

5. Rotor Design, Stability & Control and Model Flight Testing

by Dr. James Wang

SNUevtolclass@gmail.com

For students to use in the 2022 eVTOL Design Short Course at SNU,
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Topics

1. Rotor design
2. How helicopters are controlled
3. Controls for different eVTOL Aircraft
4. Aircraft stability
5. Stability & Control analysis
6. Model testing

Different Types of Rotors

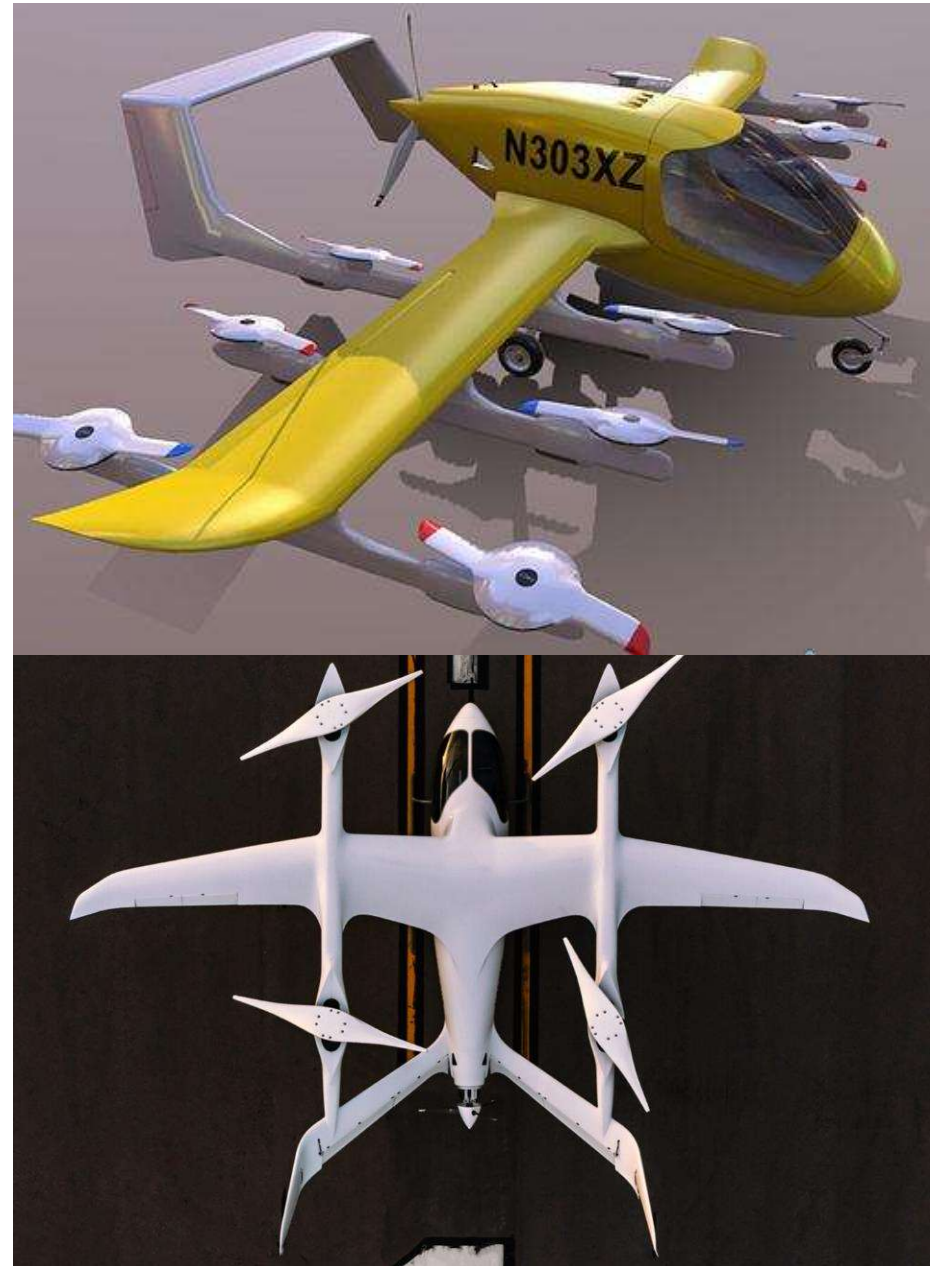
Fixed Pitch or Variable Pitch ?



Most Quadcopters and Multi-Rotors Use Fixed Pitch Rigid Propellers

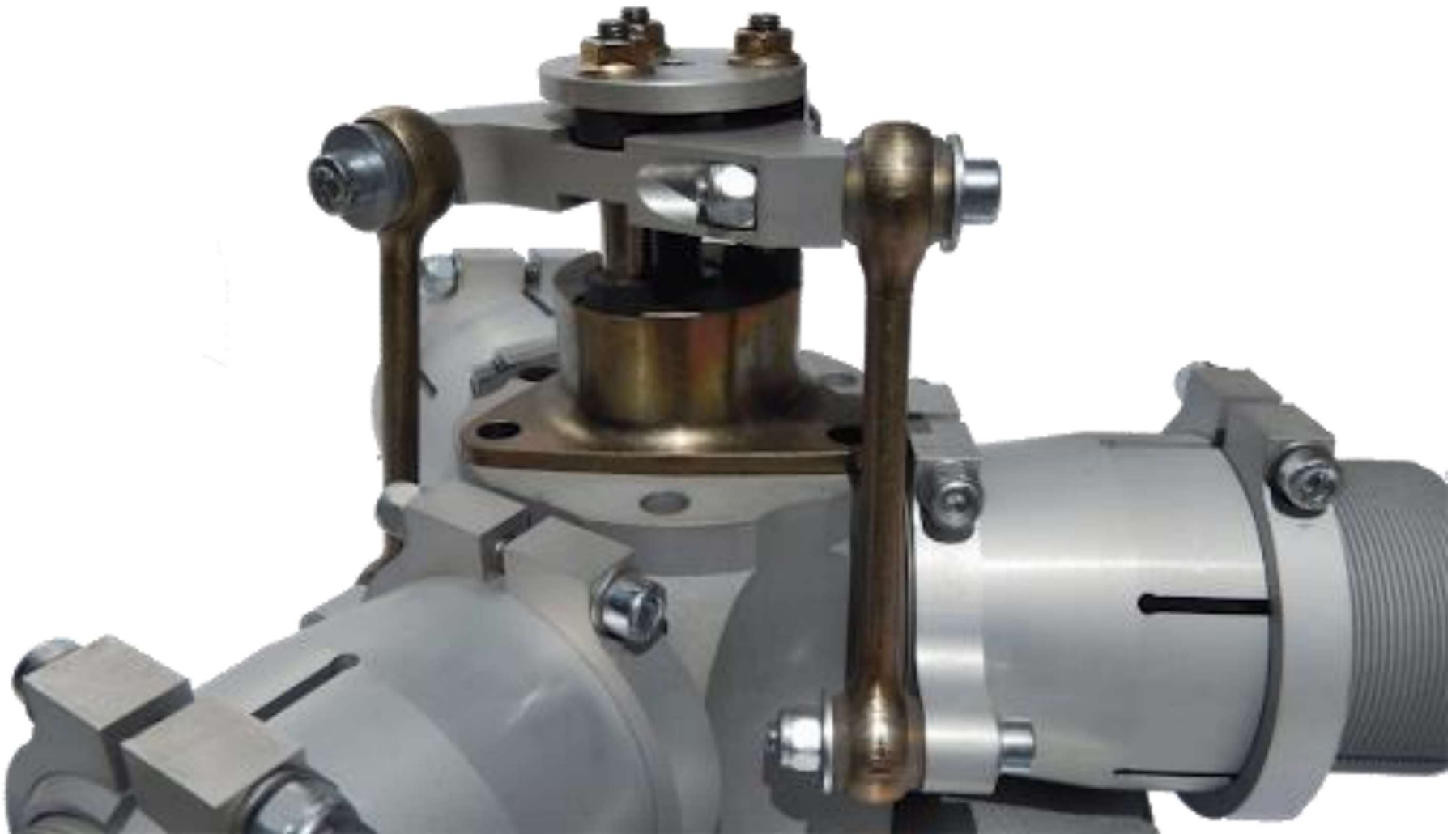


EHANG 184
Specification

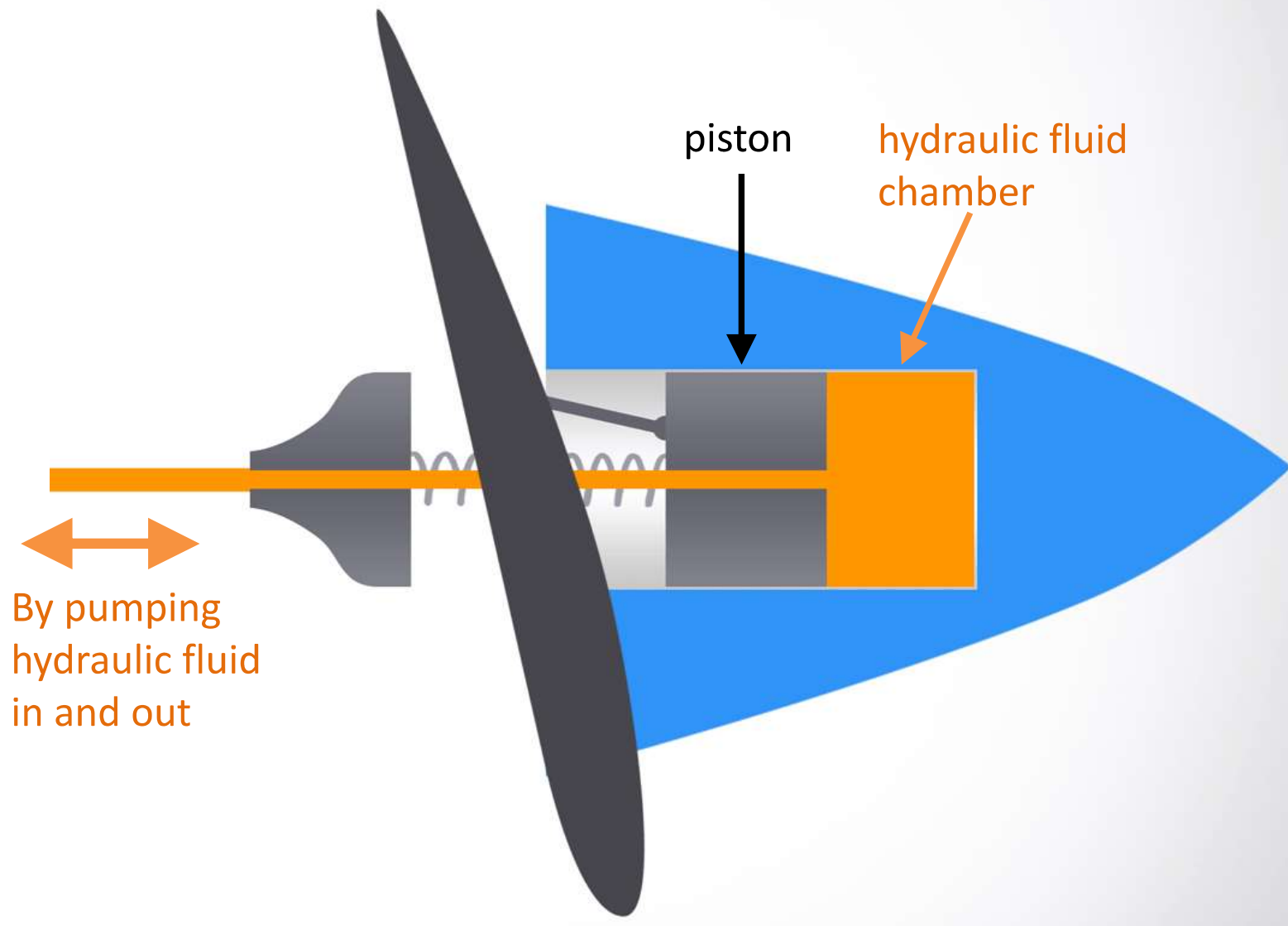


Simple Collective Pitch Change Only

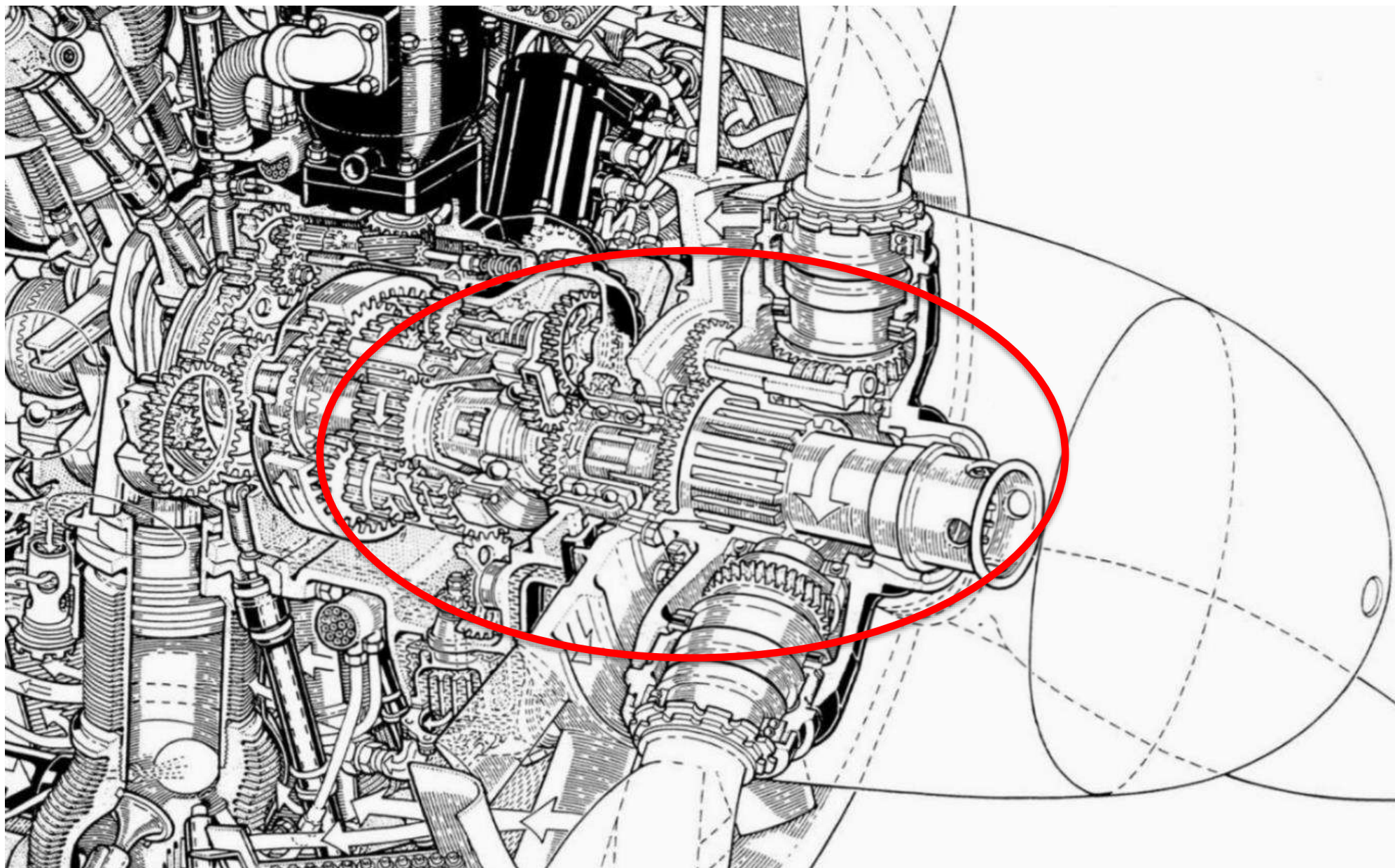
(Means all blades change pitch angle together)



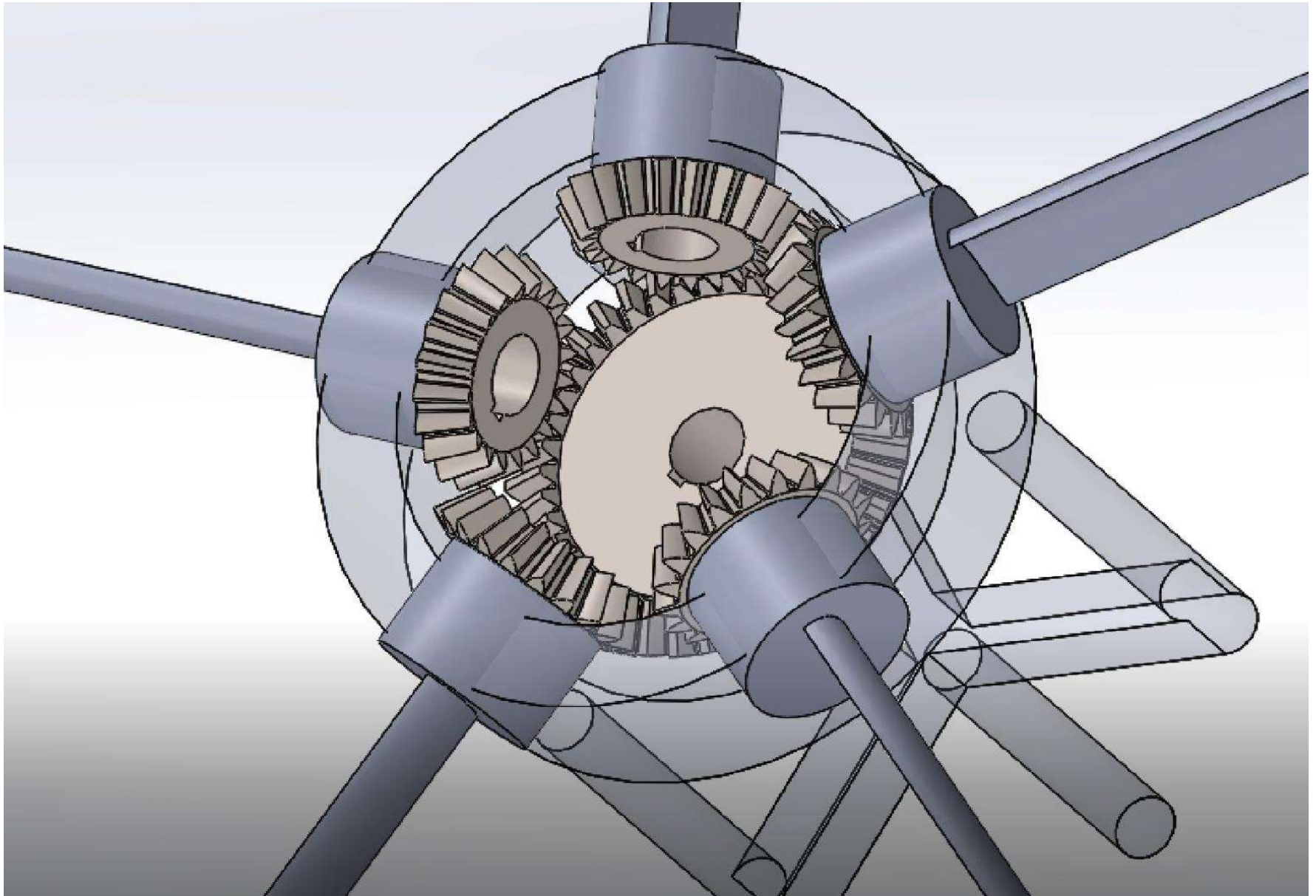
Pumping Hydraulic Fluid to Change Pitch



Using Gears to Change Pitch on FW190



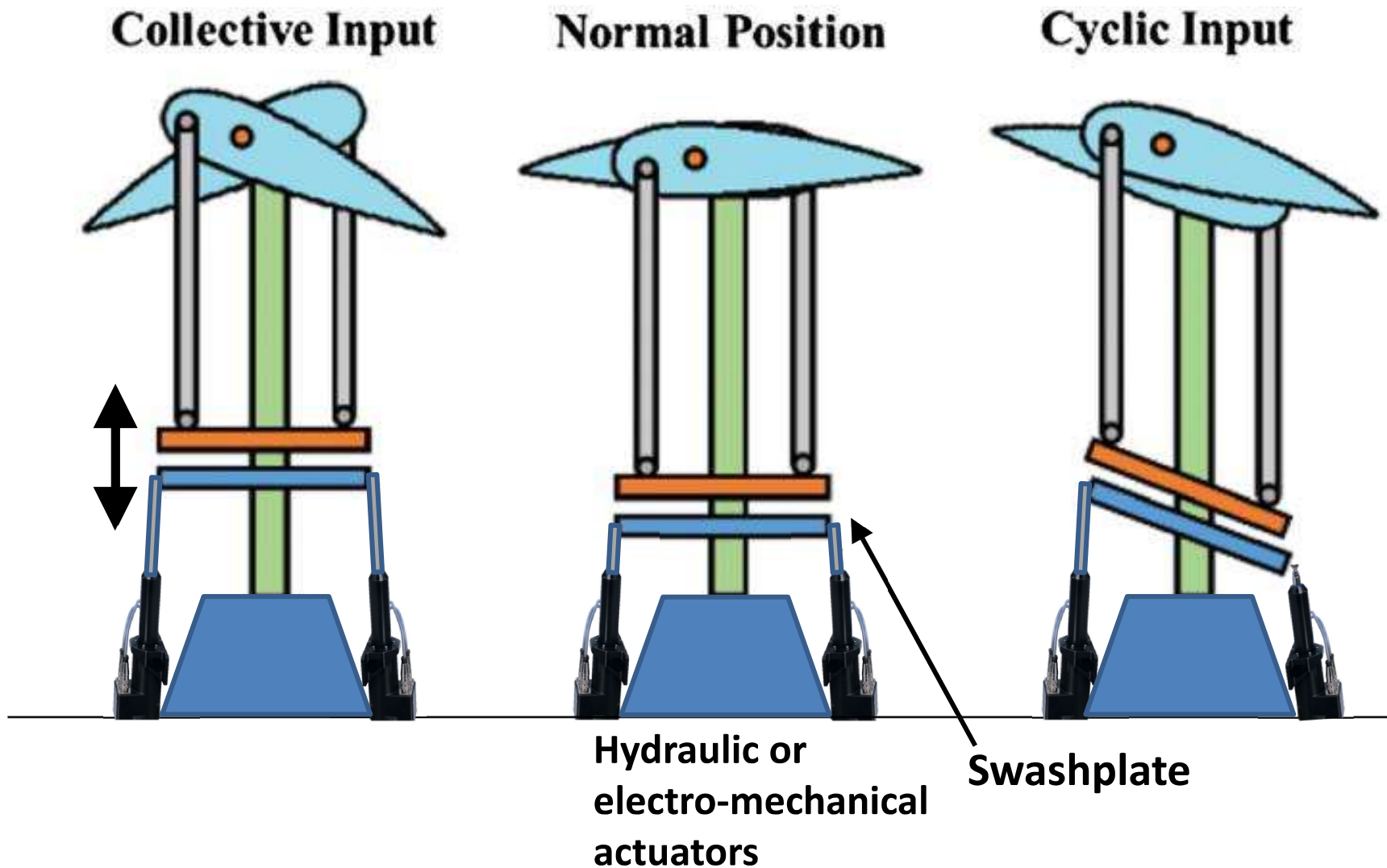
Another Design Using Gears to Change Pitch



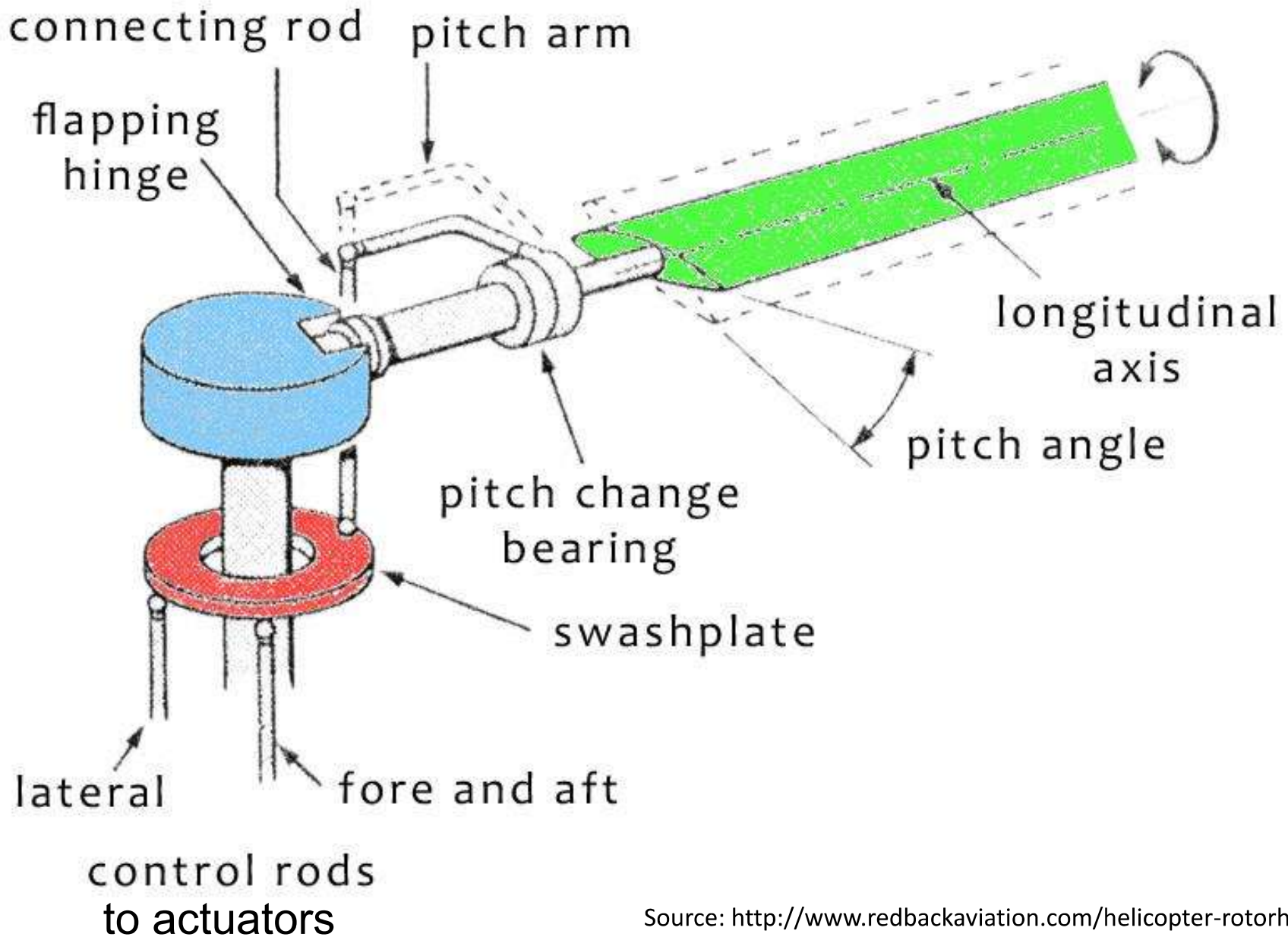
A Mechanical Actuator Through Hollow Shaft



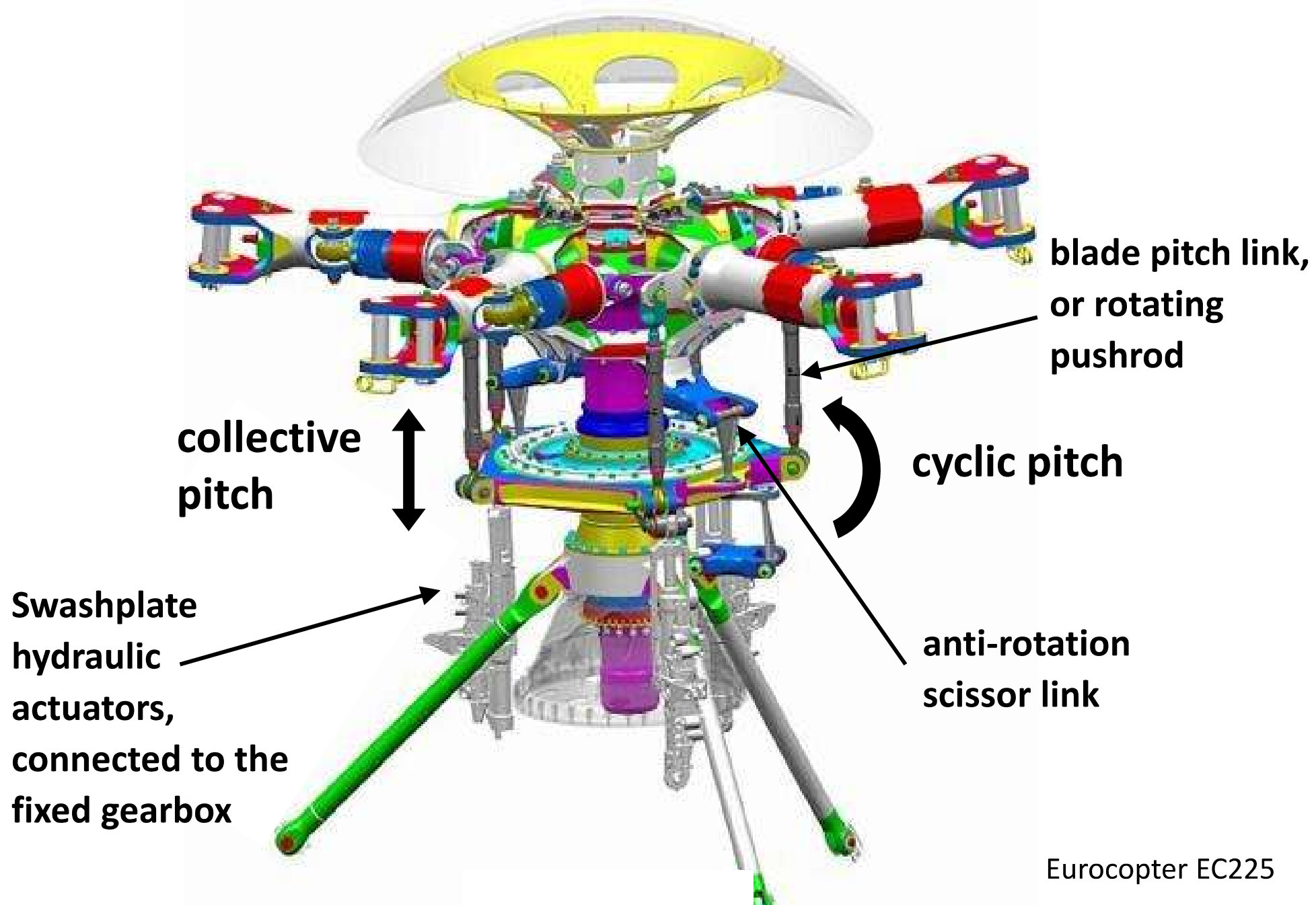
Collective + Cyclic Pitch Change



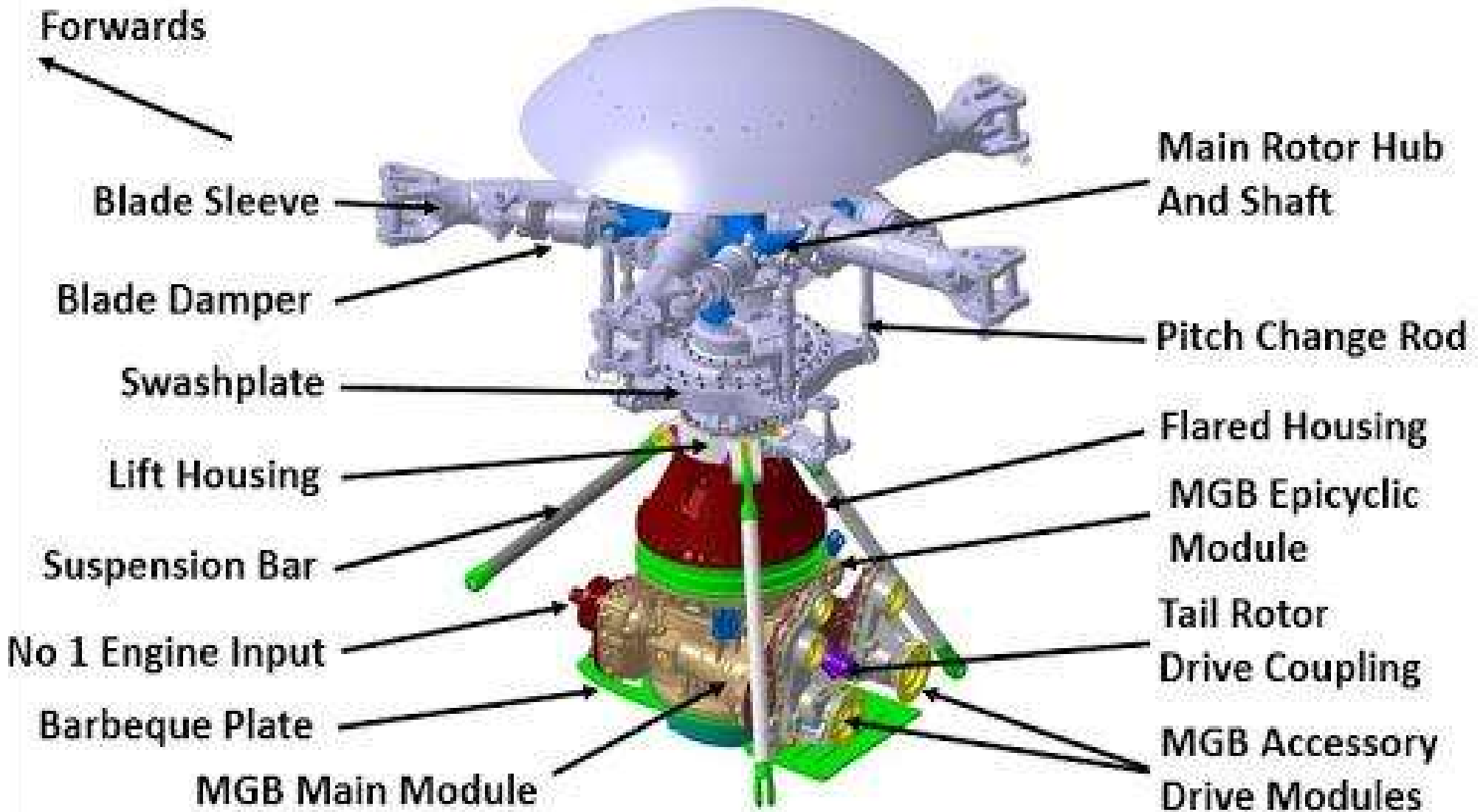
Blade Pitch Change Mechanism



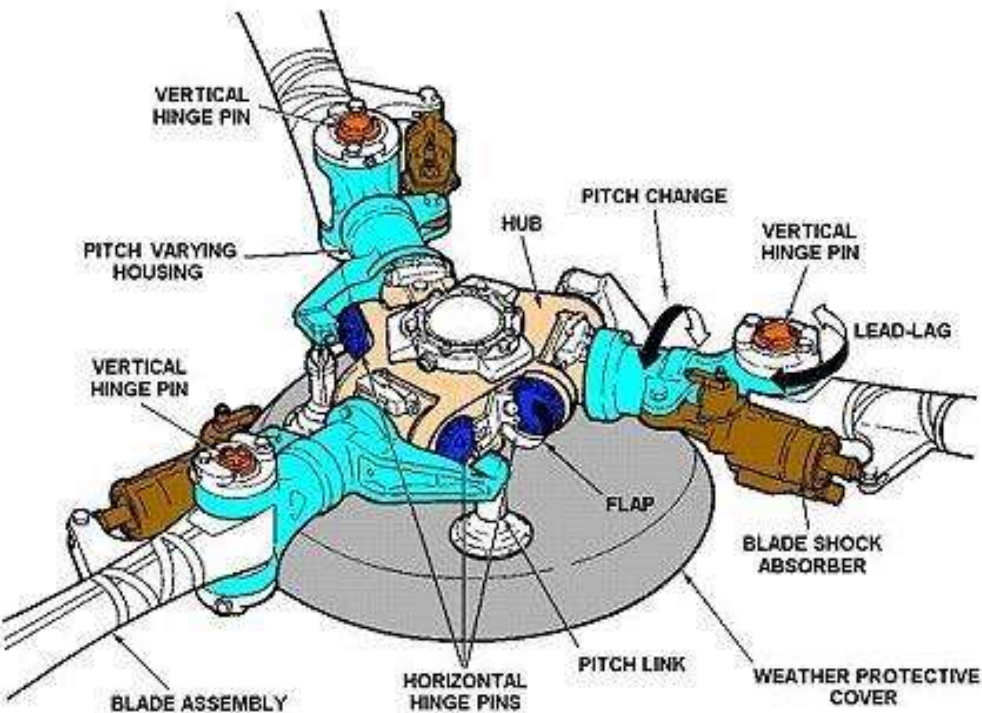
Collective + Cyclic Pitch Change



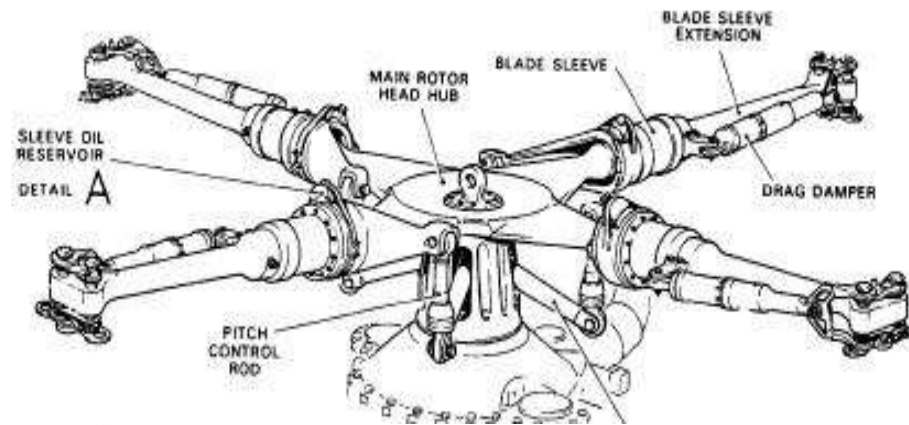
The Rotor is Mounted to the Main Gearbox



