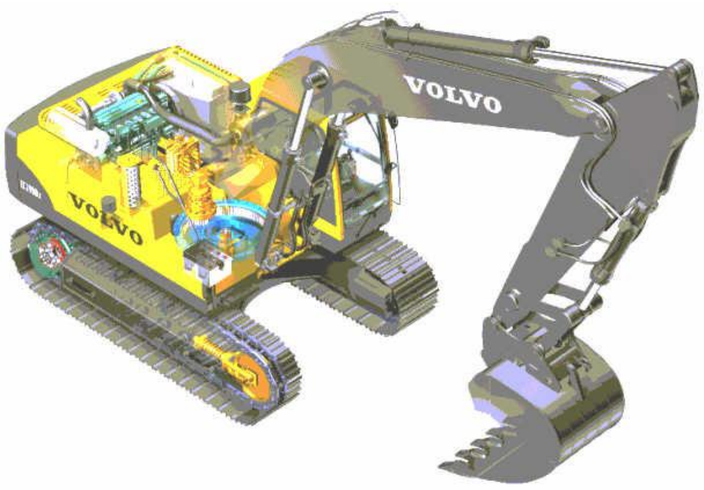
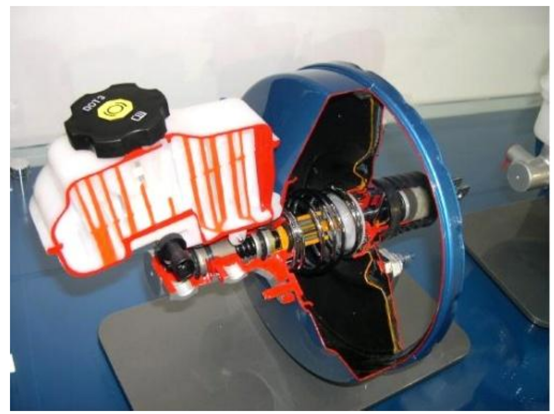
Fluid Systems

