

and combining Eqs(8) and (9)

$$\theta(x) = \frac{c_{e\text{opt}}}{a} + \frac{1}{x} \sqrt{\frac{abc_t d_{\text{opt}}}{8\pi R}}$$

 IDEAL TWIST DISTRIBUTION FOR EFFICIENT HOVER

\* tapered blade

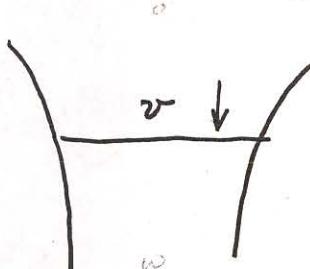
constant inflow

Velocity

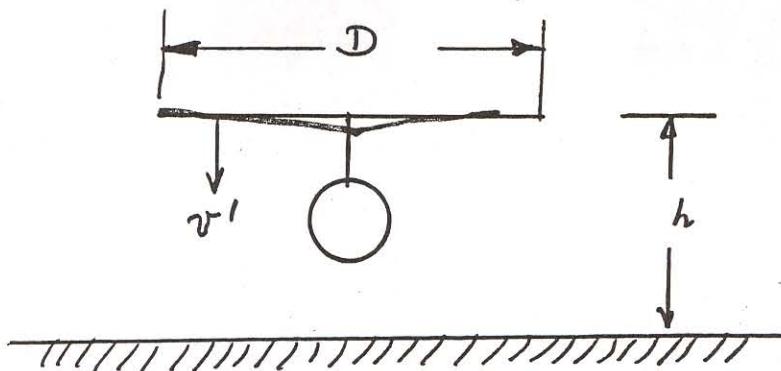
## Rotors Hovering in Ground Effect.

Consider a rotor hovering near the ground.

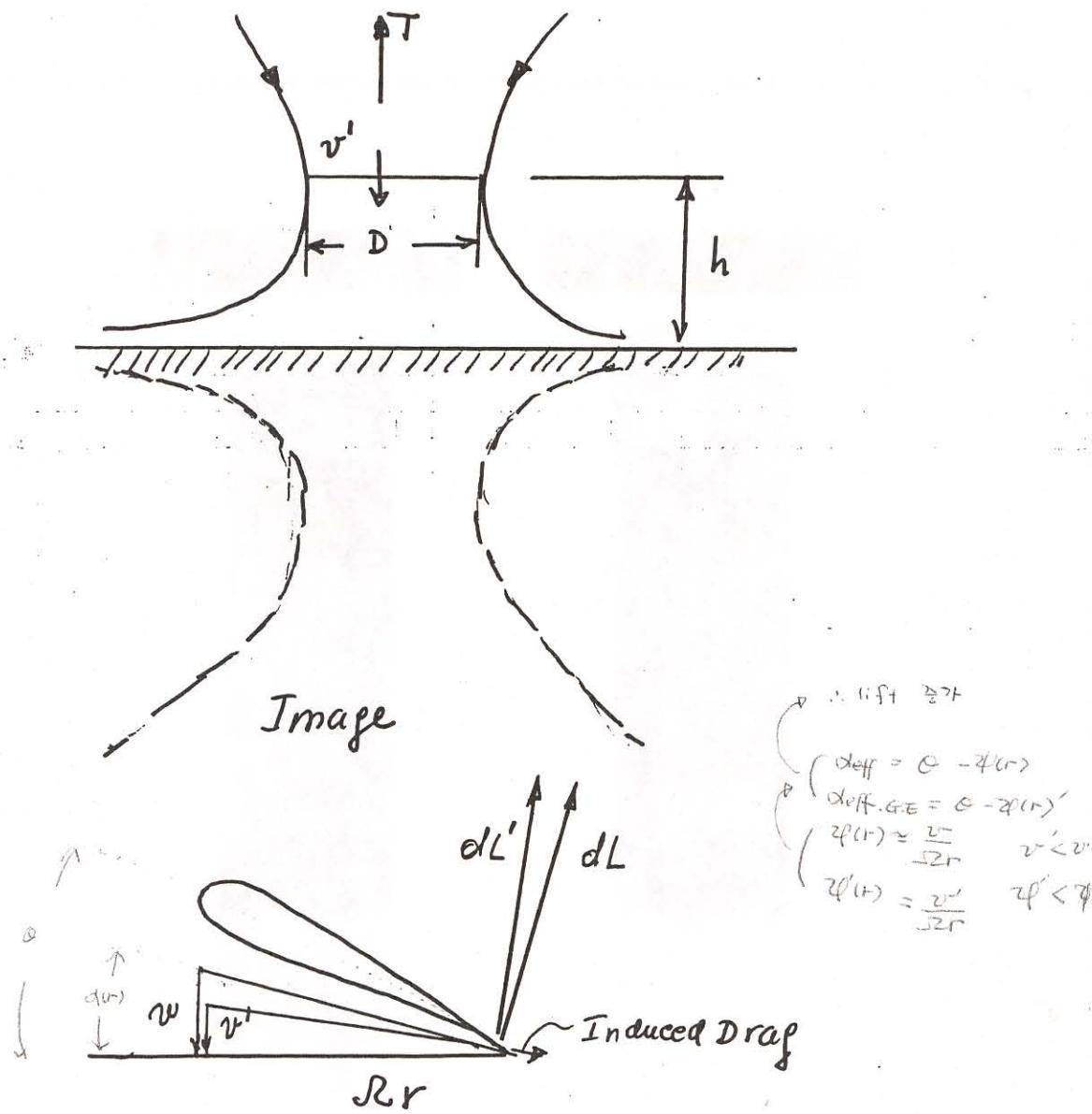
Recall that when the rotor hovers far from the ground one can obtain the inflow from momentum theory



When the rotor is near the ground

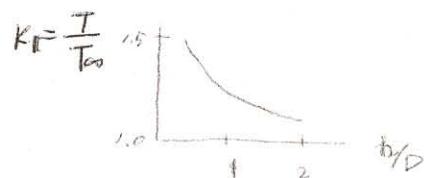


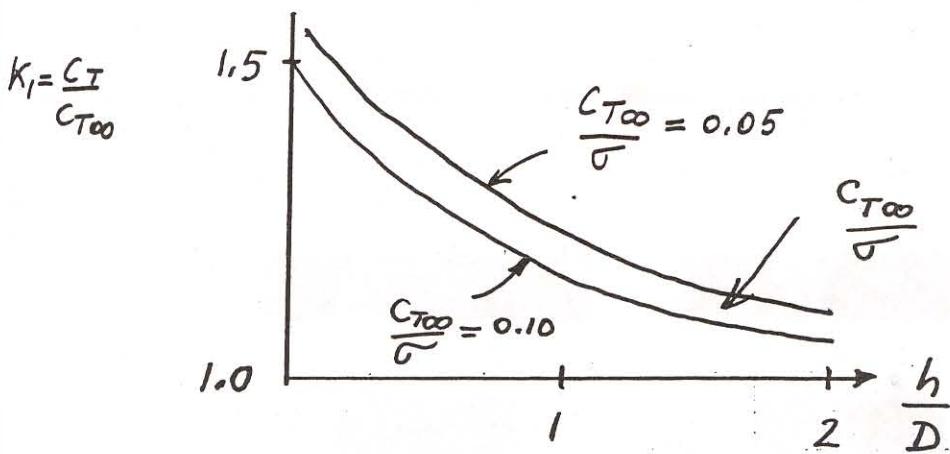
in this case one can expect the inflow  $v' < v$  for an equal amount of thrust. One can develop a fairly simple analytical model for this case by using an analytic image effect, which is schematically shown on the next page.



i.e. presence of ground reduces the size of the induced drag and therefore the power requirement is lower  
At a constant power setting descent tends to increase thrust, as shown in the Figure on the next page

$$T = K_1 T_\infty$$





out of ground effect

$$C_Q = \frac{\sigma C_{D0}}{8} + \frac{C_{T\infty}}{\sqrt{2}}^{3/2}$$

$$\lambda_H = \sqrt{\frac{C_{T\infty}}{2}}$$

In ground effect  $C_T = K_1 C_{T\infty}$  at const. power

$$C_Q = \frac{\sigma C_{D0}}{8} + \frac{C_T}{\sqrt{2}}^{3/2} K_2$$

For constant power, we can equate torque coefficients

$$\cancel{\frac{\sigma C_{D0}}{8} + \frac{1}{\sqrt{2}} (C_{T\infty})^{3/2}} = \cancel{\frac{\sigma C_{D0}}{8} + \frac{1}{\sqrt{2}}} C_T^{3/2} K_2$$

$$K_2 = \left( \frac{C_{T\infty}}{C_T} \right)^{3/2} = \left( \frac{1}{K_1} \right)^{3/2}$$

$$\boxed{\frac{C_T}{C_{D0}}} \quad \begin{array}{l} \text{P-240-B/01} \\ \text{1/10/90 (2-2)} \end{array}$$

and thus

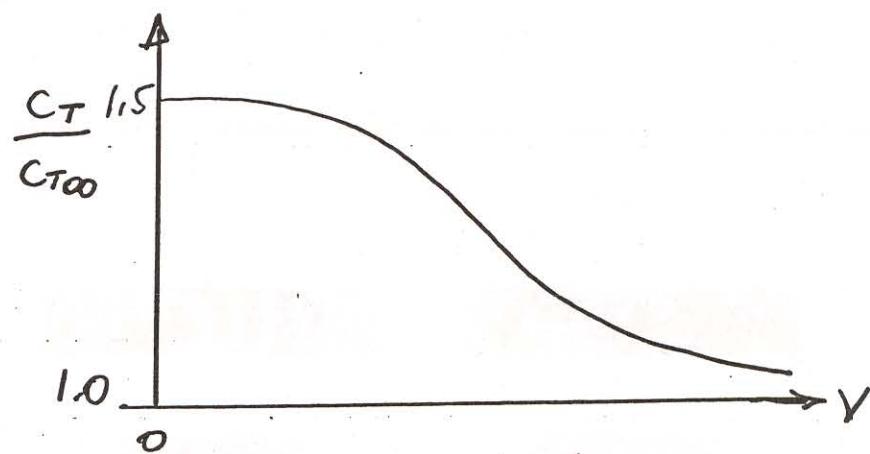
$$C_Q = \frac{\sigma C_{D0}}{8} + \frac{1}{\sqrt{2}} \left( \frac{C_T}{K_1} \right)^{3/2}$$



For forward flight the effect of  $V$ , wind or forward flight velocity has a beneficial effect, as shown in the figure on the next page.