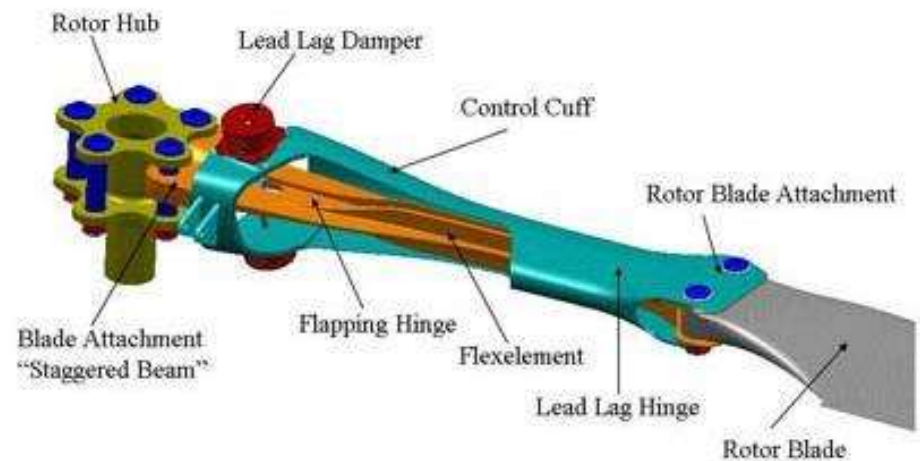
3 Major Types of Helicopter Rotor Hubs



Fully articulated rotor

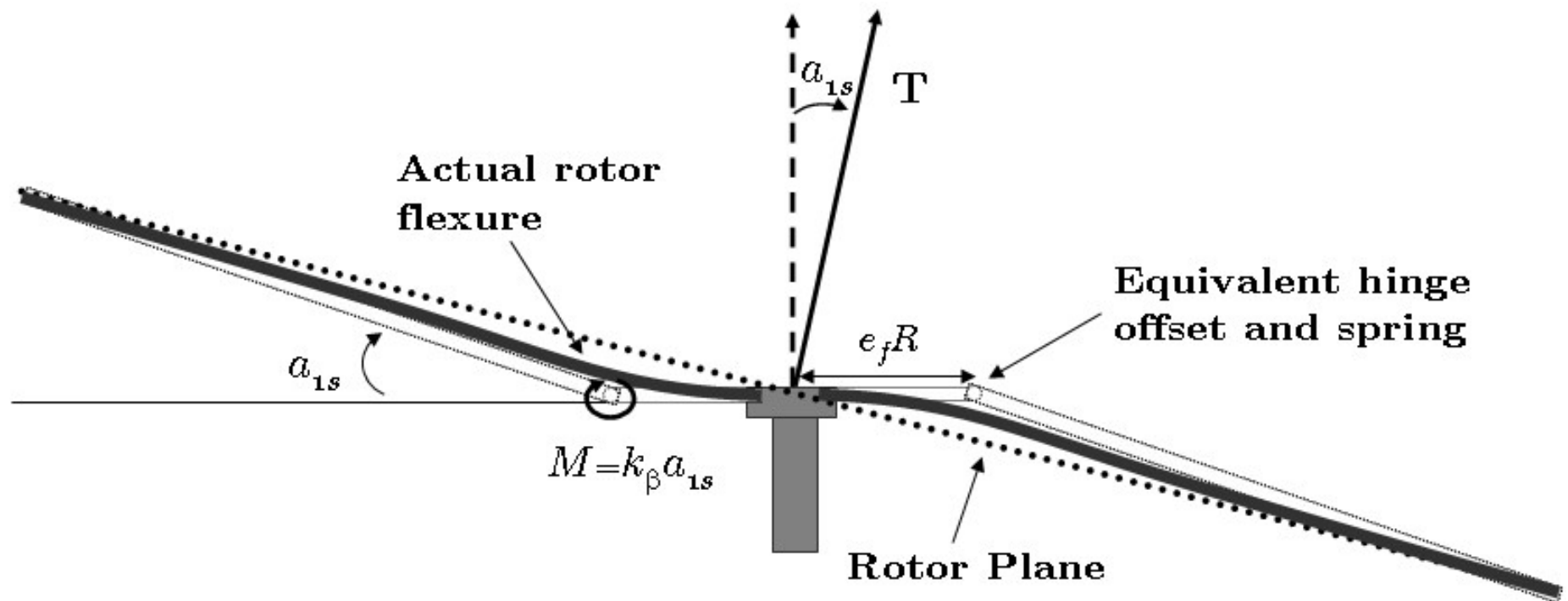


Hingeless rotor

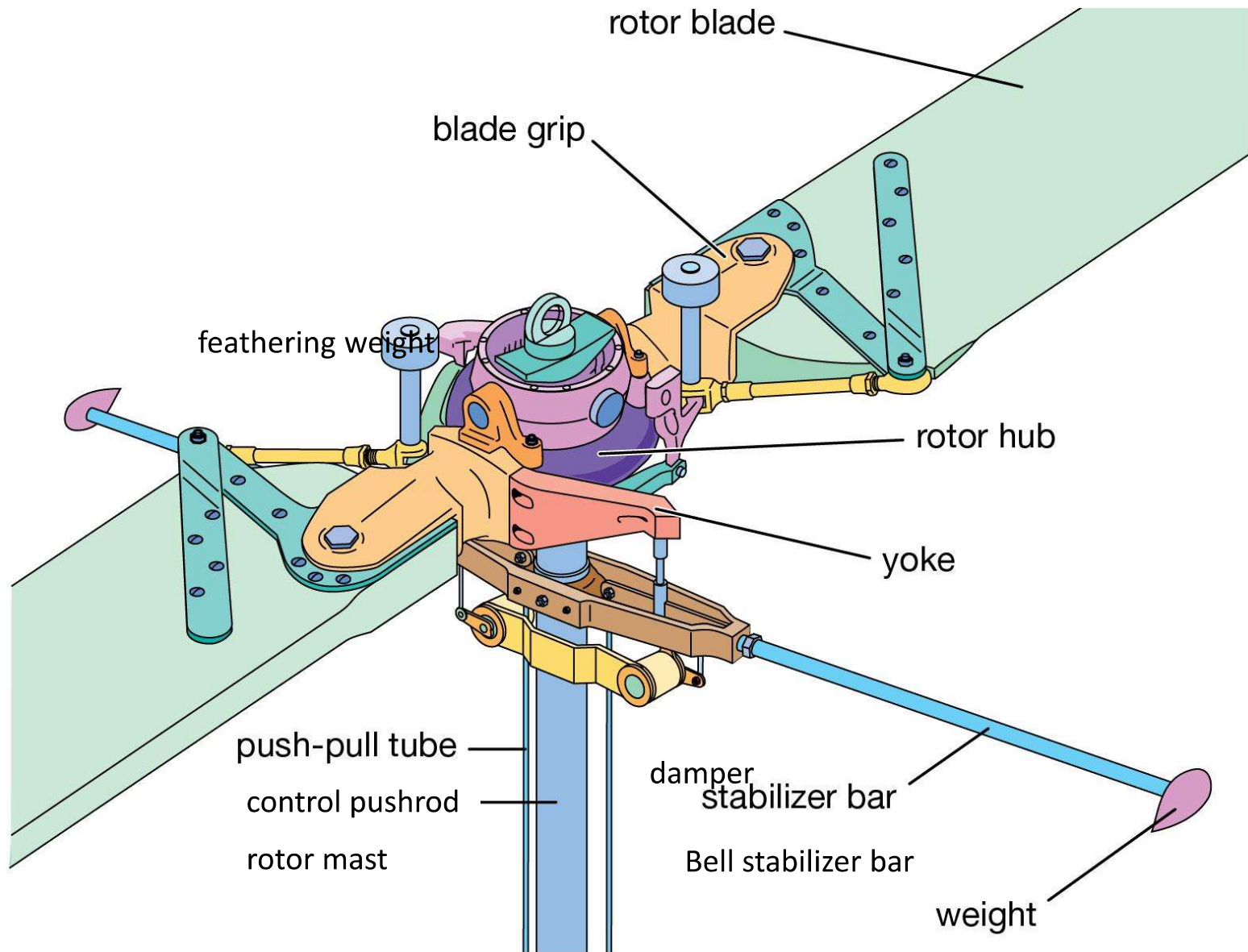


Bearingless rotor

Hingeless and Bearingless Rotors

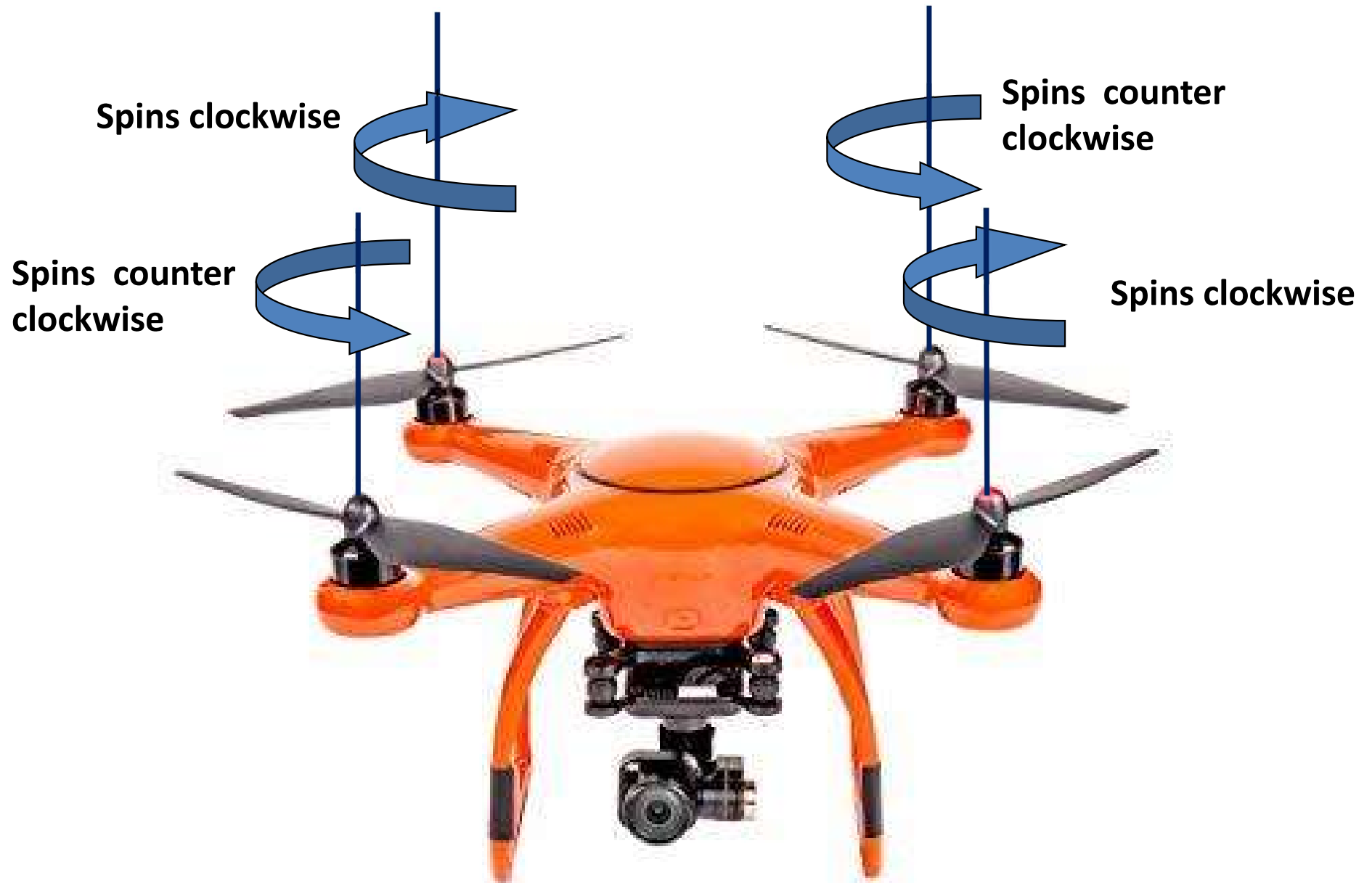


2-Bladed Teetering Rotor



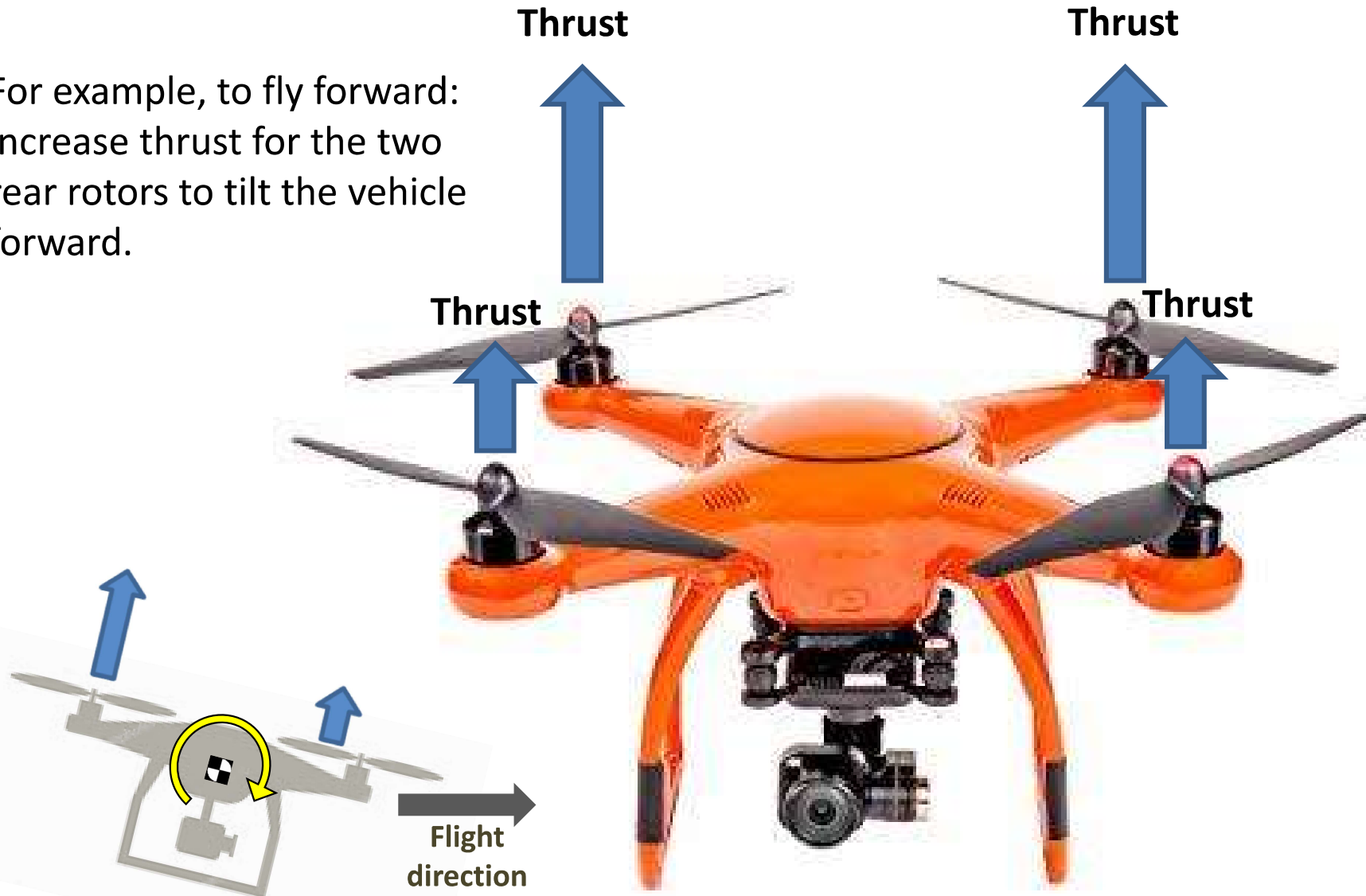
Helicopter and VTOL Flight Control

Multi-rotors are Mechanically simpler than Helicopters. Torque is Balanced When Even Number of Rotors is Used

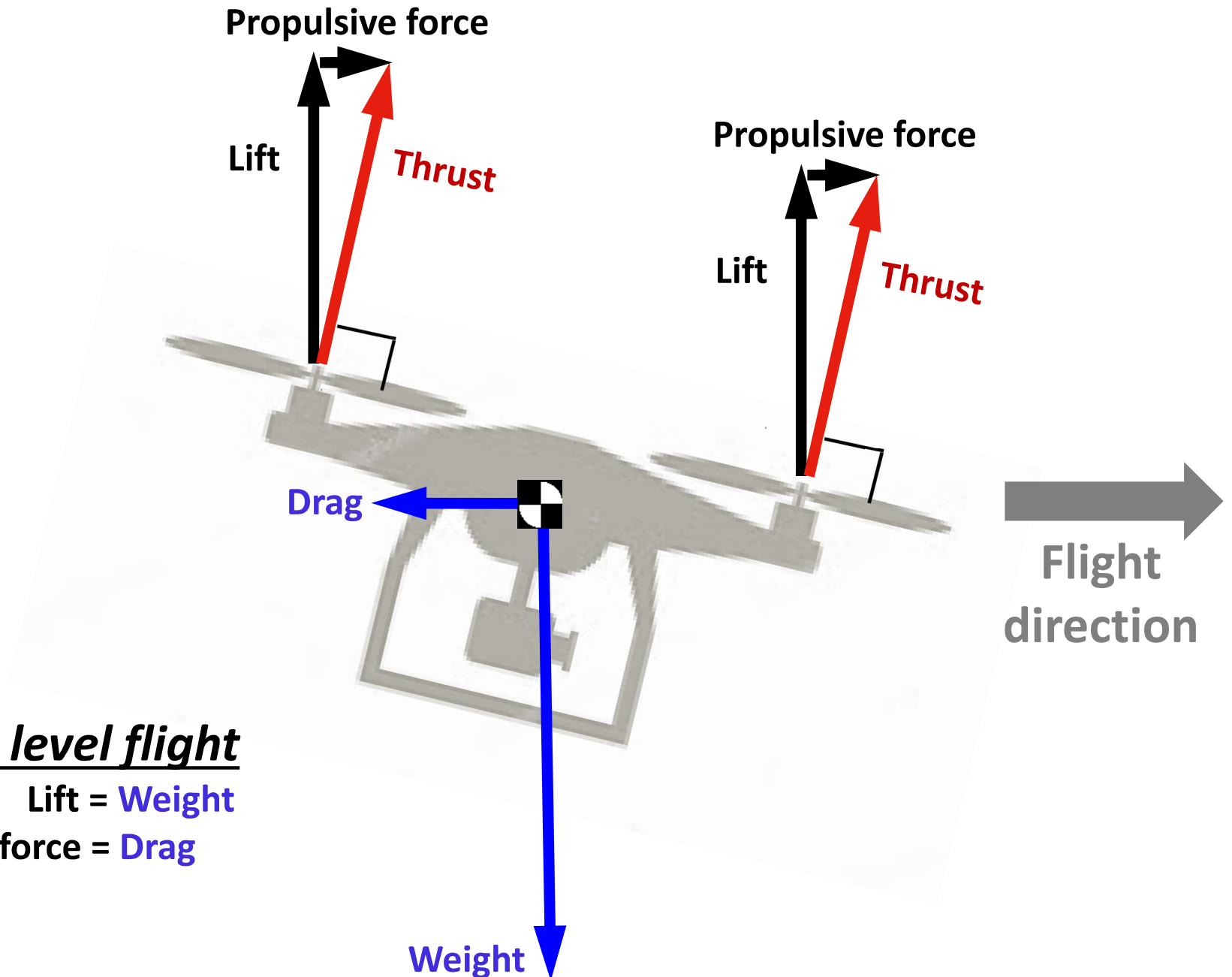


**A multi-rotor has no mechanical control. Pitching, rolling and vertical motions are controlled by varying rotor thrust/rpm.
Axi-symmetrical -> No Control Cross-coupling -> Easier Piloting**

For example, to fly forward:
increase thrust for the two
rear rotors to tilt the vehicle
forward.



Rotor Forces in Steady Forward Flight



In steady level flight

Lift = Weight

Propulsive force = Drag

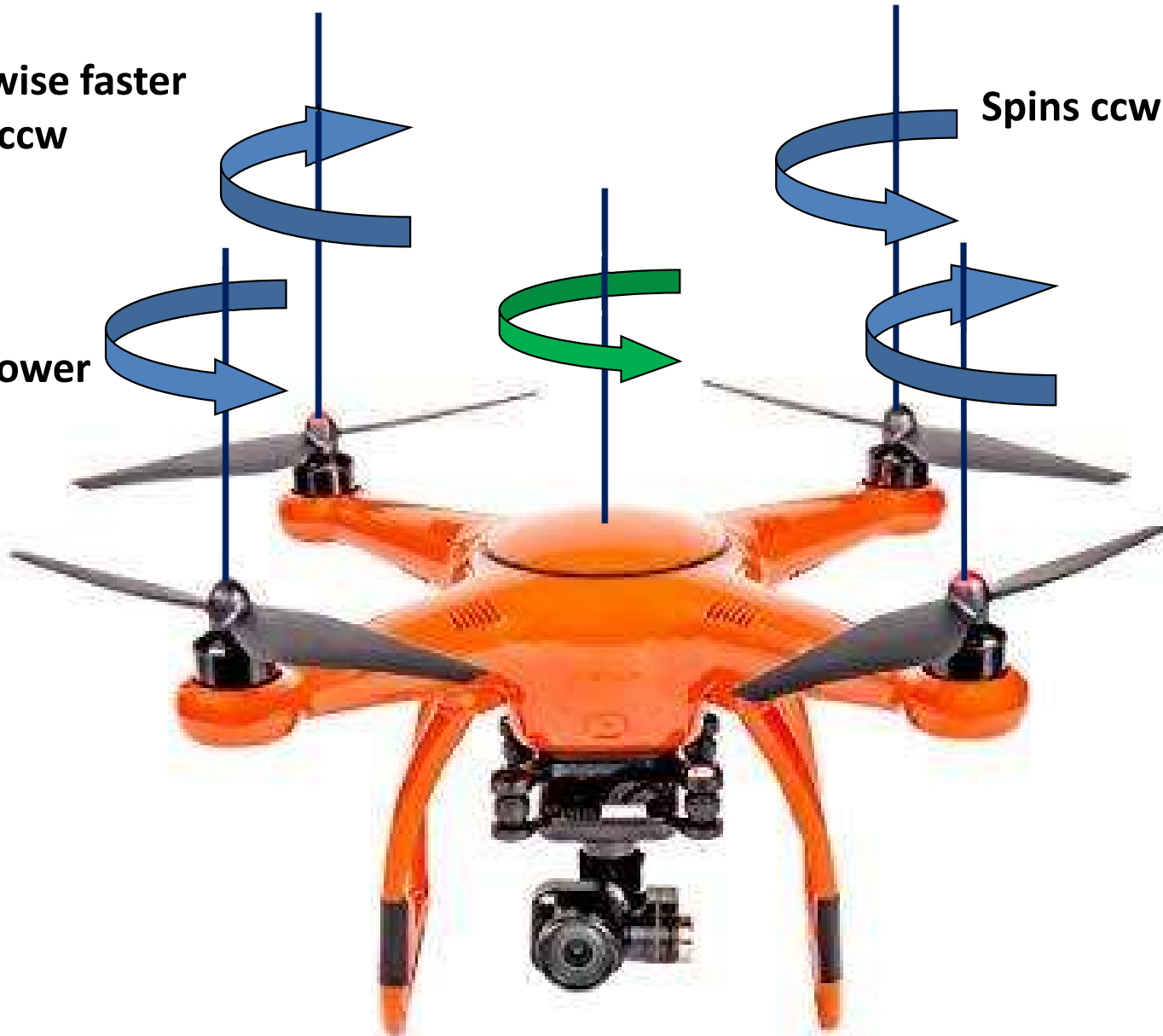
Yaw Control for Quadcopter (Ex: Yaw Left)

Spins clockwise faster
to increase ccw
reaction

Spins ccw slower

Spins ccw slower

Spins clockwise
faster to
increase ccw
reaction



Canting All 4 Rotors Improves Yaw Response by Introducing a Side Force to Spin the Drone



The DJI Phantom 3 drone introduced in April 2015

Helicopter Controls

Different Helicopter Configurations



Single Main and Tail Rotor



Tandem Rotor



Coaxial Rotor



NOTAR((NO TAIL Rotor)



Side by Side



Tip Jet



Tilt Rotor



Pusher Propeller



Quad Tilt Rotor

AW139 Helicopter Flight Controls



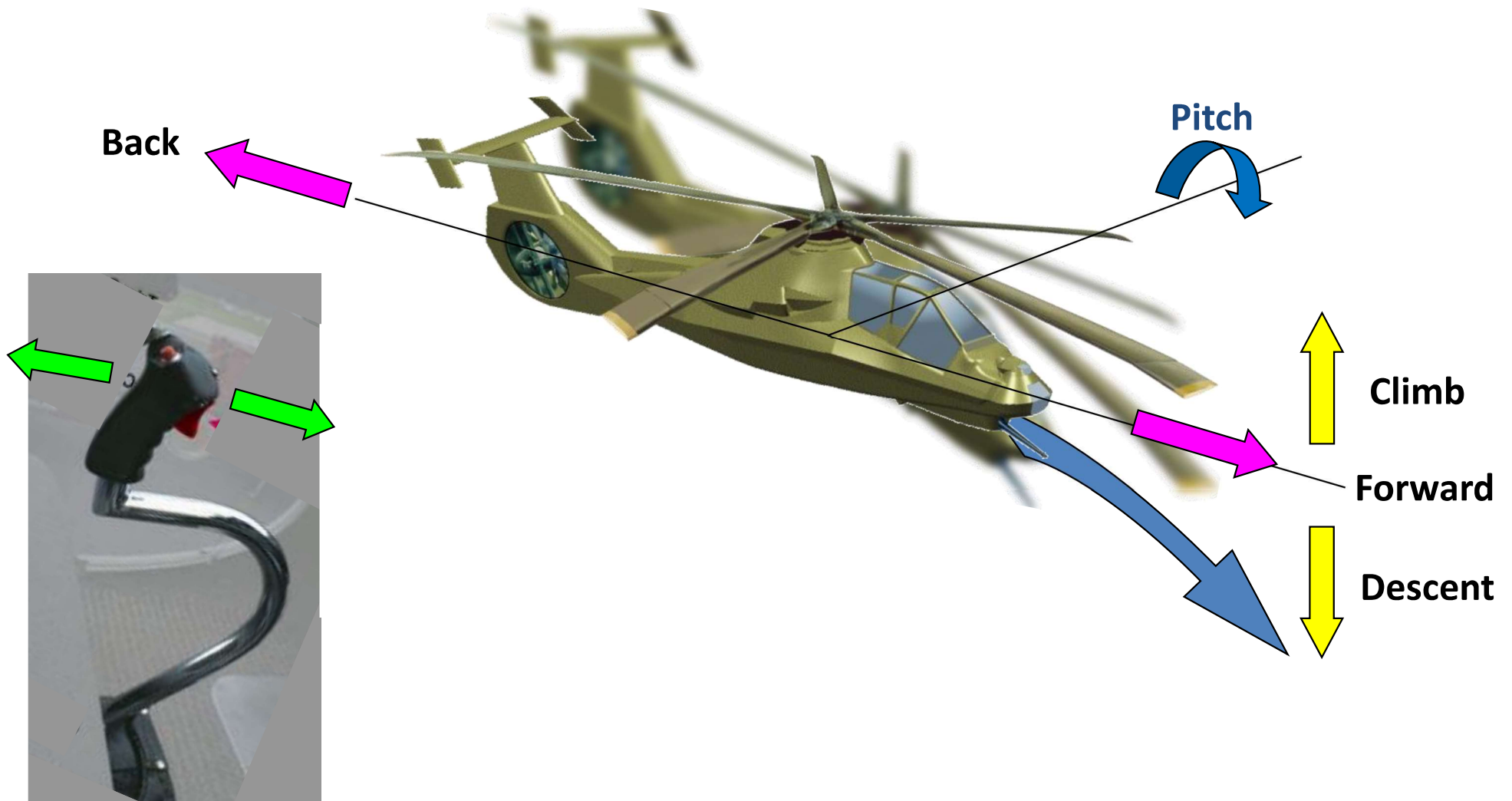
Pedals

Cyclic

Collective

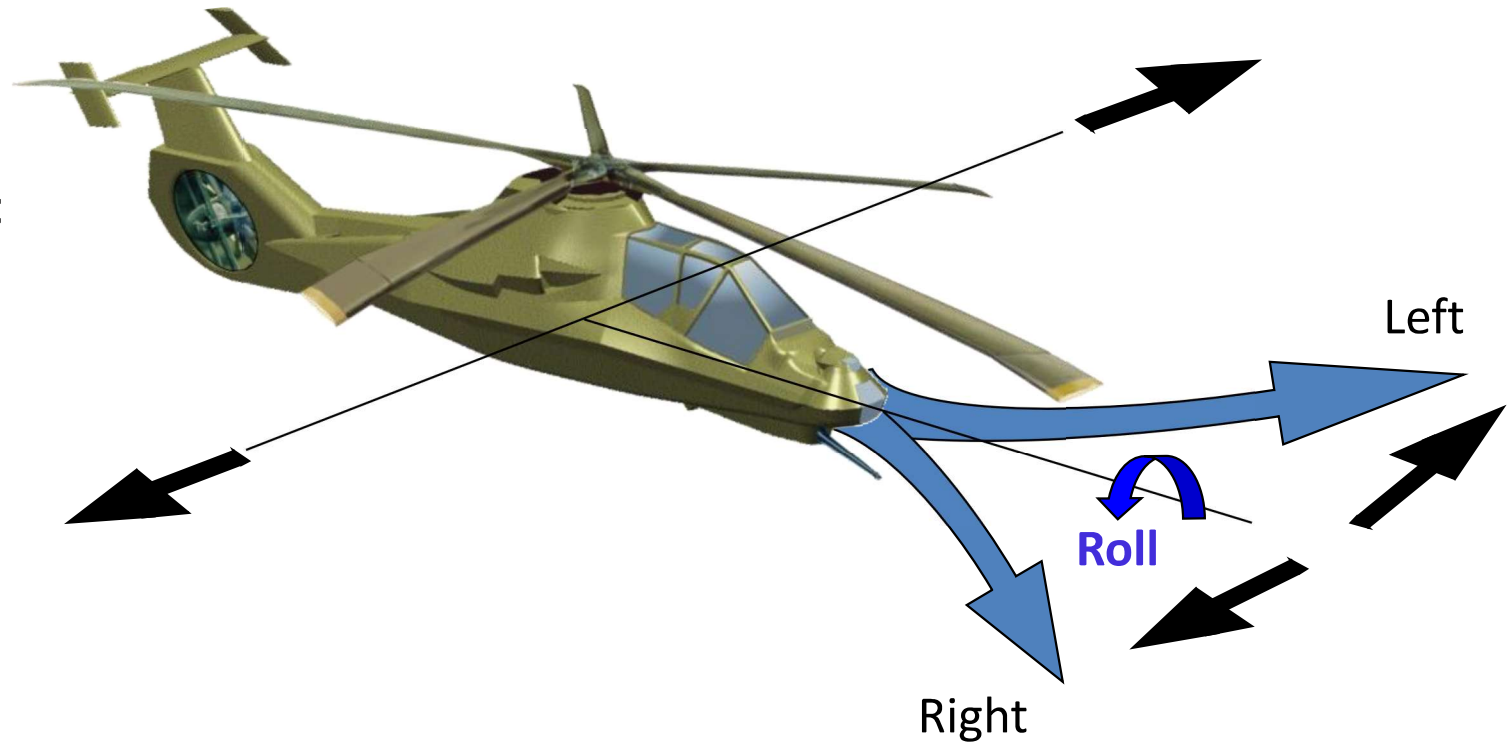
Fore/Aft Cyclic Control

Forward/aft cyclic controls pitch rotation, forward/back translation in hover, and climb/descent in forward flight.



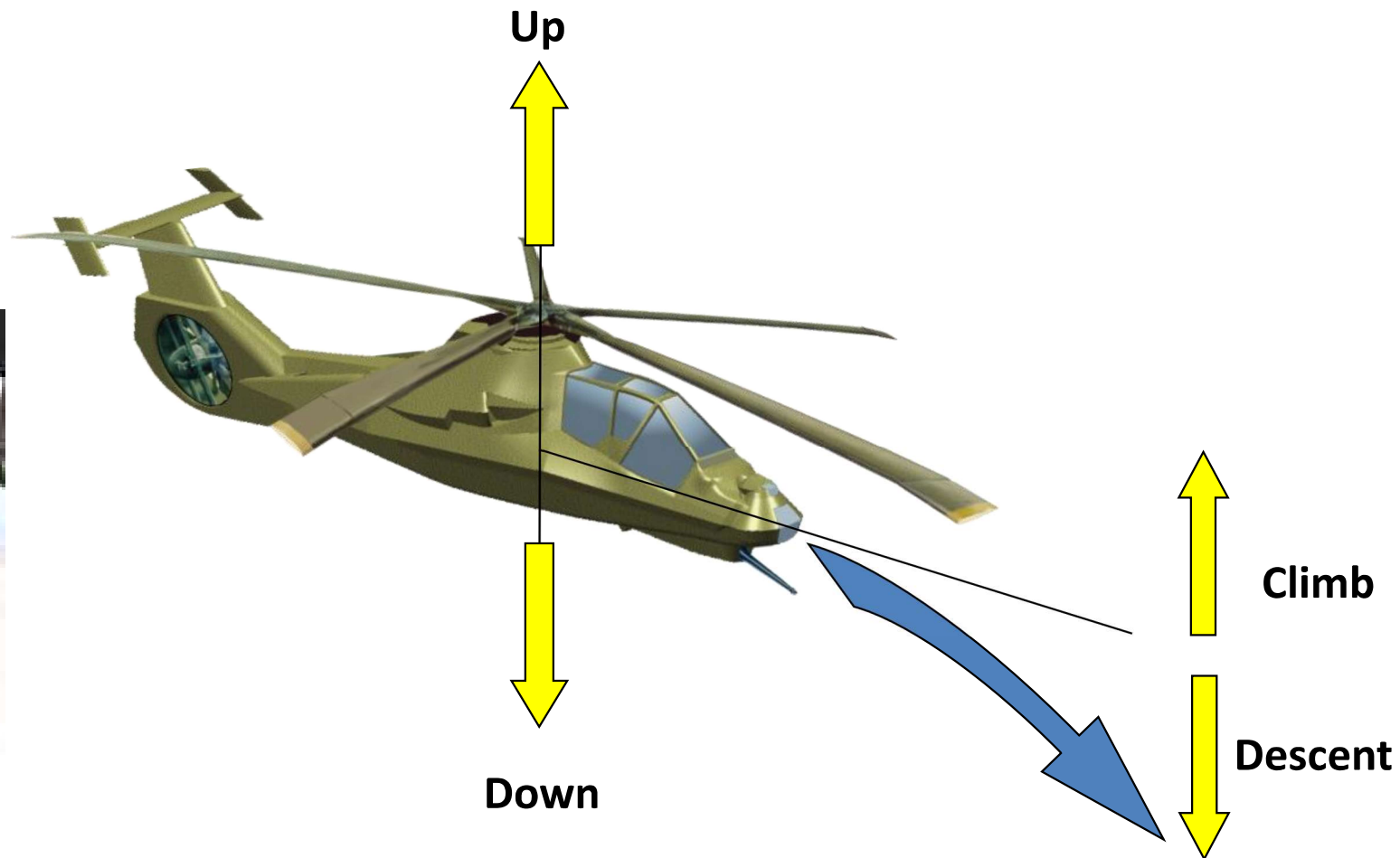
Left/Right Cyclic Control

Left/right cyclic controls roll banking, left/right translation in hover, and veer left/right in forward flight.



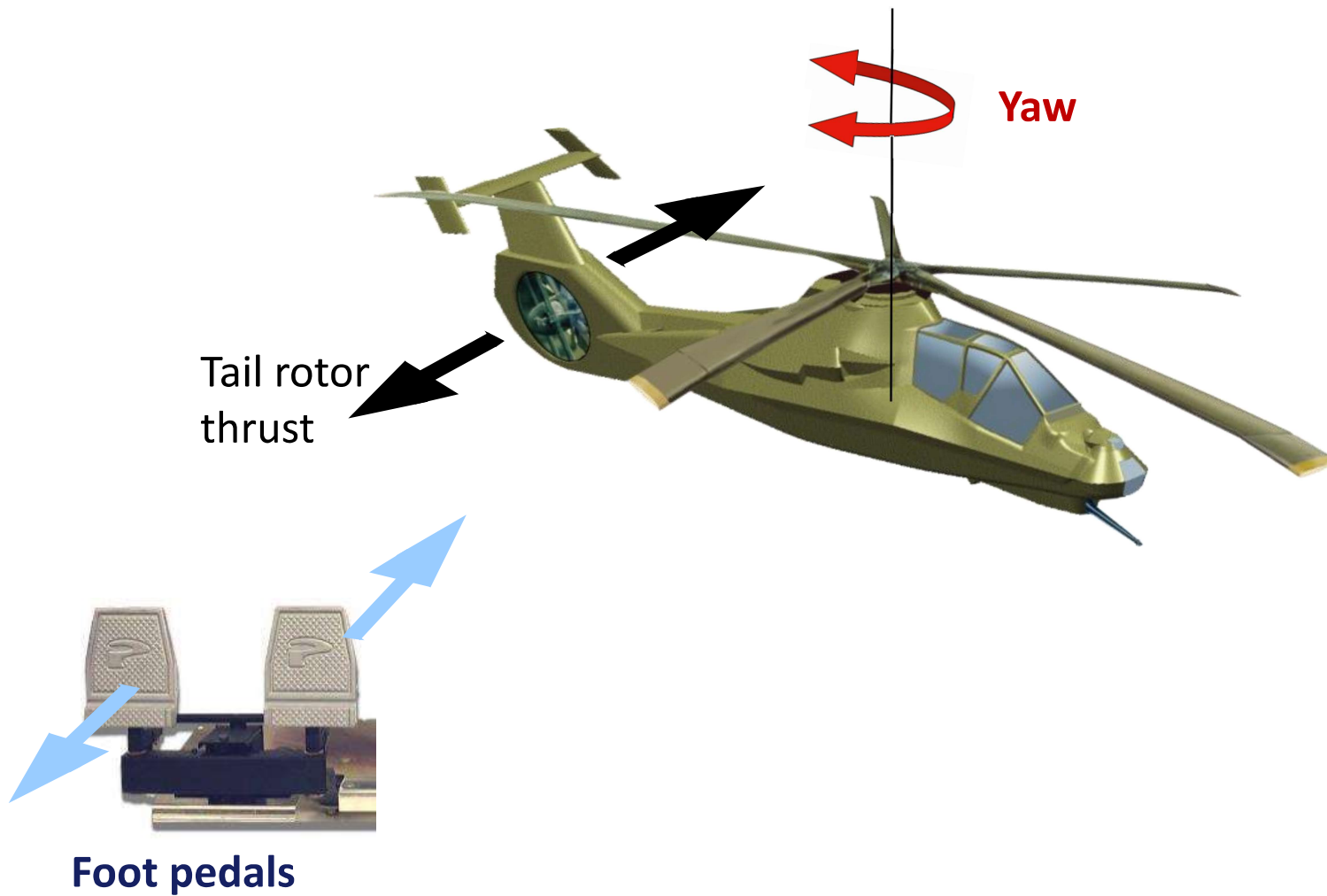
Collective Control

Collective/throttle stick controls vertical translation

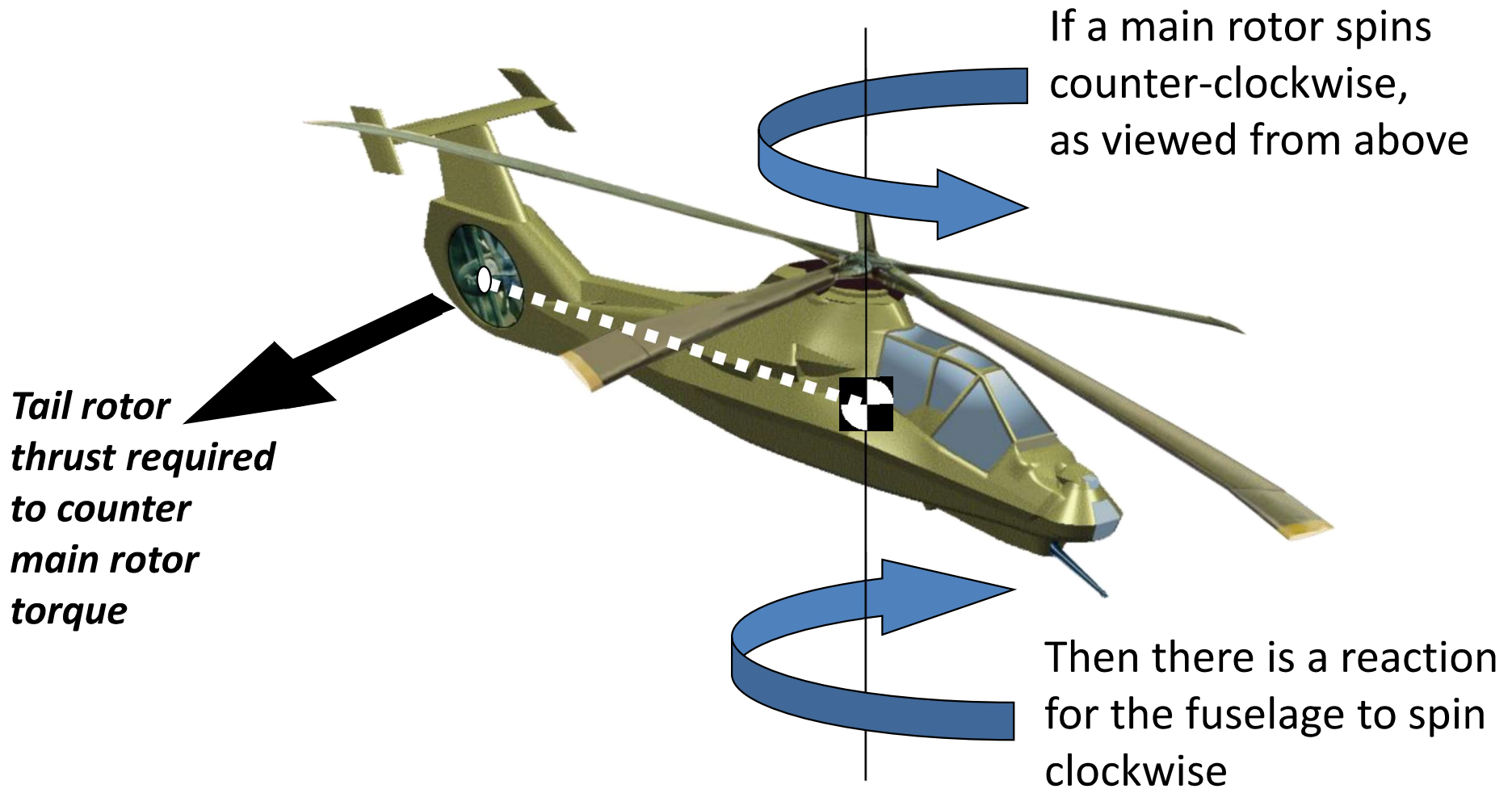


Yaw Control

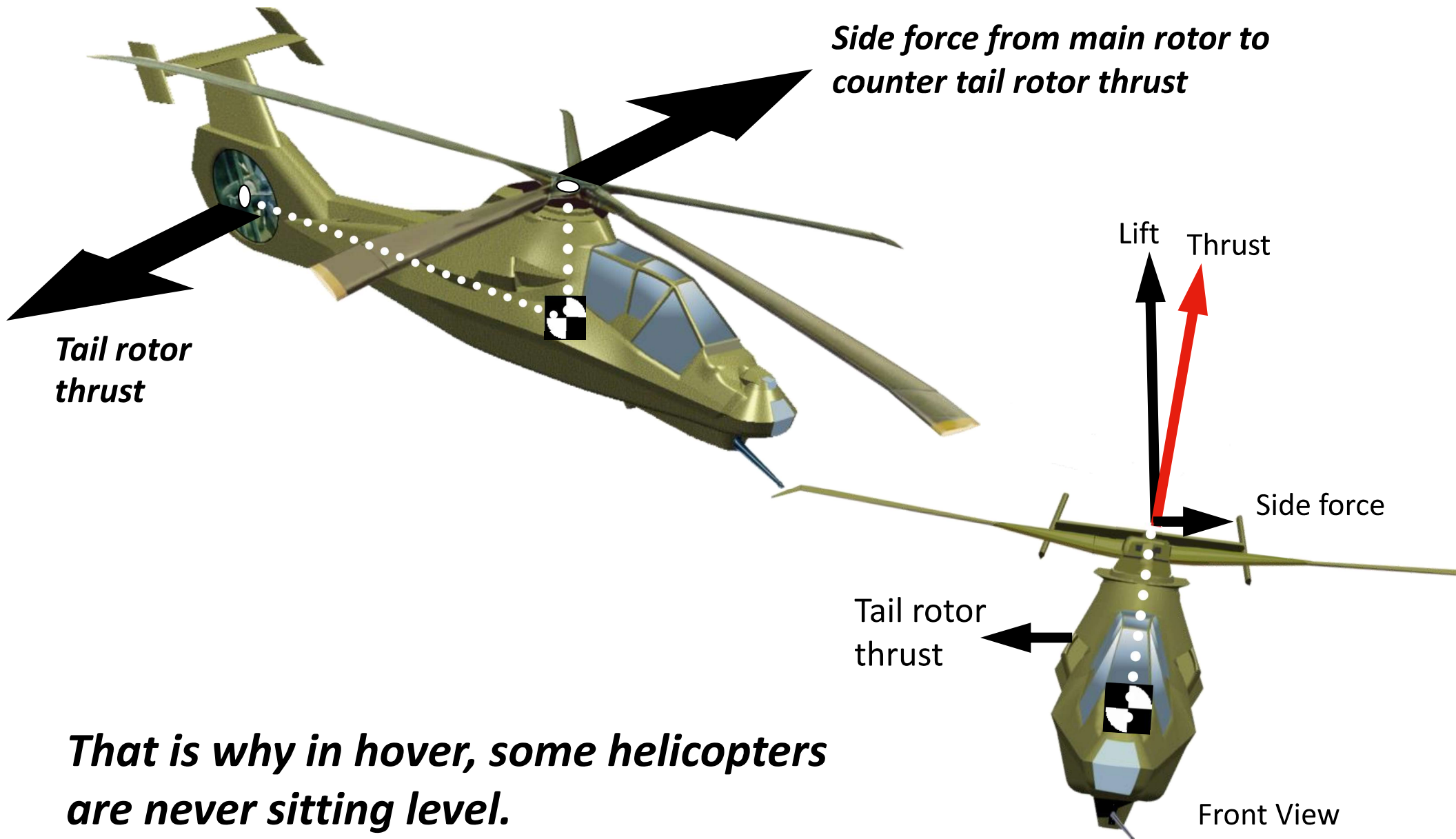
Tail rotor controls rotating left/right



Tail rotor is required to counter the main rotor torque

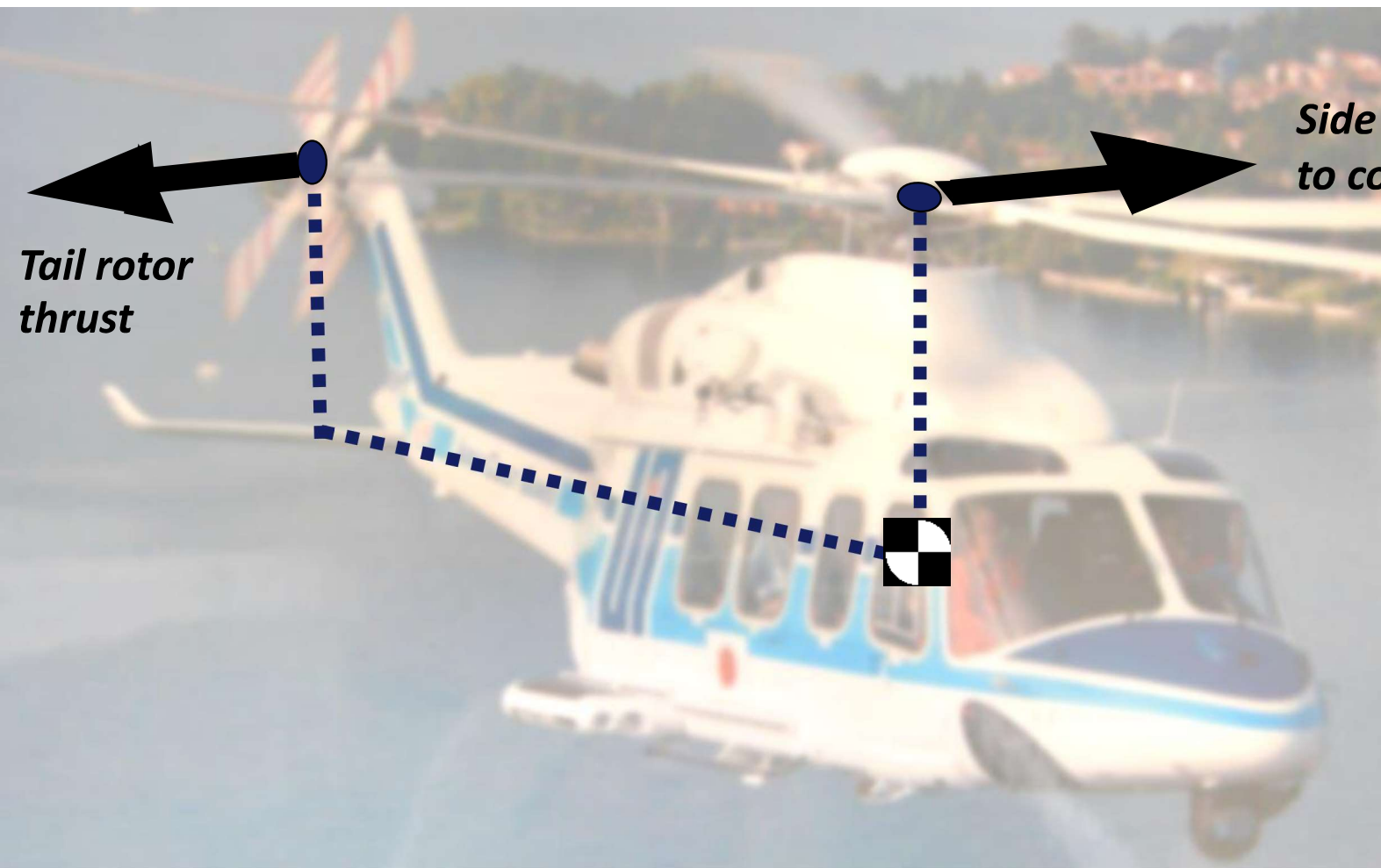


But tail rotor thrust also causes the helicopter to lean left in hover



That is why in hover, some helicopters are never sitting level.