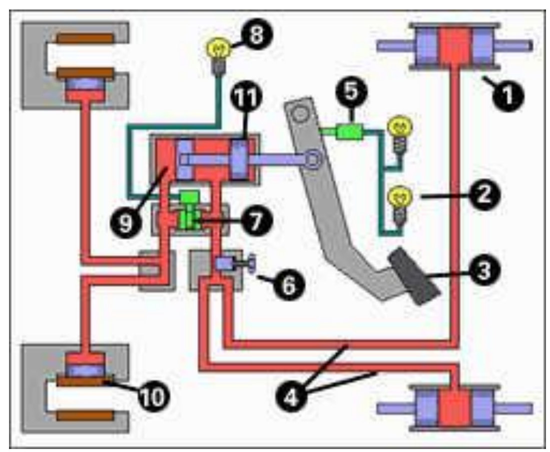
Hydraulic Systems



Excavator



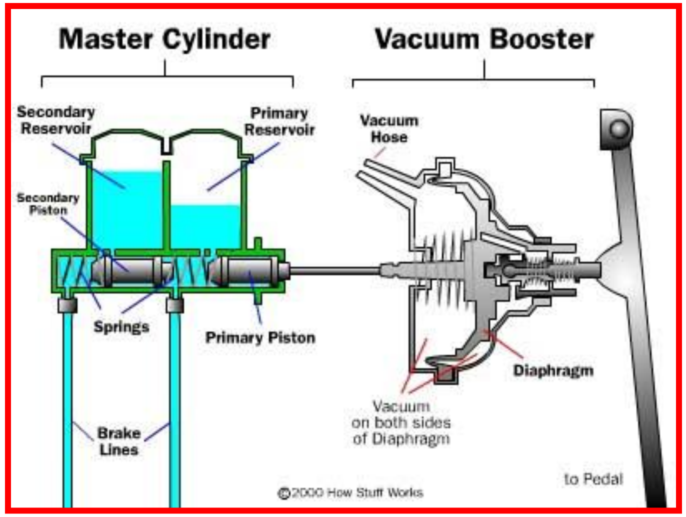
Brake System

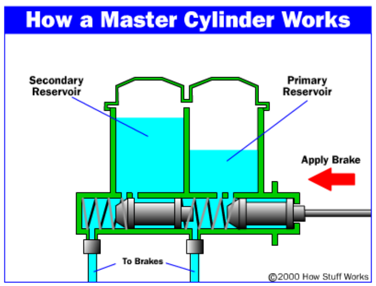


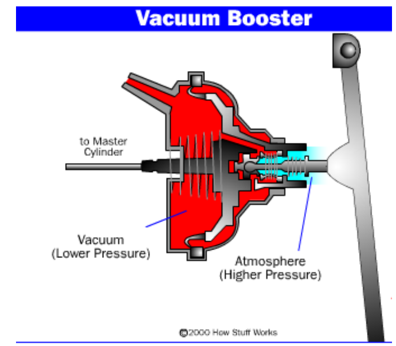
1) Wheel Cylinder
2) Brake Light
3) Brake Pedal
4) Rear Brake Lines
5) Stop Light Switch (Mechanical)
6) Front/Rear Balance Valve
7) Pressure Differentiavl Valve
8) Brake Warning Lamp
9) Brake Fluid
10) Brake Pad
11) Master Cylinder

Brake System

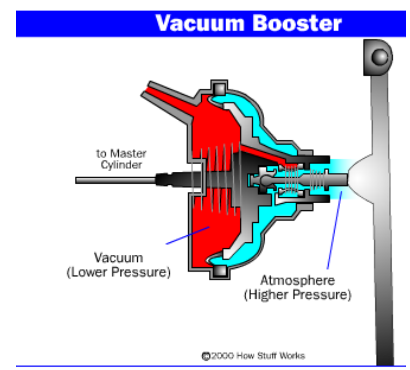
Brake Pedal → Vacuum Booster → Master Cylinder → Proportionnig Valve → Font Wheel / Rear Wheel



