Electromagnetism

Maxwell's equations

· in free space

$$\begin{cases} \nabla \cdot \epsilon_0 \bar{E} = \rho_E & : \text{Gauss' law} \\ \dfrac{\partial \bar{E}}{\partial t} = \dfrac{1}{\epsilon_0}(\nabla \times \bar{H} - \bar{J}) & : \text{Ampere's law} \\ \nabla \cdot \mu_0 \bar{H} = 0 & \\ \dfrac{\partial \bar{H}}{\partial t} = -\dfrac{1}{\mu_0}(\nabla \times \bar{E}) & : \text{Faraday's law} \end{cases}$$

Electroquasistatics

$$\nabla \times \bar{E} = -\frac{\partial \mu_0 \bar{H}}{\partial t} \simeq 0 \qquad : \quad \bar{E} = -\nabla \Phi$$

$$\nabla \cdot \epsilon_0 \bar{E} = \rho_E$$

ϵ_0 (permittivity constant) $= 8.85 \times 10^{-12}$ F/m

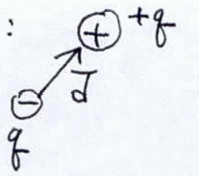
$$\nabla^2 \Phi = -\frac{\rho_E}{\epsilon_0} \qquad : \text{Poisson's equation}$$

· Insulating (dielectric) materials

charge = unpaired charge + polarization charge

$$\nabla \cdot \epsilon_0 \bar{E} = \rho_u + \rho_p$$

dipole :

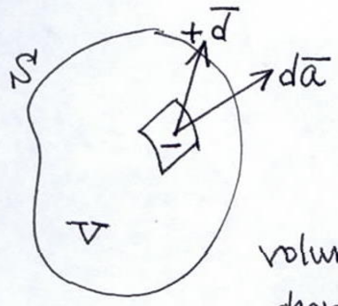


dipole moment

$$\bar{p} = q\bar{d}$$

polarization density $\quad \bar{P} = Nq\bar{d}$

N: # of <u>dipoles</u> per unit volume
(polarized particles)



volume element containing positive charges which have left negative charges on the other side of surface S

* the net charge left behind in V

$$Q = -\oint_S (qN\bar{d}) \cdot d\bar{a} = -\int_V \nabla \cdot \bar{P} \, dV$$

$$Q = \int \rho_p \, dV$$

$$\therefore \rho_p = -\nabla \cdot \bar{P}$$

* Displacement flux density

$$\bar{D} \equiv \epsilon_0 \bar{E} + \bar{P}$$

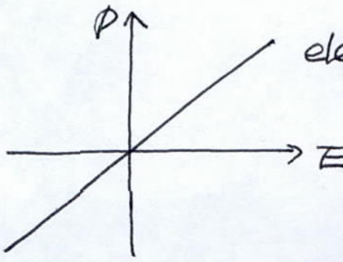
$$\nabla \cdot \epsilon_0 \bar{E} = \rho_u - \nabla \cdot \bar{P}$$

$$\nabla \cdot (\epsilon_0 \bar{E} + \bar{P}) = \rho_u$$

$$\nabla \cdot \bar{D} = \rho_u$$

* constitutive law

$$\bar{P} = \bar{P}(\bar{E})$$



electrically linear medium

$$\bar{P} = \epsilon_0 \chi_e \bar{E}$$

χ_e : dielectric susceptibility

$$\bar{D} = \epsilon_0 \bar{E} + \epsilon_0 \chi_e \bar{E} = \epsilon_0 (1 + \chi_e) \bar{E}$$

$$\bar{D} = \epsilon \bar{E}$$

- $\epsilon \equiv \epsilon_0 (1 + \chi_e)$

  : permittivity or dielectric constant

- $\frac{\epsilon}{\epsilon_0} = 1 + \chi_e$: relative dielectric constant

Medium	χ_e
air (0°C)	0.00059
water	79.1
diamond	15.5
paraffin	1.1

EQS again :

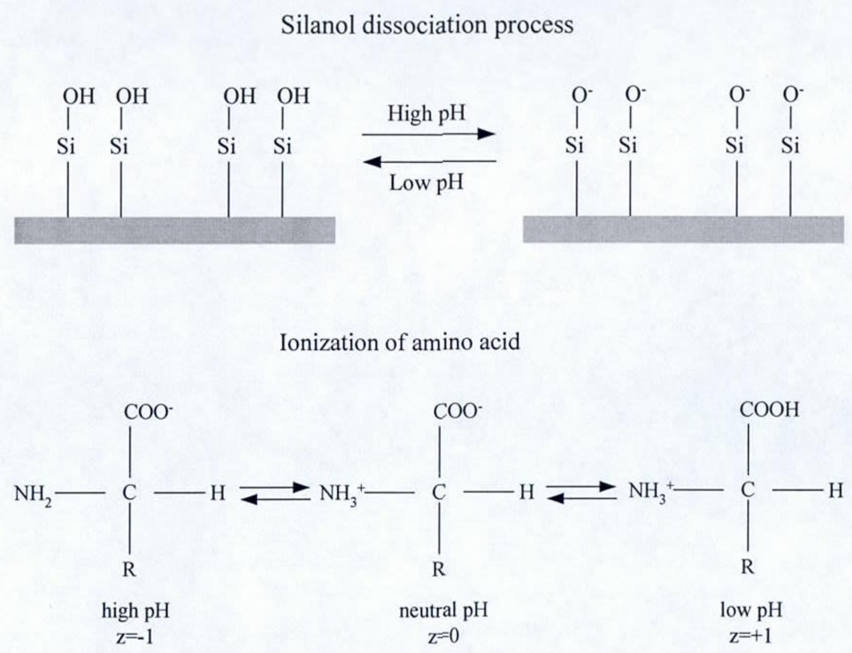
$$\nabla \cdot \epsilon \bar{E} = \rho_u$$