Solution: raise tail rotor to same height as main rotor



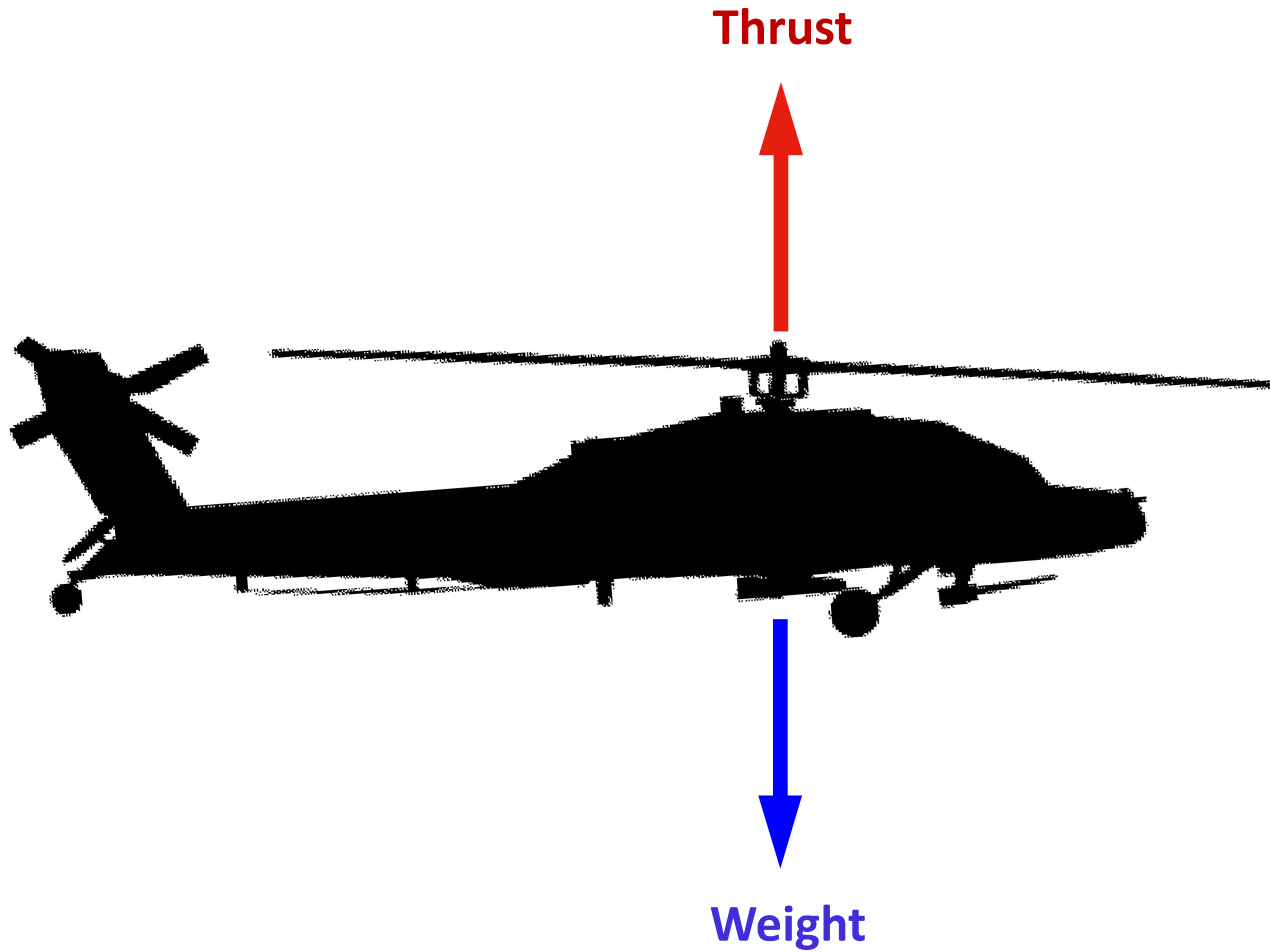
Side force from main rotor to counter Tail rotor thrust

Tail rotor thrust

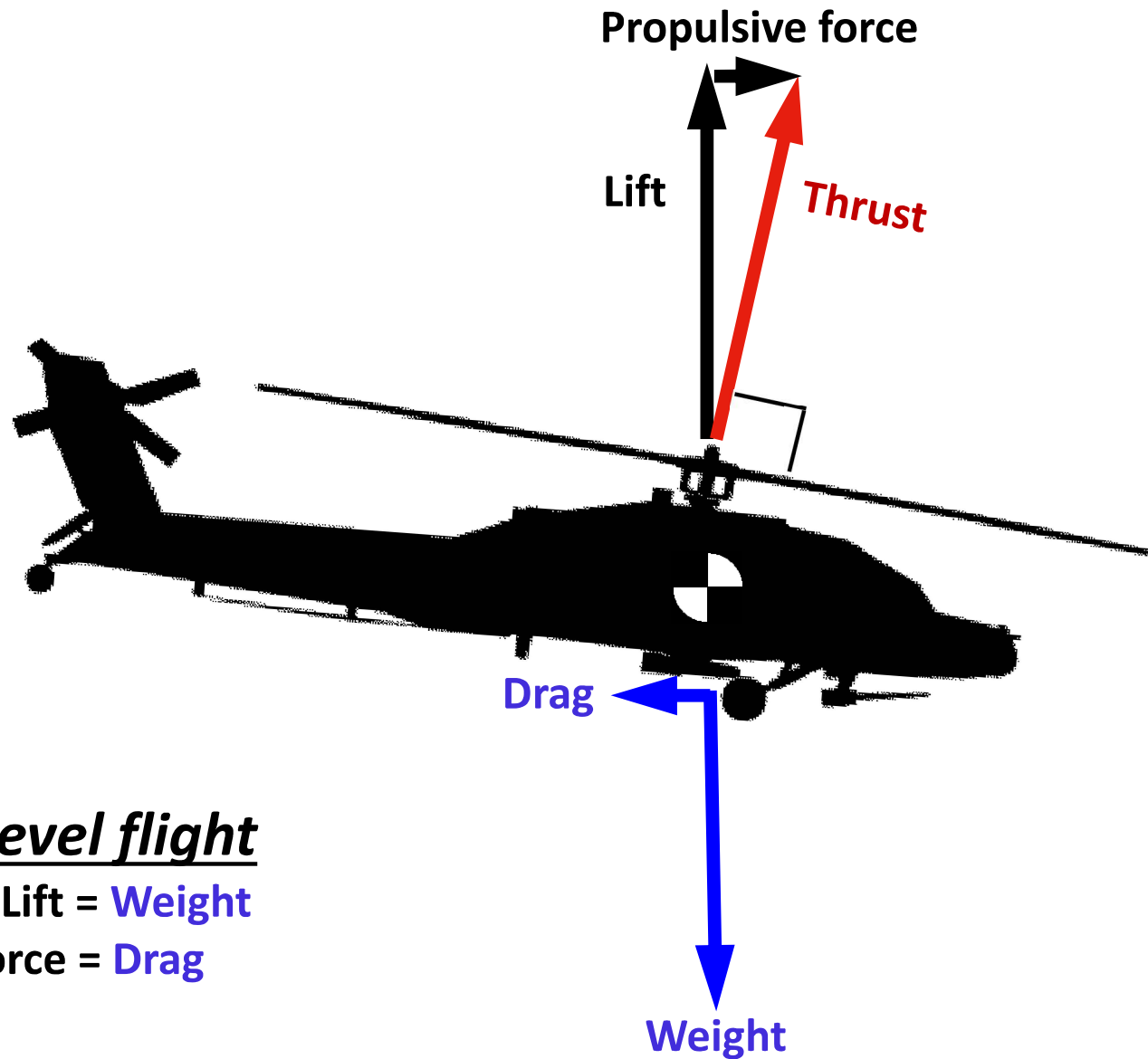
Now, forces and moments are balanced, helicopter hovers level.



Thrust Equals Weight in Hover



Rotor Forces in Steady Forward Flight



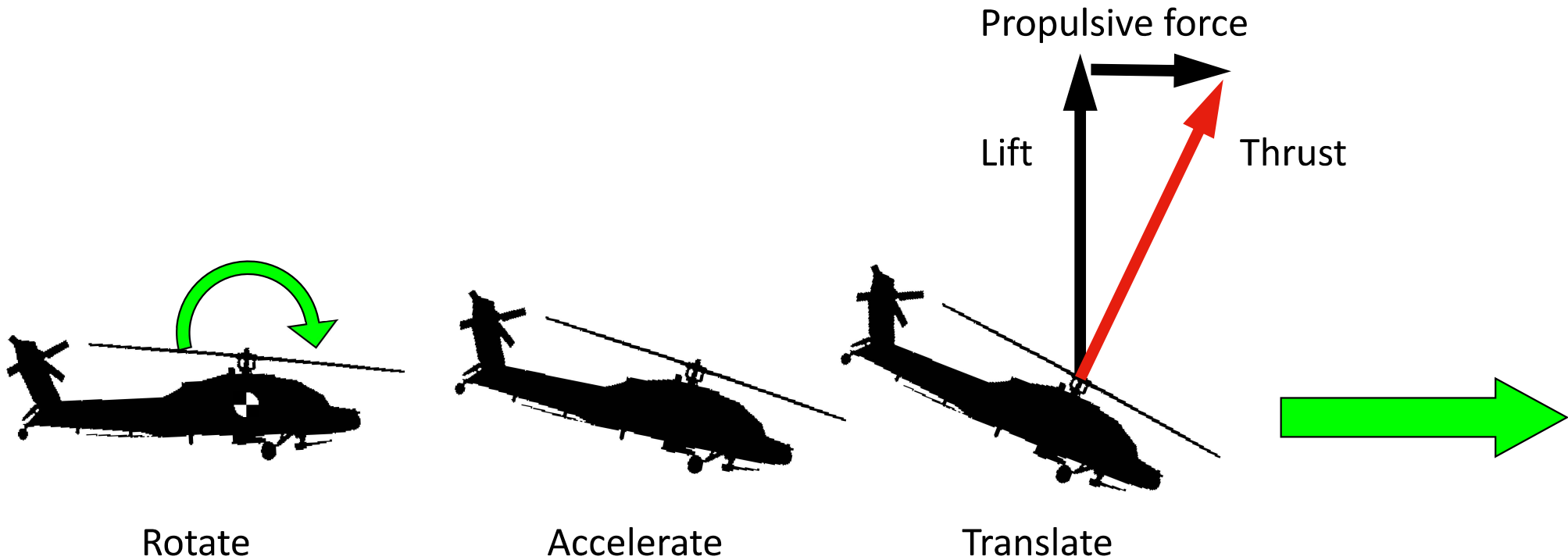
In steady level flight

Lift = Weight

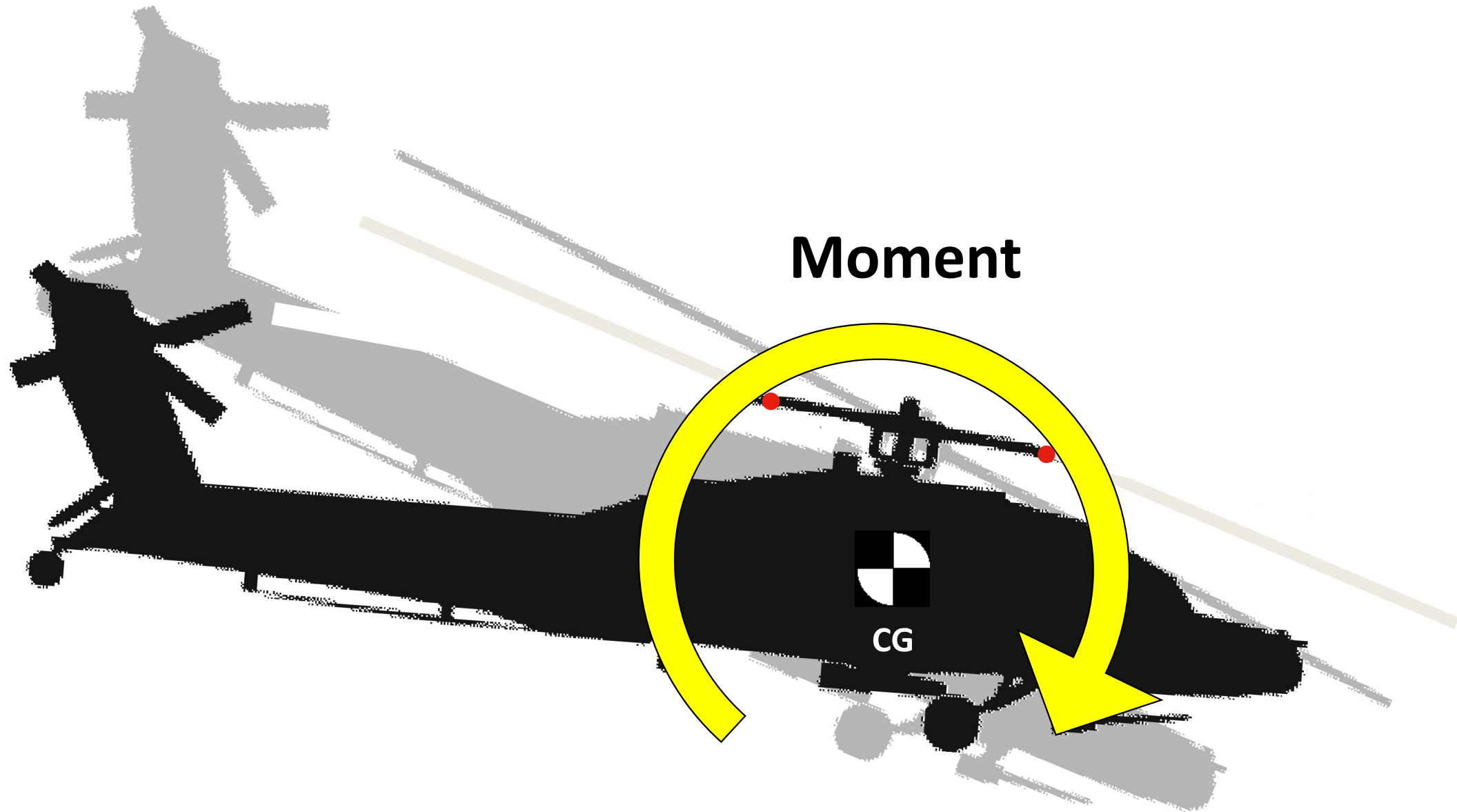
Propulsive force = Drag

How Does a Helicopter Go Into Forward Flight?

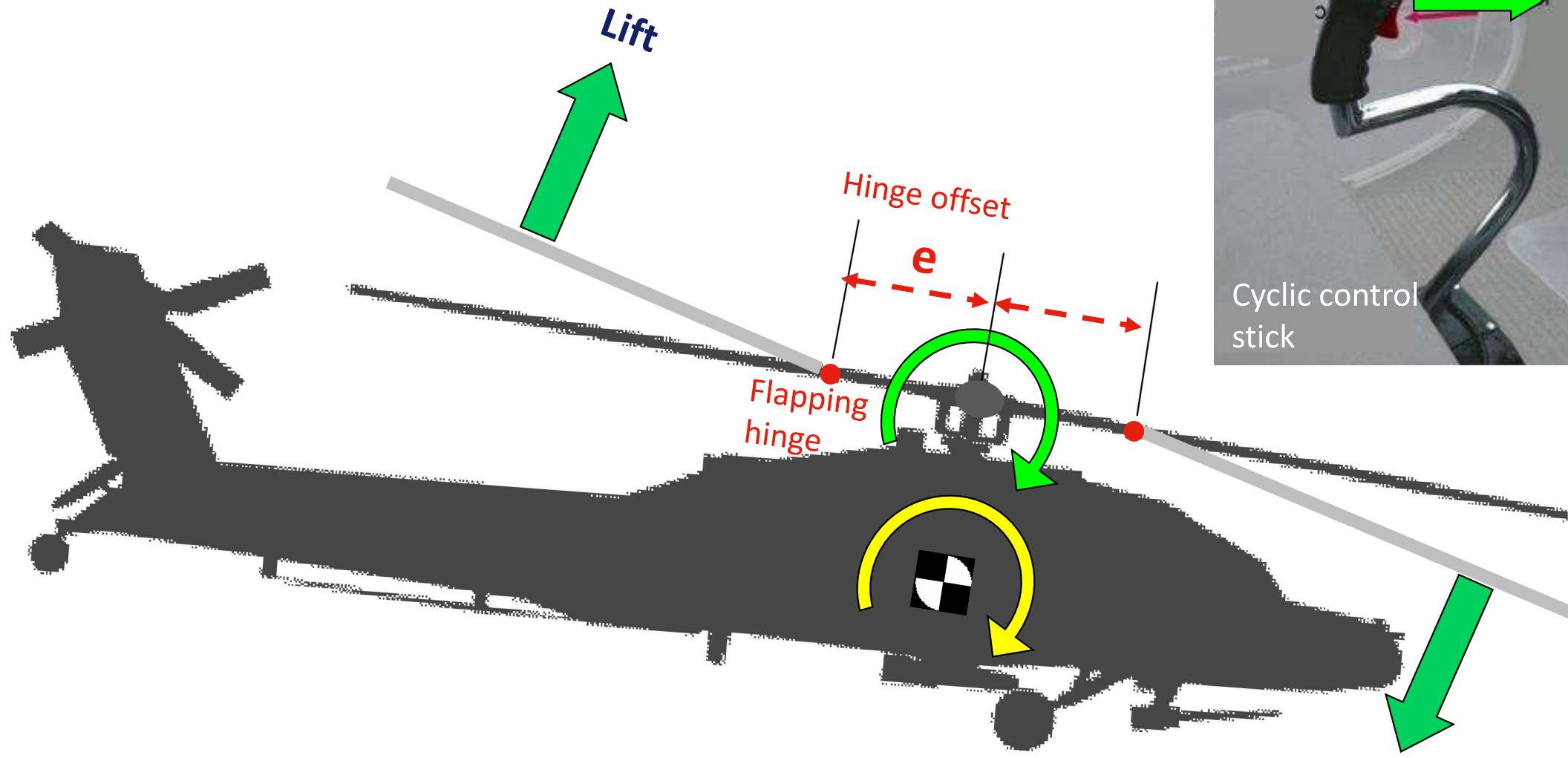
“Needs to rotate the fuselage first”



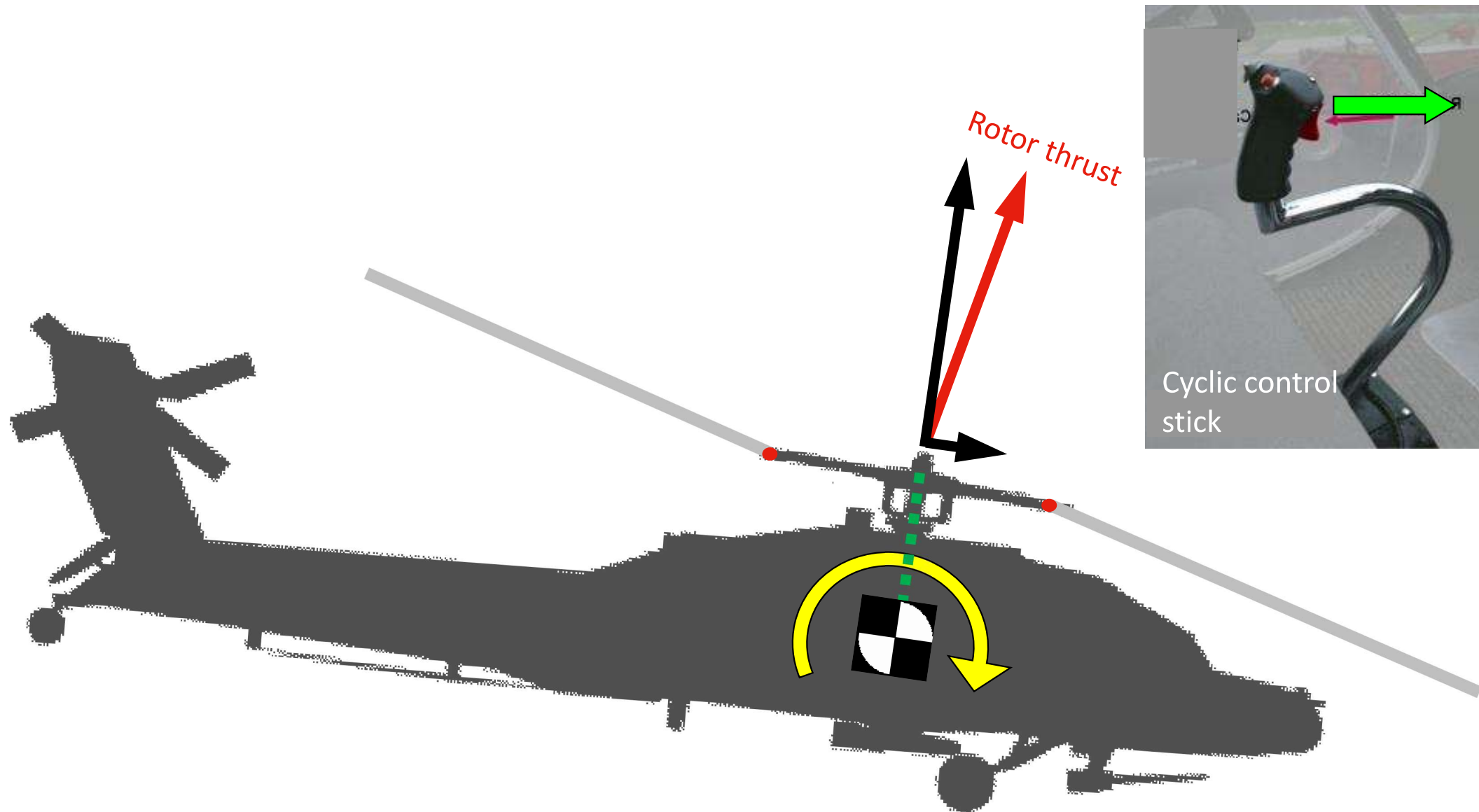
Two Ways to Produce a Moment at the Helicopter CG



1. Fuselage Moment due to a Rotor Moment at the rotor hub



2. Fuselage Moment due to a Thrust Tilt



Summary on Control Moment

- Weaker compared to moment from hinge offset
- Need at least 3 to 4% flap hinge offset “e” to have good control authority.
- For good agility and aerobatic performance, need 5 to 10% flap hinge offset.

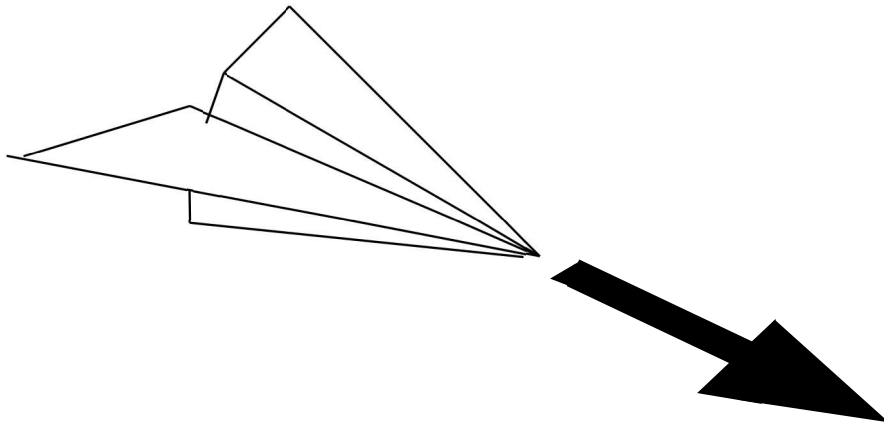
Why helicopters are difficult to fly

- (1) They are inherently unstable
- (2) They have multiple axis of instability
- (3) Each pilot control does multiple functions
- (4) Pilot has to anticipate and do 4 orders of math integration in his head

Helicopters are Difficult to Fly Because:

(1) They are inherently unstable

*Airplanes are
inherently stable*



*Helicopter are
inherently unstable*

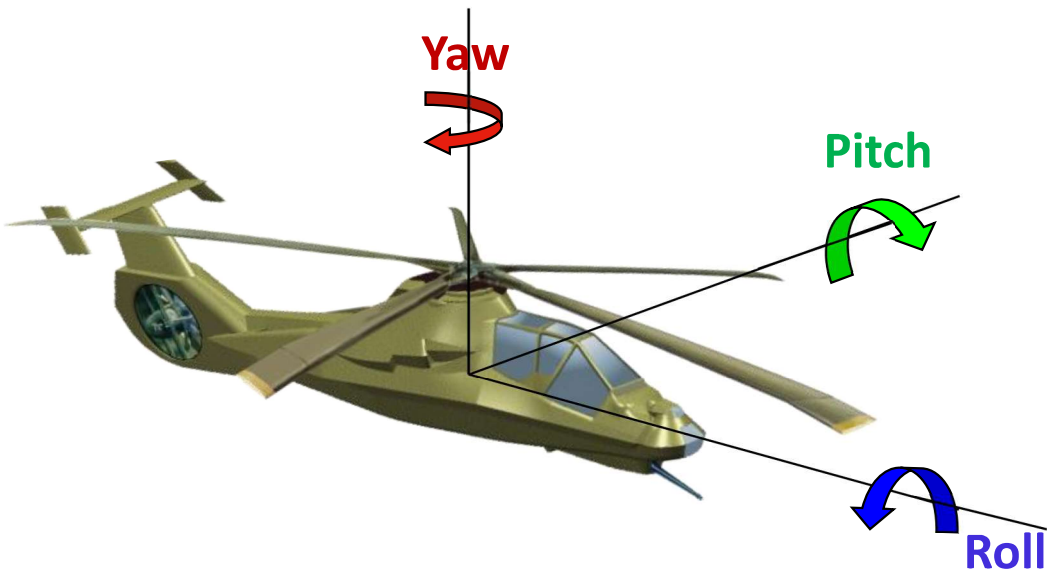


Helicopters are Difficult to Fly Because:

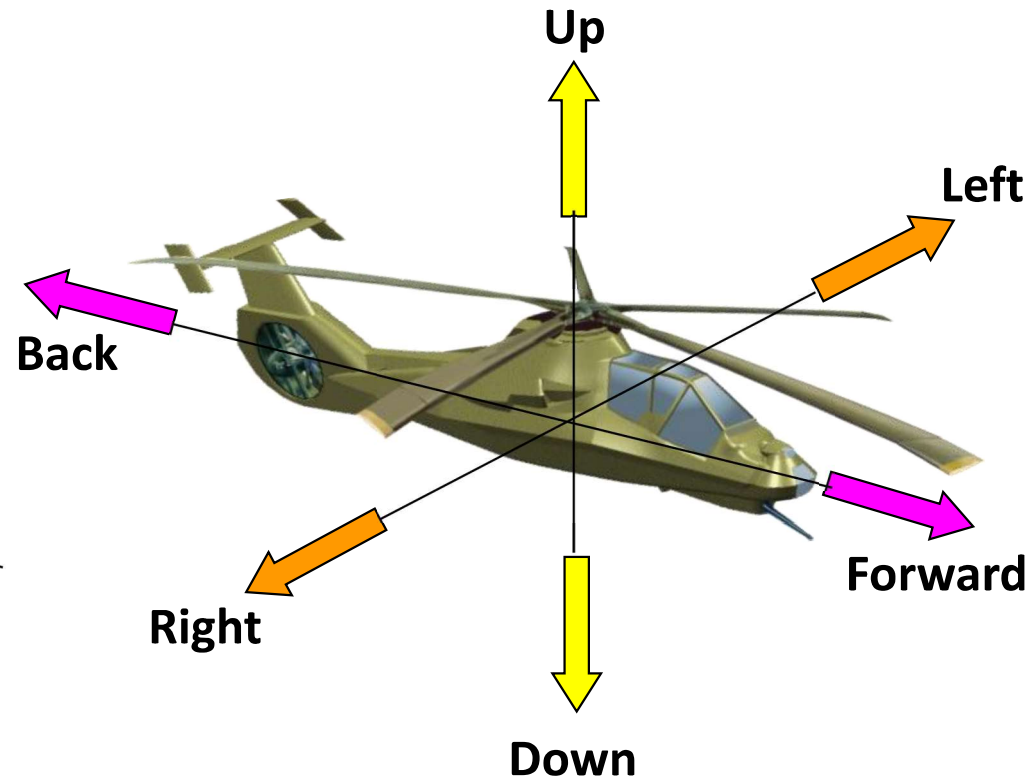
(2) They have many axis of instability

- 6 degrees of freedom (DOF)
- Human brain can easily control 2 or 3 DOF instabilities, but 6 is challenging.

3 Rotations



and 3 Translations



Helicopters have many axis of instabilities, but human brain can manage 2 or 3 DOF instabilities with practice

Balancing a broom stick is a good example



Corollary: the Smaller the More Difficult to Control

Smaller objects have a shorter *"Time Constant"*

Time constant is a measure of how fast a system reacts

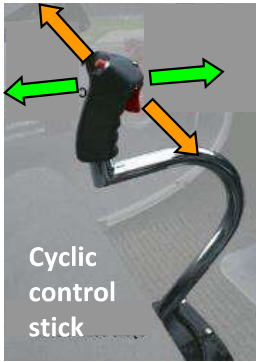


Helicopters are Difficult to Fly Because:

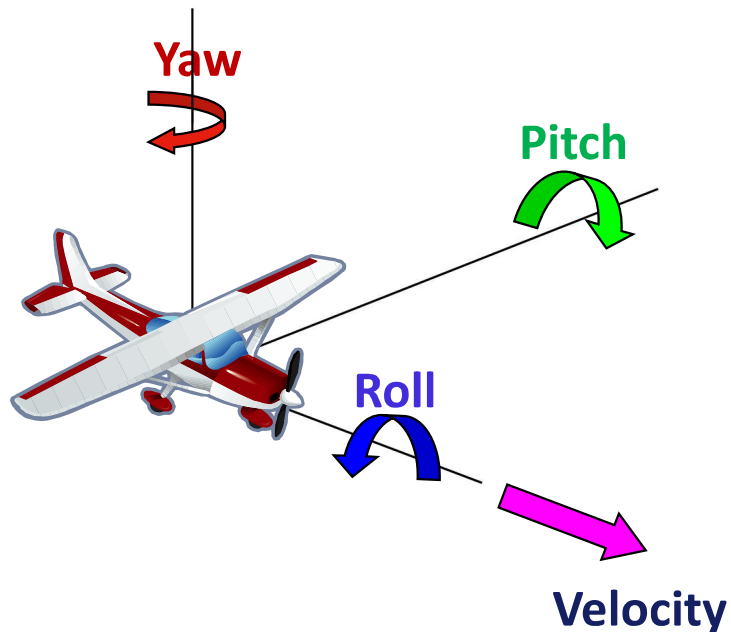
(3) Each control does multiple functions

Unlike airplane, in helicopters there are many cross-coupling

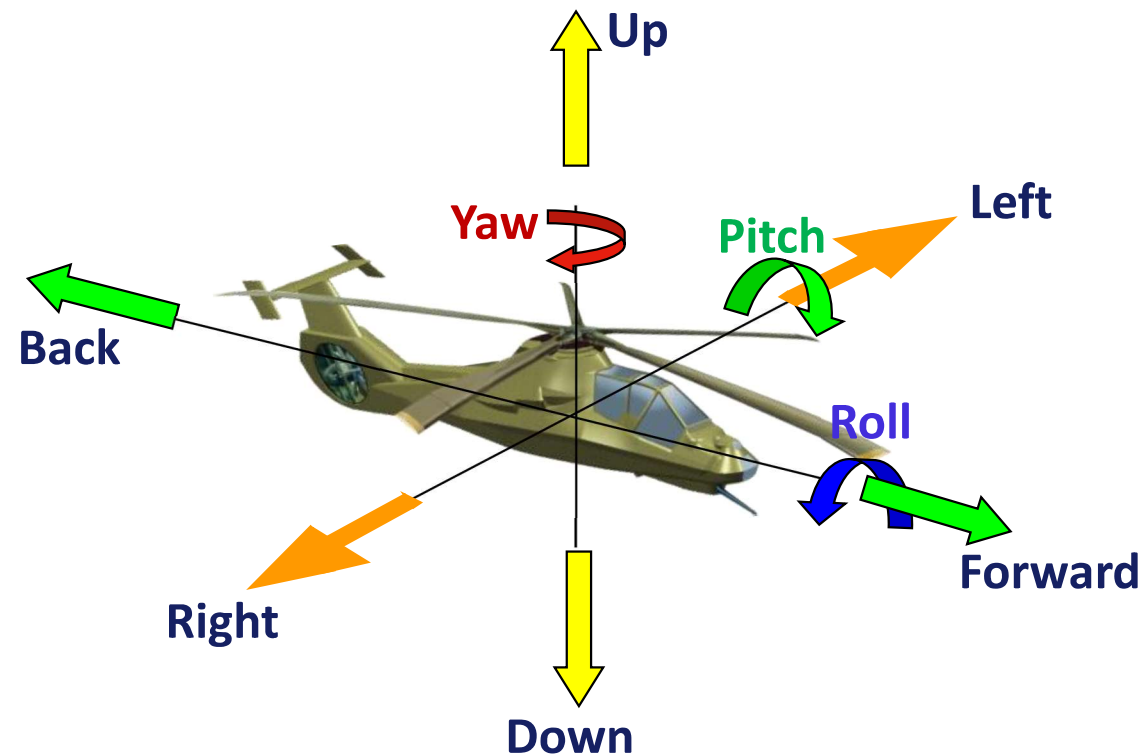
- **Forward/aft stick** controls pitch rotation, forward/back translation, climb/descent
- **Left/right cyclic** controls roll rotation and left/right translation
- **Rudder pedals** control left/right rotation



4 flight controls control 4 DOF

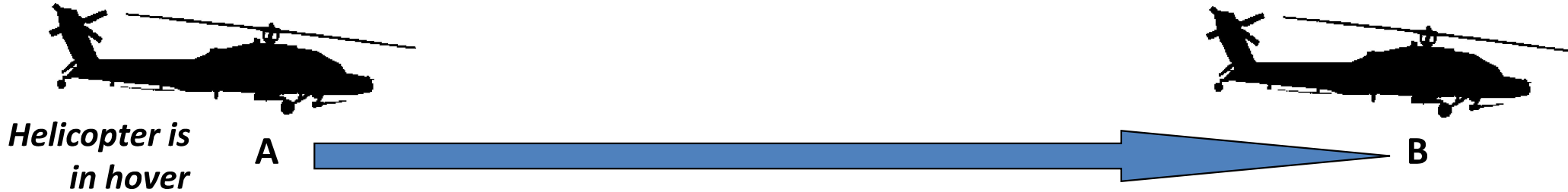


4 flight controls control 6 DOF



Helicopters are Difficult to Fly Because:

(4) Pilot Has to Anticipate Reactions in His Head



To move from point A to point B requires many unaware thought processes inside the pilot's brain



A Pilot Has to Do 4 Math Integrations in His Brain

1. Pilot input



$$x = \int_0^t \iiint f(\ddot{\theta}) dt$$



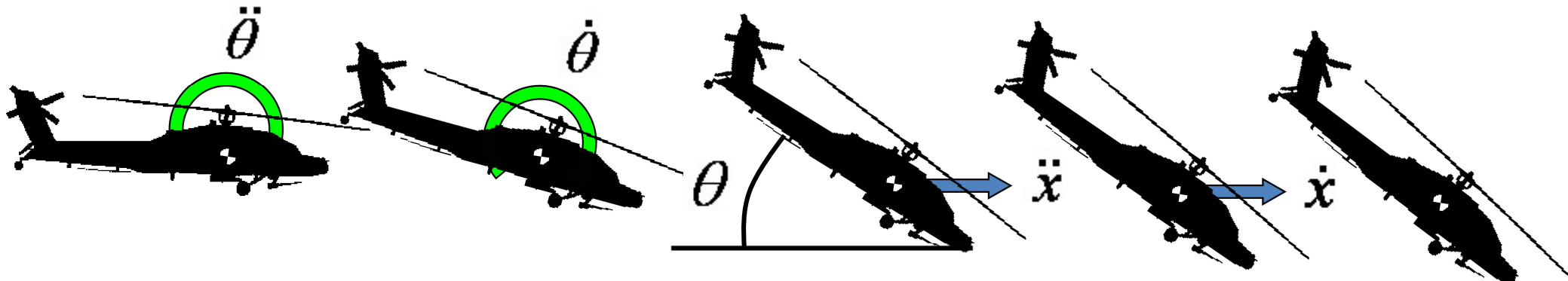
2. Fuselage Moment

3

4

5

6



A

To travel a distance x

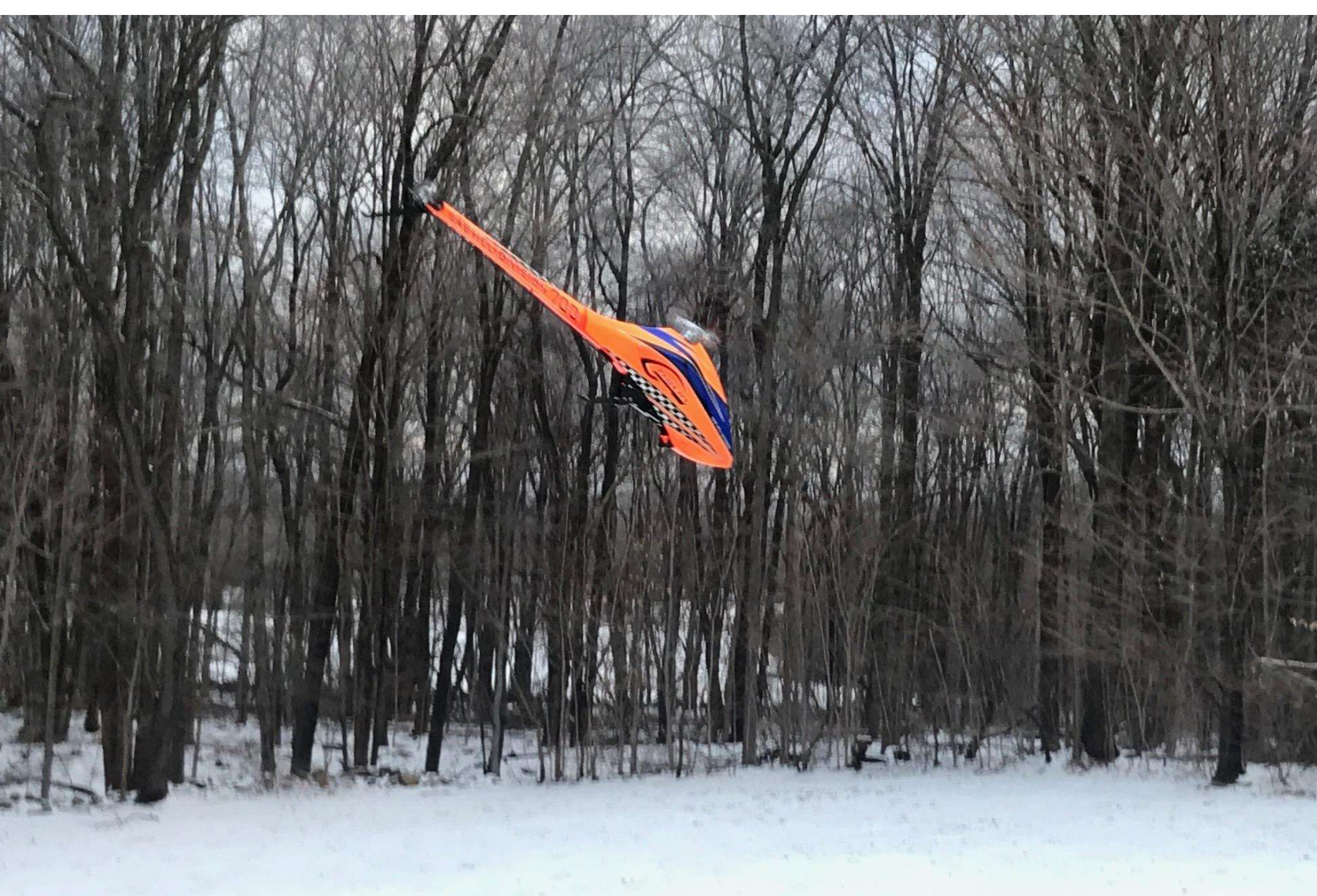
B



Starting point: steady hover



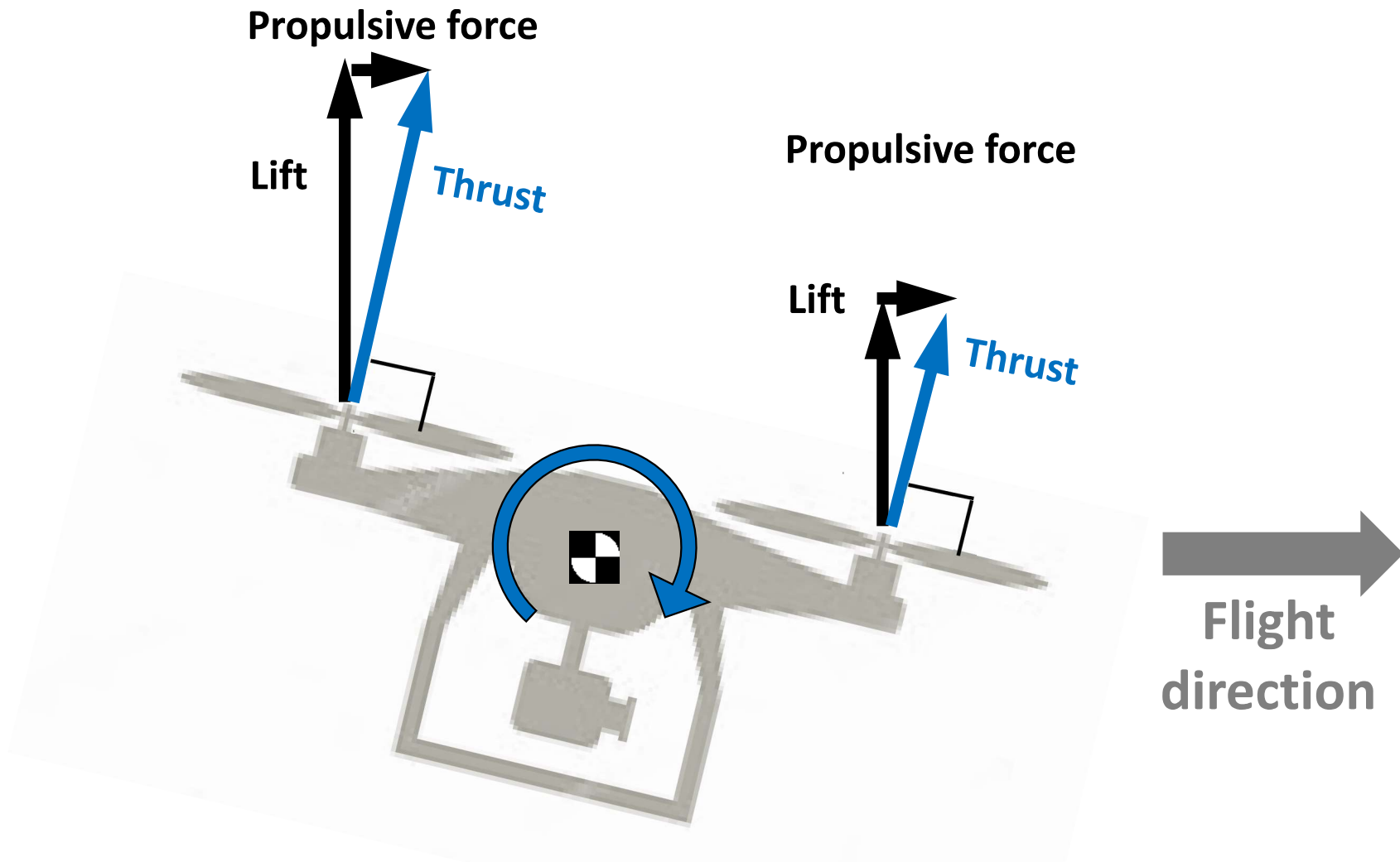
Rotated about the vehicle CG



Accelerating away in the x direction

Control Methodology and Trade Study for Other VTOL Configurations

**Quadcopters also have to rotate first then translate.
The dynamics are similar to helicopters.**



Advantage: no mechanical controls reduces design complexity

Disadvantage: tilting the vehicle in forward flight can be uncomfortable for passenger

Volocopter X2



Electric power facilitates designing vehicles with many distributed rotors and smaller motors to reduce cost



Another advantage: regardless of how many rotors, it still controls like a simple quadcopter

Basic Lift + Cruise Design



Advantages:

- (1) Eliminate heavy mechanical tilting mechanism
- (2) Simplify control laws design by separating hover and forward flight
- (3) Fuselage remains horizontal during transition and forward flight.

Disadvantages: Carrying two sets of rotors, more drag in forward flight.

Increasing the Complexity Factor by Retracting the Lift Rotors to Reduce Drag



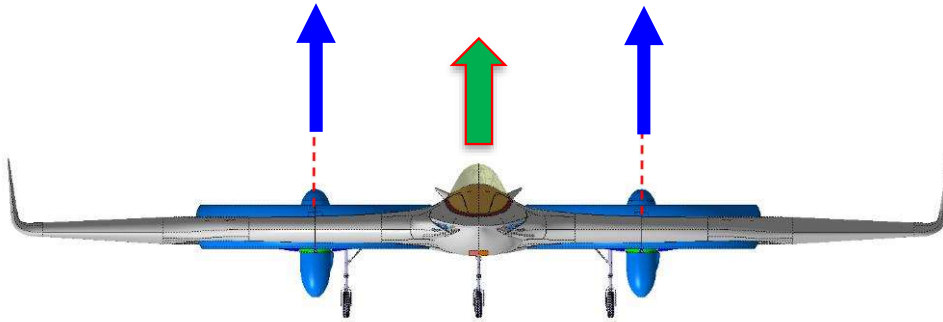
**During the Conceptual Design Phase,
It is important to analyze complexity
versus cost, schedule, and reliability**

Tiltrotors Increases Control Laws Complexity During Transition Flight Regime. But Minimizes Drag in Forward Flight.