24

36



Forward flight velocity is beneficial.

JOA

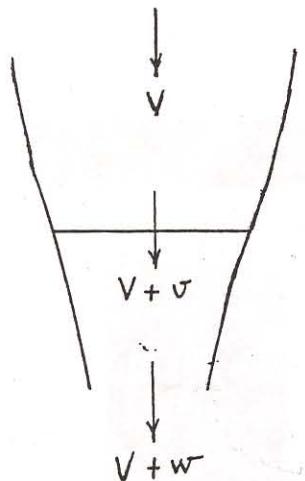
## Descent in Axial Flight

### Condition I: Power-on Vertical Climb

Established, slipstream momentum

theory applies

$$T = 2 \rho A (V + v) V$$

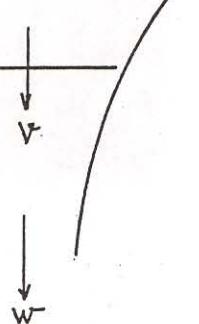


### Condition II: Hover

$$V = 0$$

Established slip stream  
momentum theory applies

$$T = 2 \rho A v^2$$



Not Computable

### Condition III: "Partial" Power Descent

Vortex Ring State, ring of vorticity stays

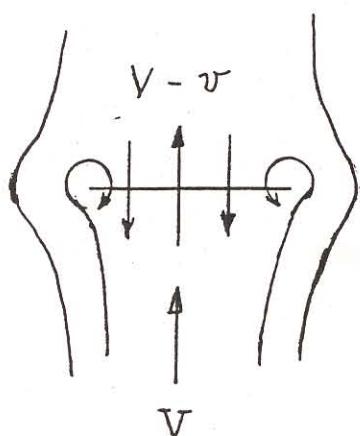
"V22" tilt rotor  
"Empirical"

with the disk, possible at an  
angle of descent, or wind.

Momentum theory not valid,

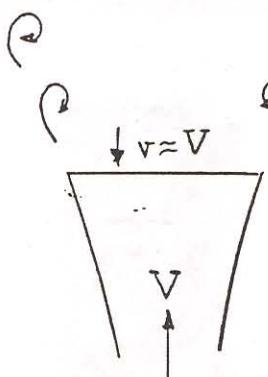
there is no slipstream. Flow

not defined both from geometry and  
stability point of view

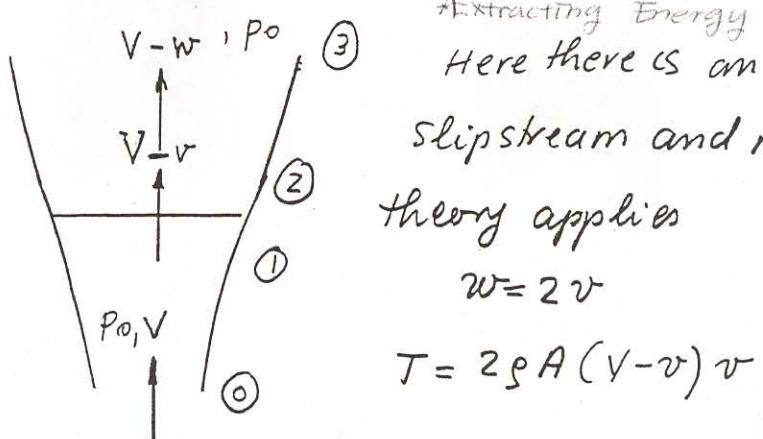


### Condition IV: Turbulent Wake State

Minimum, power-off, rate of descent. Disk approaches a state corresponding to an almost solid disk. Redefine positive  $V$  from below and  $v$  as opposed to it in direction and almost equal in magnitude. No established slipstream, momentum theory invalid. Flow state for ideal autorotation



### Condition V: Windmill Brake State (Power-off condition)



Extracting Energy from the air  
Here there is an established  
slipstream and momentum  
theory applies

$$w = 2v$$

$$T = 2\rho A (V-v)v$$

Applying momentum theory

$$\textcircled{1} \& \textcircled{1} \quad p_0 + \frac{1}{2} V^2 = p_1 + \frac{1}{2} \rho (V-v)^2$$

$$\textcircled{2} \& \textcircled{2} \quad p_2 + \frac{1}{2} \rho (V-v)^2 = p_0 + (V-w)^2$$

$$p_1 - p_2 = \frac{1}{2} \rho [V^2 - (V-w)^2] = \frac{1}{2} \rho (2Vw - w^2)$$

$$T = \rho A (V-v)w = \frac{1}{2} \rho A (2Vw - w^2)$$

$$\text{thus } w = 2v \quad \textcircled{1}$$

$$T = \frac{1}{2} \rho A (V-v)w$$

36c

and

$$T = 2A(v - u)v \rho \quad (2)$$

$$\text{or } V = \frac{T}{2\rho A} \frac{1}{v} + v \quad (3)$$

Equation (3) represents the rate of descent in the windmill brake state, from it the minimum rate of descent can be also obtained

$$\frac{dV}{dv} = 0 = -\frac{T}{2\rho A v^2} + 1 = 0$$

$$v_{min} = \sqrt{\frac{T}{2\rho A}}$$

$$V_{min} = \frac{T}{2A\rho} \sqrt{\frac{I}{2\rho A}} + \sqrt{\frac{T}{2\rho A}} = 2\sqrt{\frac{T}{2\rho A}} = \sqrt{\frac{2T}{\rho A}} \quad (4)$$

At sea level using the appropriate value of  $\rho_0 = \rho$

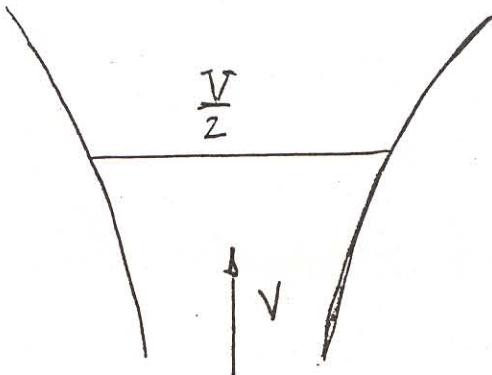
$$(\rho_0 = 0.0023769 \frac{lb sec^2}{ft^4})$$

$$V_{min} = \sqrt{\frac{2}{0.0023769}} \sqrt{\frac{I}{A}} = 29.0074 \sqrt{\frac{T}{A}} = 29 \sqrt{\frac{I}{A}} ft/sec$$

$$\text{ex) } R=15\text{ft}, T=7000\text{lb}, A=\pi R^2=3.14(15)^2=680\text{ft}^2 \Rightarrow V_{min} = 29 \sqrt{\frac{7000}{680}} = 111.29 = 428\text{ft/sec}$$

