457.309.02 Hydraulics and Laboratory

.09 Turbulent flow in rough pipe

Prepared by Jin Hwan Hwang

(2019.04.03)

Today's objectives

- Similarly to the smooth pipe case, turbulent flow in rough pipe will be understood.
- Classifying the pipes whether they are smooth or rough
- Evaluating pipe friction factors in the given condition

Turbulent flow – Rough pipes

- Pipe friction in rough pipe will be governed by the size and patterns of the roughness (like as sediment particles)
- As we learned through the last class, velocity profile follows logarithm no matter what flow is laminar or turbulent.
- When roughness is high, the viscous layer will be canceled and viscosity may not be the important parameter.
- Experiment proved that the viscosity can be replaced by the roughness in the rough wall condition.

$$\frac{u}{u_*} = 5.75 \log \frac{y}{e} + 8.5 \qquad \text{(Rough pipe, when } e \text{ is roughness)}$$

- In this condition Q (flow rate) can be determined as

$$Q = \int_0^R u(2\pi r\, dr) = 2\pi u_* \int_0^R \left[5.75 \log\left(\frac{R-r}{e} \right) + 8.5 \right] r\, dr$$

Turbulent flow – Rough pipes

- Therefore, the mean velocity is

$$\frac{V}{u_*} = 5.75\log\frac{R}{e} + 4.75$$

- Since

$$u_* = V\sqrt{\frac{f}{8}}$$

$$\frac{1}{\sqrt{f}} = 2.03\log\frac{R}{e} + 1.68$$

- With experimental adjustment,

$$\frac{1}{\sqrt{f}} = 2.0\log\frac{d}{e} + 1.14$$

Rough pipes' friction factor

Example

- The mean velocity in a 300 mm pipeline is 3m/s. The relative rough ness of the pipe is 0.002 and the kinematic viscosity of the water is 9 x 10^{-7} m^2/s. Determine the friction factor, the centerline velocity, the v elocity 50 mm from the pipe wall, and the head lost in 300 m of this pipe under the assumption that the pipe is rough.

  – Friction factor
  – Centerline velocity
  – The velocity at 50 mm from the wall
  – Head loss

Example

- Friction factor

$$\frac{1}{\sqrt{f}} = 2.0\log\frac{d}{e} + 1.14$$

- Centerline velocity

$$\frac{u_c}{u_*} = 5.75\log\frac{R}{e} + 8.5, \qquad u_* = V\sqrt{\frac{f}{8}}$$

- The velocity at 50 mm from the wall

$$\frac{u_{50}}{u_*} = 5.75\log\frac{y_{50}}{e} + 8.5$$

- Head loss

$$h_L = f\frac{l}{d}\frac{V^2}{2g_n}$$

Classification of smoothness and roughness

- What parameters do we need to compare to know whether flow is s mooth or rough?
- To cancel this, we need to see the ratio between this and that.
  - This is viscous sublayer thickness and roughness height
- What is the viscous sublayer thickness in Laminar flow?
  - Since viscosity governs overall depth in pipe, R should be the thickness.
  - Therefore, in a Laminar flow, no matter what smooth or rough,

$$f = \frac{64}{\mathrm{Re}}$$

- In turbulent flow,

$$\frac{e}{\delta_v} = \frac{e/d}{\delta_v/d} = \frac{e/d}{32.8/\mathrm{Re}\sqrt{f}} = \left(\frac{e}{d}\right)\frac{\mathrm{Re}\sqrt{f}}{32.8}$$

$$\frac{e}{d}\mathrm{Re}\sqrt{f} = 32.8\frac{e}{\delta_v}$$

Classification of smoothness and roughness

- Now we can plot the results with

$$\frac{e}{d}\mathrm{Re}\sqrt{f} \qquad \text{and} \qquad \frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e}$$