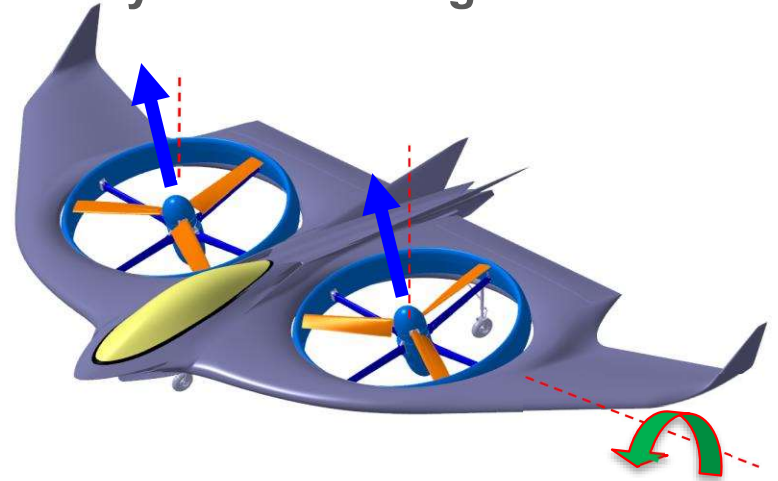


Example: Project Zero's Controls in Hover

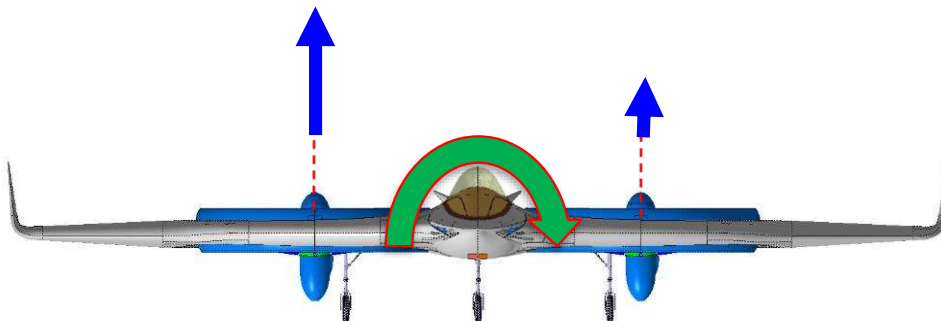
Heave: synchronous collective pitch



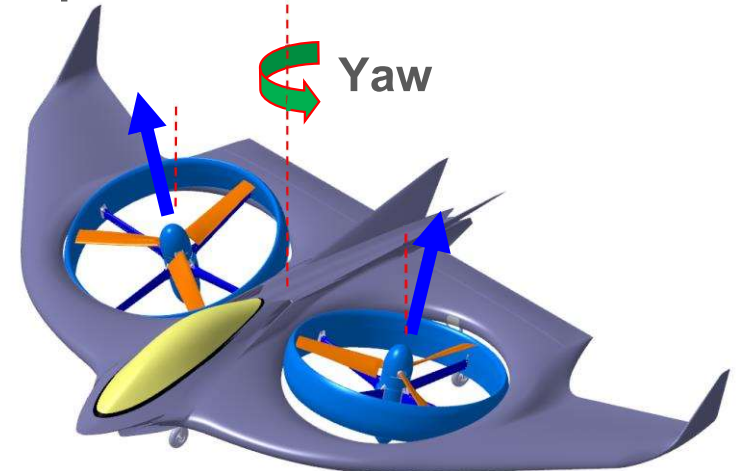
Pitch: longitudinal cyclic coupled with rotor symmetric tilting



Roll: differential collective pitch

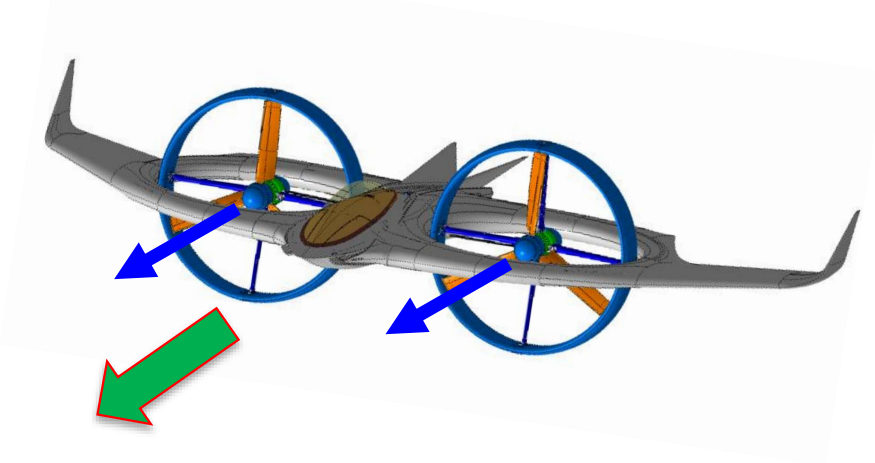


Yaw: differential longitudinal cyclic coupled with rotor differential tilting

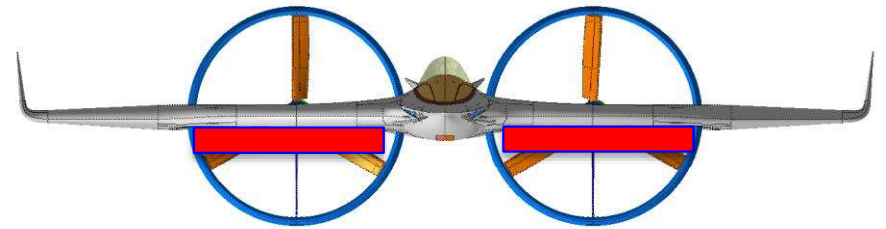


Project Zero's Controls in Forward Flight

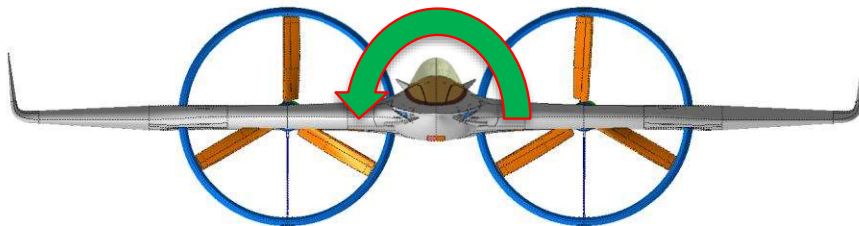
Thrust: synchronous collective pitch



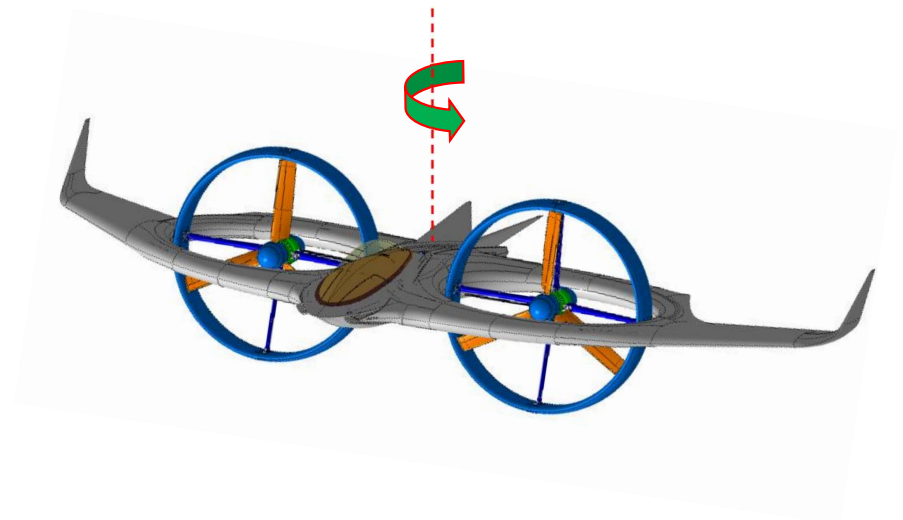
Pitch: synchronous elevon deflection



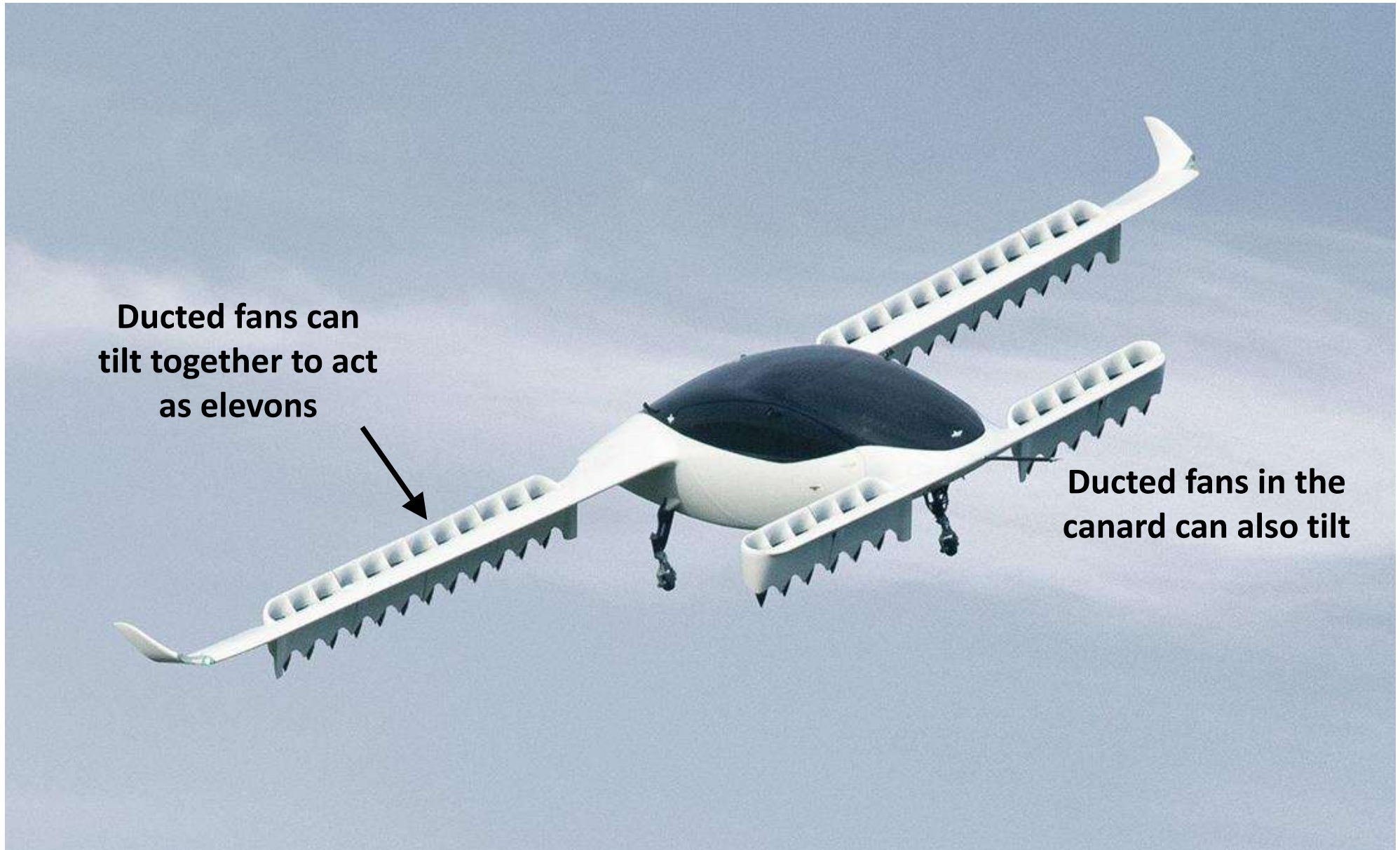
Roll: differential elevon deflection



Yaw: differential collective pitch



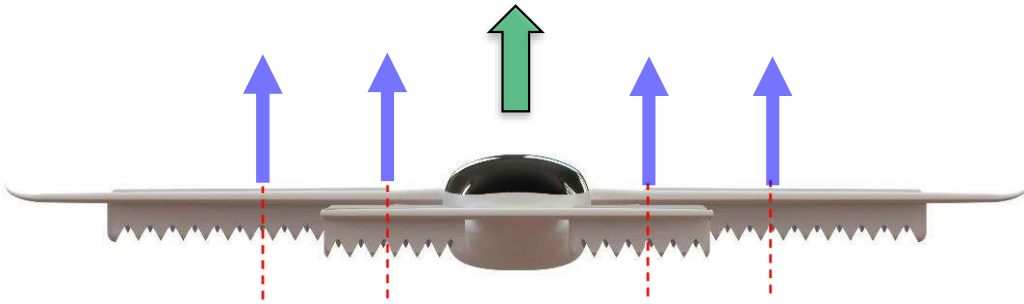
Lilium Jet's Control Surfaces



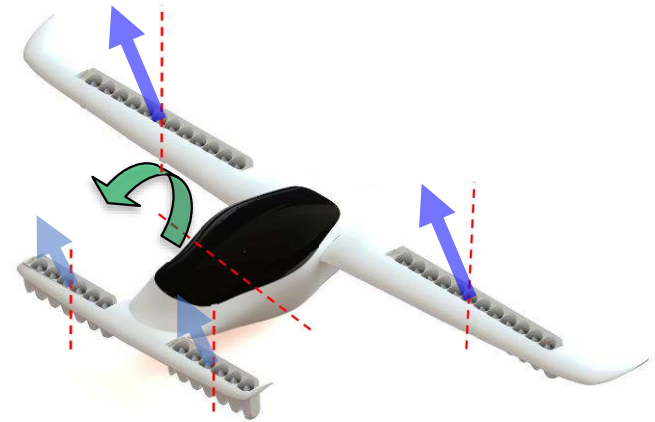
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Lilium Jet's Controls in Hover

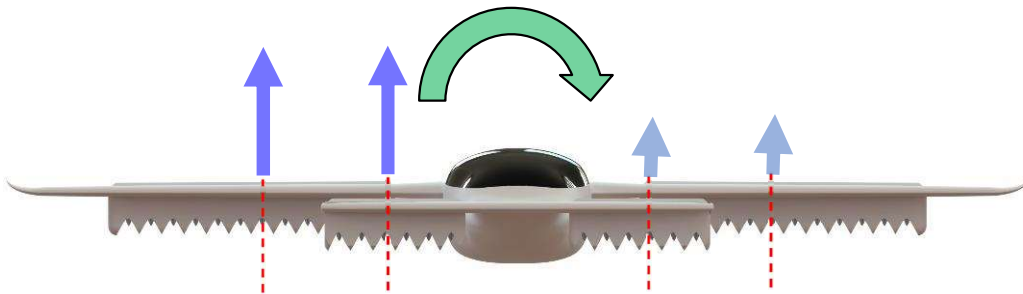
Heave: synchronous thrust change by rpm



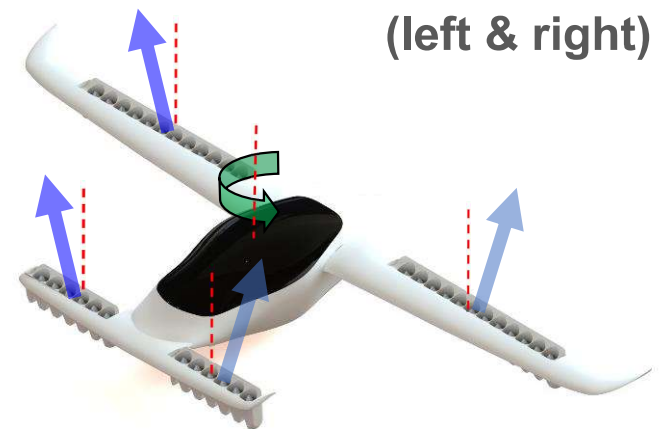
Pitch: differential thrust (fore & aft) & thrust vectoring



Roll: differential thrust by rpm (left & right)



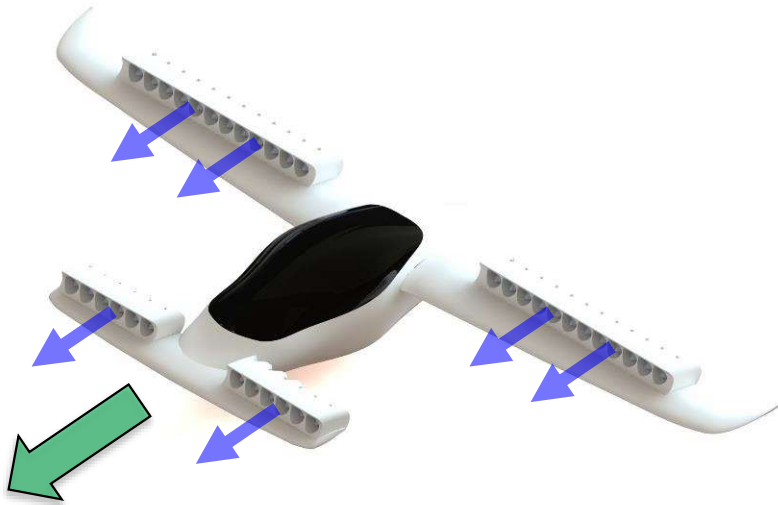
Yaw: differential thrust vectoring (left & right)



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Lilium Jet's Controls in Forward Flight

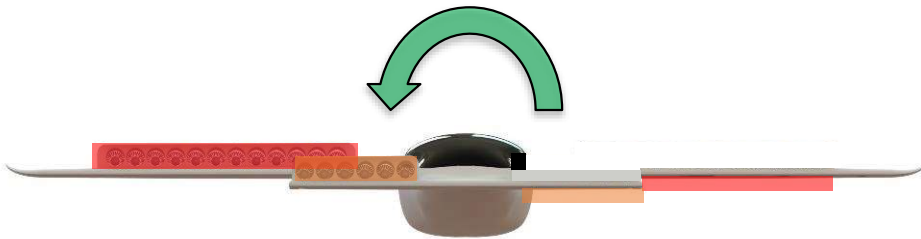
Thrust: thrust change by varying rpm



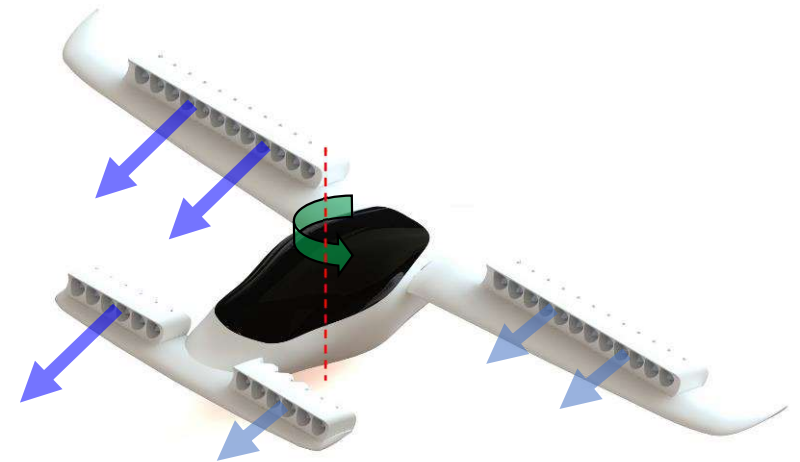
Pitch: ducted canards & wing fans acting as elevator



Roll: differential ducted canards & wing fans acting as aileron



Yaw: differential thrust (left & right)

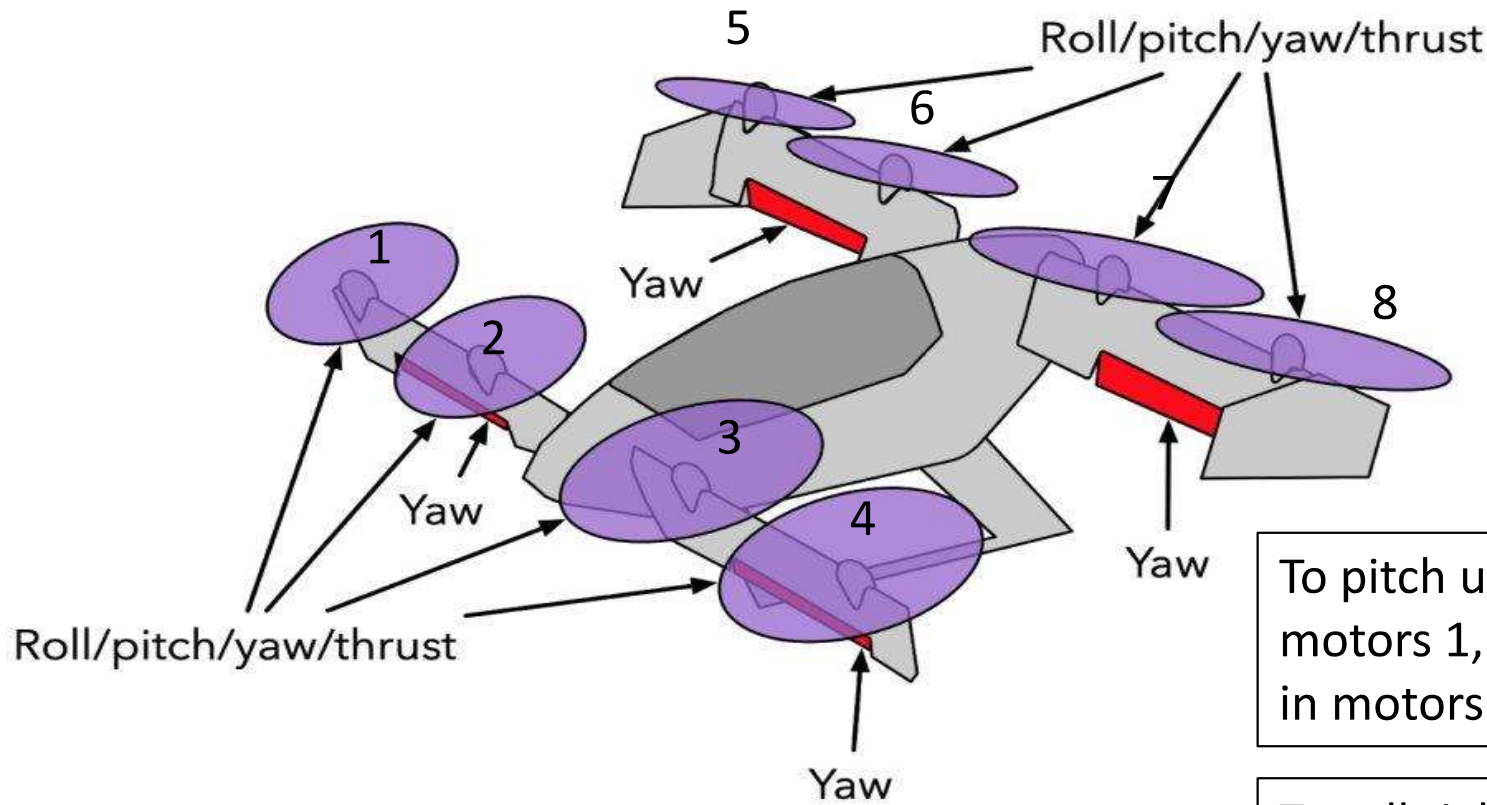


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Airbus Vahana (Tiltwing)



Vahana Controls in Hover

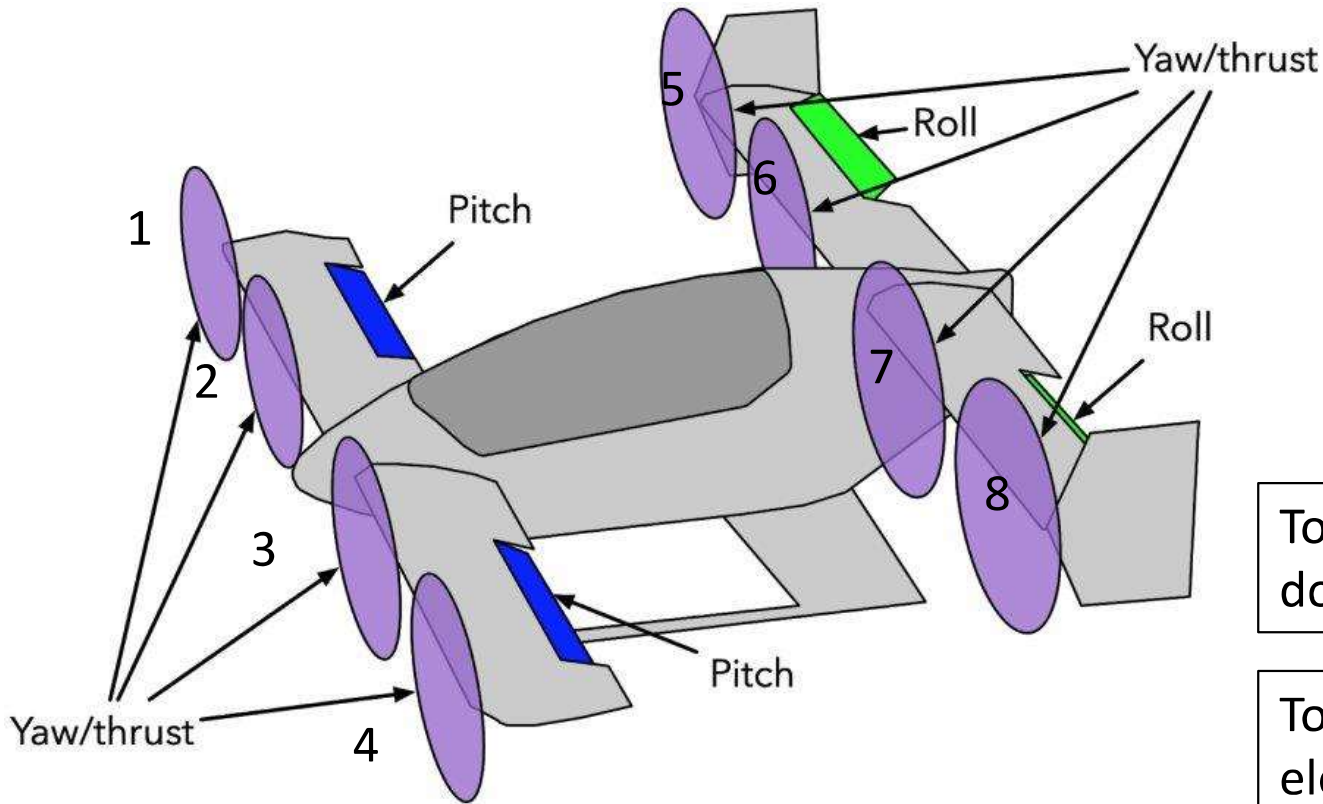


To pitch up, increase thrust in motors 1,2,3,4 and decrease thrust in motors 5,6,7,8

To roll right, increase thrust in motors 3,4,7,8 and decrease thrust in motors 1,2,5,6

To yaw right, increase thrust in motors 3,4,5,6 and decrease thrust in 1,2,7,8 (+red elevons to increase effect)

Vahana Controls in Forward Flight



To pitch up, deflect blue elevons down (both same direction)

To roll right, deflect left green elevon down + rotate right green elevon up

To yaw right, increase thrust in motors 3,4,7,8 and decrease thrust in motors 1,2,5,6

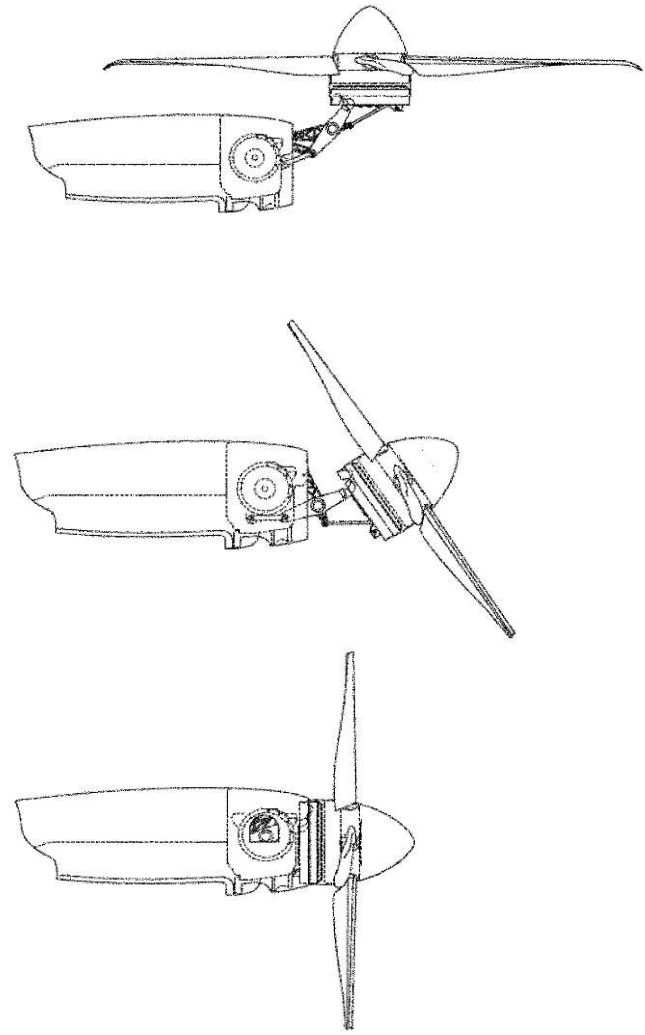
Joby S4

Ruddervators

Flaps Ailerons

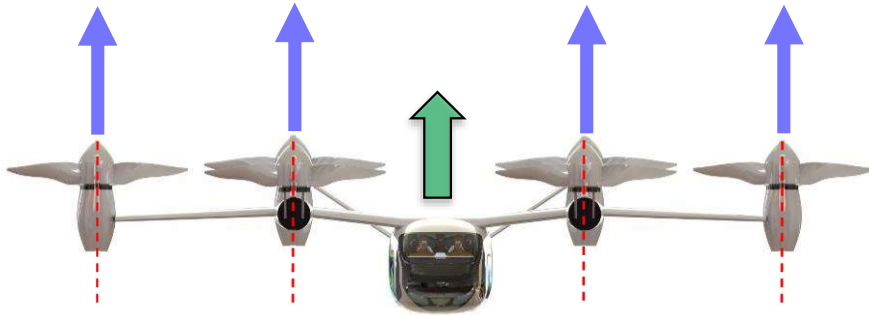
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Joby S4's Rotor Tilting Mechanism

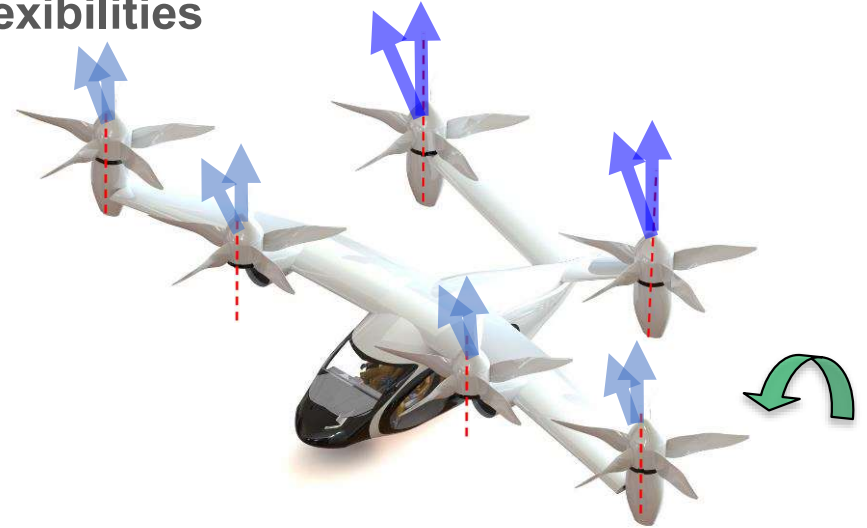


Joby S4's Controls in Hover

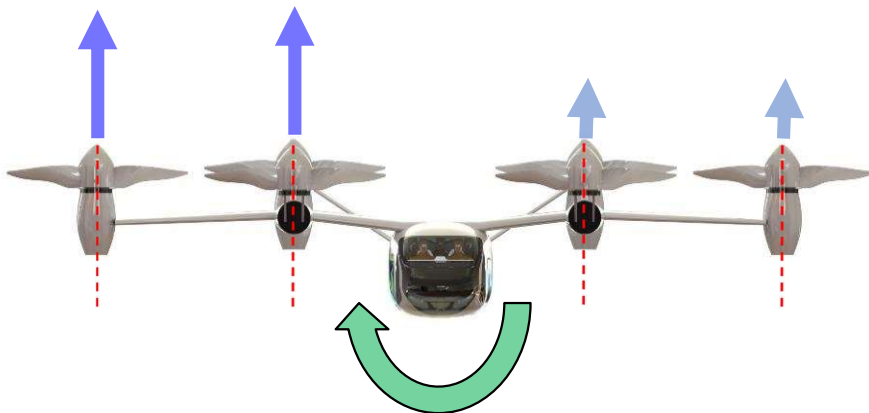
Heave: synchronous rotor thrust control by collective pitch



Pitch: differential thrust (fore & aft) & can tilt the fore and aft pylons. Many flexibilities



Roll: differential thrust (left & right)



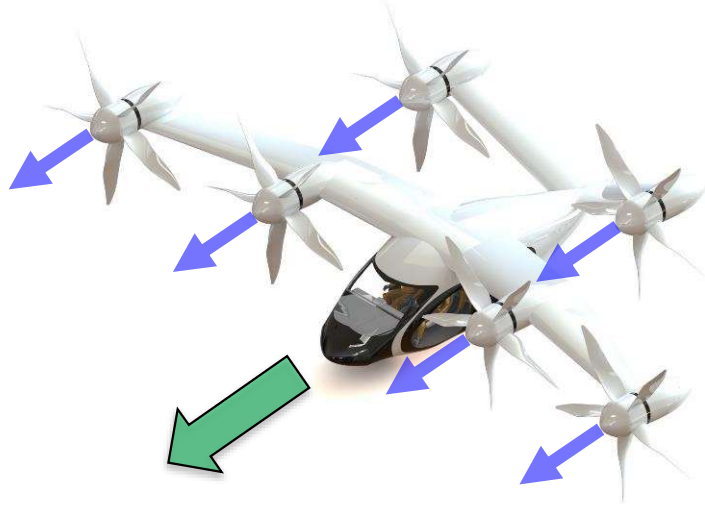
Yaw: rotor nacelle tilting (disclosed in Joby patent)



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Joby S4's Controls in Forward Flight

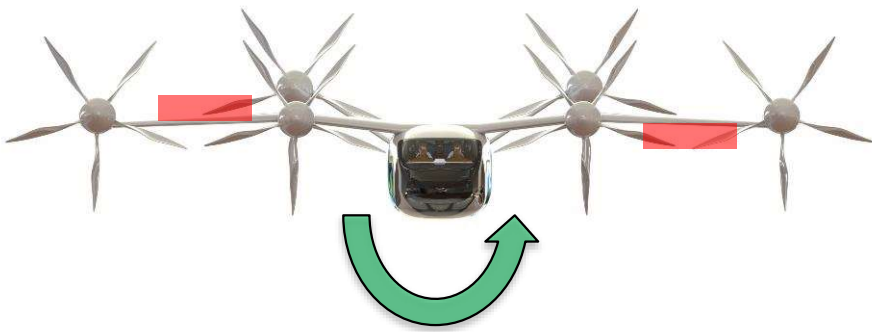
Thrust: thrust change by varying rpm & variable pitch propellers



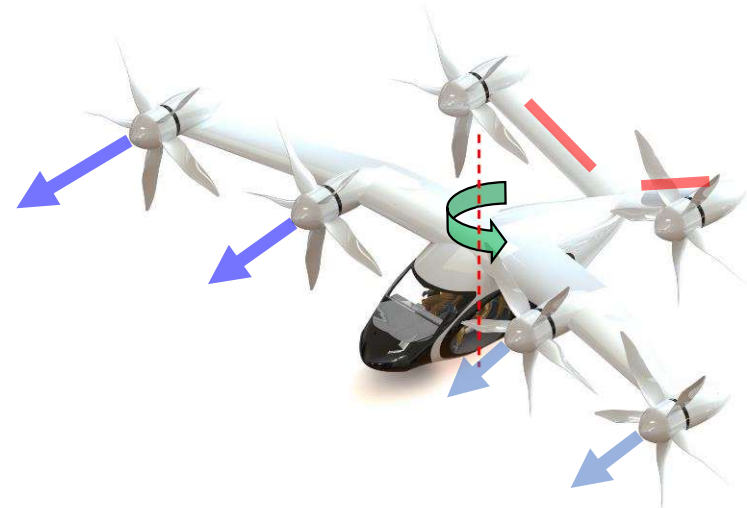
Pitch: tail ruddervators deflection



Roll: differential wing ailerons



Yaw: ruddervators & differential thrust (left & right)



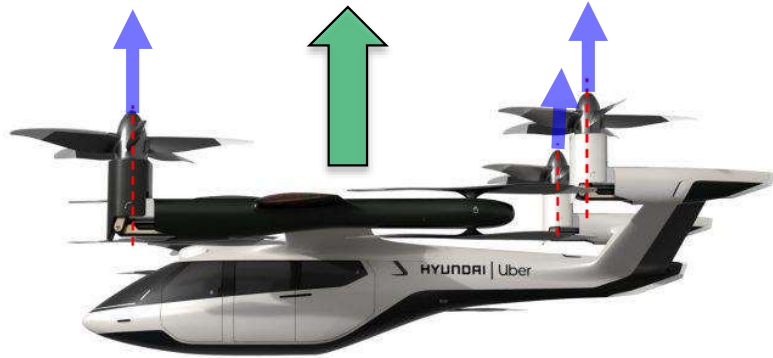
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Hyundai Supernal S-A1

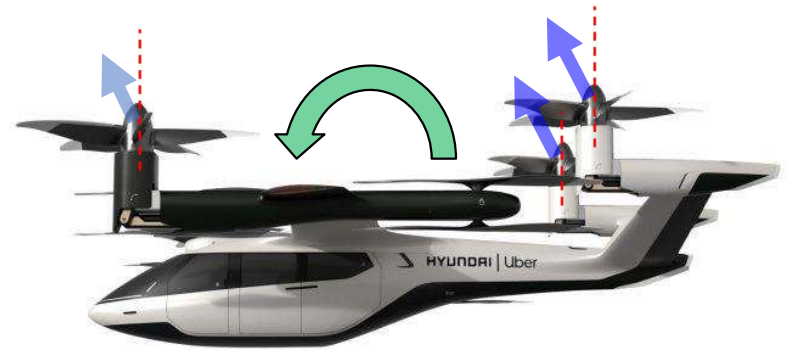


Hyundai Supernal S-A1's Controls in Hover

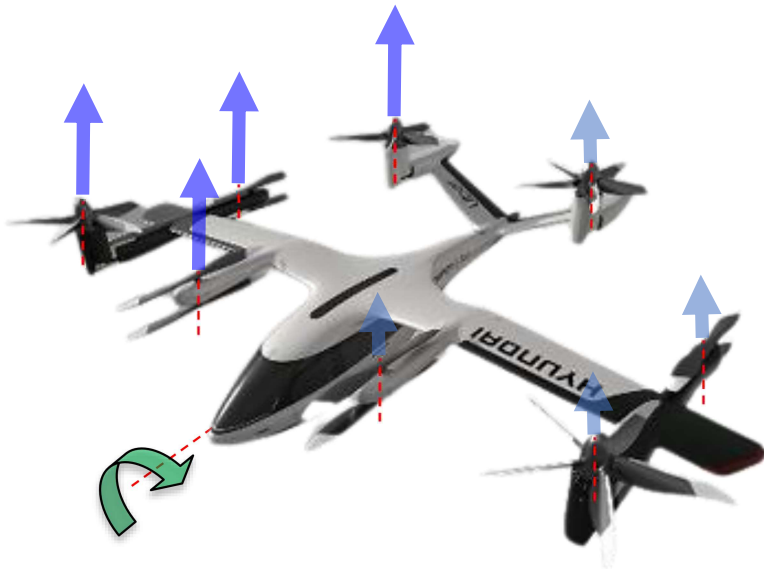
Heave: synchronous rotor thrust control by collective pitch and rpm



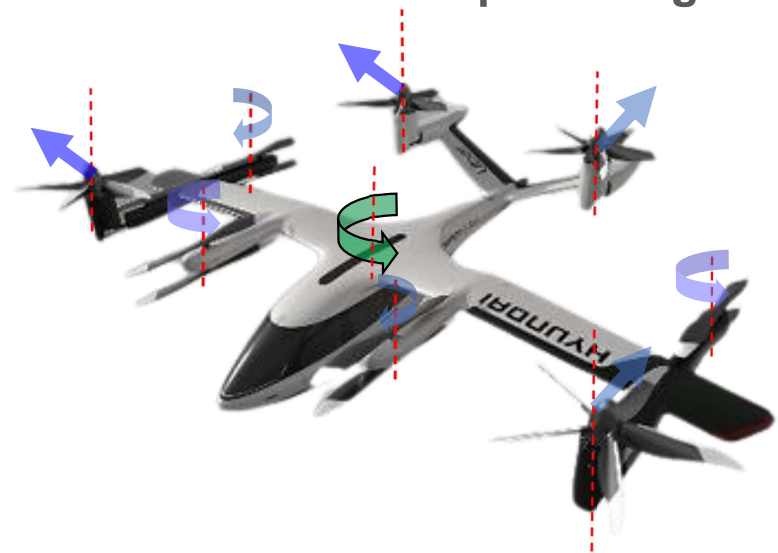
Pitch: forward tilting rotors & differential thrust (fore & aft)



Roll: differential thrust (left & right)

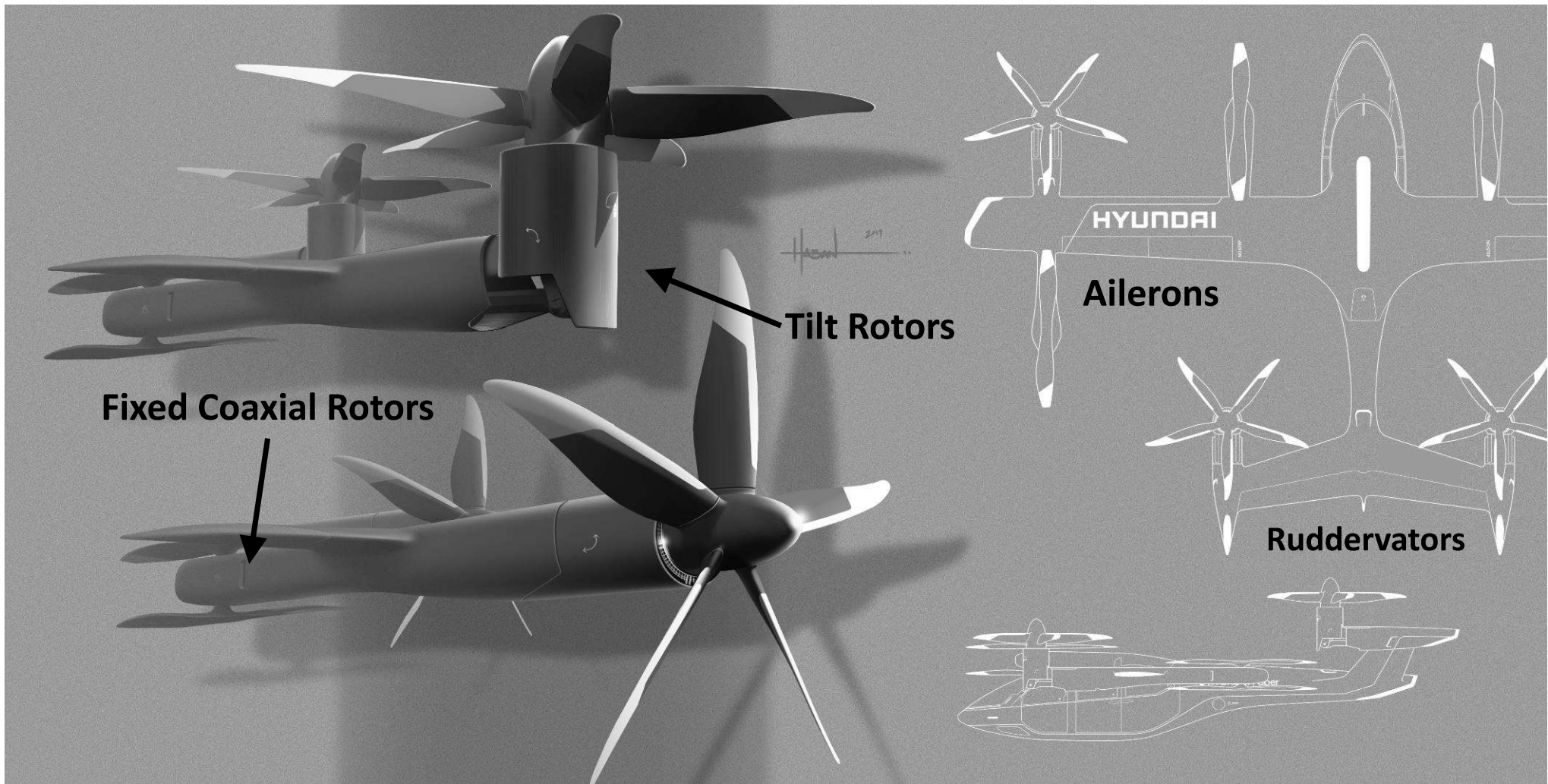


Yaw: rotor tilting angle (left & right) & differential rpm change



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

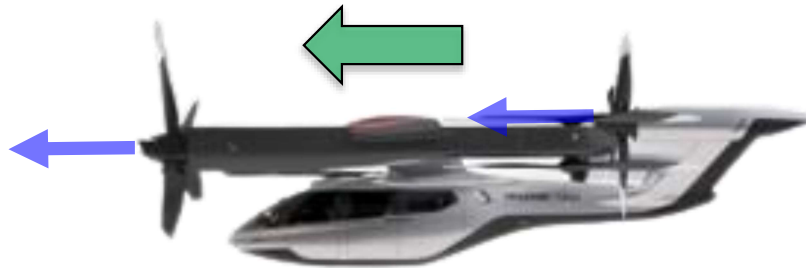
Hyundai Supernal S-A1 Control Surfaces



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Hyundai S-A1's Controls in Forward Flight

