5. Rotor Design, Stability & Control and Model Flight Testing

by Dr. James Wang

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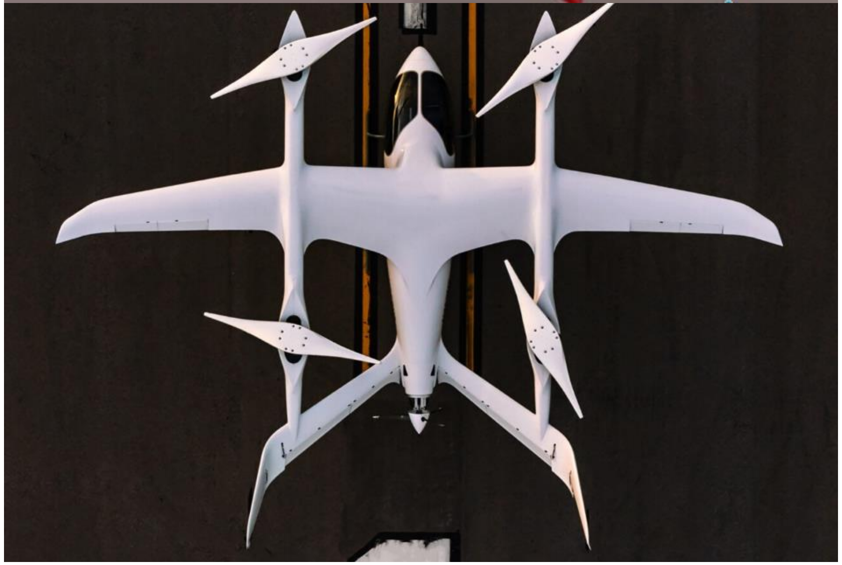
Topics

1. Rotor design
2. How helicopters are controlled
3. Controls for different eVTOL Aircraft
4. Aircraft stability
5. Stability & Control analysis
6. Model testing

Different Types of Rotors

Fixed Pitch or Variable Pitch ?

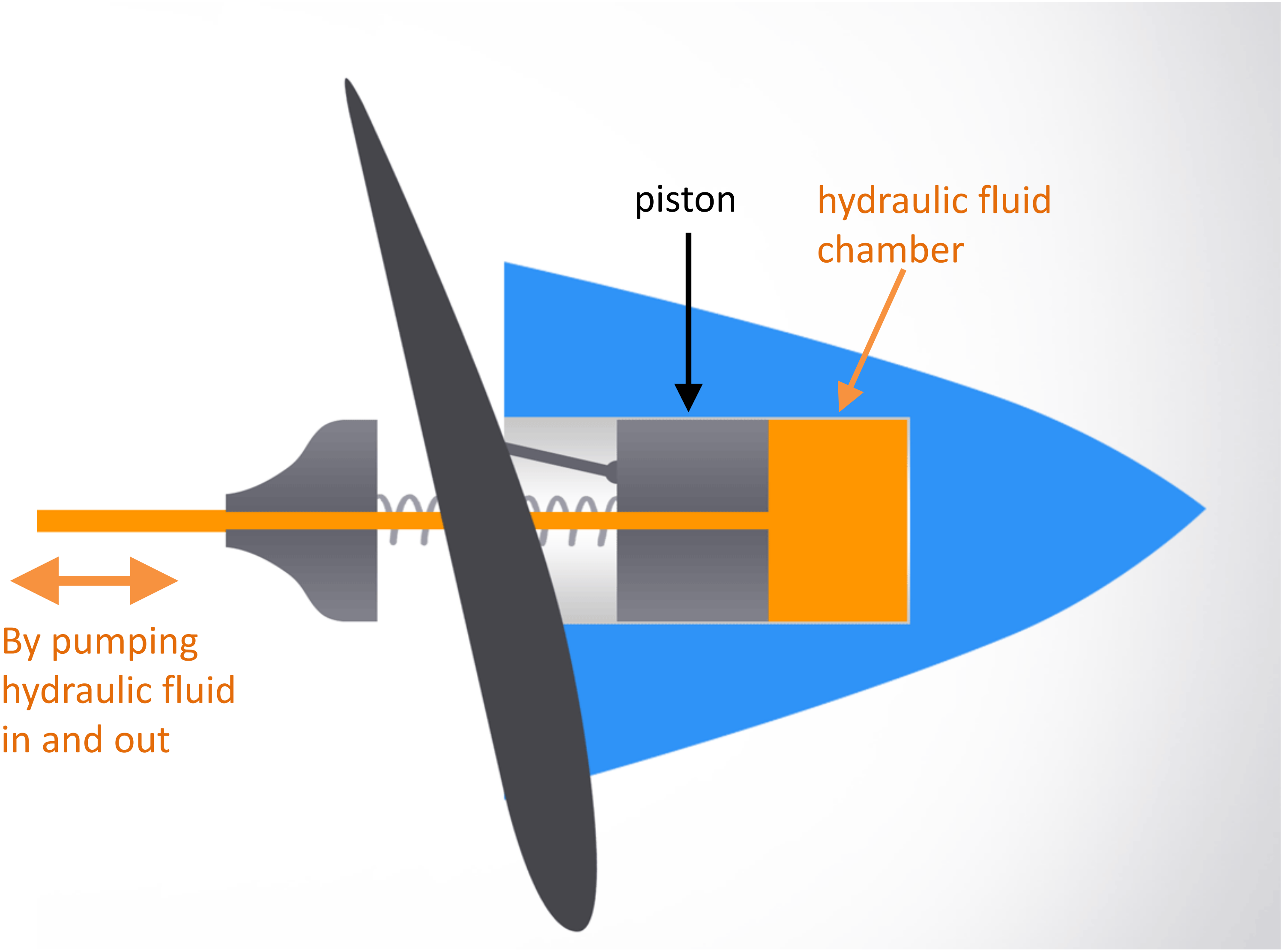
Most Quadcopters and Multi-Rotors Use Fixed Pitch Rigid Propellers



Simple Collective Pitch Change Only

(Means all blades change pitch angle together)

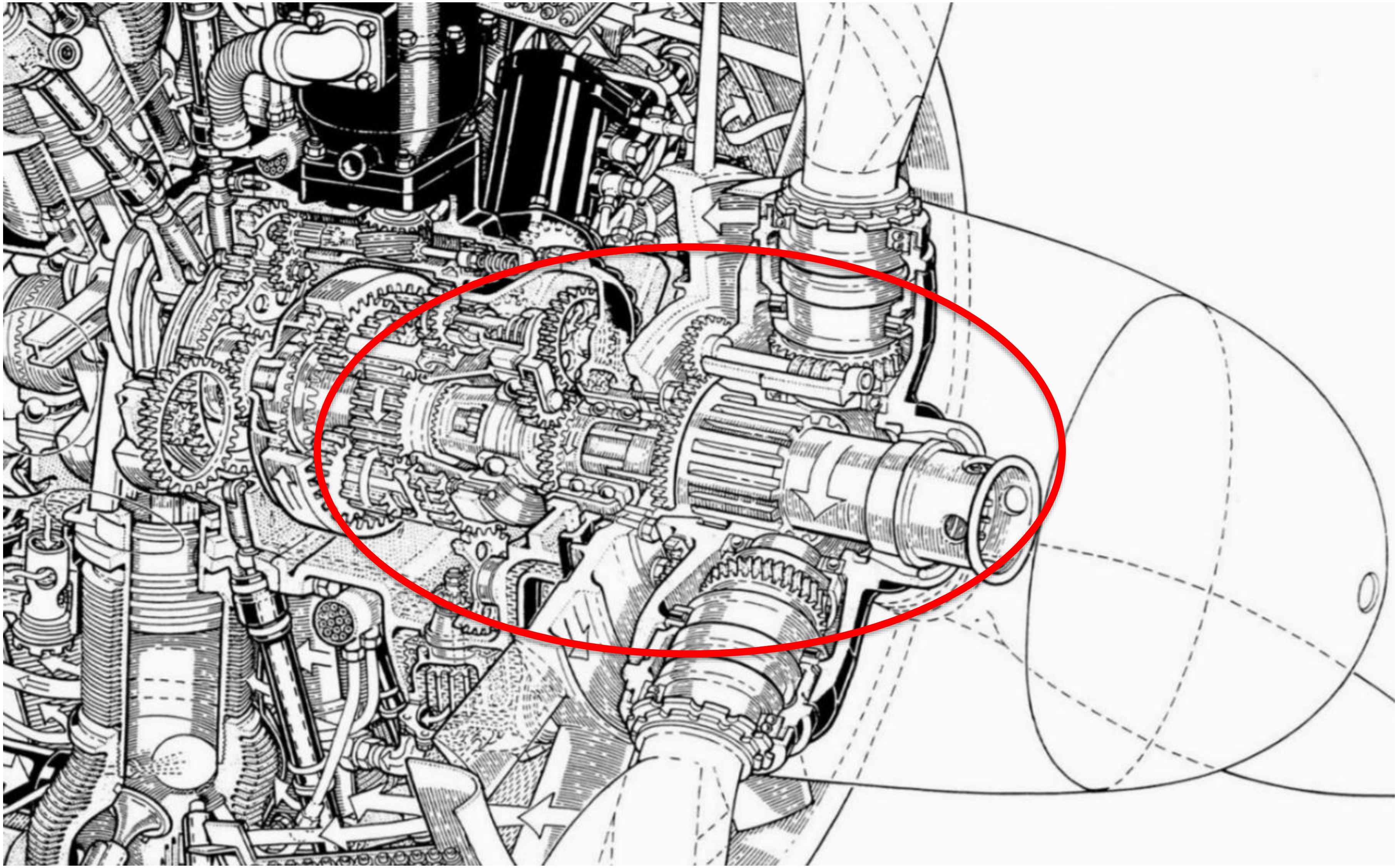
Pumping Hydraulic Fluid to Change Pitch



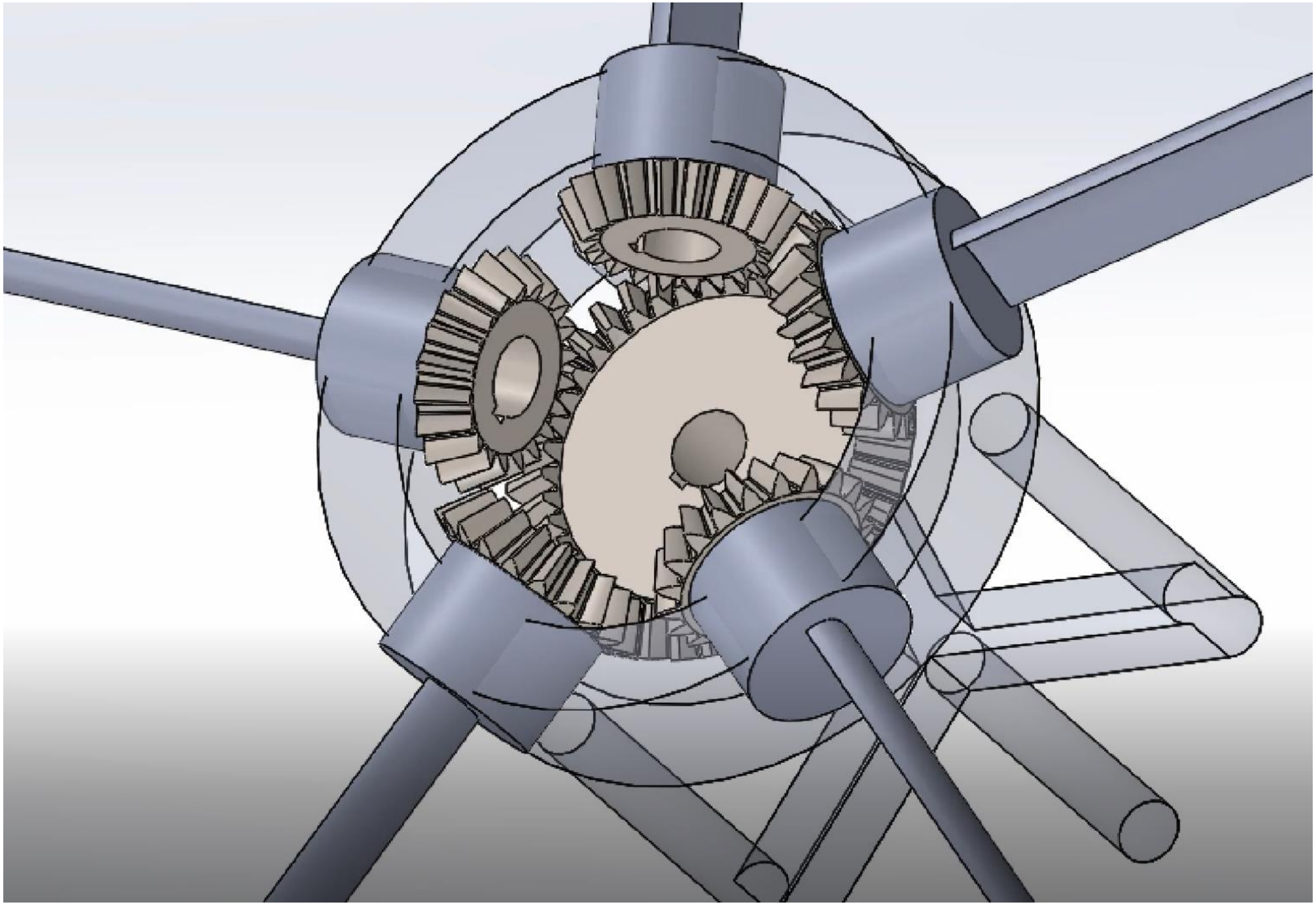
By pumping hydraulic fluid in and out

Source: https://www.boldmethod.com/learn-to-fly/aircraft-systems/how-a-constant-speed-prop-works/

Using Gears to Change Pitch on FW190

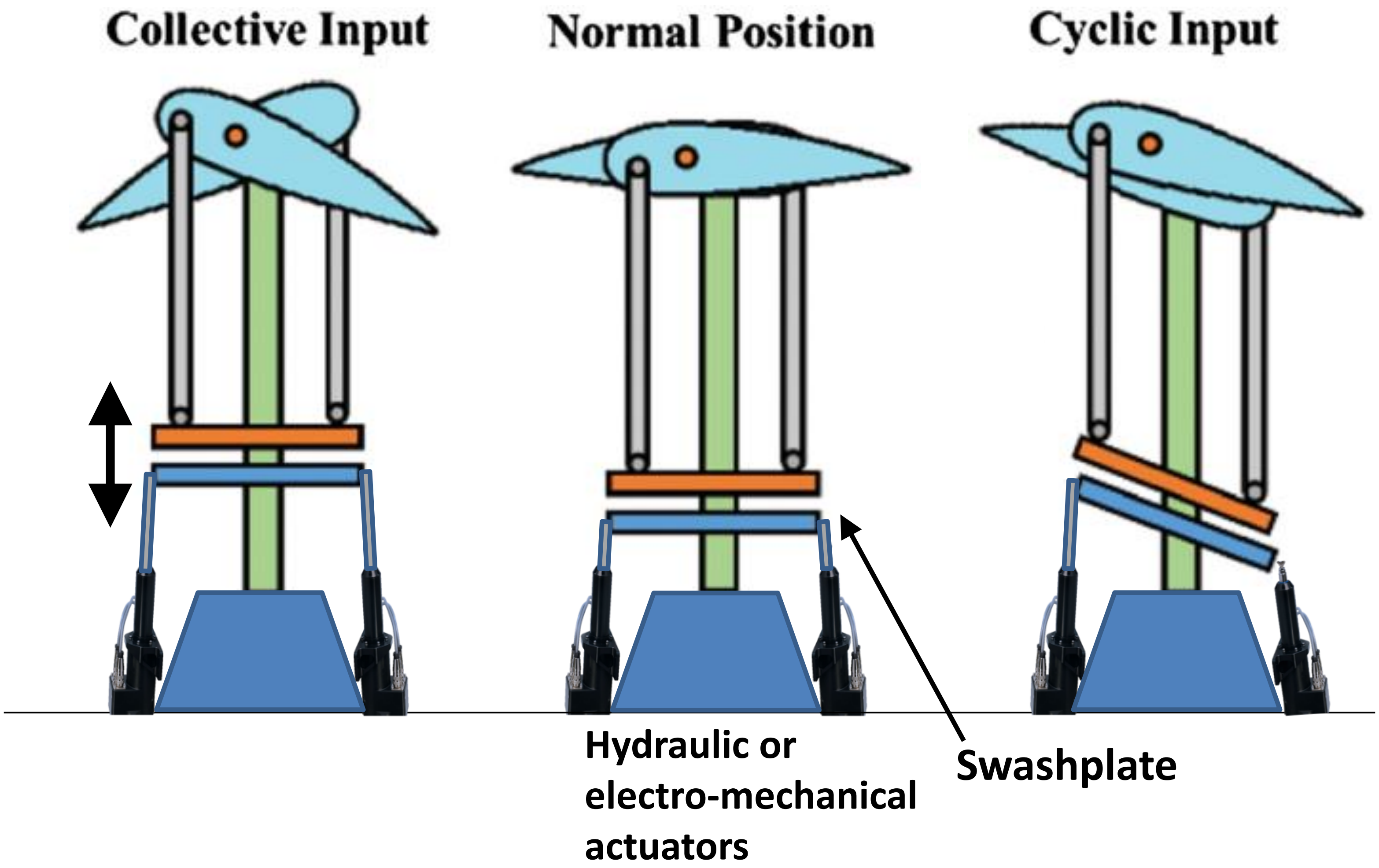


Another Design Using Gears to Change Pitch



A Mechanical Actuator Through Hollow Shaft

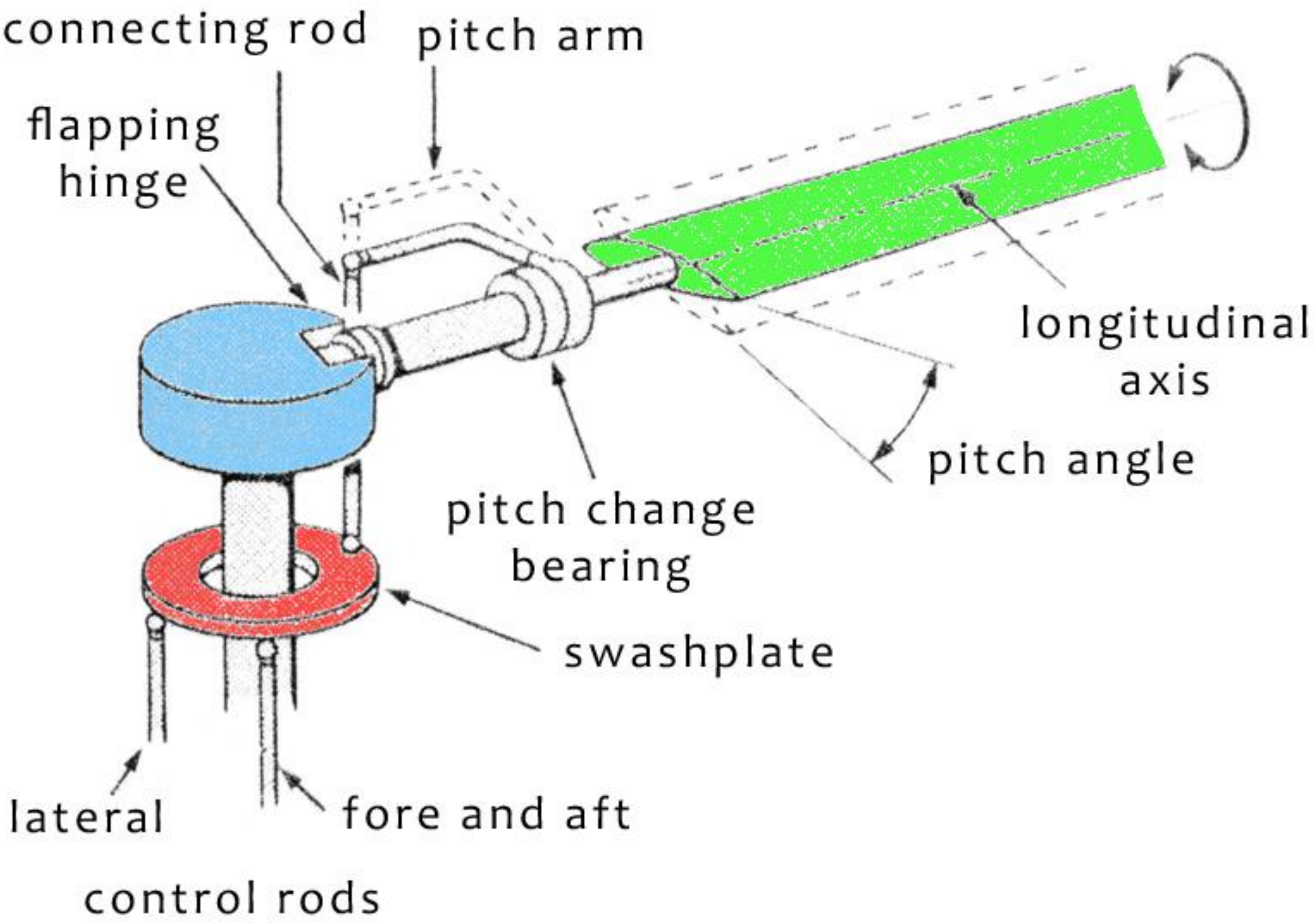
Collective + Cyclic Pitch Change



Hydraulic or electro-mechanical actuators

Swashplate

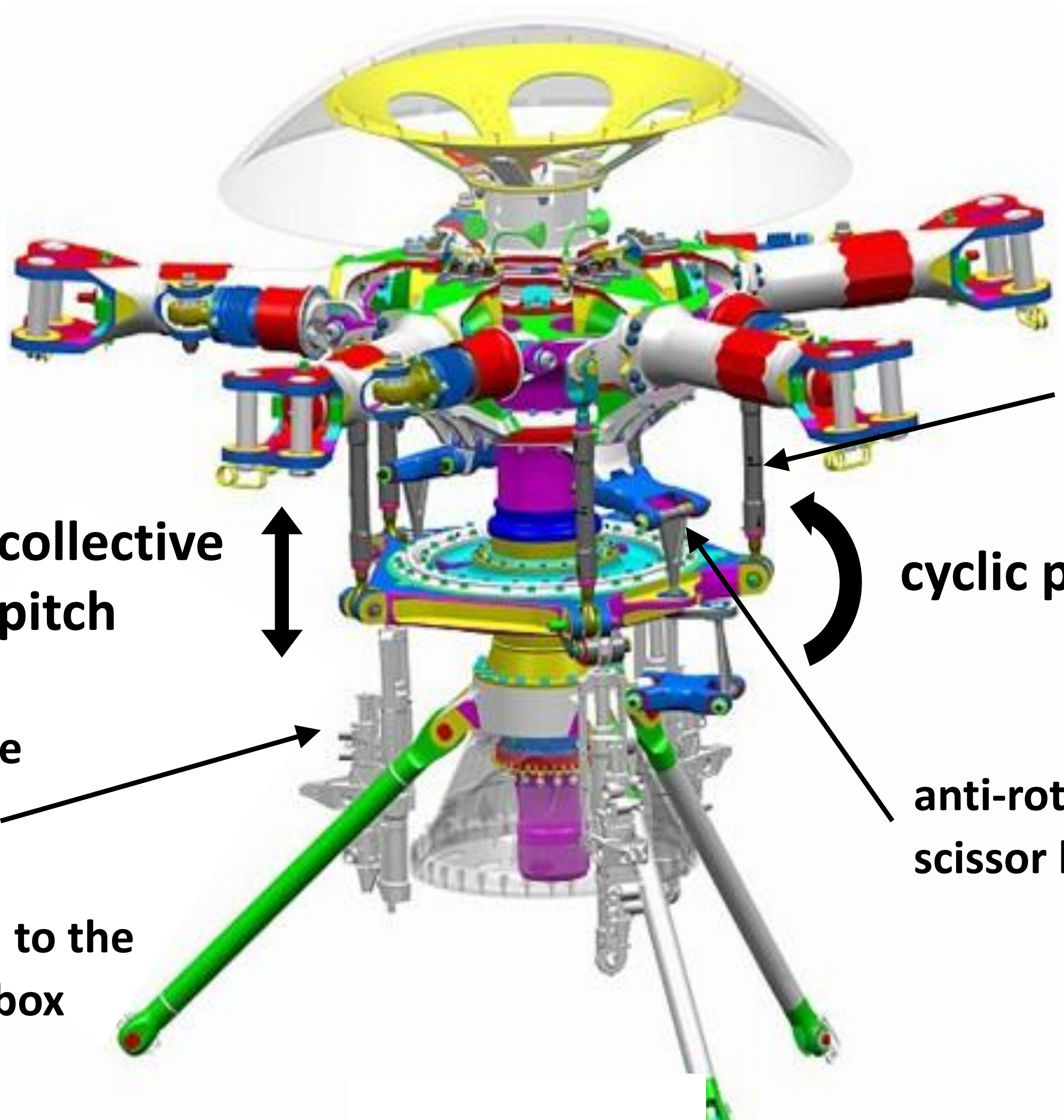
Blade Pitch Change Mechanism



to actuators

Source: http://www.redbackaviation.com/helicopter-rotorhead-design/
https://youtu.be/83h6QK-oJ4M

Collective + Cyclic Pitch Change



blade pitch link, or rotating pushrod

collective pitch

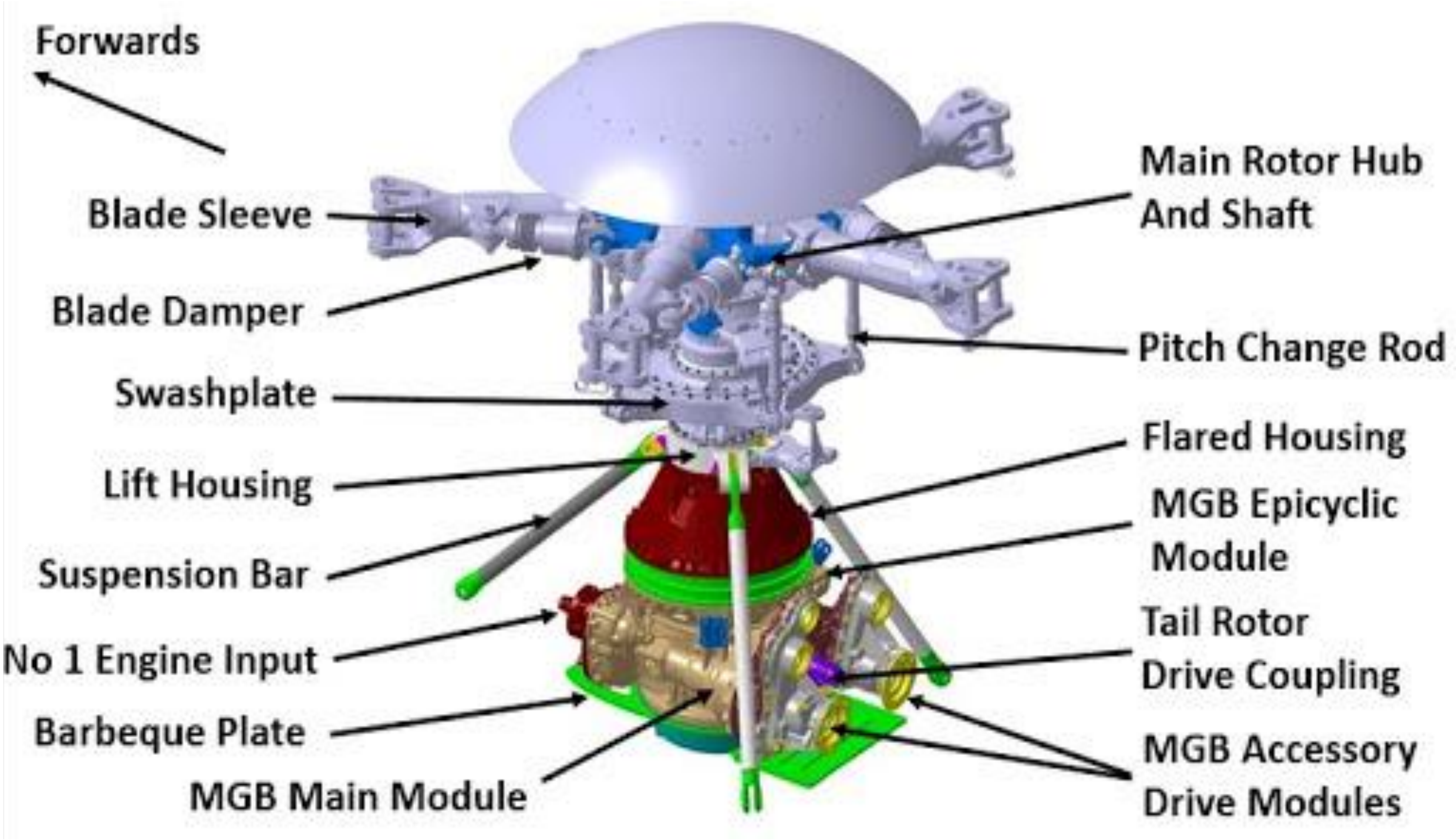
cyclic pitch

Swashplate hydraulic actuators, connected to the fixed gearbox

anti-rotation scissor link

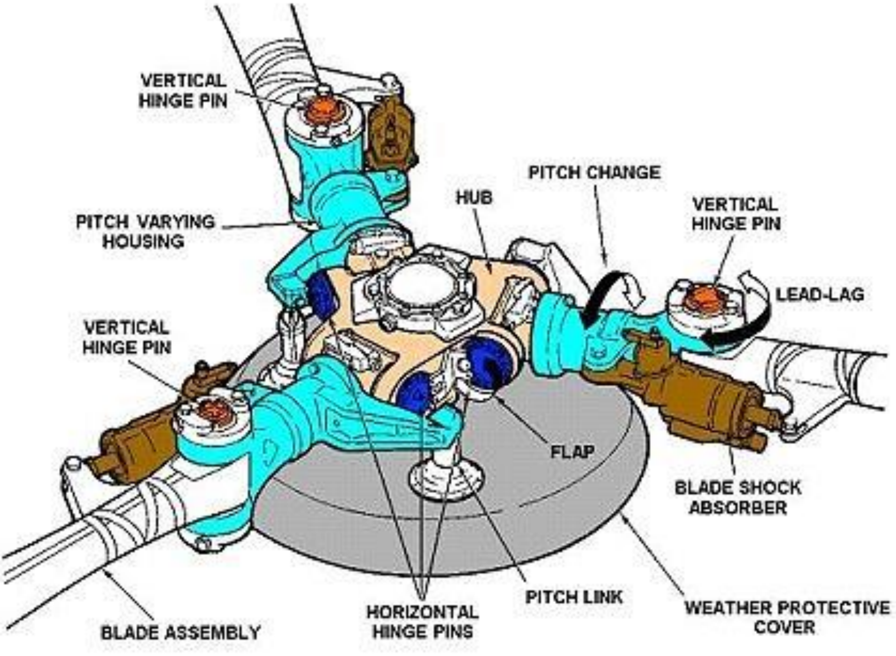
Eurocopter EC225

The Rotor is Mounted to the Main Gearbox

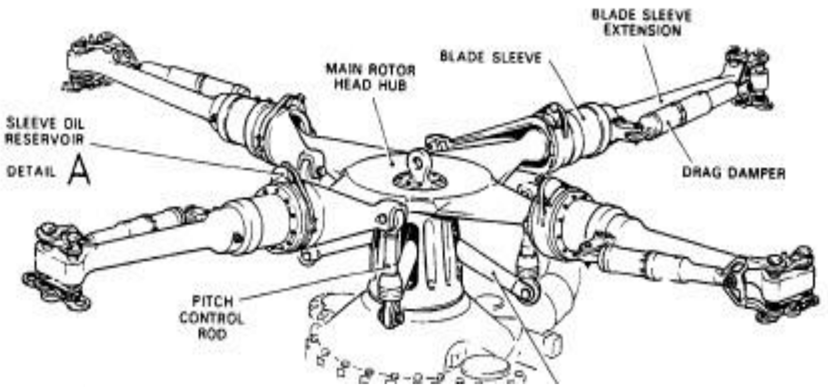


Eurocopter EC225

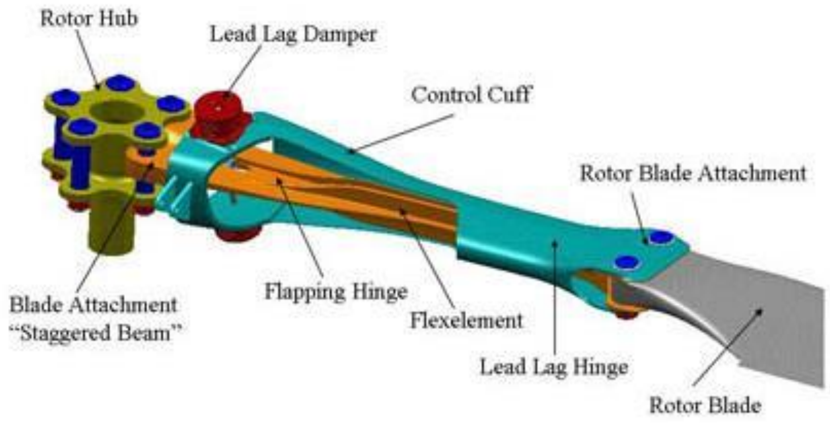
3 Major Types of Helicopter Rotor Hubs



Fully articulated rotor

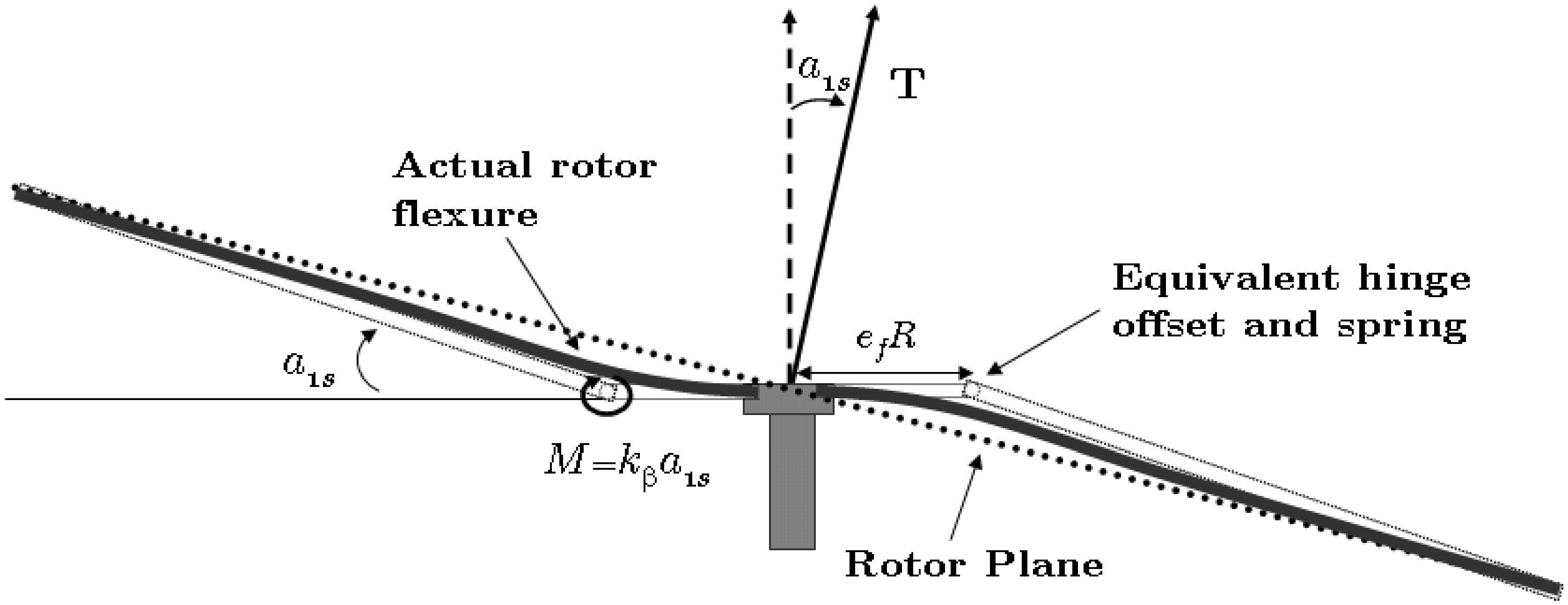


Hingeless rotor



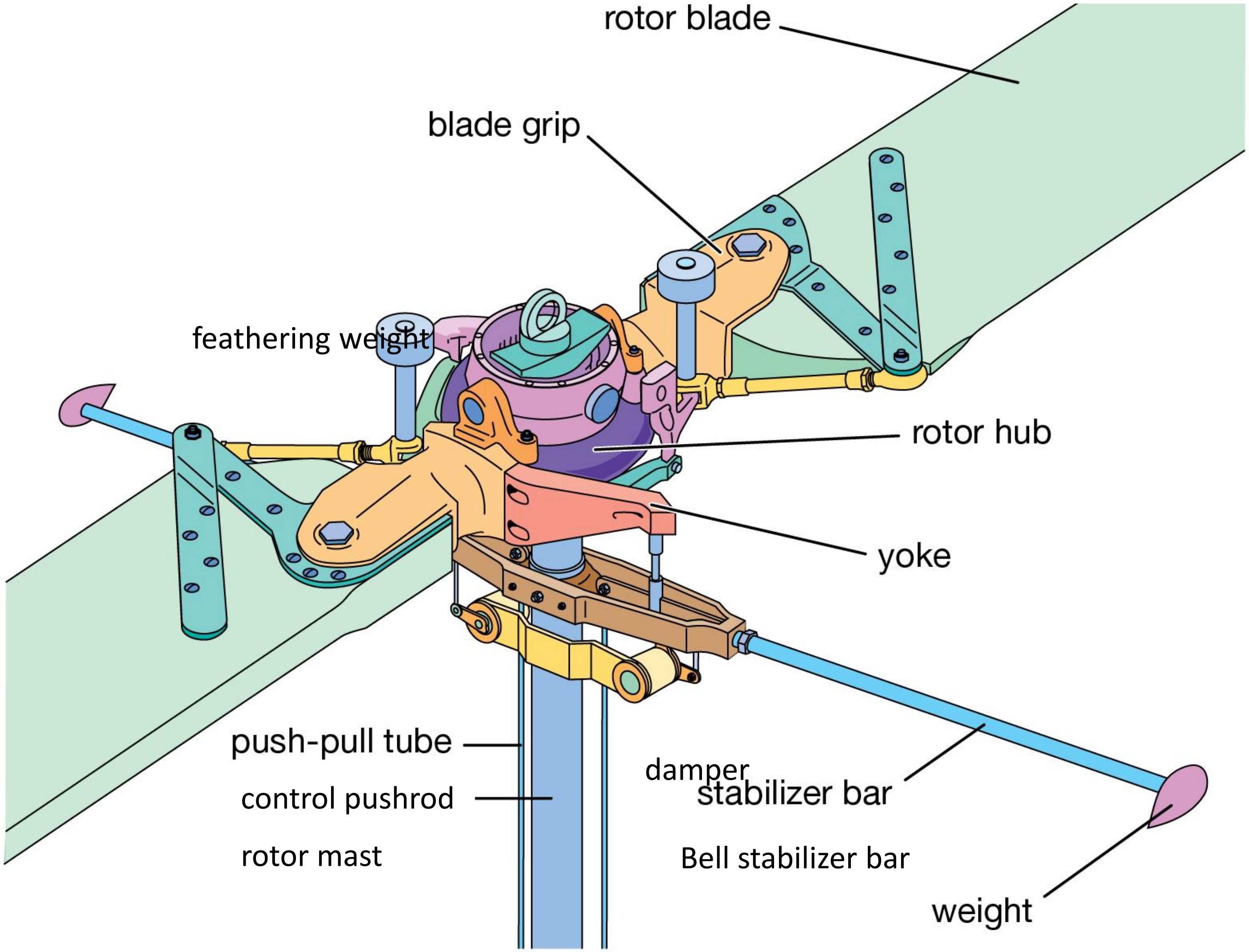
Bearingless rotor

Hingeless and Bearingless Rotors



Source: https://www.researchgate.net/figure/Diagram-representing-the-effects-of-rotor-blade-flapping-5-and-the-modeling-of-stiff_fig3_228808757

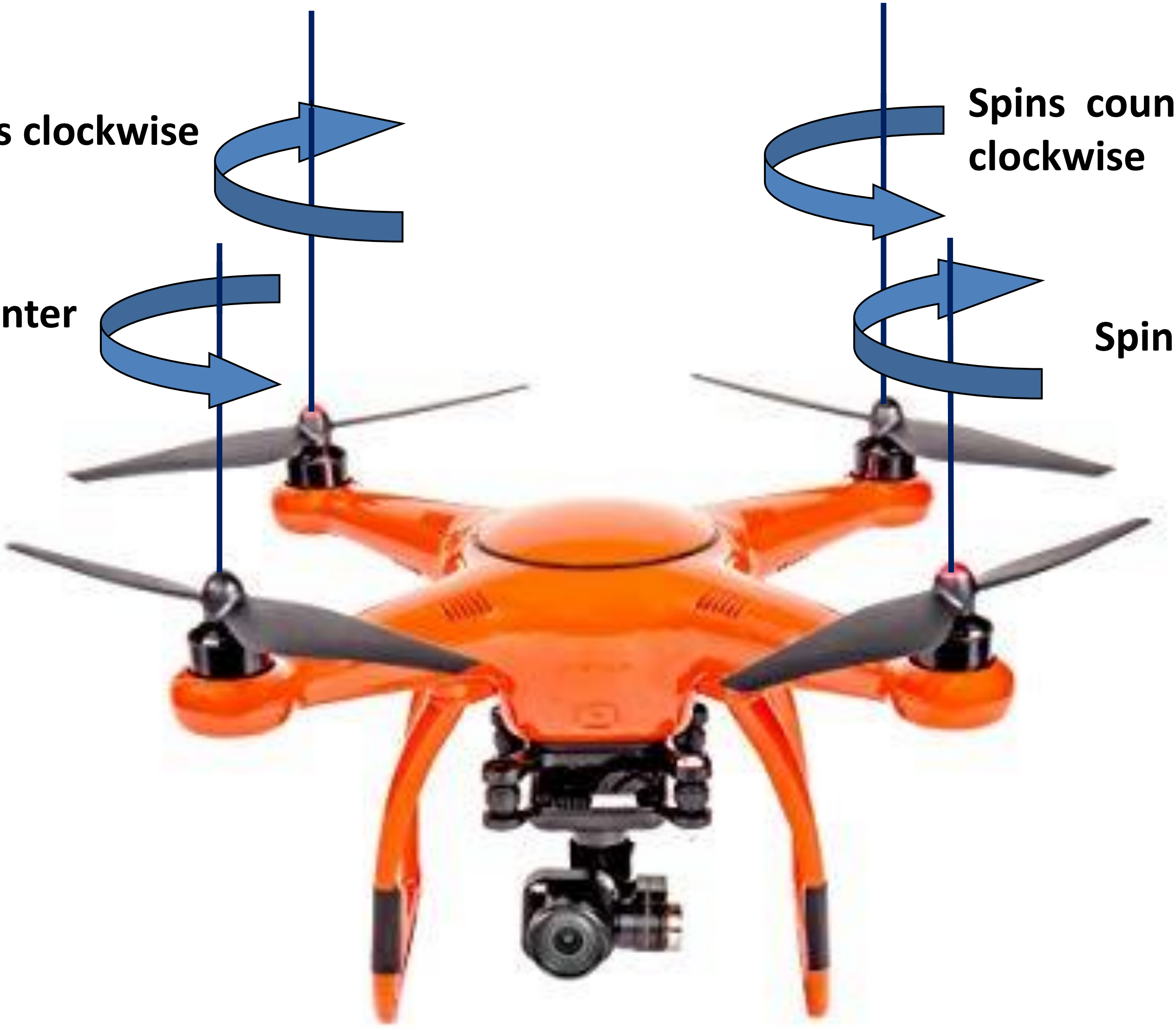
2-Bladed Teetering Rotor



Source: 2011 Encyclopedis Britannica

Helicopter and VTOL Flight Control

Multi-rotors are Mechanically simpler than Helicopters. Torque is Balanced When Even Number of Rotors is Used