This minimum rate almost represents the inverse of hover

$$V=0$$



one can also relate this to  $v_H$  inflow velocity hover

$$v_H = \sqrt{\frac{T}{2\rho A}}$$

$$v_H^2 = \frac{T}{2\rho A}$$

for vertical climb (assuming equal weight or thrust)

$$T = 2\rho A(v + v) v \quad \frac{T}{2\rho A} = (v + v) v$$

thus  $v_H^2 = (v + v) v$

or 
$$\left( \frac{V}{v_H} + \frac{v}{v_H} \right) \frac{v}{v_H} = 1 \quad (5)$$

Similarly for vertical descent, from windmill brake state Eq(2) above

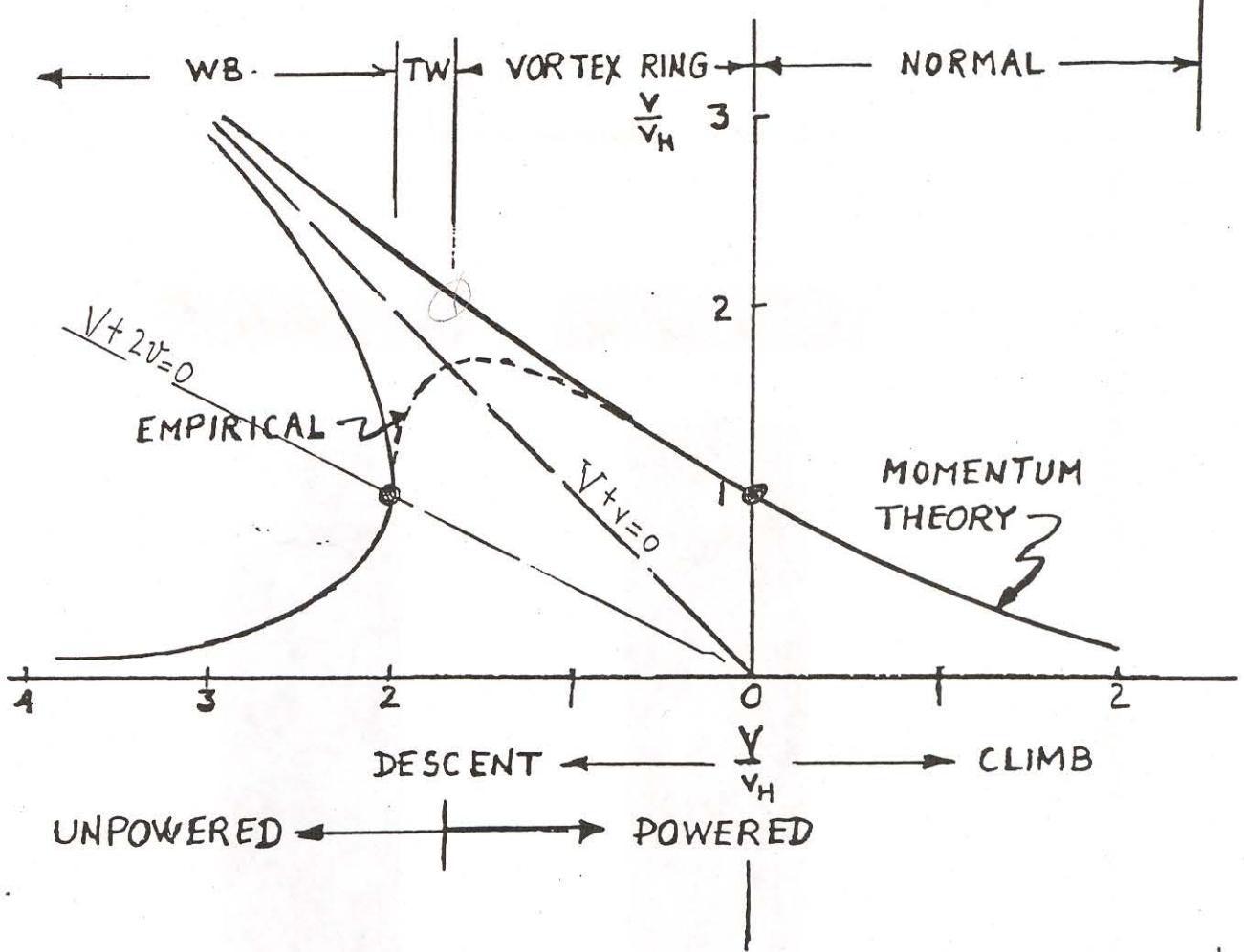
$$v_H^2 = (v - v) v$$

or 
$$\left( \frac{V}{v_H} - \frac{v}{v_H} \right) \frac{v}{v_H} = 1 \quad (6)$$

All these results can be plotted conveniently on one curve as shown on the next page

where the portion of the curve denoted by "empirical" should be assumed to be obtainable from either flight test or wind tunnel test

56J

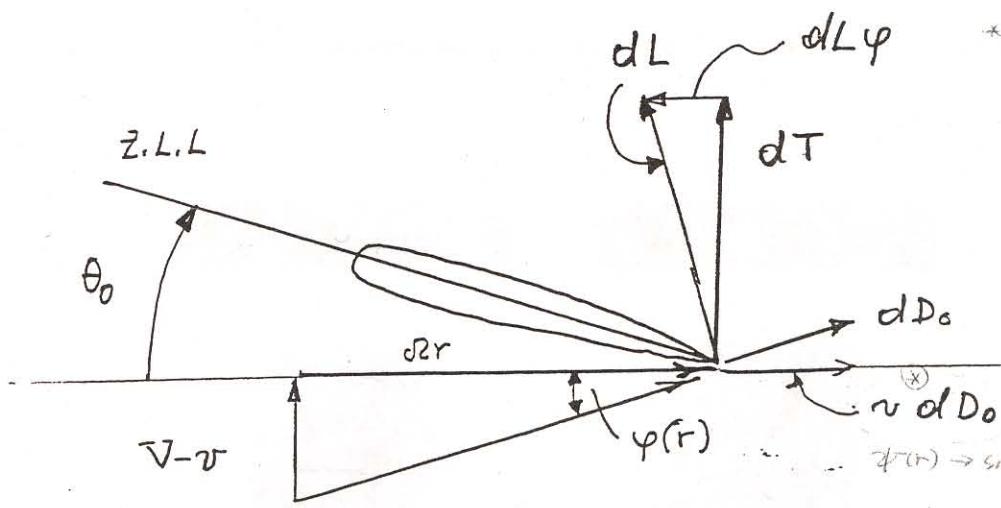


In Turbulent Wake State

$$\frac{V_{\min \text{ T.W.}}}{V_H} \approx 1.72 \quad \text{or} \quad V_{\min \text{ T.W.}} \approx 1.72 V_H = 1.72 \sqrt{\frac{T}{2\rho A}}$$

## Blade Element Theory Treatment of Descent Problem

- Climbing at  $\dot{V} = V - \dot{v}$   
 Descending case  
 \* see S.



$$\varphi(r) \rightarrow \text{small} \quad (\text{as } \varphi(r) = 1) \\ \sin \varphi(r) = \varphi(r)$$

Again assume small angles and  $C, \theta_0, V = \text{constant}$   
 then

$$dT \approx dL = \frac{1}{2} \rho a c R^2 r^2 \left[ \theta_0 + \frac{V-v}{2R} \right] dr \quad (7)$$

and we have to [redefine] the inflow ratio for vertical descent as

$$\lambda = \frac{V-v}{2R} \quad \begin{matrix} \text{inflow ratio for} \\ \text{descending flight} \end{matrix} \quad (8) \quad \text{as } \lambda_{\text{climb}} = \frac{V-v}{2R}$$

$$T = b \int_0^R dL = \frac{1}{2} \rho a b c R^2 R^3 \left[ \frac{\theta_0}{3} + \frac{\lambda}{2} \right] \quad (9)$$

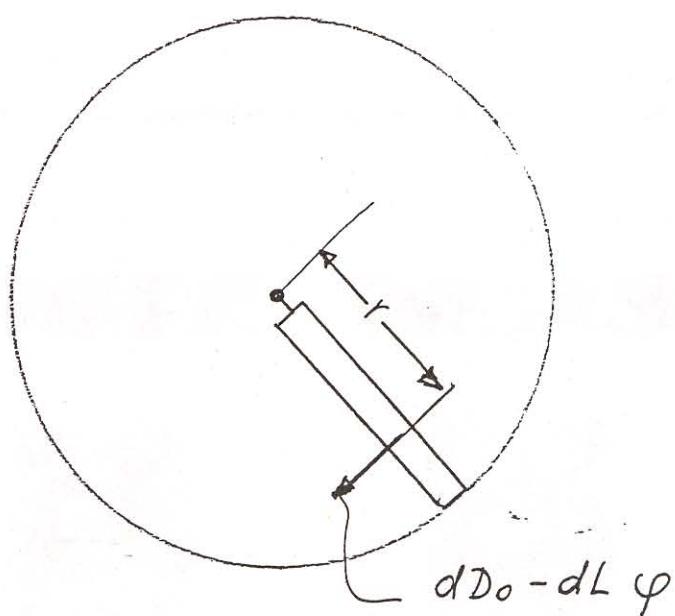
Thus the only difference between the vertical climb and descent is the sign in the square brackets in Eq(9),  
 the positive sign convention for this case is

$$\uparrow V + \downarrow v^+$$

and

$$C_T = \frac{a b}{2} \left[ \frac{\theta_0}{3} + \frac{\lambda}{2} \right] \quad (10)$$

$$C_{T_{\text{climb}}} = \frac{a b}{2} \left[ \frac{\theta_0}{3} - \frac{\lambda}{2} \right]$$