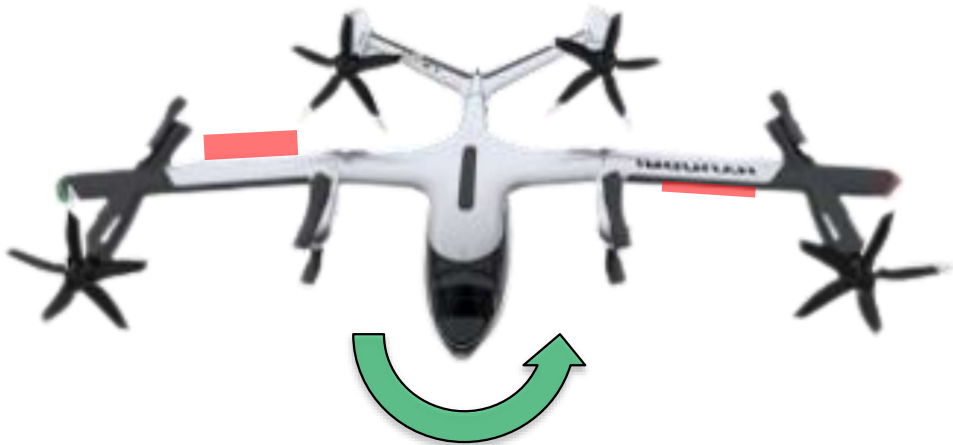
Thrust: thrust change by varying pitch



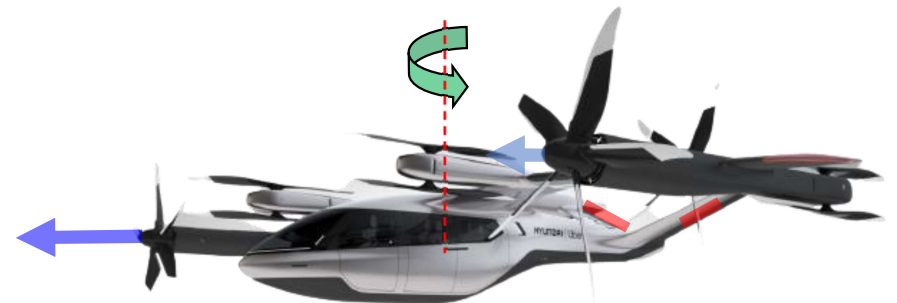
Pitch: tail ruddervators deflection



Roll: differential wing ailerons deflection

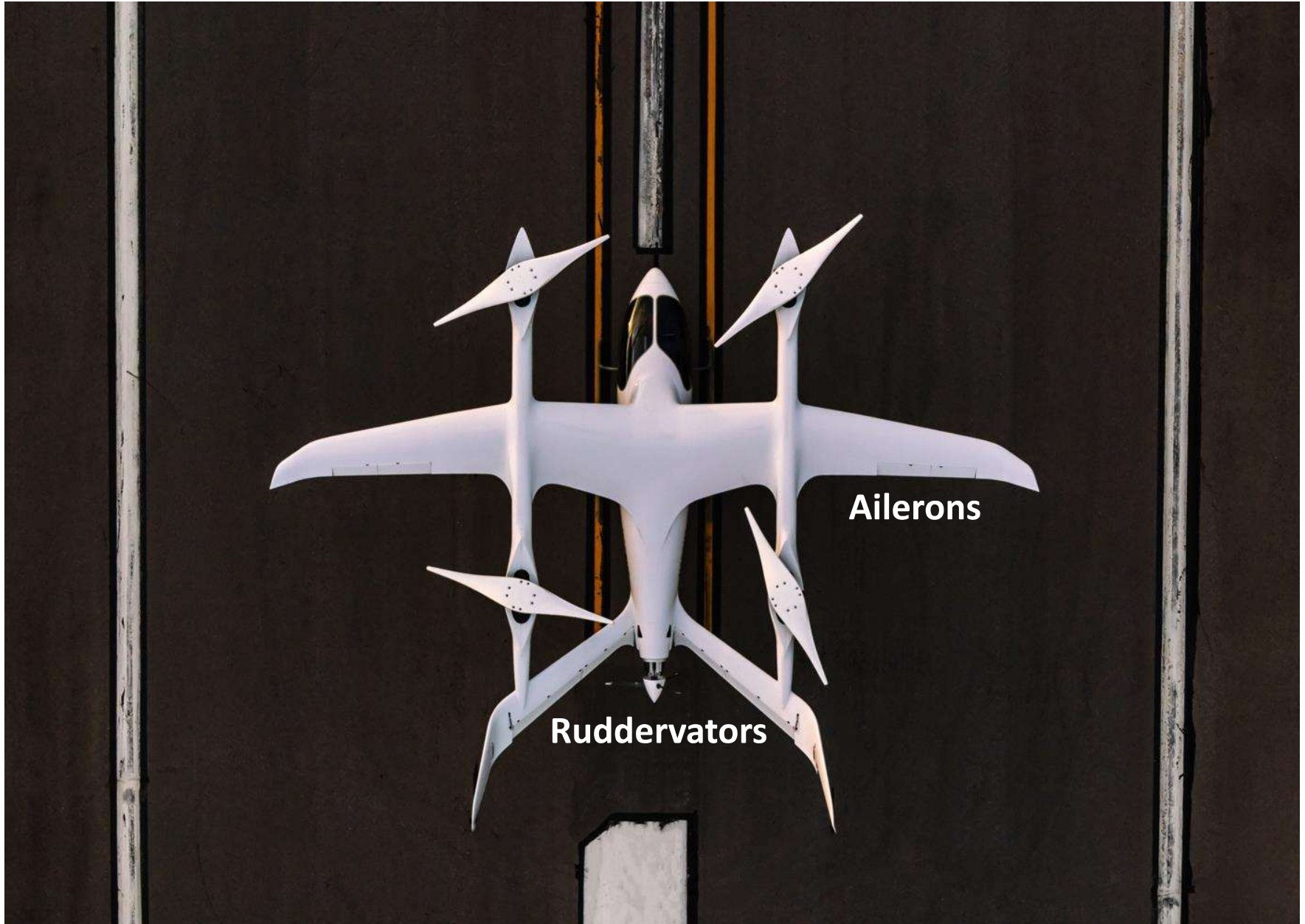


Yaw: differential thrust (left & right) & ruddervator deflection



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

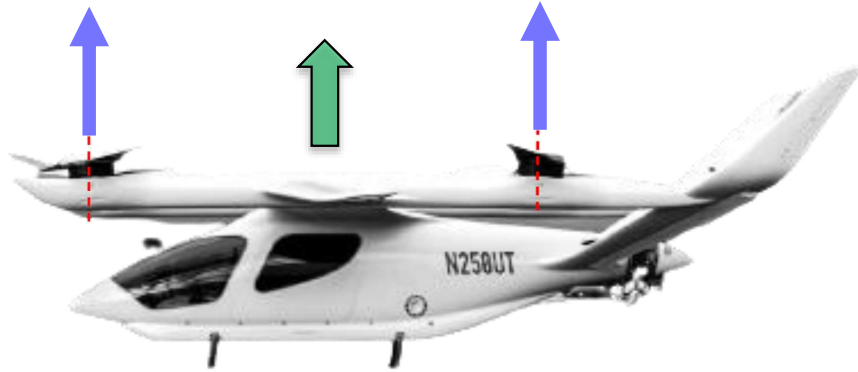
Beta Technologies Alia



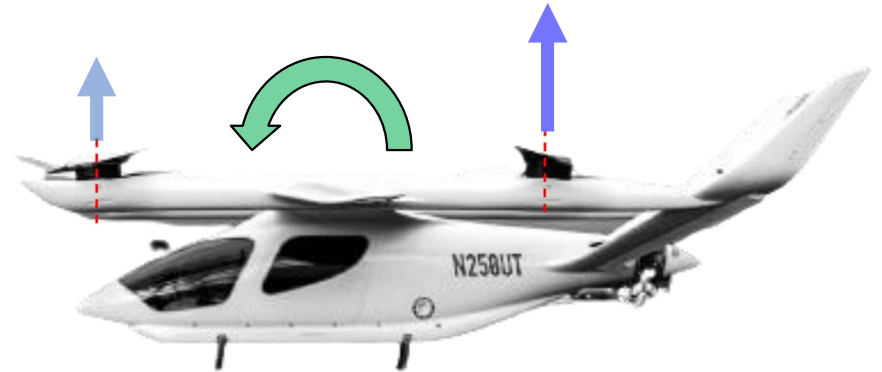
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Beta Technologies Alia's Controls in Hover

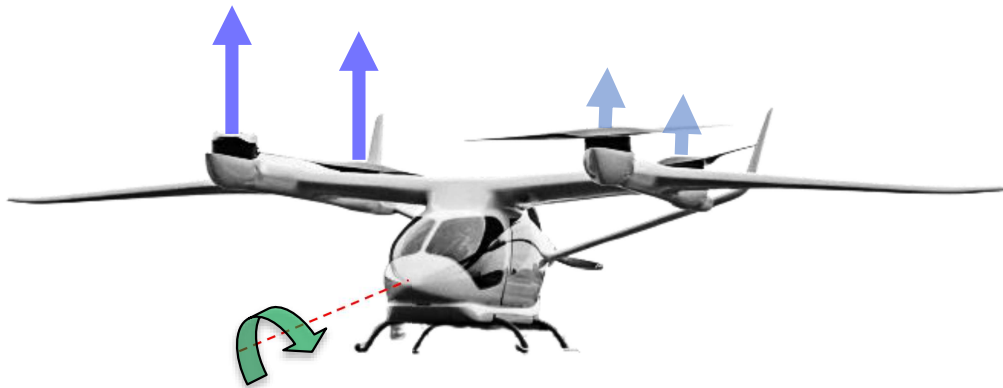
Heave: synchronous thrust change by rpm



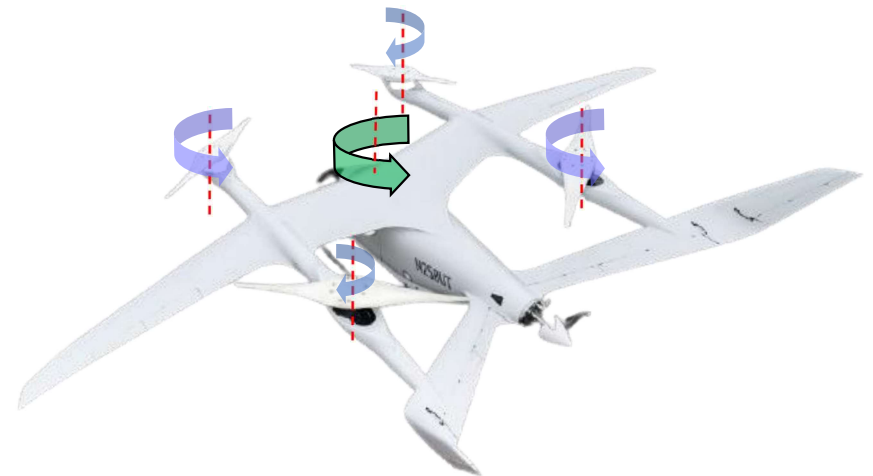
Pitch: differential thrust (fore & aft)



Roll: differential thrust (left & right)



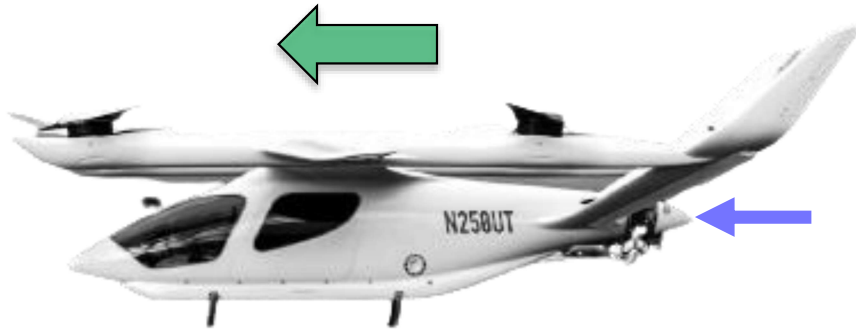
Yaw: differential rpm change



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Beta Alia's Controls in Forward Flight

Thrust: rear thrust change by varying rpm



Pitch: tail elevators deflection



Roll: differential wing aileron deflection



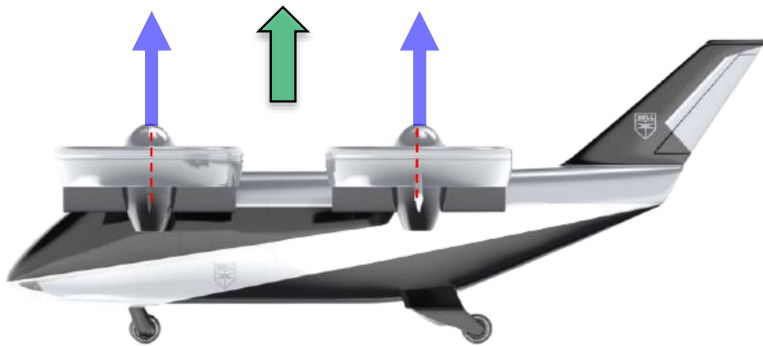
Yaw: tail rudders deflection



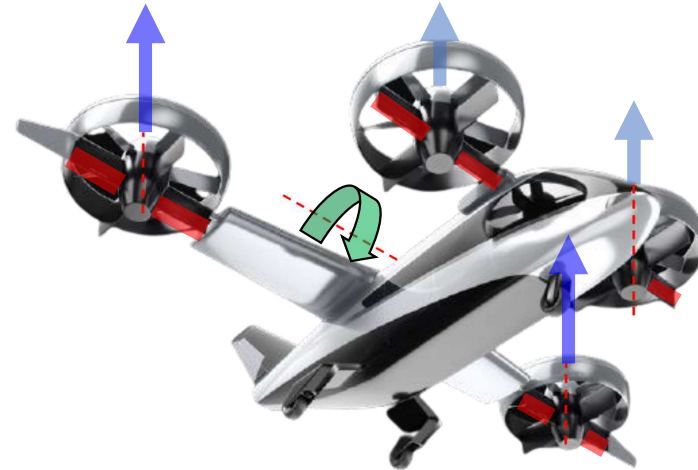
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Bell Nexus 4EX's Controls in Hover

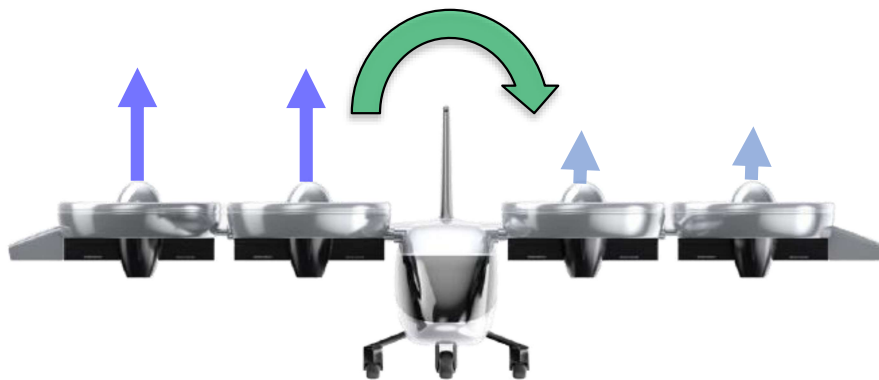
Heave: synchronous thrust change by varying pitch



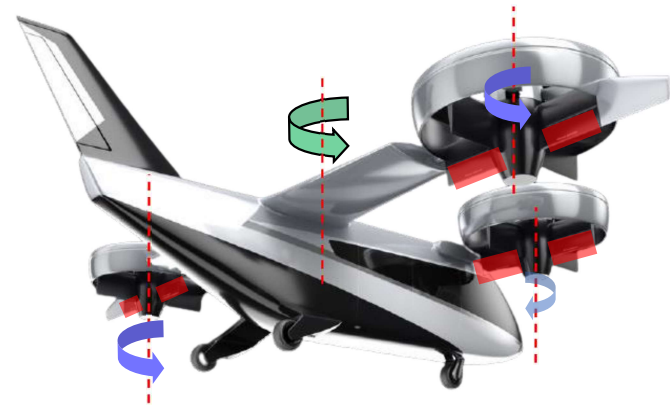
Pitch: differential thrust (fore & aft) & ducted control vanes



Roll: differential thrust (left & right)

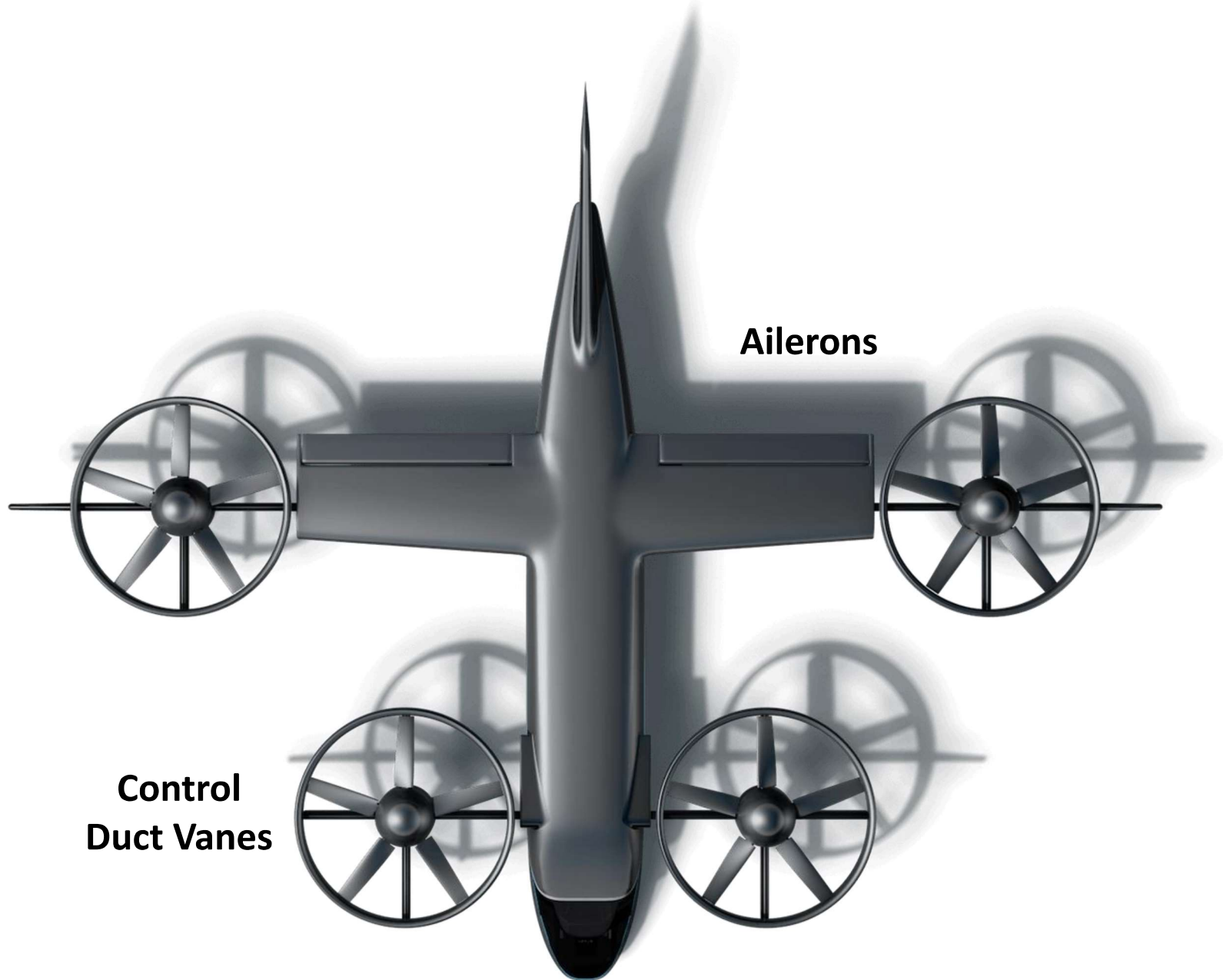


Yaw: differential rpm change & ducted control vanes



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

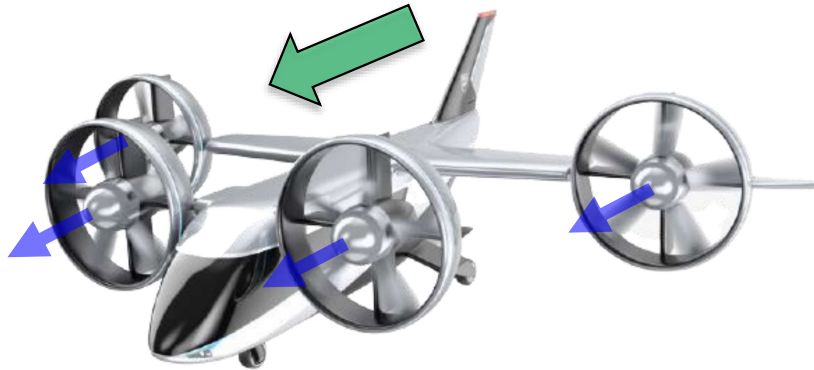
Bell Nexus 4EX's Control Surfaces



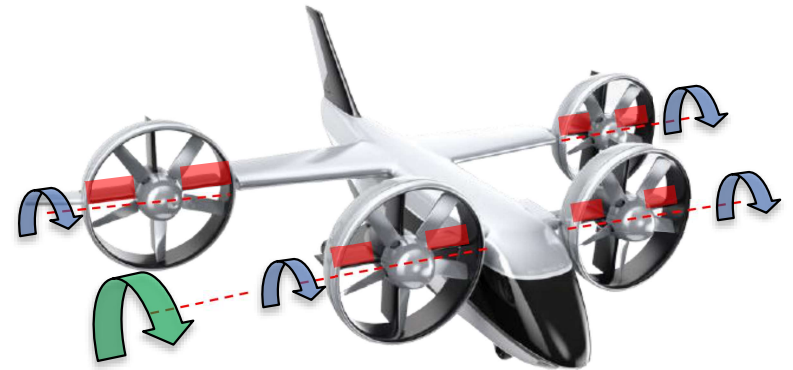
Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Bell Nexus 4EX's Controls in Forward Flight

Thrust: thrust change by varying pitch



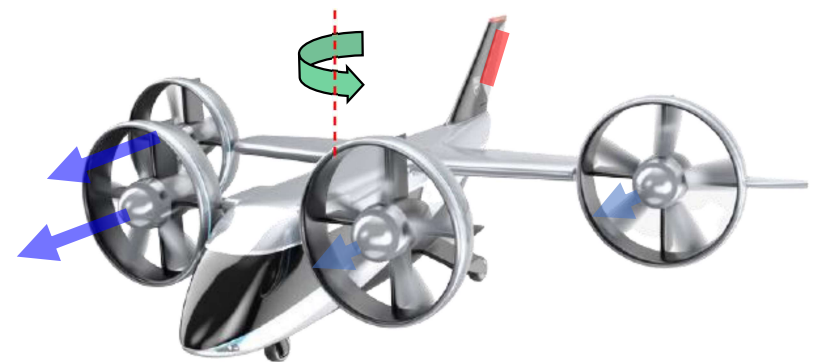
Pitch: tilting rotors & ducted vanes deflection



Roll: differential tilting rotors, ailerons & ducted vanes deflections

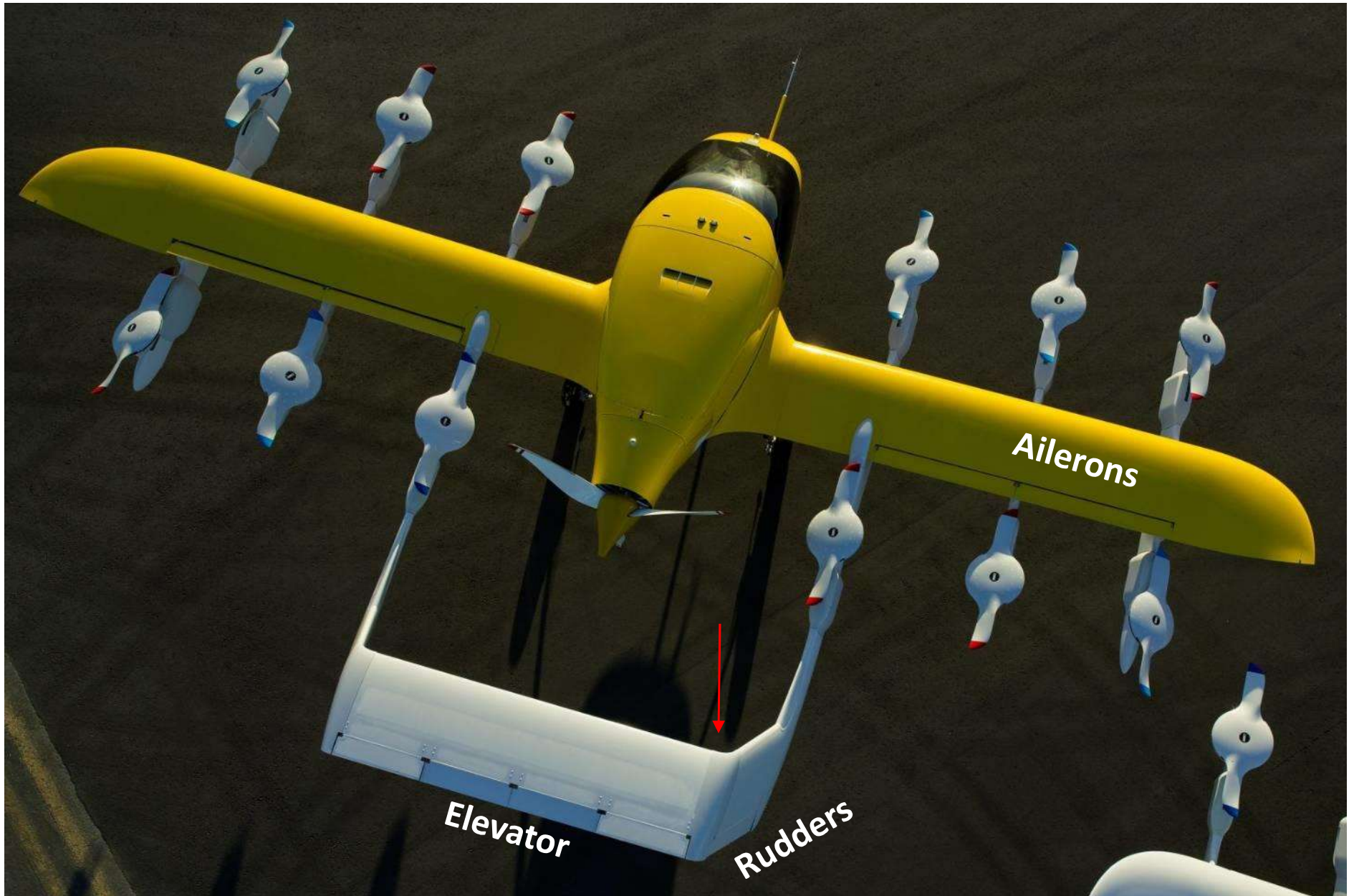


Yaw: tail rudder deflection & differential thrust (left & right)



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

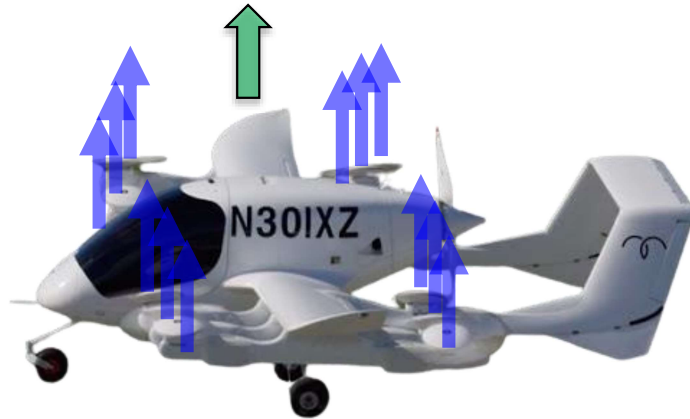
Wisk Cora Control Surfaces



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Wisk Cora Controls in Hover

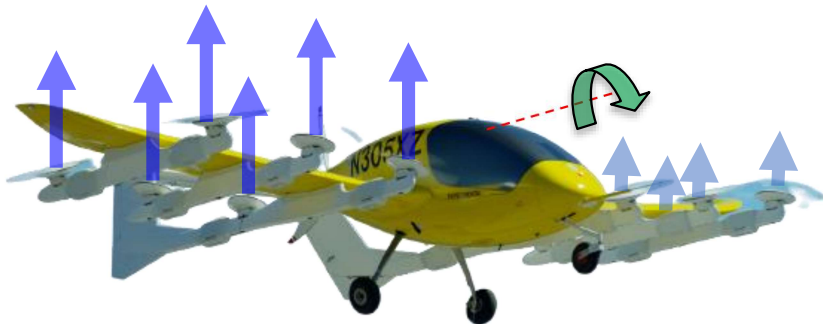
Heave: synchronous thrust change by rpm



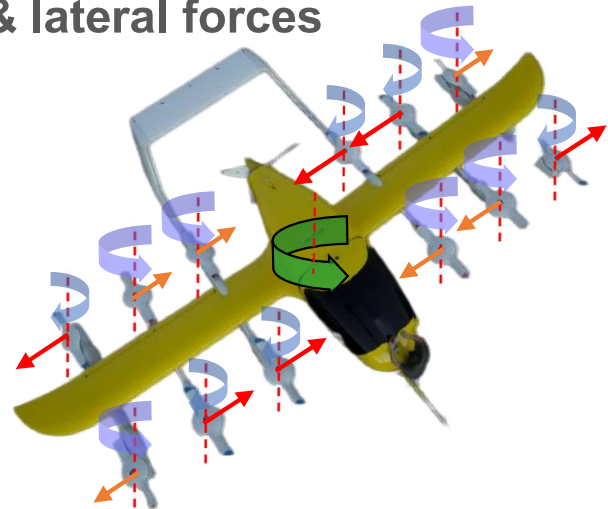
Pitch: differential thrust (fore & aft)



Roll: differential thrust (left & right)



Yaw: differential rpm change & lateral forces



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Wisk Cora Controls in Forward Flight

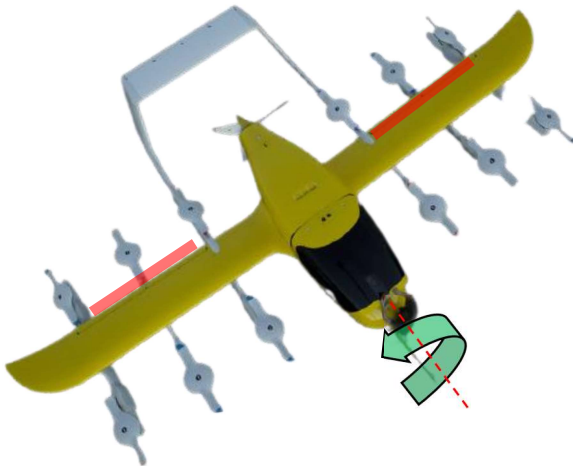
Thrust: thrust change by varying rpm



Pitch: tail elevator deflection



Roll: differential ailerons deflection



Yaw: tail rudders deflection



Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

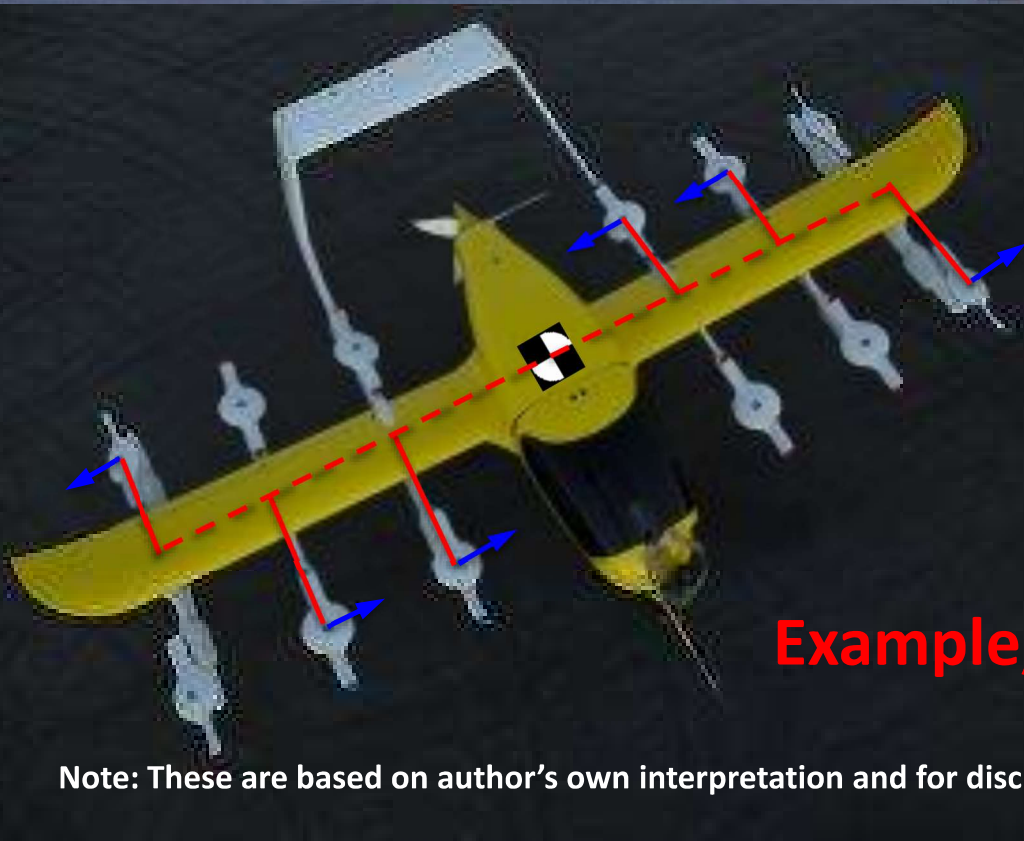
Wisk Cora Lift Rotors are all Canted



 **BOEING**

wisk

Wisk Cora Yaw Controls in Hover



Example, Yaw Left

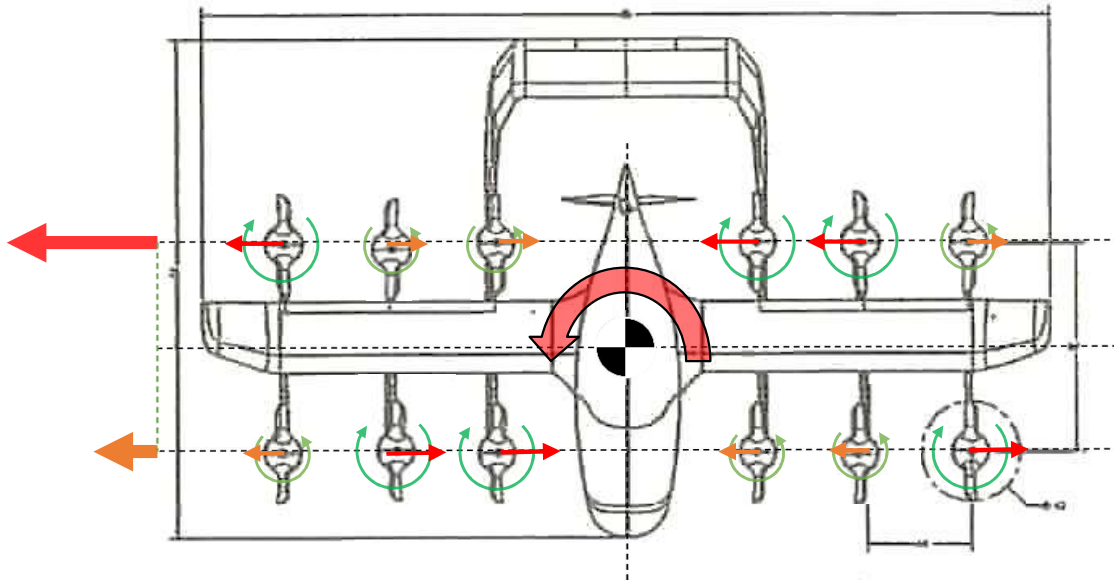


Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

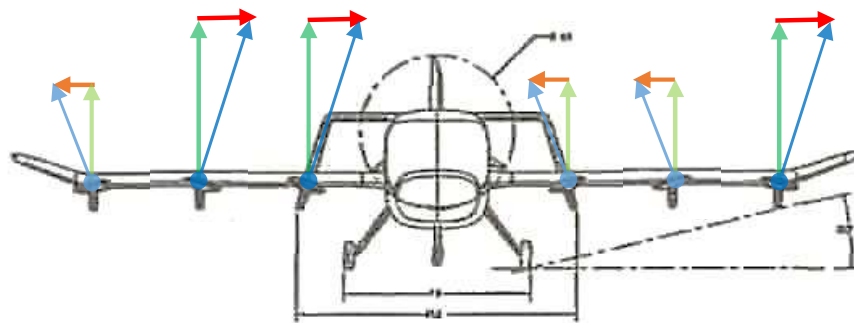
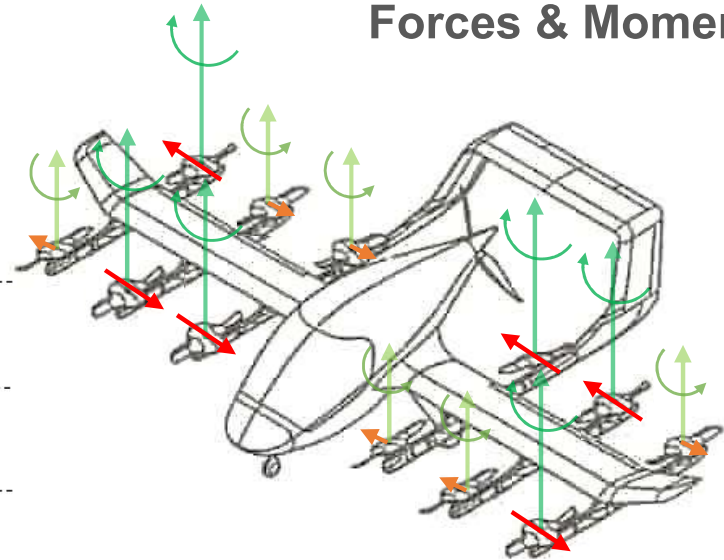
Wisk Cora's Angled Rotors

Increased yaw control effectiveness with rotor cant angle

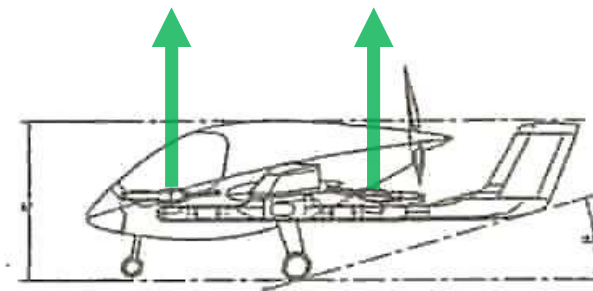
Yawing Moment Components



Forces & Moments



Thrust Vector Components



Total Vertical Thrust

Note: These are based on author's own interpretation and for discussion purpose only, and may not represent actual aircraft controls!

Boeing PAV eVTOL also tilts the lift fans for enhancing yaw controls in hover

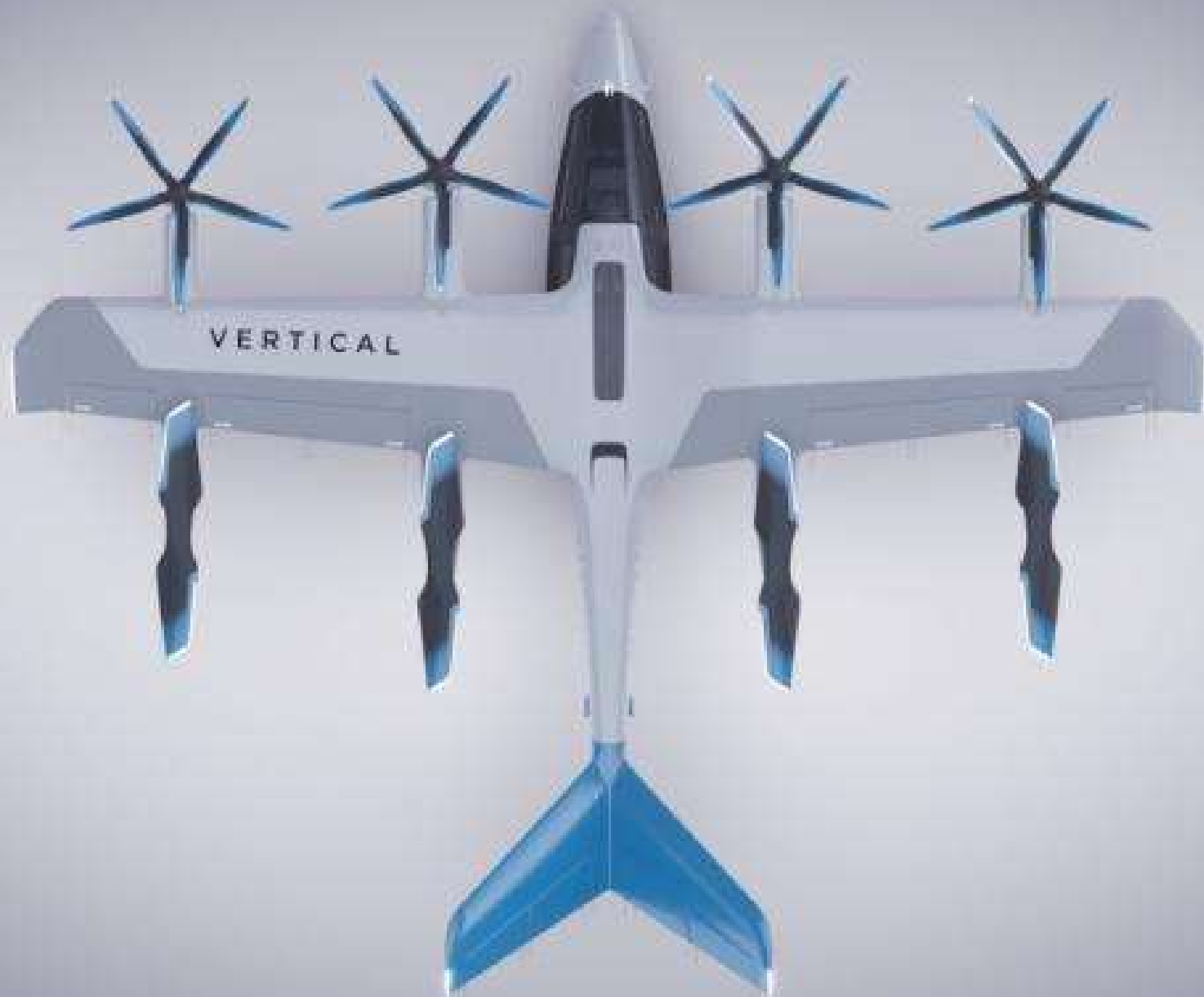


From 2019 Paris Airshow

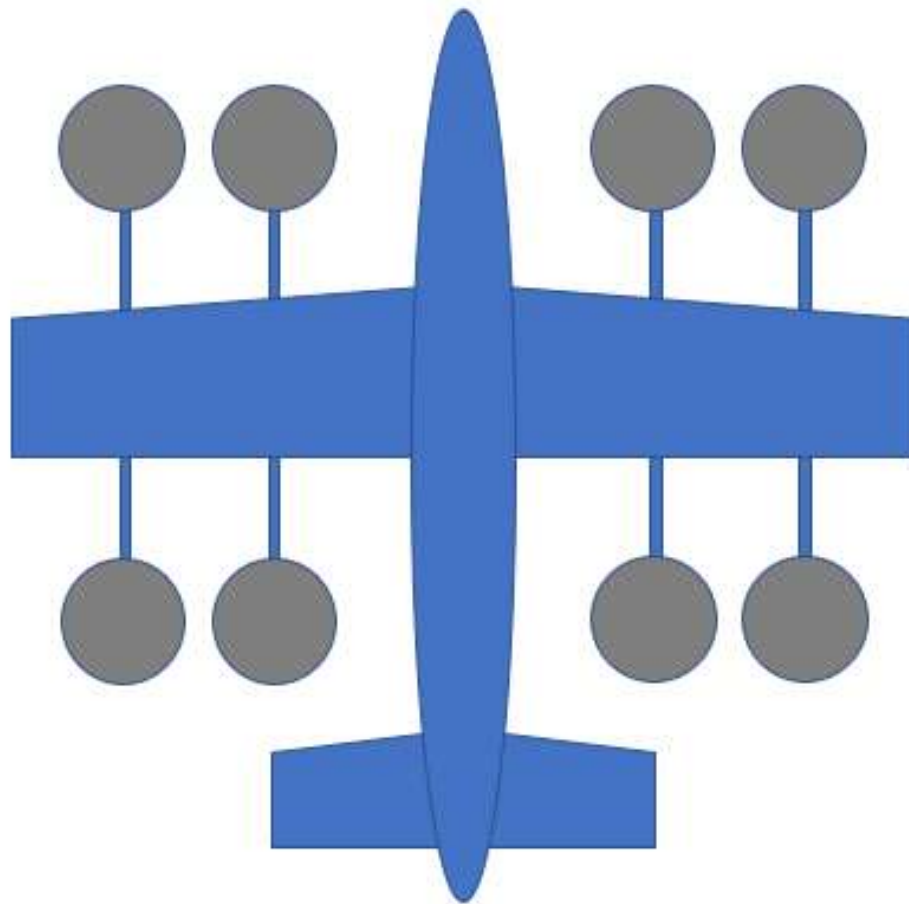
Archer Aviation Maker



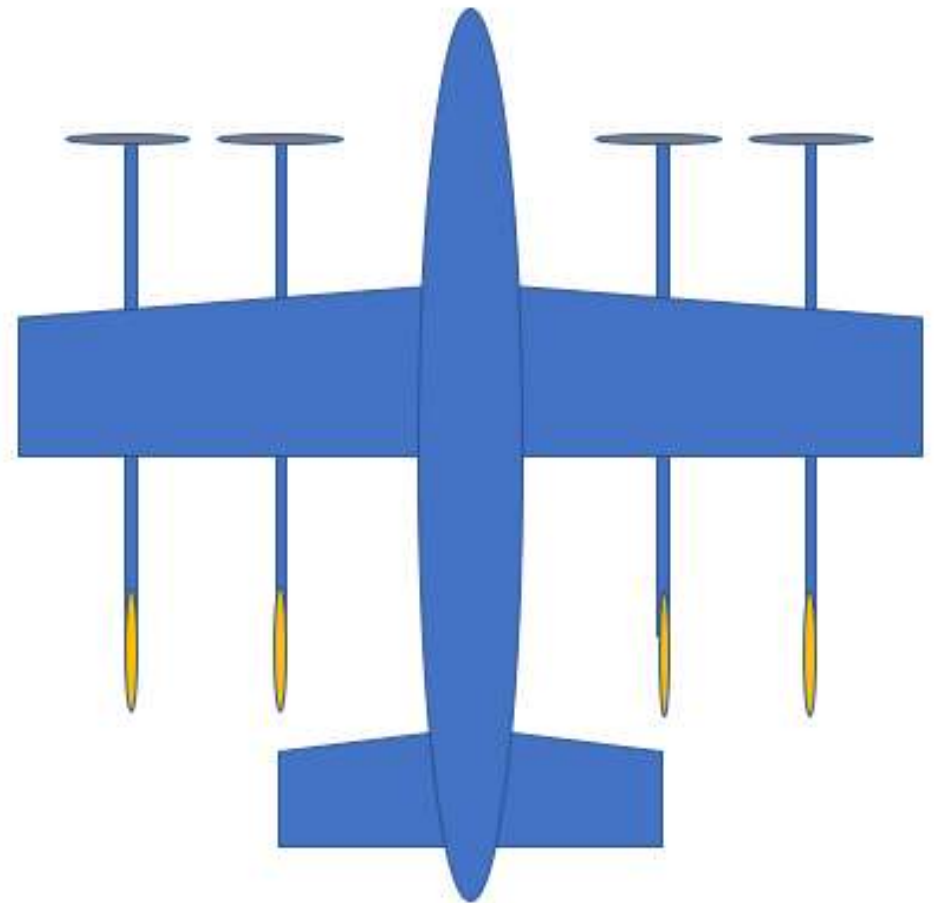
Vertical Aerospace VA-X4



Archer Aviation Maker and Vertical Aerospace VA-X4



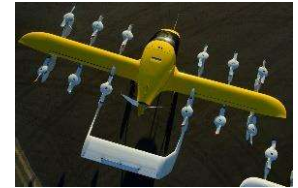
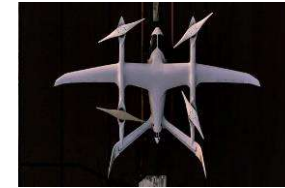
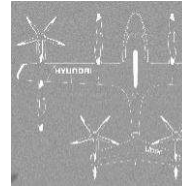
VTOL