Spins clockwise

Spins counter clockwise

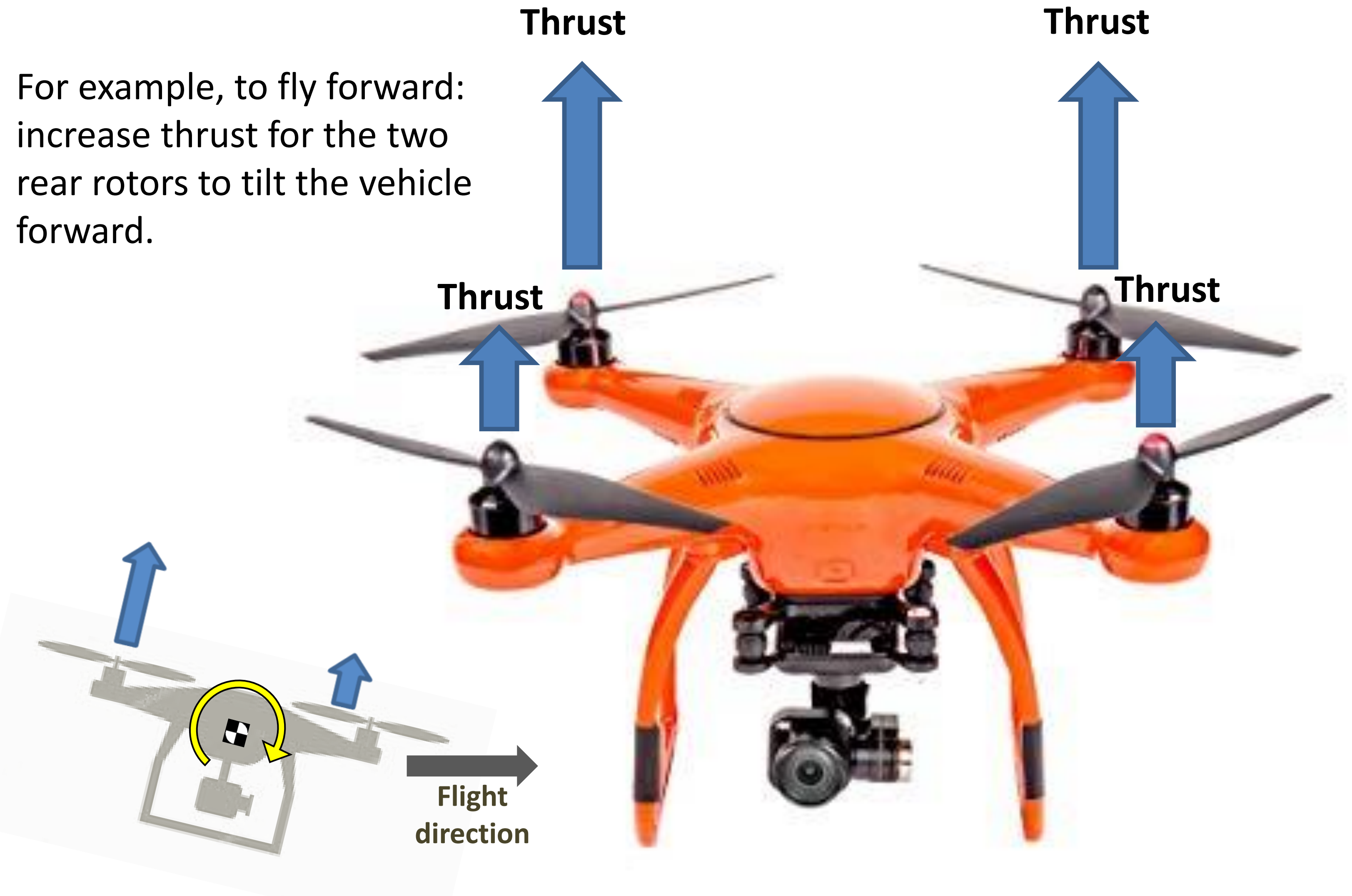
Spins counter clockwise

Spins clockwise

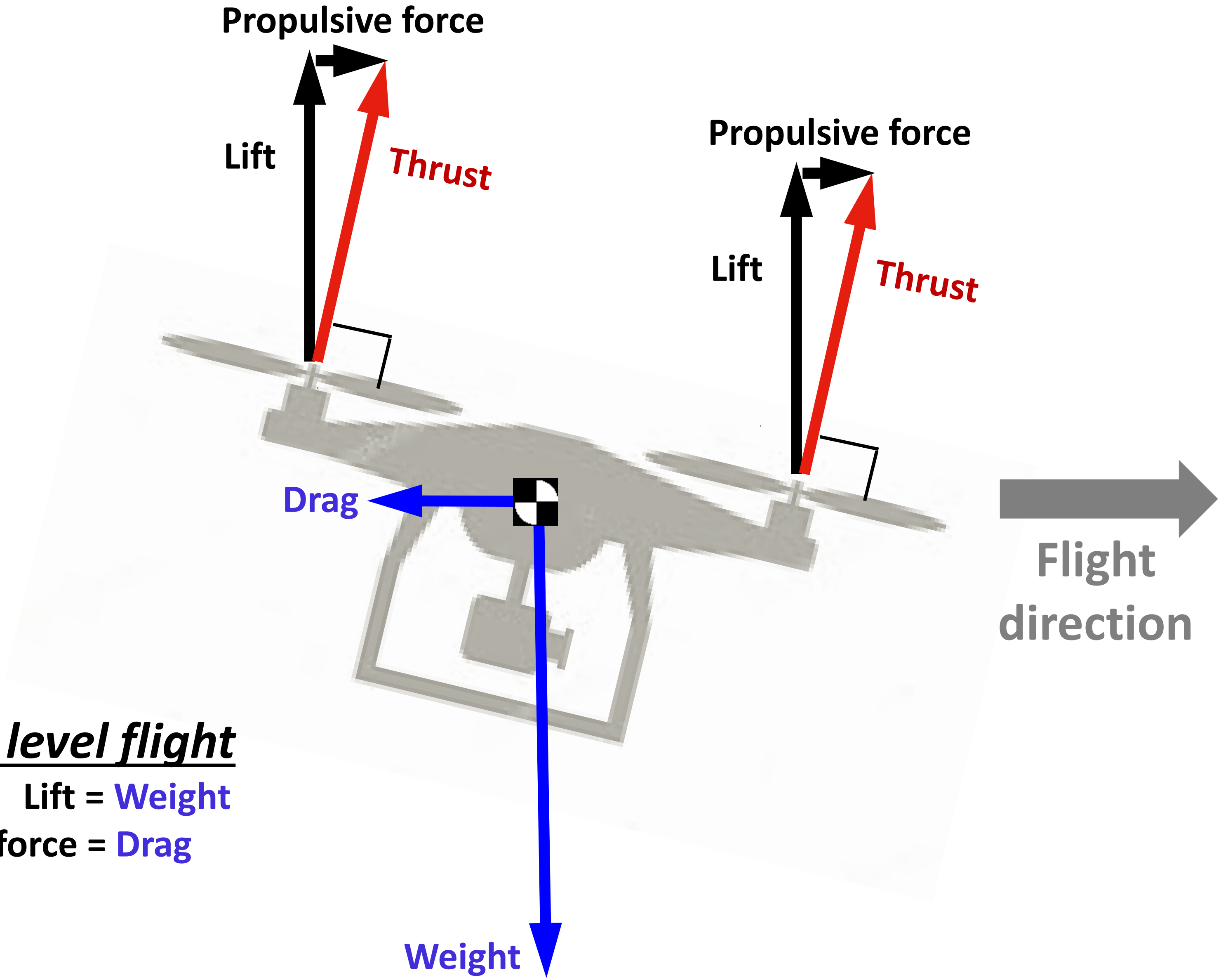
A multi-rotor has no mechanical control. Pitching, rolling and vertical motions are controlled by varying rotor thrust/rpm.

Axi-symmetrical -> No Control Cross-coupling -> Easier Piloting

For example, to fly forward: increase thrust for the two rear rotors to tilt the vehicle forward.



Rotor Forces in Steady Forward Flight



In steady level flight

Lift = Weight

Propulsive force = Drag

Yaw Control for Quadcopter (Ex: Yaw Left)

Spins clockwise faster to increase ccw reaction

Spins ccw slower

Spins ccw slower

Spins clockwise faster to increase ccw reaction

Canting All 4 Rotors Improves Yaw Response by Introducing a Side Force to Spin the Drone

The DJI Phantom 3 drone introduced in April 2015

Helicopter Controls

Different Helicopter Configurations


Single Main and Tail Rotor


Tandem Rotor


Coaxial Rotor

NOTAR((NO TAil Rotor)


Side by Side


Tip Jet


Tilt Rotor

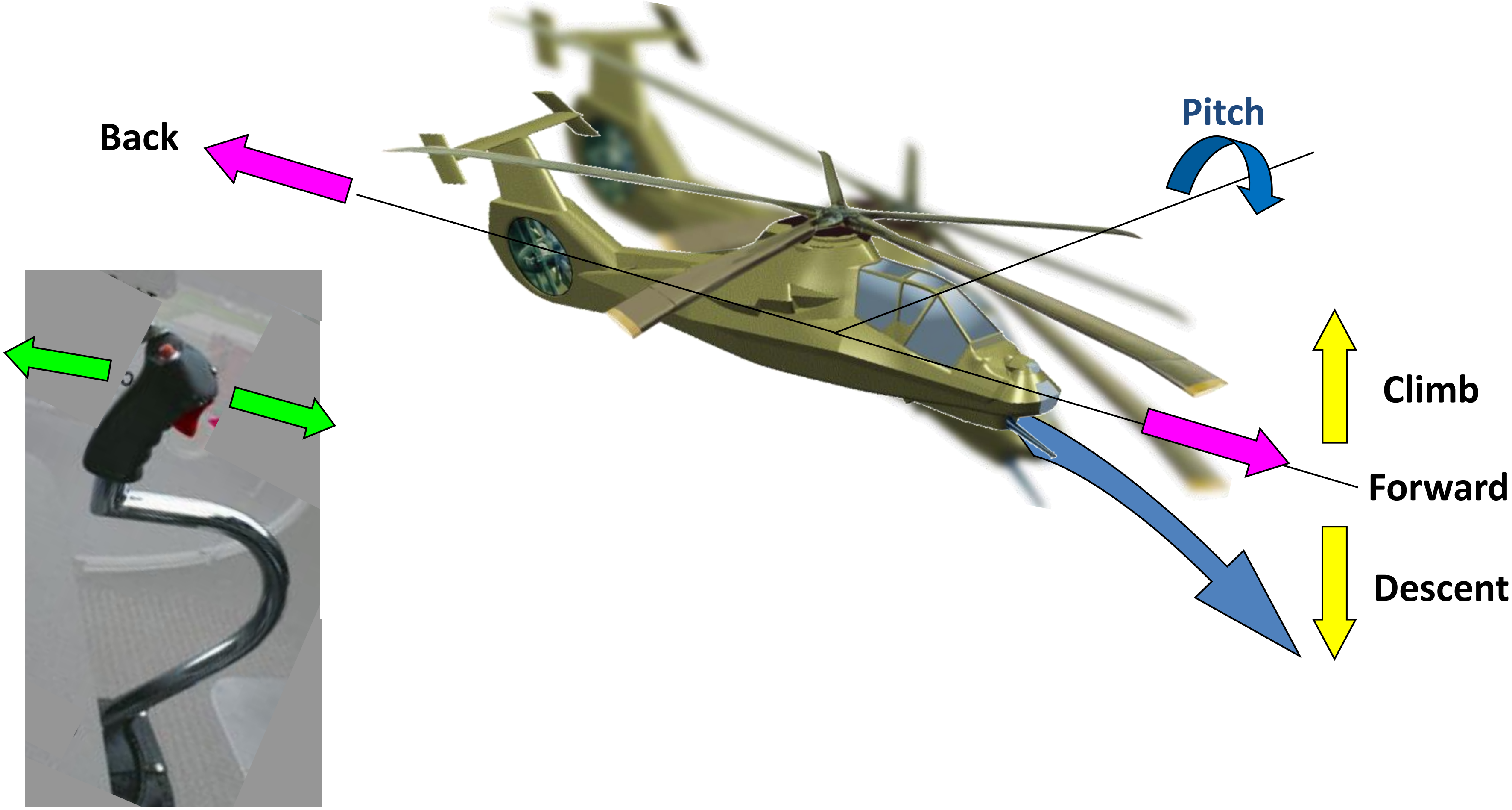

Pusher Propeller


Quad Tilt Rotor

AW139 Helicopter Flight Controls

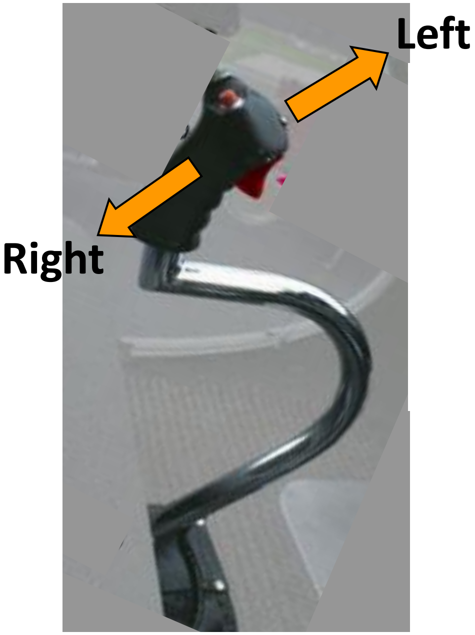
Fore/Aft Cyclic Control

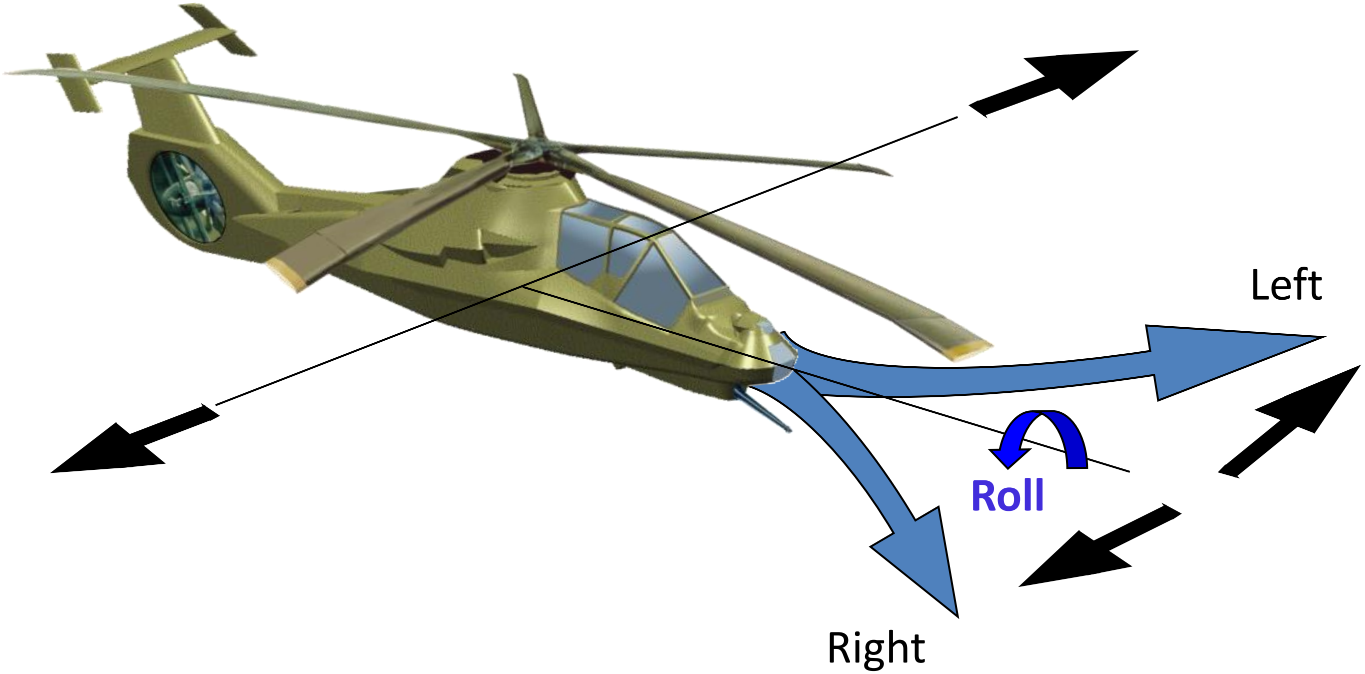
Forward/aft cyclic controls pitch rotation, forward/back translation in hover, and climb/descent in forward flight.



Left/Right Cyclic Control

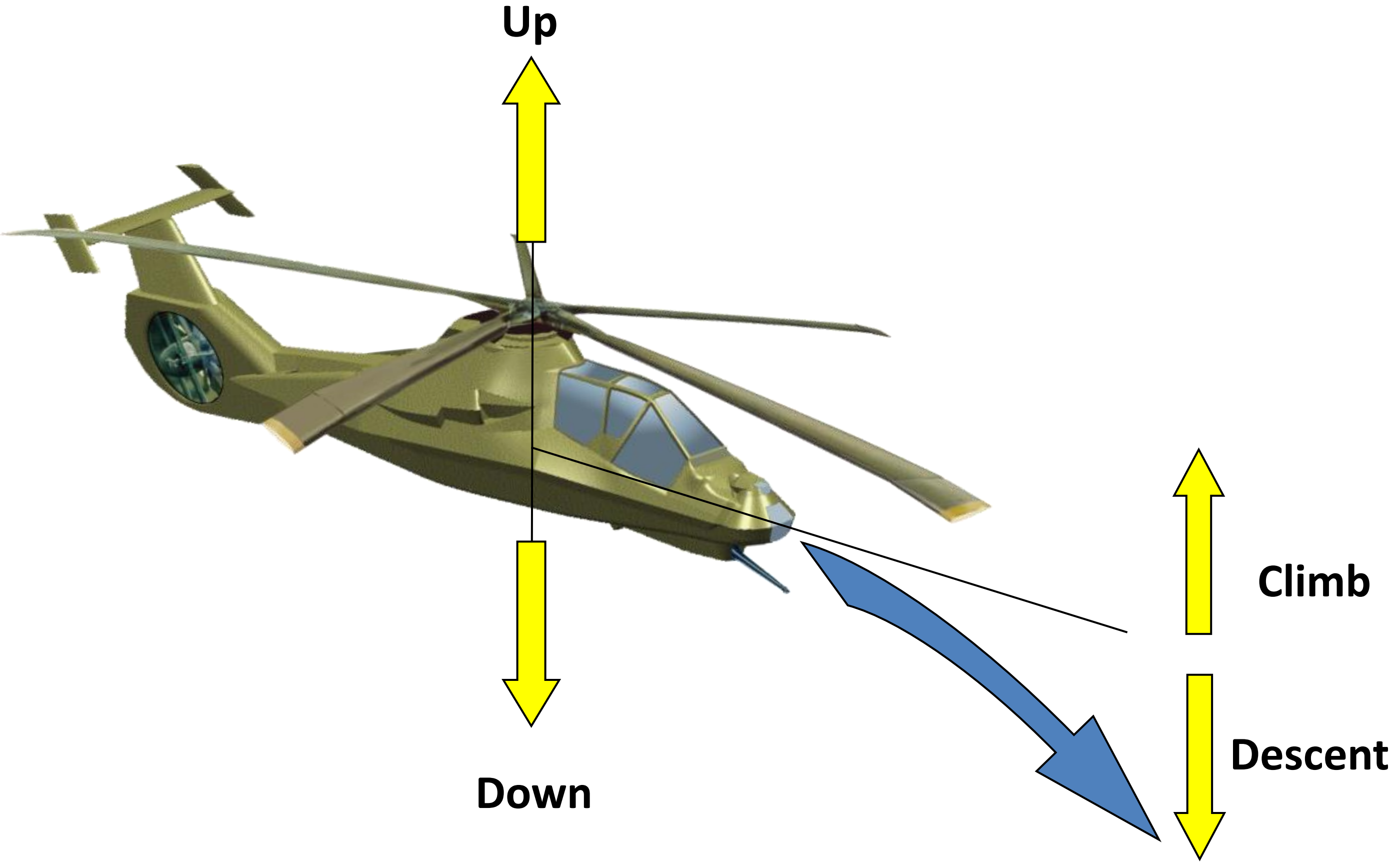
Left/right cyclic controls roll banking, left/right translation in hover, and veer left/right in forward flight.





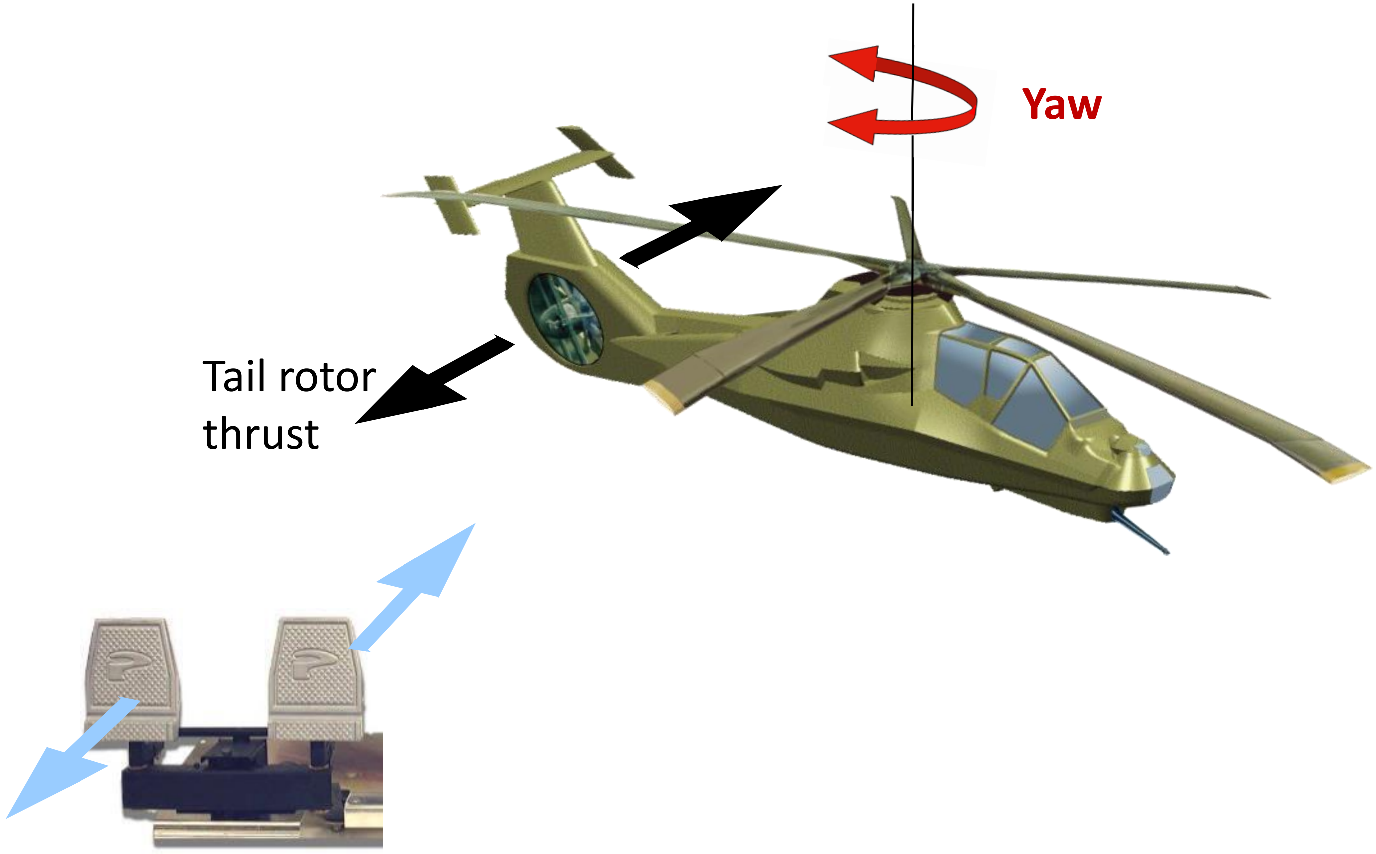
Collective Control

Collective/throttle stick controls vertical translation

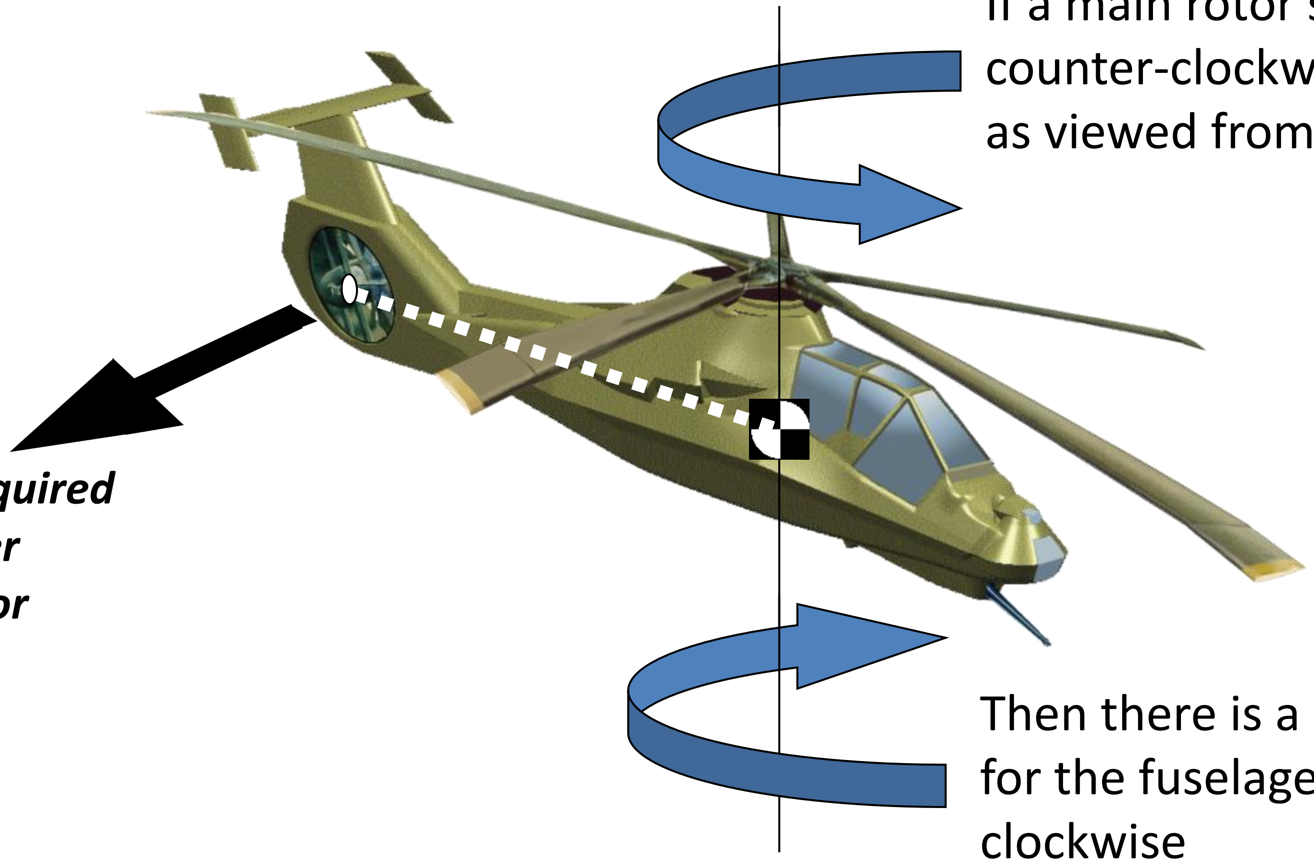


Yaw Control

Tail rotor controls rotating left/right



Tail rotor is required to counter the main rotor torque

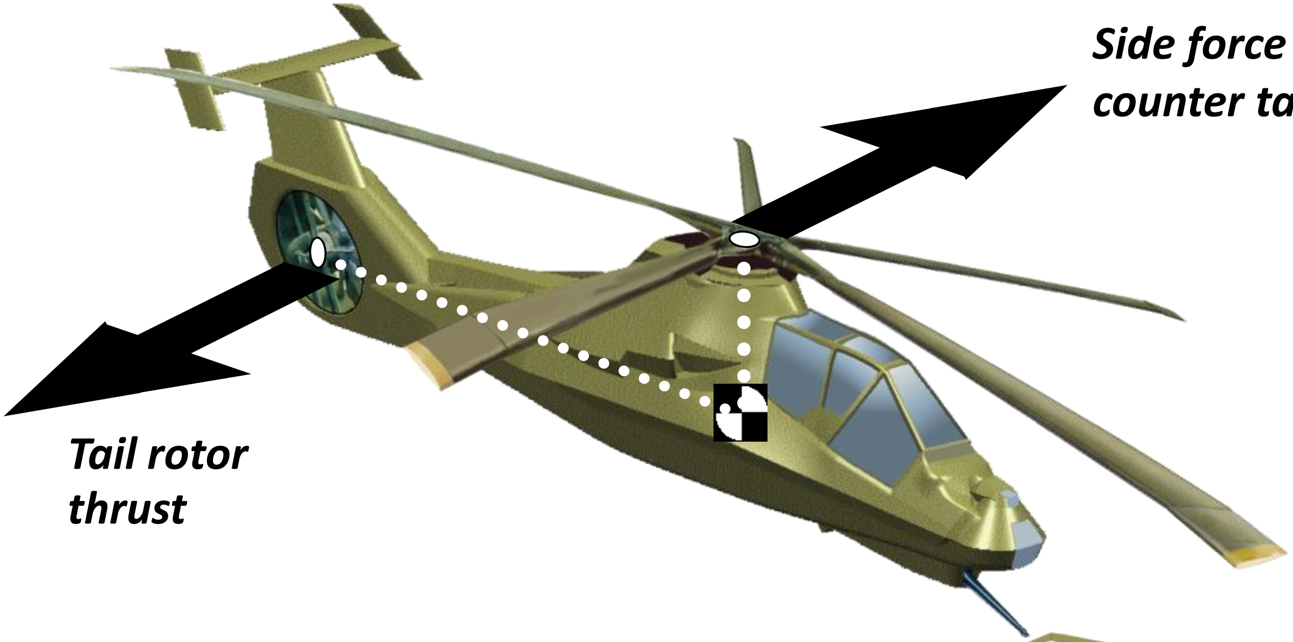


If a main rotor spins counter-clockwise, as viewed from above

Tail rotor thrust required to counter main rotor torque

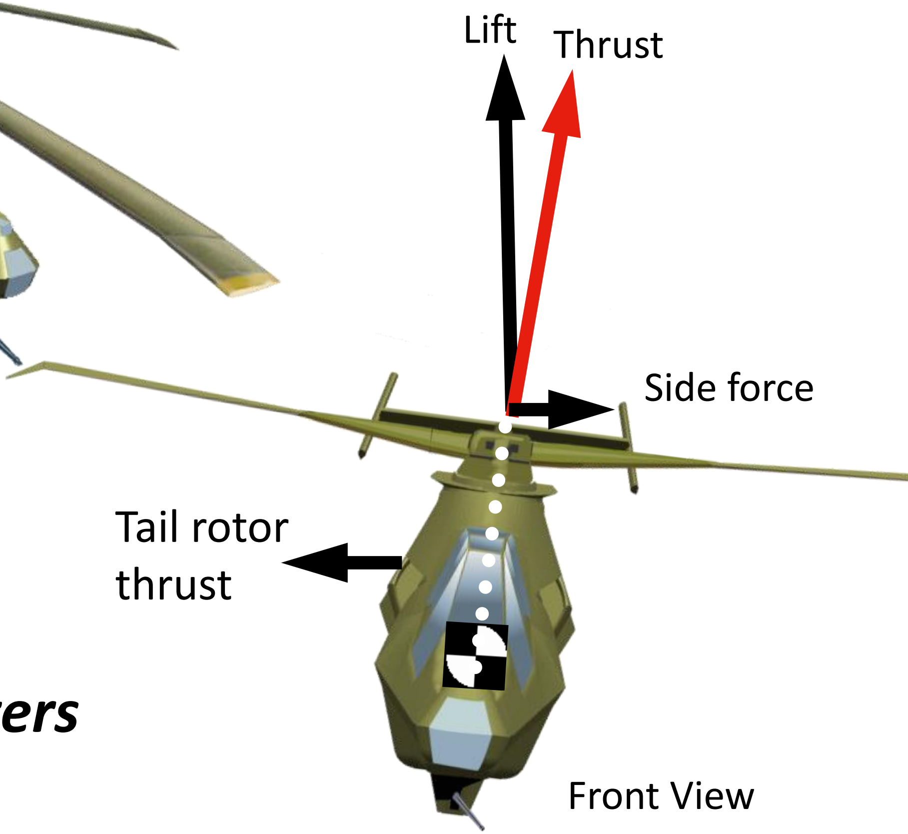
Then there is a reaction for the fuselage to spin clockwise

But tail rotor thrust also causes the helicopter to lean left in hover



Side force from main rotor to counter tail rotor thrust

Tail rotor thrust



Lift

Thrust

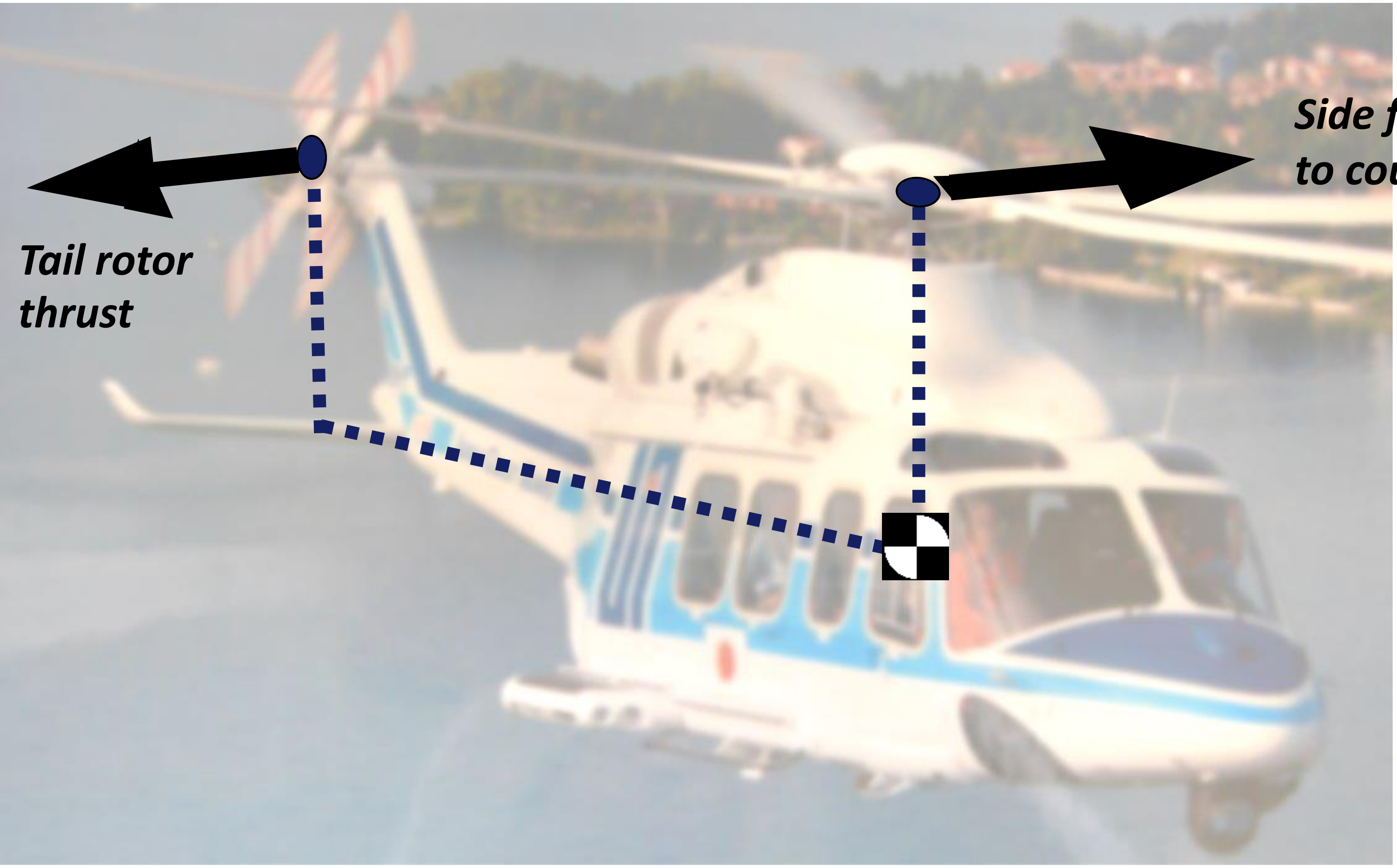
Side force

Tail rotor thrust

Front View

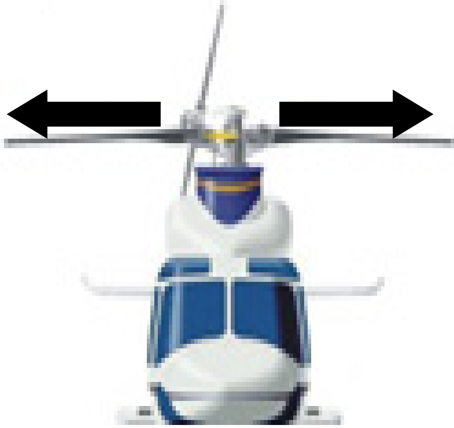
That is why in hover, some helicopters are never sitting level.

Solution: raise tail rotor to same height as main rotor



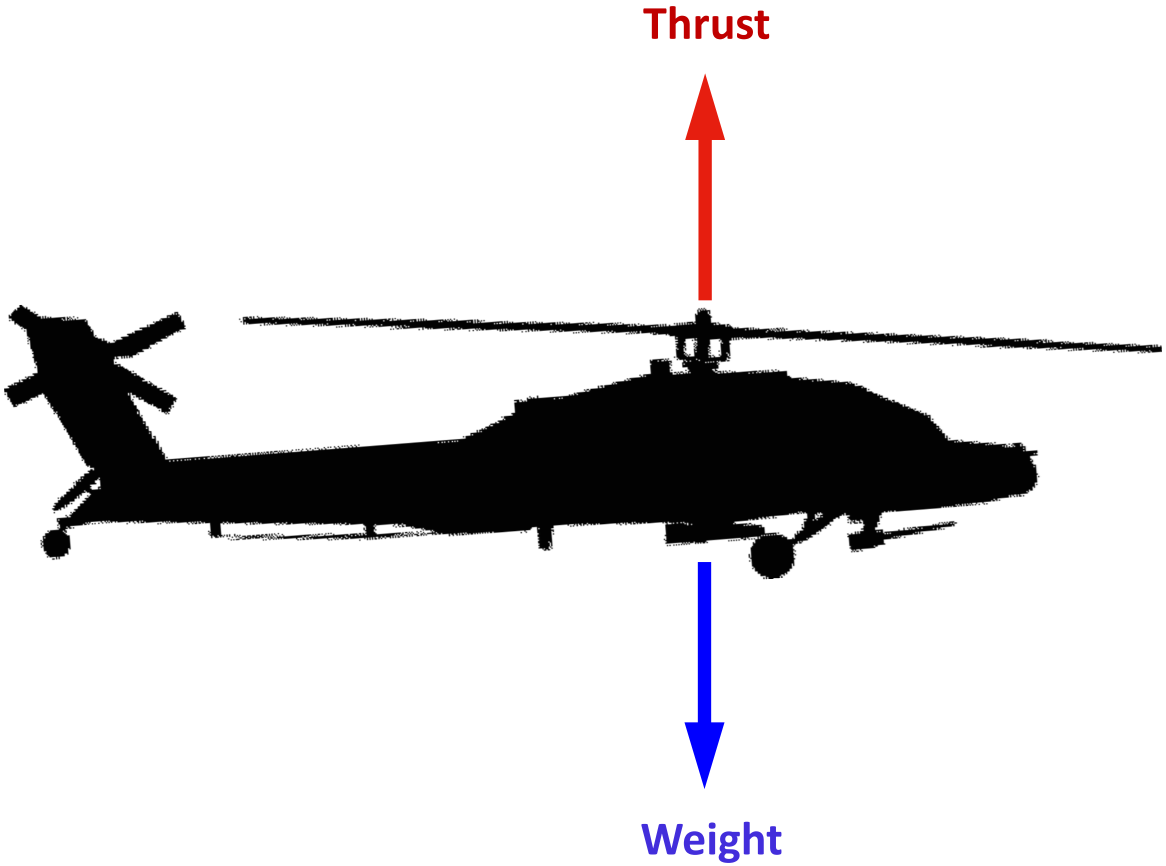
Tail rotor thrust

Side force from main rotor to counter Tail rotor thrust

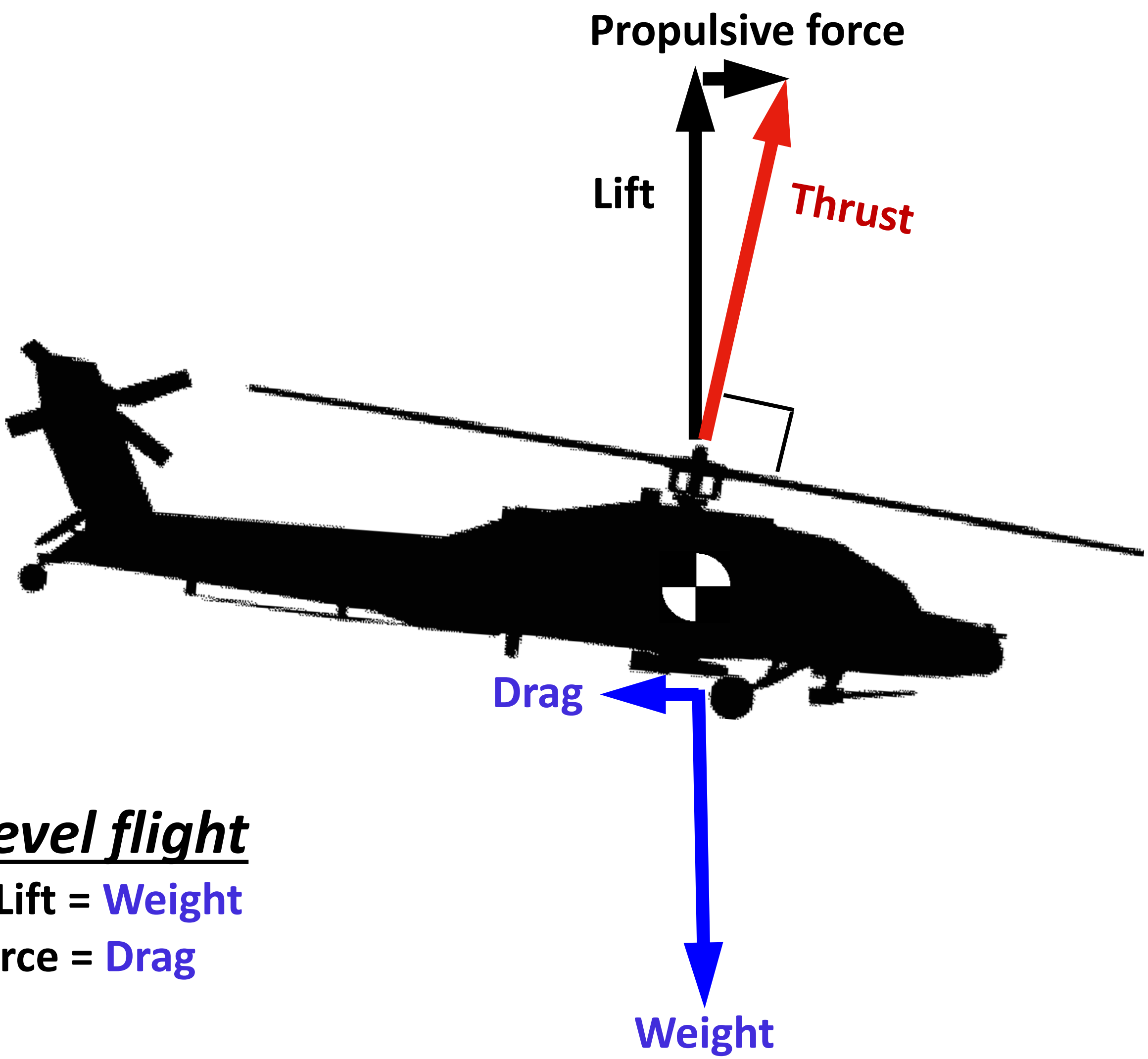


Now, forces and moments are balanced, helicopter hovers level.