From this figure it is clear that the inplane component of lift opposes the profile drag term

$$\text{Torque in descending flight} \quad dQ = r(dD_0 - \phi dL)$$

$$Q = b \int_0^R [dD_0 - \phi(r) dL] r dr$$

$$\phi(r) = \frac{V-v}{\partial r}$$

$$Q = b \int_0^R \left\{ \frac{1}{2} \rho \sigma^2 r^2 C_{D0} c dr - \frac{1}{2} \rho \sigma^2 r^2 c a [\theta_0 + \phi(r)] \phi(r) \right\} r dr$$

$$= \frac{1}{2} \rho b c \sigma^2 R^4 C_{D0} - \int_0^R \frac{1}{2} \rho abc \sigma^2 r^2 \left[ \theta_0 + \frac{V-v}{\partial r} \right] \frac{V-v}{\partial r} r dr$$

$$= \frac{1}{8} \rho b c \sigma^2 R^4 C_{D0} - \frac{1}{2} \rho abc \sigma^2 R^4 \left[ \frac{\theta_0}{3} + \frac{1}{2} \frac{V-v}{\partial R} \right] \frac{V-v}{\partial R}$$

$$C_Q = \frac{C_{D0} \sigma}{\pi R^2 (2\sigma R)^2 R}$$

$$C_Q = \frac{C_{D0} \sigma}{8} - \lambda C_T \Rightarrow$$

$$C_Q = \frac{C_{D0} \sigma}{8} - \frac{C_a}{2} \left[ \frac{\theta_0}{3} + \frac{\lambda}{2} \right] \lambda \quad (11)$$

↑ only change is the sign indicated

### Partial Powered Vertical Descent

~ 'InFlow Ratio' at  $\frac{V}{v_H} = 2$

Solve torque expression, Eq(11), for  $\lambda$

$$\lambda = \left( \frac{\overbrace{C_{d0}}^{\text{fixed}} - C_Q}{8} \right) \frac{1}{C_T} \quad (12)$$

For a given throttle setting  $C_Q$  is prescribed

$$\lambda = \frac{V-v}{v_H R} = \left( \frac{V}{v_H} - \frac{v}{v_H} \right) \frac{v_H}{v_H R}$$

and recall

$$\lambda_H = \frac{v}{v_H R} = \sqrt{\frac{C_T}{2}}$$

from momentum theory, and therefore

$$\lambda = \left( \frac{V}{v_H} - \frac{v}{v_H} \right) \sqrt{\frac{C_T}{2}} = \left( \frac{\overbrace{C_{d0}}^{\text{fixed}} - C_Q}{8} \right) \frac{1}{C_T}$$

therefore

$$\left( \frac{V}{v_H} - \frac{v}{v_H} \right) = \left( \frac{\overbrace{C_{d0}}^{\text{fixed}} - C_Q}{8} \right) \frac{\sqrt{2}}{C_T^{3/2}} \quad (13)$$

One can also replot the empirical portion of the curve shown on page <sup>36f</sup> as done on the next page

In the windmill brake range, where momentum theory is valid one can assume approximately

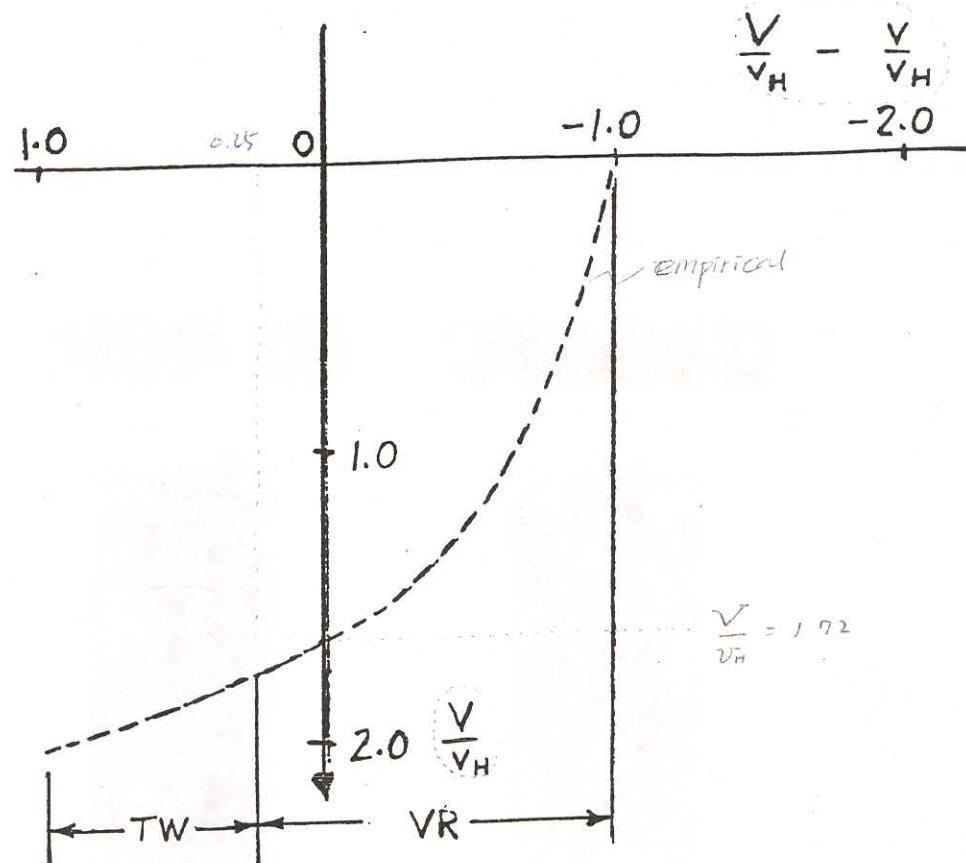
$$\left( \frac{V}{v_H} - \frac{v}{v_H} \right) \frac{v}{v_H} = 1.0 \quad \boxed{\text{Eq } 6 - p 36d}$$

On the other hand

$$\frac{V}{v_H} - \frac{v}{v_H} \approx 0.25 \quad \text{from the Fig on next page.}$$

for minimum rate of descent, power off, and

$$\frac{V}{v_H} \approx 1.72 \quad (\text{turbulent wake state})$$



Therefore

$$V \approx 1.72 V_H = 1.72 \sqrt{\frac{T}{2\rho A}} \approx 25 \sqrt{\frac{T}{A}} \quad (14)$$

at sea level

For a parachute  $C_D \approx 1.4$

$$\text{therefore } T = D \approx \frac{1}{2} \rho V^2 C_D A = \frac{1}{2} \rho (\pi R^2 C_D V^2)^{1/2}$$

$$V = \sqrt{\frac{2T}{\rho A C_D}} = \sqrt{\frac{4}{1.4}} \sqrt{\frac{T}{2\rho A}} = 1.69 \sqrt{\frac{T}{2\rho A}} = 24.5 \sqrt{\frac{T}{A}}$$

Comparing expressions (14) and (15) it is evident that they are very close to each other.

Autorotation

$\rightarrow$  Torque = zero  $\therefore$  torque here is from  
prestall drag

Engine shut down (power off), here energy balance  
between friction losses and rotational energy is important

Recall from Eq(11)

$$C_Q = \frac{G C_{D0}}{8} - \lambda C_T = 0 \quad (\text{NO POWER})$$

torque coeff.

$$\text{and } C_T = \frac{G C_{D0}}{8\lambda} \quad \textcircled{*}$$

torque coeff.

$$\text{but } C_T = \frac{G a}{2} \left( \frac{\theta_0}{3} + \frac{\lambda}{2} \right) \quad \text{descending lift from Blade Element Theory}$$

$$\frac{G a}{2} \left( \frac{\theta_0}{3} + \frac{\lambda}{2} \right) = \frac{G C_{D0}}{48\lambda}$$

$$\lambda \theta_0 + \frac{\lambda^2}{2} = \frac{C_{D0}}{4a} \quad \text{Quadratic Eq. to } \lambda$$

$$\lambda^2 + 2\theta_0\lambda - \frac{C_{D0}}{2a} = 0$$

$$\lambda = -\frac{\theta_0}{3} + \sqrt{\left(\frac{\theta_0}{3}\right)^2 + \frac{C_{D0}}{2a}} \quad (16)$$

Knowing  $\lambda$  get  $C_T = \frac{T}{\rho A (J2R)^2}$ , from Eq(\*) above

From  $\left(\frac{T}{A}\right) \rightarrow$  get  $\rightarrow$  ORR  $\rightarrow$  return to definition of  $\lambda$

$$\lambda = \frac{V - v}{J2R} = \left( \frac{V}{v_H} - \frac{v}{v_H} \right) \sqrt{\frac{C_T}{2}}$$

or

$$\left( \frac{V}{v_H} - \frac{v}{v_H} \right) = \frac{\lambda}{\sqrt{\frac{C_T}{2}}} \quad (17)$$

Using Eq(17) one can use the Fig on p. 40 to  
determine in which region one is. If one is in TW  
state, need to use empirical part, if in

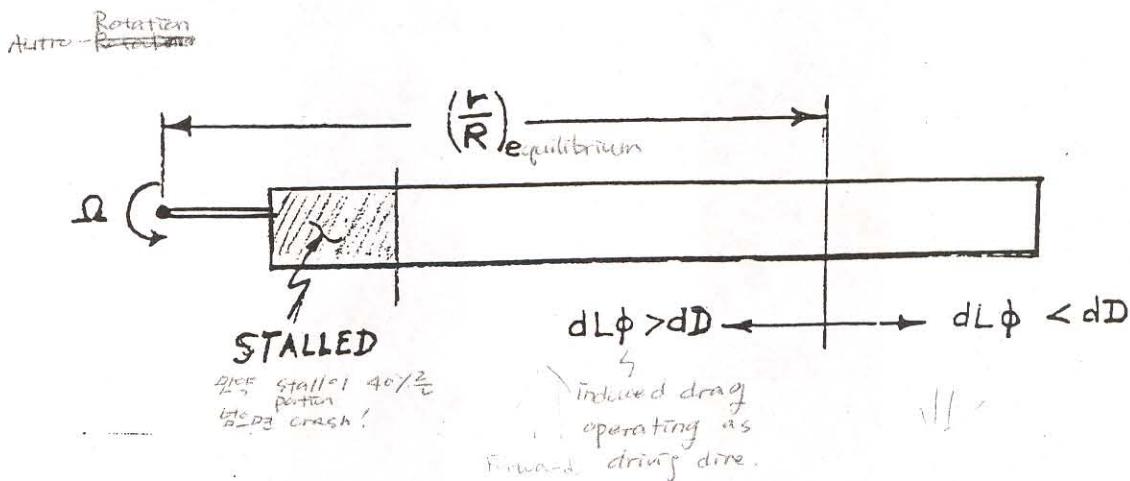
windmill brake state use momentum theory