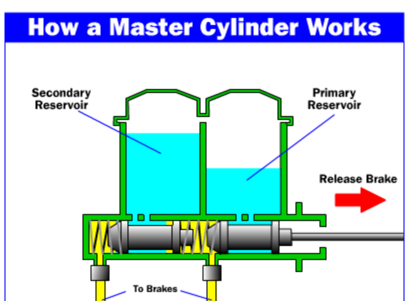




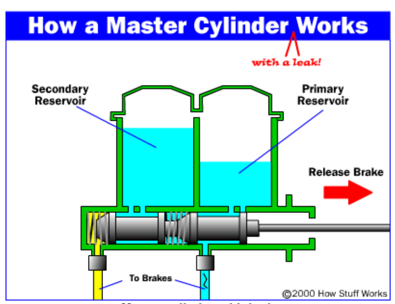
Apply brake



w/o leak



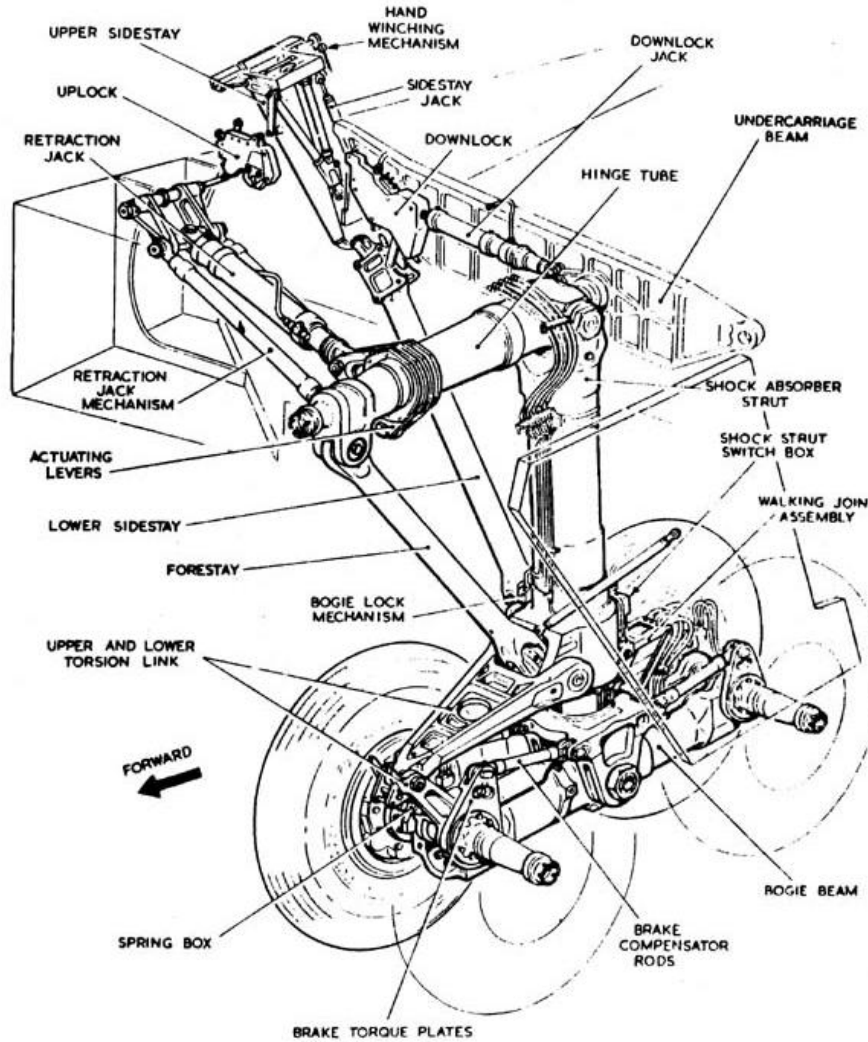
with leak



Master cylinder with leak

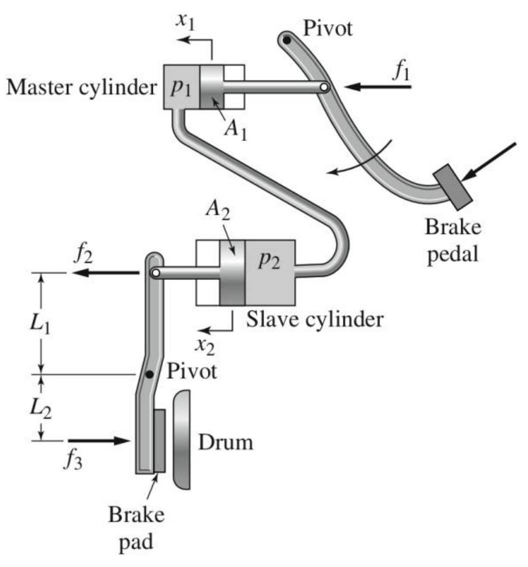
Hydraulic Systems : Landing Gear System

Landing gear system of AIRBUS A330



Conservation of Mass

$$\dot{m} = q_{mi} - q_{mo}$$



$$i)\ A_1 dx_1 = A_2 dx_2$$

$$ii)\ p_1 = p_2 + \rho g h$$

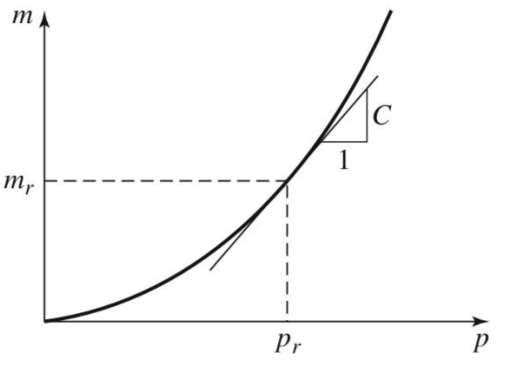
$$f_1 =$$

$$f_2 =$$

$$f_2 = \qquad dx_2 =$$

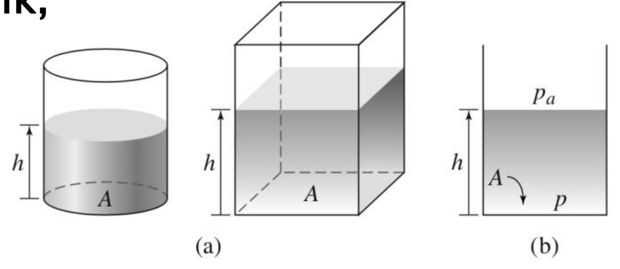
Fluid Capacitance

$C =$



General fluid capacitance

For a tank,



$$C = \rho A \frac{dh}{dp} = \rho A \frac{1}{\rho g} = \frac{A}{g}$$

$$\frac{dm}{dt} =$$

$$C \frac{dp}{dt} = \rho A \frac{dh}{dt} = q_{mi} - q_{mo}$$

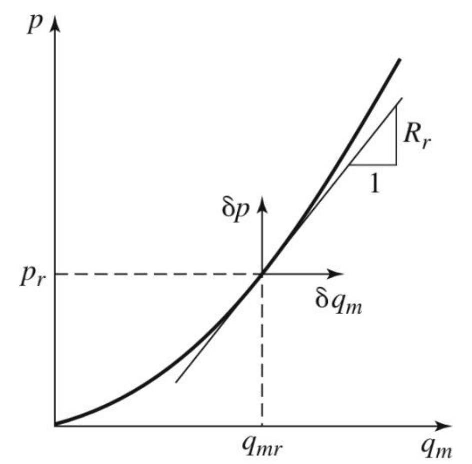
For liquid level system, C is defined as

$$C = \frac{change\ in\ liquid\ stored}{change\ in\ head} \frac{m^3}{m} = \rho A$$

use h(pressure head) , instead of p.