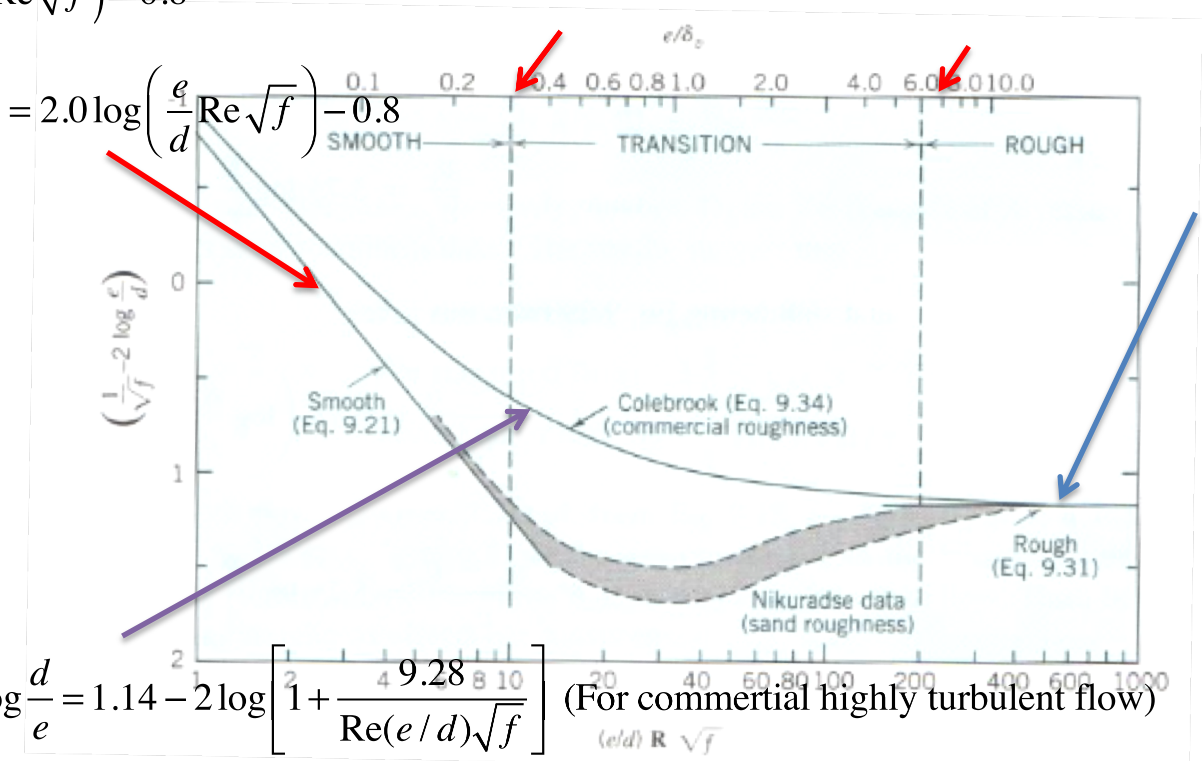
$$\frac{1}{\sqrt{f}} = 2.0\log\left(\mathrm{Re}\sqrt{f}\right) - 0.8$$

$$\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 2.0\log\left(\frac{e}{d}\mathrm{Re}\sqrt{f}\right) - 0.8$$

Smooth flow

$$\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 1.14$$

Fully rough flow



$$\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 1.14 - 2\log\left[1 + \frac{9.28}{\mathrm{Re}(e/d)\sqrt{f}}\right]$$ (For commercial highly turbulent flow)

Classification of smoothness and roughness

- Classifications

For smooth flow: $\frac{e}{d}\mathrm{Re}\sqrt{f} \le 10$

For transition flow: $10 < \frac{e}{d}\mathrm{Re}\sqrt{f} < 200$

For rough flow: $200 \le \frac{e}{d}\mathrm{Re}\sqrt{f}$

Classification based on velocity profile

- Another means of classification of the roughness effects is to use the velocity profiles directly. In all pipes (page 331),

$$\frac{u_c - u}{u_*} = 5.75 \log \frac{R}{y}$$

- In rough pipes,

$$\frac{u}{u_*} = 5.75 \log \frac{y}{e} + 8.5$$

$$\frac{u_c}{u_*} = 5.75 \log \frac{R}{e} + 8.5 \quad \text{(At center)}$$

$$\frac{u_c - u}{u_*} = 5.75 \left(\log \frac{R}{e} - \log \frac{y}{e} \right)$$

$$= 5.75 \log \frac{R}{y}$$

Classification based on velocity profile

- Now let's modify the equation
  - For both (smooth and rough)