Two Eqs with two unknowns, one has to pick  $\theta_0$  so as to get the condition one wants

$$\text{Recall } dL \approx \frac{1}{2} \rho \bar{U}^2 r^2 c [ \theta_0 + \varphi(r) ] c dr$$

$$dD_0 = \frac{1}{2} \rho \bar{U}^2 r^2 c C_{D0} dr$$

$$\varphi(r) = \frac{V - U}{\bar{U} r}$$



At some station  $r_e$

$$dD_0 - \varphi_e dL = 0 \quad \varphi_e = \frac{dD_0}{dL}$$

inplane component of profile drag balances inplane component of lift. Further ~~if~~ <sup>in</sup> the lift is greater and in the outboard direction the opposite is true. At equilibrium

$$dL \varphi(r_e) = dD_0$$

$$\frac{1}{2} \rho \bar{U}^2 r^2 c [ \theta_0 + \varphi_e(r) ] \varphi_e'(r) dr = \frac{1}{2} \rho C_{D0} \bar{U}^2 r^2 c dr$$

which yields a quadratic equation for  $\varphi_e(r)$

$$\varphi_e(r) = -\frac{\theta_0}{2} + \sqrt{\left(\frac{\theta_0}{2}\right)^2 + \frac{C_{D0}}{c}}$$

$$\varphi_e(r) = \frac{V - v}{\omega r_e} = \frac{V - v}{\omega R} \left( \frac{R}{r_e} \right)$$

or  $\frac{r_e}{R} = \frac{\lambda}{\varphi_e(r)}$  this determines  $\frac{r_e}{R}$  location of  
the equilibrium section  
 $\alpha(r) = \theta_0 + \varphi(r)$

angle of attack increases as one goes inboard on an autorotating blade. In such a blade stall occurs inboard. If stall exceeds 40% of the blade the vehicle can be destroyed.

### Pre-Lecture

$$\lambda = \sqrt{\frac{C_r}{2}}$$

$$M = \frac{V \cos \lambda R}{\omega R}$$

$$\lambda = \mu \tan \mu + \frac{C_r}{2\mu^2 + \lambda^2}$$

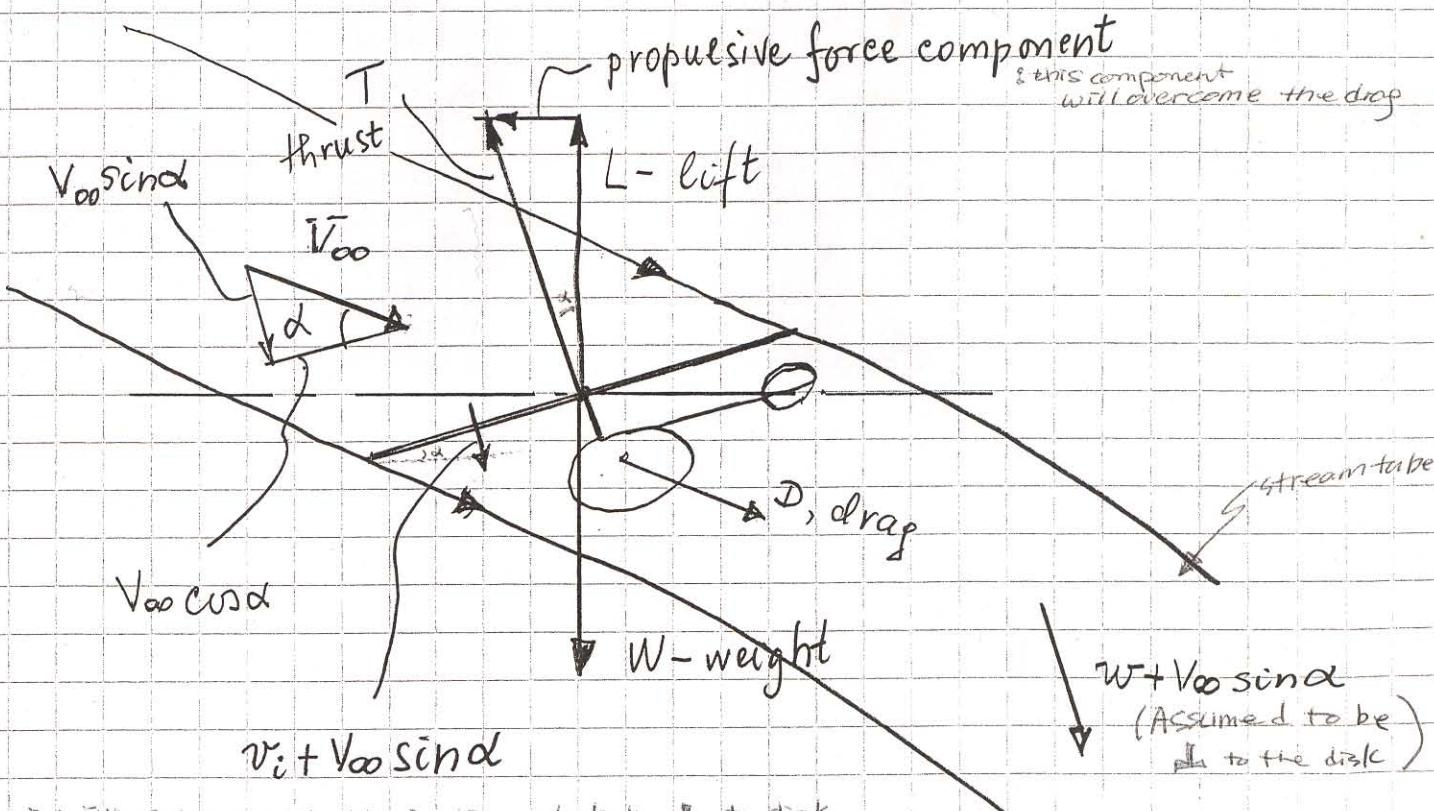
or Accurate static stall condition

$$\text{if } M=0 \quad \lambda = \sqrt{\frac{C_r}{2}}$$

## Rotors in Non-Axial Forward Flight - or

### Momentum Analysis in Forward Flight

Consider a rotor in forward flight. In this case there is an edge wise component of velocity, as illustrated below, because the thrust vector has to be tilted so as to provide a force component in the direction of flight.



$v_i$ : INDUCED VELOCITY distribution at disk,  $\perp$  to disk

and assumed to be uniformly distributed over the disk

Slipstream

Under these conditions the axisymmetric flow that was used in the derivation of the momentum theory in coaxial flight is lost. However, by

(436)

assumptions originally introduced by Glauert it is possible to construct a momentum theory that allows one to calculate rotor performance in forward flight.  $\Sigma V^2 = \text{resultant velocity} = V_{\perp \rightarrow \text{disk}}^2 + V_{\parallel}^2 =$

At the rotor disk the resultant velocity is

$$\begin{aligned} V &= \left[ (V_\infty \cos \alpha)^2 + (V_\infty \sin \alpha + v_i)^2 \right]^{1/2} = \sim (*) \\ &= \left[ V_\infty^2 (\cos^2 \alpha + \sin^2 \alpha) + 2V_\infty \sin \alpha v_i + v_i^2 \right]^{1/2} = \sim (\$) \\ &= \left[ V_\infty^2 + 2V_\infty v_i \sin \alpha + v_i^2 \right] \quad (1) \end{aligned}$$

From momentum theory, normal to the disk

$$T = \dot{m}(\bar{w} + V_\infty \sin \alpha) - \dot{m}V_\infty \sin \alpha = \dot{m}\bar{w} \quad (2)$$

Using conservation of energy

$$\begin{aligned} P &= T(v_i + V_\infty \sin \alpha) = \frac{1}{2} \dot{m} (V_\infty \sin \alpha + \bar{w})^2 - \frac{1}{2} \dot{m} V_\infty^2 \sin^2 \alpha \\ &= \frac{1}{2} \dot{m} (2V_\infty \sin \alpha \bar{w} + \bar{w}^2) \quad (3) \end{aligned}$$

From Eqs (2) and (3) we have

$$\cancel{\dot{m}\bar{w}(v_i + V_\infty \sin \alpha)} = \cancel{\frac{1}{2}\dot{m}(2V_\infty \sin \alpha \bar{w} + \bar{w}^2)}$$