Cruise

Summary of eVTOL Models Studied

Assumed
Control
Capabilities



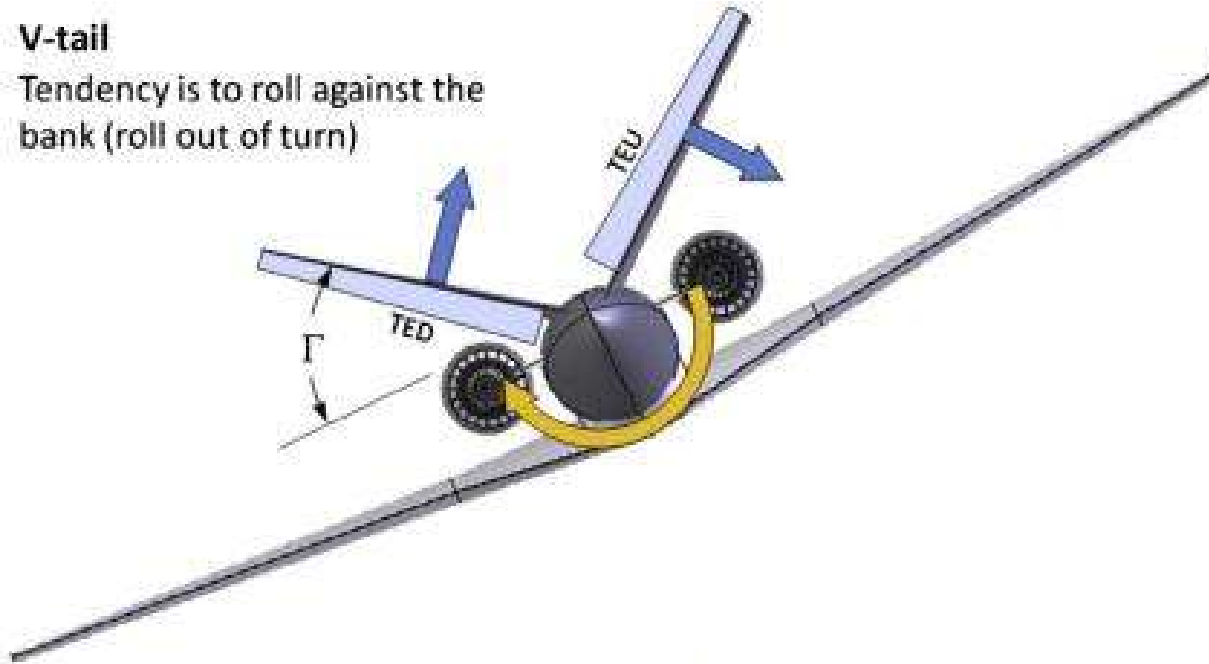
Control Strategy	Lilium Jet	Joby S4	Hyundai SA-1	Bell Nexus 4EX	Beta Alias	Wisk Cora
Hover Controls	Multicopter + Thrust Vectoring Capabilities	Multicopter + Thrust Vectoring Capabilities	Multicopter + Thrust Vectoring Capabilities	Multicopter + Thrust Vectoring Capabilities	Multicopter	Multicopter
Forward Flight Controls	Tilting ducted rotors on canard and wing like control surfaces	Ailerons & Ruddervators	Ailerons & Ruddervators	Ailerons & Control Vanes	Ailerons & Ruddervators	Ailerons, Rudders Elevator
Transition Modes	Rotor Tilting Vectored Thrust	Rotor Tilting Vectored Thrust	Rotor Tilting Vectored Thrust & Lift + Cruise	Rotor Tilting Vectored Thrust	Lift + Cruise	Lift + Cruise
Weight Challenge	Many motors, inverters, wires	6 tilting rotor, variable pitch, +control surfaces	4 tilting rotor, boom structures, +control surfaces	4 massive duct tilting rotor, +control surfaces	Boom structures and 4 lift rotors for hover only	12 rotors and pylons for hover only
Control Challenges	In hover rely on rpm change, could be delay	Require automatic scheduling nacelle tilting vs airspeed	Require automatic scheduling nacelle tilting vs airspeed	Require automatic scheduling duct tilting vs airspeed	Transfer/blend multicopter and airplane controls during transition	Transfer/blend multicopter and airplane controls during transition

V-Tail

Left ruddervator also causes roll right

V-tail

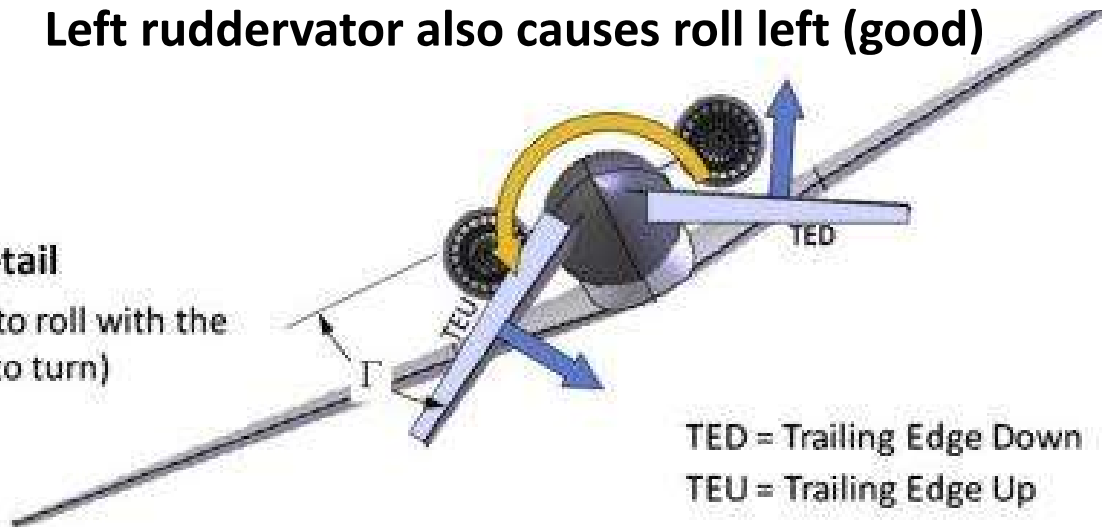
Tendency is to roll against the bank (roll out of turn)



Left ruddervator also causes roll left (good)

Inverted V-tail

Tendency is to roll with the bank (roll into turn)



TED = Trailing Edge Down
TEU = Trailing Edge Up



Taylor Mini IMP

Aircraft Forward Flight Stability

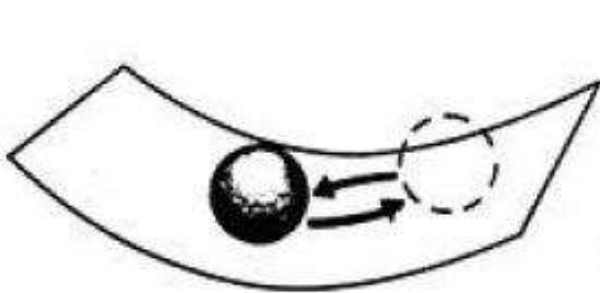
I recommend design your eVTOL aircraft to be stable in forward flight first. Preferably it can be flown in forward flight without any SAS (stability augmentation system).



Then use SAS to stabilize hover. Helicopters can hover without SAS, but it is almost surely your eVTOL aircraft will require SAS for hovering.

Static Stability

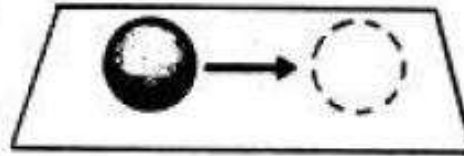
Static stability of a body is an initial tendency of that body to return to its equilibrium state after a disturbance.



Positive static stability



Means it will return to the original state after a disturbance



Neutral static stability



Neutral does not mean it is stable because it does not return to its original state. But it does not diverge

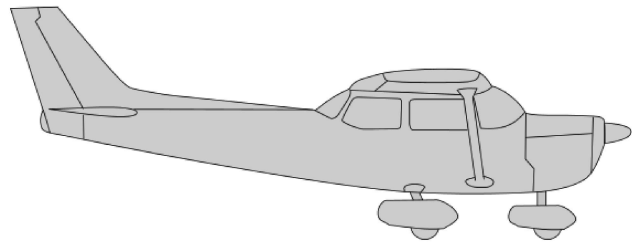


Negative static stability



It is unstable and diverges

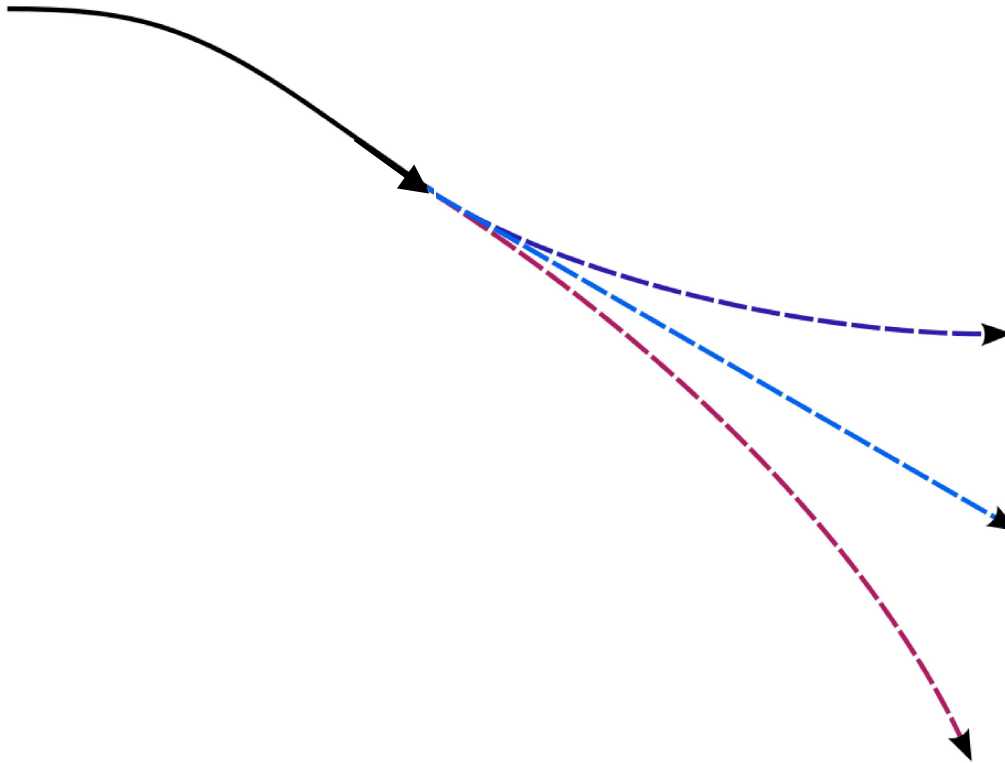
Static Stability



In a steady equilibrium flight



Suddenly experienced a pitch-down disturbance



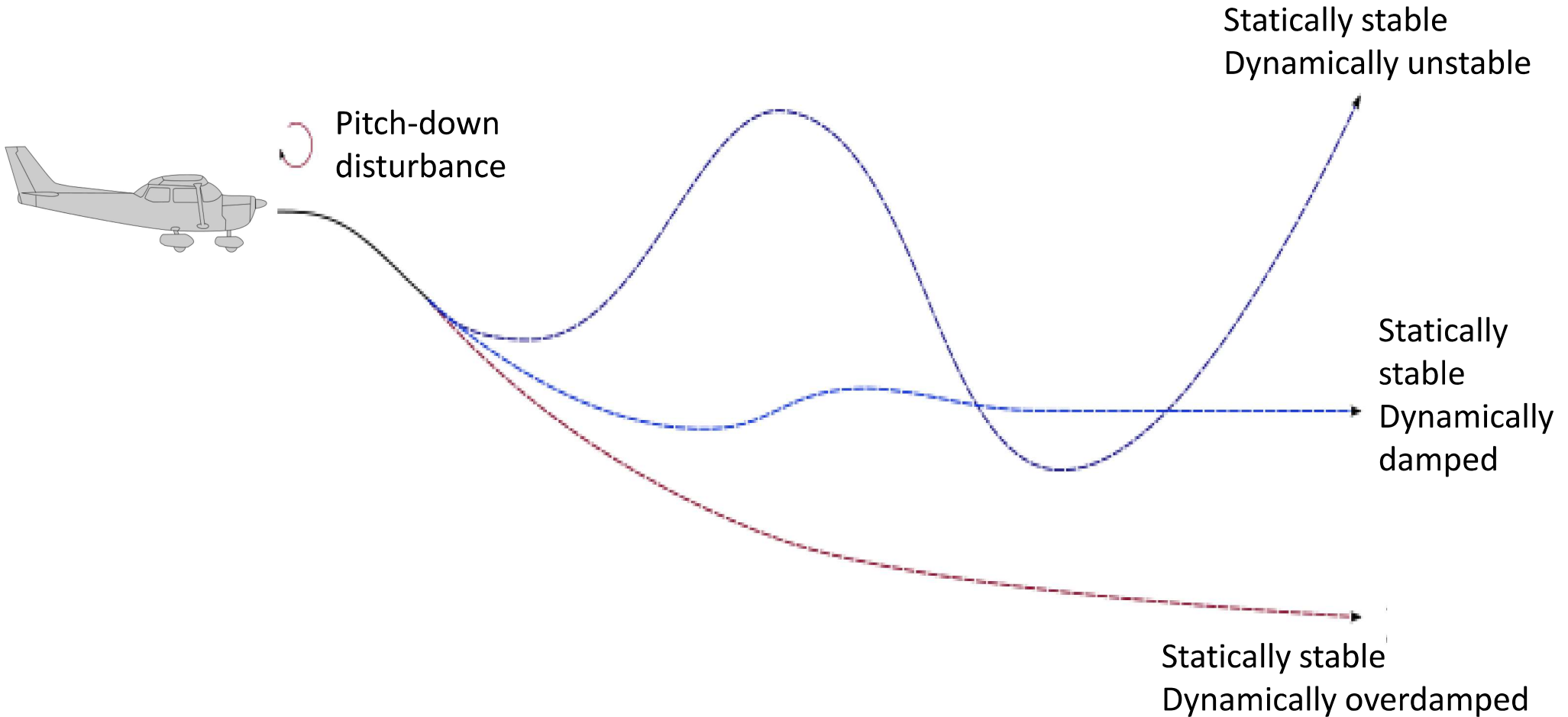
Positive static stability
(returns to level)

Neutral static stability
(continues)

Negative static stability
(diverges)

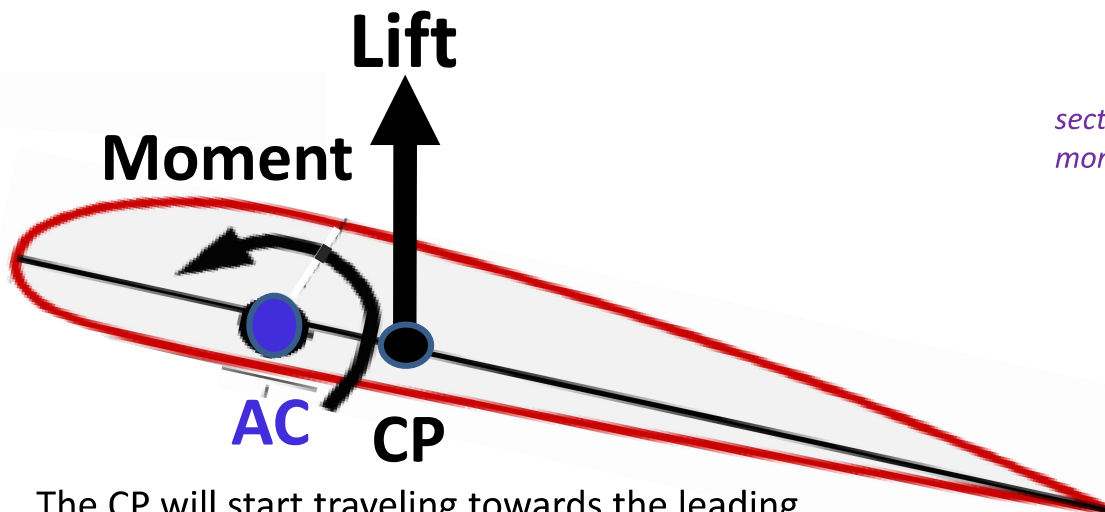
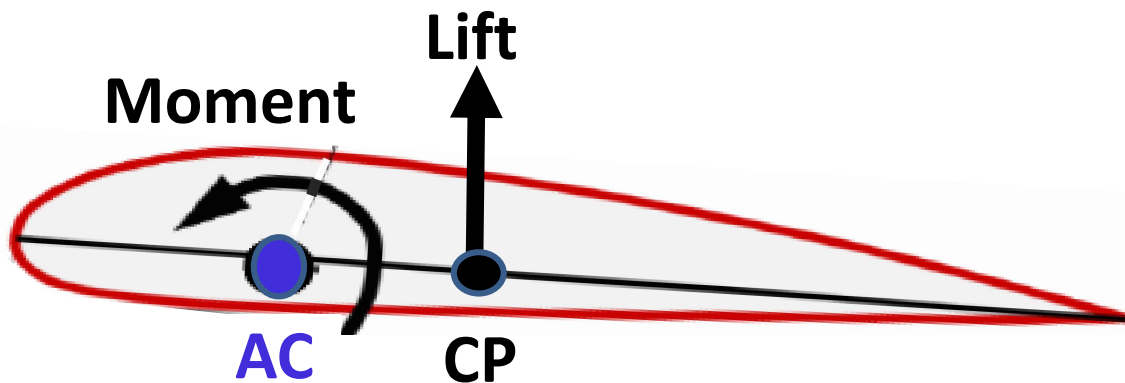
Dynamic Stability

An aircraft is said to be dynamically stable if, after a disturbance, it eventually returns to its equilibrium state.



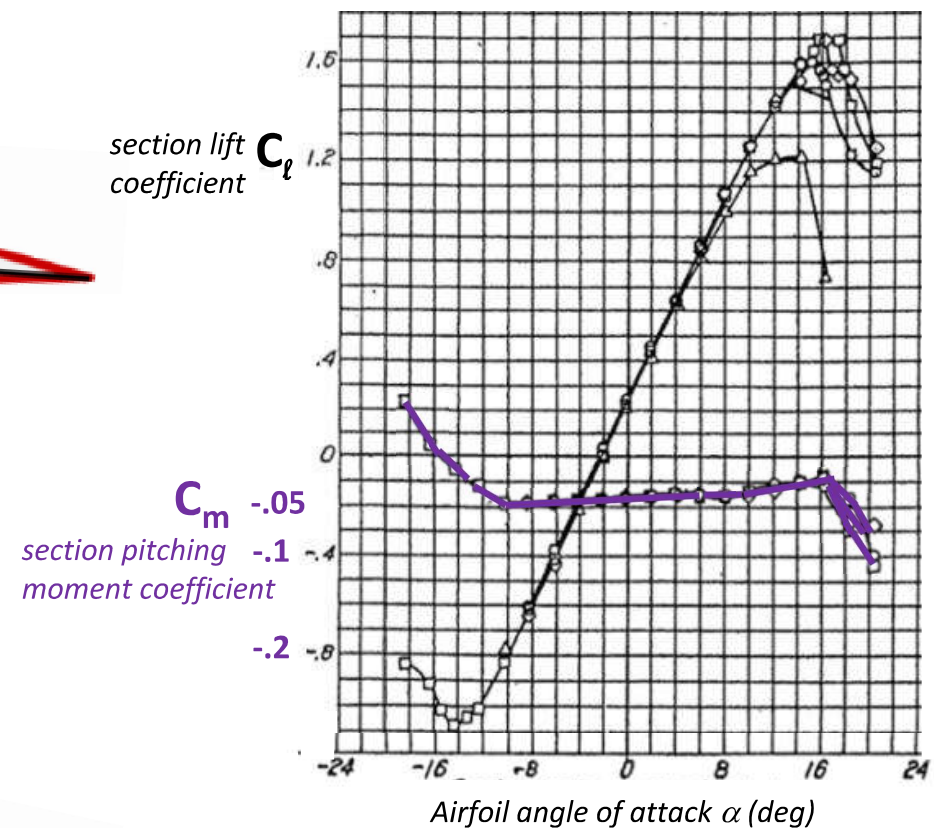
Aerodynamic Center of an Airfoil

The aerodynamic center (AC) of an airfoil, or a wing, is defined as that point on the airfoil where the aerodynamic moment (M) does not depend on angle of attack (α)



The CP will start traveling towards the leading edge and closer to the $\frac{1}{4}$ chord when α increased

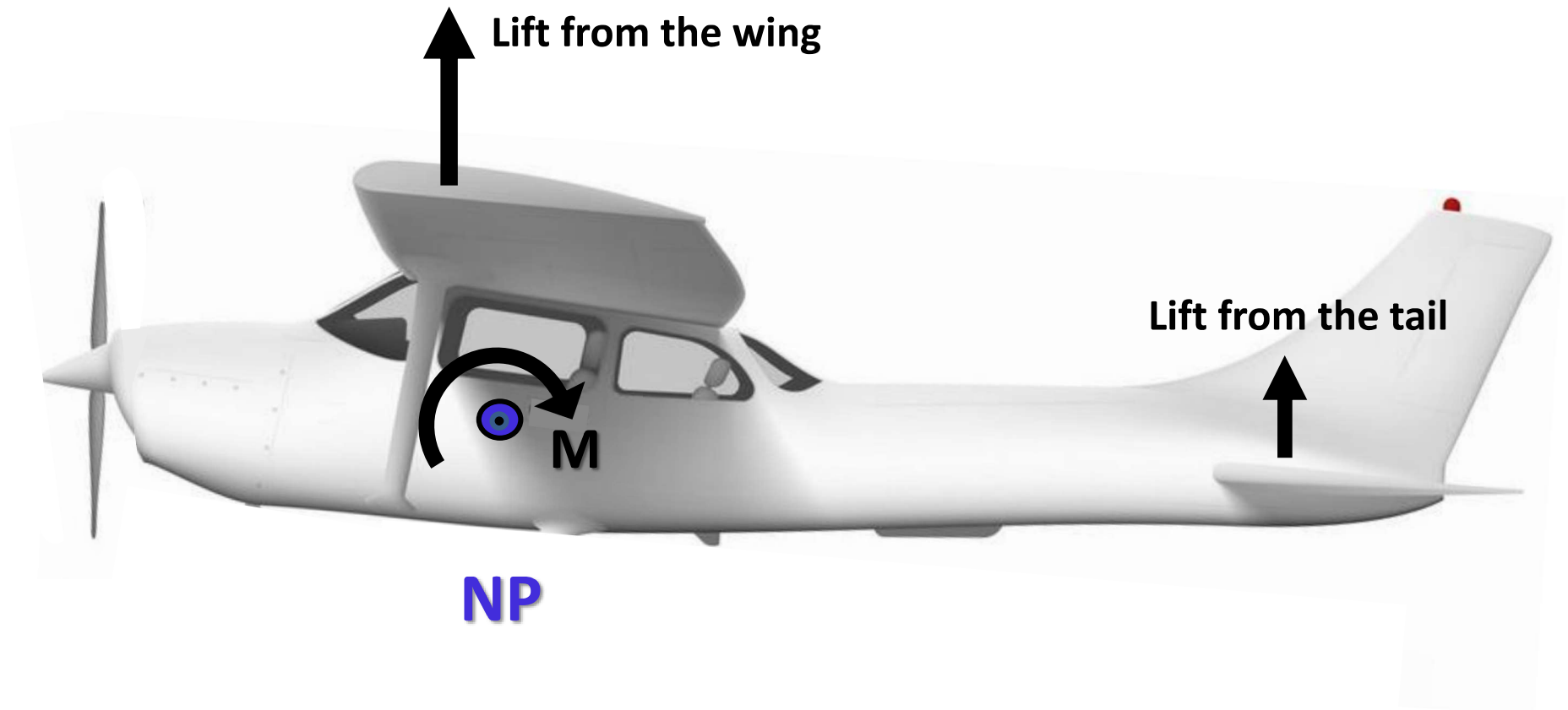
NACA 2412 airfoil data



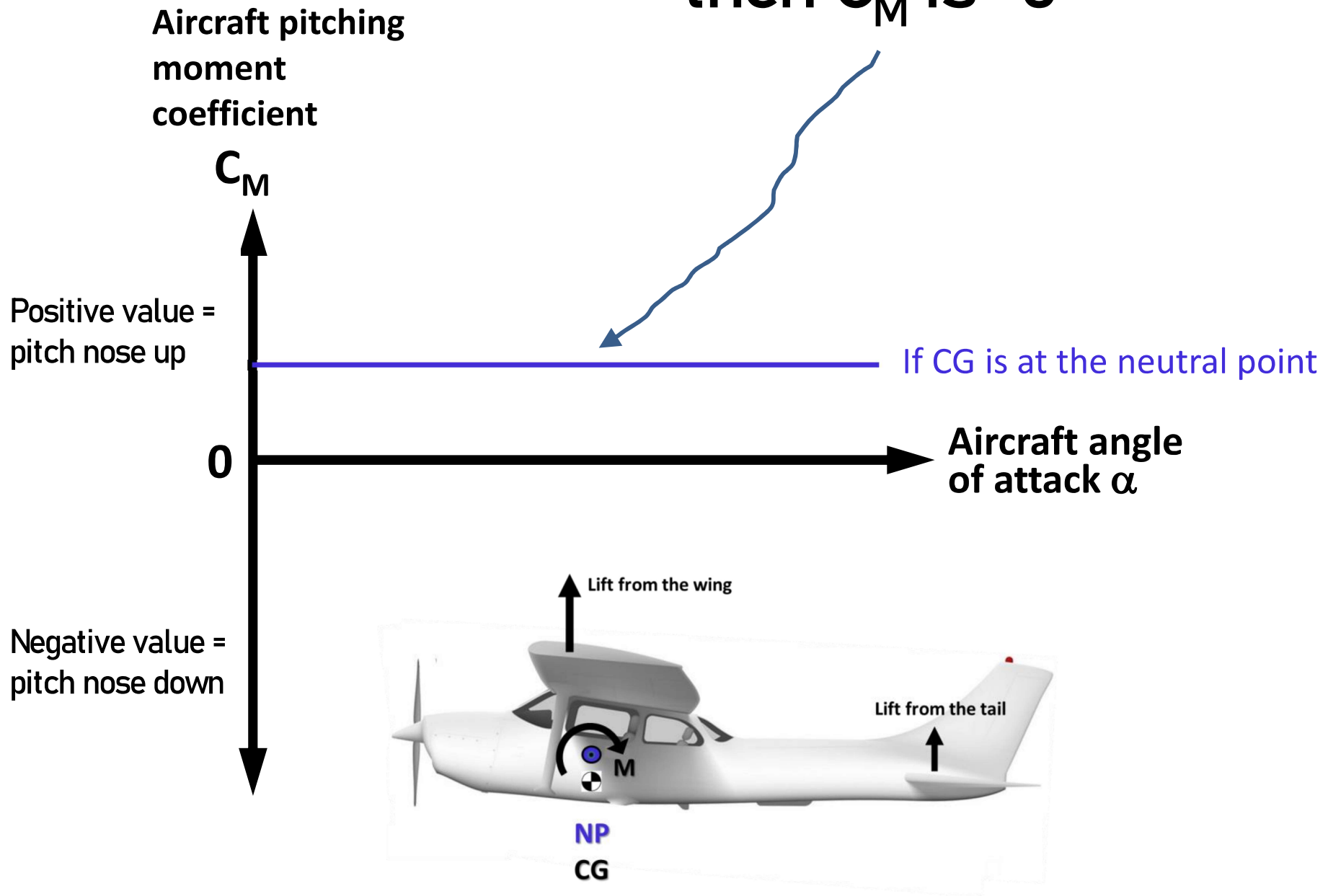
Notice the C_m curve is nearly flat at around -0.04 for $\alpha = -10^\circ$ to $+10^\circ$

Neutral Point of an Aircraft

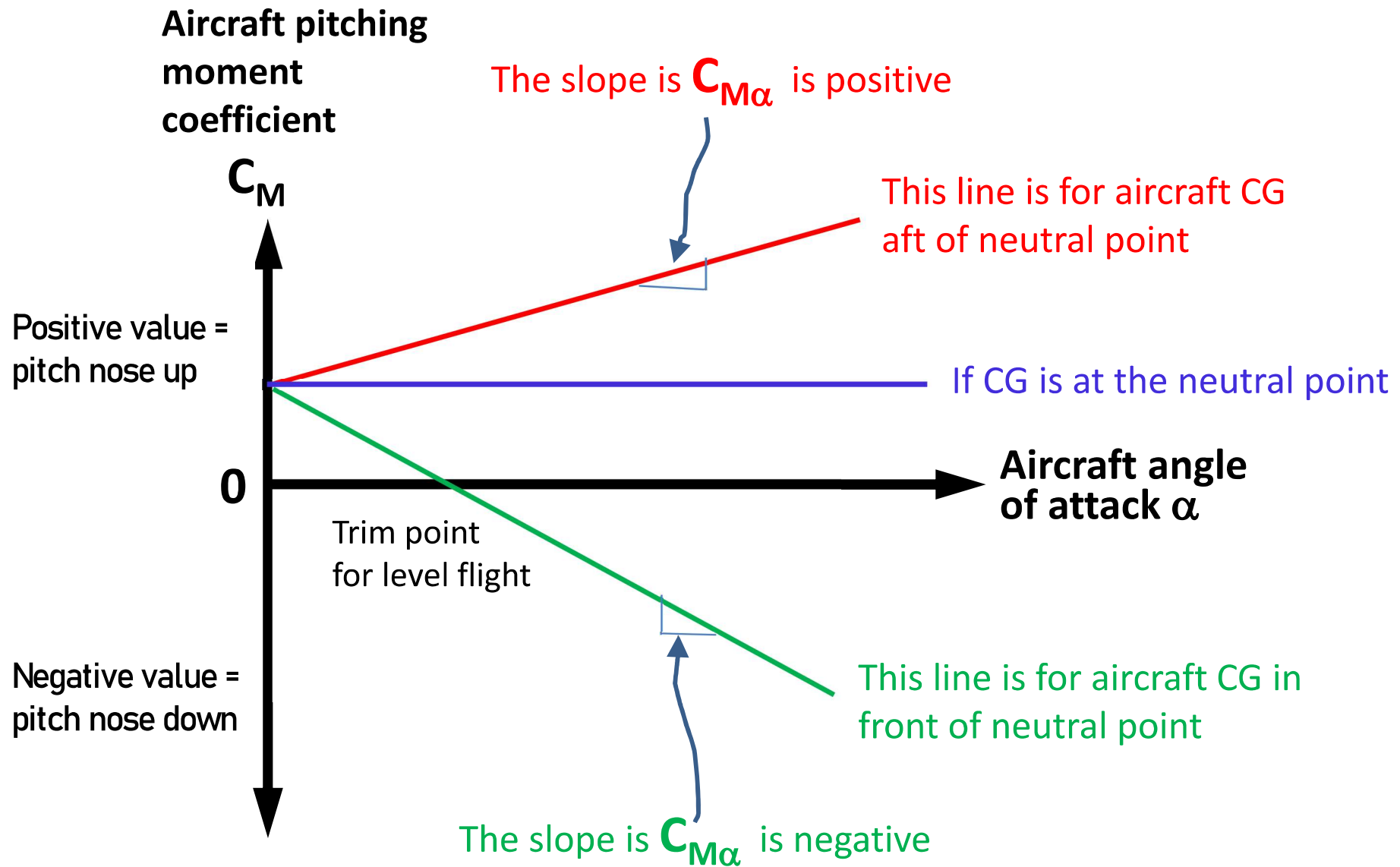
The neutral point (NP) is defined as that point on the longitudinal axis of an aircraft where the aerodynamic moment (M) does not depend on angle of attack α of the aircraft.



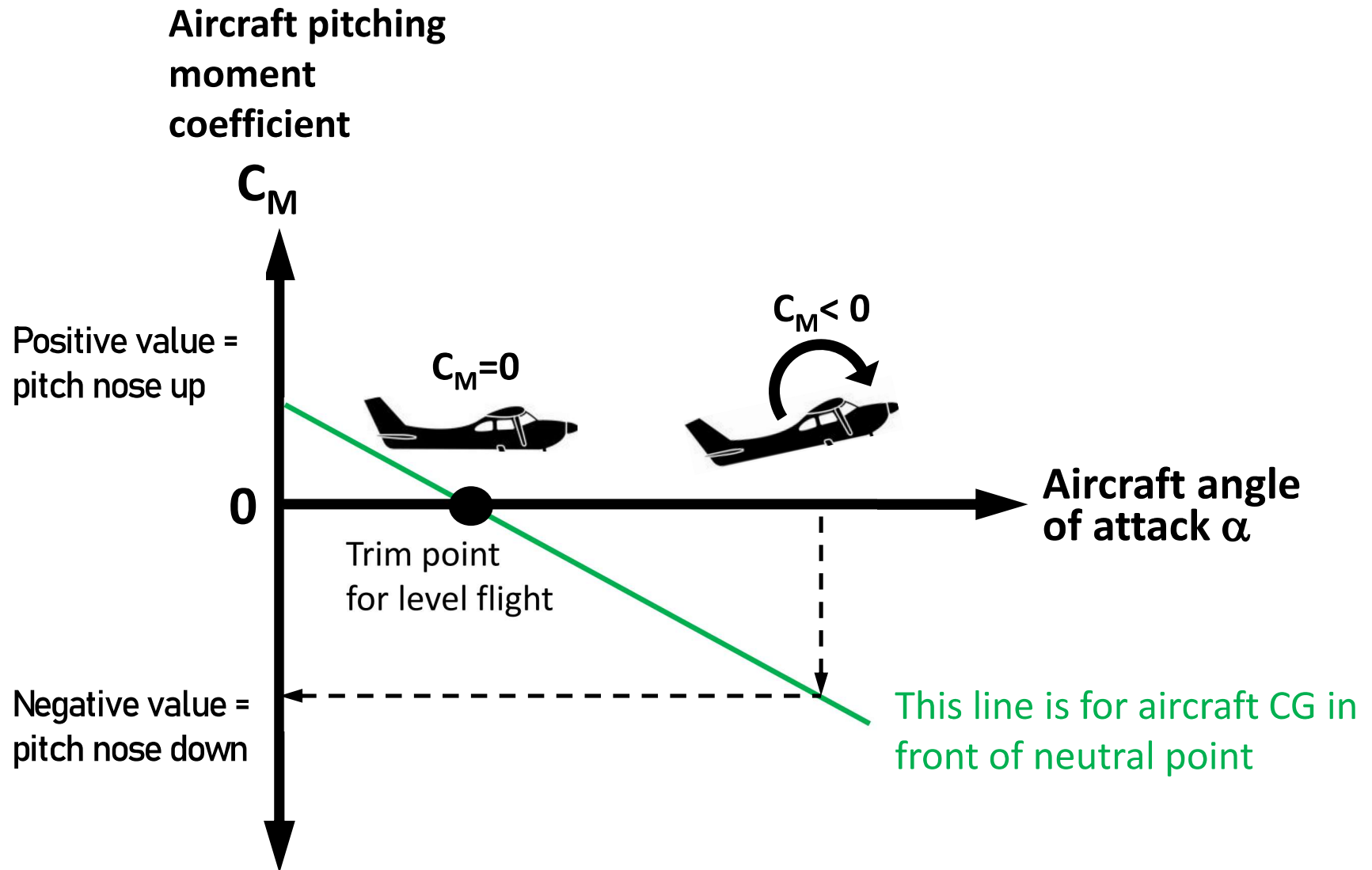
If the aircraft CG is located at the neutral point, then C_M is >0



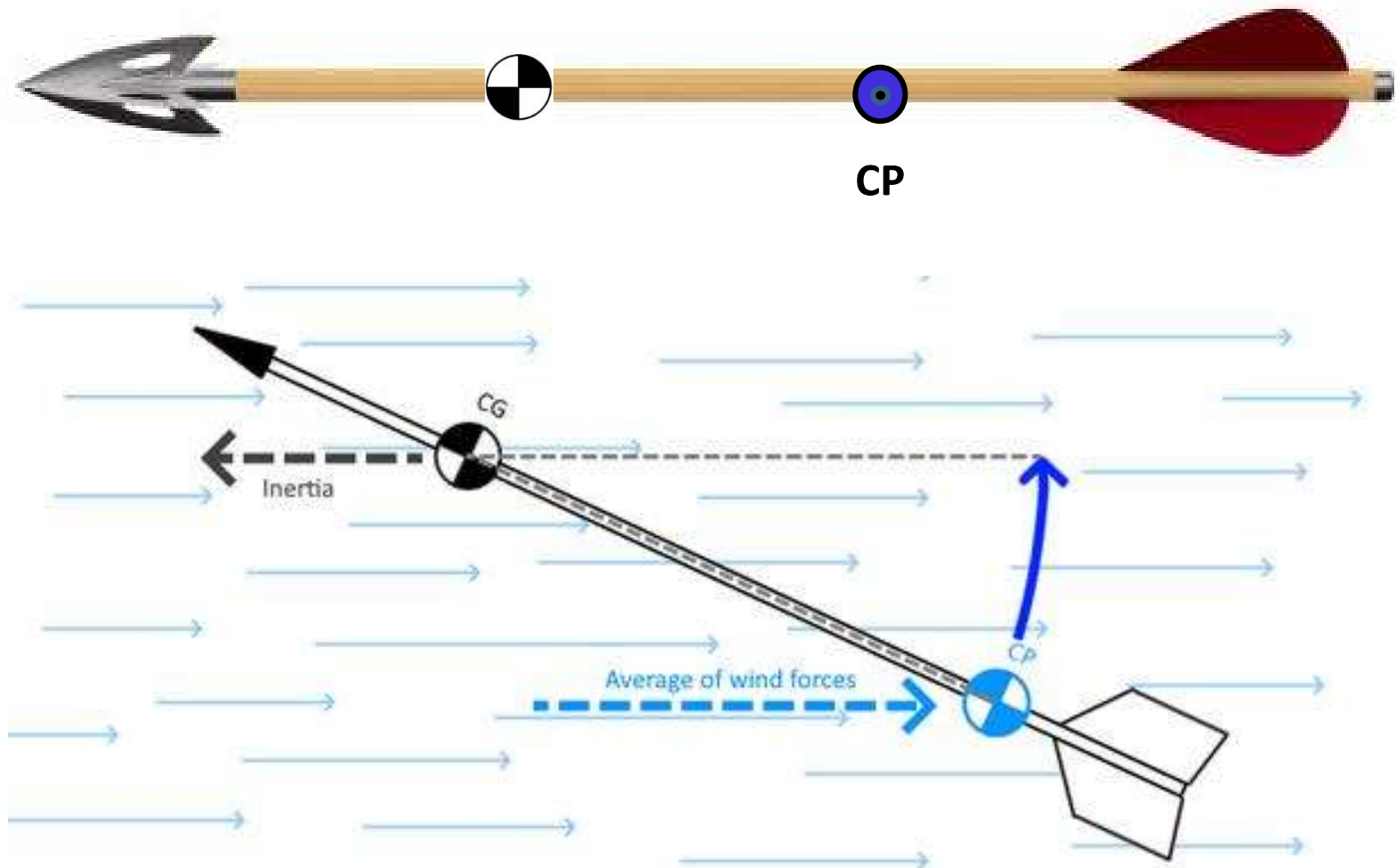
To have a stable aircraft, $C_{M\alpha}$ must be negative



To have a stable aircraft, $C_{M\alpha}$ must be negative



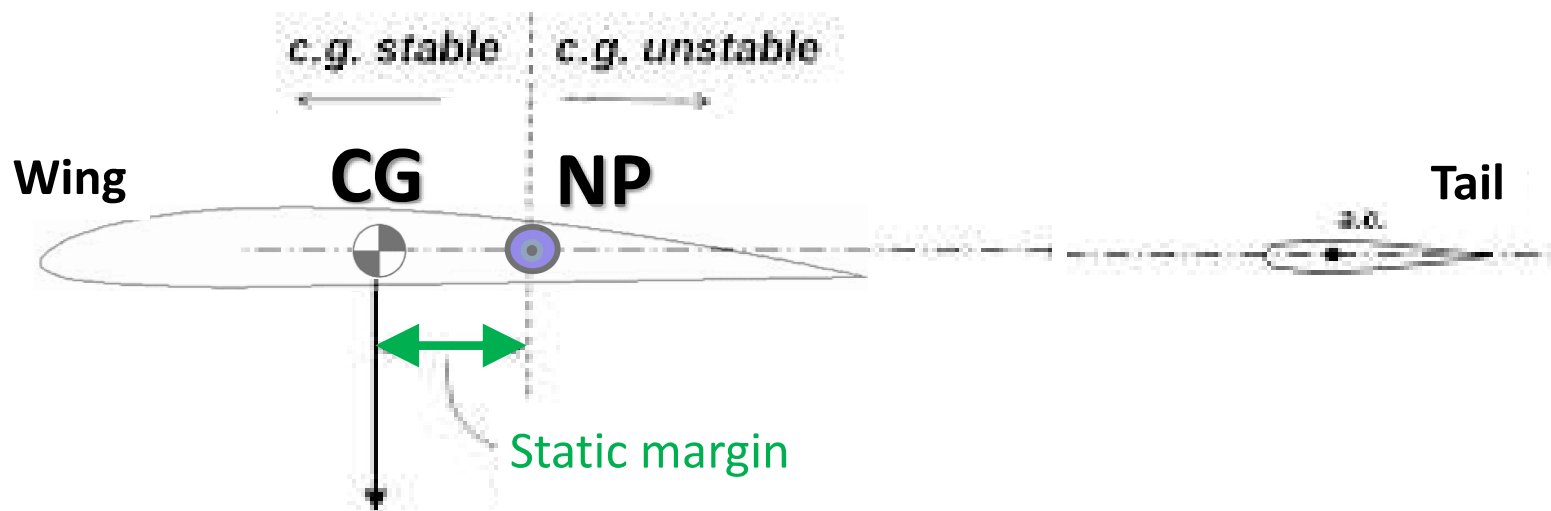
Even for Arrow, Wants CG to be Ahead of CP



The center of pressure (CP) is the point where the total sum of a pressure field acts on a body.

Static Margin

- The static margin is the distance between the center of gravity (CG) and the neutral point (NP). It is usually quoted as a percentage of the Mean Aerodynamic Chord. A desirable value is between 0.05 to 0.30



- The center of gravity must lie ahead of the neutral point for positive stability (positive static margin).

Static Margin for Several Aircraft

Cessna 172	0.19
Learjet 35	0.13
Boeing 747	0.27
North American P-51	0.05
Convair F-106	0.07
General Dynamics F-16A	-0.02
General Dynamics F-16C	0.01
Grumman X-29	-0.33
Your eVTOL	?



Case Study: Tandem Wing eVTOL Configuration



Challenging to design



Summary on Static Margin

- Some combat aircraft have the CG behind the NP and they are intentionally designed to be longitudinally unstable so they will be highly maneuverable. They rely on flight computer to stabilize the aircraft.
- There are many eVTOL aircraft configurations, some of these configuration may have poor static margin and must rely on flight computer to make the aircraft controllable.
- Ultimately, the position of the center of gravity relative to the neutral point determines the stability, control forces, and controllability of the vehicle.

