force : F_d

dU (variation of interfacial energy)

$$= \left[(\gamma_{SL} - \gamma_{SG})_B - (\gamma_{SL} - \gamma_{SG})_A \right] dx \quad < 0$$

$$F_d = -\frac{dU}{dx} = +(\gamma_{SG} - \gamma_{SL})_B - (\gamma_{SG} - \gamma_{SL})_A \quad > 0.$$

(2) Resisting force : F_v

$$F_v = \int_A^B \tau_w \, dx$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}.$$

$$0 = -\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} \qquad (h \ll L)$$

: thin drop
lubrication approximation

$$\text{B.C.} \begin{cases} y=0: & u=0 \\ y=h: & \frac{\partial u}{\partial y}=0 \end{cases}$$

$$u = \frac{1}{2\mu}\frac{dP}{dx}(y^2 - 2yh). \qquad \frac{dP}{dx} \to V$$

$$V = \frac{1}{h}\int_0^h u \, dy = \frac{1}{h} \cdot \frac{1}{2\mu}\frac{dP}{dx}\int_0^h (y^2 - 2hy)\, dy$$

$$= \frac{1}{h}\frac{1}{2\mu}\left(\frac{dP}{dx}\right)\left(\frac{h^3}{3} - h^3\right) = \frac{1}{3}h^2\frac{1}{\mu}\left(-\frac{dP}{dx}\right)$$

$$-\frac{dp}{dx} = 3\mu \frac{V}{h^2}$$

$$u = \frac{1}{2\mu}\cdot 3\mu \frac{V}{h^2}(2yh - y^2)$$

$$u = \frac{3}{2}\frac{V}{h^2}(-y^2 + 2hy)$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu \frac{3}{2}\frac{V}{h^2}(-2y + 2h)\Big|_{y=0}$$

$$\tau_w = 3\mu\left(\frac{V}{h}\right).$$

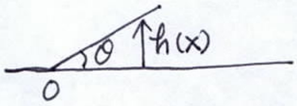
$$F_V = \int_A^B 3\mu\left(\frac{V}{h}\right)dx$$ //

(3) Force balance : $F_d = F_V$

$$3\mu V \int_A^B \frac{1}{h}dx = +(\gamma_{SG} - \gamma_{SL})_B - (\gamma_{SG} - \gamma_{SL})_A$$

Young's eq : $\gamma_{SG} - \gamma_{SL} = +\gamma\cos\theta_e$

$$3\mu V \int_A^B \frac{dx}{h(x)} = \gamma(\cos\theta_{e,B} - \cos\theta_{e,A})$$



$h(x) = x\tan\theta \simeq x\theta$

$$\int_A^B \frac{dx}{h} \simeq 2\int_0^{\Lambda} \frac{dx}{x\theta} \approx \frac{2}{\theta}\ln\left(\frac{\Lambda}{\lambda}\right)$$

λ : cutoff length

ex.) $\lambda = 10^{-9}$, $\Lambda = 10^{-3}$

$\ln\left(\frac{\Lambda}{\lambda}\right) \approx 14 = \Gamma$

$$6\mu\frac{V}{\theta}\Gamma = \gamma(\cos\theta_{e,B} - \cos\theta_{e,A})$$

* Spreading coefficient S

$$S = \gamma_{SG} - \gamma_{SL} - \gamma_{LG}. \qquad \begin{array}{l} >0 \ : \text{complete wetting} \\ <0 \ : \text{partial wetting.} \end{array}$$

Young $\rightarrow$ $\gamma_{SG} = \gamma_{SL} + \gamma \cos\theta_e$

$$\gamma_{SG} - \gamma_{SL} = \gamma \cos\theta_e$$

$$S = \gamma(-1 + \cos\theta_e) \qquad S = -\gamma + \gamma\cos\theta_e$$

$$-1 + \cos\theta_e = \frac{S}{\gamma}$$

$$\underbrace{\cos\theta_e}_{} = 1 + \frac{S}{\gamma}$$

$$\Downarrow$$

$$1 - 2\sin^2\frac{\theta_e}{2} = 1 + \frac{S}{\gamma}$$

$$\sin^2\frac{\theta_e}{2} = -\frac{S}{2\gamma}$$

$$\sin\frac{\theta_e}{2} = \left(-\frac{S}{2\gamma}\right)^{1/2}$$

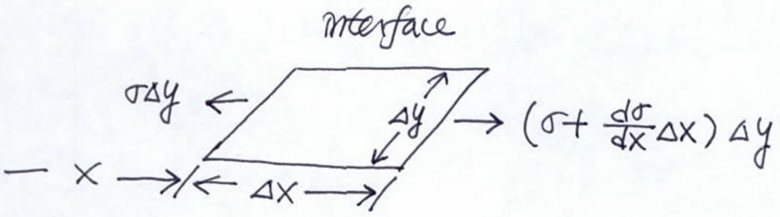
$$F_d = \gamma(\cos\theta_{e,B} - \cos\theta_{e,A})$$