~~$$2\bar{w}v_i + 2V_\infty \bar{w} \sin \alpha = 2V_\infty \sin \alpha \bar{w} + \bar{w}^2$$~~

$$\bar{w} = 2v_i \quad (4)$$

Momentum theory  
Mod 23, Hoverlet At Forward Flight  
Date 2014-09-14

which is the same result that was obtained previously for momentum theory in axial flight (Hover)

43c

From Eq (2) and (4) the thrust is given by

$$\bar{T} = 2\dot{m} v_i \quad (5)$$

$$\dot{m} \approx \rho A U \quad (6) \text{ this is an assumption recall}$$

$$\bar{T} = 2\rho A v_i \sqrt{V_\infty^2 + 2V_\infty v_i \sin \alpha + v_i^2} \quad (7)$$

$$\bar{T} = 2\rho A U v_i = 2\rho A v_i \left[ (V_\infty \cos \alpha)^2 + (V_\infty \sin \alpha + v_i)^2 \right]^{1/2} \quad (8)$$

Next introduce the following definitions

$$\mu = \text{advance ratio} = \frac{V_\infty \cos \alpha}{\sigma R} \quad (9)$$

Def. of inflow ration in forward flight

$$\lambda = \frac{V_\infty \sin \alpha + v_i}{\sigma R} = \frac{V_\infty \cos \alpha}{\sigma R} \frac{\sin \alpha}{\cos \alpha} + \frac{v_i}{\sigma R}$$

$$\text{Remember! } \lambda = \mu \tan \alpha + \frac{v_i}{\sigma R} \quad (10)$$

$$\text{From (8)} \quad C_T = \frac{\bar{T}}{\rho (\sigma R)^2 A} = 2 \left( \frac{v_i}{\sigma R} \right) \left[ \mu^2 + \lambda^2 \right]^{1/2}$$

$$\text{thus} \quad \frac{v_i}{\sigma R} = \frac{C_T}{2 \sqrt{\mu^2 + \lambda^2}} \quad (11)$$

Combining Eqs (10) and (11) yields

$$\lambda = \mu \tan \alpha + \frac{C_T}{2 \sqrt{\mu^2 + \lambda^2}} \quad (12)$$

Alternative Derivation for the forward Flight ~~Inducement Relation~~  
Rotors in Non-axial Forward Flight

Circular Wing  
~~Elliptical~~

Consider an elliptical monoplane (fixed-wing) wing. The air influenced by the wing can be viewed as part of a stream tube having a diameter  $2R$ .

Due to this effect the stream tube is bent downwards

From lifting line theory (Wing Theory)

$$L = \rho \pi R^2 V (w - v)$$

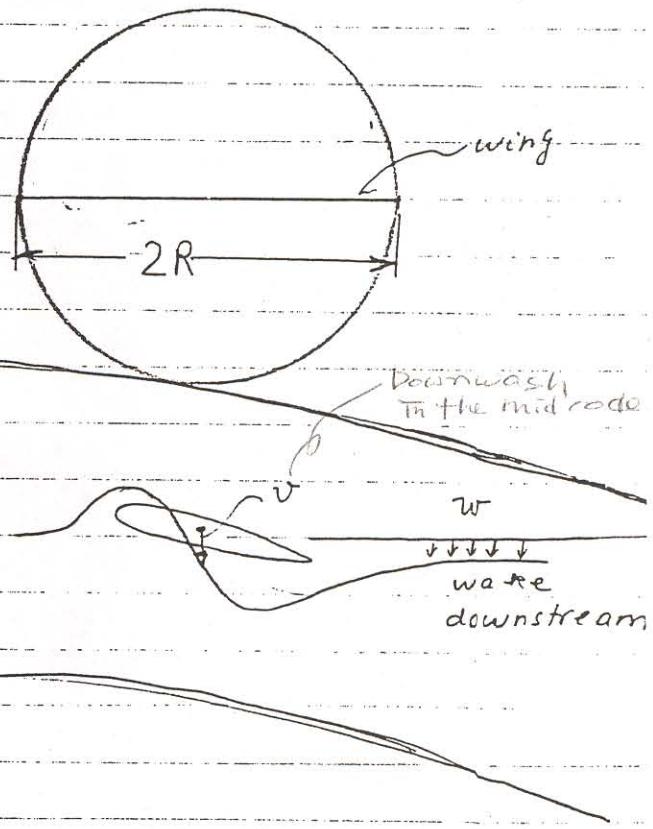
mass flow

initial velocity

It can be shown by lifting line theory that

$$w = 2v \quad \textcircled{*}$$

$$\text{Also } V \gg v \text{ and } \sqrt{V^2 + v^2} \approx V$$



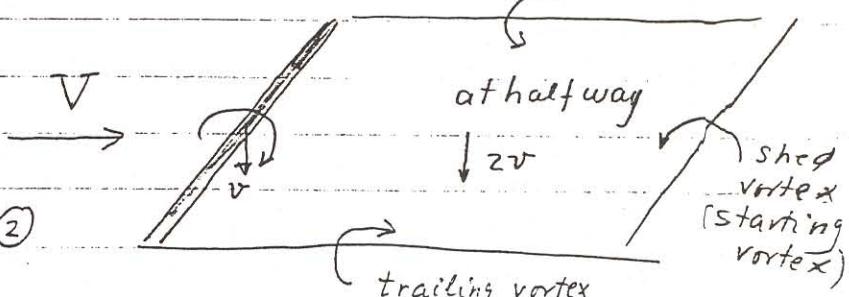
Wing theory  $\rightarrow L = \rho \pi R^2 V w \quad \textcircled{1}$

(Circular wing and using Eqs  $\textcircled{*}$  and  $\textcircled{1}$  with Elliptical loading) yields

$$L = 2 \rho \pi R^2 V v \quad \textcircled{2}$$

Wing theory for a circular wing

for  $v \ll V$ .

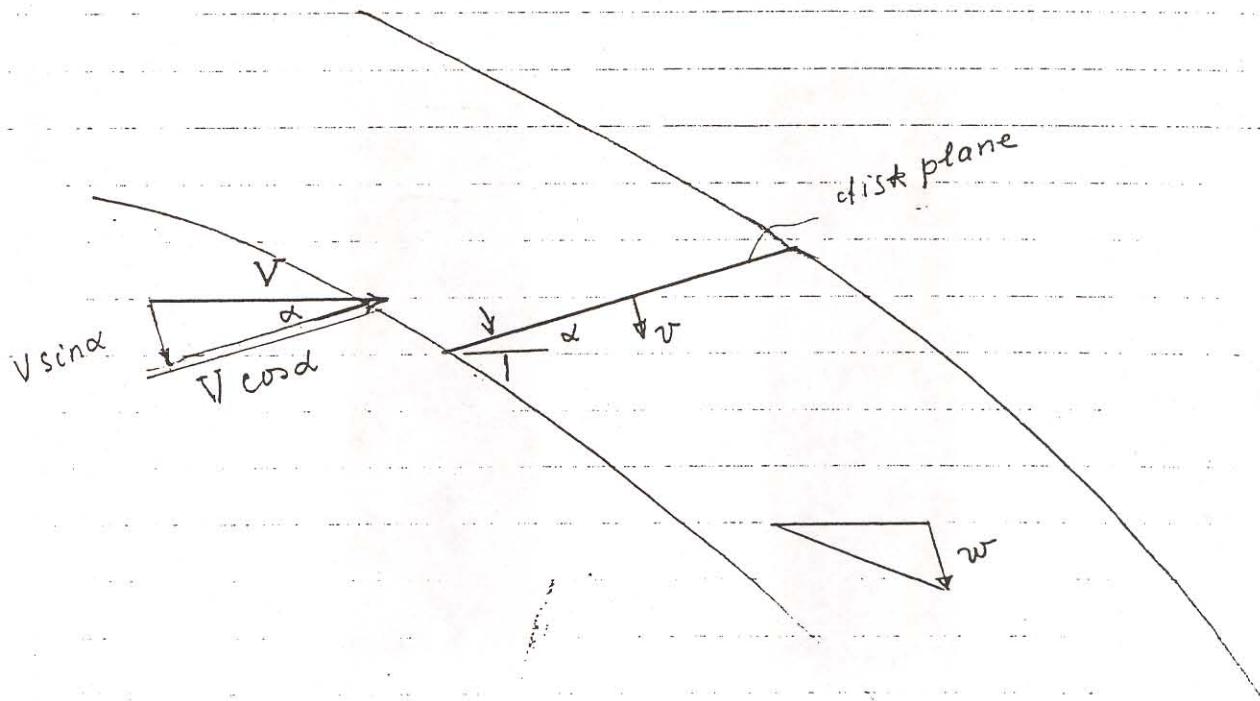


Also recall that for a helicopter rotor in axial flight momentum theory provided the following expression for thrust

momentum theory { Thrust of rotor in axial flight

$$T = 2 \rho \pi R^2 (V_a + v) v \quad \textcircled{3}$$

Consider a rotor operating at a velocity of forward flight  $V$ , with an angle of attack  $\alpha$  between the free stream velocity and the rotor disk



Resolve  $V$  parallel and perpendicular to the disk

Glaauert postulated that for a rotor in forward flight at an incidence  $\alpha$

$$T = 2 \rho \pi R^2 V' v \quad (4)$$

where  $V'$  is the resultant velocity at the disk, and  $v$  is uniform over the disk! From the geometry  $V'$  the resultant velocity is

$$V' = \sqrt{(V \sin \alpha + v)^2 + (V \cos \alpha)^2} = \sqrt{v^2 \sin^2 \alpha + v^2 + 2 V v \sin \alpha + V^2 \cos^2 \alpha} = \sqrt{V^2 + 2 V v \sin \alpha + v^2} \quad (5)$$

Note that the assumption of uniform induced velocity distribution over the disk in forward flight is not as good as the same assumption for the case of hover. At high forward flight speeds the induced velocity is small compared to the other velocity components at the rotor blade. At low forward flight speeds the variation of inflow velocity over the disk can be important. Also note that the uniformly loaded actuator disk which has been used to represent hover is replaced here (essentially) by an actuator disk in forward flight which is viewed as a circular wing. A circular wing can be also viewed as a special case of an elliptic wing.