Thrust Equals Weight in Hover



Rotor Forces in Steady Forward Flight



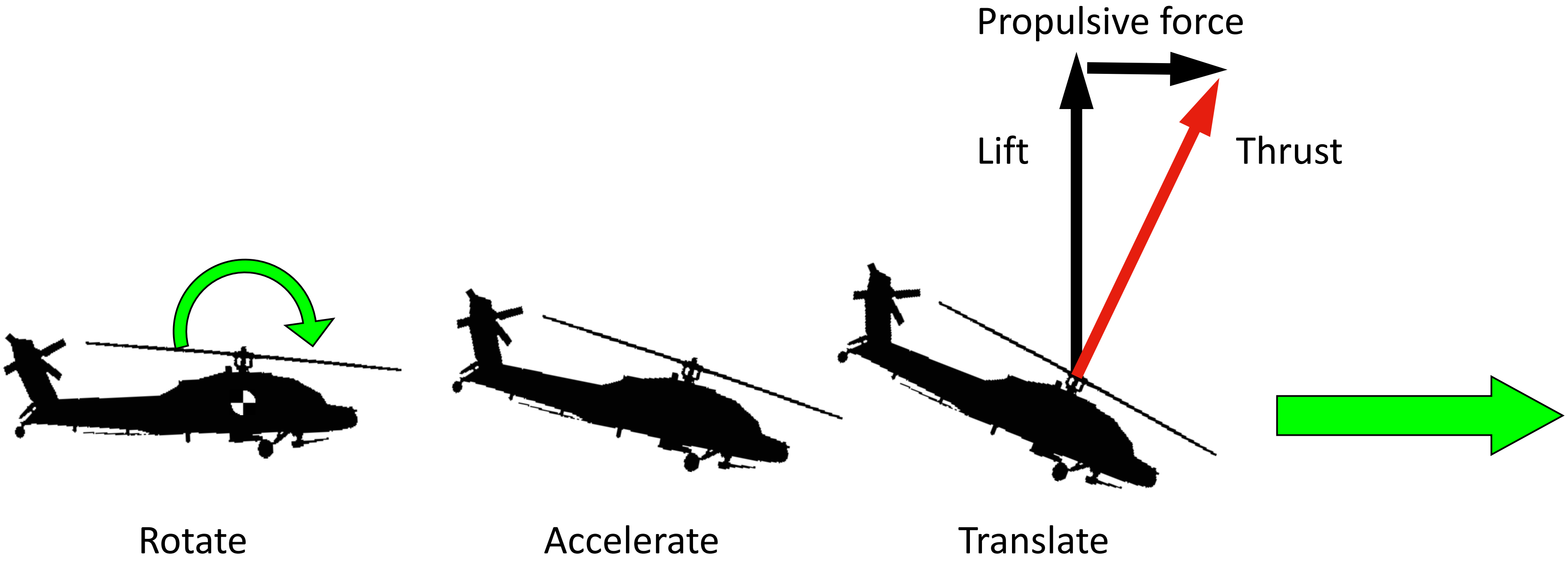
In steady level flight

Lift = Weight

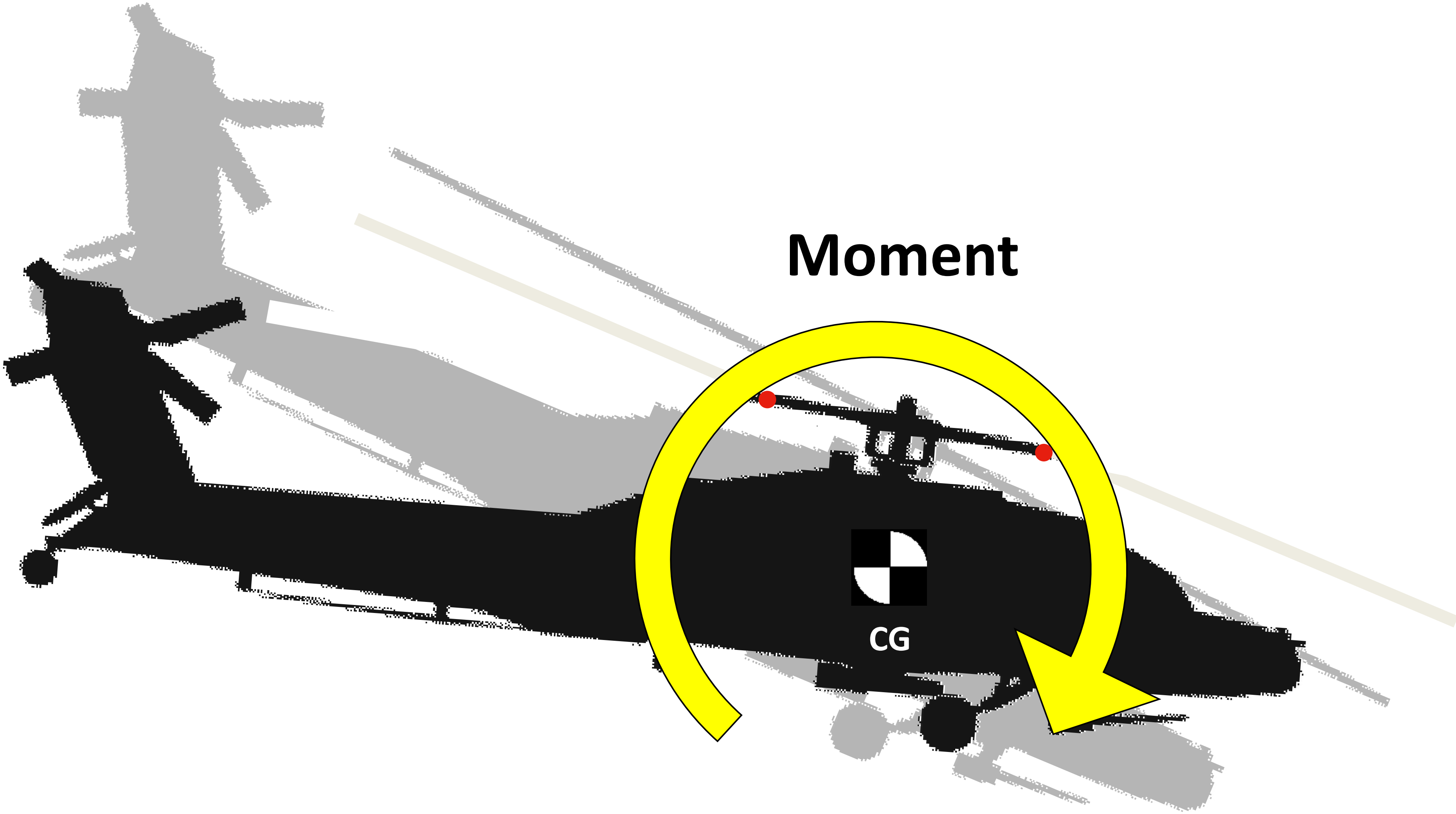
Propulsive force = Drag

How Does a Helicopter Go Into Forward Flight?

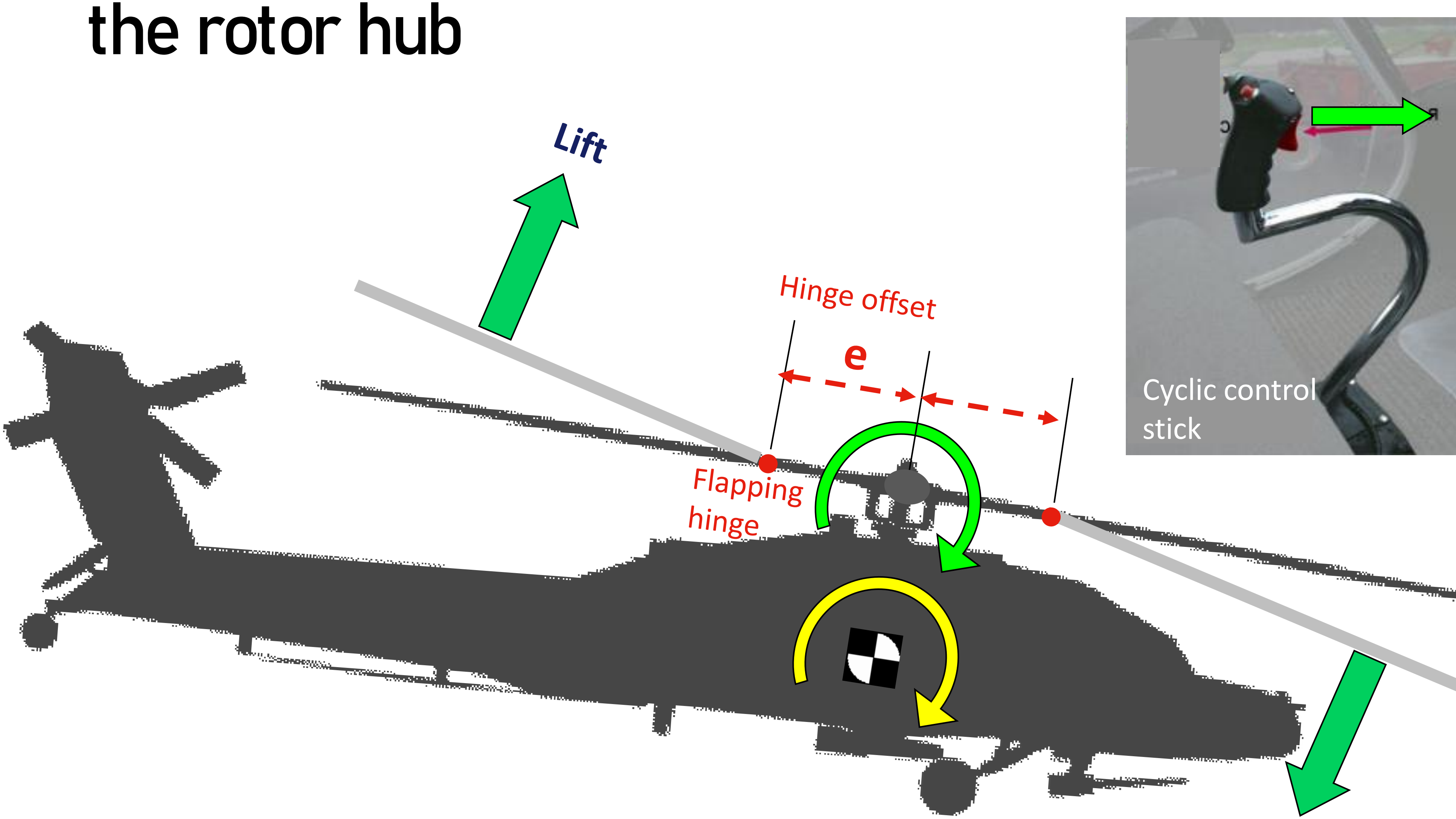
"Needs to rotate the fuselage first"



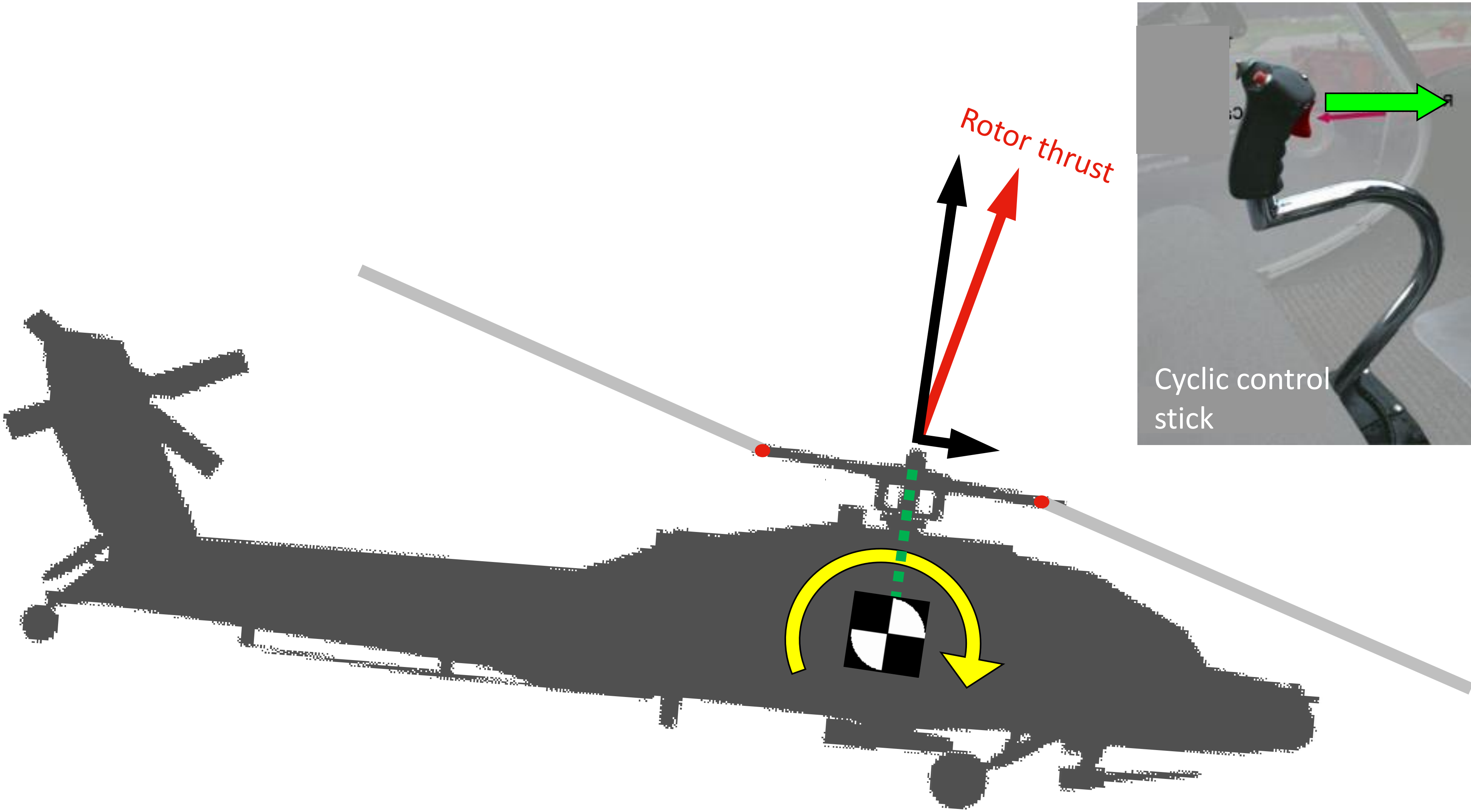
Two Ways to Produce a Moment at the Helicopter CG



1. Fuselage Moment due to a Rotor Moment at the rotor hub



2. Fuselage Moment due to a Thrust Tilt



Summary on Control Moment

- Weaker compared to moment from hinge offset
- Need at least 3 to 4% flap hinge offset “e” to have good control authority.
- For good agility and aerobatic performance, need 5 to 10% flap hinge offset.

Why helicopters are difficult to fly

(1) They are inherently unstable

(2) They have multiple axis of instability

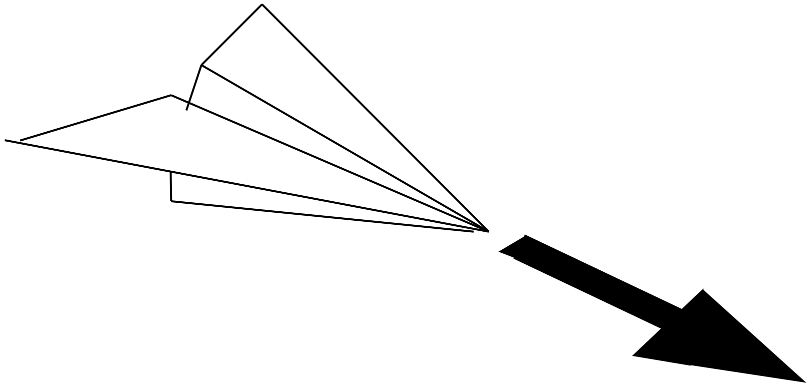
(3) Each pilot control does multiple functions

(4) Pilot has to anticipate and do 4 orders of math integration in his head

Helicopters are Difficult to Fly Because:

(1) They are inherently unstable

Airplanes are inherently stable



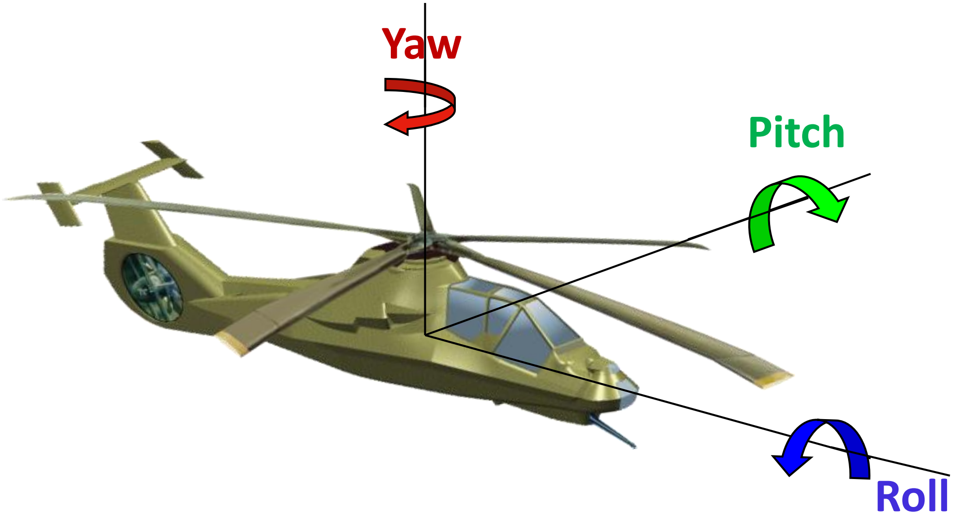
Helicopter are inherently unstable

Helicopters are Difficult to Fly Because:

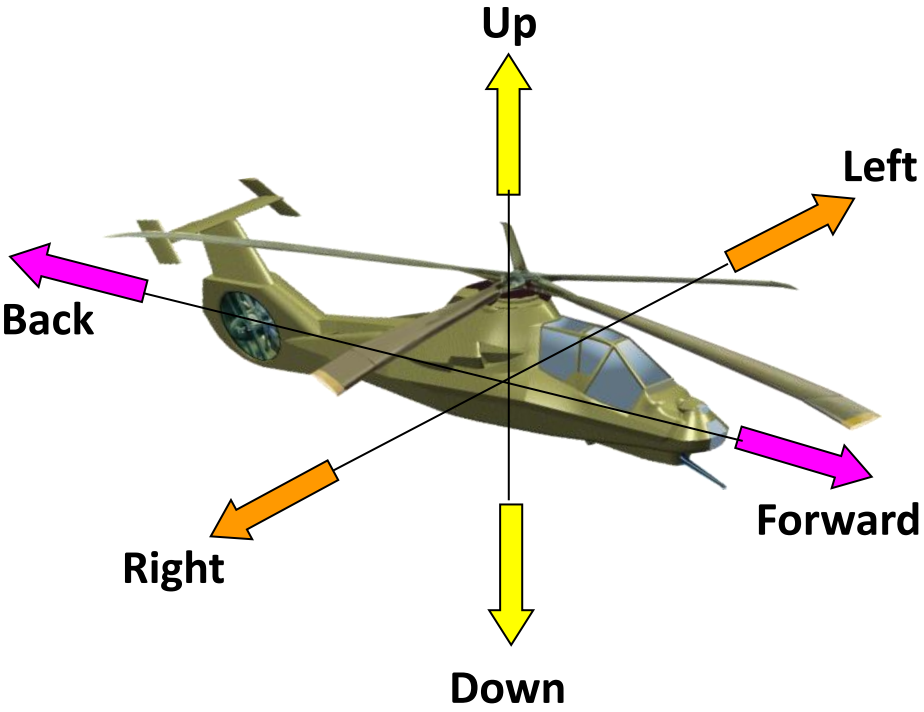
(2) They have many axis of instability

- 6 degrees of freedom (DOF)
- Human brain can easily control 2 or 3 DOF instabilities, but 6 is challenging.

3 Rotations



and 3 Translations



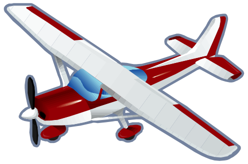
Helicopters have many axis of instabilities, but human brain can manage 2 or 3 DOF instabilities with practice

Balancing a broom stick is a good example

Corollary: the Smaller the More Difficult to Control

Smaller objects have a shorter *“Time Constant ”*

***Time constant* is a measure of how fast a system reacts**

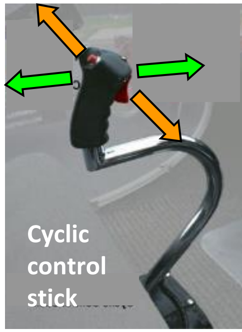


Helicopters are Difficult to Fly Because:

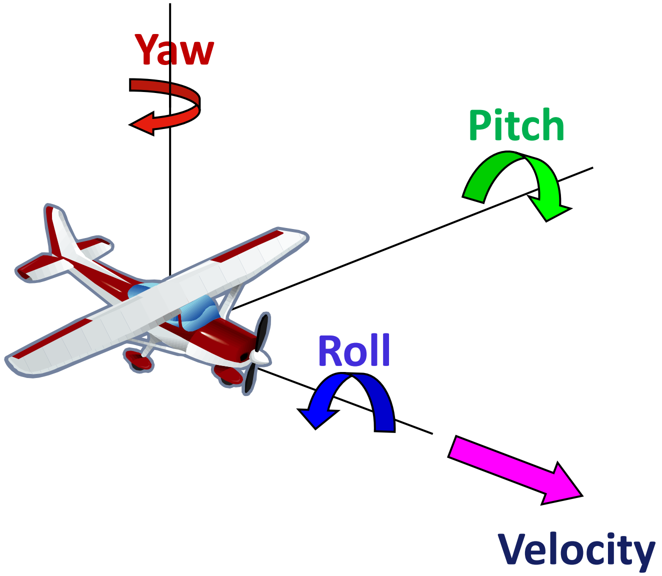
(3) Each control does multiple functions

Unlike airplane, in helicopters there are many cross-coupling

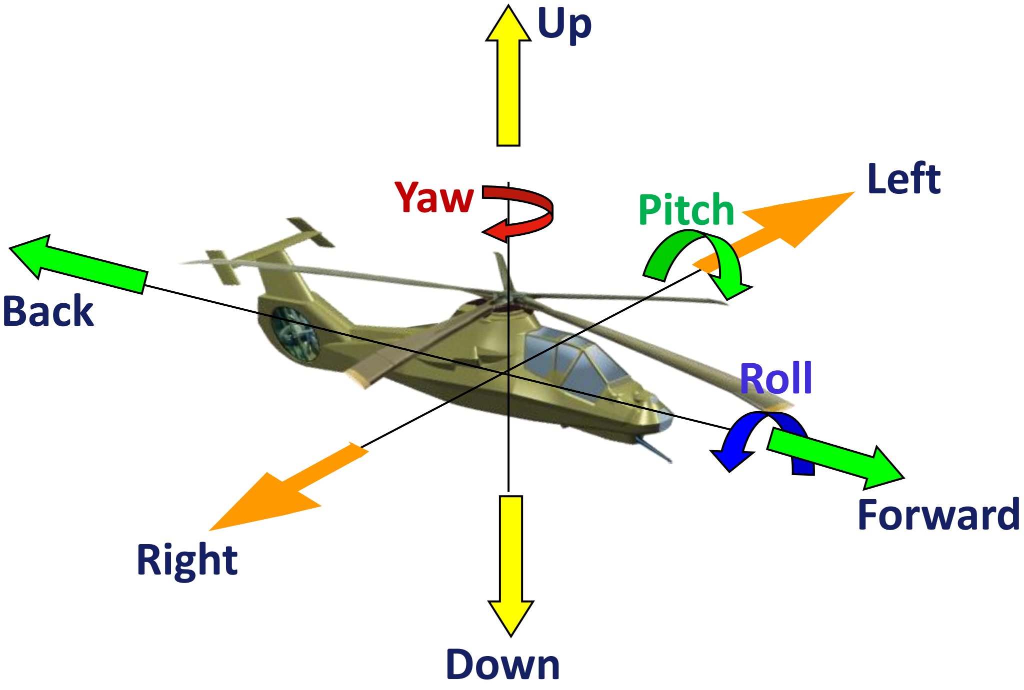
- **Forward/aft stick** controls pitch rotation, forward/back translation, climb/descent
- **Left/right cyclic** controls roll rotation and left/right translation
- **Rudder pedals** control left/right rotation



4 flight controls control 4 DOF

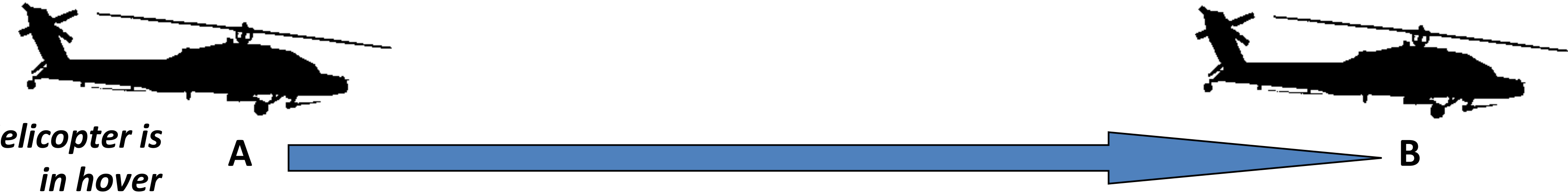


4 flight controls control 6 DOF



Helicopters are Difficult to Fly Because:

(4) Pilot Has to Anticipate Reactions in His Head



Helicopter is in hover

A

B

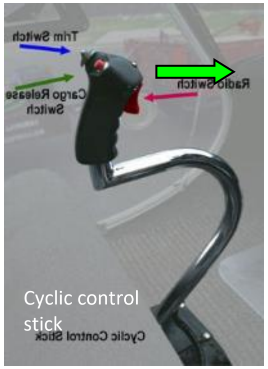
To move from point ***A*** *to point* ***B*** *requires many unaware thought processes inside the pilot's brain*

$$x = \int_0^t \iiint f(\ddot{\theta})dt$$

$$\theta, \dot{\theta}, \ddot{\theta}$$

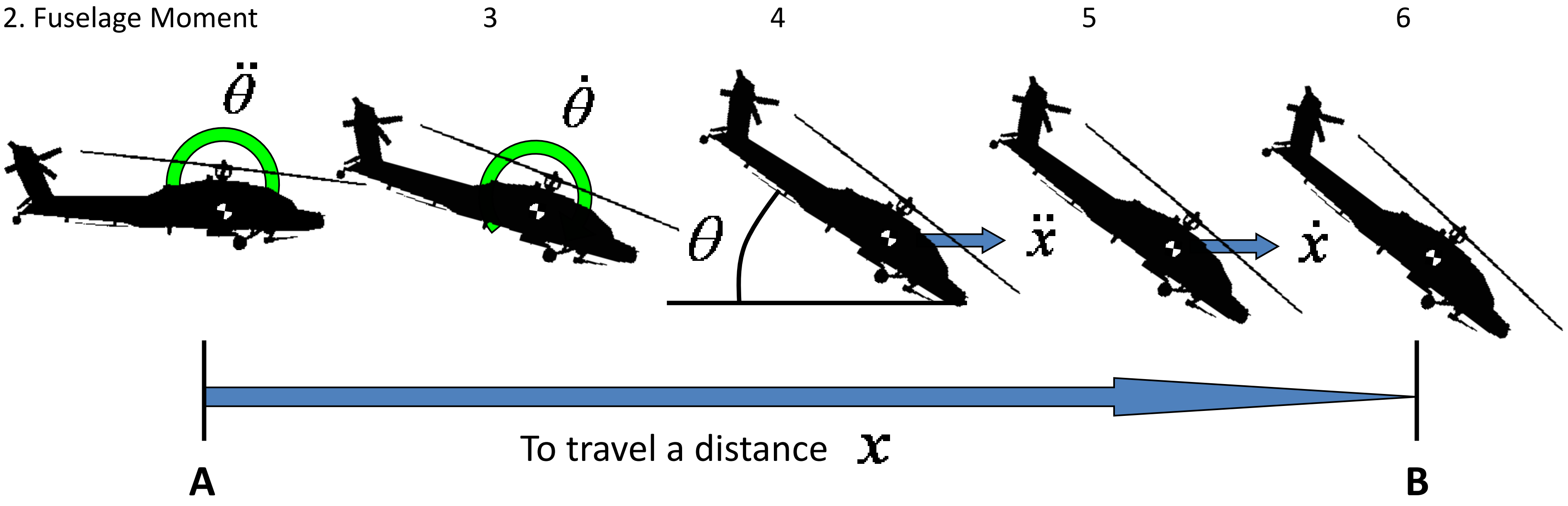
A Pilot Has to Do 4 Math Integrations in His Brain

1. Pilot input



$$x = \int_0^t \iiint f(\ddot{\theta})dt$$

2. Fuselage Moment



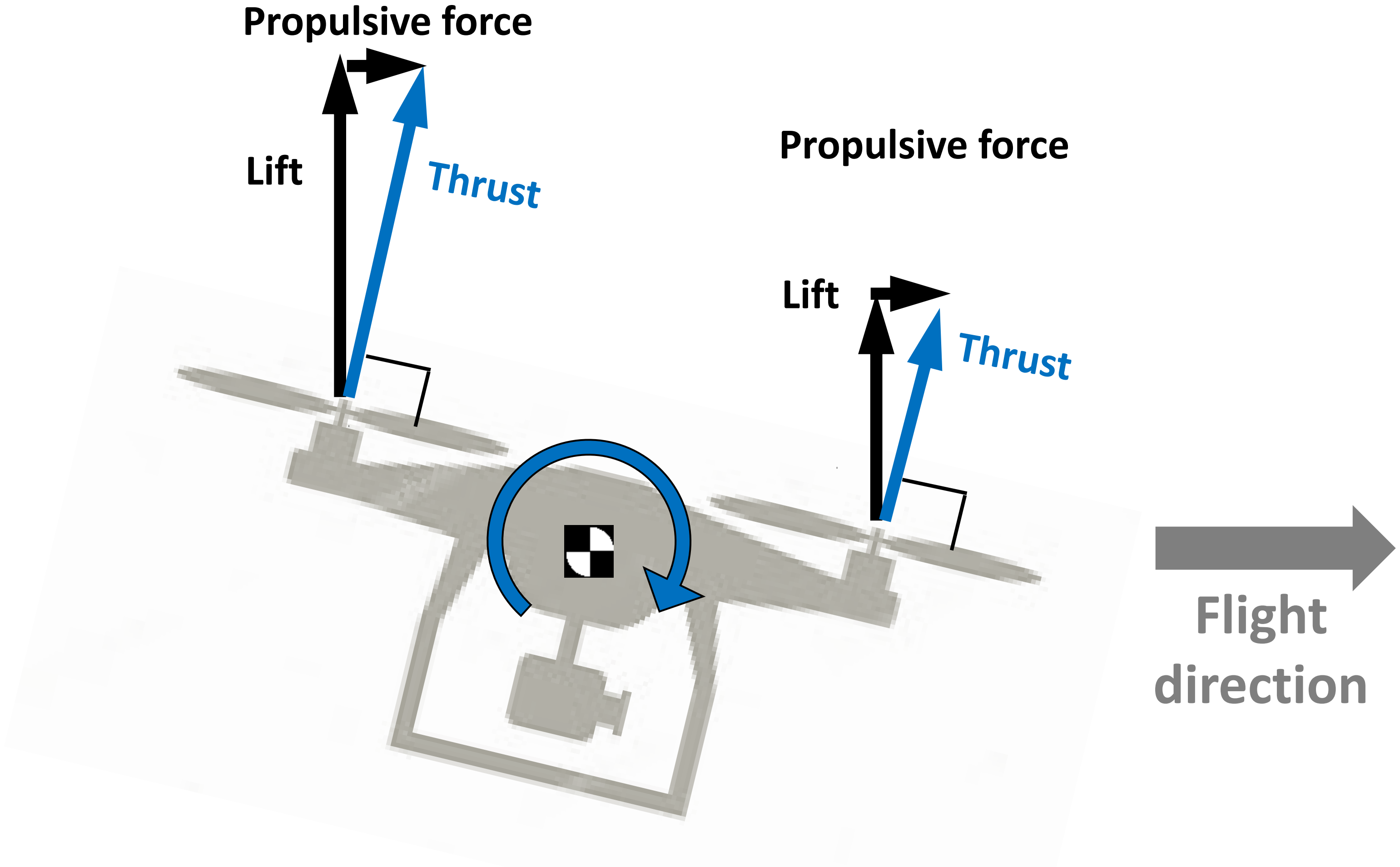
Starting point: steady hover

Rotated about the vehicle CG

Accelerating away in the x direction

Control Methodology and Trade Study for Other VTOL Configurations

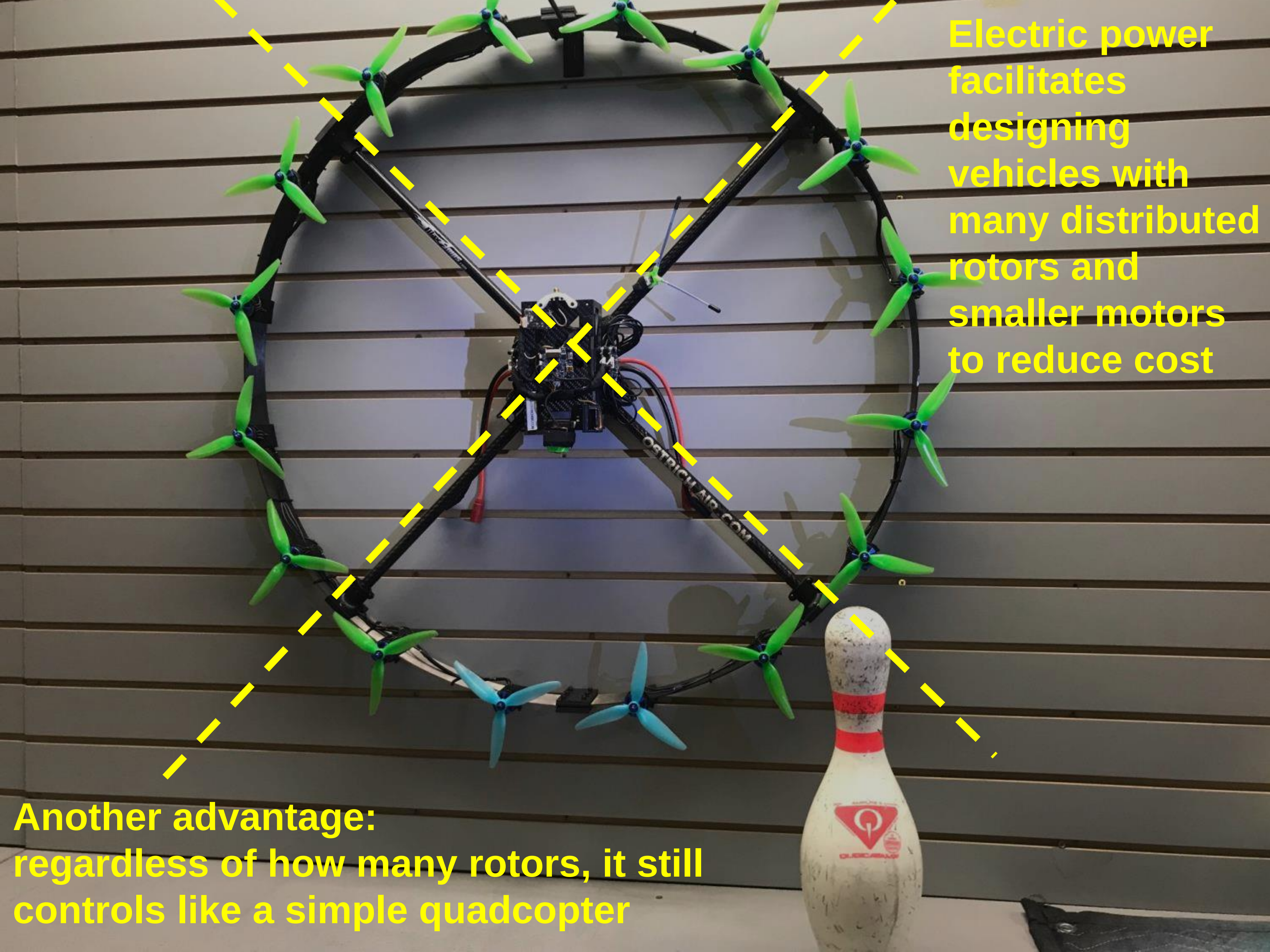
Quadcopters also have to rotate first then translate. The dynamics are similar to helicopters.



Advantage: no mechanical controls reduces design complexity

Disadvantage: tilting the vehicle in forward flight can be uncomfortable for passenger

Volocopter X2



Electric power facilitates designing vehicles with many distributed rotors and smaller motors to reduce cost

Another advantage:
regardless of how many rotors, it still controls like a simple quadcopter

Basic Lift + Cruise Design

Propellers for lift only

Propeller for forward flight only

Advantages:
(1) Eliminate heavy mechanical tilting mechanism
(2) Simplify control laws design by separating hover and forward flight
(3) Fuselage remains horizontal during transition and forward flight.
Disadvantages: Carrying two sets of rotors, more drag in forward flight.

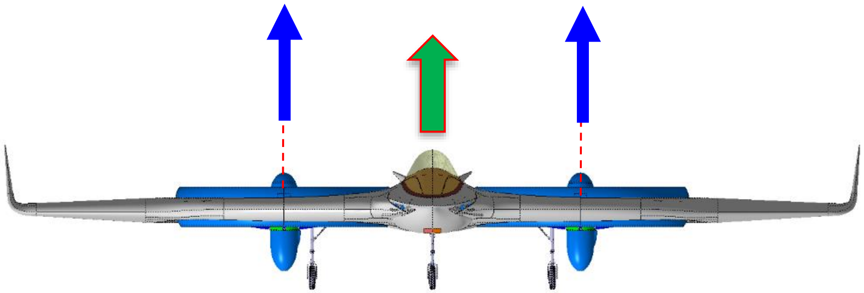
Increasing the Complexity Factor by Retracting the Lift Rotors to Reduce Drag

During the Conceptual Design Phase,
It is important to analyze complexity
versus cost, schedule, and reliability

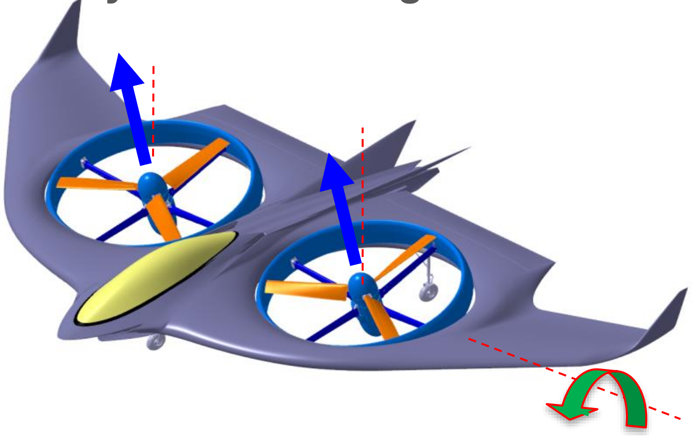
Tiltrotors Increases Control Laws Complexity During Transition Flight Regime. But Minimizes Drag in Forward Flight.