$$= \int_A^B \frac{dS}{dx}\,dx \underset{\substack{if \\ S'=const.}}{=} S'\underbrace{(X_B - X_A)}_{=\,l} = S'l \qquad //$$

$$6\mu\frac{V}{\theta}\Gamma = S'l$$

$$V = \frac{l\theta}{6\mu\Gamma}S'$$

Flows driven by surface tension gradient (Marangoni flow)



shear force due to surf. tension gradient per unit area

$$\tau_s = \frac{d\sigma}{dx}$$

$$\bar{\tau}_s = \nabla_s \sigma$$

- Causes of spatial gradients in surface tension

  - temperature variation : thermocapillary
  - chemical impurity / additives : diffusocapillary
  - electric charge / surface potential : electrocapillary

- Boundary condition at the interface

  – tangential stress

$$\mu \frac{\partial u}{\partial y}\Big|_{interface} = \frac{\partial \sigma}{\partial x}$$

- $\sigma(T) = \sigma_0 + \frac{\partial \sigma}{\partial T}(T - T_0)$

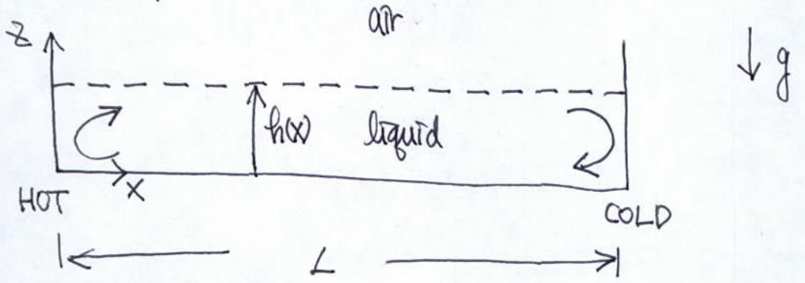
$$\frac{\partial \sigma}{\partial T} < 0 \quad \text{in general}$$

water: $\sigma = 0.072\ N/m$

$$\frac{\partial \sigma}{\partial T} = -0.15\ mN/m \cdot K$$

* Thermocapillary motion

Ex. shallow pan



$h \ll L$

$$u(x, z) = ?$$

governing eq

· x-dir: $\dfrac{\partial p}{\partial x} = \mu \dfrac{\partial^2 u}{\partial z^2}$

· z-dir: $\dfrac{\partial p}{\partial z} = -\rho g \quad : \quad p = p_a + \rho g (h - z)$

$$\mu \frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} z + c_1$$

$$\mu u = \frac{1}{2} \frac{\partial p}{\partial x} z^2 + c_1 z + c_2$$

B.C. $u(z=0) = 0 \ : \quad c_2 = 0.$

$z = h: \ \mu \dfrac{\partial u}{\partial z} = \dfrac{d\sigma}{dx}$

$$\frac{\partial p}{\partial x} h + c_1 = \frac{d\sigma}{dx} \qquad c_1 = \frac{d\sigma}{dx} - h \frac{\partial p}{\partial x}$$

$$\mu u = \frac{1}{2} \frac{\partial p}{\partial x} z^2 + \left(\frac{d\sigma}{dx} - h \frac{\partial p}{\partial x} \right) z$$

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial h}{\partial x} = \rho g \frac{dh}{dx}$$

$$Q = \int_0^h u(z)\, dz = 0.$$

$$\frac{1}{6}\frac{\partial p}{\partial x}h^3 + \left(\frac{d\sigma}{dx} - h\frac{\partial p}{\partial x}\right)\frac{1}{2}h^2 = 0.$$