Also note that in equation (4) we no longer assume that  $v \ll V$ .

Next we define two new quantities :

$$\lambda = \text{inflow ratio in forward flight} = \frac{V \sin \alpha + v}{\sqrt{2} R} \quad (6)$$

$$\text{Advance ratio} \quad \mu = \frac{V \cos \alpha}{\sqrt{2} R} \quad (7)$$

Using Eqs (5), (6) and (7)

$$\frac{V'}{\sqrt{2} R} = \sqrt{\lambda^2 + \mu^2} \quad (8) \quad V' = \sqrt{2} R \sqrt{\lambda^2 + \mu^2}$$

Combining Eqs (4) and (8) yields

$$T = 2 \rho \pi R^2 (\sqrt{2} R)^2 \sqrt{\lambda^2 + \mu^2} \frac{v}{\sqrt{2} R}$$

$$\begin{aligned} \frac{T}{\rho \pi R^2 (\sqrt{2} R)^2} &= C_T = 2 \sqrt{\mu^2 + \lambda^2} \frac{v}{\sqrt{2} R} \\ &= 2 \frac{V'}{\sqrt{2} R} \frac{v}{\sqrt{2} R} \end{aligned}$$

(9)

$$\frac{v}{\partial R} = \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

analysis

To illustrate the validity of this relation, Eq(9) consider two limiting cases.

(a)  $v \ll V$  and  $\alpha$  - small

$\frac{v}{V}$  small,  $\alpha$  small,

$$L = 2\rho\pi R^2 V v$$

fixed wing  
result

$$T = 2\rho\pi R^2 v \bar{V} \left[ 1 + 2\left(\frac{v}{V}\right) \sin \alpha + \left(\frac{v}{V}\right)^2 \right]^{1/2}$$

limitation

These are second order terms, because  
 $\sin \alpha$  small and  $\frac{v}{V} \ll 1$

$$\text{Thus } T = 2\rho\pi R^2 V v$$

which is the same as for elliptical fixed wing.

(b) For  $\alpha = 90^\circ$  Rotor in axial flight

$$T = 2\rho\pi R^2 (V + v) v$$

which is the correct expression for axial flight

Next by using Eqs (6) (7) and (9)

$$\lambda = \mu \tan \alpha + \frac{v}{\partial R}$$

$$\lambda = \frac{V \sin \alpha}{\partial R} + \frac{v}{\partial R} =$$

$$= \frac{\sin \alpha}{\cos \alpha} \frac{V \cos \alpha}{\partial R} + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

(10)

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

For a very lightly loaded at high  $\mu$  ( $\mu > 0.10$  and  $C_T \approx 0.005$ ) one can use the approximation

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\mu}$$

It is important to emphasize that this is a completely intuitive approach which produces good results. More accurate analysis by Heyson in NACA TN D-240, has demonstrated the validity of this expression (Eq.10)

In general, it is necessary to solve a quartic equation for  $\lambda$ , based on Eq(10). An iterative procedure for calculating  $\lambda$  may be developed by considering a Newton-Raphson solution of  $f(\lambda) = 0$

$$f(\lambda) = \lambda - \mu \tan \alpha - \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} = 0 \quad (11)$$

namely  $\lambda_{n+1} = \lambda_n - \left( \frac{f}{f'} \right)_n$

or

$$\lambda_{n+1} = \left( \frac{\mu \tan \alpha + \frac{C_T}{2} \frac{(\mu^2 + \lambda^2)^2}{(\mu^2 + \lambda^2)^{3/2}}}{1 + \frac{C_T}{2} \frac{\lambda}{(\mu^2 + \lambda^2)^{3/2}}} \right)_n \quad (12)$$

Three or four iterations are usually sufficient, starting from the value

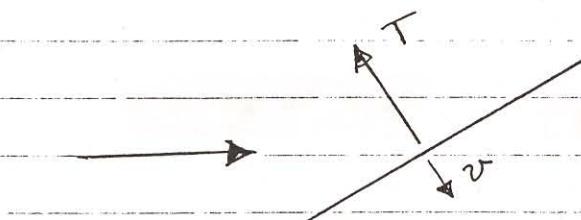
$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + \left(\frac{C_T}{2}\right)^2}}$$

## Performance Calculation Using Energy Concepts

### Forward Flight Performance Using Energy Concepts

~~Assumption of infinite aspect ratio~~ — Neglected "Two"

Induced power for rotor in forward flight



- ① Neglecting "Dynamic Stall"
- ② Neglecting "Compressibility"

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

$$P_i = T v \quad \text{induced Power due to (lift & thrust) } = T v$$

$$C_{Pi} \approx \frac{T v}{\rho \pi R^2 (\Omega R)^3} = C_T \frac{v}{\Omega R} = \frac{C_T^2}{2\sqrt{\mu^2 + \lambda^2}} \quad (13)$$

$$\left( \frac{v}{\Omega R} = \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} \right)$$

high advance ratio, low thrust corresponding to cruise

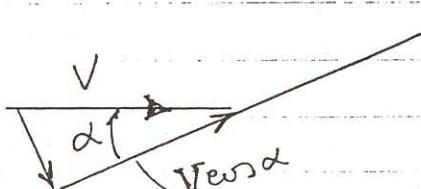
$$C_{Pi} \approx C_T^2 / 2\mu \quad \mu > 0.10 \quad C_T \leq 0.005$$

### Profile Drag Power

Power required to overcome the profile drag

$$V \cos \alpha$$

$$\mu > 0.10 \quad C_T \leq 0.005$$



Consider figure

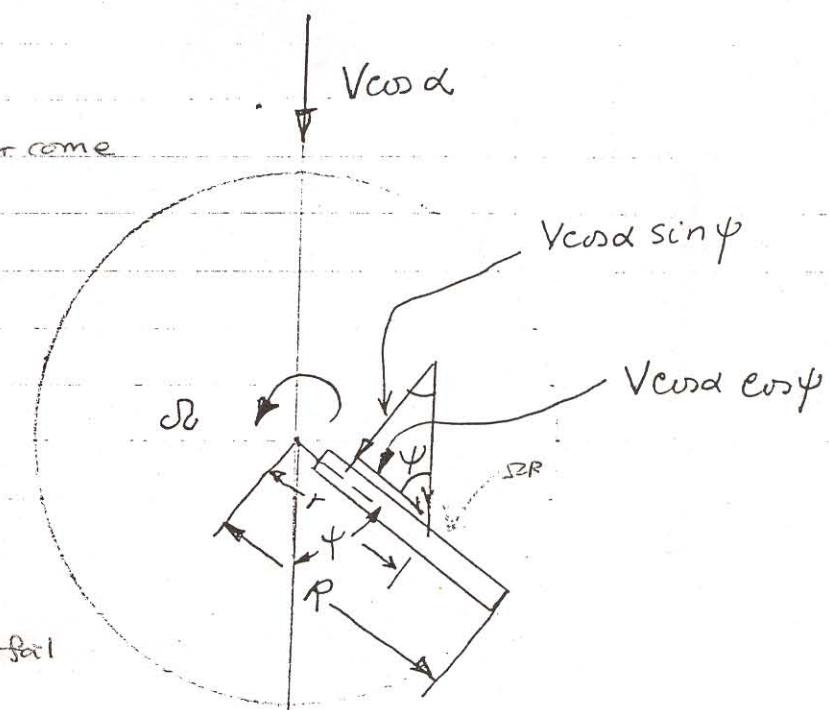
$U_T$ : a velocity oriented along the airfoil

$$* U_T = V \cos \alpha \sin \psi + \times \Omega R =$$

$$\times \Omega R = r^2 \left(\frac{r}{R}\right) \cdot R = \Omega R$$

$$= \Omega R (x + \mu \sin \psi) \quad (14)$$

$$* V_R = \text{radial component} = V \cos \alpha \cos \psi = \Omega R \mu \cos \psi \quad (15)$$



$U_p$  = much smaller than  $U_T, U_R$  in resultant velocity  
i.e.  $U_p \ll U_T, U_R$

In simple power calculations  $U_R$  is frequently neglected, however it is a considerable error.