Example: Project Zero's Controls in Hover

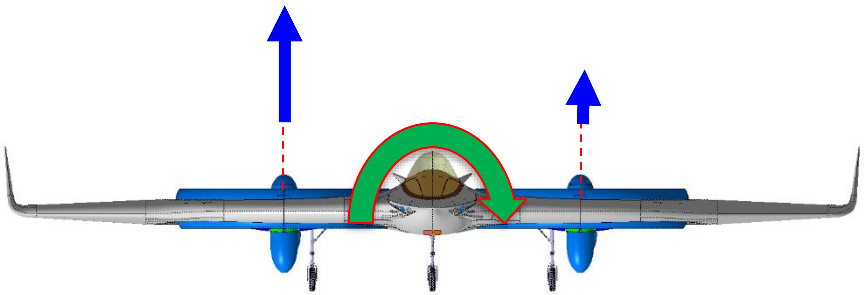
Heave: synchronous collective pitch



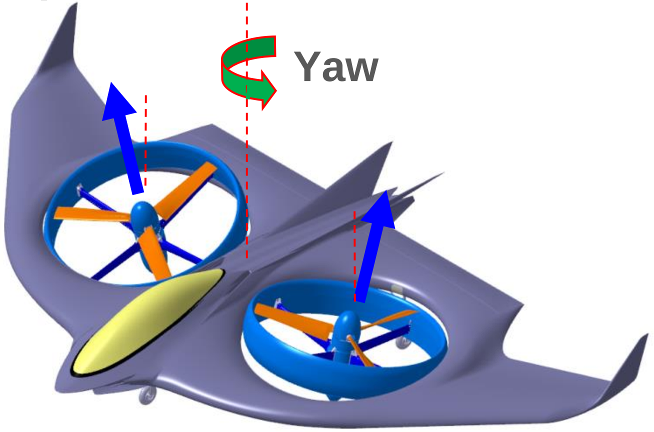
Pitch: longitudinal cyclic coupled with rotor symmetric tilting



Roll: differential collective pitch

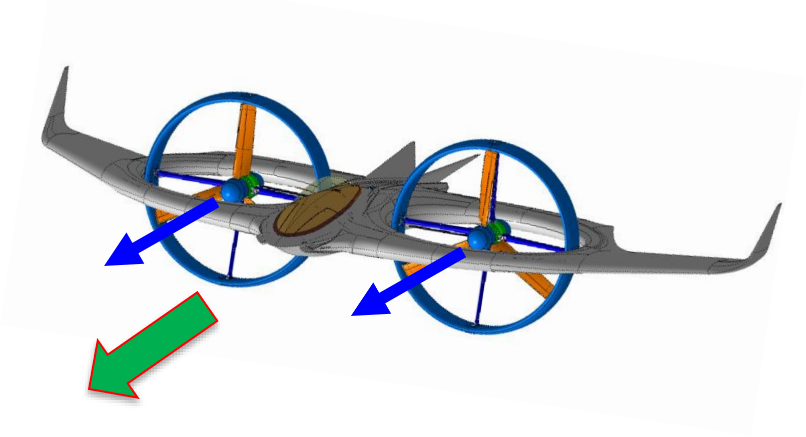


Yaw: differential longitudinal cyclic coupled with rotor differential tilting

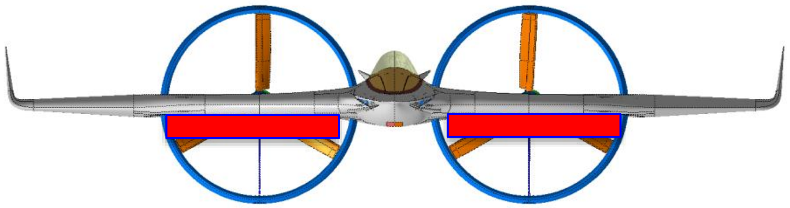


Project Zero's Controls in Forward Flight

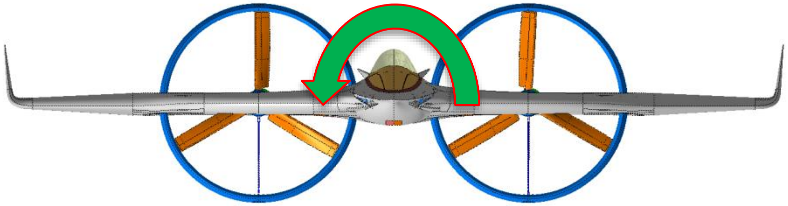
Thrust: synchronous collective pitch



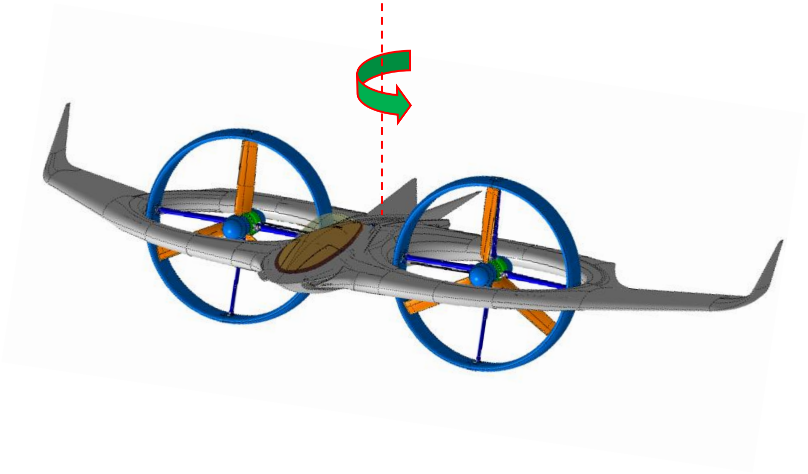
Pitch: synchronous elevon deflection



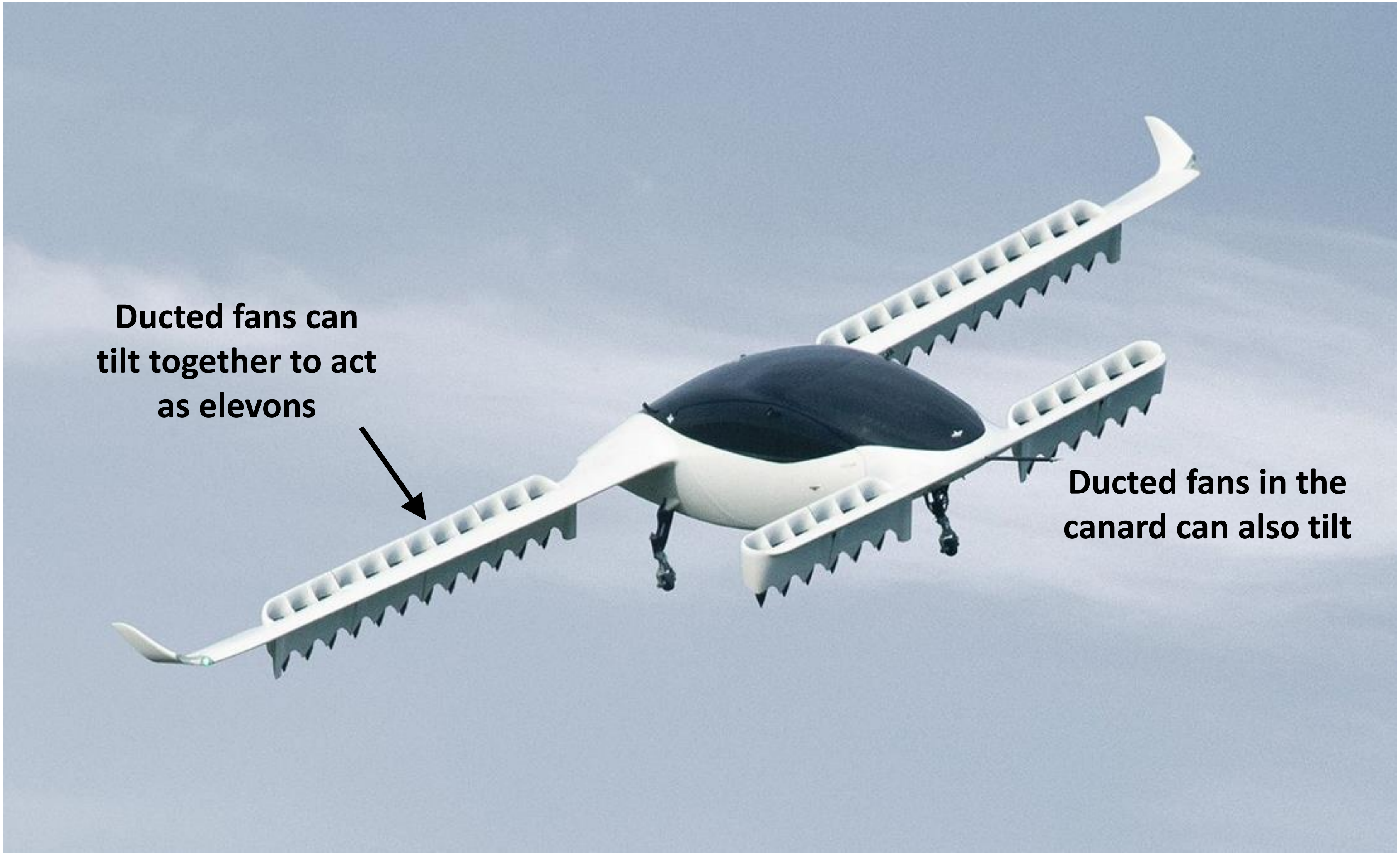
Roll: differential elevon deflection



Yaw: differential collective pitch



Lilium Jet's Control Surfaces



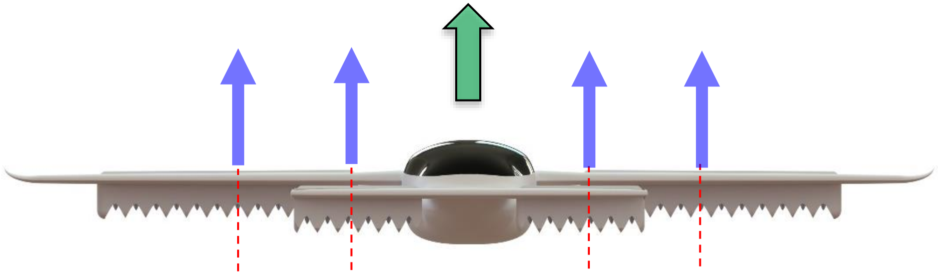
Ducted fans can tilt together to act as elevons

Ducted fans in the canard can also tilt

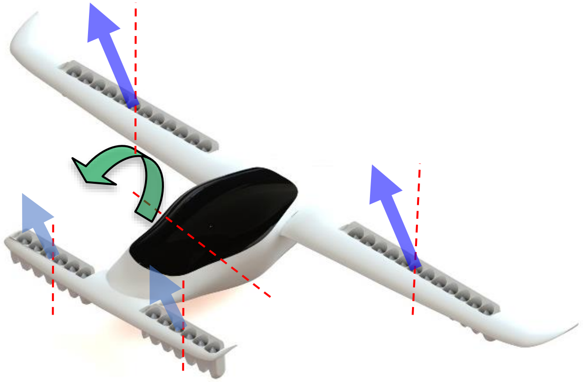
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Lilium Jet's Controls in Hover

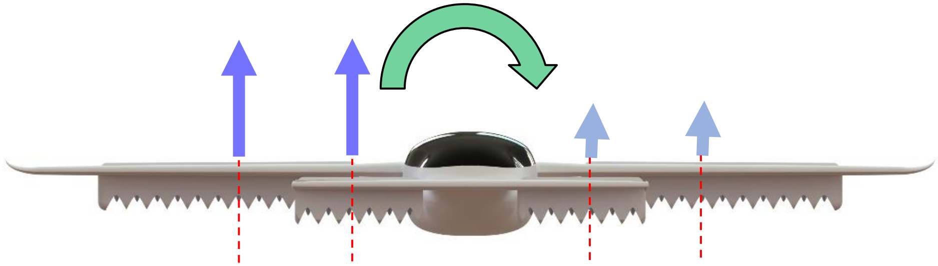
Heave: synchronous thrust change by rpm



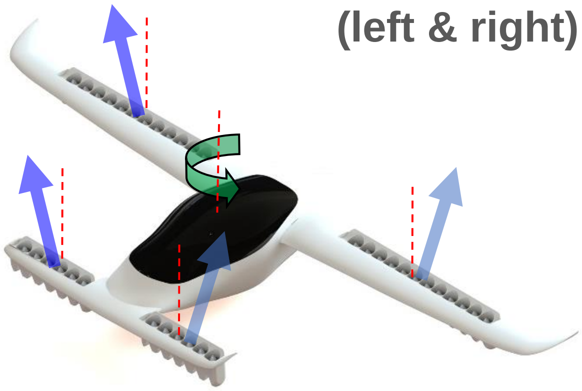
Pitch: differential thrust (fore & aft) & thrust vectoring



Roll: differential thrust by rpm (left & right)



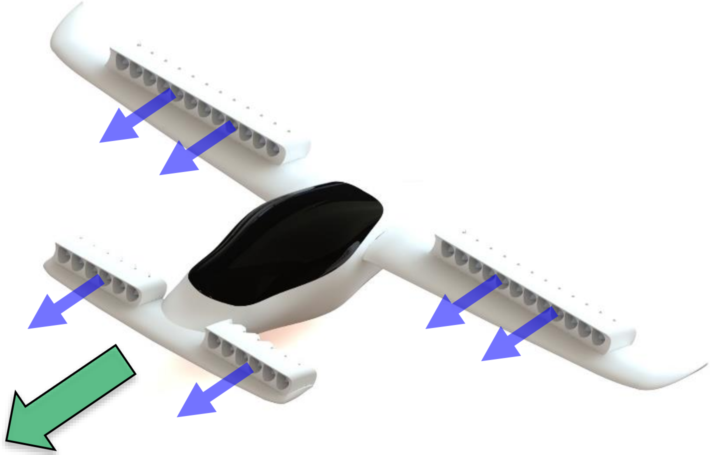
Yaw: differential thrust vectoring (left & right)



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

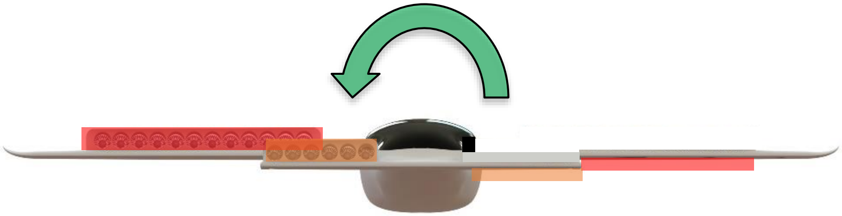
Lilium Jet's Controls in Forward Flight

Thrust: thrust change by varying rpm

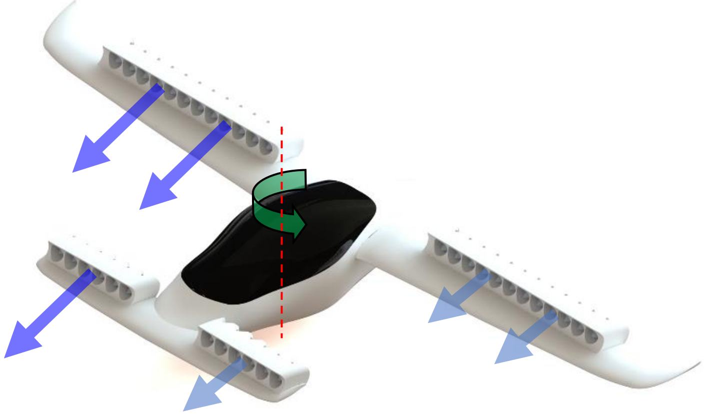


Pitch: ducted canards
& wing fans acting as elevator

Roll: differential ducted canards & wing
fans acting as aileron



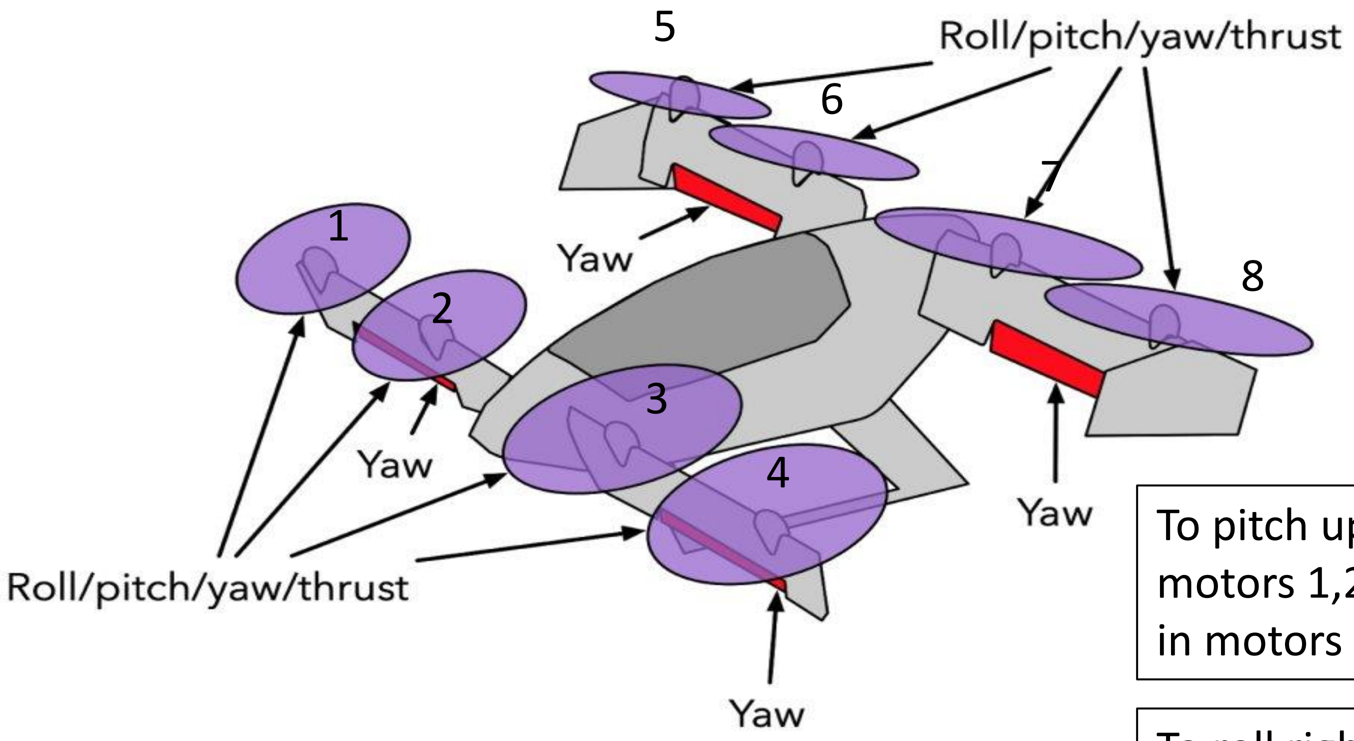
Yaw: differential thrust (left & right)



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Airbus Vahana (Tiltwing)

Vahana Controls in Hover



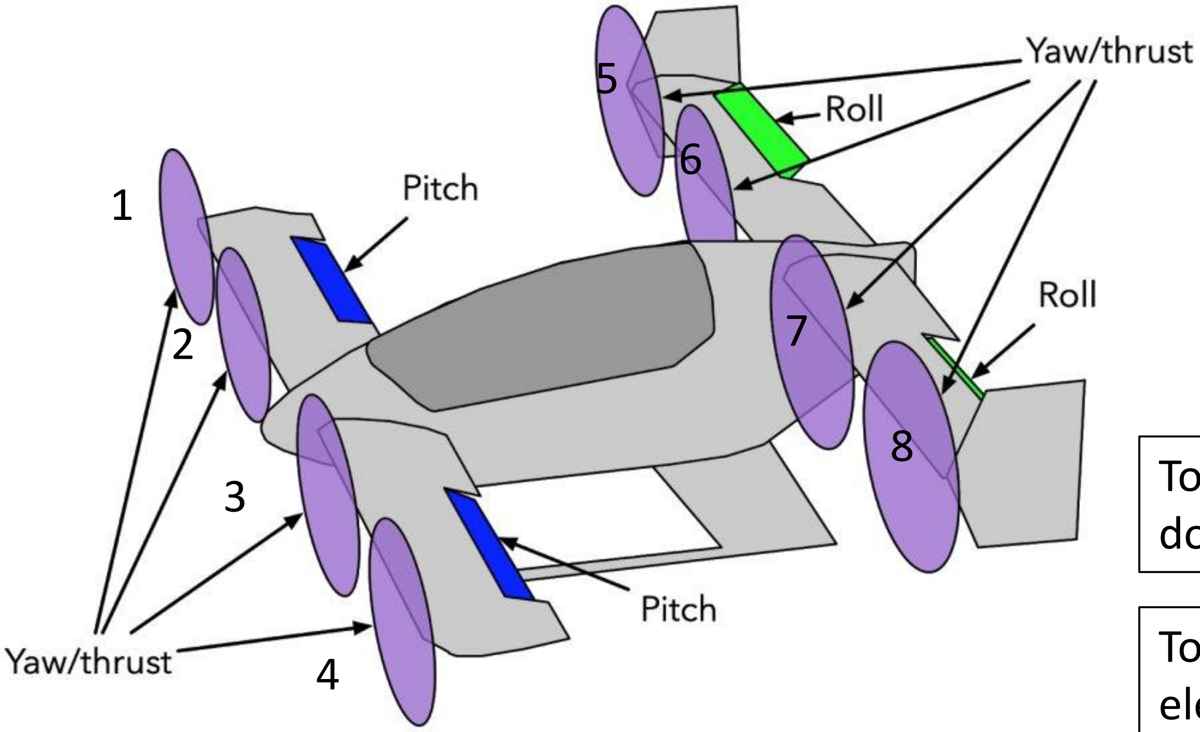
To pitch up, increase thrust in motors 1,2,3,4 and decrease thrust in motors 5,6,7,8

To roll right, increase thrust in motors 3,4,7,8 and decrease thrust in motors 1,2,5,6

To yaw right, increase thrust in motors 3,4,5,6 and decrease thrust in 1,2,7,8 (+red elevons to increase effect)

Source: https://acubed.airbus.com/blog/vahana/exploring-control-allocation-for-e-vtol-vehicles/

Vahana Controls in Forward Flight



To pitch up, deflect blue elevons down (both same direction)

To roll right, deflect left green elevon down + rotate right green elevon up

To yaw right, increase thrust in motors 3,4,7,8 and decrease thrust in motors 1,2,5,6

Source: https://acubed.airbus.com/blog/vahana/exploring-control-allocation-for-e-vtol-vehicles/

Joby S4

The picture can't be displayed.

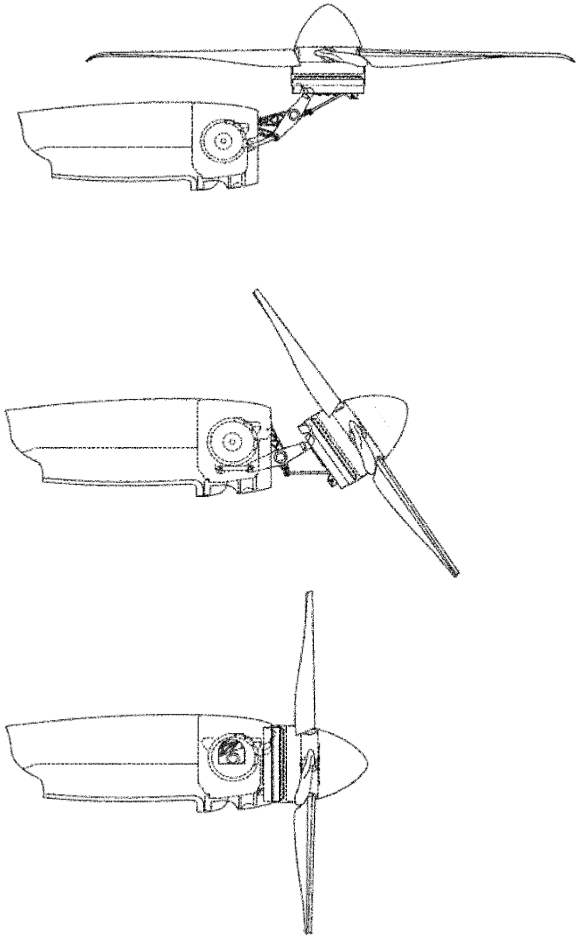
Ruddervators

Flaps

Ailerons

Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

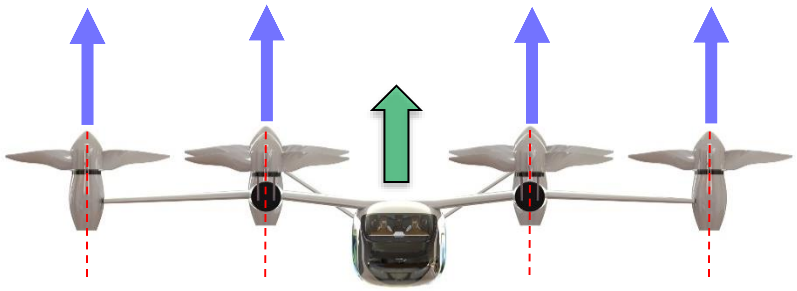
Joby S4's Rotor Tilting Mechanism



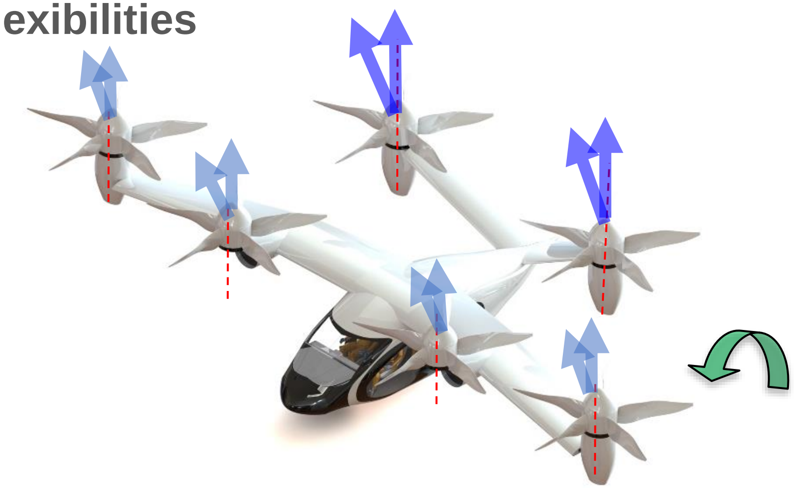
Source: VFS Nov 2020 eVTOL News, Sustainableskies and Joby patent

Joby S4's Controls in Hover

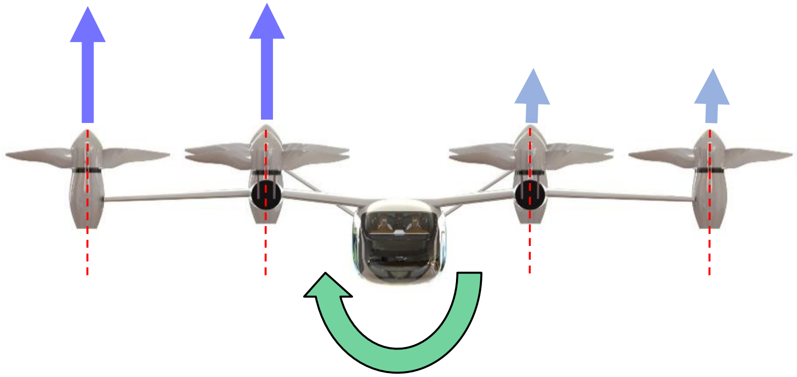
Heave: synchronous rotor thrust control by collective pitch



Pitch: differential thrust (fore & aft) & can tilt the fore and aft pylons. Many flexibilities



Roll: differential thrust (left & right)

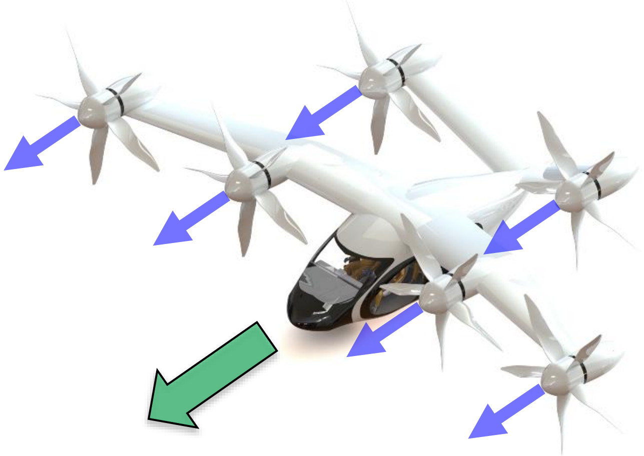


Yaw: rotor nacelle tilting (disclosed in Joby patent)

Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

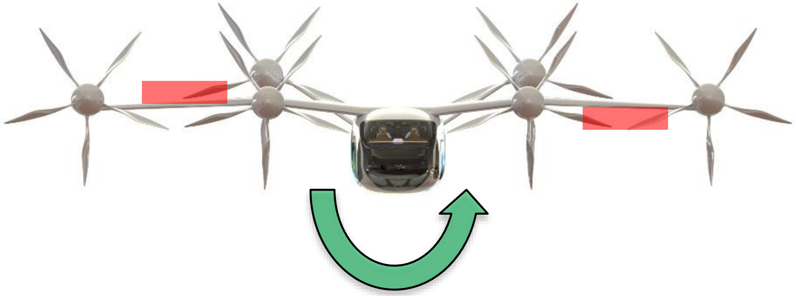
Joby S4's Controls in Forward Flight

Thrust: thrust change by varying rpm & variable pitch propellers

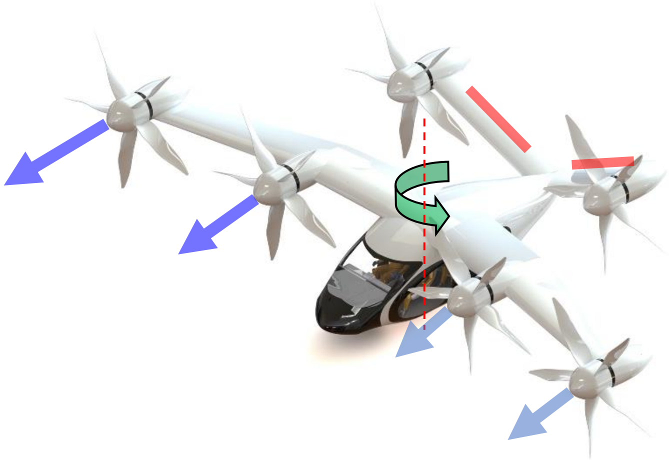


Pitch: tail ruddervators deflection

Roll: differential wing ailerons



Yaw: ruddervators & differential thrust (left & right)

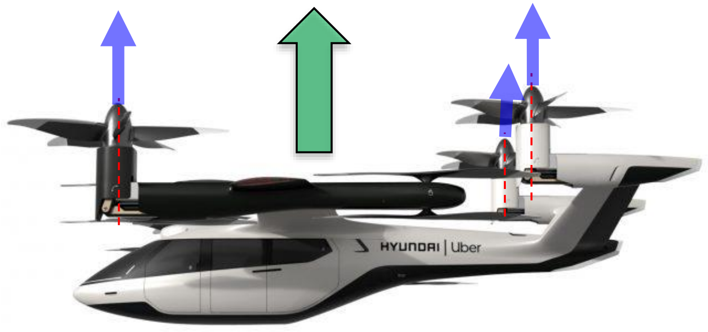


Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

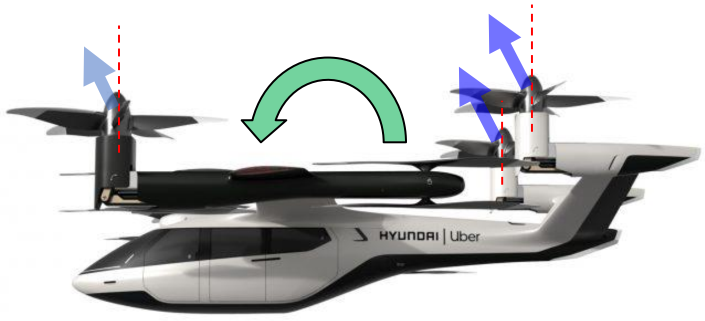
Hyundai Supernal S-A1

Hyundai Supernal S-A1's Controls in Hover

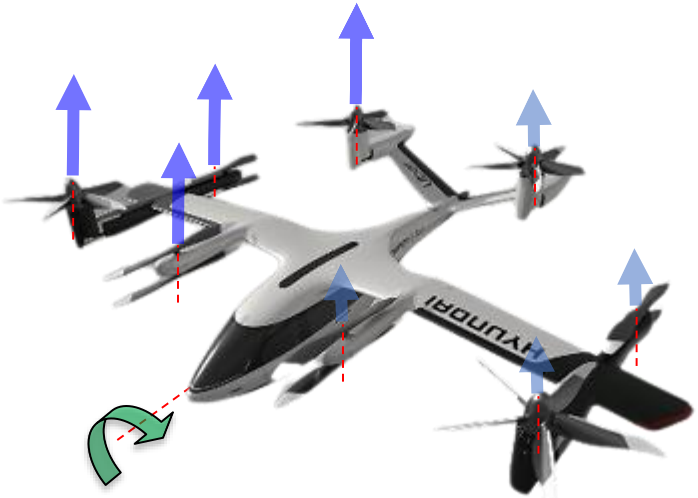
Heave: synchronous rotor thrust control by collective pitch and rpm



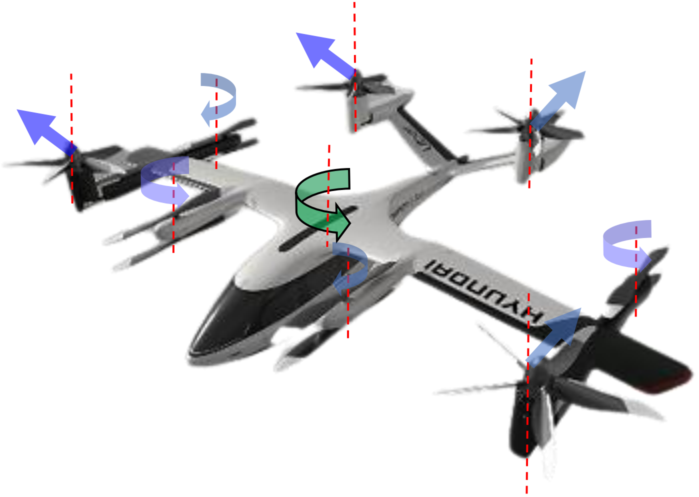
Pitch: forward tilting rotors & differential thrust (fore & aft)



Roll: differential thrust (left & right)

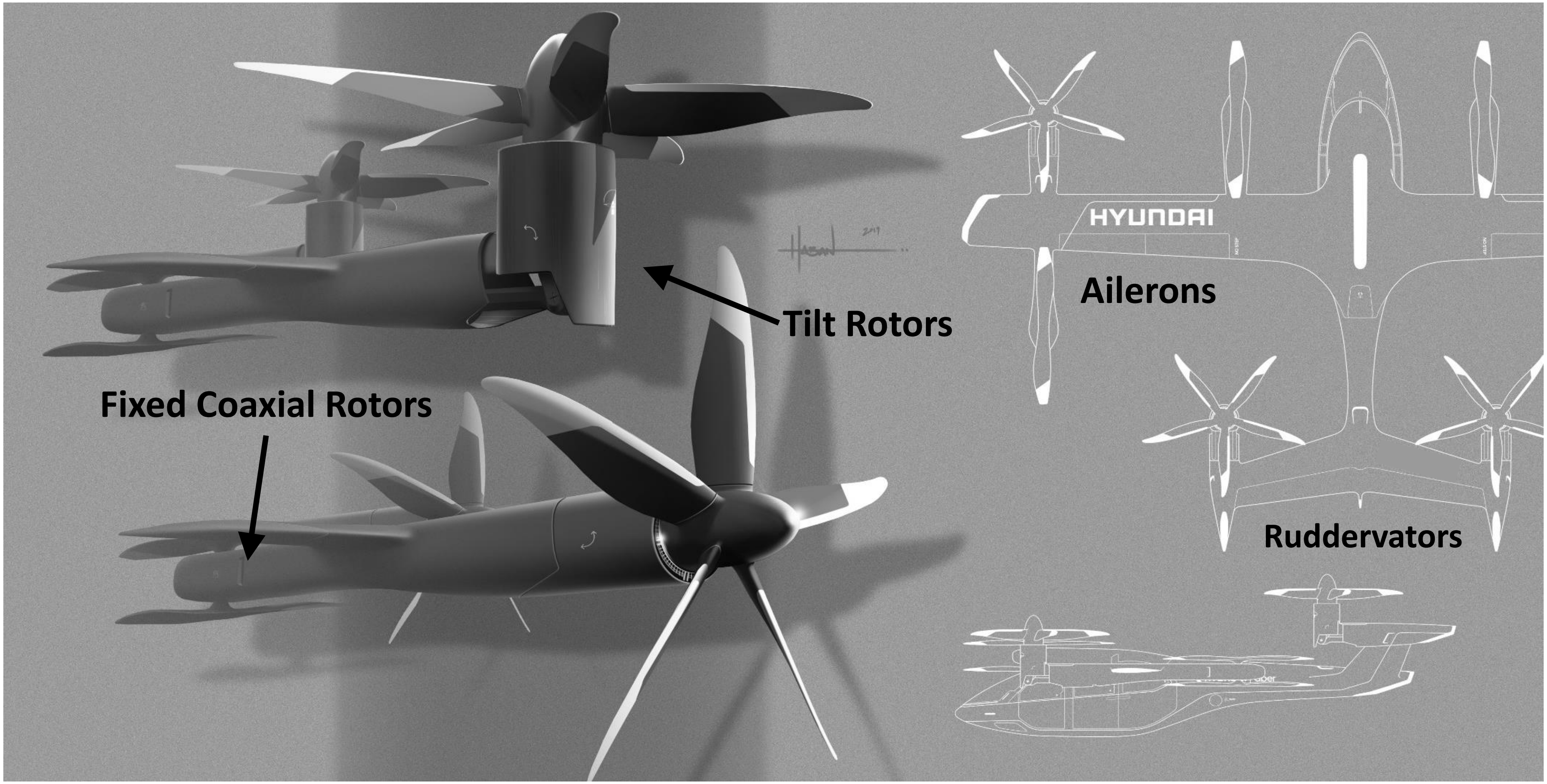


Yaw: rotor tilting angle (left & right) & differential rpm change



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

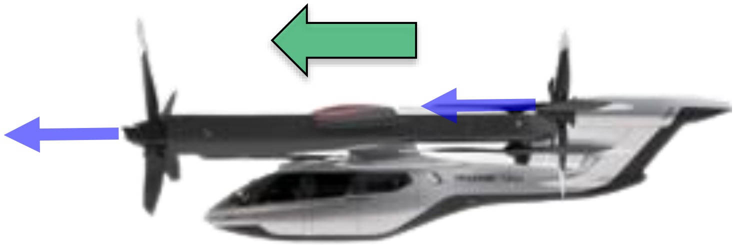
Hyundai Supernal S-A1 Control Surfaces



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

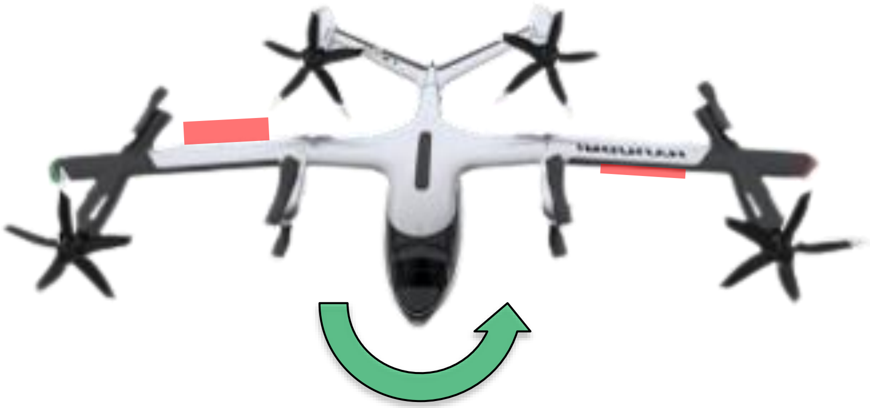
Hyundai S-A1's Controls in Forward Flight

Thrust: thrust change by varying pitch

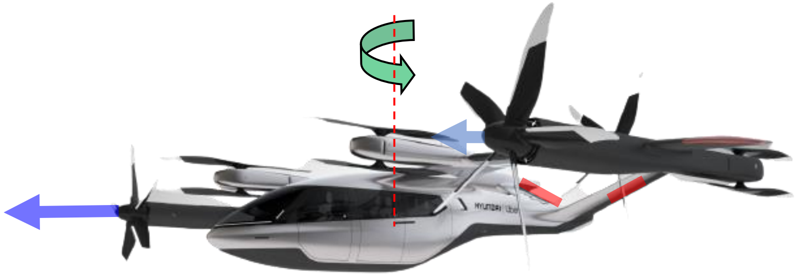


Pitch: tail ruddervators deflection

Roll: differential wing ailerons deflection

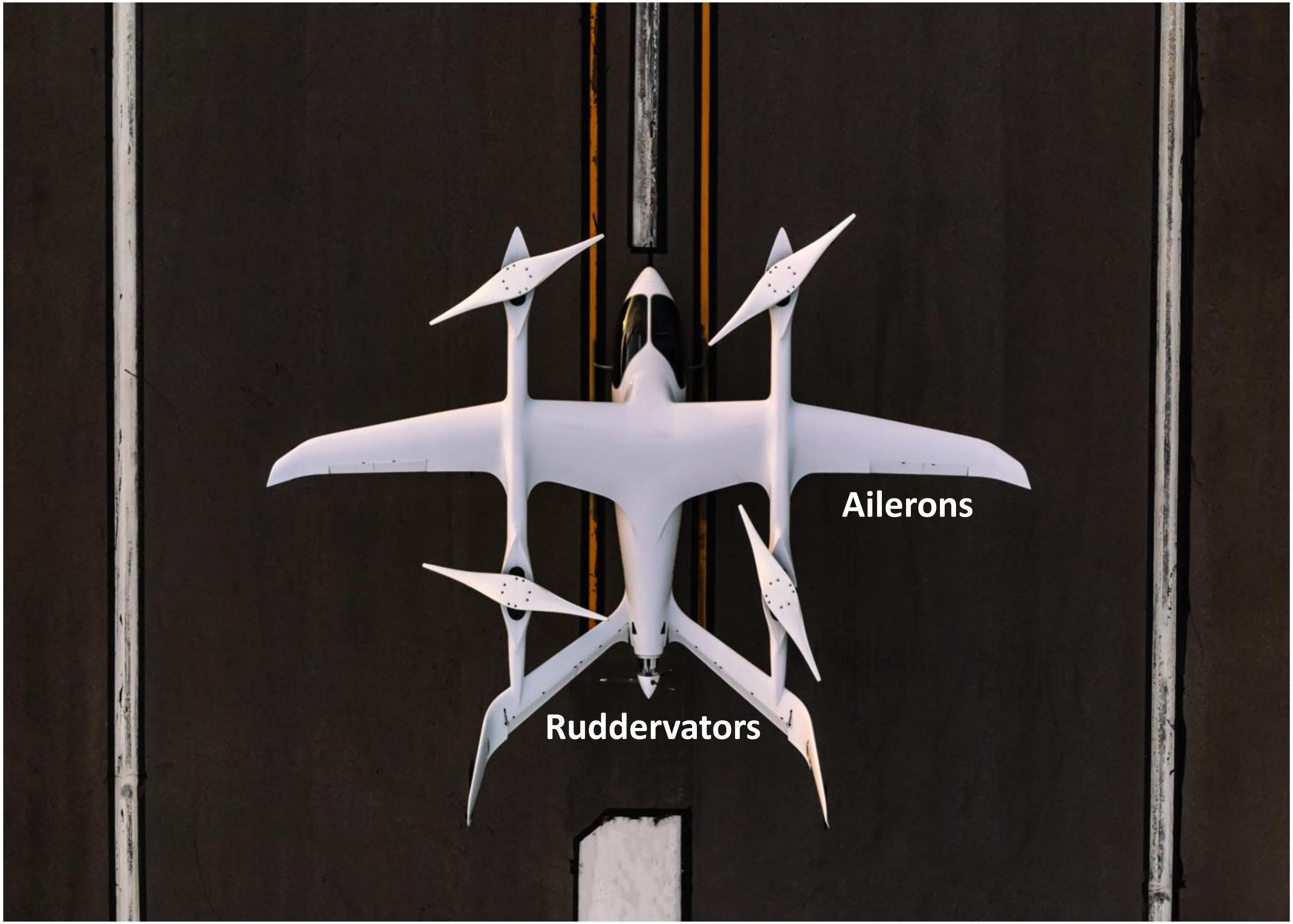


Yaw: differential thrust (left & right) & ruddervator deflection



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

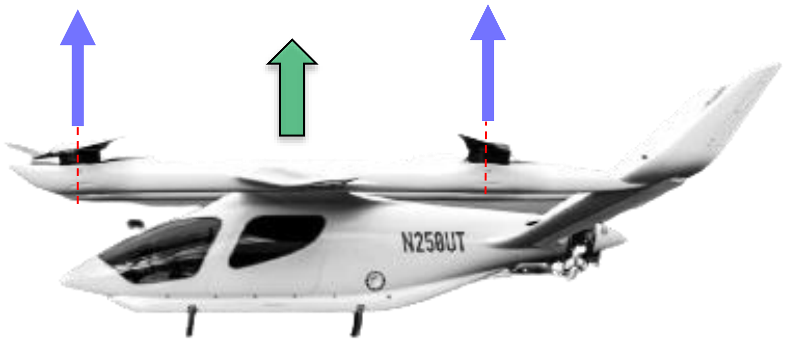
Beta Technologies Alia



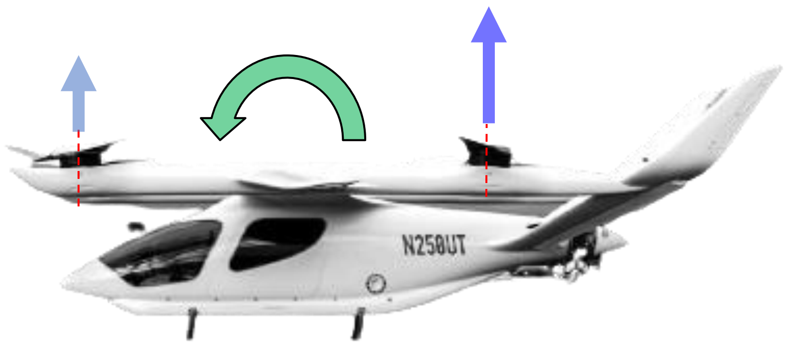
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Beta Technologies Alia's Controls in Hover

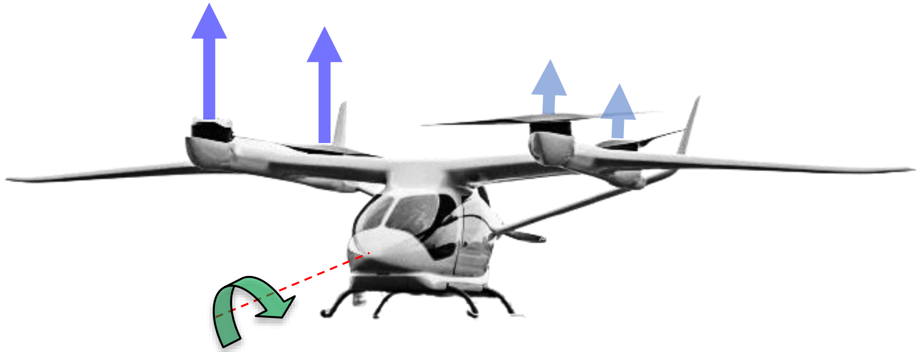
Heave: synchronous thrust change by rpm



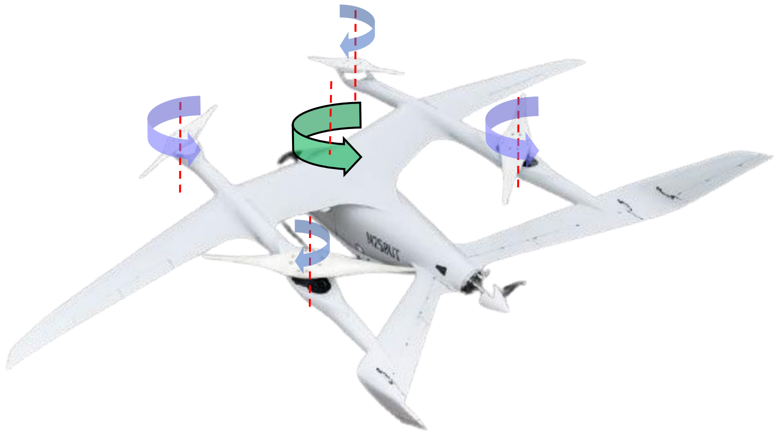
Pitch: differential thrust (fore & aft)



Roll: differential thrust (left & right)



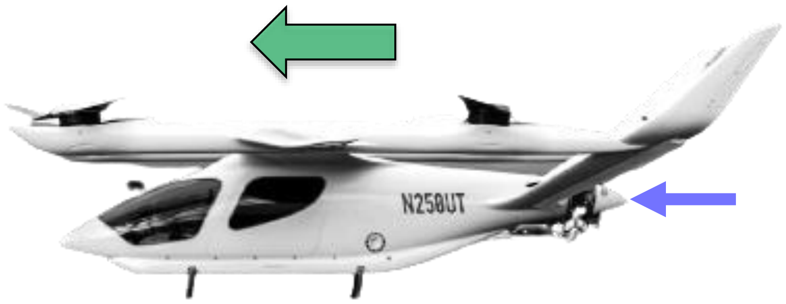
Yaw: differential rpm change



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Beta Alia's Controls in Forward Flight