Fluid Resistance

$R = $



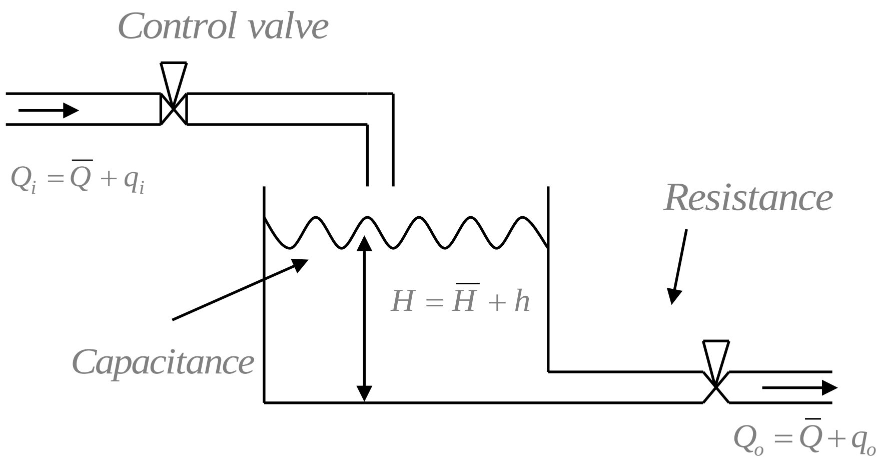
$$q_m = \frac{p}{R}$$ For laminar flow

$$q_m = \sqrt{\frac{p}{R_l}}$$ For turbulent flow

For liquid level system, C is defined as

$$R = \frac{change\ in\ level\ difference}{change\ in\ flow\ rate} \frac{m}{m^3/s} = \frac{dh}{dq_m}$$

Liquid Level Systems



$$R = \frac{\text{change in level difference}}{\text{change in flow rate}} \quad \frac{m}{m^3/s}$$

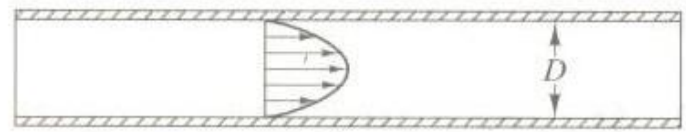
$$C = \frac{\text{change in liquid stored}}{\text{change in head}} \quad \frac{m^3}{m}$$

$$\text{Steady state}: \quad Q_i = \bar{Q} = Q_0 =$$

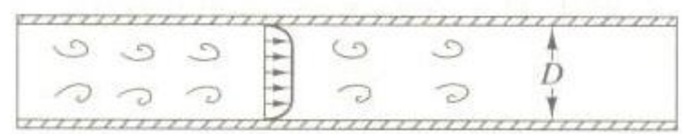
$$\text{consider}, \quad Q_o = \frac{H}{R}, \qquad Q_i - Q_o =$$

Basic Concepts

Laminar flow : When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.



Turbulent flow : It can be interpreted that when the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent.



Reynolds number (Re) : The ratio of inertial forces to viscous forces.

It is used to determine whether a flow will be laminar or turbulent.