Power consumed by one blade element

$$dP_0 = \frac{1}{2} \rho U^2 C_d \cdot C \cdot dr \cdot U$$

where  $U = \sqrt{U_T^2 + U_R^2}$ , for  $b$ -blades

$$U_T = U_T(2\pi)$$

$$U_R = U_R(2\pi)$$

$$2\pi = 2\pi t$$

$$P_0 = b \int_0^R \frac{dP_0}{dr} dr$$

The average power (power over ~~over~~ a revolution)

$$P_0 = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dP_0}{dr} dr d\psi \Rightarrow \text{the resulting expression can not be integrated in closed form}$$

Integration can be performed numerically

$$P_0 = \frac{1}{8} \rho C_d b c \Omega^3 R^4 (1 + 4.6 \mu^2) \quad 0 < \mu < 0.50$$

$$C_{P_0} = \frac{P_0}{(\rho \Omega^3 R^4) (2\pi)^2}$$

$$C_{P_0} = \frac{\rho C_d}{8} (1 + 4.6 \mu^2)$$

(16)  $\approx$  radial flow component is neglected

Note this equation (16), includes radial flow effects; without radial flow effects one has

$$C_{P_0} = \frac{\rho C_d}{8} [1 + 3 \mu^2] \quad (17)$$

$C_{Q_0}$  = power required to turn the rotor is different from this expression and is given by

$$C_{Q_0} = \frac{\rho C_d}{8} [1 + \mu^2]$$

and it different from both Eqs (16) and (17)

Consider a detailed evaluation of  $P_0$  with and without the radial flow terms:

$$P_0 = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dP_0}{dr} dr d\psi =$$

$$= \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{1}{2} \rho v^3 c_{D0} c dr d\psi$$

$$= \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{1}{2} \rho c c_{D0} \rho \left[ (\partial R \mu \cos \psi)^2 + \partial R \left( \frac{r}{R} + \mu \sin \psi \right)^2 \right]^{3/2} dr d\psi$$

$$= \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{1}{2} \rho c c_{D0} \left[ \partial^2 R^2 (\mu^2 \cos^2 \psi + \mu^2 \sin^2 \psi) + \partial^2 R^2 \left( \frac{r}{R} \right)^2 + \right.$$

$$\left. + 2 \partial R \left( \frac{r}{R} \right) \mu \sin \psi \right]^{3/2} dr d\psi =$$

$$= \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{1}{2} \rho c c_{D0} \left[ \partial^2 R^2 \mu^2 + \partial^2 R^2 \left( \frac{r}{R} \right)^2 + 2 \partial R \left( \frac{r}{R} \right) \mu \sin \psi \right]^{3/2} dr d\psi$$

$$= \left( \frac{b}{2\pi} \right) \frac{1}{2} \rho c c_{D0} \partial^2 R^3 \int_0^{2\pi} \int_0^R \left[ \mu^2 + \left( \frac{r}{R} \right)^2 + 2 \left( \frac{r}{R} \right) \mu \sin \psi \right]^{3/2} dr d\psi$$

$$C_{P0} = \frac{1}{\rho \pi R^2 (\partial R)^3} \left( \frac{b}{2\pi} \right) \frac{1}{2} \rho c c_{D0} \partial^2 R^3 \int_0^{2\pi} \int_0^R \left[ \mu^2 + \left( \frac{r}{R} \right)^2 + 2 \frac{r}{R} \frac{1}{\partial R} \mu \sin \psi \right]^{3/2} dr d\psi$$

$$= \frac{1}{4} \underline{c_{D0} R} \int_0^{2\pi} \int_0^R \left[ \mu^2 + \left( \frac{r}{R} \right)^2 + 2 \frac{r}{R} \frac{1}{\partial R} \mu \sin \psi \right]^{3/2} dr d\psi$$

If the radial flow component is neglected :

$$P_0 = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dP_0}{dr} dr d\psi = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{1}{2} \rho v^3 c_{D0} c dr d\psi$$

$$= \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{1}{2} \rho c_{D0} c \cdot \partial^2 R^3 \left( \frac{r}{R} + \mu \sin \psi \right)^3 dr d\psi$$

$$= \frac{b}{2\pi} \frac{1}{2} \rho c_{D0} c \partial^2 R^3 \int_0^{2\pi} \int_0^R \left[ \left( \frac{r}{R} \right)^3 + \mu^3 \sin^3 \psi + 3 \left( \frac{r}{R} \right)^2 \mu \sin \psi + \right.$$

$$\left. + 3 \left( \frac{r}{R} \right) \mu^2 \sin^2 \psi \right] dr d\psi = - \frac{b}{2\pi} \frac{1}{2} \rho c_{D0} c \partial^2 R^4 \int_0^{2\pi} \int_0^R \left[ x^3 \right.$$

$$\left. + \mu^3 \sin^3 \psi + 3 x^2 \mu \sin \psi + 3 x \mu^2 \sin^2 \psi \right] dx dy$$

52

$$\begin{aligned} &= \frac{b}{2\pi} \frac{1}{2} \rho c c_{D0} \sigma^3 R^4 \left[ \frac{x^4}{4} 2\pi + 0 + 0 + \frac{3}{2} x^2 \mu^2 \pi \right] \Big|_0^1 \\ &= \frac{b}{2\pi} \frac{1}{2} \rho c c_{D0} \sigma^3 R^4 \left[ \frac{1}{2} \pi + \frac{3}{2} \mu^2 \pi \right] = \\ &= \frac{b}{8} \rho c c_{D0} \sigma^3 R^4 (1 + 3\mu^2) \end{aligned}$$

$$\frac{C_{p0}}{C_0} = \frac{P_0}{\rho \pi R^2 (2R)^3} = \frac{bc}{\pi R} \frac{c_{D0}}{8} (1 + 3\mu^2) = \frac{5c_{D0}}{8} (1 + 3\mu^2)$$

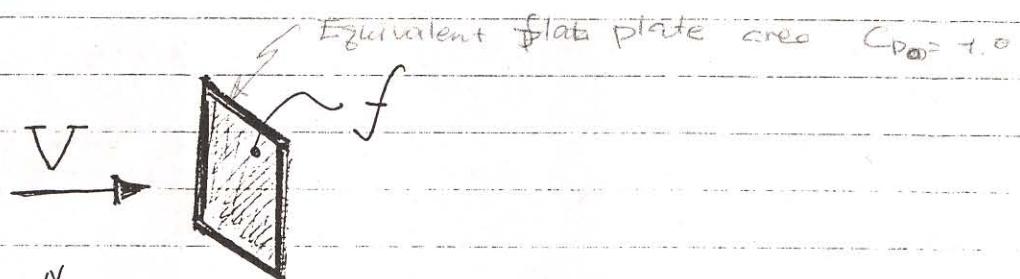
where we have used the relations

$$\int_0^{2\pi} \sin \psi d\psi = \int_0^{2\pi} \sin^3 \psi d\psi = 0$$

$$\int_0^{2\pi} \sin^2 \psi d\psi = \int_0^{2\pi} \cos^2 \psi d\psi = \pi$$

### Parasite Power (Fuselage Equivalent Drag Factor)

Let  $f$  be an equivalent flat plate area representative of a helicopter fuselage. Also recall that the drag of a flat plate, perpendicular to the flow can be represented by a drag coefficient  $C_D \approx 1.0$



$$1.f = \sum_{n=1}^N C_{Dn} A_n \quad \text{this expression can be built up, using conventional drag coefficient } C_{Dn}$$

based on the appropriate areas of the components. Interference can be also accounted for in a conventional way.

$$P_p = D_p V = \frac{1}{2} \rho V^2 1.f V = \frac{1}{2} \rho V^3 f$$

$$C_{pp} = \frac{f}{2\pi R^2} \left( \frac{V}{\delta R} \right)^3 = \frac{f}{2\pi R^2} \left( \frac{\mu}{\cos \alpha} \right)^3 \approx \frac{f}{2\pi R^2} \mu^3 \quad (18)$$

recall

$$\frac{V \cos \alpha}{\delta R} = \mu \quad \alpha < 10^\circ$$

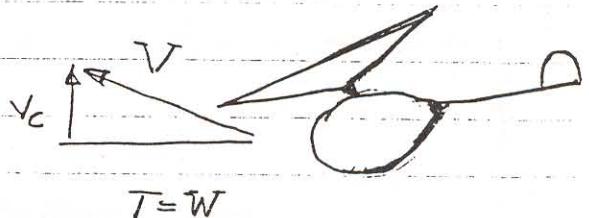
$$0 < \alpha < 10^\circ$$

A reasonable number for a helicopter is

$$\boxed{-\frac{f}{\pi R^2} \approx 0.01}$$

Climb Power

$$P_c = W \frac{dh}{dt} = W V_c$$



$$C_{Pc} = C_T \frac{V_c}{\rho R} = C_T \delta \quad (1g)$$

where  $\delta = \frac{V_c}{\rho R}$   $\equiv$  nondimensional rate of climb

Total Power

$$C_p \approx \underbrace{\frac{C_{D0}}{8} [1 + 4.6 \mu^2]}_{\text{profile power}} + \underbrace{\frac{C_T^2}{2\mu}}_{\text{approx. induced}} + \underbrace{\frac{f}{2\pi R^2} \mu^3}_{\text{parasite power}} + C_T \delta \quad (20)$$

(radial flow is included)

These equations all neglect compressibility and stall effects which are not included in a simple energy method