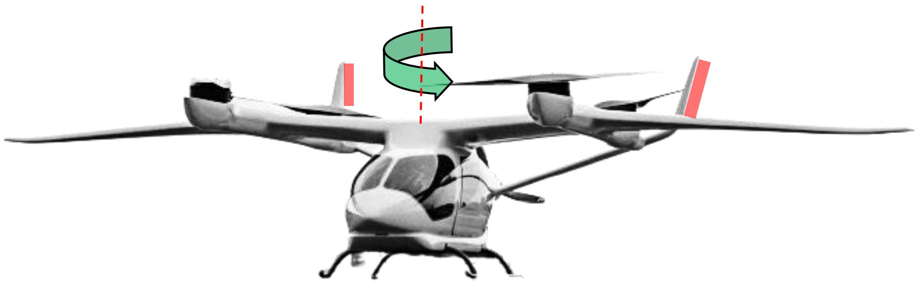
Thrust: rear thrust change by varying rpm



Pitch: tail elevators deflection

Roll: differential wing aileron deflection

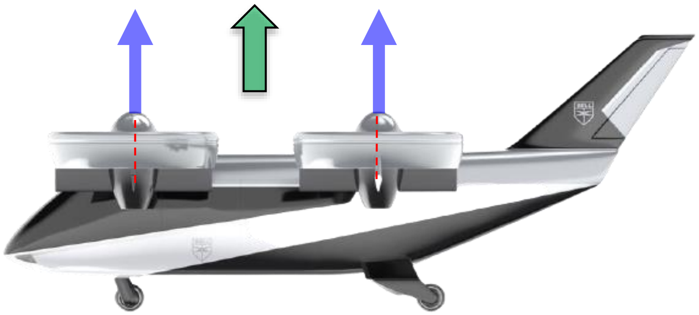
Yaw: tail rudders deflection



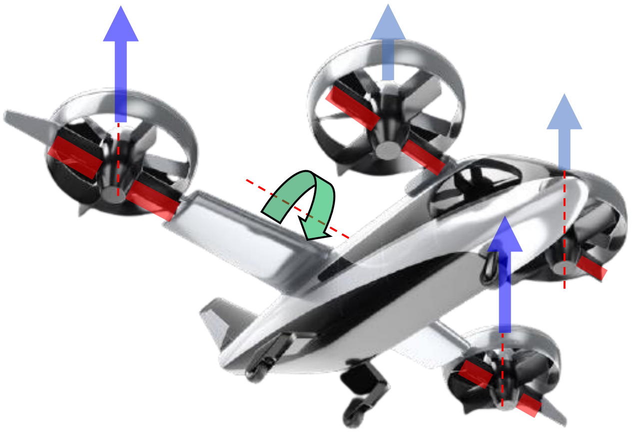
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Bell Nexus 4EX's Controls in Hover

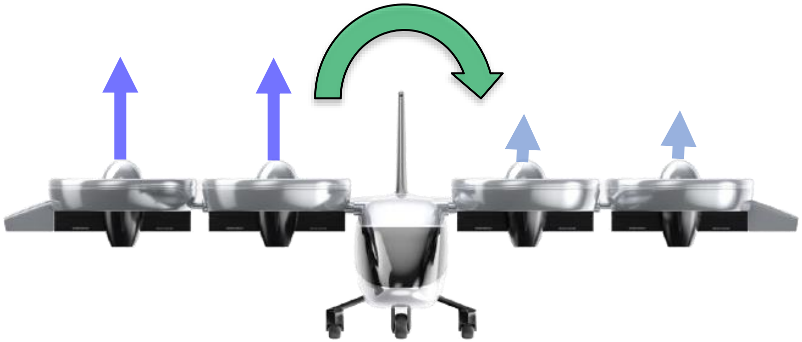
Heave: synchronous thrust change by varying pitch



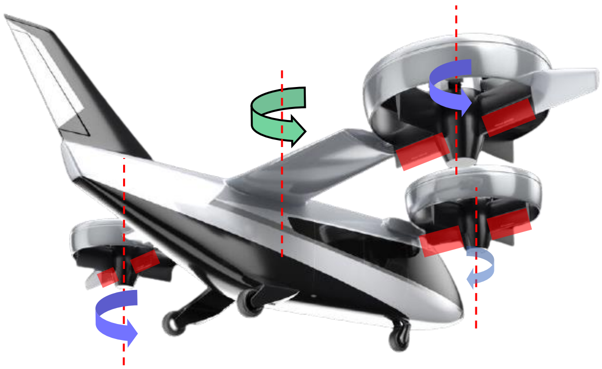
Pitch: differential thrust (fore & aft) & ducted control vanes



Roll: differential thrust (left & right)

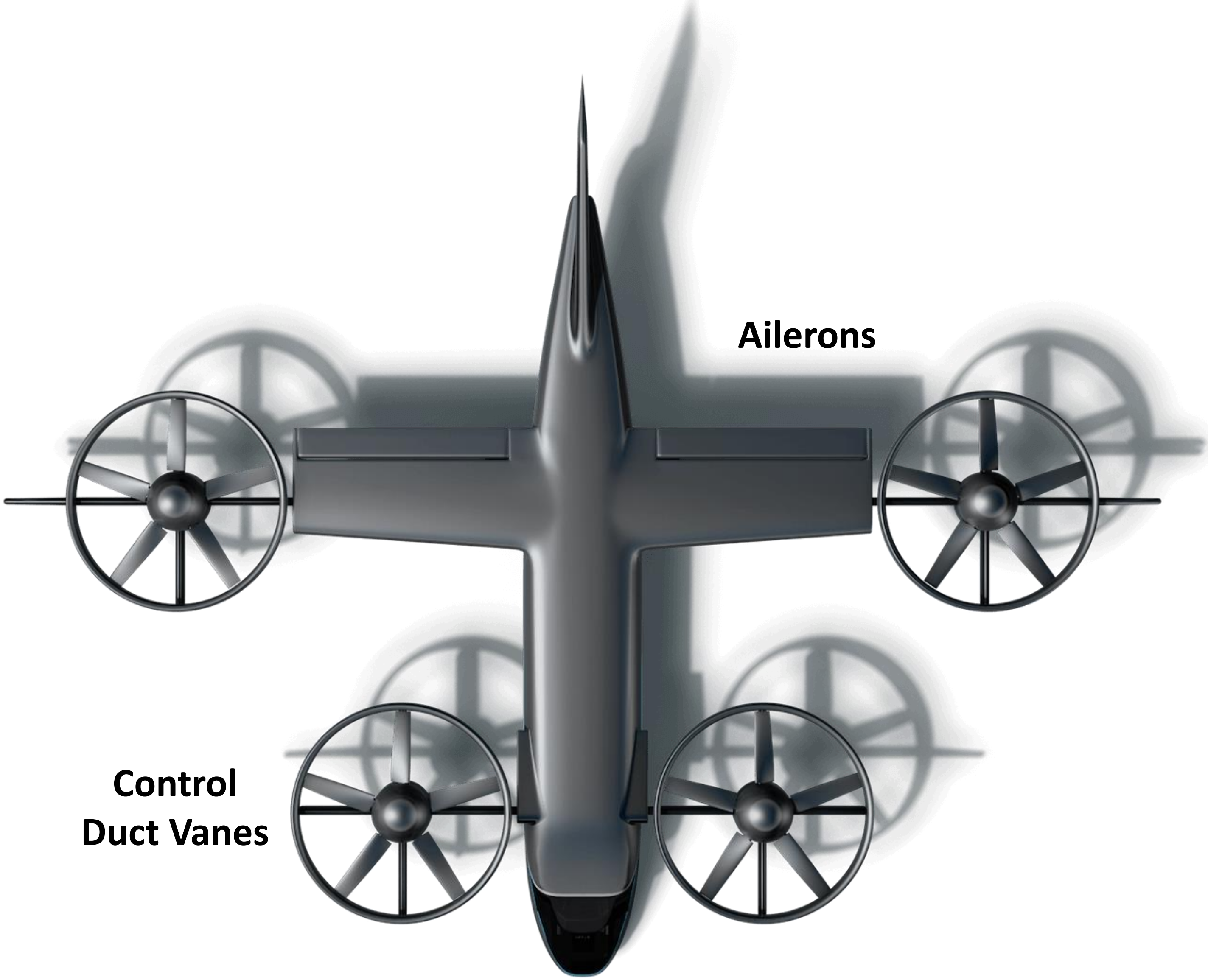


Yaw: differential rpm change & ducted control vanes



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

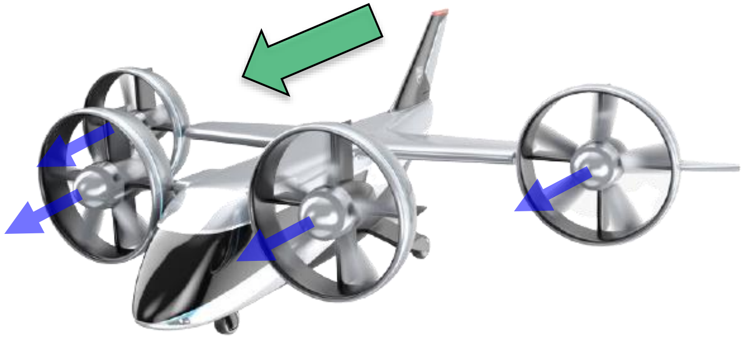
Bell Nexus 4EX's Control Surfaces



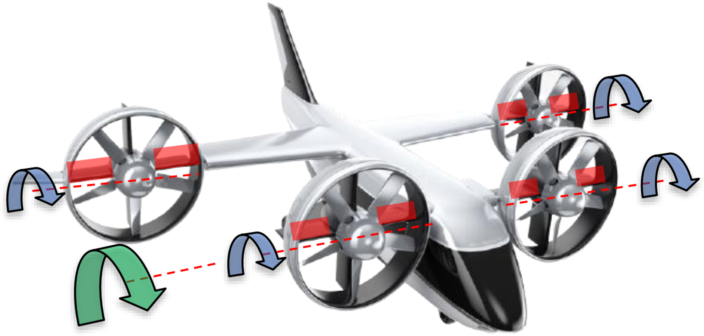
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Bell Nexus 4EX's Controls in Forward Flight

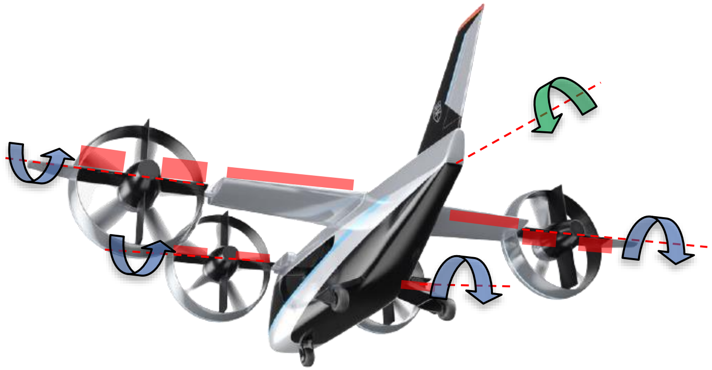
Thrust: thrust change by varying pitch



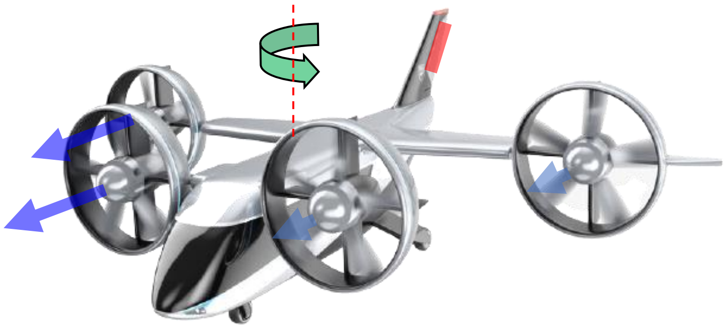
Pitch: tilting rotors & ducted vanes deflection



Roll: differential tilting rotors, ailerons & ducted vanes deflections

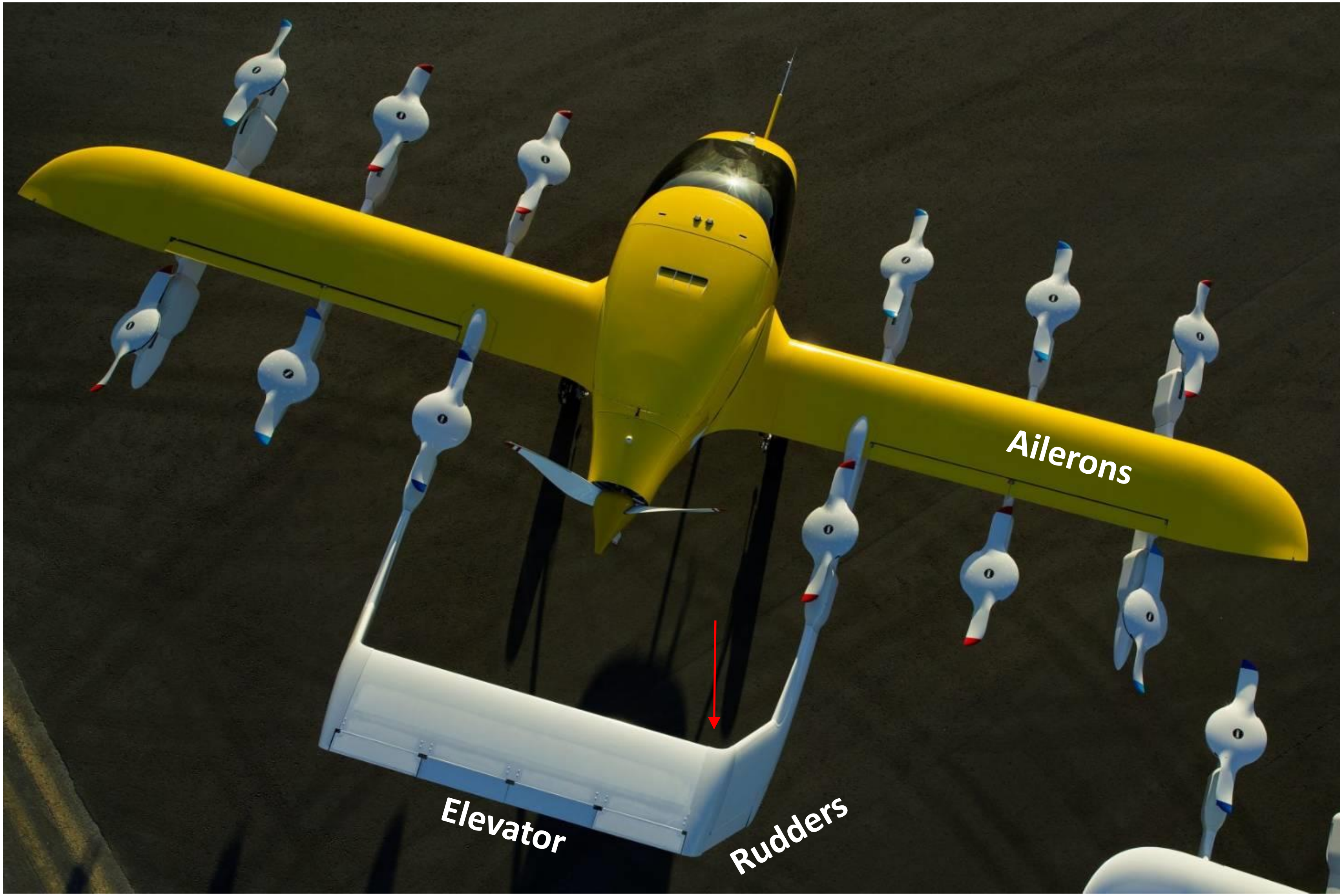


Yaw: tail rudder deflection & differential thrust (left & right)



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

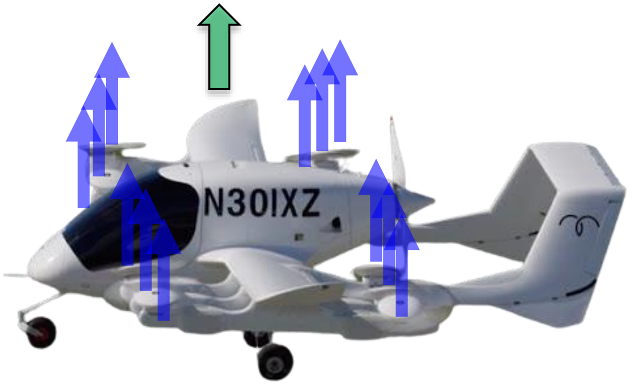
Wisk Cora Control Surfaces



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

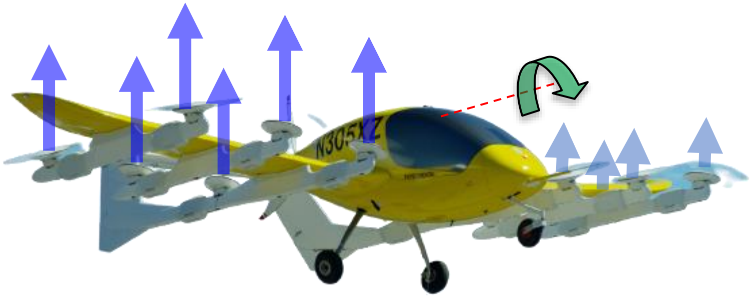
Wisk Cora Controls in Hover

Heave: synchronous thrust change by rpm

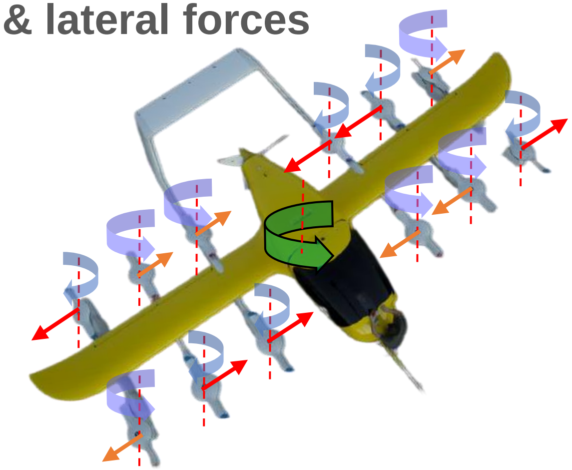


Pitch: differential thrust (fore & aft)

Roll: differential thrust (left & right)



Yaw: differential rpm change & lateral forces



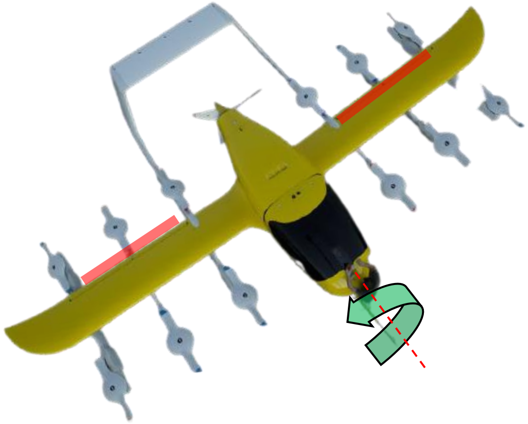
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Wisk Cora Controls in Forward Flight

Thrust: thrust change by varying rpm

Pitch: tail elevator deflection

Roll: differential ailerons deflection

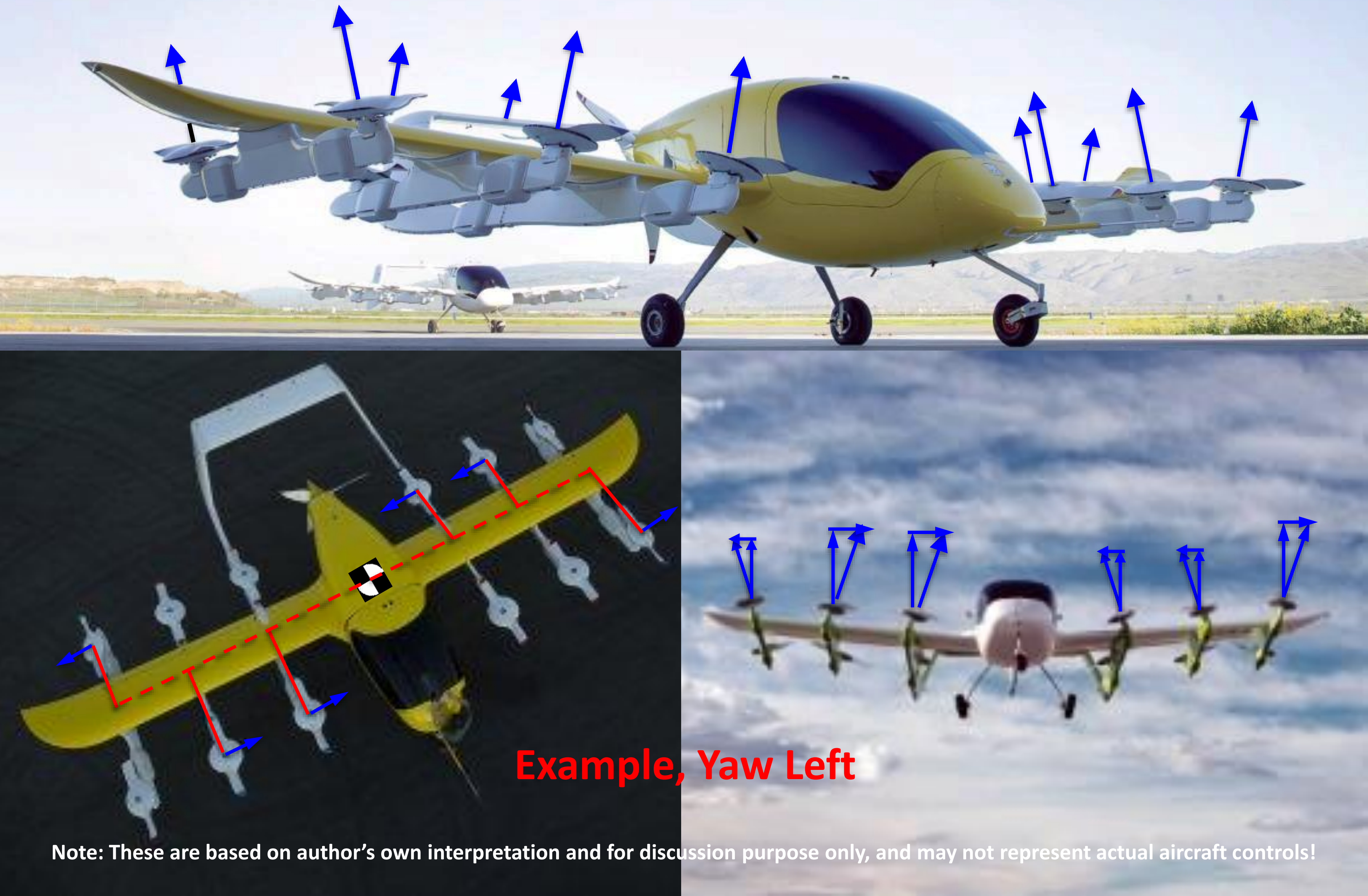


Yaw: tail rudders deflection

Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Wisk Cora Lift Rotors are all Canted

Wisk Cora Yaw Controls in Hover



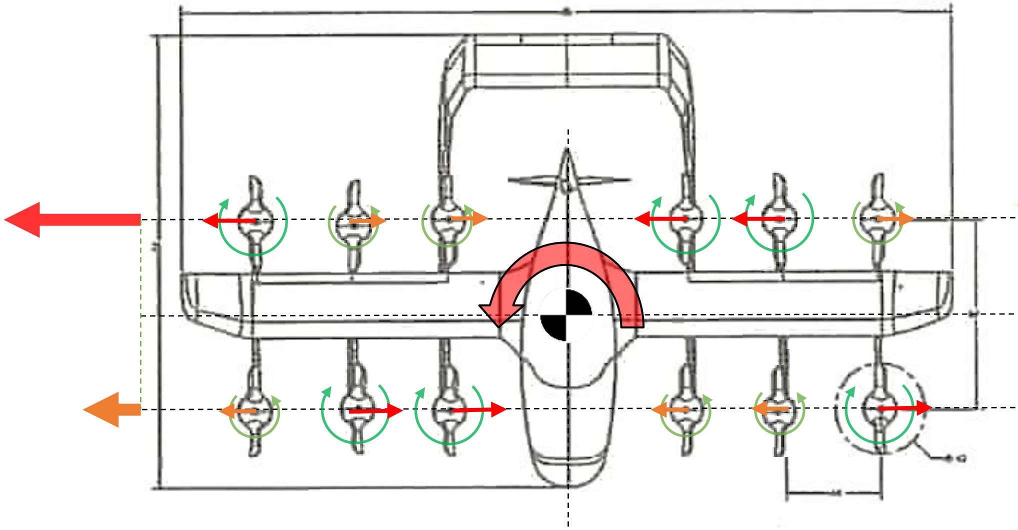
Example, Yaw Left

Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

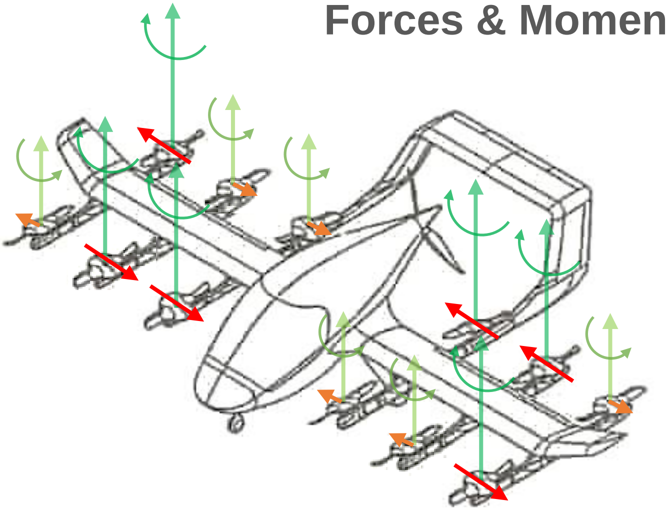
Wisk Cora's Angled Rotors

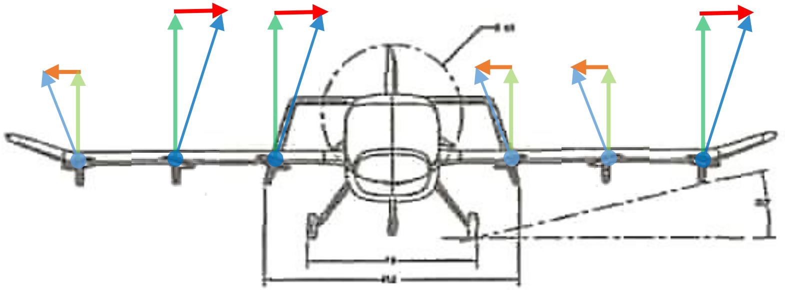
Increased yaw control effectiveness with rotor cant angle

Yawing Moment Components

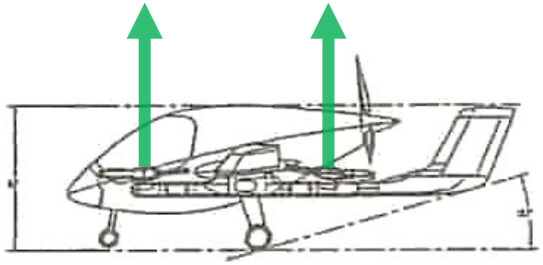


Forces & Moments





Thrust Vector Components



Total Vertical Thrust

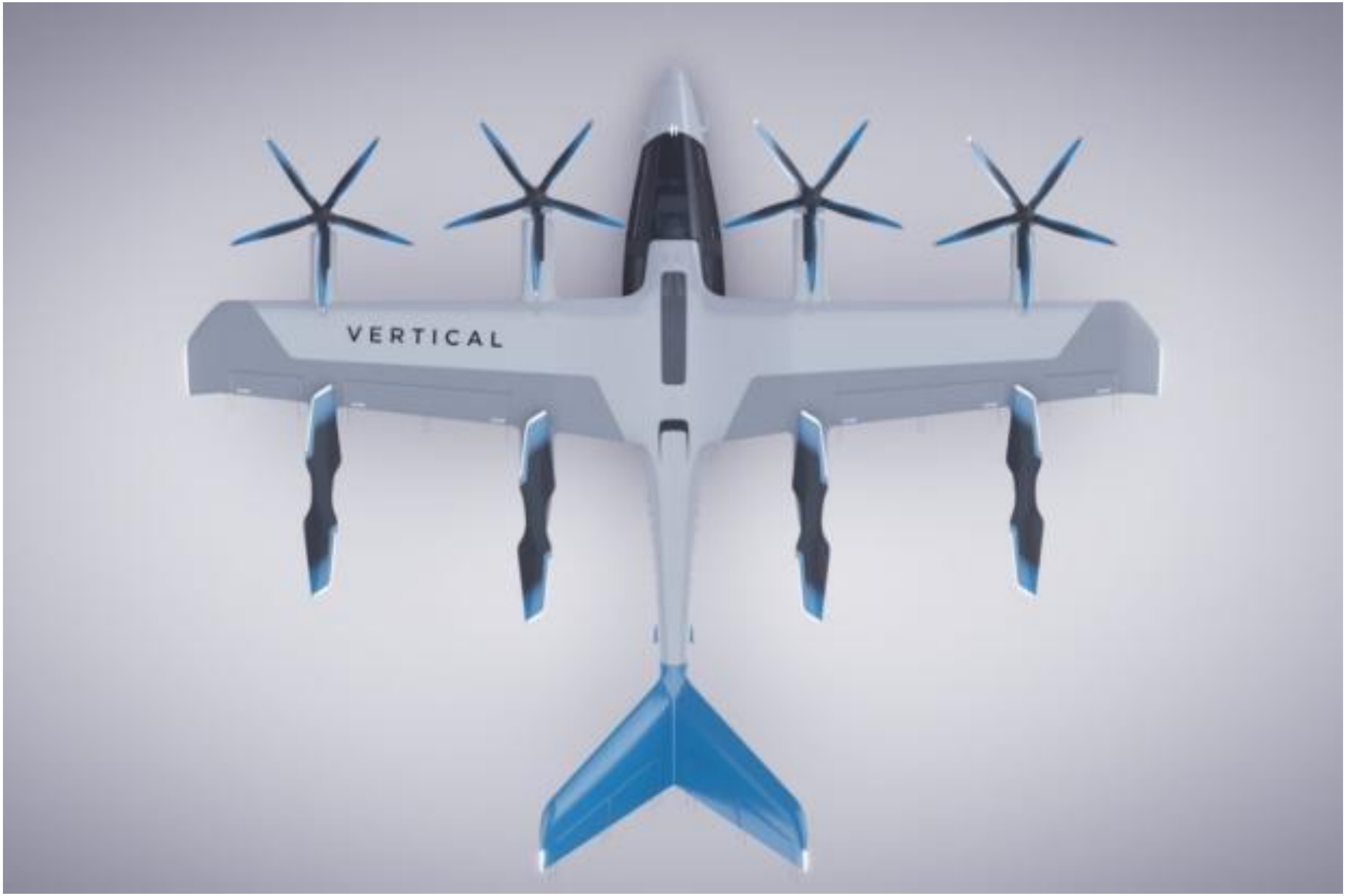
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Boeing PAV eVTOL also tilts the lift fans for enhancing yaw controls in hover

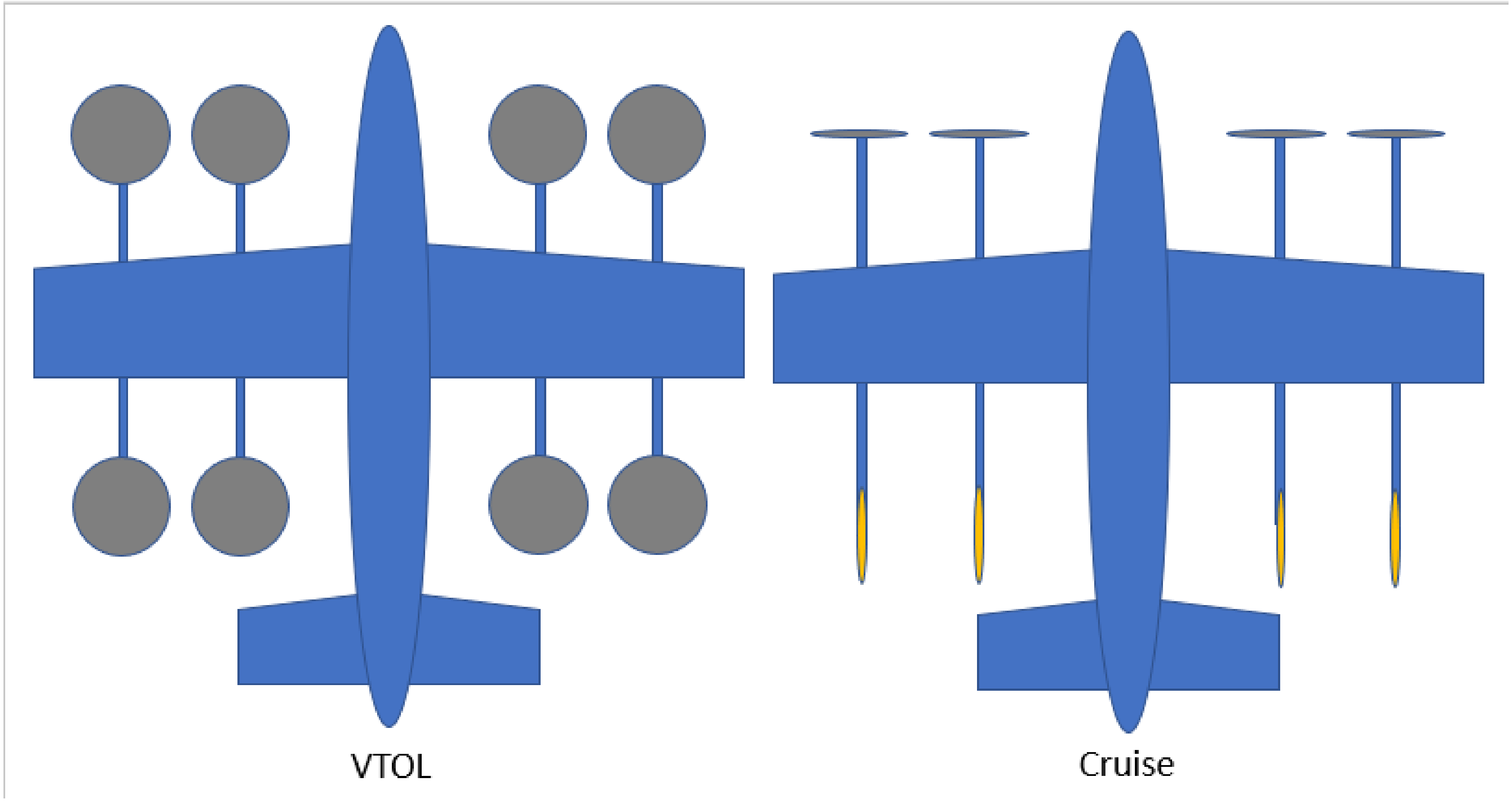
From 2019 Paris Airshow

Archer Aviation Maker

Vertical Aerospace VA-X4

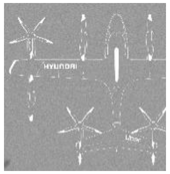


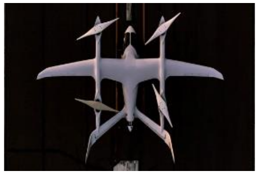
Archer Aviation Maker and Vertical Aerospace VA-X4



Summary of eVTOL Models Studied

Assumed Control Capabilities

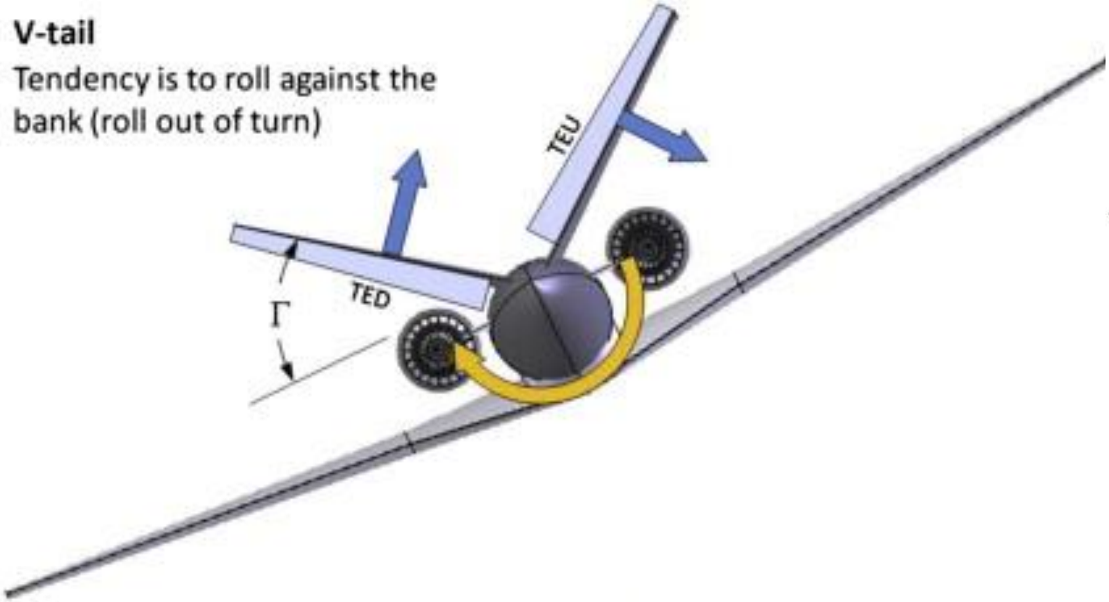




Control Strategy	**Lilium Jet**	**Joby S4**	**Hyundai SA-1**	**Bell Nexus 4EX**	**Beta Alias**	**Wisk Cora**
Hover Controls	Multirotor + Thrust Vectoring Capabilities	Multirotor + Thrust Vectoring Capabilities	Multirotor + Thrust Vectoring Capabilities	Multirotor + Thrust Vectoring Capabilities	Multirotor	Multirotor
Forward Flight Controls	Tilting ducted rotors on canard and wing like control surfaces	Ailerons & Ruddervators	Ailerons & Ruddervators	Ailerons & Control Vanes	Ailerons & Ruddervators	Ailerons, Rudders Elevator
Transition Modes	Rotor Tilting Vectored Thrust	Rotor Tilting Vectored Thrust	Rotor Tilting Vectored Thrust & Lift + Cruise	Rotor Tilting Vectored Thrust	Lift + Cruise	Lift + Cruise
Weight Challenge	Many motors, inverters, wires	6 tilting rotor, variable pitch, +control surfaces	4 tilting rotor, boom structures, +control surfaces	4 massive duct tilting rotor, +control surfaces	Boom structures and 4 lift rotors for hover only	12 rotors and pylons for hover only
Control Challenges	In hover rely on rpm change, could be delay	Require automatic scheduling nacelle tilting vs airspeed	Require automatic scheduling nacelle tilting vs airspeed	Require automatic scheduling duct tilting vs airspeed	Transfer/blend multirotor and airplane controls during transition	Transfer/blend multirotor and airplane controls during transition

V-Tail

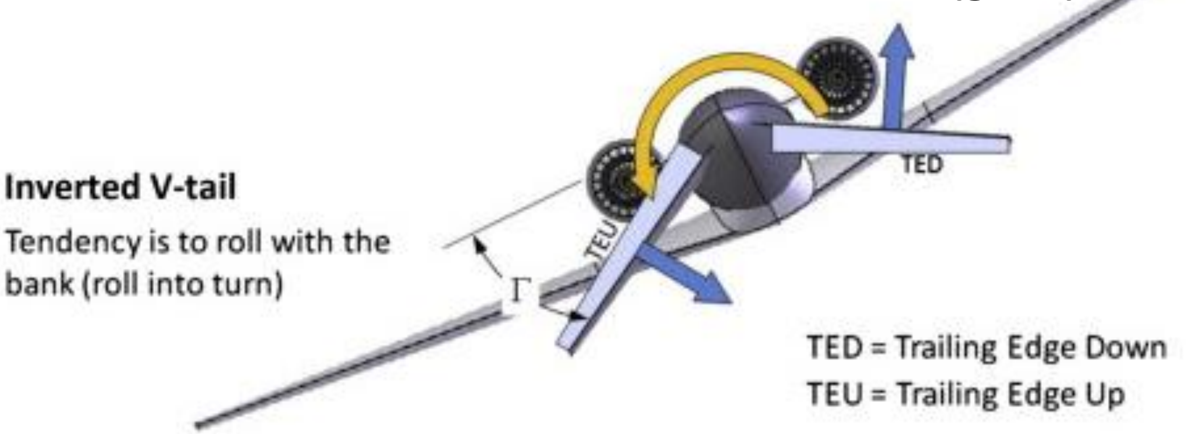
Left ruddervator also causes roll right



V-tail

Tendency is to roll against the bank (roll out of turn)

Left ruddervator also causes roll left (good)

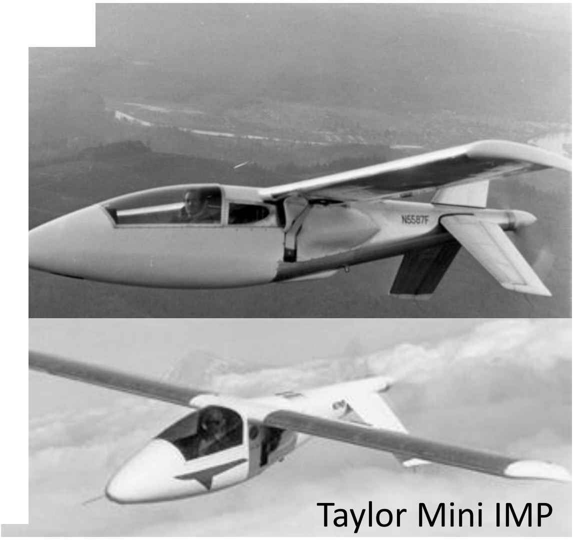


Inverted V-tail

Tendency is to roll with the bank (roll into turn)

TED = Trailing Edge Down

TEU = Trailing Edge Up



Taylor Mini IMP

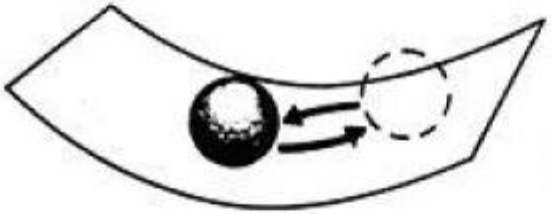
Aircraft Forward Flight Stability

I recommend design your eVTOL aircraft to be stable in forward flight first. Preferably it can be flown in forward flight without any SAS (stability augmentation system).

Then use SAS to stabilize hover. Helicopters can hover without SAS, but it is almost surely your eVTOL aircraft will require SAS for hovering.

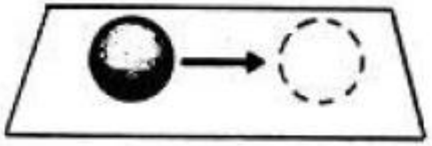
Static Stability

Static stability of a body is an initial tendency of that body to return to its equilibrium state after a disturbance.



