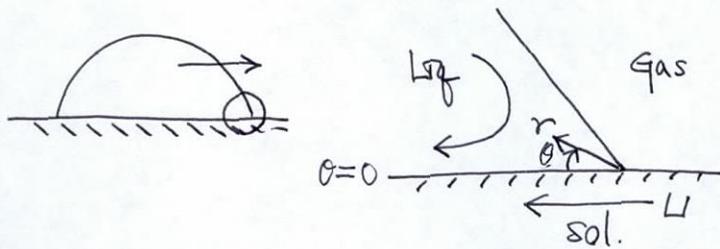


# Wedge dissipation on partially wettable solids

$$0^\circ \ll \theta \ll 180^\circ$$



cylindrical coordinate

## Corner flow

$$\nabla^4 \psi = 0.$$

$$v_r = -\frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad v_\theta = +\frac{\partial \psi}{\partial r}$$

$$\text{B.C.} \quad \theta = 0: \quad v_\theta = 0, \quad -\frac{1}{r} \frac{\partial \psi}{\partial \theta} = U$$

$$\theta = \theta_0: \quad v_\theta = 0, \quad \tau_{r\theta} = 0.$$

$$\tau_{r\theta} = \frac{\mu}{r} \left[ \frac{\partial v_r}{\partial \theta} - v_\theta \right]$$

$$\tau_{r\theta}|_{\theta=\theta_0} = 0: \quad \frac{\partial v_r}{\partial \theta} = \frac{\partial}{\partial \theta} \left( \frac{\partial \psi}{\partial r} \right) = 0.$$

$$\psi = r(a \sin \theta + c \cos \theta + b \theta + d \theta^2)$$

$$v_\theta = \frac{\partial \psi}{\partial r} = a \sin \theta + b \theta + c \cos \theta + d \theta^2$$

$$v_\theta(\theta=0) = b = 0$$

$$v_\theta(\theta=\theta_0) = a \sin \theta_0 + c \cos \theta_0 + d \theta_0^2 = 0. \quad \dots (1)$$

$$\frac{\partial v_r}{\partial \theta} = r(a \cos \theta - b \sin \theta + c \sin \theta + 2d \theta) = r(a \cos \theta + c \sin \theta + 2d \theta)$$

$$\frac{\partial v_r}{\partial \theta}|_{\theta=0} = r(a + d) = -rU. \quad a + d = -U \quad \dots (2)$$

$$\frac{\partial V_r}{\partial \theta} = \frac{\partial}{\partial \theta} \left( -\frac{1}{r} \frac{\partial \psi}{\partial \theta} \right) = 0 \quad \text{at } \theta = \theta_0$$

$$\frac{1}{r} \frac{\partial^2 \psi}{\partial \theta^2} = -a \sin \theta + c \cos \theta + c \cos \theta - c \theta \sin \theta - d \sin \theta - d \sin \theta - d \theta \cos \theta$$

$$\left. \frac{\partial^2 \psi}{\partial \theta^2} \right|_{\theta_0} \Rightarrow -a \sin \theta_0 + c (2 \cos \theta_0 - \theta_0 \sin \theta_0) - d (2 \sin \theta_0 + \theta_0 \cos \theta_0) = 0 \quad \dots (3)$$

$$(2): a = -d - U$$

$$(1) \rightarrow (-d - U) \sin \theta_0 + c \theta_0 \sin \theta_0 + d \theta_0 \cos \theta_0 = 0$$

$$d (\theta_0 \cos \theta_0 - \sin \theta_0) + c \theta_0 \sin \theta_0 = U \sin \theta_0$$

$$(3) \rightarrow (d + U) \sin \theta_0 + c (2 \cos \theta_0 - \theta_0 \sin \theta_0) - d (2 \sin \theta_0 + \theta_0 \cos \theta_0) = 0$$

$$-d (\theta_0 \cos \theta_0 + \sin \theta_0) + c (2 \cos \theta_0 - \theta_0 \sin \theta_0) = U \sin \theta_0$$

$$\begin{bmatrix} \theta_0 \sin \theta_0 & \theta_0 \cos \theta_0 - \sin \theta_0 \\ 2 \cos \theta_0 - \theta_0 \sin \theta_0 & -\theta_0 \cos \theta_0 - 3 \sin \theta_0 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} U \sin \theta_0 \\ U \sin \theta_0 \end{bmatrix}$$

$$c = \frac{\begin{vmatrix} U \sin \theta_0 & \theta_0 \cos \theta_0 - \sin \theta_0 \\ U \sin \theta_0 & -\theta_0 \cos \theta_0 - 3 \sin \theta_0 \end{vmatrix}}{\begin{vmatrix} \theta_0 \sin \theta_0 & \theta_0 \cos \theta_0 - \sin \theta_0 \\ 2 \cos \theta_0 - \theta_0 \sin \theta_0 & -\theta_0 \cos \theta_0 - 3 \sin \theta_0 \end{vmatrix}}$$