$$Re = \frac{\rho v D}{\mu}$$

μ : *the dynamic viscousity of the fluid*

ρ : *density of the fluid*

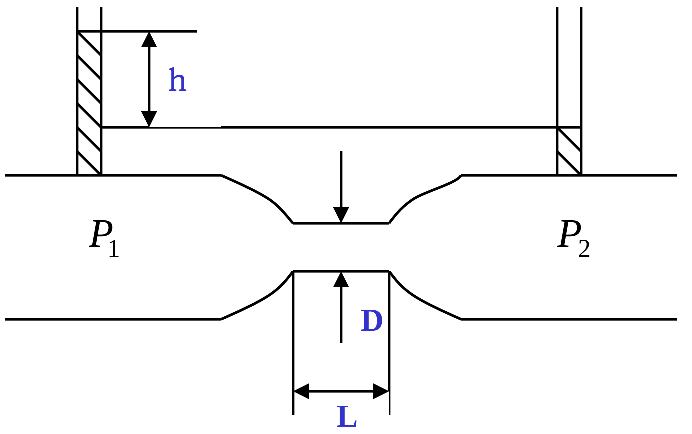
D : *diameter*

$Re < 2000$: always laminar

$Re > 4000$: always turbulent

Laminar Flow

Cylindrical pipe



$$P_1 - P_2 = \rho g h, \quad Q\frac{128\nu L}{g\pi D^4} = h$$

$\nu : viscosity,$

$L : length\ of\ \ pipe$

$D : diameter\ of\ \ pipe$

$$Q = \frac{h}{R} = K_l\, h, \qquad R = \frac{128\nu L}{g\pi D^4} \quad [s/m^2]$$

$Q\ \ : steady-state\ \ flow\ rate$

$K_l\ : constant$

$h\ \ : steady-state\ \ head$

Laminar Flow

Liquid level dynamics :

$$C\frac{dH}{dt}=$$

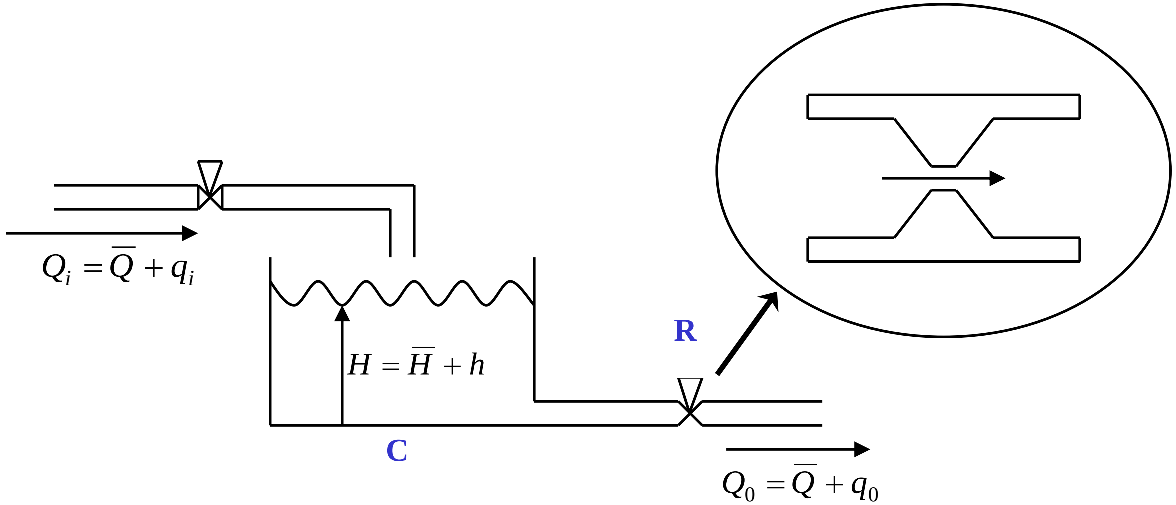
$$=\bar{Q}+q_i-(\bar{Q}+q_o)=q_i-q_o$$

$$\frac{dH}{dt}=\frac{d}{dt}(\bar{H}+h)=\frac{dh}{dt}$$

$$Q_o = ($$

$$\rightarrow \frac{dh}{dt}=$$

Turbulent Flow



$$Q = C_d \cdot a \cdot \sqrt{\frac{2}{\rho}(P_1 - P_2)}$$

$\rho : density, \quad a : area, \quad C_d : discharge\ coefficient$

$$steady\ state:\ Q_i = \overline{Q} = Q_0 = K_t \sqrt{\overline{H}}$$

Turbulent Flow

Liquid level dynamics

$$C\frac{dH}{dt}=Q_i-Q_0=\overline{Q}+q_i-K\sqrt{H}$$

$$\frac{dH}{dt}=\frac{1}{C}Q_i-\frac{1}{C}K\sqrt{H}=f(Q_i,H)$$

$$f(Q_i,H)=f(\overline{Q}_i,\overline{H})+\left.\frac{\partial f}{\partial Q_i}\right|_{\overline{Q},\overline{H}}(Q_i-\overline{Q})+\frac{1}{2!}\frac{\partial^2 f}{\partial Q_i^2}(Q_i-\overline{Q})^2+\cdots$$