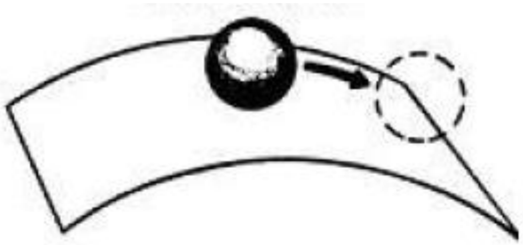
Positive static stability

Means it will return to the original state after a disturbance



Neutral static stability

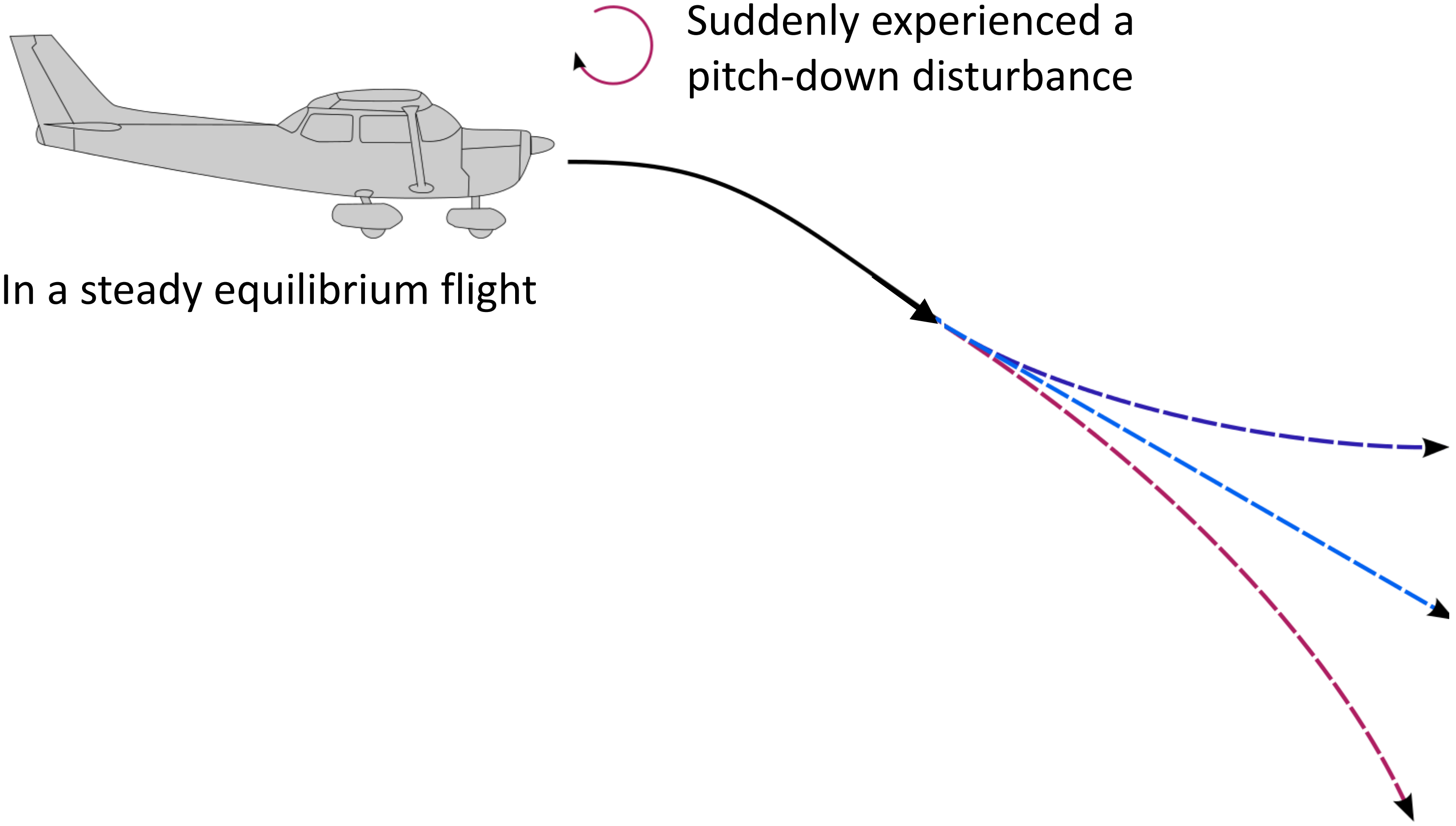
Neutral does not mean it is stable because it does not return to its original state. But it does not diverge



Negative static stability

It is unstable and diverges

Static Stability



Suddenly experienced a pitch-down disturbance

In a steady equilibrium flight

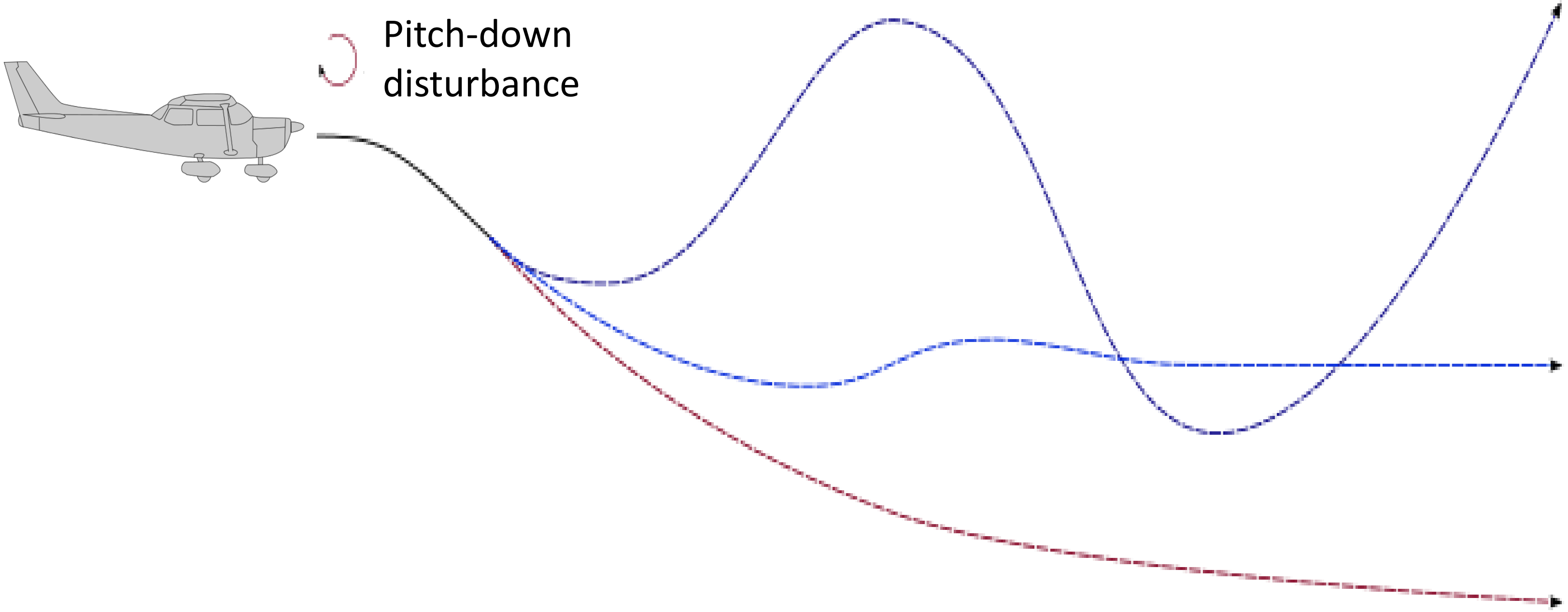
Positive static stability
(returns to level)

Neutral static stability
(continues)

Negative static stability
(diverges)

Dynamic Stability

An aircraft is said to be dynamically stable if, after a disturbance, it eventually returns to its equilibrium state.



Pitch-down
disturbance

Statically stable
Dynamically unstable

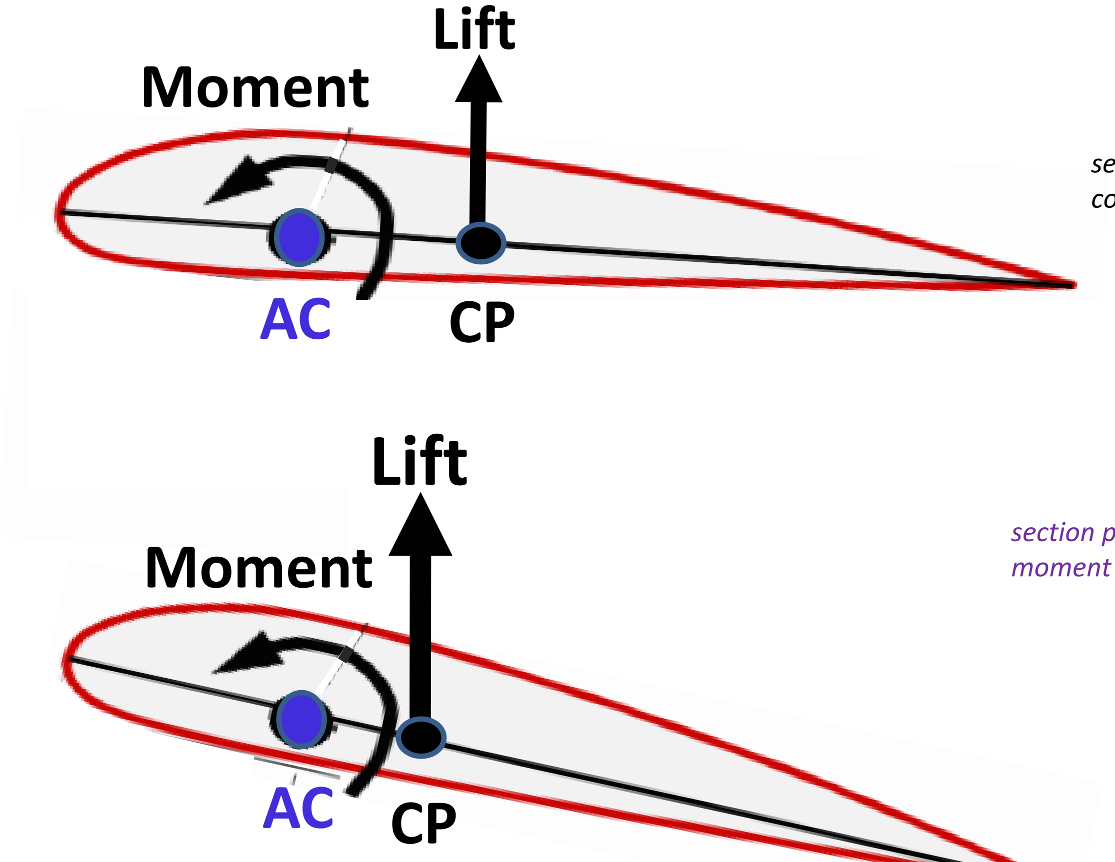
Statically
stable
Dynamically
damped

Statically stable
Dynamically overdamped

Source: Olivier Cleynen and Werneuchen

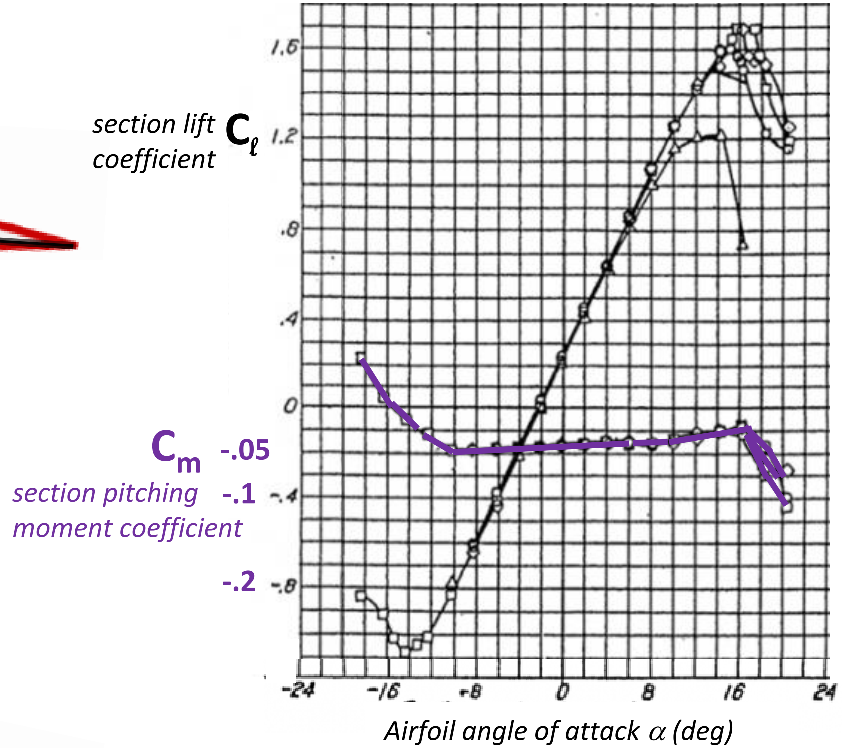
Aerodynamic Center of an Airfoil

The aerodynamic center (AC) of an airfoil, or a wing, is defined as that point on the airfoil where the aerodynamic moment (M) does not depend on angle of attack (α)



The CP will start traveling towards the leading edge and closer to the ¼ chord when α increased

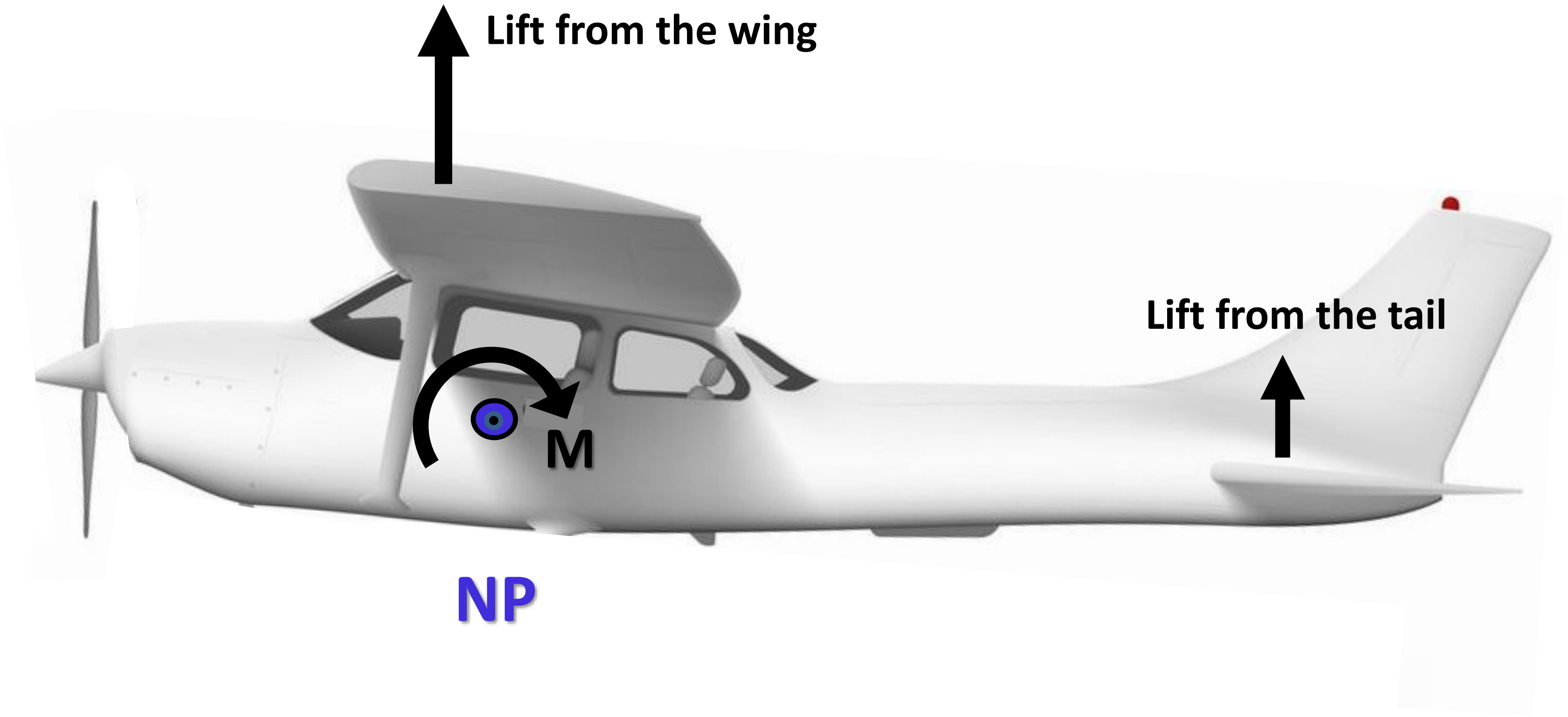
NACA 2412 airfoil data



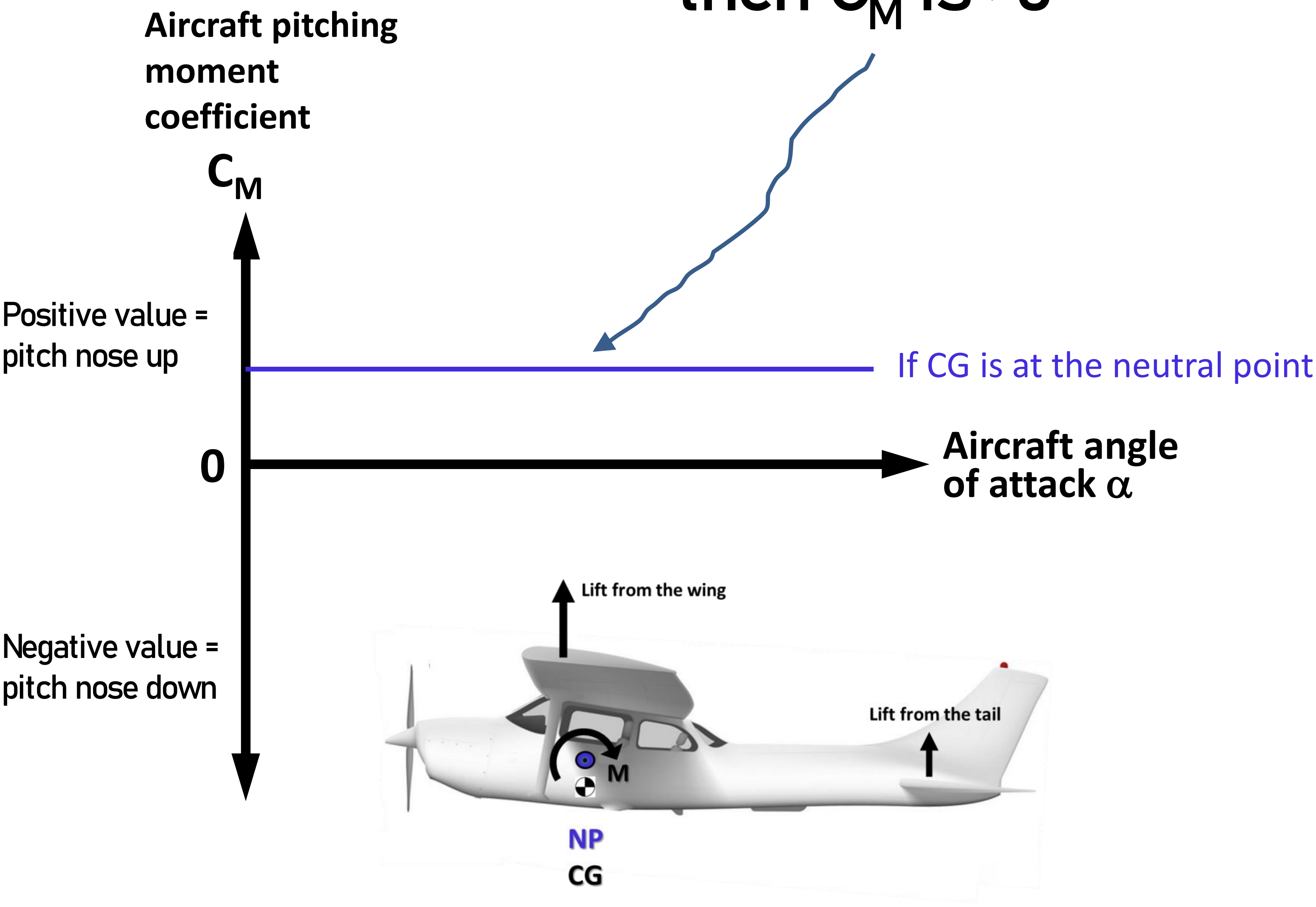
Notice the Cm curve is nearly flat at around -.04 for α = -10° to +10°

Neutral Point of an Aircraft

The neutral point (NP) is defined as that point on the longitudinal axis of an aircraft where the aerodynamic moment (M) does not depend on angle of attack α of the aircraft.



If the aircraft CG is located at the neutral point, then C_M is >0



Aircraft pitching moment coefficient

C_M

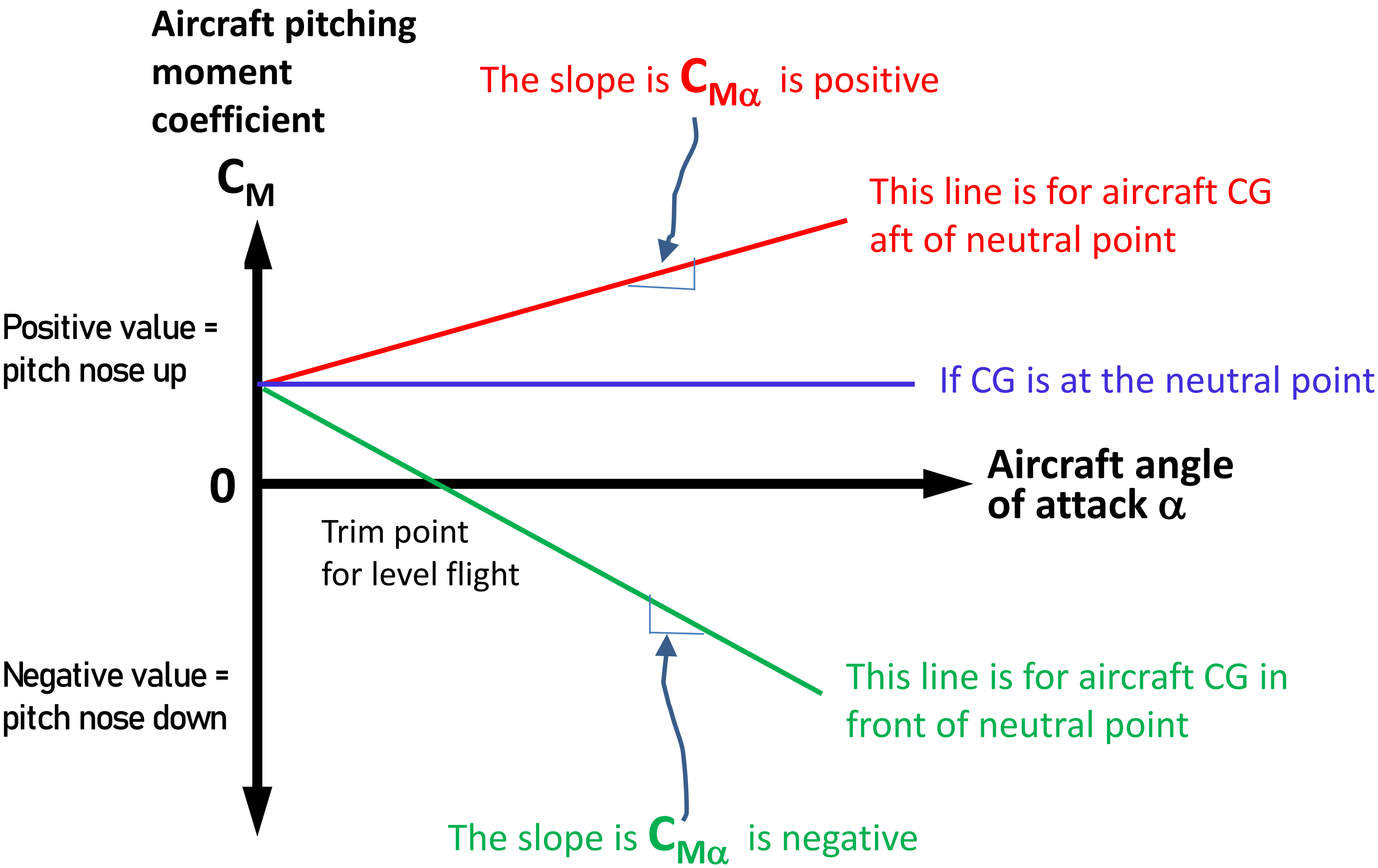
Positive value = pitch nose up

Negative value = pitch nose down

If CG is at the neutral point

Aircraft angle of attack α

To have a stable aircraft, $C_{M\alpha}$ must be negative



Aircraft pitching moment coefficient

C_M

The slope is $C_{M\alpha}$ is positive

This line is for aircraft CG aft of neutral point

Positive value = pitch nose up

If CG is at the neutral point

0

Aircraft angle of attack α

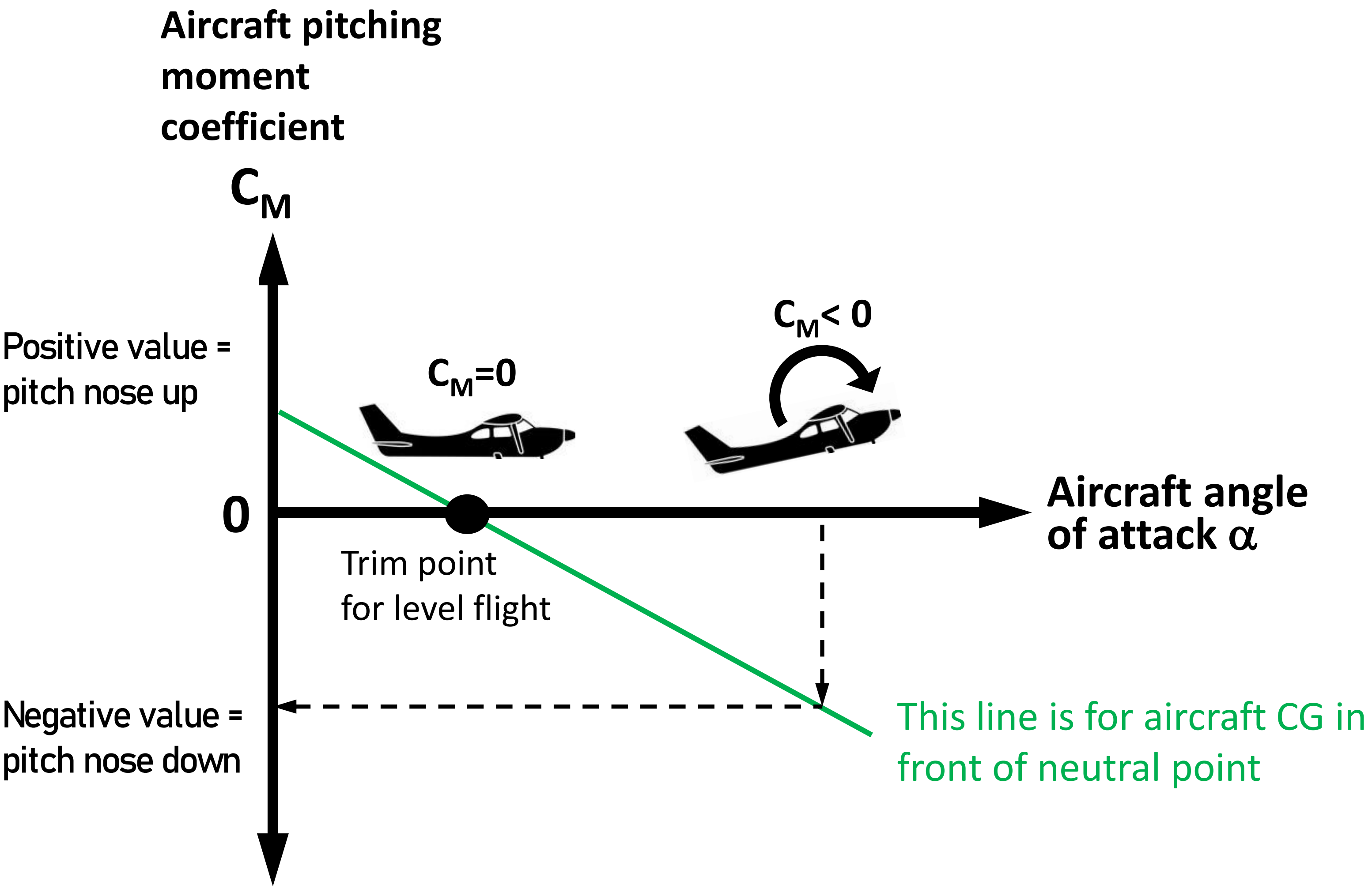
Trim point for level flight

Negative value = pitch nose down

This line is for aircraft CG in front of neutral point

The slope is $C_{M\alpha}$ is negative

To have a stable aircraft, $C_{M\alpha}$ must be negative



Aircraft pitching moment coefficient

C_M

Positive value = pitch nose up

0

Negative value = pitch nose down

$C_M=0$

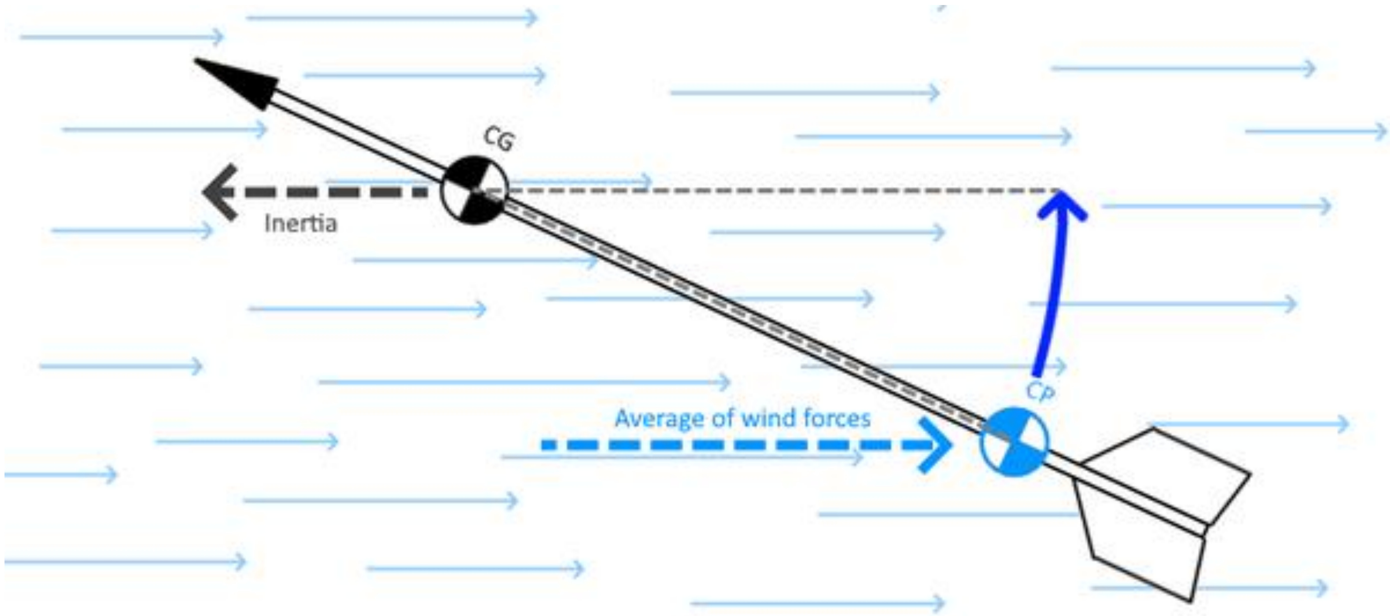
$C_M< 0$

Aircraft angle of attack α

Trim point for level flight

This line is for aircraft CG in front of neutral point

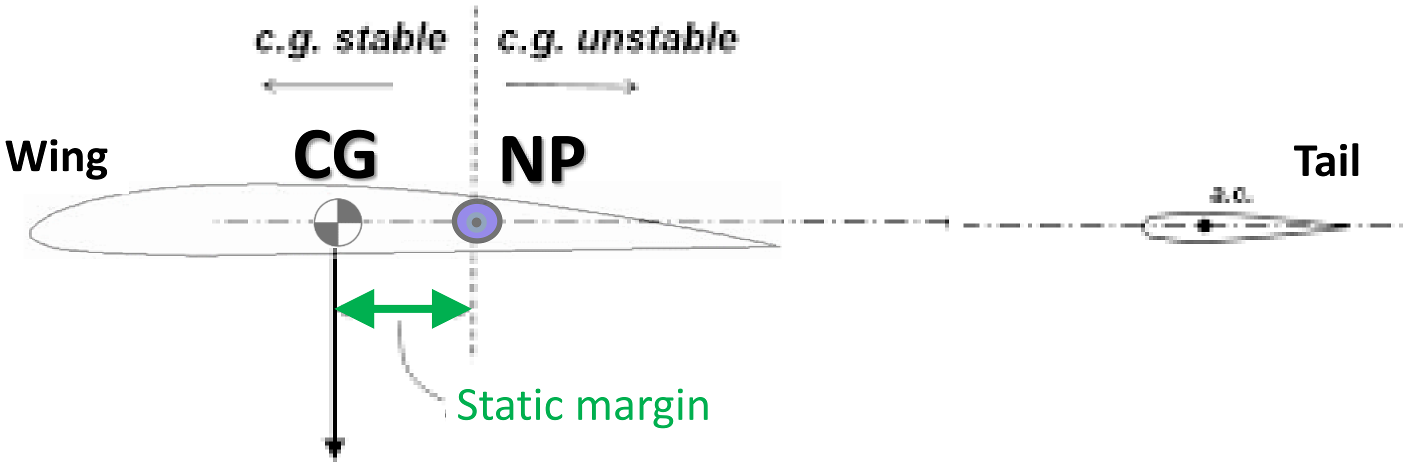
Even for Arrow, Wants CG to be Ahead of CP



The center of pressure (CP) is the point where the total sum of a pressure field acts on a body.

Static Margin

- The static margin is the distance between the center of gravity (CG) and the neutral point (NP). It is usually quoted as a percentage of the Mean Aerodynamic Chord. A desirable value is between 0.05 to 0.30



- The center of gravity must lie ahead of the neutral point for positive stability (positive static margin).

Static Margin for Several Aircraft

Cessna 172	0.19
Learjet 35	0.13
Boeing 747	0.27
North American P-51	0.05
Convair F-106	0.07
General Dynamics F-16A	-0.02
General Dynamics F-16C	0.01
Grumman X-29	-0.33
Your eVTOL	?


X-29

Case Study: Tandem Wing eVTOL Configuration

Challenging to design

Summary on Static Margin

- Some combat aircraft have the CG behind the NP and they are intentionally designed to be longitudinally unstable so they will be highly maneuverable. They rely on flight computer to stabilize the aircraft.

- There are many eVTOL aircraft configurations, some of these configuration may have poor static margin and must rely on flight computer to make the aircraft controllable.

- Ultimately, the position of the center of gravity relative to the neutral point determines the stability, control forces, and controllability of the vehicle.

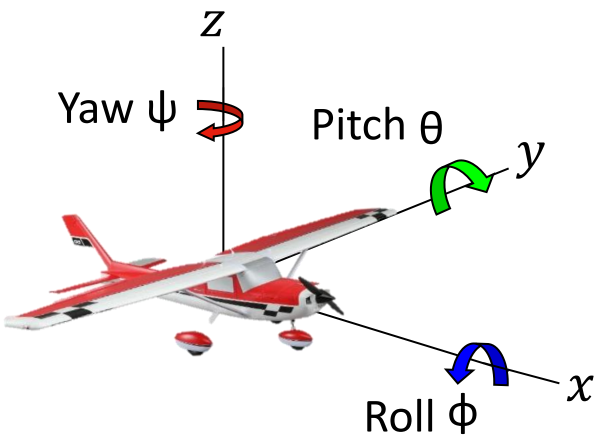
Stability & Control Analysis and Control Laws

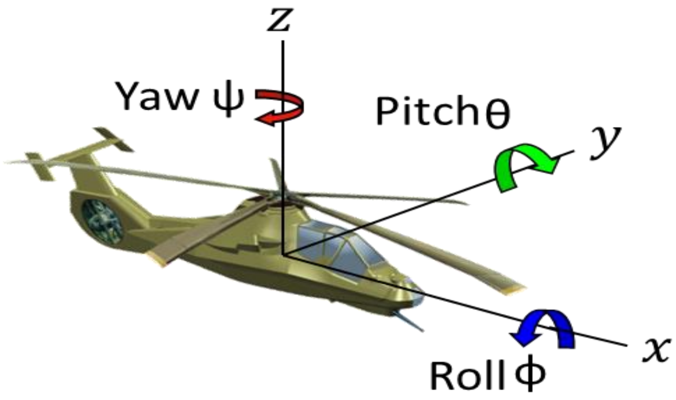
- Equations of Motions (EOM)
- Stability derivatives
- Feedback loop

Let’s Derive the Equations of Motion for a Helicopter

Aircraft's Equations of Motion (EOM)

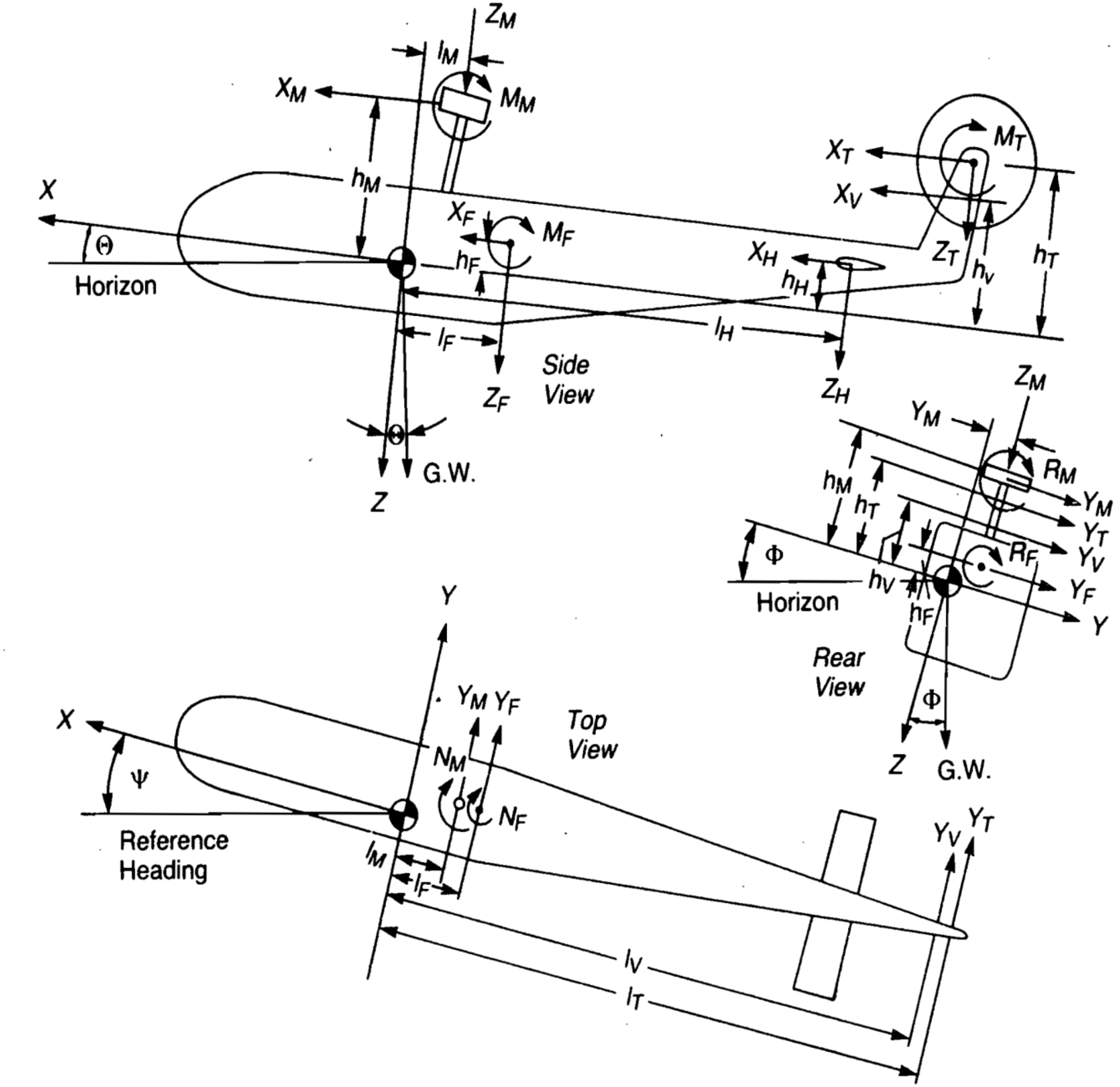
- The forces and moments acting on any aircraft can be described by 6 EOM and 6 degrees of freedom (three x, y, z translations and three rotations: pitch, roll, yaw).





- The forces and moments acting on the aircraft are produced by aerodynamics acting on the wing, horizontal stabilizer, vertical fin, fuselage, main rotor, tail rotor and any other surfaces.

1st Step, Draw Force and Moment Diagrams



2nd Write the 3 Forces and 3 Moments Equations

Due to Main rotor

Due to Tail rotor

Due to Horizontal stabilizer

Vertical fin

Due to Fuselage

Gravity

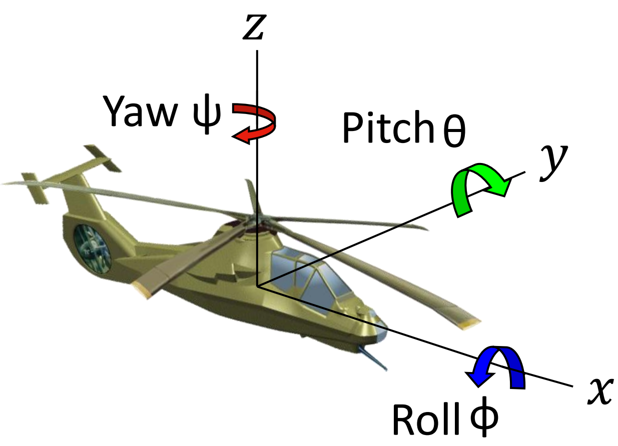
Aircraft Weight

Aircraft pitch angle

Longitudinal force (forward)	$X_M + X_T + X_H + X_V + X_F = \text{G.W. } \sin \Theta$
Lateral force (right)	$Y_M + Y_T + Y_V + Y_F = -\text{G.W. } \sin \Phi$
Vertical force (down)	$Z_M + Z_T + Z_H + Z_V + Z_F = -\text{G.W. } \cos \Theta$
Rolling moment (down to right)	$R_M + Y_M h_M + Z_M y_M + Y_T h_T + Y_V h_V + Y_F h_F + R_F = 0$
Pitching moment (nose-up)	$M_M - X_M h_M + Z_M l_M + M_T - X_T h_T + Z_T l_T - X_H h_H + Z_H l_H - X_V h_V + M_F + Z_F l_F - X_F h_F = 0$
Yawing moment (nose to right)	$N_M - Y_M l_M - Y_T l_T - Y_V l_V + N_F - Y_F l_F = 0$

Stability Derivatives

- Measure of how much a force or moment changes when there is a small change in flight condition parameters
- 10 flight parameters can be considered

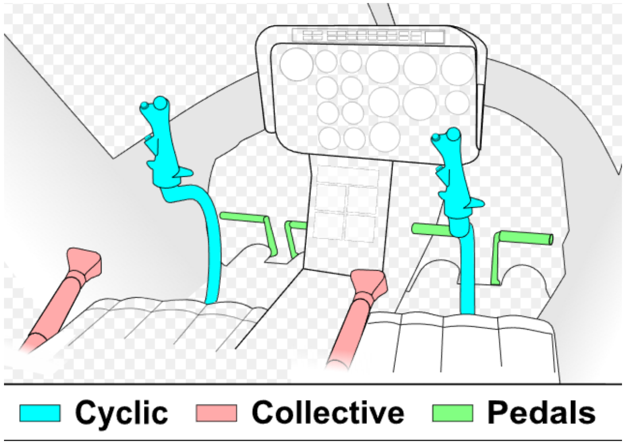


6 DOF

– pitch angle, θ	Longitudinal translation, x
– roll angle, ϕ	Lateral translation, y
– Yaw angle, ψ	Vertical translation, z

4 pilot control inputs

- Longitudinal Cyclic Pitch, B_1
- Lateral Cyclic Pitch, A_1
- Collective Pitch, Θ_{0_M}
- Tail Rotor Pitch, Θ_{0_T}



Stability Derivatives

Force in X direction

$$X = \overbrace{\frac{\partial X}{\partial \dot{x}}\dot{x} + \frac{\partial X}{\partial \dot{y}}\dot{y} + \frac{\partial X}{\partial \dot{z}}\dot{z} + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial p}p + \frac{\partial X}{\partial r}r}^{\text{Relate to the 6 DOF}} + \frac{\partial X}{\partial \theta_{0_M}}\theta_{0_M}$$

$$+ \underbrace{\frac{\partial X}{\partial \theta_{0_T}}\theta_{0_T} + \frac{\partial X}{\partial A_1}A_1 + \frac{\partial X}{\partial B_1}B_1}_{\text{Relate to the 4 pilot control inputs}}$$

How to interpret these stability derivatives physically?

Example: $\frac{\partial X}{\partial \dot{x}}$ is a measure of how much a small change in longitudinal velocity affects the longitudinal force experienced by the helicopter.