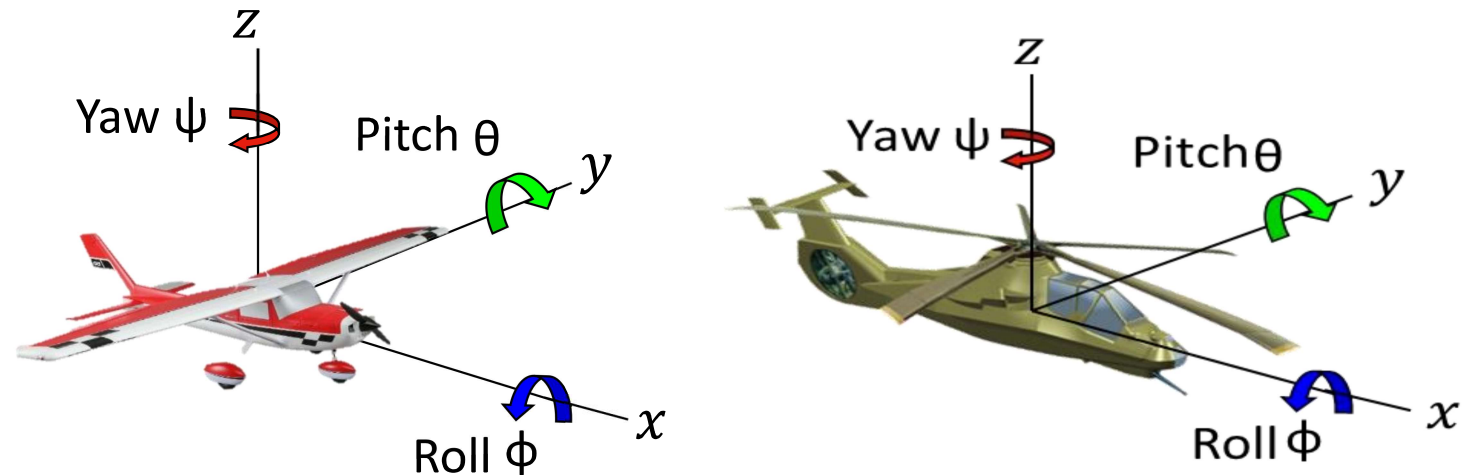
Stability & Control Analysis and Control Laws

- Equations of Motions (EOM)
- Stability derivatives
- Feedback loop

Let's Derive the Equations of Motion for a Helicopter

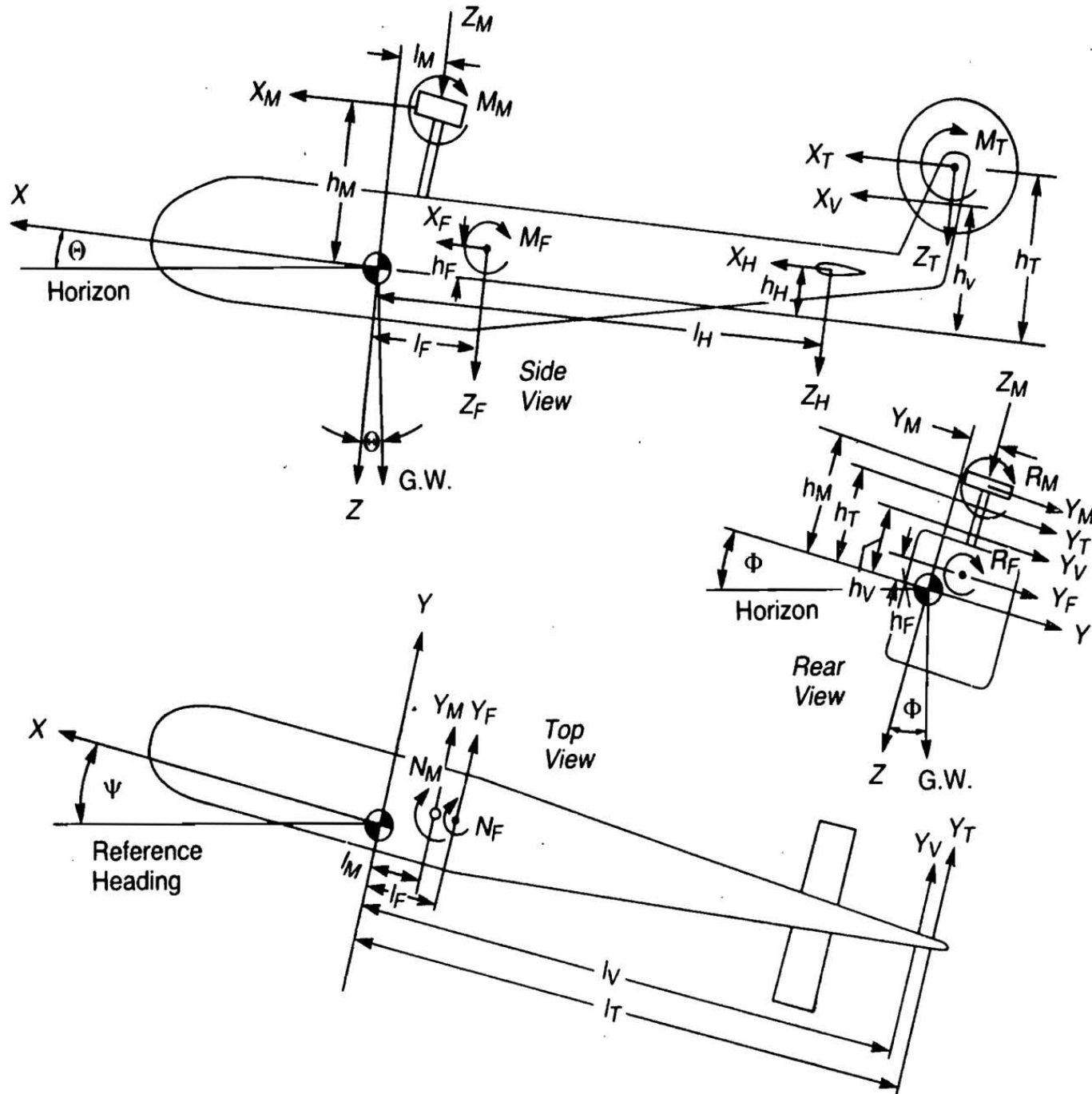
Aircraft's Equations of Motion (EOM)

- The forces and moments acting on any aircraft can be described by 6 EOM and 6 degrees of freedom (three x , y , z translations and three rotations: pitch, roll, yaw).

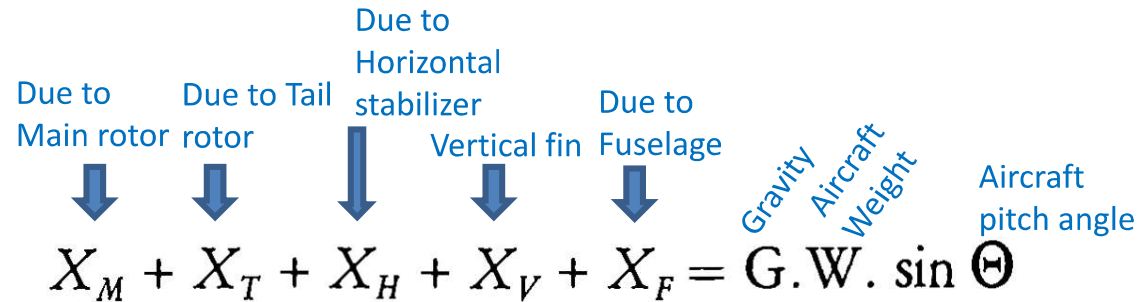


- The forces and moments acting on the aircraft are produced by aerodynamics acting on the wing, horizontal stabilizer, vertical fin, fuselage, main rotor, tail rotor and any other surfaces.

1st Step, Draw Force and Moment Diagrams



2nd Write the 3 Forces and 3 Moments Equations



Longitudinal force (forward)

$$X_M + X_T + X_H + X_V + X_F = G.W. \sin \Theta$$

Lateral force (right)

$$Y_M + Y_T + Y_V + Y_F = -G.W. \sin \Phi$$

Vertical force (down)

$$Z_M + Z_T + Z_H + Z_V + Z_F = -G.W. \cos \Theta$$

Rolling moment (down to right)

$$R_M + Y_M b_M + Z_M y_M + Y_T b_T + Y_V b_V + Y_F b_F + R_F = 0$$

Pitching moment (nose-up)

$$M_M - X_M b_M + Z_M l_M + M_T - X_T b_T + Z_T l_T - X_H b_H + Z_H l_H - X_V b_V + M_F + Z_F l_F - X_F b_F = 0$$

Yawing moment (nose to right)

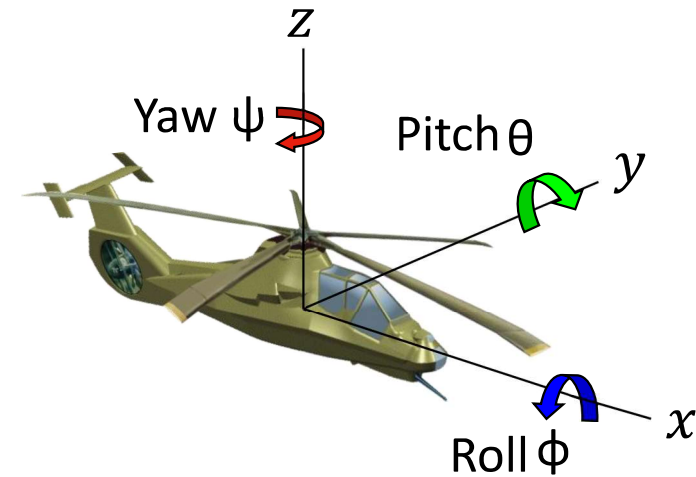
$$N_M - Y_M l_M - Y_T l_T - Y_V l_V + N_F - Y_F l_F = 0$$

Stability Derivatives

- Measure of how much a force or moment changes when there is a small change in flight condition parameters
- 10 flight parameters can be considered

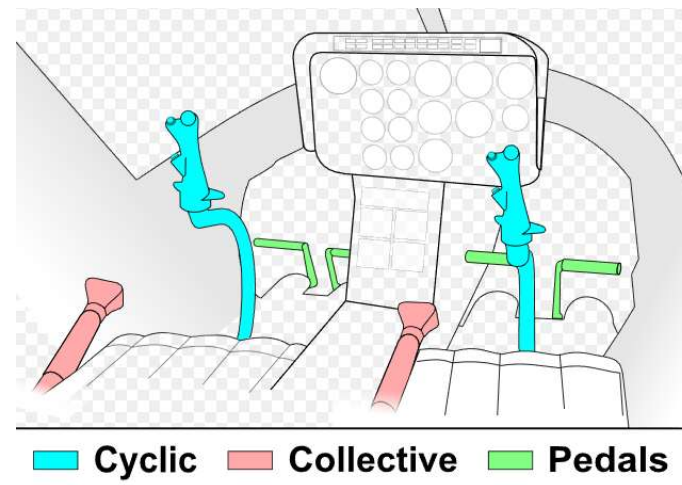
6 DOF

- pitch angle, θ Longitudinal translation, x
- roll angle, ϕ Lateral translation, y
- Yaw angle, ψ Vertical translation, z



4 pilot control inputs

- Longitudinal Cyclic Pitch, B_1
- Lateral Cyclic Pitch, A_1
- Collective Pitch, Θ_{0M}
- Tail Rotor Pitch, Θ_{0T}



Stability Derivatives

Force in X direction

Relate to the 6 DOF

$$X = \frac{\partial X}{\partial \dot{x}} \dot{x} + \frac{\partial X}{\partial \dot{y}} \dot{y} + \frac{\partial X}{\partial \dot{z}} \dot{z} + \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial p} p + \frac{\partial X}{\partial r} r + \frac{\partial X}{\partial \theta_{0M}} \theta_{0M} \\ + \frac{\partial X}{\partial \theta_{0T}} \theta_{0T} + \frac{\partial X}{\partial A_1} A_1 + \frac{\partial X}{\partial B_1} B_1$$

Relate to the 4 pilot control inputs

How to interpret these stability derivatives physically?

Example: $\frac{\partial X}{\partial \dot{x}}$ is a measure of how much a small change in longitudinal velocity affects the longitudinal force experienced by the helicopter.

Matrix form for the 6 EoM

	X	Z	Θ	Y	Φ	Ψ
	LONGITUDINAL					
X	$-\frac{G.W.}{g} s^2 + \frac{\partial X}{\partial \dot{x}} s$	$\frac{\partial X}{\partial \dot{z}} s$	$\left(\frac{\partial X}{\partial q} - \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s - G.W.$	$\frac{\partial X}{\partial j} s$	$\frac{\partial X}{\partial p} s$	$\frac{\partial X}{\partial r} s$
Z	$\frac{\partial Z}{\partial \dot{x}} s$	$\left(\frac{\partial Z}{\partial \dot{z}} - \frac{G.W.}{g} \right) s^2 + \frac{\partial Z}{\partial \dot{z}} s$	$\left(\frac{\partial Z}{\partial q} + \frac{G.W.}{g} \bar{V} \right) s$	$\frac{\partial Z}{\partial j} s$	$\frac{\partial Z}{\partial p} s$	$\frac{\partial Z}{\partial r} s$
M	$\frac{\partial M}{\partial \dot{x}} s$	$\frac{\partial M}{\partial \dot{z}} s^2 + \frac{\partial M}{\partial \dot{z}} s$	$-I_{yy} s^2 + \frac{\partial M}{\partial q} s$	$\frac{\partial M}{\partial j} s$	$\frac{\partial M}{\partial p} s$	$\frac{\partial M}{\partial r} s$
				LATERAL-DIRECTIONAL		
Y	$\frac{\partial Y}{\partial \dot{x}} s$	$\frac{\partial Y}{\partial \dot{z}} s$	$\frac{\partial Y}{\partial q} s$	$-\frac{G.W.}{g} s^2 + \frac{\partial Y}{\partial j} s$	$\left(\frac{\partial Y}{\partial p} + \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s + G.W.$	$\left(\frac{\partial Y}{\partial r} - \frac{G.W.}{g} \bar{V} \right) s$
R	$\frac{\partial R}{\partial \dot{x}} s$	$\frac{\partial R}{\partial \dot{z}} s$	$\frac{\partial R}{\partial q} s$	$\frac{\partial R}{\partial j} s$	$-I_{xx} s^2 + \frac{\partial R}{\partial p} s$	$\frac{\partial R}{\partial r} s$
N	$\frac{\partial N}{\partial \dot{x}} s$	$\frac{\partial N}{\partial \dot{z}} s$	$\frac{\partial N}{\partial q} s$	$\frac{\partial N}{\partial j} s$	$\frac{\partial N}{\partial p} s$	$-I_{zz} s^2 + \frac{\partial N}{\partial r} s$

$$F = ma = M\ddot{x} + C\dot{x} + Kx$$

$$M = I\ddot{\Theta} = (Ms^2 + Cs + K)\Theta$$

Matrix form for the 6 EoM

	X	Z	Θ	Y	Φ	Ψ
	<i>LONGITUDINAL</i>					
X	$-\frac{G.W.}{g} s^2 + \frac{\partial X}{\partial \dot{x}} s$	$\frac{\partial X}{\partial \dot{z}} s$	$\left(\frac{\partial X}{\partial q} - \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s - G.W.$	$\frac{\partial X}{\partial j} s$	$\frac{\partial X}{\partial p} s$	$\frac{\partial X}{\partial r} s$
Z	$\frac{\partial Z}{\partial \dot{x}} s$	$\left(\frac{\partial Z}{\partial \dot{z}} - \frac{G.W.}{g} \right) s^2 + \frac{\partial Z}{\partial \dot{z}} s$	$\left(\frac{\partial Z}{\partial q} + \frac{G.W.}{g} \bar{V} \right) s$	$\frac{\partial Z}{\partial j} s$	$\frac{\partial Z}{\partial p} s$	$\frac{\partial Z}{\partial r} s$
M	$\frac{\partial M}{\partial \dot{x}} s$	$\frac{\partial M}{\partial \dot{z}} s^2 + \frac{\partial M}{\partial \dot{z}} s$	$-I_{yy} s^2 + \frac{\partial M}{\partial q} s$	$\frac{\partial M}{\partial j} s$	$\frac{\partial M}{\partial p} s$	$\frac{\partial M}{\partial r} s$
	<i>LATERAL-DIRECTIONAL</i>					
Y	$\frac{\partial Y}{\partial \dot{x}} s$	$\frac{\partial Y}{\partial \dot{z}} s$	$\frac{\partial Y}{\partial q} s$	$-\frac{G.W.}{g} s^2 + \frac{\partial Y}{\partial j} s$	$\left(\frac{\partial Y}{\partial p} + \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s + G.W.$	$\left(\frac{\partial Y}{\partial r} - \frac{G.W.}{g} \bar{V} \right) s$
R	$\frac{\partial R}{\partial \dot{x}} s$	$\frac{\partial R}{\partial \dot{z}} s$	$\frac{\partial R}{\partial q} s$	$\frac{\partial R}{\partial j} s$	$-I_{xx} s^2 + \frac{\partial R}{\partial p} s$	$\frac{\partial R}{\partial r} s$
N	$\frac{\partial N}{\partial \dot{x}} s$	$\frac{\partial N}{\partial \dot{z}} s$	$\frac{\partial N}{\partial q} s$	$\frac{\partial N}{\partial j} s$	$\frac{\partial N}{\partial p} s$	$-I_{zz} s^2 + \frac{\partial N}{\partial r} s$

X (Longitudinal Translation)

Z (Vertical Translation)

M (Pitching Moment)

→ Contribute majorly in longitudinal motion

Matrix form for the 6 EoM

	X	Z	Θ	Y	Φ	Ψ
	LONGITUDINAL					
X	$-\frac{G.W.}{g} s^2 + \frac{\partial X}{\partial \dot{x}} s$	$\frac{\partial X}{\partial \dot{z}} s$	$\left(\frac{\partial X}{\partial q} - \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s - G.W.$	$\frac{\partial X}{\partial j} s$	$\frac{\partial X}{\partial p} s$	$\frac{\partial X}{\partial r} s$
Z	$\frac{\partial Z}{\partial \dot{x}} s$	$\left(\frac{\partial Z}{\partial \dot{z}} - \frac{G.W.}{g} \right) s^2 + \frac{\partial Z}{\partial \dot{z}} s$	$\left(\frac{\partial Z}{\partial q} + \frac{G.W.}{g} \bar{V} \right) s$	$\frac{\partial Z}{\partial j} s$	$\frac{\partial Z}{\partial p} s$	$\frac{\partial Z}{\partial r} s$
M	$\frac{\partial M}{\partial \dot{x}} s$	$\frac{\partial M}{\partial \dot{z}} s^2 + \frac{\partial M}{\partial \dot{z}} s$	$-I_{yy} s^2 + \frac{\partial M}{\partial q} s$	$\frac{\partial M}{\partial j} s$	$\frac{\partial M}{\partial p} s$	$\frac{\partial M}{\partial r} s$
				LATERAL-DIRECTIONAL		
Y	$\frac{\partial Y}{\partial \dot{x}} s$	$\frac{\partial Y}{\partial \dot{z}} s$	$\frac{\partial Y}{\partial q} s$	$-\frac{G.W.}{g} s^2 + \frac{\partial Y}{\partial j} s$	$\left(\frac{\partial Y}{\partial p} + \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s + G.W.$	$\left(\frac{\partial Y}{\partial r} - \frac{G.W.}{g} \bar{V} \right) s$
R	$\frac{\partial R}{\partial \dot{x}} s$	$\frac{\partial R}{\partial \dot{z}} s$	$\frac{\partial R}{\partial q} s$	$\frac{\partial R}{\partial j} s$	$-I_{xx} s^2 + \frac{\partial R}{\partial p} s$	$\frac{\partial R}{\partial r} s$
N	$\frac{\partial N}{\partial \dot{x}} s$	$\frac{\partial N}{\partial \dot{z}} s$	$\frac{\partial N}{\partial q} s$	$\frac{\partial N}{\partial j} s$	$\frac{\partial N}{\partial p} s$	$-I_{zz} s^2 + \frac{\partial N}{\partial r} s$

Y (Lateral Translation)
 R (Rolling Moment)
 N (Yawing Moment)

→ Contribute majorly in lateral motion

Matrix form for the 6 EoM

	X	Z	Θ	Y	Φ	Ψ
	LONGITUDINAL					
X	$-\frac{G.W.}{g} s^2 + \frac{\partial X}{\partial \dot{x}} s$	$\frac{\partial X}{\partial \dot{z}} s$	$\left(\frac{\partial X}{\partial q} - \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s - G.W.$	$\frac{\partial X}{\partial j} s$	$\frac{\partial X}{\partial p} s$	$\frac{\partial X}{\partial r} s$
Z	$\frac{\partial Z}{\partial \dot{x}} s$	$\left(\frac{\partial Z}{\partial \dot{z}} - \frac{G.W.}{g} \right) s^2 + \frac{\partial Z}{\partial \dot{z}} s$	$\left(\frac{\partial Z}{\partial q} + \frac{G.W.}{g} \bar{V} \right) s$	$\frac{\partial Z}{\partial j} s$	$\frac{\partial Z}{\partial p} s$	$\frac{\partial Z}{\partial r} s$
M	$\frac{\partial M}{\partial \dot{x}} s$	$\frac{\partial M}{\partial \dot{z}} s^2 + \frac{\partial M}{\partial \dot{z}} s$	$-I_{yy} s^2 + \frac{\partial M}{\partial q} s$	$\frac{\partial M}{\partial j} s$	$\frac{\partial M}{\partial p} s$	$\frac{\partial M}{\partial r} s$
				LATERAL-DIRECTIONAL		
Y	$\frac{\partial Y}{\partial \dot{x}} s$	$\frac{\partial Y}{\partial \dot{z}} s$	$\frac{\partial Y}{\partial q} s$	$-\frac{G.W.}{g} s^2 + \frac{\partial Y}{\partial j} s$	$\left(\frac{\partial Y}{\partial p} + \frac{G.W.}{g} \bar{V} \bar{\Theta} \right) s + G.W.$	$\left(\frac{\partial Y}{\partial r} - \frac{G.W.}{g} \bar{V} \right) s$
R	$\frac{\partial R}{\partial \dot{x}} s$	$\frac{\partial R}{\partial \dot{z}} s$	$\frac{\partial R}{\partial q} s$	$\frac{\partial R}{\partial j} s$	$-I_{xx} s^2 + \frac{\partial R}{\partial p} s$	$\frac{\partial R}{\partial r} s$
N	$\frac{\partial N}{\partial \dot{x}} s$	$\frac{\partial N}{\partial \dot{z}} s$	$\frac{\partial N}{\partial q} s$	$\frac{\partial N}{\partial j} s$	$\frac{\partial N}{\partial p} s$	$-I_{zz} s^2 + \frac{\partial N}{\partial r} s$

Minimal contribution to longitudinal & lateral motion (coupled effects)

- Can be usually ignored
- With computer analysis, can be left in

The 6x6 Matrix

- models the natural characteristics of the aircraft/helicopter (plant)
- The eigenvalues of the 6x6 matrix reveals the natural stability of the plant.
- Can plot the eigenvalues (poles and zeros) on a root locus plot → The plant is stable if all poles are on the left half side of the root locus plot.

Matrix form for 4 Control inputs that generates the Rotor Forces and Moments

	θ_{oM}	θ_{oT}	A_1	B_1	
$x(s)$	$-\frac{\partial X}{\partial \theta_{oM}}$	$-\frac{\partial X}{\partial \theta_{oT}}$	$-\frac{\partial X}{\partial A_1}$	$-\frac{\partial X}{\partial B_1}$	θ_{oM}
$z(s)$	$-\frac{\partial Z}{\partial \theta_{oM}}$	$-\frac{\partial Z}{\partial \theta_{oT}}$	$-\frac{\partial Z}{\partial A_1}$	$-\frac{\partial Z}{\partial B_1}$	θ_{oT}
$\Theta(s)$	$-\frac{\partial M}{\partial \theta_{oM}}$	$-\frac{\partial M}{\partial \theta_{oT}}$	$-\frac{\partial M}{\partial A_1}$	$-\frac{\partial M}{\partial B_1}$	A_1
$y(s)$	$-\frac{\partial Y}{\partial \theta_{oM}}$	$-\frac{\partial Y}{\partial \theta_{oT}}$	$-\frac{\partial Y}{\partial A_1}$	$-\frac{\partial Y}{\partial B_1}$	B_1
$\Phi(s)$	$-\frac{\partial R}{\partial \theta_{oM}}$	$-\frac{\partial R}{\partial \theta_{oT}}$	$-\frac{\partial R}{\partial A_1}$	$-\frac{\partial R}{\partial B_1}$	
$\Psi(s)$	$-\frac{\partial N}{\partial \theta_{oM}}$	$-\frac{\partial N}{\partial \theta_{oT}}$	$-\frac{\partial N}{\partial A_1}$	$-\frac{\partial N}{\partial B_1}$	

Vertical = heli main rotor pitch or airplane throttle

Yaw = heli tail rotor pitch or airplane rudder control

Roll = heli left/right cyclic or airplane aileron control

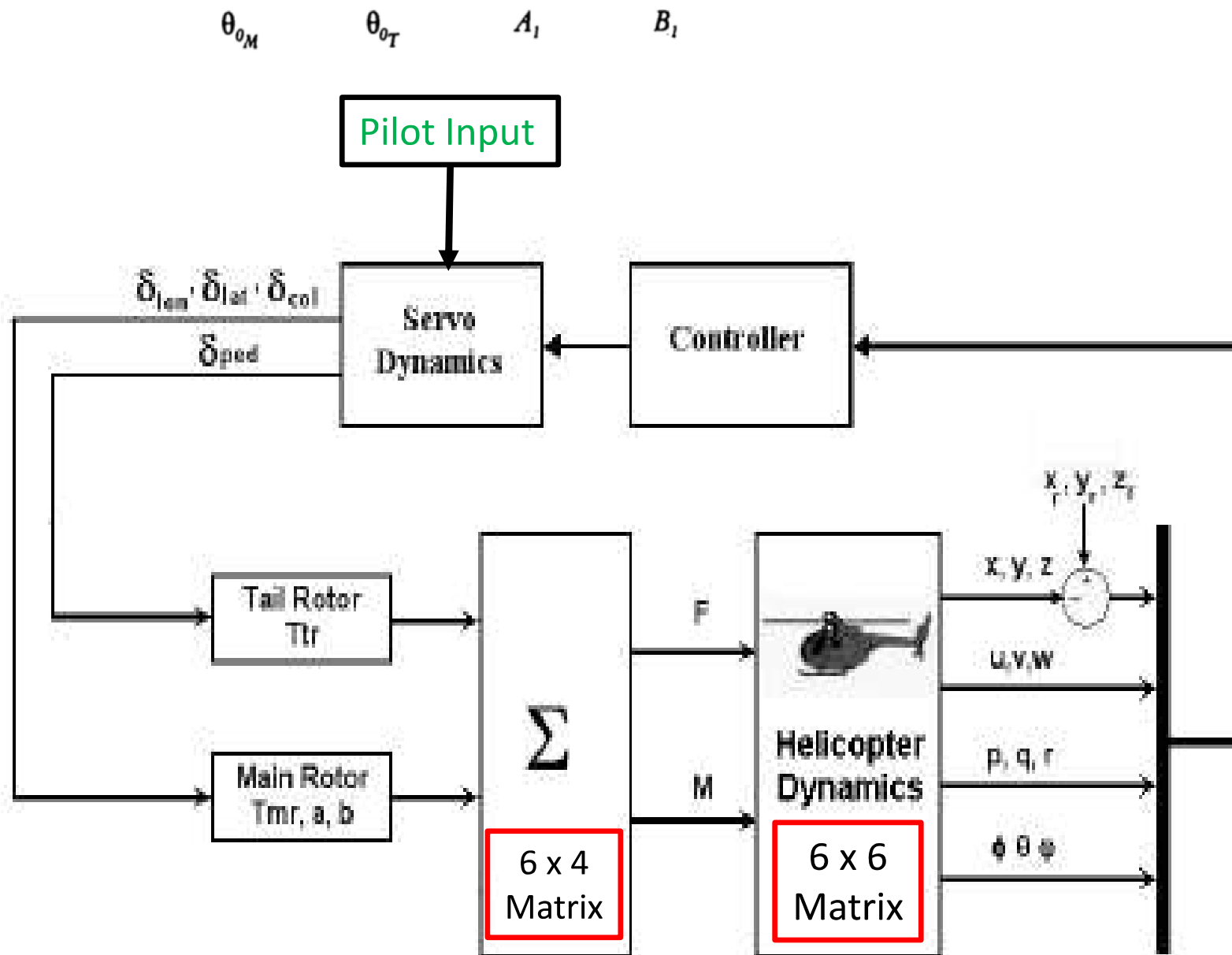
Pitch = heli fore/aft cyclic or airplane elevator control

6 DOF

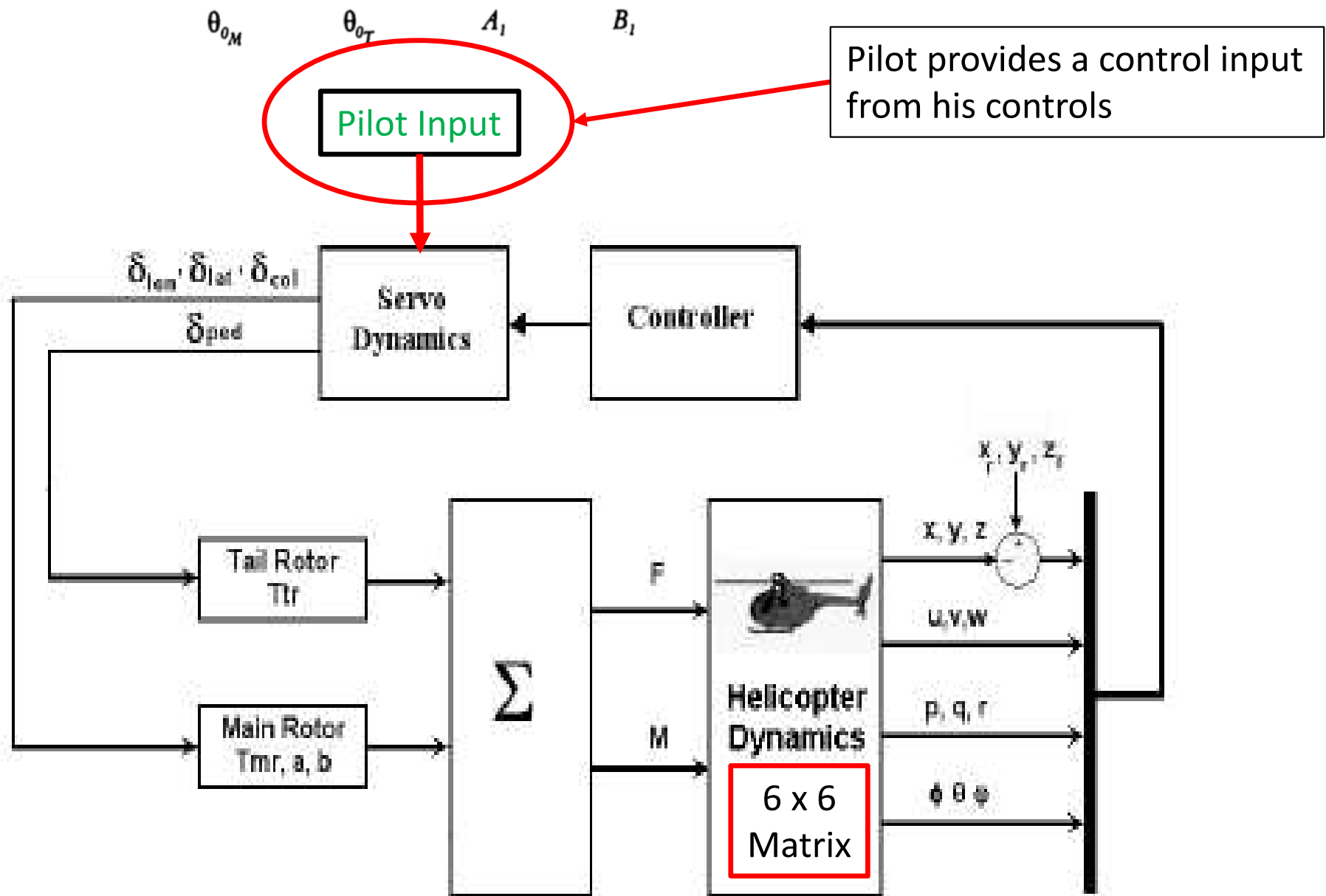
6x4 matrix of Control derivatives

4 pilot controls

Feedback loop (Block Diagram)

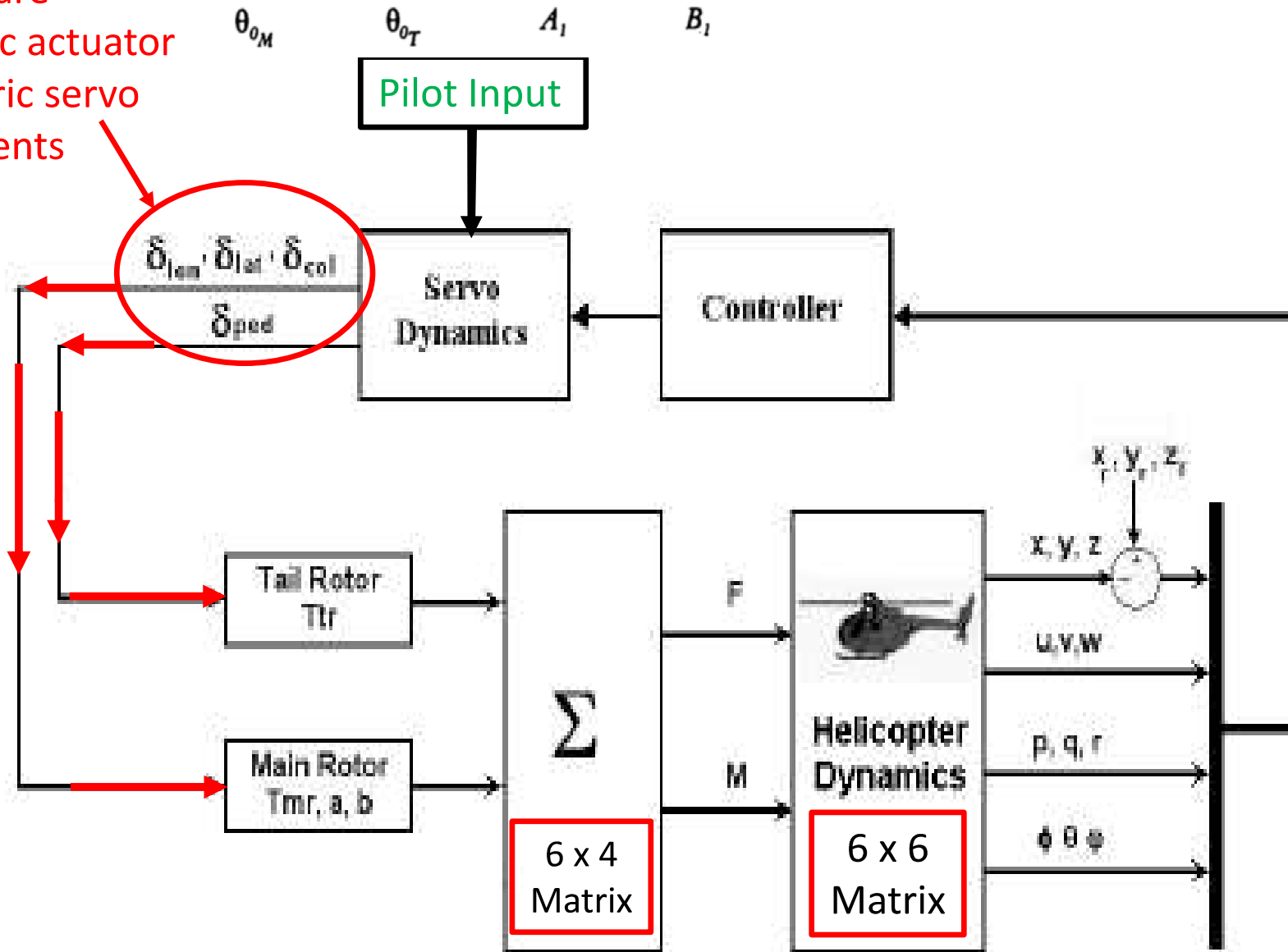


Feedback loop (Block Diagram)

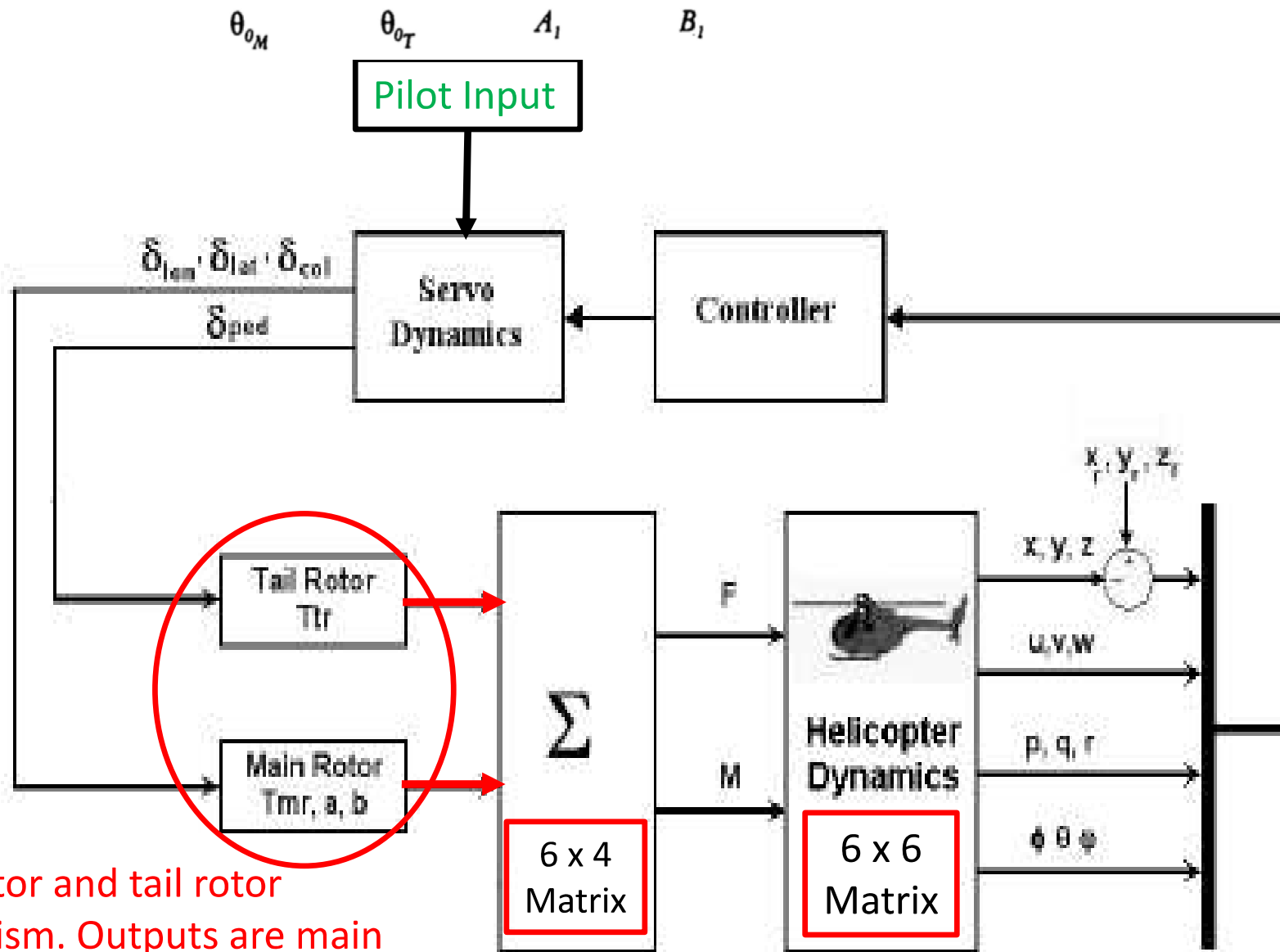


Feedback loop (Block Diagram)

Output are hydraulic actuator or electric servo movements

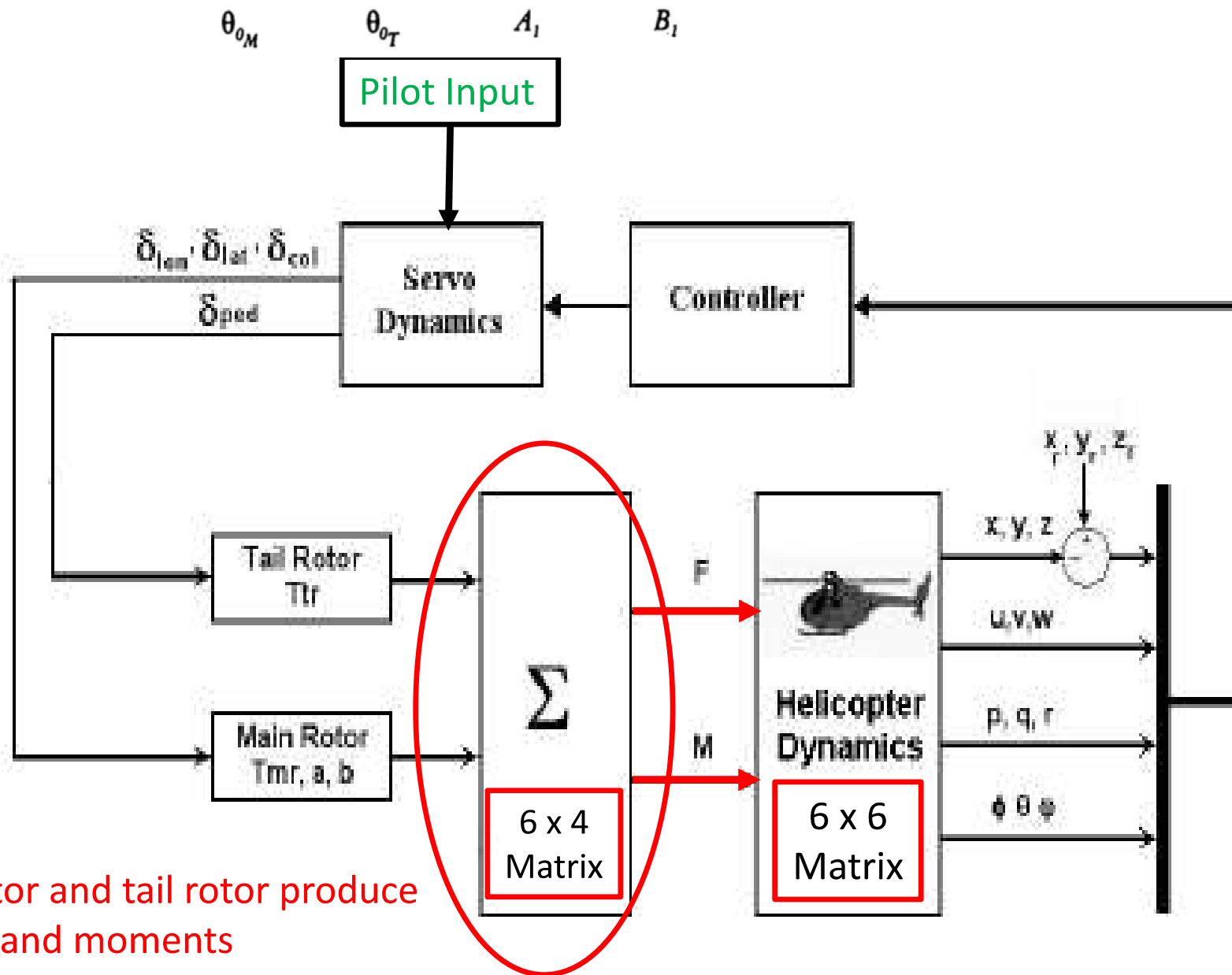


Feedback loop (Block Diagram)