$$\cdots+\left.\frac{\partial f}{\partial H}\right|_{\overline{Q},\overline{H}}(H-\overline{H})+\frac{1}{2!}\frac{\partial^2 f}{\partial H^2}(H-\overline{H})^2+\cdots$$

$$=\frac{1}{C}\overline{Q}-\frac{1}{C}K\sqrt{\overline{H}}+\left(\frac{1}{C}q_i-\frac{K}{C\cdot 2\sqrt{\overline{H}}}h\right)+\ high\ \ order\ \ term$$

Turbulent Flow

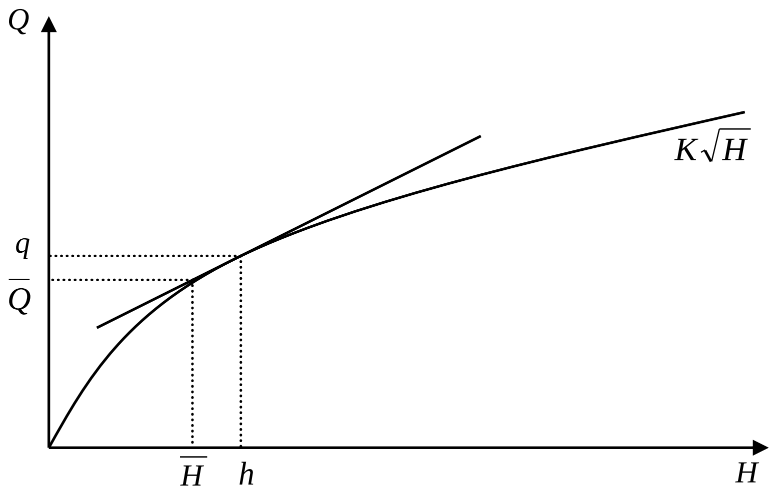
$$\frac{dH}{dt} = \frac{d\bar{H}}{dt} + \frac{dh}{dt}$$

$$\Rightarrow \qquad \frac{dh}{dt} = \frac{1}{C} q_i - \frac{K}{C \cdot 2\sqrt{\bar{H}}} h$$

$$q_o = \frac{h}{R}, \qquad R = \frac{2\sqrt{\bar{H}}}{K} = \frac{2\bar{H}}{\bar{Q}}, \qquad \left(\bar{Q} = K\sqrt{\bar{H}} \right)$$

$$\therefore \begin{cases} \dfrac{dh}{dt} = -\dfrac{1}{CR} h + \dfrac{1}{C} q_i \\ q_o = \dfrac{h}{R} \end{cases} \qquad \textit{for small} \quad q_i$$

Linearization



$$Q = \overline{Q} + q = K\sqrt{\overline{H}} + (\quad)\cdot h$$

$$y = f(x), \qquad \overline{y} = f(\overline{x}), \qquad x = \overline{x} + \Delta x$$

$$y = f(\overline{x}) + \left.\frac{\partial f}{\partial x}\right|_{\overline{x}} \cdot \Delta x + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial x^2}\right|_{\overline{x}} \cdot \Delta x^2 + \cdots$$

$$\approx \overline{y} + \left.\frac{\partial f}{\partial x}\right|_{\overline{x}} \cdot \Delta x$$

$$y - \overline{y} = \left.\frac{\partial f}{\partial x}\right|_{\overline{x}} (x - \overline{x}) = K(x - \overline{x})$$

$$y = f(x_1, x_2) \approx f(\overline{x_1}, \overline{x_2}) + \left.\frac{\partial f}{\partial x_1}\right|_{\overline{x}_1, \overline{x}_2} (x_1 - \overline{x_2}) + \left.\frac{\partial f}{\partial x_2}\right|_{\overline{x}_1, \overline{x}_2} (x_2 - \overline{x_2})$$

$$= \overline{y} + K_1(x_1 - \overline{x_1}) + K_2(x_2 - \overline{x_2})$$

$$\therefore\ y - \overline{y} = K_1(x_1 - \overline{x_1}) + K_2(x_2 - \overline{x_2})$$