$$\nabla \times \bar{E} = 0 \quad , \quad \bar{E} = -\nabla \Phi$$

$$\boxed{\nabla^2 \Phi = -\frac{\rho_u}{\epsilon}}$$ Poisson's equation

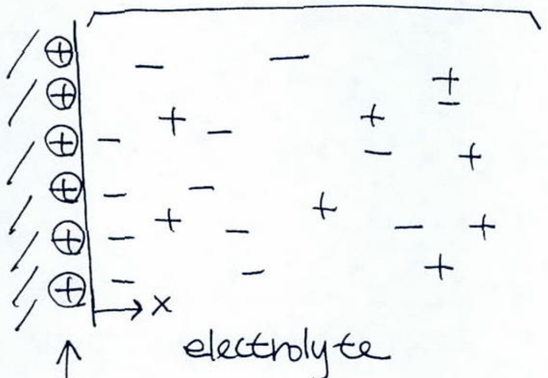
Charging of a solid surface in a liquid

- Ionization or dissociation of surface groups



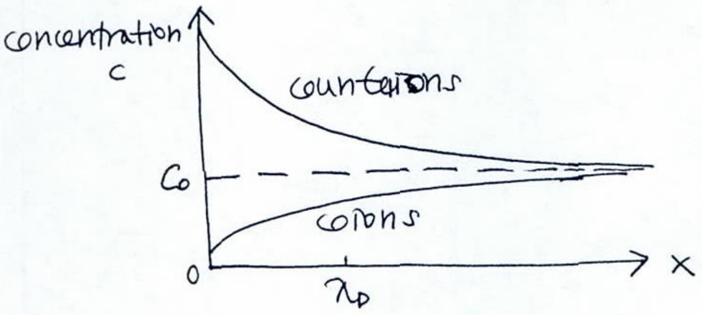
- Adsorption (binding) of ions from solution onto a previously uncharged surface

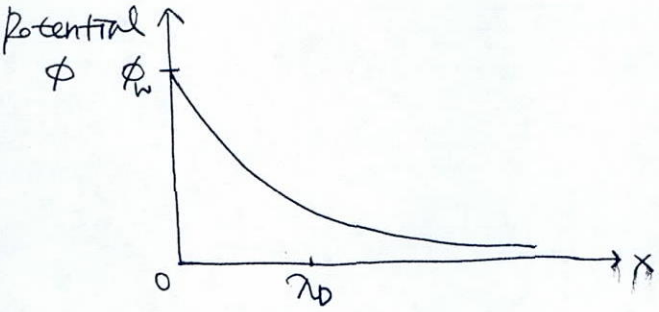
Electrical Double Layer



Acquired surface change

: ionization, ion adsorption, ion dissolution





Thickness of the double layer

· assume ① symmetrical salt in solution

$$z_+ = -z_- = z$$

② $\bar{E} = E\,\hat{i}_x$

③ no coions in the layer

* Balancing : electric potential energy ~ kT (thermal motion)

Poisson's eq

$$\nabla^2 \phi = -\frac{\rho_E}{\varepsilon}$$ ~ permittivity

$$\rho_E = F z c$$

- F = Faraday's constant (charge of 1 mole of singly ionized molecules)

$= N_A e = 9.65 \times 10^4$ C/mol

$N_A = 6.022 \times 10^{23}$ /mole

$e = 1.602 \times 10^{-19}$ C

- z = charge number

$$\frac{d^2\phi}{dx^2} = \frac{Fzc}{\varepsilon}$$

- Electric potential energy $W = -Fz\phi$

$$\frac{d\phi}{dx} = \frac{Fzc}{\varepsilon}x \qquad \Delta\phi = \frac{Fzc}{2\varepsilon}x^2$$

$$\therefore \Delta W = -\frac{F^2 z^2 c x^2}{2\varepsilon} \qquad [J/mole]$$

$$\Delta W = RT$$

$$x \equiv \lambda_D = \left(\frac{\varepsilon RT}{2F^2 z^2 c}\right)^{1/2}$$: Debye length

For an aqueous sol. of a symmetric electrolyte at 25°C

$$\lambda_D = \frac{9.61 \times 10^{-9}}{(z^2 c)^{1/2}} \qquad [m]$$

$z=1$ (univalent) : $c = 10^2$ mol/m³ ⇒ $\lambda_D = 1$ nm

$c = 1$ mol/m³ ⇒ $\lambda_D = 10$ nm