Matrix form for the 6 EoM

LONGITUDINAL

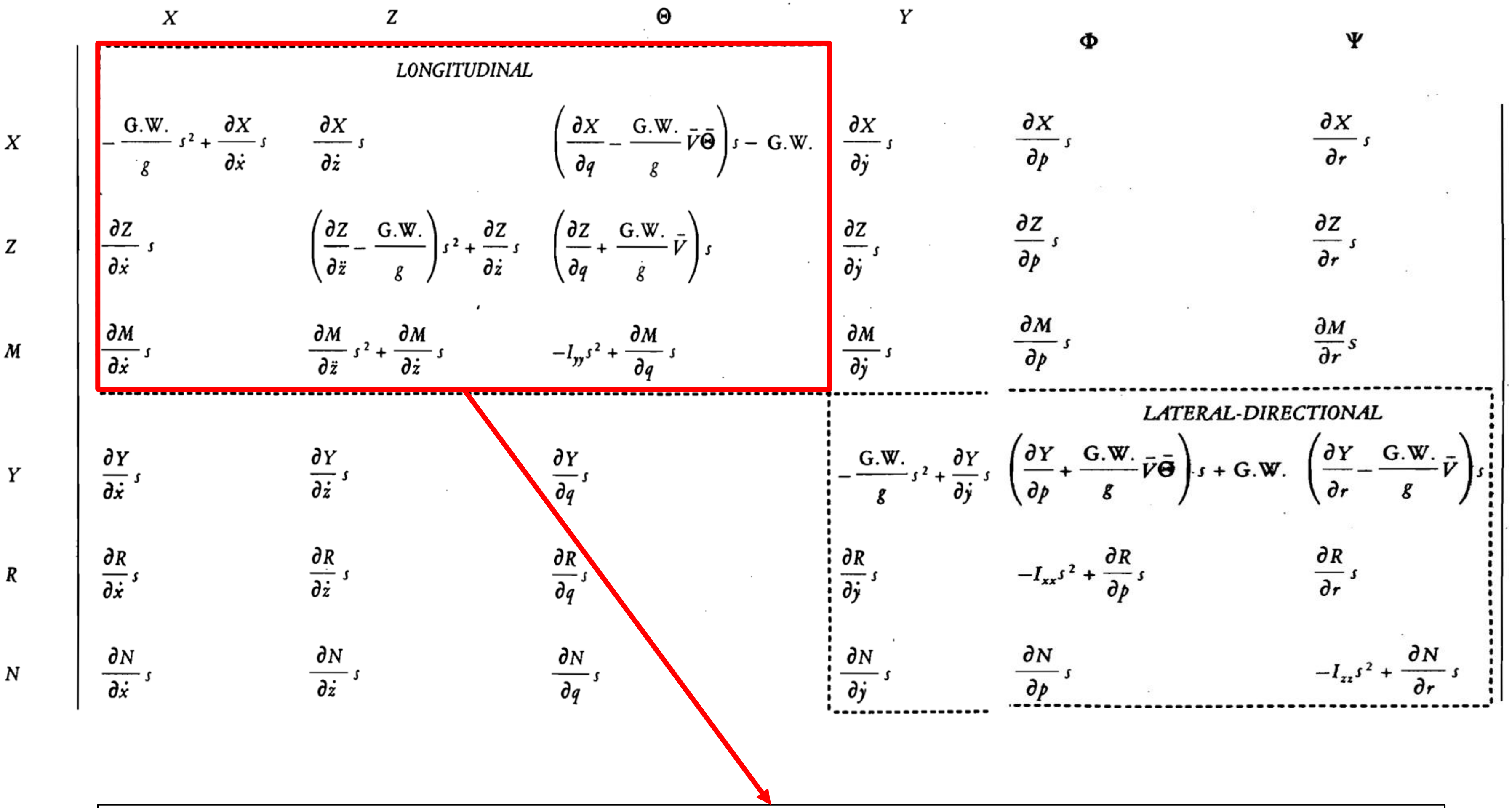
$$
\left.\begin{array}{c|cccccc}
 & X & Z & \Theta & Y & \Phi & \Psi \\
\hline
X & -\frac{\text{G.W.}}{g}s^2+\frac{\partial X}{\partial \dot{x}}s & \frac{\partial X}{\partial \dot{z}}s & \left(\frac{\partial X}{\partial q}-\frac{\text{G.W.}}{g}\bar{V}\bar{\Theta}\right)s-\text{G.W.} & \frac{\partial X}{\partial \dot{y}}s & \frac{\partial X}{\partial p}s & \frac{\partial X}{\partial r}s \\
Z & \frac{\partial Z}{\partial \dot{x}}s & \left(\frac{\partial Z}{\partial \ddot{z}}-\frac{\text{G.W.}}{g}\right)s^2+\frac{\partial Z}{\partial \dot{z}}s & \left(\frac{\partial Z}{\partial q}+\frac{\text{G.W.}}{g}\bar{V}\right)s & \frac{\partial Z}{\partial \dot{y}}s & \frac{\partial Z}{\partial p}s & \frac{\partial Z}{\partial r}s \\
M & \frac{\partial M}{\partial \dot{x}}s & \frac{\partial M}{\partial \ddot{z}}s^2+\frac{\partial M}{\partial \dot{z}}s & -I_{yy}s^2+\frac{\partial M}{\partial q}s & \frac{\partial M}{\partial \dot{y}}s & \frac{\partial M}{\partial p}s & \frac{\partial M}{\partial r}s \\
Y & \frac{\partial Y}{\partial \dot{x}}s & \frac{\partial Y}{\partial \dot{z}}s & \frac{\partial Y}{\partial q}s & -\frac{\text{G.W.}}{g}s^2+\frac{\partial Y}{\partial \dot{y}}s & \left(\frac{\partial Y}{\partial p}+\frac{\text{G.W.}}{g}\bar{V}\bar{\Theta}\right)s+\text{G.W.} & \left(\frac{\partial Y}{\partial r}-\frac{\text{G.W.}}{g}\bar{V}\right)s \\
R & \frac{\partial R}{\partial \dot{x}}s & \frac{\partial R}{\partial \dot{z}}s & \frac{\partial R}{\partial q}s & \frac{\partial R}{\partial \dot{y}}s & -I_{xx}s^2+\frac{\partial R}{\partial p}s & \frac{\partial R}{\partial r}s \\
N & \frac{\partial N}{\partial \dot{x}}s & \frac{\partial N}{\partial \dot{z}}s & \frac{\partial N}{\partial q}s & \frac{\partial N}{\partial \dot{y}}s & \frac{\partial N}{\partial p}s & -I_{zz}s^2+\frac{\partial N}{\partial r}s
\end{array}\right|
\begin{array}{c} x \\ z \\ \theta \\ y \\ \phi \\ \psi \end{array}
$$

LATERAL-DIRECTIONAL

$$F = ma = Mx + Cx + Kx$$

$$M = I\ddot{\theta} = (Ms^2 + Cs + K)\theta$$

Matrix form for the 6 EoM



$$
\begin{array}{c|cccccc|}
 & X & Z & \Theta & Y & \Phi & \Psi \\
\hline
X & -\frac{G.W.}{g}s^2+\frac{\partial X}{\partial \dot{x}}s & \frac{\partial X}{\partial \dot{z}}s & \left(\frac{\partial X}{\partial q}-\frac{G.W.}{g}\bar{V}\bar{\Theta}\right)s-G.W. & \frac{\partial X}{\partial \dot{y}}s & \frac{\partial X}{\partial p}s & \frac{\partial X}{\partial r}s \\
Z & \frac{\partial Z}{\partial \dot{x}}s & \left(\frac{\partial Z}{\partial \ddot{z}}-\frac{G.W.}{g}\right)s^2+\frac{\partial Z}{\partial \dot{z}}s & \left(\frac{\partial Z}{\partial q}+\frac{G.W.}{g}\bar{V}\right)s & \frac{\partial Z}{\partial \dot{y}}s & \frac{\partial Z}{\partial p}s & \frac{\partial Z}{\partial r}s \\
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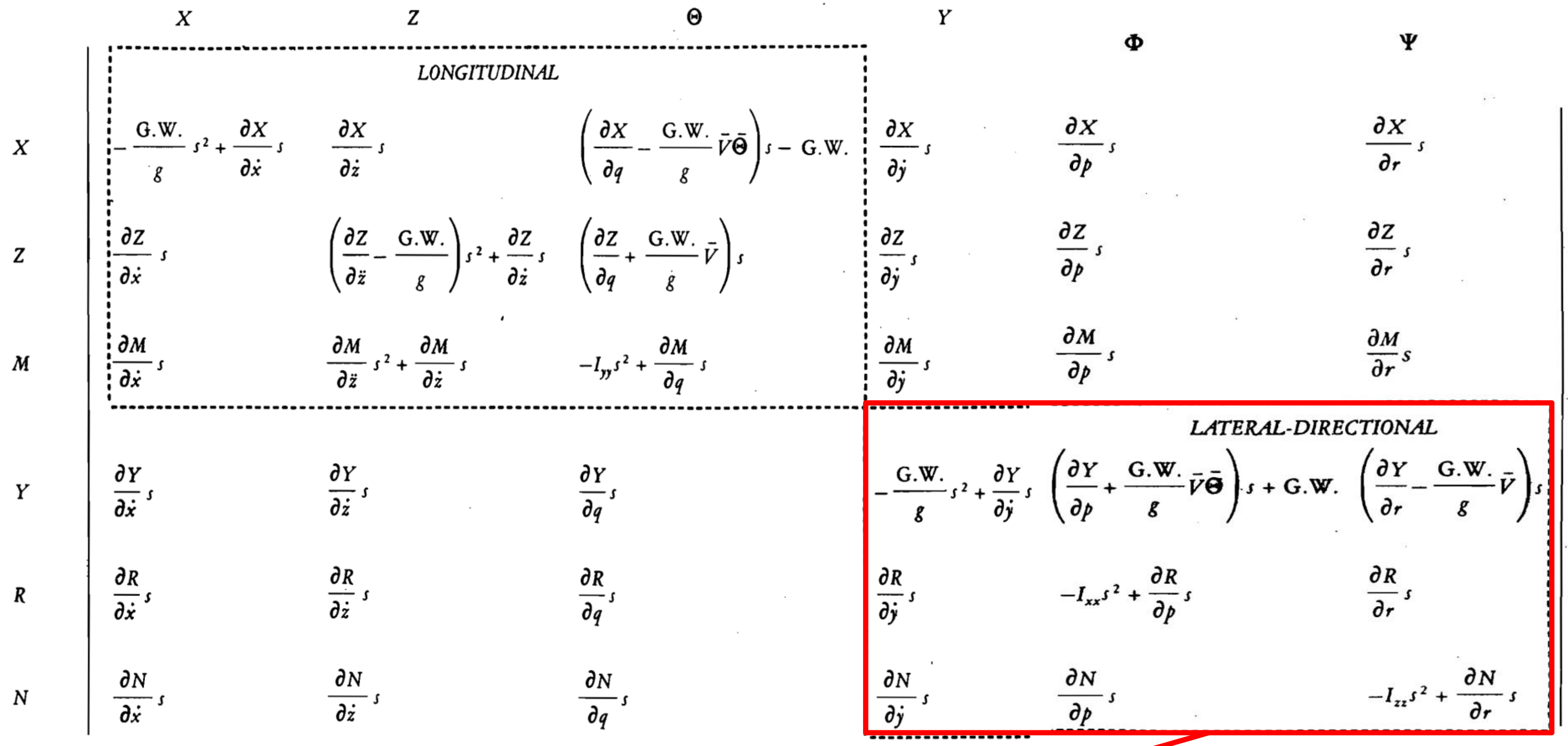
Y & \frac{\partial Y}{\partial \dot{x}}s & \frac{\partial Y}{\partial \dot{z}}s & \frac{\partial Y}{\partial q}s & -\frac{G.W.}{g}s^2+\frac{\partial Y}{\partial \dot{y}}s & \left(\frac{\partial Y}{\partial p}+\frac{G.W.}{g}\bar{V}\bar{\Theta}\right)s+G.W. & \left(\frac{\partial Y}{\partial r}-\frac{G.W.}{g}\bar{V}\right)s \\
R & \frac{\partial R}{\partial \dot{x}}s & \frac{\partial R}{\partial \dot{z}}s & \frac{\partial R}{\partial q}s & \frac{\partial R}{\partial \dot{y}}s & -I_{xx}s^2+\frac{\partial R}{\partial p}s & \frac{\partial R}{\partial r}s \\
N & \frac{\partial N}{\partial \dot{x}}s & \frac{\partial N}{\partial \dot{z}}s & \frac{\partial N}{\partial q}s & \frac{\partial N}{\partial \dot{y}}s & \frac{\partial N}{\partial p}s & -I_{zz}s^2+\frac{\partial N}{\partial r}s
\end{array}
$$

X (Longitudinal Translation)
Z (Vertical Translation)
M (Pitching Moment)

→ Contribute majorly in <u>longitudinal</u> motion

Matrix form for the 6 EoM



LONGITUDINAL

LATERAL-DIRECTIONAL

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\begin{array}{c|cccccc|}
 & X & Z & \Theta & Y & \Phi & \Psi \\
\hline
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Z & \frac{\partial Z}{\partial \dot{x}}s & \left(\frac{\partial Z}{\partial \ddot{z}}-\frac{G.W.}{g}\right)s^2+\frac{\partial Z}{\partial \dot{z}}s & \left(\frac{\partial Z}{\partial q}+\frac{G.W.}{g}\bar{V}\right)s & \frac{\partial Z}{\partial \dot{y}}s & \frac{\partial Z}{\partial p}s & \frac{\partial Z}{\partial r}s \\
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R & \frac{\partial R}{\partial \dot{x}}s & \frac{\partial R}{\partial \dot{z}}s & \frac{\partial R}{\partial q}s & \frac{\partial R}{\partial \dot{y}}s & -I_{xx}s^2+\frac{\partial R}{\partial p}s & \frac{\partial R}{\partial r}s \\
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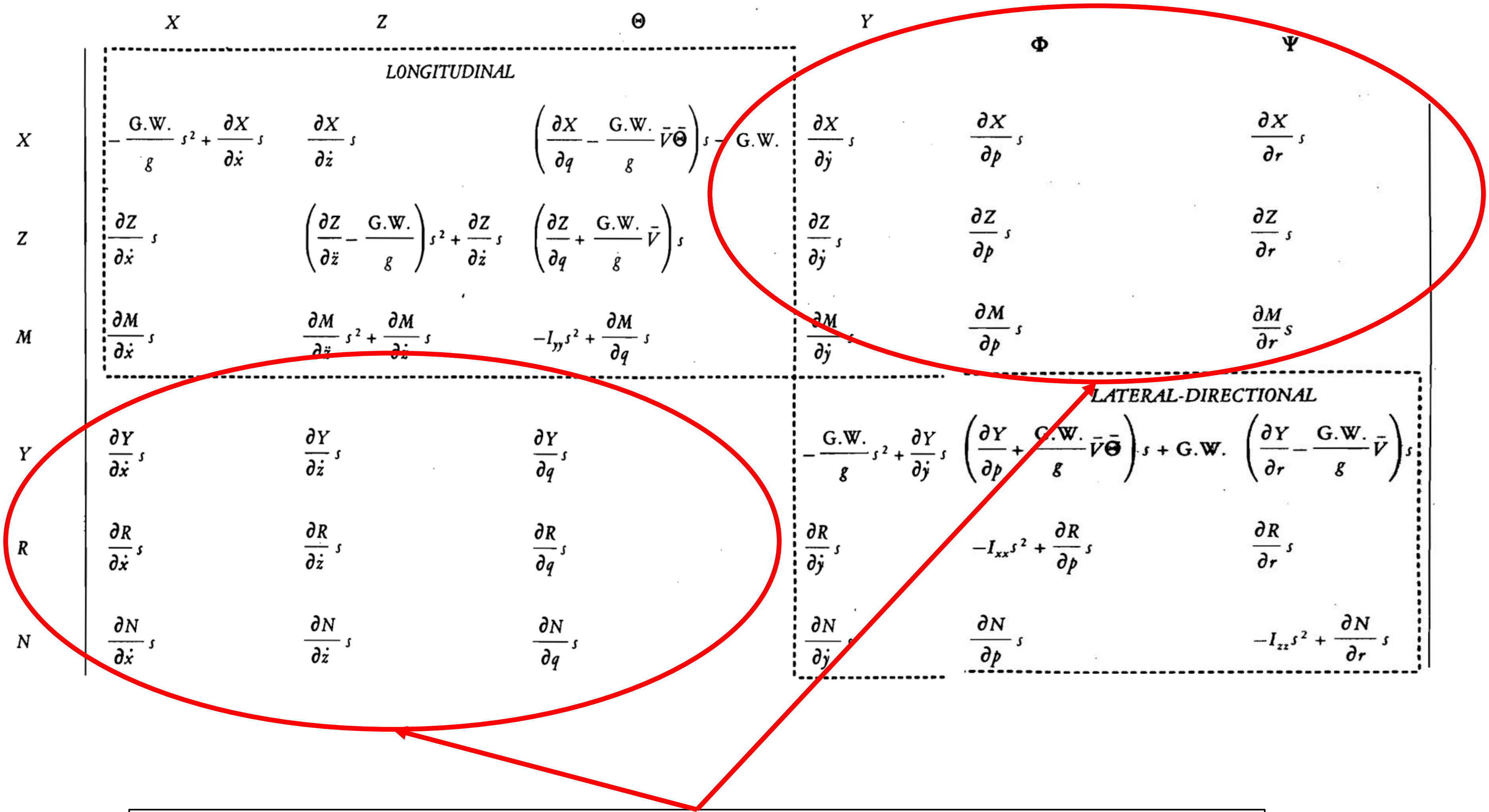
\end{array}
$$

Y (Lateral Translation)
R (Rolling Moment)
N (Yawing Moment)

→ Contribute majorly in lateral motion

Matrix form for the 6 EoM



$$
\begin{array}{c|cccccc|}
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\hline
X & -\frac{G.W.}{g}s^2+\frac{\partial X}{\partial \dot{x}}s & \frac{\partial X}{\partial \dot{z}}s & \left(\frac{\partial X}{\partial q}-\frac{G.W.}{g}\bar{V}\bar{\Theta}\right)s-G.W. & \frac{\partial X}{\partial \dot{y}}s & \frac{\partial X}{\partial p}s & \frac{\partial X}{\partial r}s \\
Z & \frac{\partial Z}{\partial \dot{x}}s & \left(\frac{\partial Z}{\partial \ddot{z}}-\frac{G.W.}{g}\right)s^2+\frac{\partial Z}{\partial \dot{z}}s & \left(\frac{\partial Z}{\partial q}+\frac{G.W.}{g}\bar{V}\right)s & \frac{\partial Z}{\partial \dot{y}}s & \frac{\partial Z}{\partial p}s & \frac{\partial Z}{\partial r}s \\
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\end{array}
$$

Minimal contribution to longitudinal & lateral motion (coupled effects)

- Can be usually ignored
- With computer analysis, can be left in

The 6x6 Matrix

- models the natural characteristics of the aircraft/helicopter (plant)
- The eigenvalues of the 6x6 matrix reveals the natural stability of the plant.
- Can plot the eigenvalues (poles and zeros) on a root locus plot → The plant is stable if all poles are on the left half side of the root locus plot.

Matrix form for 4 Control inputs that generates the Rotor Forces and Moments

$$\left|\begin{matrix} x(s)\\[2ex] z(s)\\[2ex] \Theta(s)\\[2ex] y(s)\\[2ex] \Phi(s)\\[2ex] \Psi(s)\end{matrix}\right| = \begin{array}{cccc} \theta_{0_M} & \theta_{0_T} & A_1 & B_1 \\ \left|-\frac{\partial X}{\partial \theta_{0_M}}\right. & -\frac{\partial X}{\partial \theta_{0_T}} & -\frac{\partial X}{\partial A_1} & \left.-\frac{\partial X}{\partial B_1}\right| \\ \left|-\frac{\partial Z}{\partial \theta_{0_M}}\right. & -\frac{\partial Z}{\partial \theta_{0_T}} & -\frac{\partial Z}{\partial A_1} & \left.-\frac{\partial Z}{\partial B_1}\right| \\ \left|-\frac{\partial M}{\partial \theta_{0_M}}\right. & -\frac{\partial M}{\partial \theta_{0_T}} & -\frac{\partial M}{\partial A_1} & \left.-\frac{\partial M}{\partial B_1}\right| \\ \left|-\frac{\partial Y}{\partial \theta_{0_M}}\right. & -\frac{\partial Y}{\partial \theta_{0_T}} & -\frac{\partial Y}{\partial A_1} & \left.-\frac{\partial Y}{\partial B_1}\right| \\ \left|-\frac{\partial R}{\partial \theta_{0_M}}\right. & -\frac{\partial R}{\partial \theta_{0_T}} & -\frac{\partial R}{\partial A_1} & \left.-\frac{\partial R}{\partial B_1}\right| \\ \left|-\frac{\partial N}{\partial \theta_{0_M}}\right. & -\frac{\partial N}{\partial \theta_{0_T}} & -\frac{\partial N}{\partial A_1} & \left.-\frac{\partial N}{\partial B_1}\right| \end{array} \left|\begin{matrix} \theta_{0_M}\\[2ex] \theta_{0_T}\\[2ex] A_1\\[2ex] B_1 \end{matrix}\right|$$

Vertical = heli main rotor pitch or airplane throttle

Yaw = heli tail rotor pitch or airplane rudder control

Roll = heli left/right cyclic or airplane aileron control

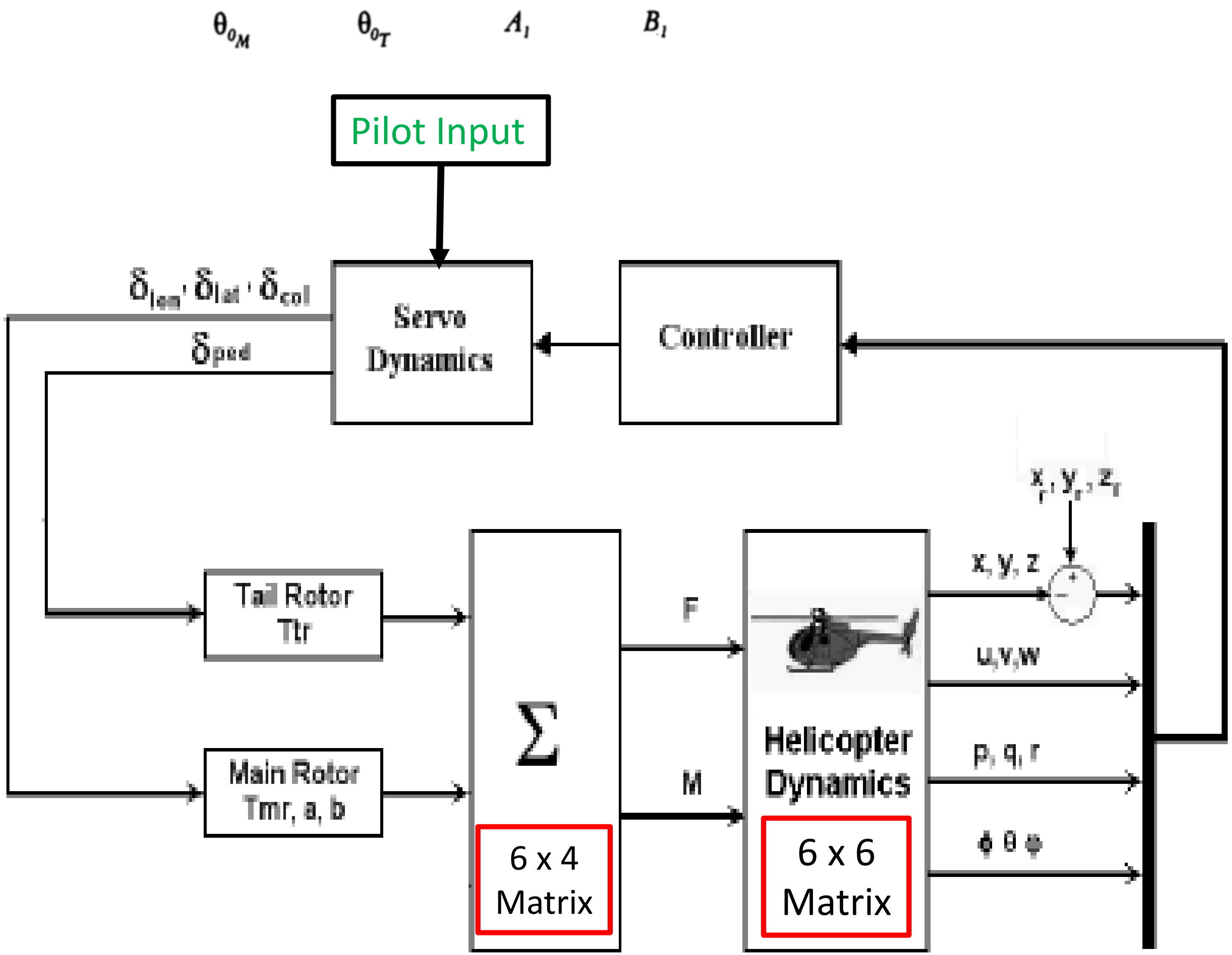
Pitch = heli fore/aft cyclic or airplane elevator control

6 DOF

6x4 matrix of Control derivatives

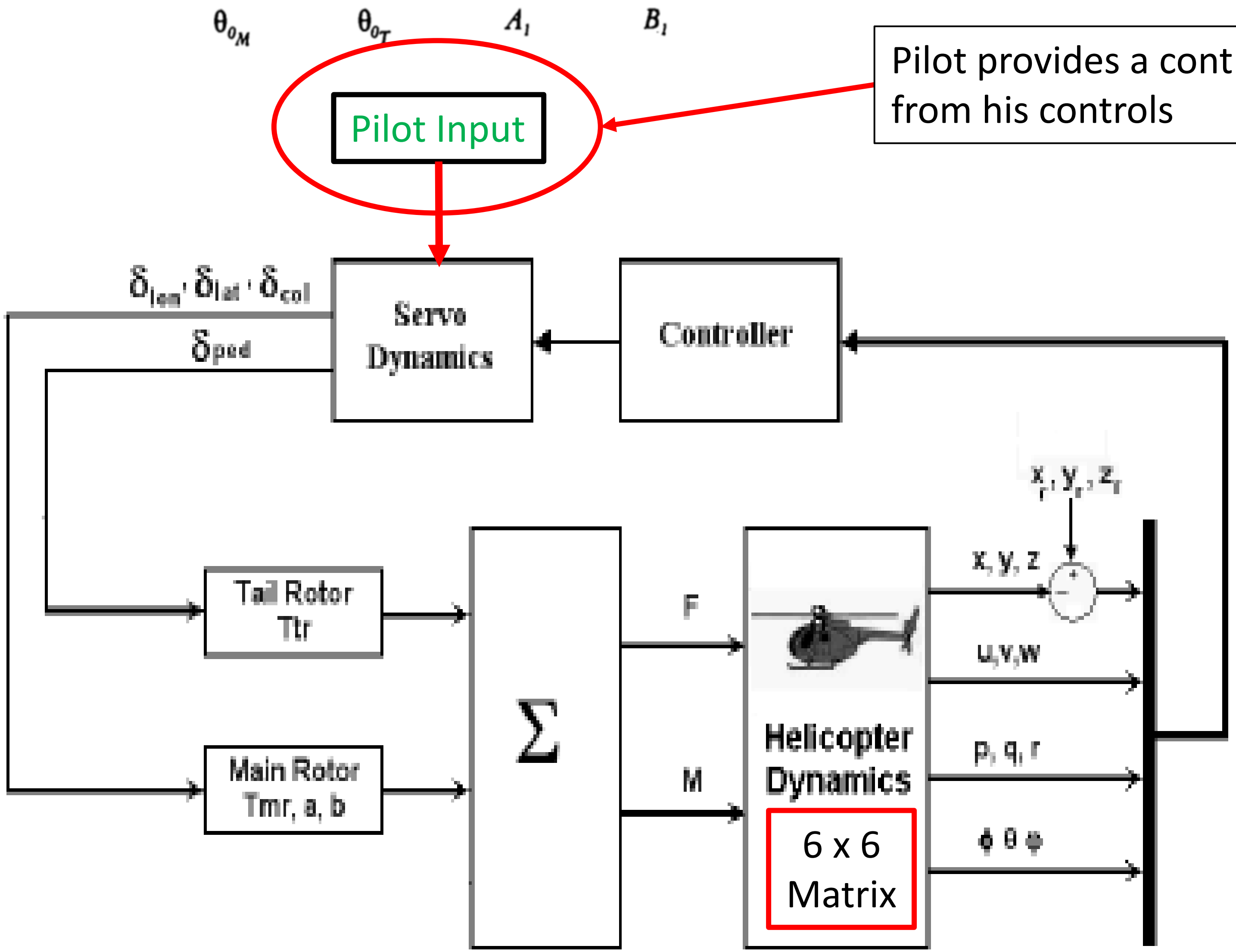
4 pilot controls

Feedback loop (Block Diagram)



Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

Feedback loop (Block Diagram)

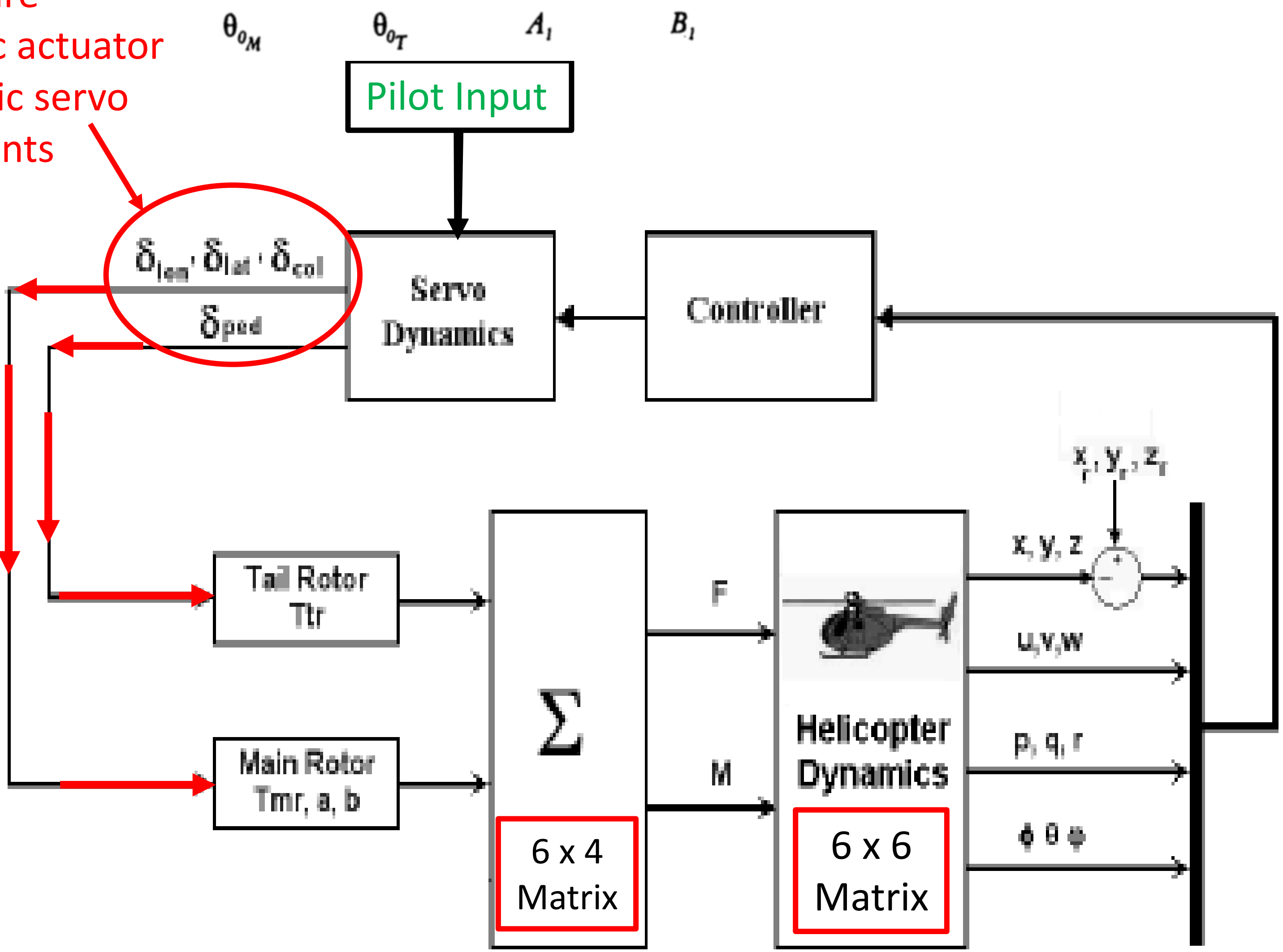


Pilot provides a control input from his controls

Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

Feedback loop (Block Diagram)

Output are hydraulic actuator or electric servo movements



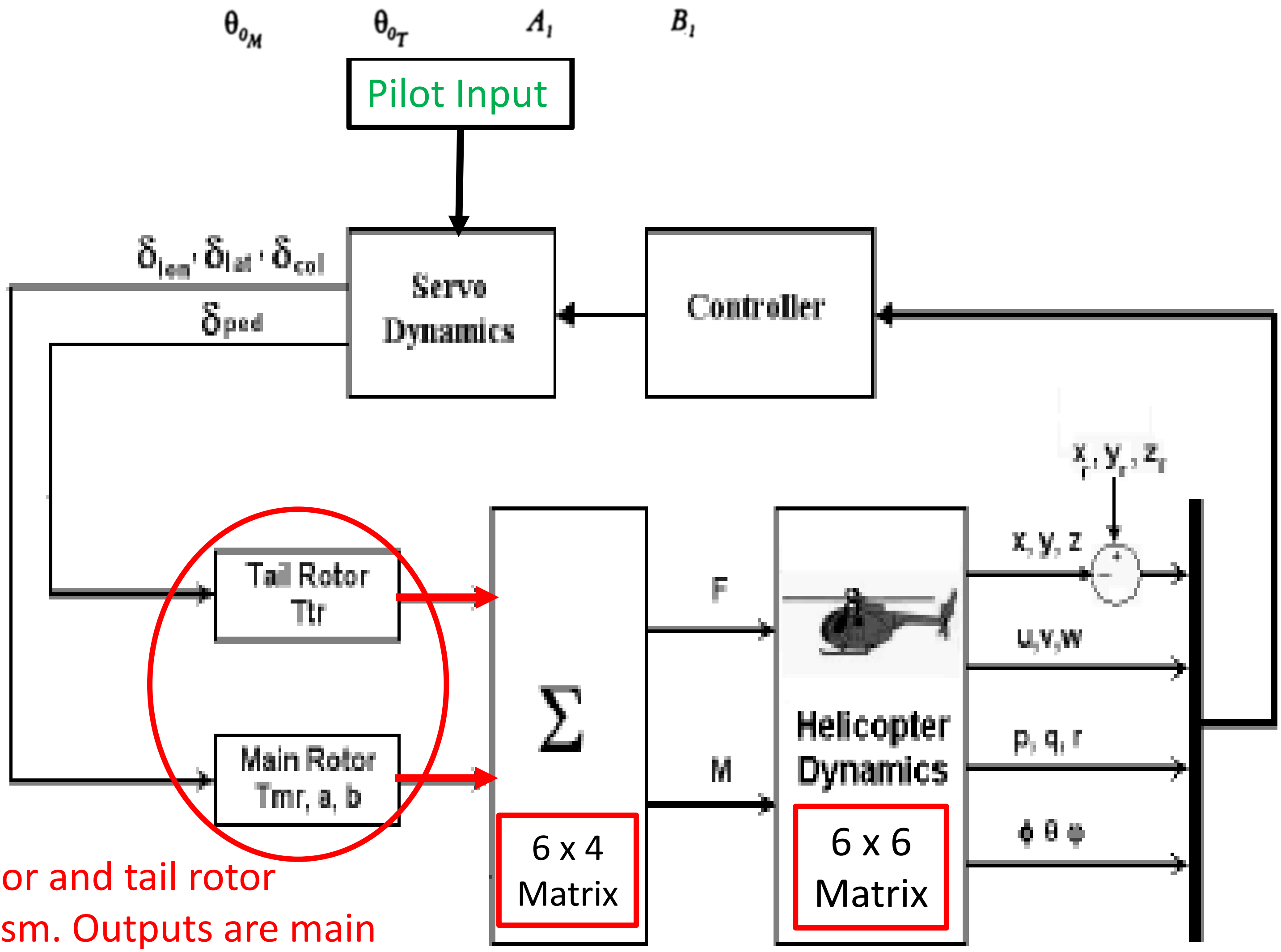
Pilot Input

6 x 4 Matrix

6 x 6 Matrix

Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

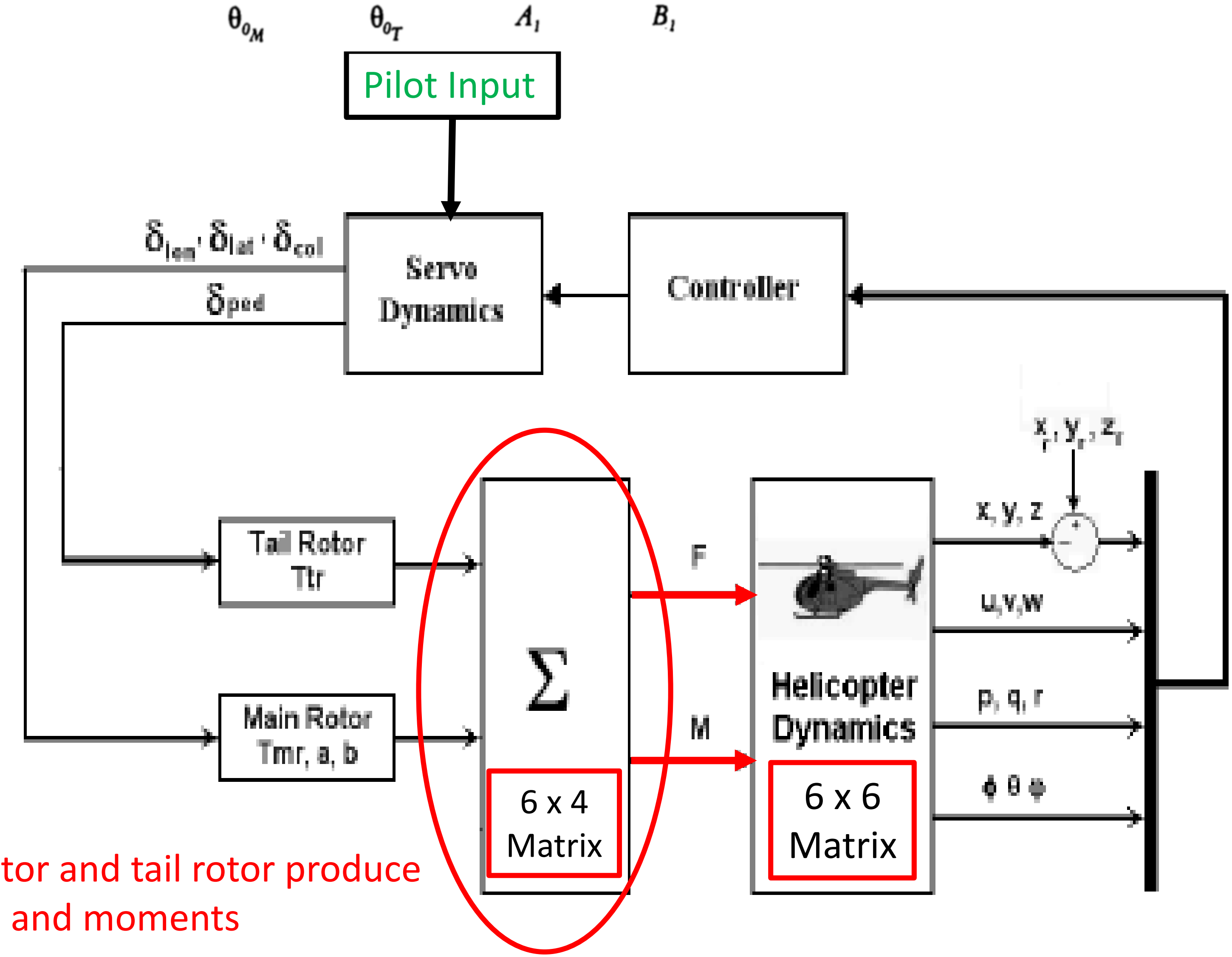
Feedback loop (Block Diagram)



Main rotor and tail rotor mechanism. Outputs are main and tail blade pitch angle changes

Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

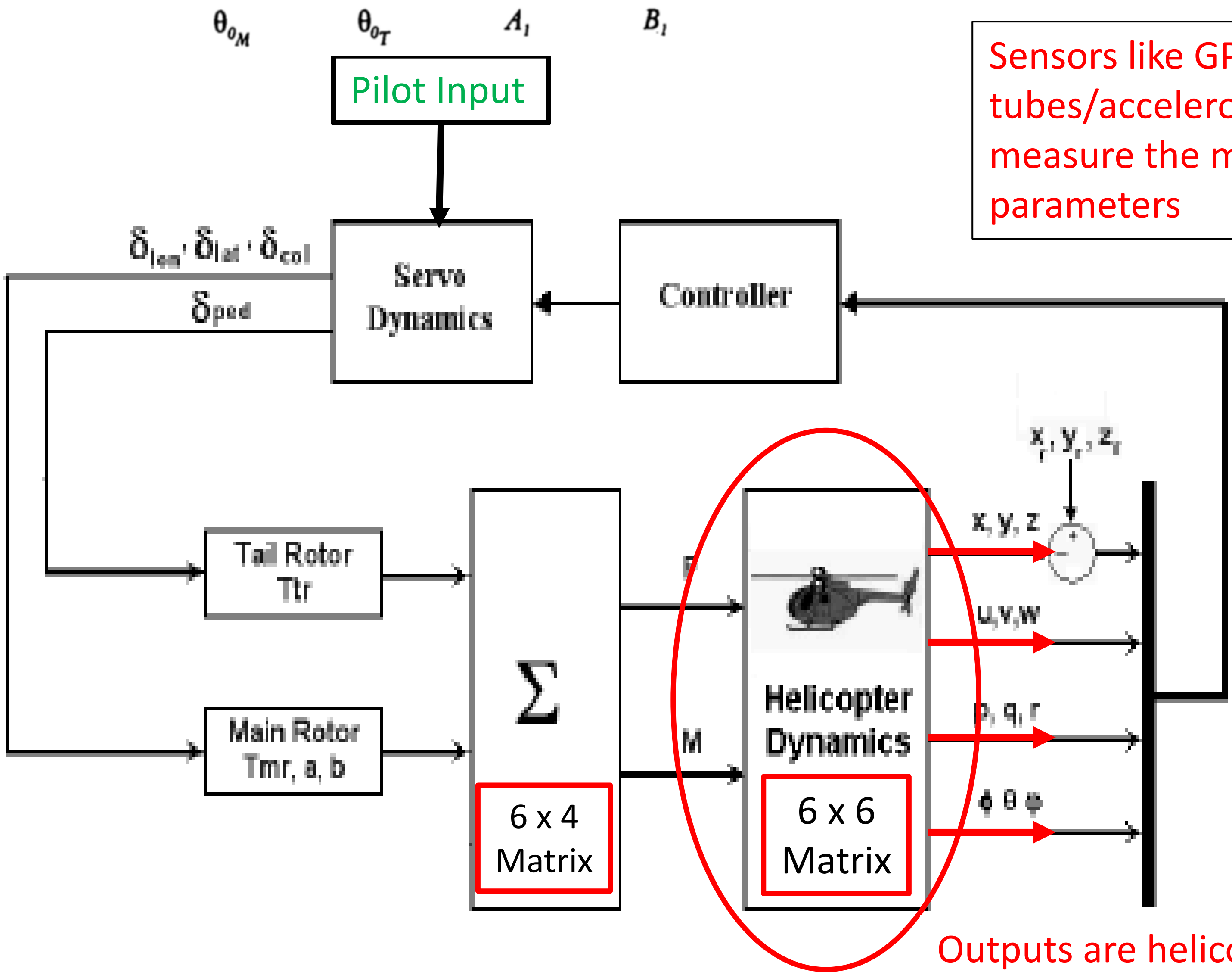
Feedback loop (Block Diagram) [4]



Main rotor and tail rotor produce 3 forces and moments

Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

Feedback loop (Block Diagram)

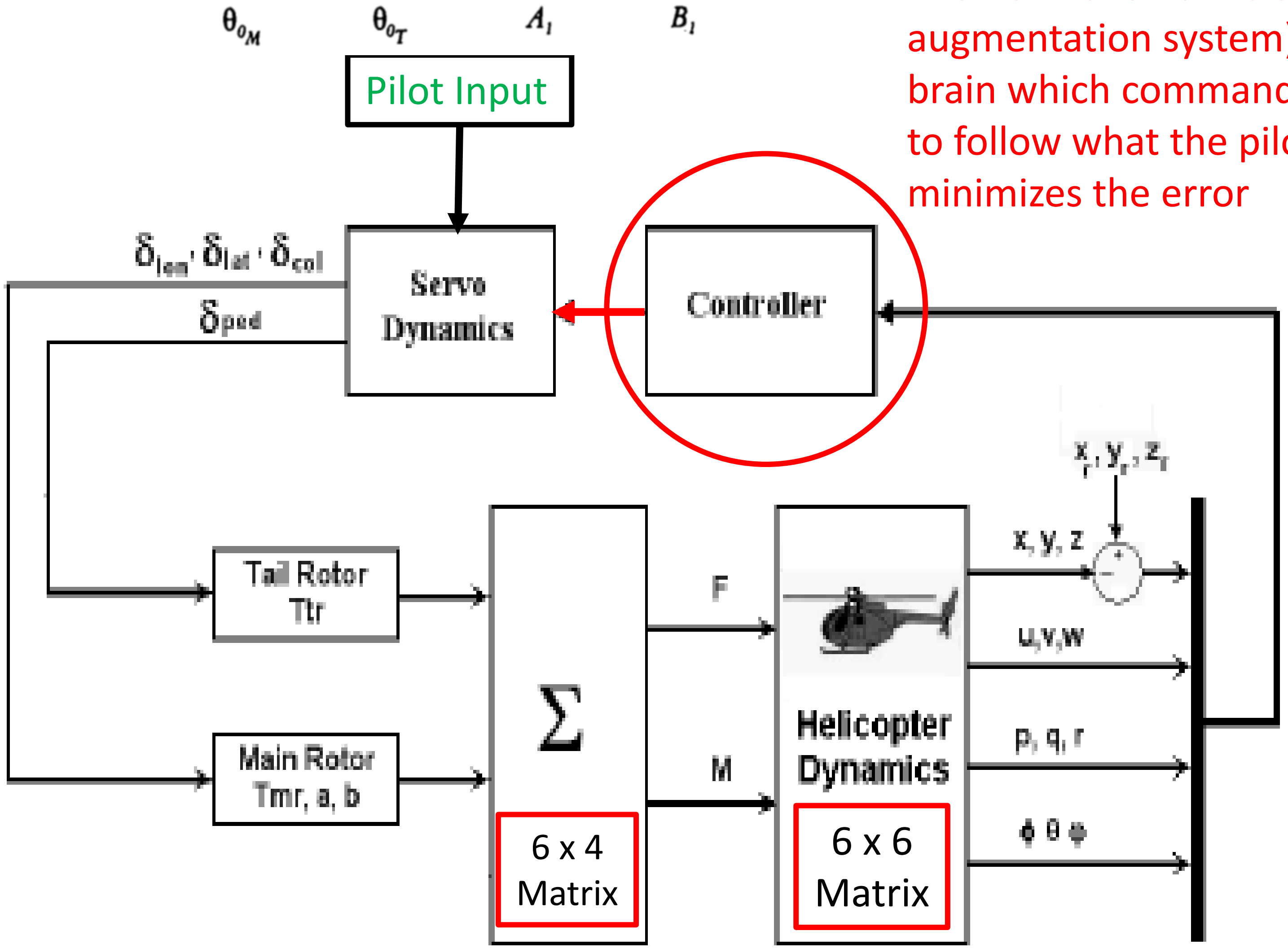


Sensors like GPS/pitot tubes/accelerometers measure the motion parameters

Outputs are helicopter motions

Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

Feedback loop (Block Diagram)



The Controller is the SAS (stability augmentation system) and the brain which commands the servos to follow what the pilot wants and minimizes the error

Source: https://www.researchgate.net/figure/Block-diagram-of-the-controlled-helicopter-dynamics_fig1_229034236

How To Implement the Theory and do Flight Simulation

- There are off-the-shelf software to do stability and feedback analysis.