$$\frac{u}{u_*} = \frac{u_c}{u_*} + 5.75\log\frac{y}{R}$$

$$= \frac{u_c}{u_*} + 5.75\log\frac{e}{R} + 5.75\log\frac{y}{e}$$

$$= A + 5.75\log\frac{y}{e}$$

$$A = \frac{u_c}{u_*} + 5.75\log\frac{e}{R}$$

$$\frac{u}{u_*} = 5.75\log\frac{u_* y}{\nu} + 5.5 \quad \text{(Smooth pipe)}$$

$$= 5.5 + 5.75\log\frac{u_* e}{\nu} + 5.75\log\frac{y}{e}$$

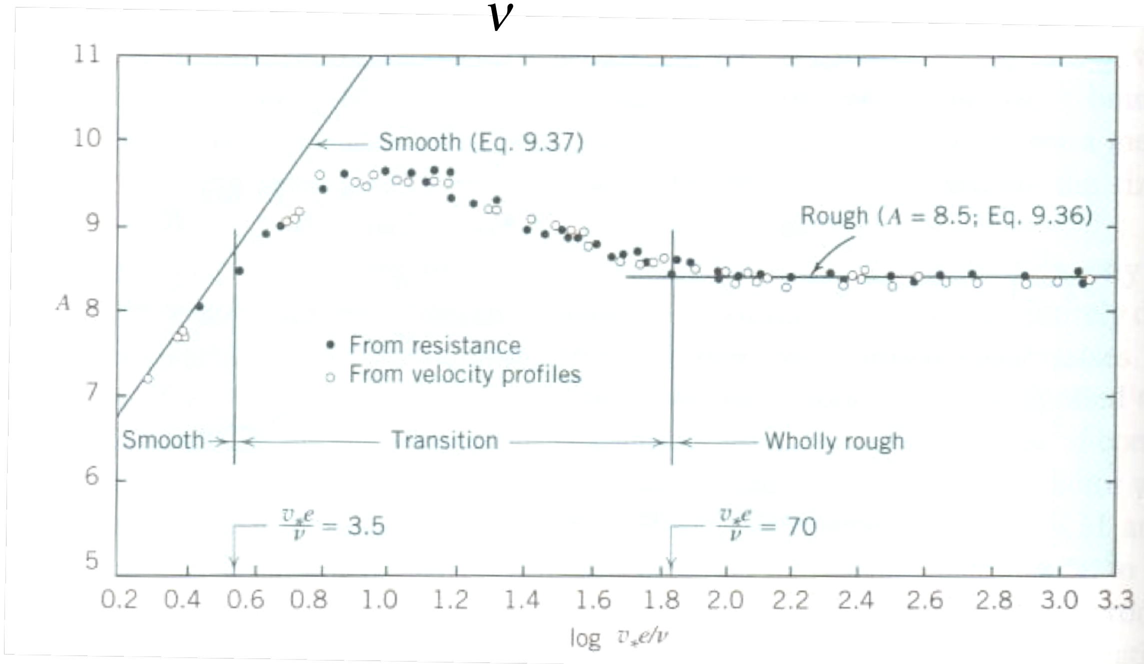
$$= A + 5.75\log\frac{y}{e}$$

$$A = 5.5 + 5.75\log\frac{u_* e}{\nu}$$

- In smooth flow, A=8.5 from experiment.
- Check the page 332, equation 9.17,

Classification based on velocity profile

- In the previous equation $\dfrac{u_* e}{\nu}$ is ***the Roughness Reynolds Number***.



$$\left(11.6/\delta_v\right)e = u_* e/\nu \le 3.5 \qquad \text{Smooth flow}$$

$$3.5 < \left(11.6/\delta_v\right)e = u_* e/\nu < 70 \qquad \text{Transition flow}$$

$$70 \le \left(11.6/\delta_v\right)e = u_* e/\nu \qquad \text{Wholly rough flow}$$