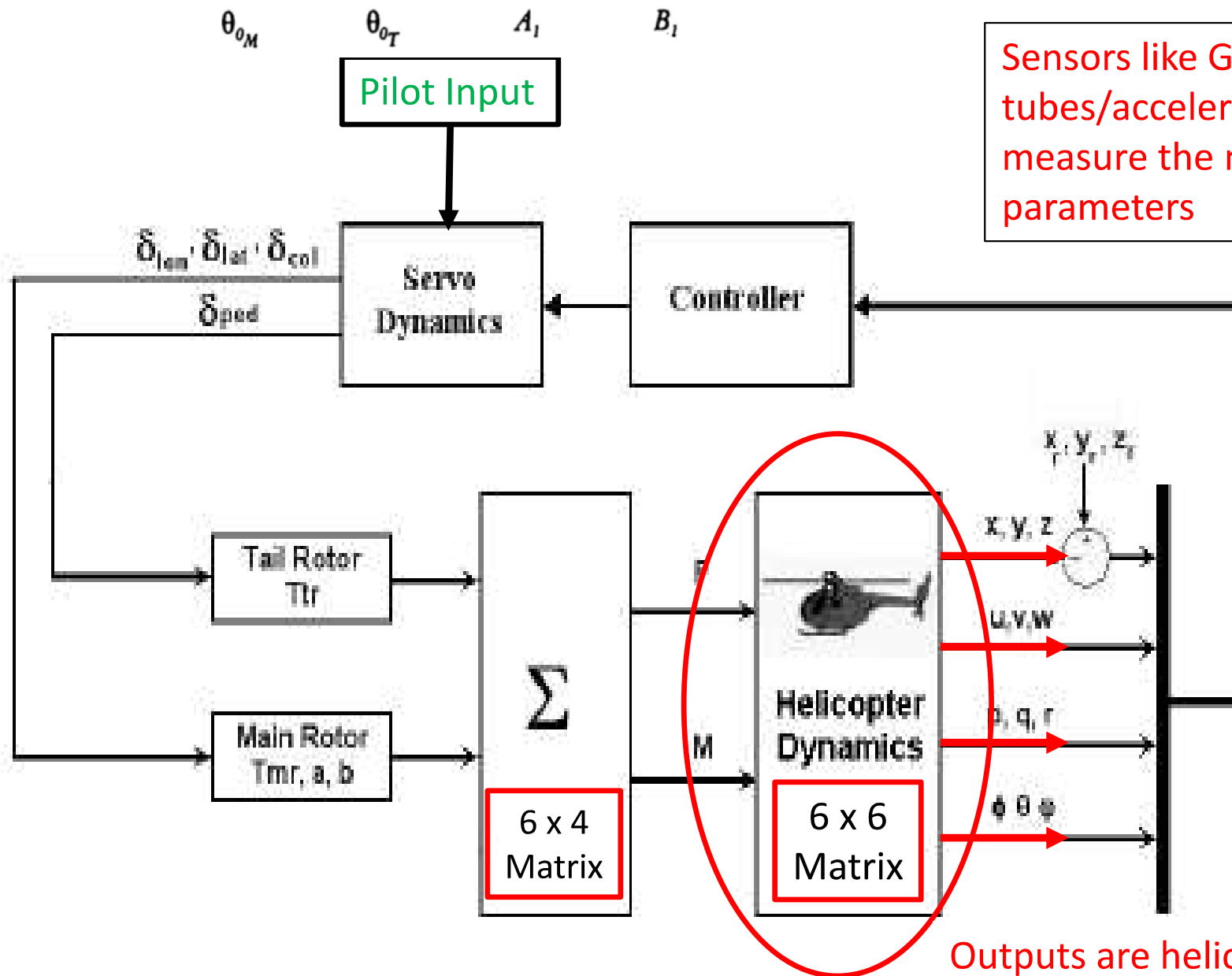
Main rotor and tail rotor mechanism. Outputs are main and tail blade pitch angle changes

Feedback loop (Block Diagram) [4]



Main rotor and tail rotor produce 3 forces and moments

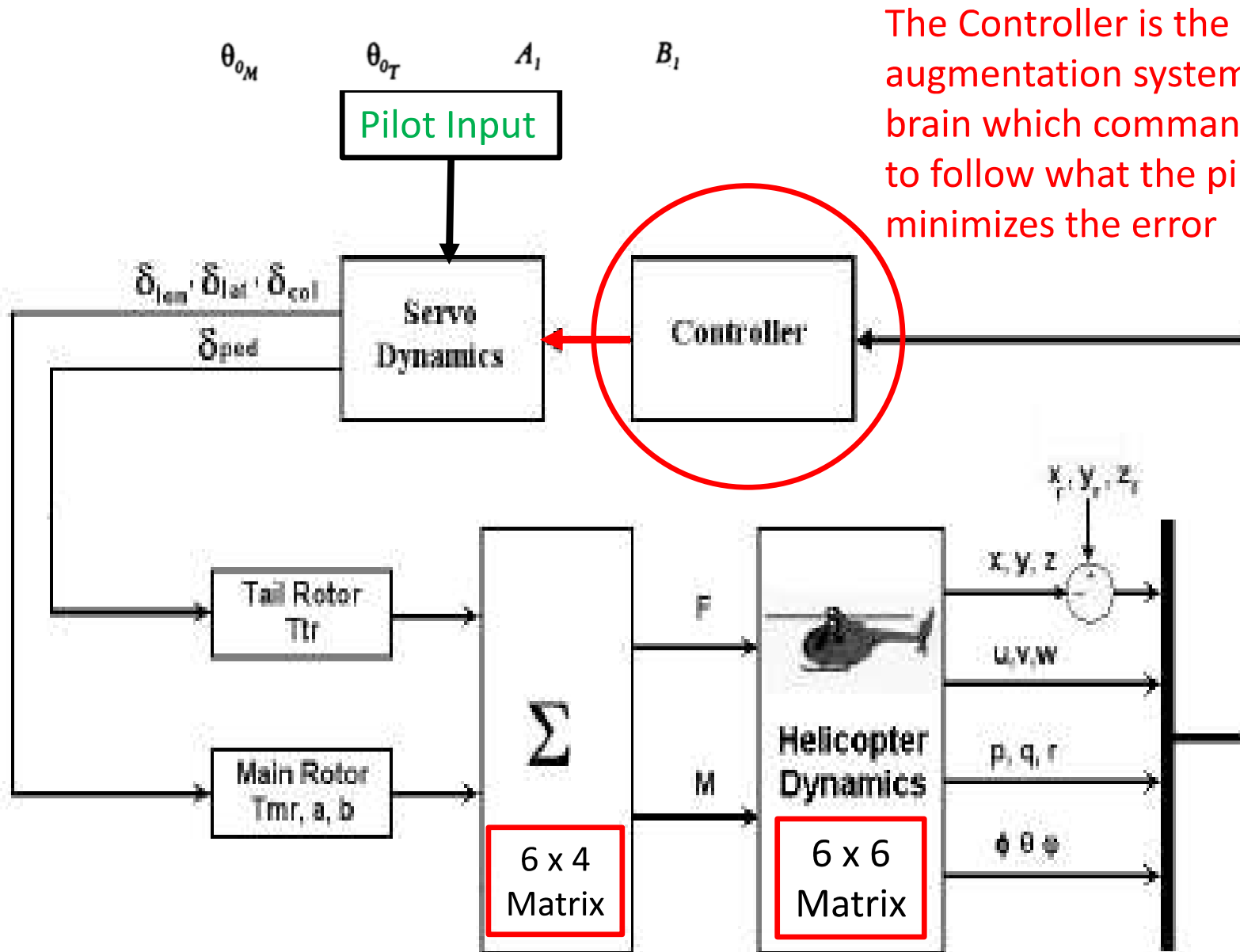
Feedback loop (Block Diagram)



Sensors like GPS/pitot tubes/accelerometers measure the motion parameters

Outputs are helicopter motions

Feedback loop (Block Diagram)



The Controller is the SAS (stability augmentation system) and the brain which commands the servos to follow what the pilot wants and minimizes the error

How To Implement the Theory and do Flight Simulation

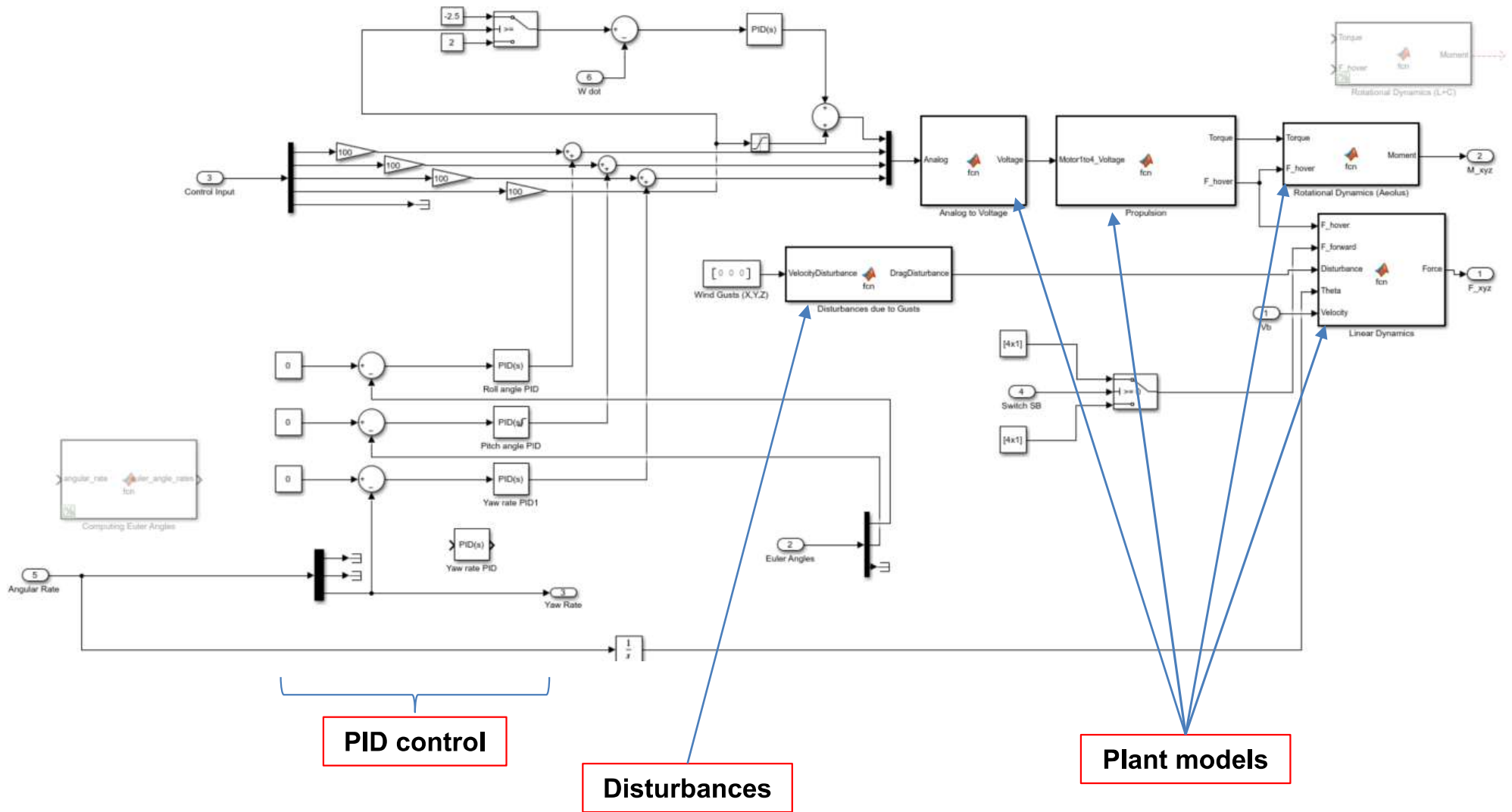
- There are off-the-shelf software to do stability and feedback analysis.
- Can model the equations in **Matlab**, and use **Simulink** to do a simulation of the system.
- Then link **Simulink** to **FlightGear** to do the graphic display
- **FlightLab** is a professional software developed by ART to model the complete flight mechanics of different helicopters and airplanes.

FlightGear Simulator

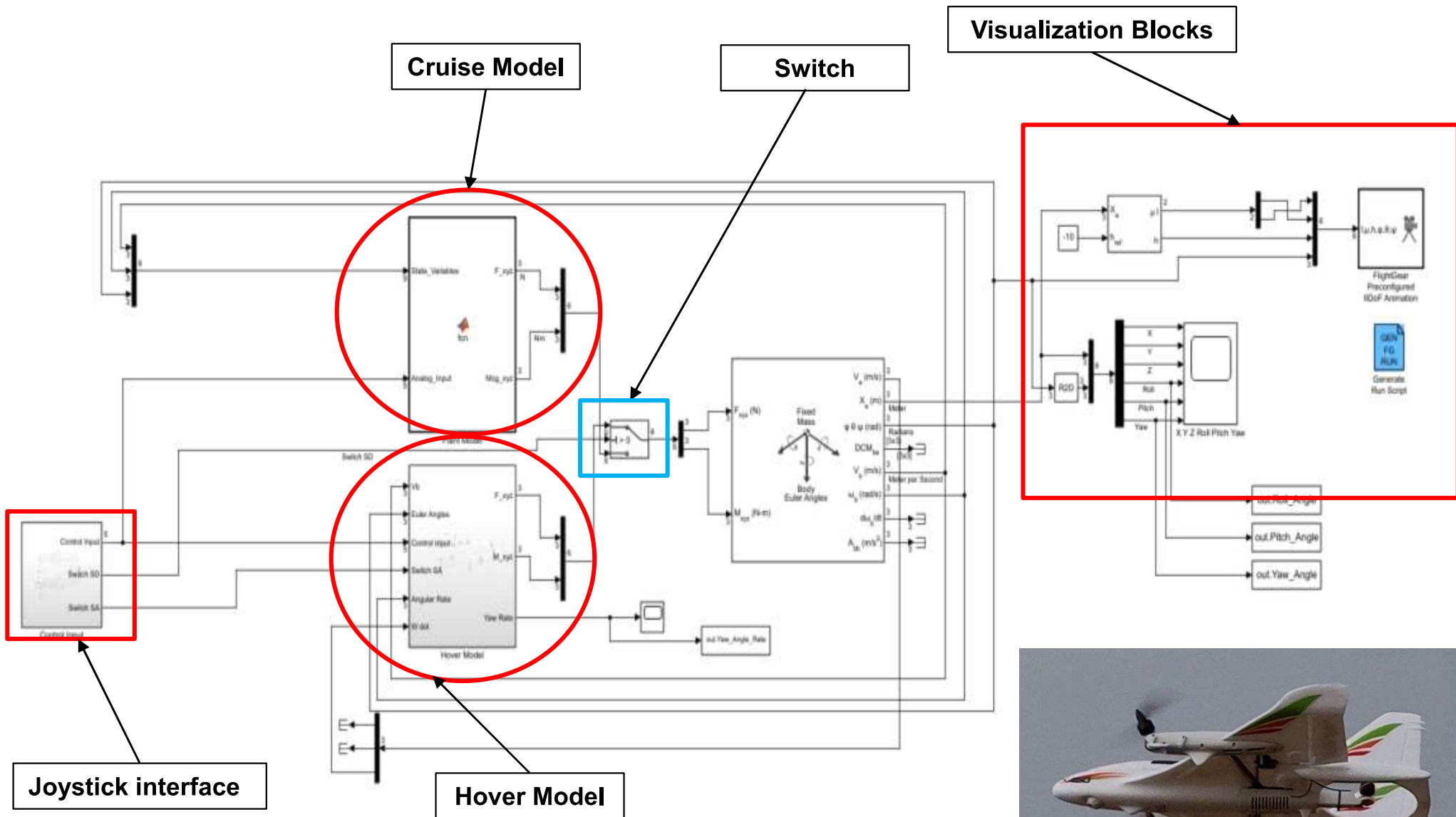


- Free open source flight simulator
- Visualise position and attitude of aircraft model, whilst rendering surrounding environment
- Information in Simulink model sent to FlightGear for 3D rendering/visualisation
- Customised aircraft model exported from Blender (graphic software) into FlightGear and simulated using transition model in Simulink

Example Simulink Model for an Hover eVTOL

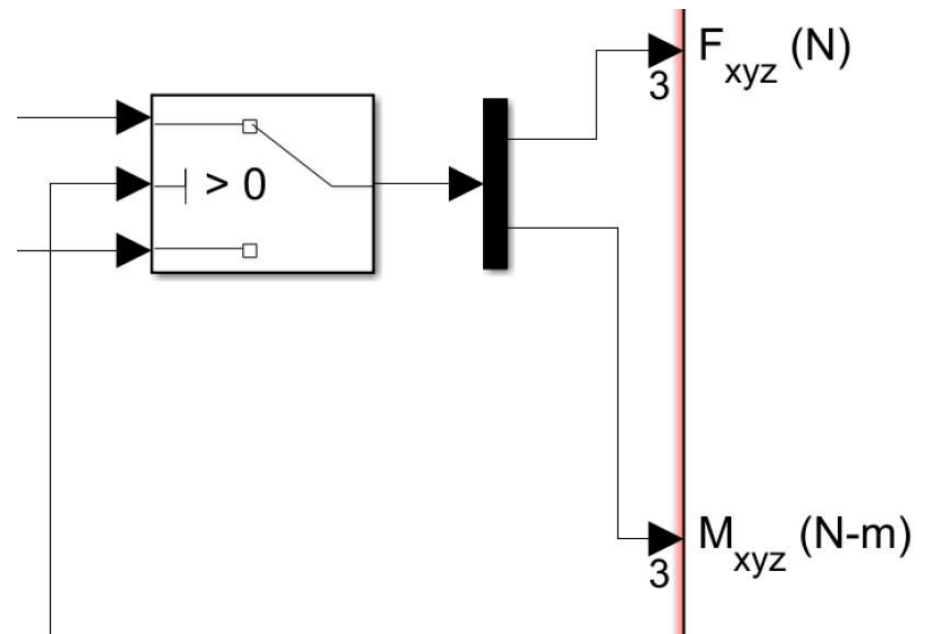


Components of Transition Model Using Switch



Switch tool

- Switches between cruise and hover modes, depending on the control input provided from controller
- A 6x1 matrix (F & M) from both cruise and hover models are input to this switch (top & bottom)
- Middle input is from controller

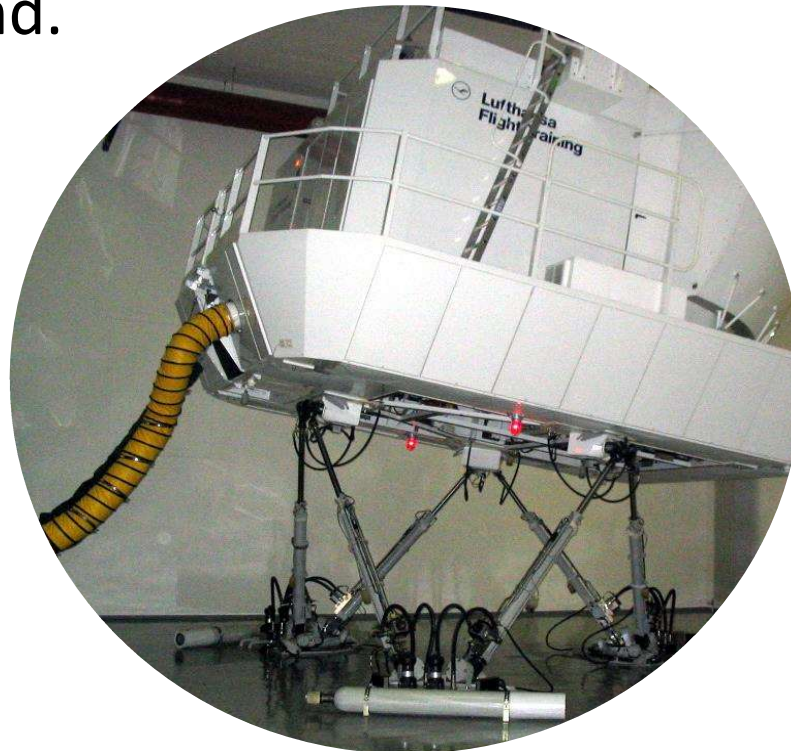


X-Plane Flight Simulator Software

- Latest release: X-Plane 11.55 (2021)
- X-Plane differentiates itself from other simulators by incorporating **Blade Element Theory** to compute lift forces
- “Plane Maker” software allows users to customise their aircraft and fly without input equations
 - Shape body of aircraft
 - Work with aircraft system
 - Modify aircraft properties
 - Perform flight tests
 - Starting at under \$100 for a personal version

Physical Flight Simulators

- Flight simulators have different level of qualifications depending its complexity
- It ranges from Level A to D where A has a motion system with at least 3 degrees of freedom and D has all degrees of freedom with a digital display of at least 150 degrees which provides realistic visuals and sound.



**Use models for flight testing and
for validating configuration**

Small RC Models Can provide Insights



Transition Mode with Rotor Thrust Line at 45°

Modified the flight controller to allow inflight changing rotor tilt to any angle



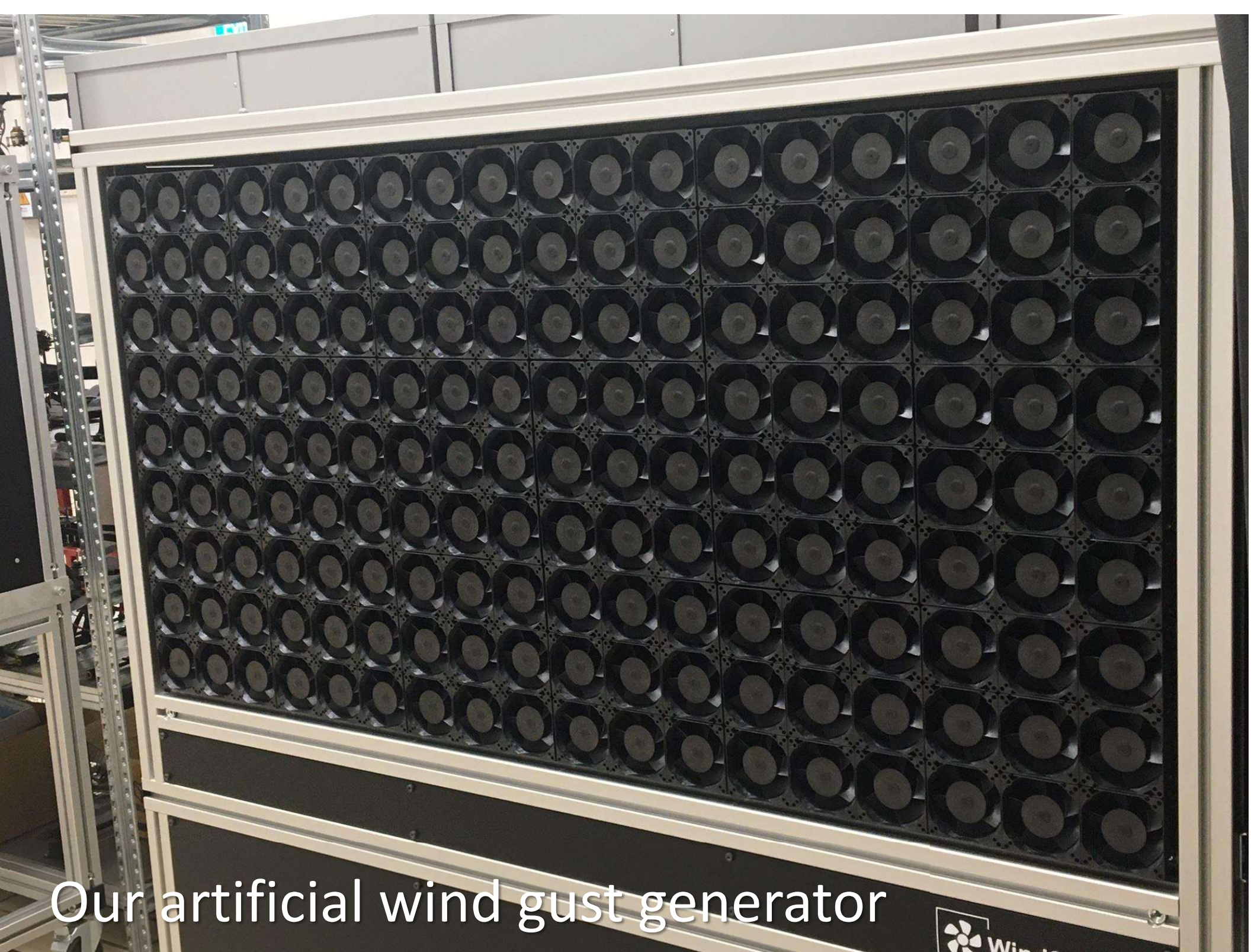
Hovering with Rotor Thrust Line at 45°



**With a wing, an eVTOL becomes
much more sensitive to gust,
especially from the lateral
direction**

Our Indoor Testing Room





Our artificial wind gust generator



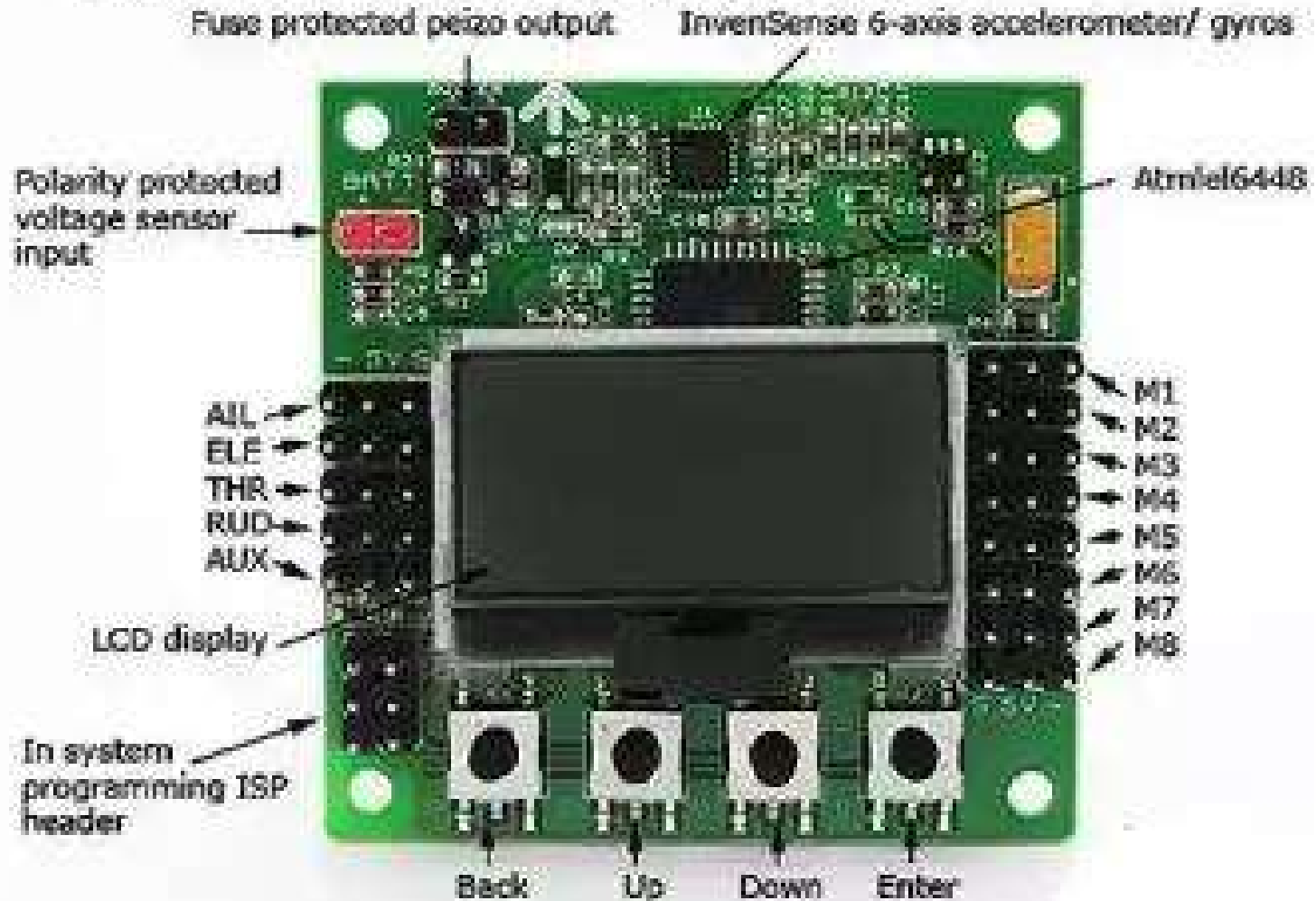
Arm with 6-Axis Balance



Flight Controllers

KK2 Controller (about US\$25)

KK 2.1.5 Multi-Rotor control Board

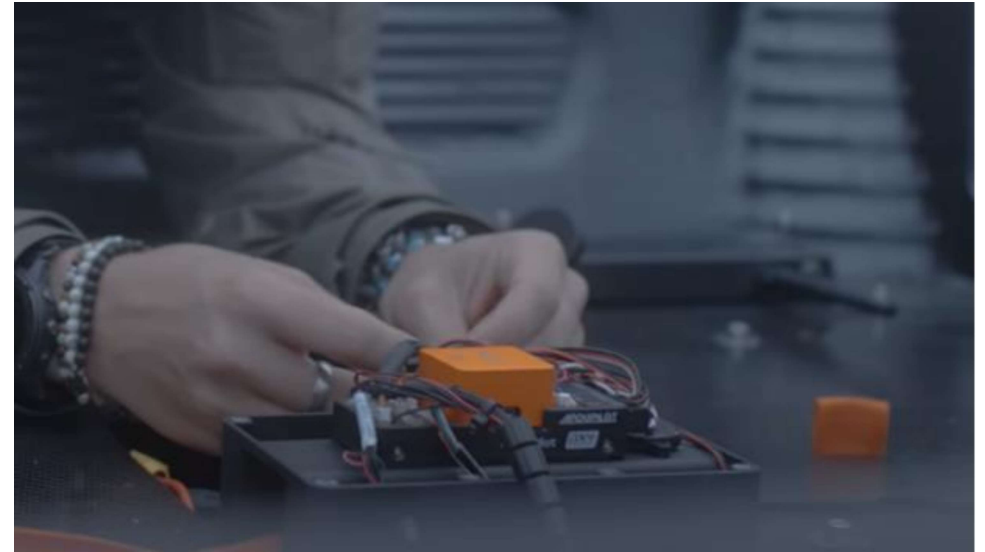
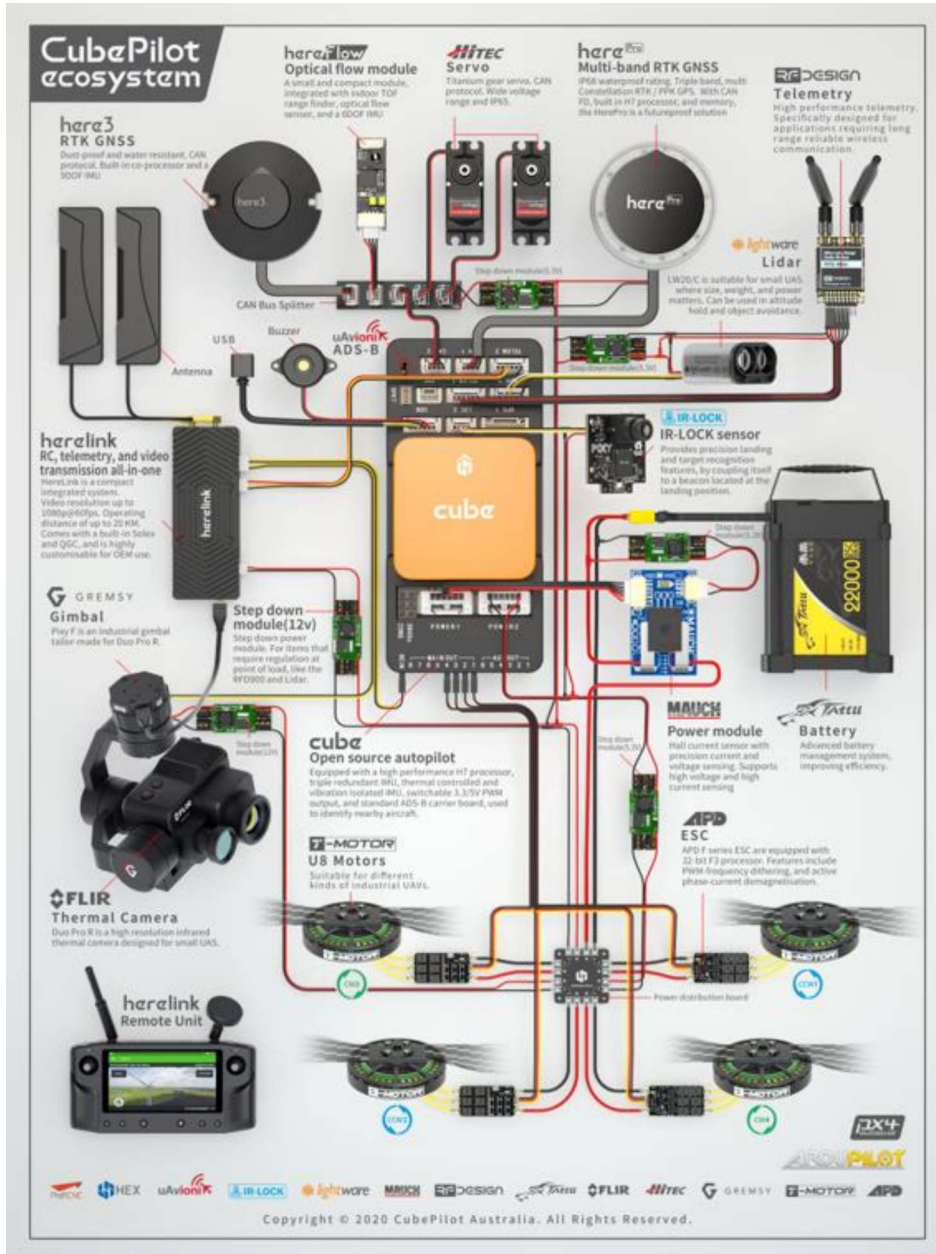


Pixhawk 4 Controller (about US\$400)

Main FMU Processor	STM32F765 32 Bit Arm Cortex-M7, 216Mhz, 2MB Memory, 512KB RAM
IO Processor	STM32F100 32 Bit Arm Cortex-M3, 24Mhz 8KB SRAM
Sensors	2 3-axis Accel/Gyro 1 Magnetometer 1 Barometer 1 GPS (External Module)
Interfaces	8-16 PWM outputs (8 IO, 8 FMU) 3 dedicated PWM/Capture inputs on FMU 5 general purpose serial ports
Open Source AP	ArduPilot, PX4, Paparazzi



Many Companies Make Pixhawk Hardware



Source: [Pixhawk Orange Cube](https://www.pixhawk.org/)

PX4 is the Control Laws Software



- An open source flight control software for drones and other unmanned vehicles
- Runs on NuttX RTOS
- Software can be setup using Ground Station software or built using an IDE or console

PX4 Built-in Configurations

The screenshot displays the PX4 Airframe Setup interface. The top menu bar includes 'File', 'Widgets', and a 'Manual' link. The left sidebar contains navigation options: 'Vehicle Setup', 'Summary', 'Firmware', 'Airframe' (selected), 'Radio', 'Sensors', 'Flight Modes', 'Power', 'Safety', 'Tuning', 'Camera', and 'Parameters'. The main area is titled 'Airframe Setup' and contains the following text: 'Airframe Setup is used to select the airframe which matches your vehicle. This will in turn set up the various tuning values for flight parameters. Airframe is not set. To change this configuration, select the desired airframe below then click "Apply and Restart".' An 'Apply and Restart' button is located in the top right corner.

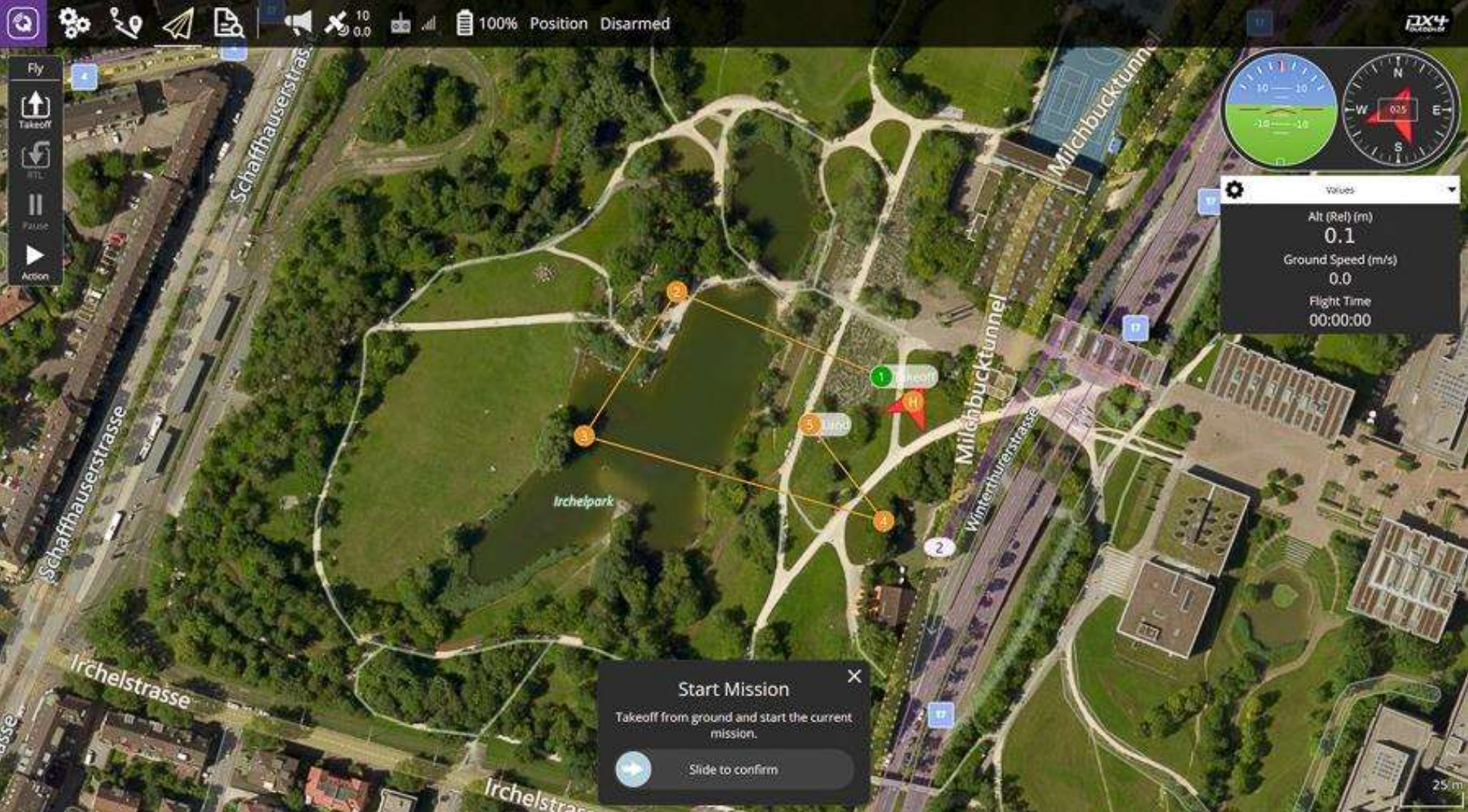
The interface is organized into a grid of airframe categories, each with a representative icon and a dropdown menu:

- Coaxial Helicopter:** Esky (Big) Lama v4
- Flying Wing:** TBS Caipirinha
- Helicopter:** Blade 130X
- Hexarotor +:** Generic Hexarotor + geometry
- Hexarotor Coaxial:** Generic Hexa coaxial geometry
- Hexarotor x:** Generic Hexarotor x geometry
- Octo Coax Wide:** Steadidrone MAVRIK
- Octorotor +:** Generic Octocopter + geometry
- Octorotor Coaxial:** Generic 10" Octo coaxial geometry
- Octorotor x:** Generic Octocopter X geometry
- Plane A-Tail:** Applied Aeronautics Albatross
- Quadrotor +:** Generic 10" Quad + geometry
- Quadrotor Wide:** Team Blacksheep Discovery Endu... (highlighted)
- Quadrotor x:** DJI Flame Wheel F450
- Rover:** Axial Racing AX10
- Simulation:** (empty)
- Standard Plane:** (empty)
- Tricopter Y+:** (empty)
- Tricopter Y-:** (empty)

A dropdown menu is open for the 'Team Blacksheep Discovery Endu...' configuration, showing the following options:

- Team Blacksheep Discovery Endurance
- Team Blacksheep Discovery
- 3DR Iris Quadrotor
- Steadidrone QU4D

PX4 QGroundControl



PX4 QGroundControl



- Ground Station Software for MAVLink Protocol
- Firmware installation
- Calibration and Setup of Drone
- Allows you to send commands to drone from the ground
- Functions such as: Waypoints, GPS, Flightpath
- Provides various telemetry data

PX4 QGroundControl

The screenshot displays the PX4 QGroundControl interface. At the top, a status bar shows various icons including a gear, location, paper plane, document, speaker, and a battery icon with '100%' and 'Hold Disarmed'. The 'PX4 autopilot pro' logo is in the top right. A left sidebar contains menu items: Vehicle Setup, Summary, Firmware, Airframe, Radio, Sensors, Flight Modes, Power, Safety, Tuning, Camera, and Parameters (highlighted in yellow). The main area features a search bar and a 'Clear' button. Below is a table of battery parameters.

Component #	Parameter	Value	Description
1	BAT_A_PER_V	15.39103031	Battery current per volt (A/V)
*Default Group	BAT_CAPACITY	-1 mA	Battery capacity
Battery Calibration	BAT_CNT_V_CURR	0.00080566	Scaling from ADC counts to volt on the ADC input (battery current)
Camera trigger	BAT_CNT_V_VOLT	0.00080566	Scaling from ADC counts to volt on the ADC input (battery voltage)
Circuit Breaker	BAT_CRIT_THR	7 %	Critical threshold
Commander	BAT_EMERGEN_THR	5 %	Emergency threshold
Data Link Loss	BAT_LOW_THR	15 %	Low threshold
EKF2	BAT_N_CELLS	3S Battery	Number of cells
FW Attitude Control	BAT_R_INTERNAL	-1.000 Ohms	Explicitly defines the per cell internal resistance
FW L1 Control	BAT_SOURCE	Power Module	Battery monitoring source
FW Launch detection	BAT_V_CHARGED	4.05 V	Full cell voltage (5C load)
FW TECS	BAT_V_DIV	10.17793941	Battery voltage divider (V divider)
Follow target	BAT_V_EMPTY	3.40 V	Empty cell voltage (5C load)
GPS Failure Navigation	BAT_V_LOAD_DROP	0.30 V	Voltage drop per cell on full throttle
Geofence	BAT_V_OFFS_CURR	0.00000000	Offset in volt as seen by the ADC input of the current sensor
Land Detector			

PX4 Logging Telemetry

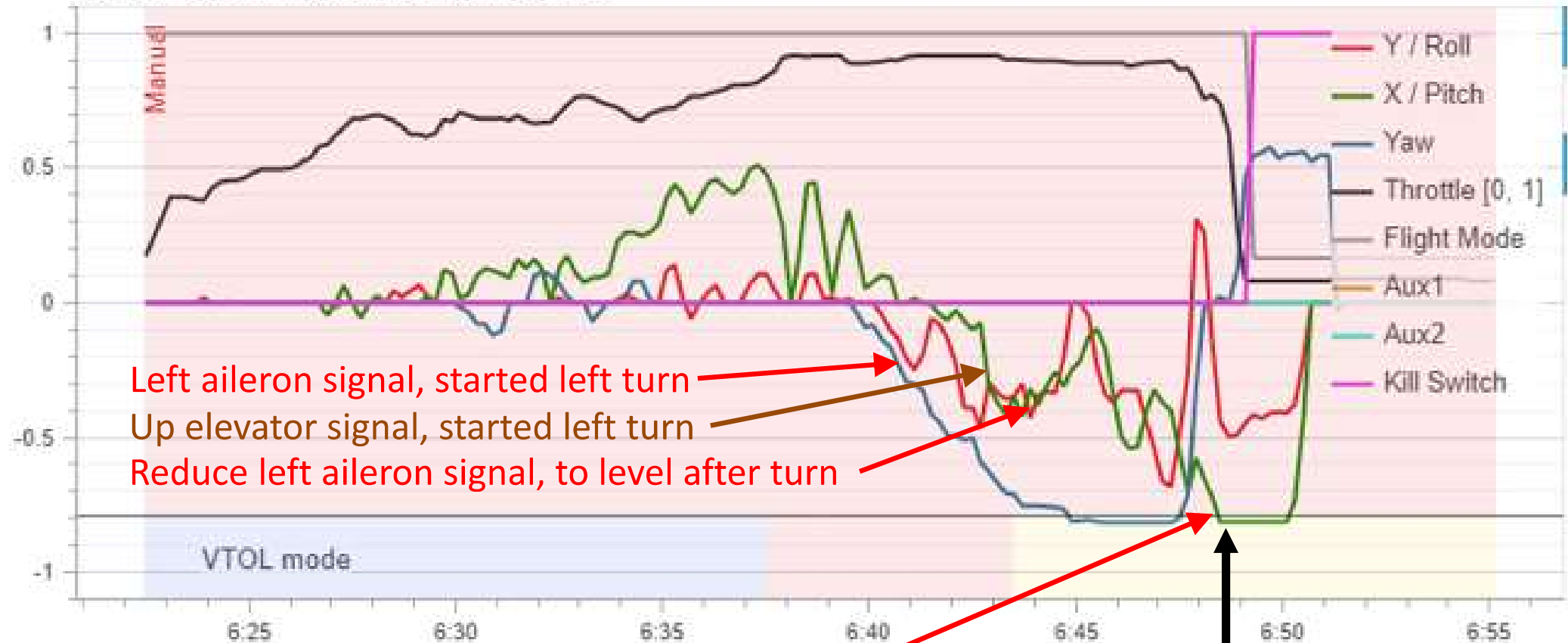
- Flight Data containing aircraft state and sensor data are automatically logged and stored on the Pixhawk SD card.
- Can be downloaded via QGroundControl
- Log Analysis access is available when log file is uploaded through PX4 flight review tool (<http://logs.px4.io>)

ArduPilot is another flight control software and also comes with a mission planner tool



PX4 Recorded Flight Controls of a L+C Model

Manual Control Inputs (Radio or Joystick)



Left aileron signal, started left turn

Up elevator signal, started left turn

Reduce left aileron signal, to level after turn

More Left aileron signal, to turn left, but plane did not respond

Aircraft crashed here

5. Rotor Design, Stability & Control and Model Flight Testing

by Dr. James Wang

SNUevtolclass@gmail.com

For students to use in the 2022 eVTOL Design Short Course at SNU,
please do not reproduce or distribute