- Can model the equations in **Matlab**, and use **Simulink** to do a simulation of the system.

- Then link **Simulink** to **FlightGear** to do the graphic display

- **FlightLab** is a professional software developed by ART to model the complete flight mechanics of different helicopters and airplanes.

FlightGear Simulator

FlightGear 2020.3.8

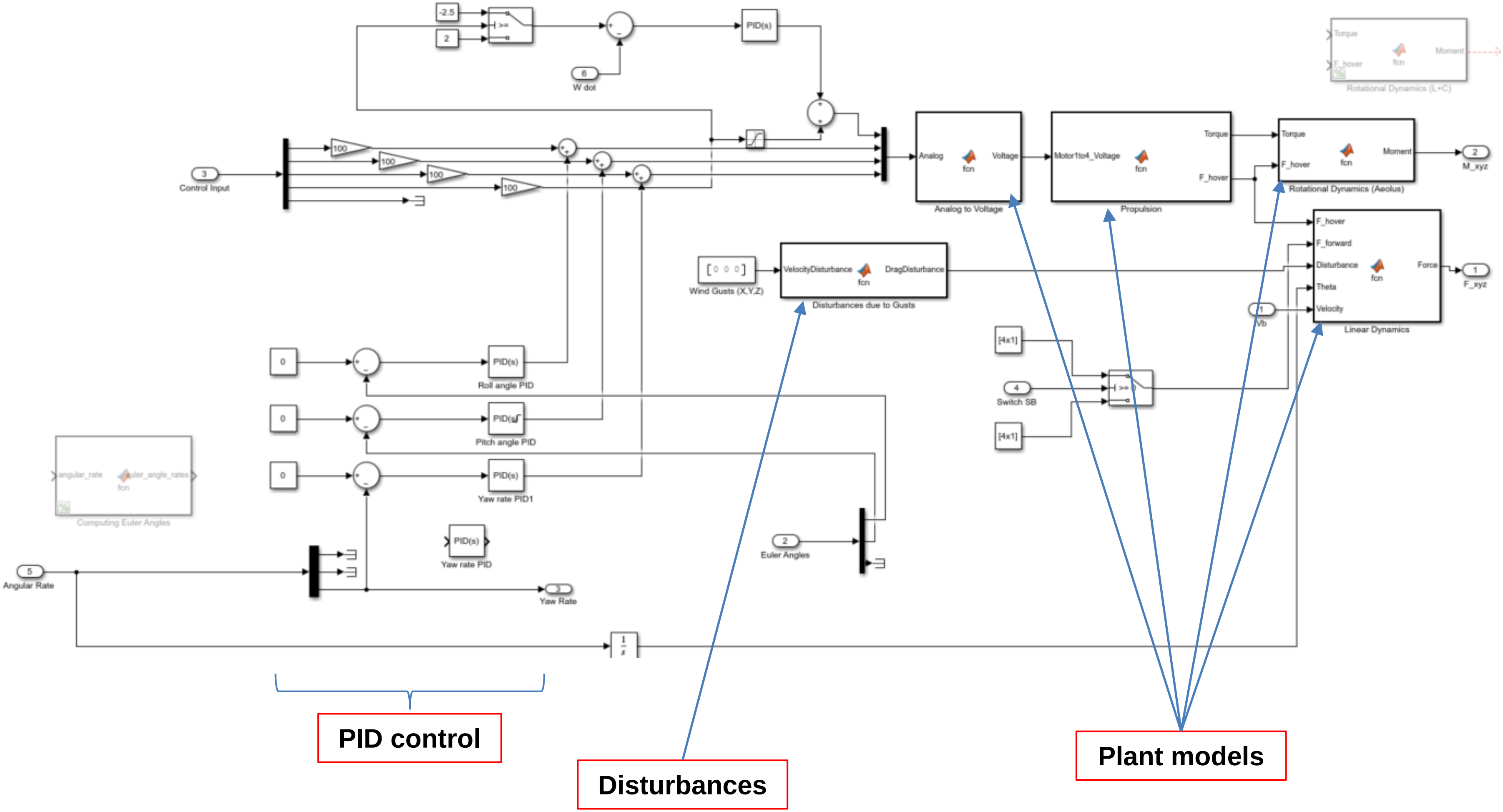
Drag while holding the right mouse button to look around

Cessna 172P Skyhawk (1982)

initializing subsystems

- Free open source flight simulator
- Visualise position and attitude of aircraft model, whilst rendering surrounding environment
- Information in Simulink model sent to FlightGear for 3D rendering/visualisation
- Customised aircraft model exported from Blender (graphic software) into FlightGear and simulated using transition model in Simulink

Example Simulink Model for an Hover eVTOL

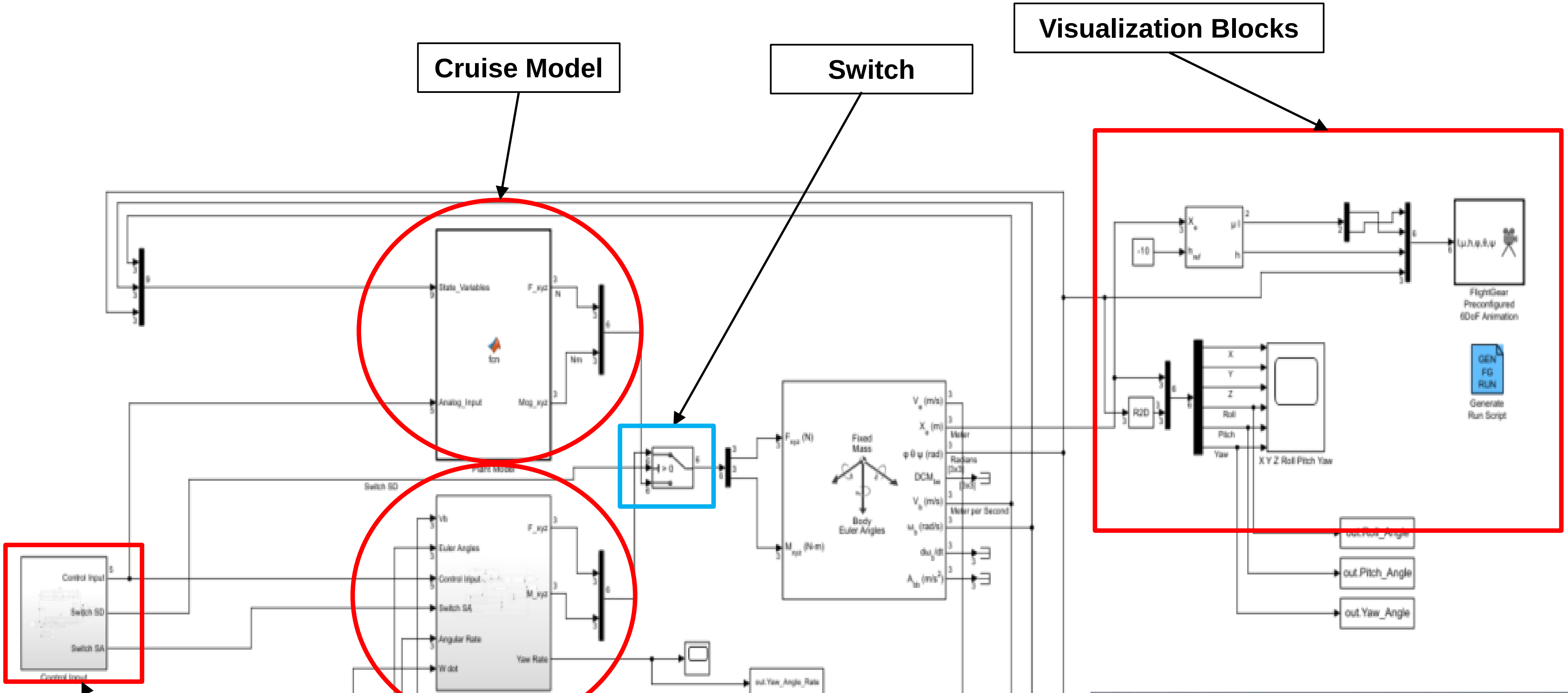


PID control

Disturbances

Plant models

Components of Transition Model Using Switch



Cruise Model

Switch

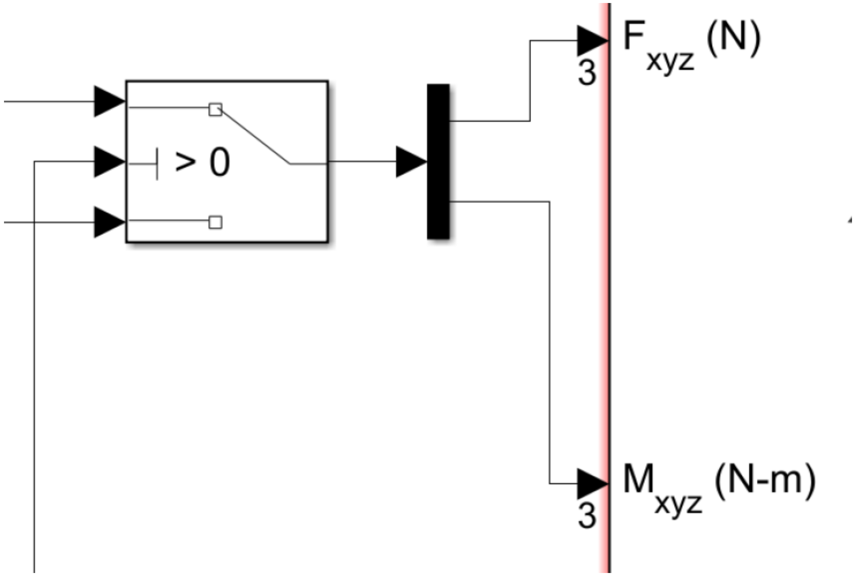
Visualization Blocks

Joystick interface

Hover Model

Switch tool

- Switches between cruise and hover modes, depending on the control input provided from controller
- A 6x1 matrix (F & M) from both cruise and hover models are input to this switch (top & bottom)
- Middle input is from controller



X-Plane Flight Simulator Software

- Latest release: X-Plane 11.55 (2021)
- X-Plane differentiates itself from other simulators by incorporating **Blade Element Theory** to compute lift forces
- “Plane Maker” software allows users to customise their aircraft and fly without input equations

➢Shape body of aircraft

➢Work with aircraft system

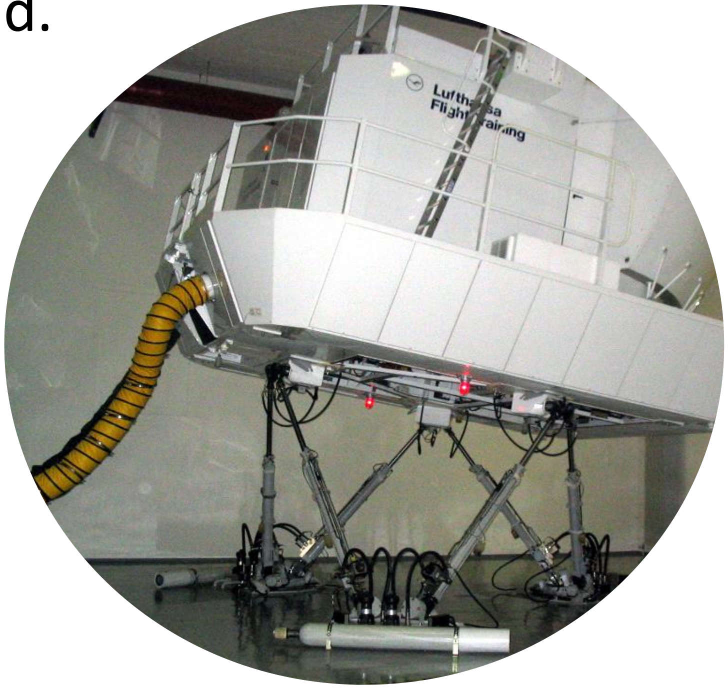
➢Modify aircraft properties

➢Perform flight tests

➢Starting at under $100 for a personal version

Physical Flight Simulators

- Flight simulators have different level of qualifications depending its complexity
- It ranges from Level A to D where A has a motion system with at least 3 degrees of freedom and D has all degrees of freedom with a digital display of at least 150 degrees which provides realistic visuals and sound.



Use models for flight testing and for validating configuration

Small RC Models Can provide Insights

Transition Mode with Rotor Thrust Line at 45º

Modified the flight controller to allow inflight changing rotor tilt to any angle

Hovering with Rotor Thrust Line at 45º

With a wing, an eVTOL becomes much more sensitive to gust, especially from the lateral direction

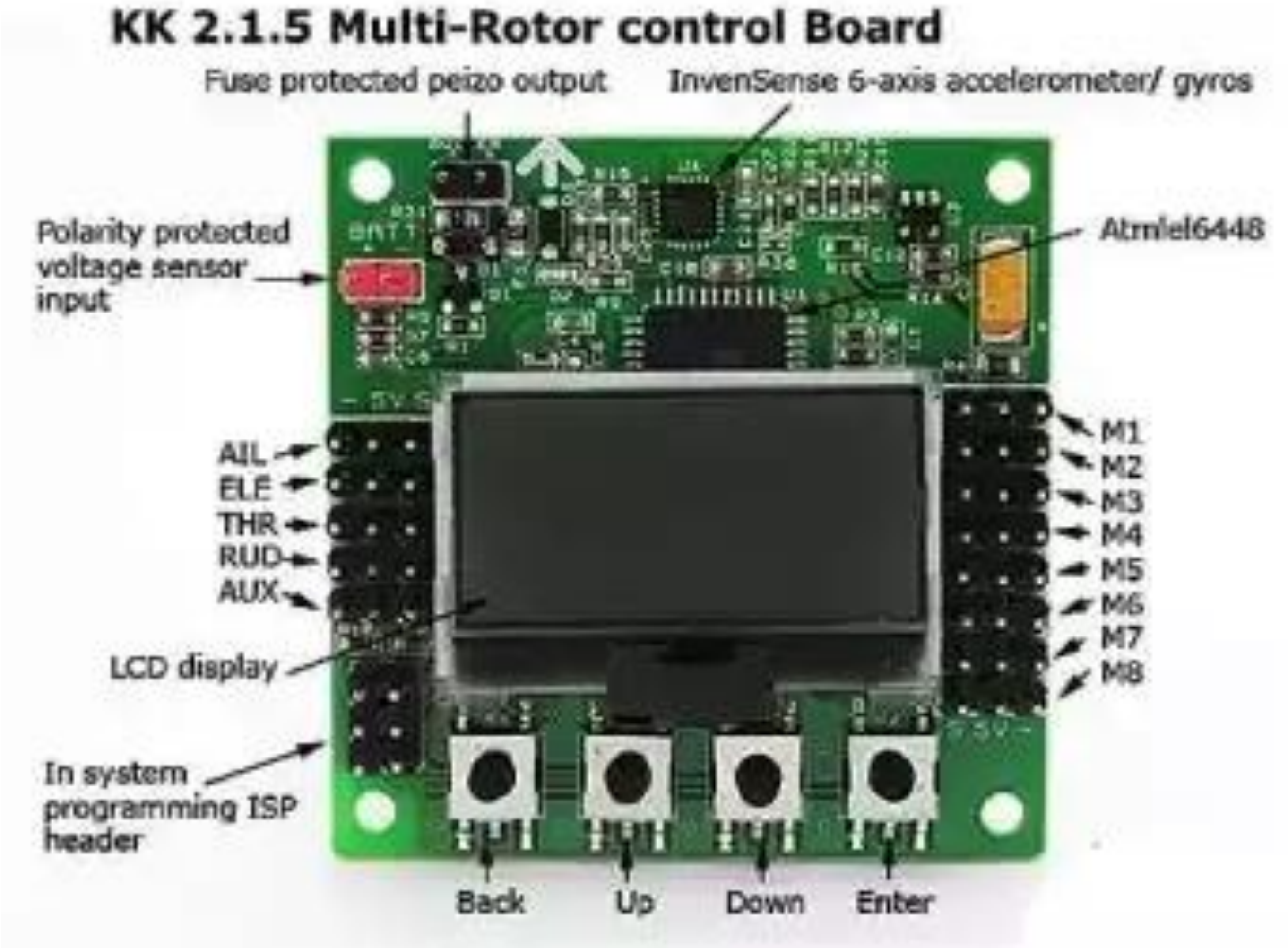
Our Indoor Testing Room

Our artificial wind gust generator

Arm with 6-Axis Balance

Flight Controllers

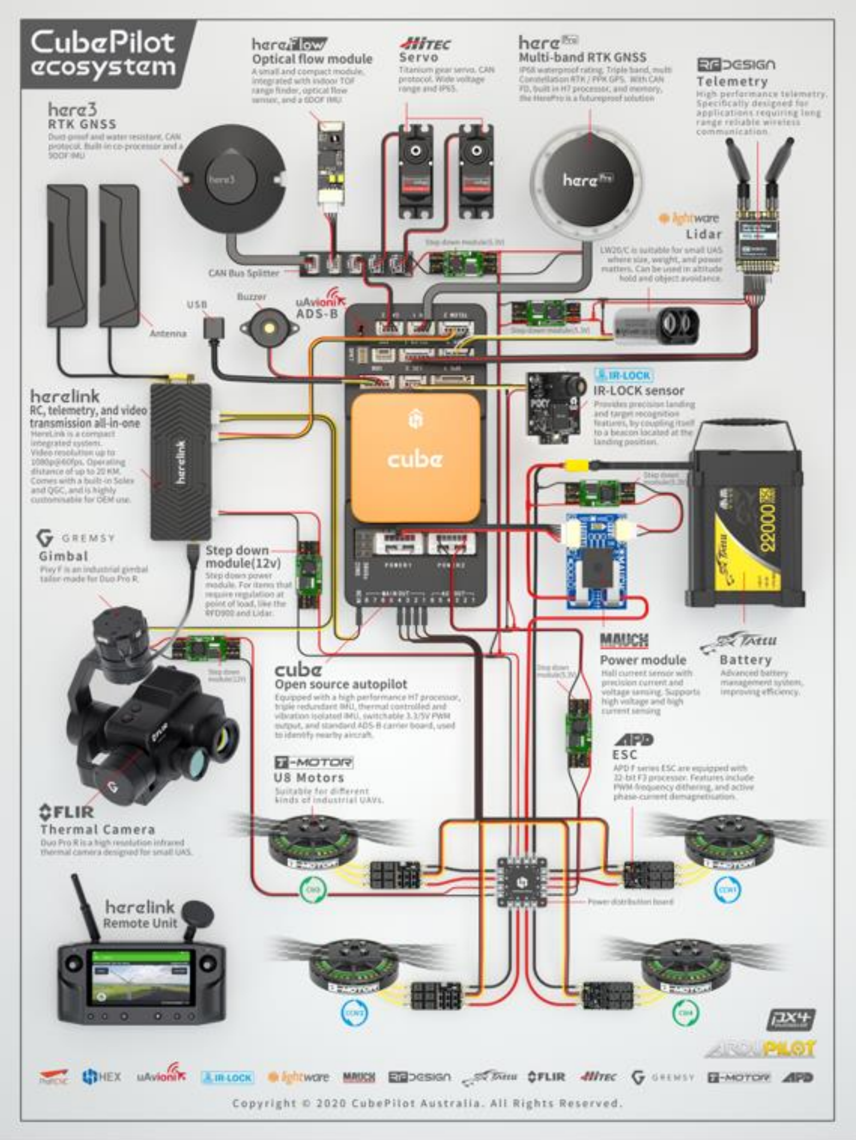
KK2 Controller (about US$25)



Pixhawk 4 Controller (about US$400)

Main FMU Processor	STM32F765 32 Bit Arm Cortex-M7, 216Mhz, 2MB Memory, 512KB RAM
IO Processor	STM32F100 32 Bit Arm Cortex-M3, 24Mhz 8KB SRAM
Sensors	2 3-axis Accel/Gyro 1 Magnetometer 1 Barometer 1 GPS (External Module)
Interfaces	8-16 PWM outputs (8 IO, 8 FMU) 3 dedicated PWM/Capture inputs on FMU 5 general purpose serial ports
Open Source AP	ArduPilot, PX4, Paparazzi

Many Companies Make Pixhawk Hardware



here3 RTK GNSS
Dust-proof and water resistant, CAN protocol. Built-in co-processor and a 9DOF IMU

hereFlow Optical flow module
A small and compact module, integrated with indoor TOF range finder, optical flow sensor, and a 6DOF IMU

HITEC Servo
Titanium gear servo, CAN protocol. Wide voltage range and IP65.

herePro Multi-band RTK GNSS
IP68 waterproof rating. Triple band, multi Constellation RTK / PPK GPS. With CAN FD, built in H7 processor, and memory, the HerePro is a futureproof solution

RFDESIGN Telemetry
High performance telemetry. Specifically designed for applications requiring long range reliable wireless communication.

lightware Lidar
LW20/C is suitable for small UAS where size, weight, and power matters. Can be used in altitude hold and object avoidance.

herelink RC, telemetry, and video transmission all-in-one
HereLink is a compact integrated system. Video resolution up to 1080p@60fps. Operating distance of up to 20 KM. Comes with a built-in Solex and QGC, and is highly customisable for OEM use.

IR-LOCK sensor
Provides precision landing and target recognition features, by coupling itself to a beacon located at the landing position.

GREMSY Gimbal
Pixy F is an industrial gimbal tailor-made for Duo Pro R.

Step down module(12v)
Step down power module. For items that require regulation at point of load, like the RFD900 and Lidar.

cube Open source autopilot
Equipped with a high performance H7 processor, triple redundant IMU, thermal controlled and vibration isolated IMU, switchable 3.3/5V PWM output, and standard ADS-B carrier board, used to identify nearby aircraft.

MAUCH Power module
Hall current sensor with precision current and voltage sensing. Supports high voltage and high current sensing

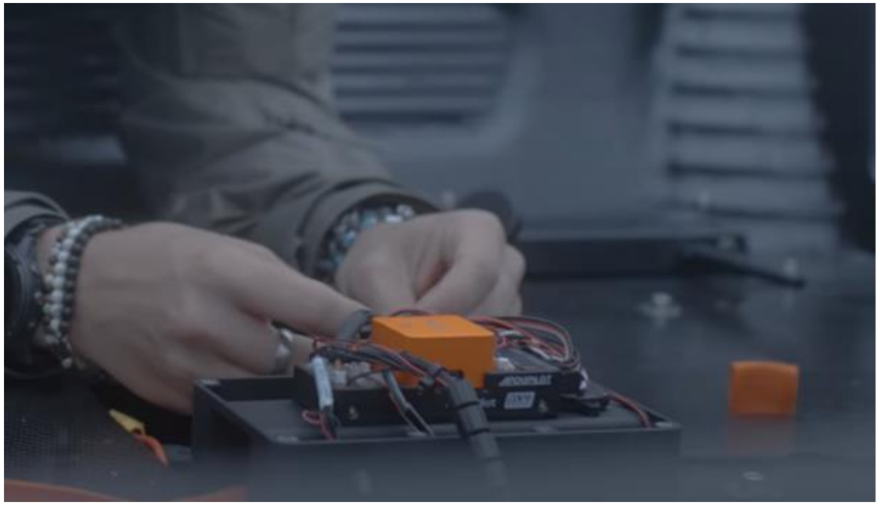
TATTU Battery
Advanced battery management system, improving efficiency.

T-MOTOR U8 Motors
Suitable for different kinds of industrial UAVs.

APD ESC
APD F series ESC are equipped with 32-bit F3 processer. Features include PWM-frequency dithering, and active phase-current demagnetisation.

FLIR Thermal Camera
Duo Pro R is a high resolution infrared thermal camera designed for small UAS.

herelink Remote Unit

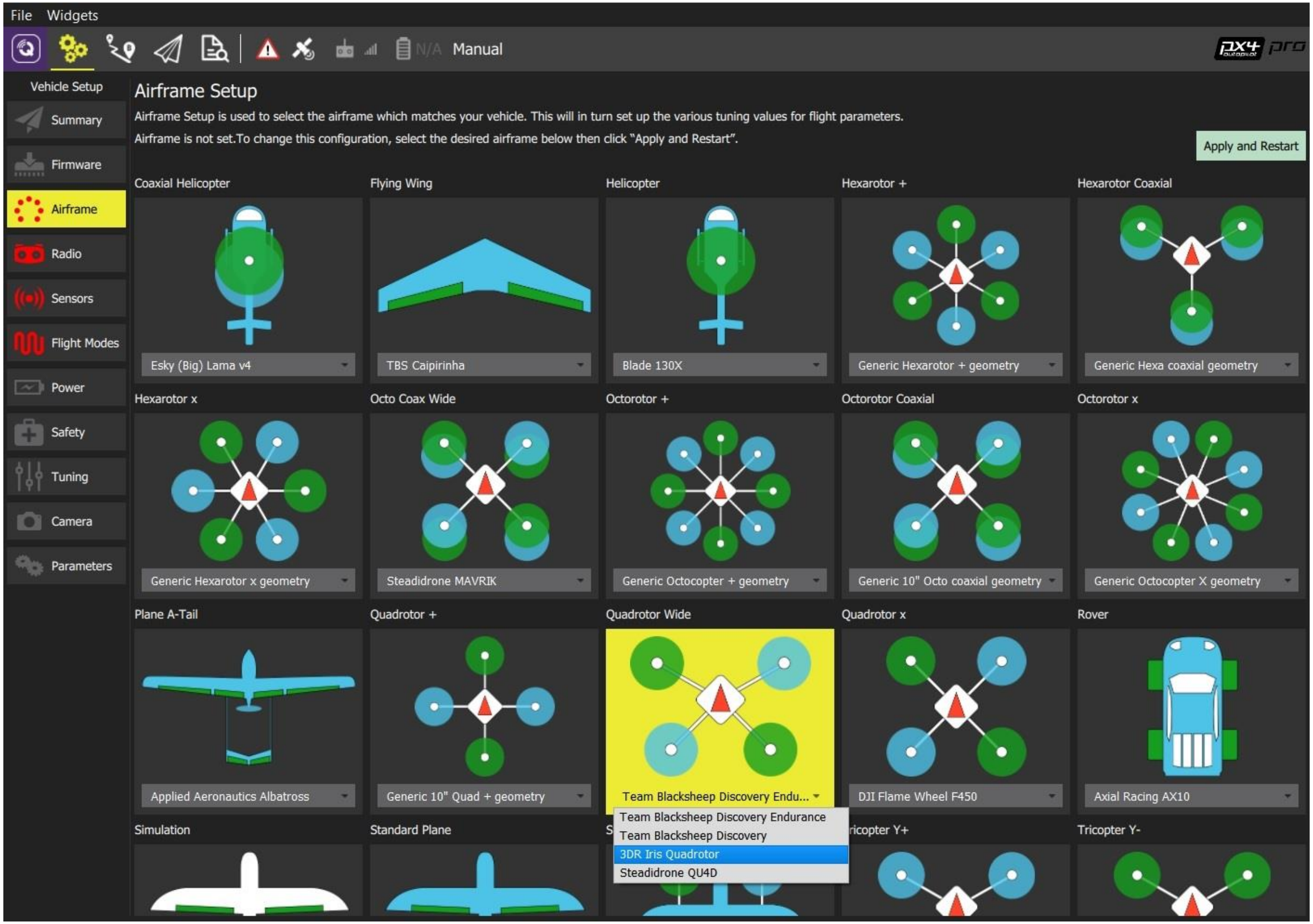


Source: [Pixhawk Orange Cube]

PX4 is the Control Laws Software

- An open source flight control software for drones and other unmanned vehicles
- Runs on NuttX RTOS
- Software can be setup using Ground Station software or built using an IDE or console

PX4 Built-in Configurations



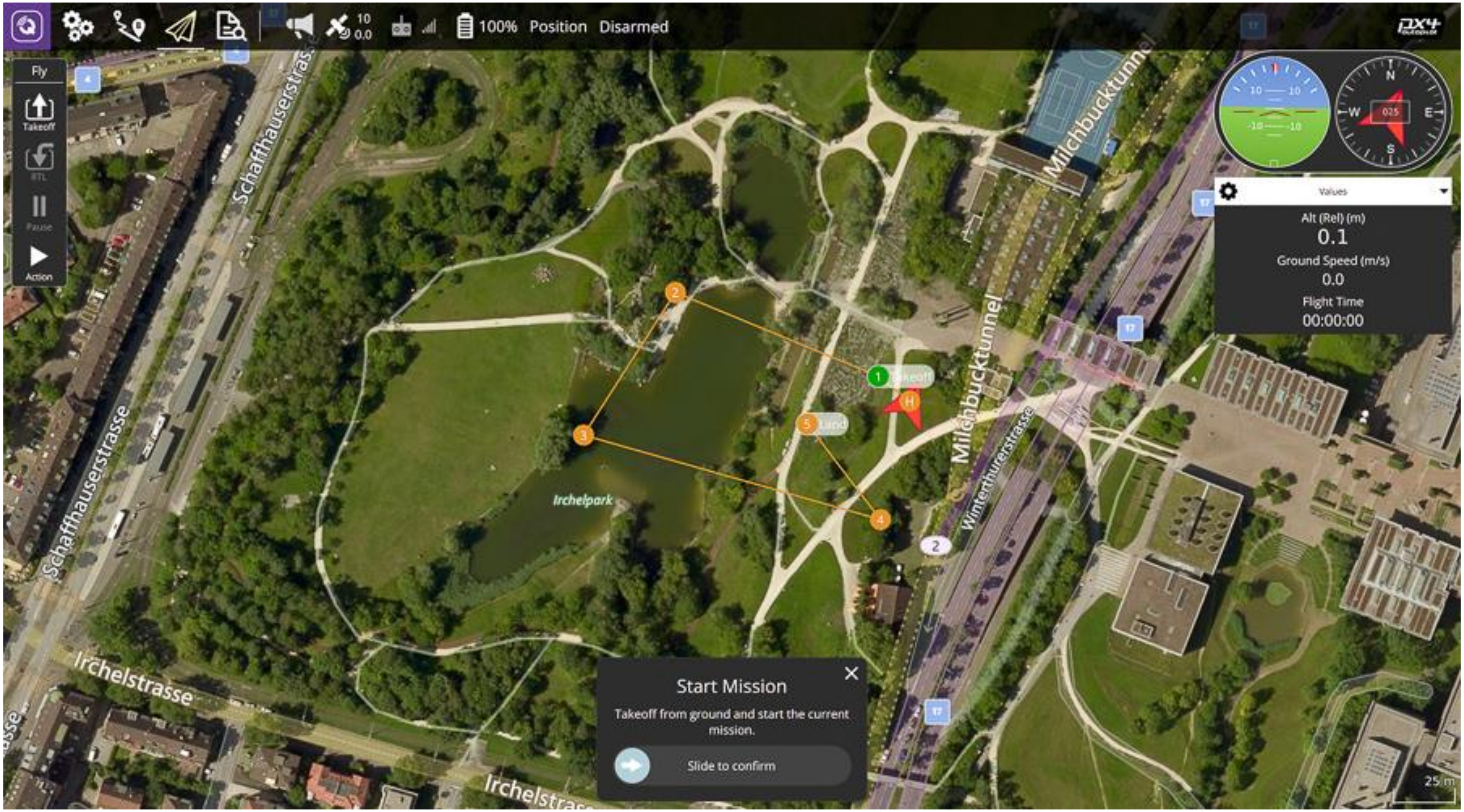
Airframe Setup

Airframe Setup is used to select the airframe which matches your vehicle. This will in turn set up the various tuning values for flight parameters.

Airframe is not set.To change this configuration, select the desired airframe below then click "Apply and Restart".

Coaxial Helicopter: Esky (Big) Lama v4
Flying Wing: TBS Caipirinha
Helicopter: Blade 130X
Hexarotor +: Generic Hexarotor + geometry
Hexarotor Coaxial: Generic Hexa coaxial geometry
Hexarotor x: Generic Hexarotor x geometry
Octo Coax Wide: Steadidrone MAVRIK
Octorotor +: Generic Octocopter + geometry
Octorotor Coaxial: Generic 10" Octo coaxial geometry
Octorotor x: Generic Octocopter X geometry
Plane A-Tail: Applied Aeronautics Albatross
Quadrotor +: Generic 10" Quad + geometry
Quadrotor Wide: Team Blacksheep Discovery Endu...
Quadrotor x: DJI Flame Wheel F450
Rover: Axial Racing AX10
Simulation
Standard Plane
S
ricopter Y+
Tricopter Y-

PX4 QGroundControl



Alt (Rel) (m)
0.1

Ground Speed (m/s)
0.0

Flight Time
00:00:00

Start Mission

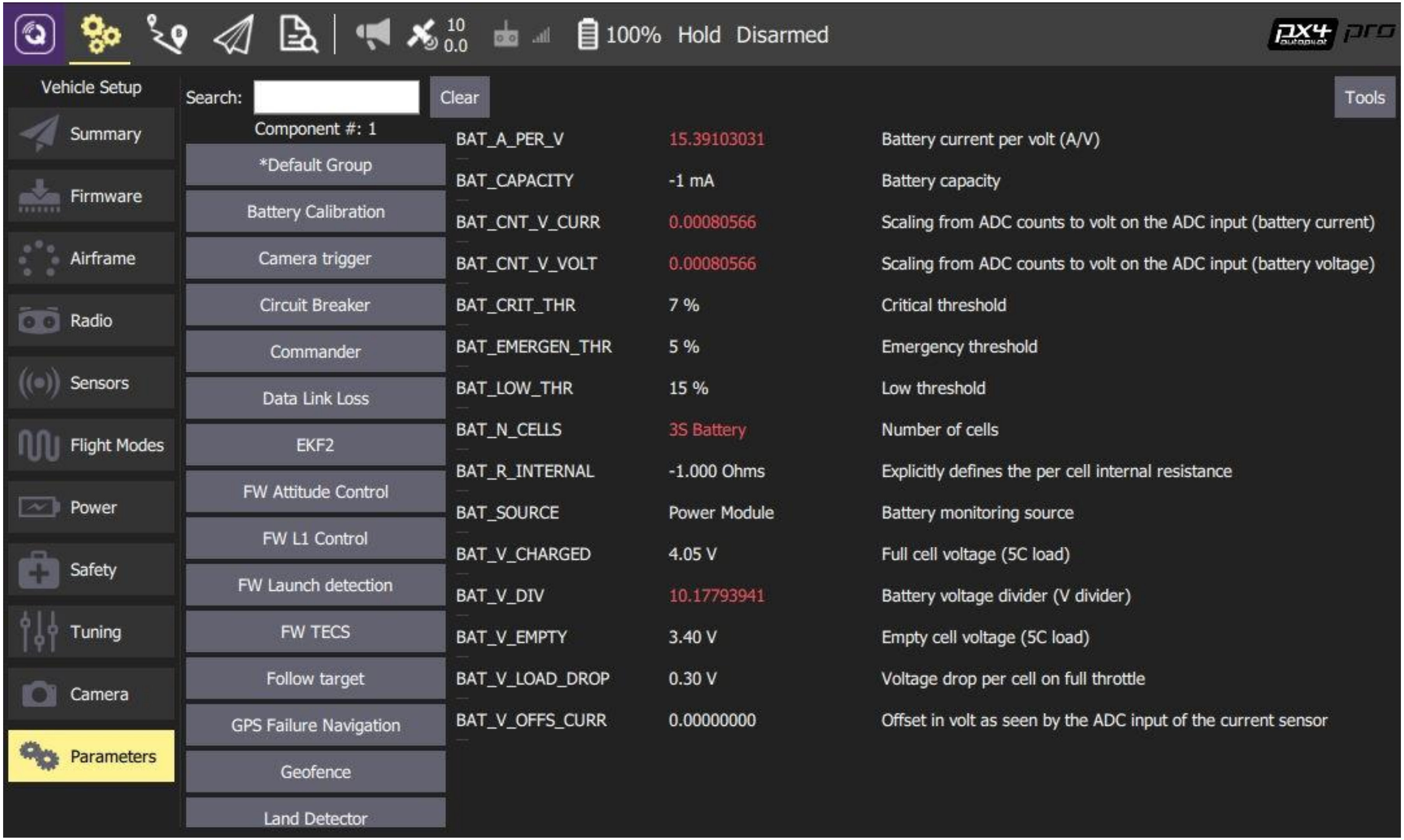
Takeoff from ground and start the current mission.

Slide to confirm

PX4 QGroundControl

- Ground Station Software for MAVLink Protocol
- Firmware installation
- Calibration and Setup of Drone
- Allows you to send commands to drone from the ground
- Functions such as: Waypoints, GPS, Flightpath
- Provides various telemetry data

PX4 QGroundControl



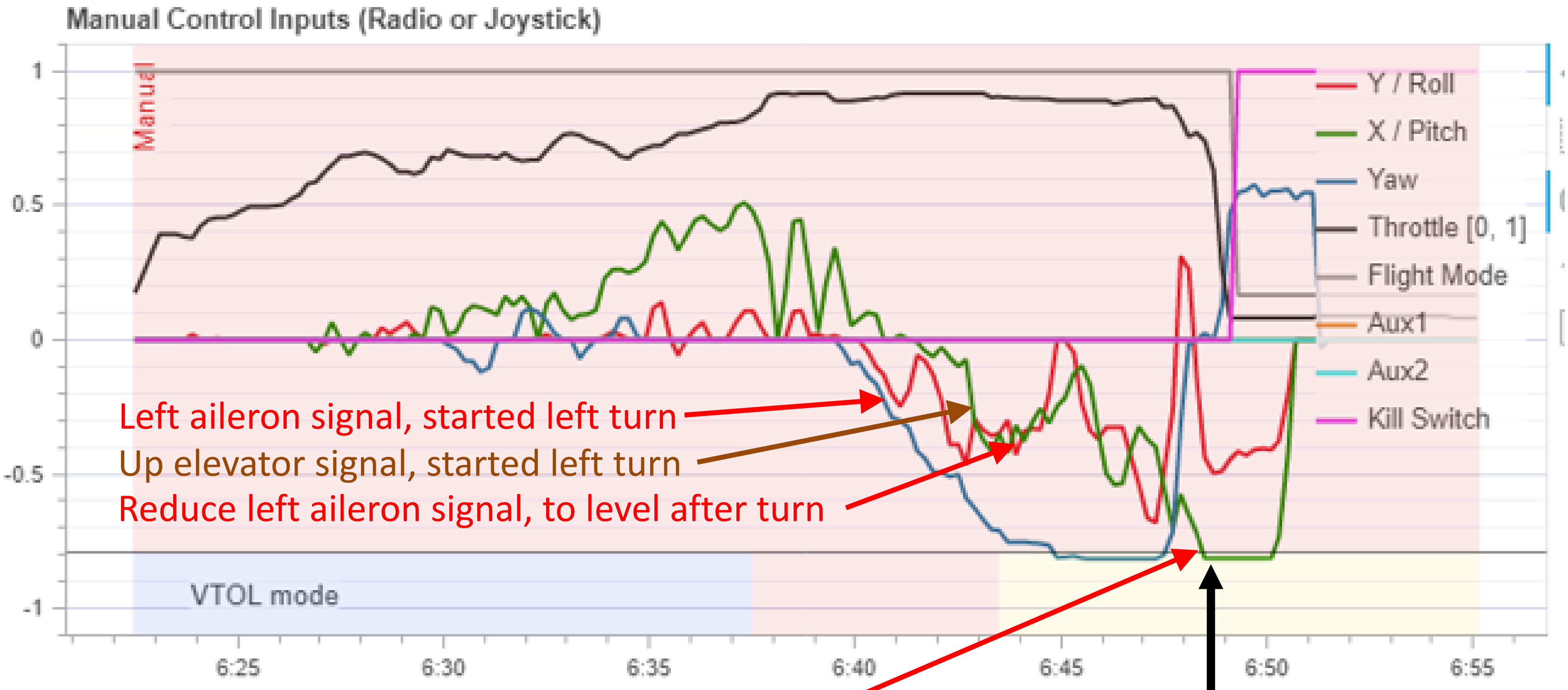
BAT_A_PER_V	15.39103031	Battery current per volt (A/V)
BAT_CAPACITY	-1 mA	Battery capacity
BAT_CNT_V_CURR	0.00080566	Scaling from ADC counts to volt on the ADC input (battery current)
BAT_CNT_V_VOLT	0.00080566	Scaling from ADC counts to volt on the ADC input (battery voltage)
BAT_CRIT_THR	7 %	Critical threshold
BAT_EMERGEN_THR	5 %	Emergency threshold
BAT_LOW_THR	15 %	Low threshold
BAT_N_CELLS	3S Battery	Number of cells
BAT_R_INTERNAL	-1.000 Ohms	Explicitly defines the per cell internal resistance
BAT_SOURCE	Power Module	Battery monitoring source
BAT_V_CHARGED	4.05 V	Full cell voltage (5C load)
BAT_V_DIV	10.17793941	Battery voltage divider (V divider)
BAT_V_EMPTY	3.40 V	Empty cell voltage (5C load)
BAT_V_LOAD_DROP	0.30 V	Voltage drop per cell on full throttle
BAT_V_OFFS_CURR	0.00000000	Offset in volt as seen by the ADC input of the current sensor

PX4 Logging Telemetry

- Flight Data containing aircraft state and sensor data are automatically logged and stored on the Pixhawk SD card.
- Can be downloaded via QGroundControl
- Log Analysis access is available when log file is uploaded through PX4 flight review tool (http://logs.px4.io)

ArduPilot is another flight control software and also comes with a mission planner tool

PX4 Recorded Flight Controls of a L+C Model



Left aileron signal, started left turn

Up elevator signal, started left turn

Reduce left aileron signal, to level after turn

More Left aileron signal, to turn left, but plane did not respond

Aircraft crashed here

5. Rotor Design, Stability & Control and Model Flight Testing

by Dr. James Wang

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