$$= \frac{U \sin \theta_0 (-\theta_0 \cos \theta_0 - 3 \sin \theta_0 - \theta_0 \cos \theta_0 + \sin \theta_0)}{-\theta_0^2 \sin \theta_0 \cos \theta_0 - 3 \theta_0 \sin^2 \theta_0 - (2 \theta_0 \cos^2 \theta_0 - \theta_0^2 \sin \theta_0 \cos \theta_0 - 2 \sin \theta_0 \cos \theta_0 + \theta_0 \sin^2 \theta_0)}$$

$$= \frac{-2 U \sin \theta_0 (\theta_0 \cos \theta_0 + \sin \theta_0)}{-4 \theta_0 \sin^2 \theta_0 - 2 \theta_0 \cos^2 \theta_0 + 2 \sin \theta_0 \cos \theta_0} = \frac{-U \sin \theta_0 (\theta_0 \cos \theta_0 + \sin \theta_0)}{-\theta_0 \sin^2 \theta_0 - \theta_0 + \sin \theta_0 \cos \theta_0}$$

$$= \frac{-2 \theta_0 \sin^2 \theta_0 - 2 \theta_0 (\sin^2 \theta_0 + \cos^2 \theta_0)}{-\theta_0 \sin^2 \theta_0 - 2 \theta_0}$$

$$\left\{ \begin{aligned} a &= \frac{-U\theta_0}{\theta_0 - \sin\theta_0 \cos\theta_0} = U\tilde{\alpha}(\theta_0) \\ c &= \frac{U\rho \sin^2\theta_0}{\theta_0 - \sin\theta_0 \cos\theta_0} = U\tilde{c}(\theta_0) \\ d &= \frac{U \sin\theta_0 \cos\theta_0}{\theta_0 - \sin\theta_0 \cos\theta_0} = U\tilde{d}(\theta_0) \end{aligned} \right.$$

$$v_\theta = a \sin\theta + c \cos\theta + d \cos\theta$$

$$v_r = -[a \cos\theta + c(\sin\theta + \theta \cos\theta) + d(\cos\theta - \rho \sin\theta)]$$

Viscous shear stress

$$\tau_{r\theta} = \mu \left[ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] = \frac{\mu}{r} \left( \frac{\partial v_r}{\partial \theta} - v_\theta \right)$$

$$= \frac{\mu}{r} (-2c \cos\theta + 2d \sin\theta)$$

$$= \frac{2\mu}{r} (-c \cos\theta + d \sin\theta)$$

$$\tau_{rr} = 2\mu \frac{\partial v_r}{\partial r} = 0$$

$$\tau_{\theta\theta} = 2\mu \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) = 0.$$

→ at the wall

$$\tau_{r\theta}(\theta=0) = -\frac{2\mu}{r} U \frac{\rho \sin^2\theta_0}{\theta_0 - \sin\theta_0 \cos\theta_0}$$

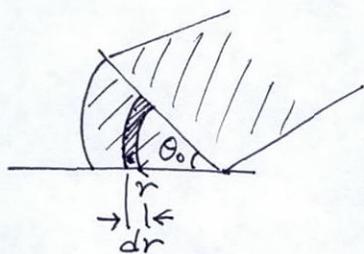
Energy dissipation  $\Phi$  [W/m<sup>3</sup>]

In cylindrical coordinates,

$$\Phi = 2\mu \left[ \left( \frac{\partial v_r}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)^2 + \frac{1}{2} \left( \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 \right]$$

$$= \mu \frac{4U^2}{r^2} (\tilde{c} \cos\theta - \tilde{d} \sin\theta)^2$$

Energy dissipation over a finite volume.



$$\begin{aligned}
 \int_{V'} \Phi dV' &= \int_0^{\lambda} \int_0^{\theta_0} \Phi r d\theta dr \quad [W/m] \\
 &= \int_0^{\lambda} \int_0^{\theta_0} \mu \frac{4U^2}{r} (\bar{c} \cos \theta - \bar{d} \sin \theta)^2 d\theta dr \\
 &= 4\mu U^2 \int_0^{\lambda} \frac{dr}{r} \underbrace{\int_0^{\theta_0} (\bar{c} \cos \theta - \bar{d} \sin \theta)^2 d\theta}_{= f(\theta_0)} \\
 &= 4\mu U^2 f(\theta_0) \ln\left(\frac{\lambda}{r}\right)
 \end{aligned}$$