Summary of Liquid Level Systems

Laminar flow

$$\frac{dh}{dt} = -\frac{1}{CR}h + \frac{1}{C}q_i$$

$$\bar{Q} = K \cdot \bar{H} \qquad R = \frac{\bar{H}}{\bar{Q}}$$

$$R = \frac{128\nu L}{g\pi D^4}$$

$$q_o = \frac{h}{R}$$

Turbulent flow

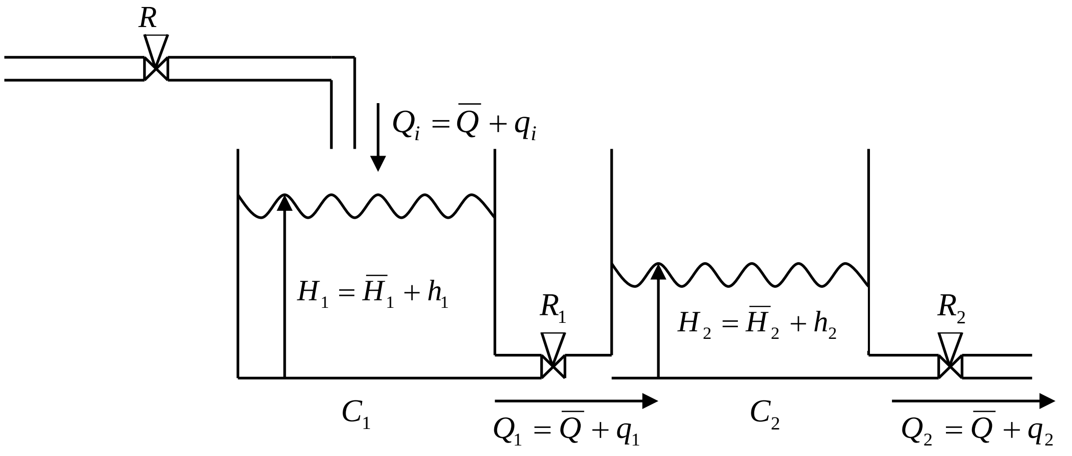
$$\frac{dh}{dt} = -\frac{1}{CR}h + \frac{1}{C}q_i$$

$$q_o = \frac{h}{R}$$

$$R = \frac{2\bar{H}}{\bar{Q}} = \frac{2\sqrt{\bar{H}}}{K} = \frac{2\sqrt{\bar{H}}}{C_d a\sqrt{2g}}$$

$$\left(\because \bar{Q} = K\sqrt{\bar{H}} = C_d a\sqrt{\frac{2}{\rho}\rho g\bar{H}} = C_d a\sqrt{2g}\sqrt{\bar{H}} \right)$$

Liquid Level Systems with Interaction



Steady state :

$$Q_i = \bar{Q} = Q_1 = Q_2$$

$$= \frac{\bar{H}_1 - \bar{H}_2}{R_1} = \frac{\bar{H}_2}{R_2}$$

Liquid level dynamics :

$$C_1 \frac{dH_1}{dt} = \bar{Q} + q_i - \left(\bar{Q} + q_1\right) \rightarrow C_1 \frac{dh_1}{dt} = q_i - q_1$$

$$Q_1 = \bar{Q} + q_1 = \frac{1}{R_1}\left(\bar{H}_1 + h_1 - (\bar{H}_2 + h_2)\right) \rightarrow q_1 = \frac{h_1 - h_2}{R_1}$$

$$C_2 \frac{dH_2}{dt} = \bar{Q} + q_1 - \left(\bar{Q} + q_2\right) \rightarrow C_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$Q_2 = \bar{Q} + q_2 = \frac{\bar{H}_2 + h_2}{R_2} \rightarrow q_2 = \frac{h_2}{R_2}$$

Liquid Level Systems with Interaction

$$\therefore \begin{cases} \dfrac{dh_1}{dt} = -\dfrac{1}{C_1}\left(\dfrac{h_1 - h_2}{R_1}\right) + \dfrac{1}{C_1} q_i \\ \\ \dfrac{dh_2}{dt} = \dfrac{1}{C_2}\left(\dfrac{h_1 - h_2}{R_1}\right) - \dfrac{1}{C_2}\dfrac{h_2}{R_2} \end{cases}$$

$$Transfer\ Function = \frac{Q_2(S)}{Q_i(S)} = \frac{1}{R_1C_1R_2C_2S^2 + \left(R_1C_1 + R_2C_2 + R_2C_1\right)S + 1}$$