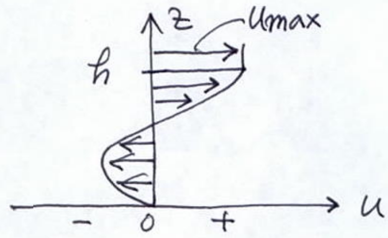
$$\frac{1}{6}\frac{\partial p}{\partial x}h - \frac{1}{2}h\frac{\partial p}{\partial x} + \frac{1}{2}\frac{d\sigma}{dx} = 0$$

$$\frac{1}{2}\frac{d\sigma}{dx} = \frac{1}{3}h\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = \frac{3}{2}\frac{1}{h}\frac{d\sigma}{dx}.$$

$$\mu u = \frac{1}{2}\frac{3}{2}\frac{1}{h}\frac{d\sigma}{dx}z^2 + \underbrace{\left(1-\frac{3}{2}\right)}_{=-\frac{1}{2}}\frac{d\sigma}{dx}z$$

$$u = \frac{z}{2\mu}\left(\frac{3}{2}\frac{z}{h} - 1\right)\frac{d\sigma}{dx}$$



$$u_{max} = \frac{h}{4\mu}\frac{d\sigma}{dx}.$$

Scaling: $\tau \sim \mu\frac{U}{h} \sim \frac{\Delta\sigma}{L}$

$$U \sim \frac{h}{\mu}\frac{\Delta\sigma}{L}$$

height: $\frac{\partial p}{\partial x} = \rho g\frac{dh}{dx} = \frac{3}{2}\frac{1}{h}\frac{d\sigma}{dx}$

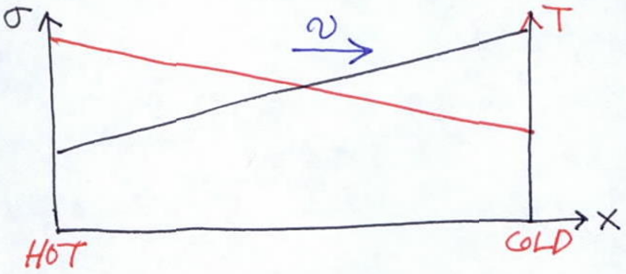
$$h\frac{dh}{dx} = \frac{3}{2}\frac{1}{\rho g}\frac{d\sigma}{dx} \quad \text{OR} \quad \frac{d\sigma}{dx} = \frac{2}{3}\rho g h\frac{dh}{dx}$$

integrating $\sigma - \sigma_1 = \frac{\rho g}{3}(h^2 - h_1^2)$

(at $x=0$: $h = h_1$, $\sigma = \sigma_1$)

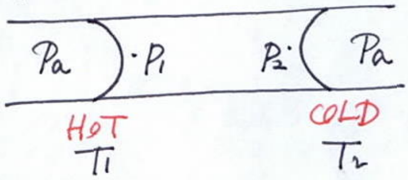
· Direction of thermocapillary flow

* interface : hot → cold



* drop / bubble

– hydrophilic :



$$P_1 = P_a - \sigma_1 \cdot K(\theta_1)$$

$$P_2 = P_a - \sigma_2 \cdot K(\theta_2)$$

assuming $\theta_1 = \theta_2$

$$P_1 - P_2 = K(\sigma_2 - \sigma_1)$$

$$T_1 > T_2$$

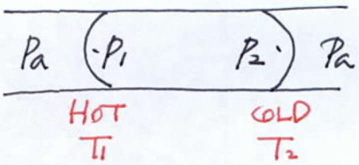
$$\sigma_1 < \sigma_2$$

$$\sigma_2 - \sigma_1 > 0$$

$$P_1 > P_2$$

∴ hot → cold

– hydrophobic



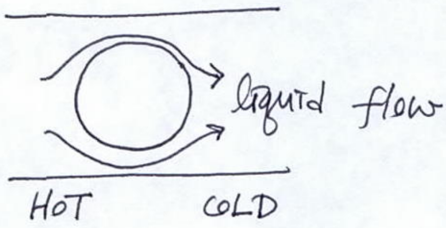
$P_1 = P_a + \sigma_1 K$ $\qquad T_1 > T_2$

$P_2 = P_a + \sigma_2 K.$ $\qquad \sigma_1 < \sigma_2$

$$P_2 > P_1$$

∴ cold → hot

* Bubble immersed in liquid

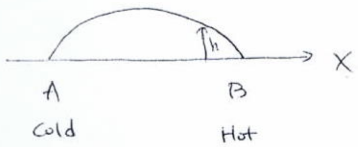


bubble : cold → hot

4/14. 월.

4.23. 수 Midterm Exam

Motion of droplets induced by thermal gradient.



U : avg. vel.

infinitely long strip : 2-D drop.

$$\frac{dT}{dx} > 0$$

$$\frac{dS}{dx} > 0 \qquad S \cdot \text{ spreading coefficient.}$$

$$\frac{d\gamma}{dx} < 0$$

· Driving force

$$
\begin{aligned}
F_d &= -\frac{dE}{dx} \\
&= (\gamma_{SG} - \gamma_{SL})_B - (\gamma_{SG} - \gamma_{SL})_A \\
&= \gamma_B \cos\theta_{e,B} - \gamma_A \cos\theta_{e,A} \\
&= (S+\gamma)_B - (S+\gamma)_A \\
&= \frac{d}{dx}(S^{\circ}+\gamma^{\circ}) \cdot l \\
&= (S'+\gamma') \cdot l
\end{aligned}
$$

· Resisting force

thin drop $(h \ll l)$

$$\frac{dP}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$

B.C. $\begin{cases} y=0 : u=0 \\ y=h : \mu \frac{\partial u}{\partial y} = \frac{d\gamma}{dx} \end{cases}$

$$\mu u = \frac{1}{2}\frac{dP}{dx}y^2 + C_1 y + \cancel{C_2}^{\,0} \quad \text{B.C. 1}$$

$$\mu \frac{\partial u}{\partial y}\Big|_{y=h} = \frac{d\gamma}{dx} = \frac{dP}{dx}h + C_1$$

$$C_1 = -\frac{dP}{dx}h + \frac{d\gamma}{dx}$$

$$\therefore u = \frac{1}{2\mu}\frac{dP}{dx}y^2 + \frac{1}{\mu}\left(-\frac{dP}{dx}h + \frac{d\gamma}{dx}\right)y$$

$$Q = Uh = \int_0^h u\,dy$$

$$= \frac{1}{\mu}\left(-\frac{1}{3}h^3\frac{dP}{dx} + \frac{d\gamma}{dx}\frac{h^2}{2}\right)$$

$$\frac{dP}{dx} = \frac{3}{h^2}\left(\frac{h}{2}\frac{d\gamma}{dx} - \mu U\right)$$

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = -\frac{dP}{dx}h + \frac{d\gamma}{dx}$$

$$= -\frac{3}{h}\left(\frac{h}{2}\frac{d\gamma}{dx} - \mu U\right) + \frac{d\gamma}{dx}$$

$$= 3\mu\frac{U}{h} - \frac{1}{2}\frac{d\gamma}{dx}$$

$$F_v = \int_A^B \tau_w\,dx$$

$$= \underbrace{\int_A^B \cancel{3\mu\frac{U}{h(x)} - \frac{1}{2}\frac{d\gamma}{dx}}\; 3\mu\frac{U}{h(x)}dx}_{\text{same approach as before (chemical gradient)}} - \frac{1}{2}\overbrace{(\gamma_B - \gamma_A)}^{\gamma'\cdot l}$$

$$= 6\mu\frac{U}{\theta}\Gamma$$

$$\tau_w = 3\mu \frac{V}{h} - \frac{1}{2}\frac{d\gamma}{dx}$$

$$F_V = \int_A^B \tau_w \, dx = \underbrace{\int_A^B 3\mu \frac{V}{h(x)} dx}_{6\mu \frac{V}{\theta}\Gamma} - \frac{1}{2}\overbrace{(\gamma_B - \gamma_A)}^{= \gamma' l}$$

same approach as before (chemical)

$6\mu \frac{V}{\theta}\Gamma$

- Force balance

$$(S' + \gamma')\, l = 6\mu \frac{V}{\theta}\Gamma - \frac{1}{2}\gamma' l$$

$$6\mu \frac{V}{\theta}\Gamma = (S' + \frac{3}{2}\gamma')\, l$$

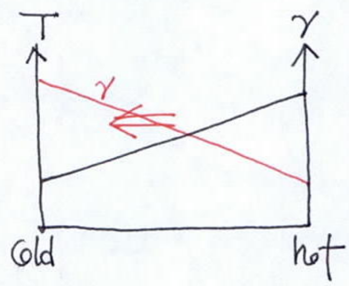
$$V = \frac{\theta}{6\mu\Gamma}(S' + \frac{3}{2}\gamma')\, l$$

OR $\quad S' = (\gamma_{sg} - \gamma_{SL})' - \gamma'$

$$V = \frac{\theta}{6\mu\Gamma}\left[(\gamma_{sg} - \gamma_{SL})' + \frac{1}{2}\gamma'\right] l$$

if $(\gamma_{sg} - \gamma_{SL})' = 0.$ OR $\frac{d(\gamma_{sg} - \gamma_{SL})}{dT} = 0.$

$V \sim \gamma'$



* drop on solid: hot → cold