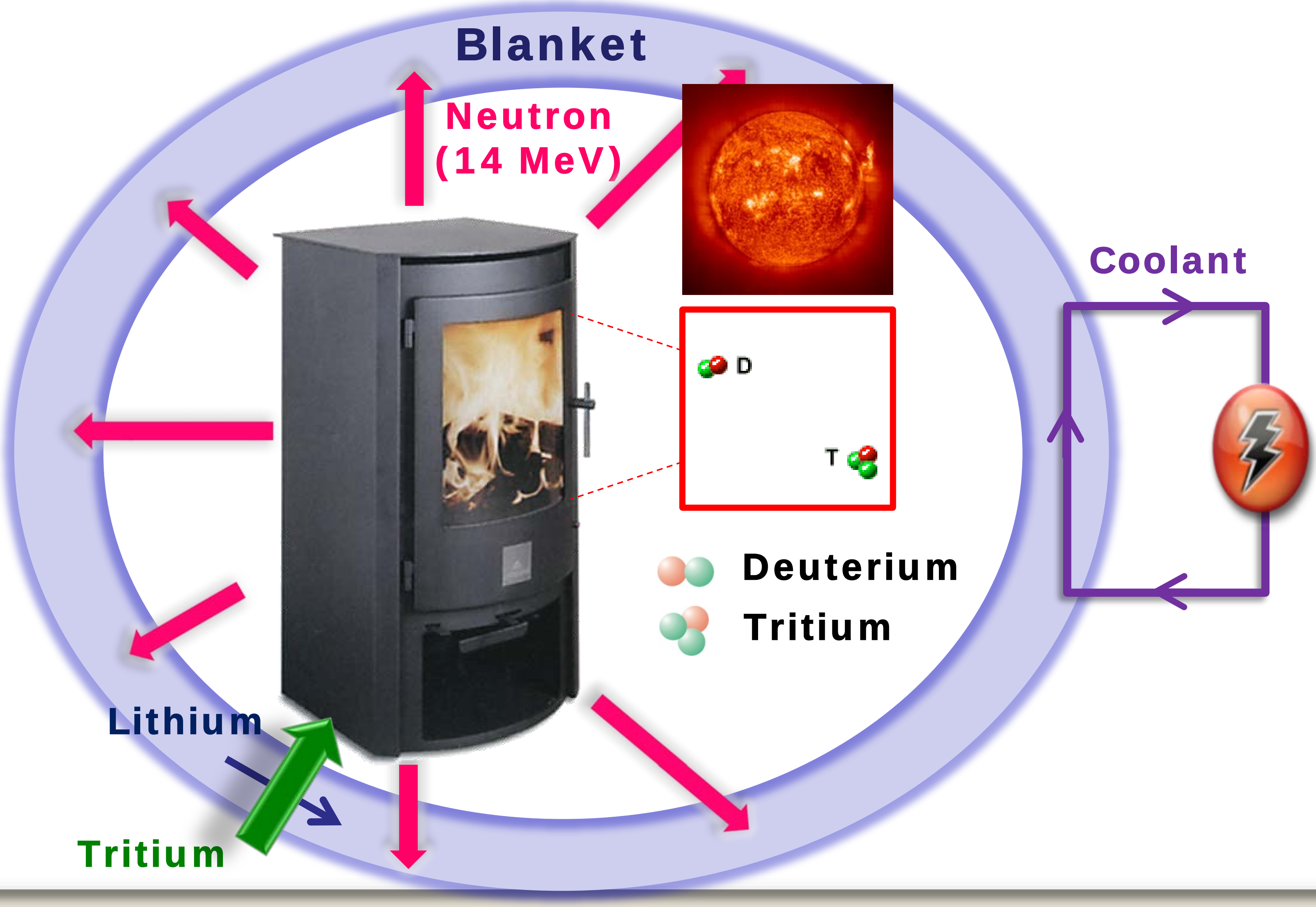
Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

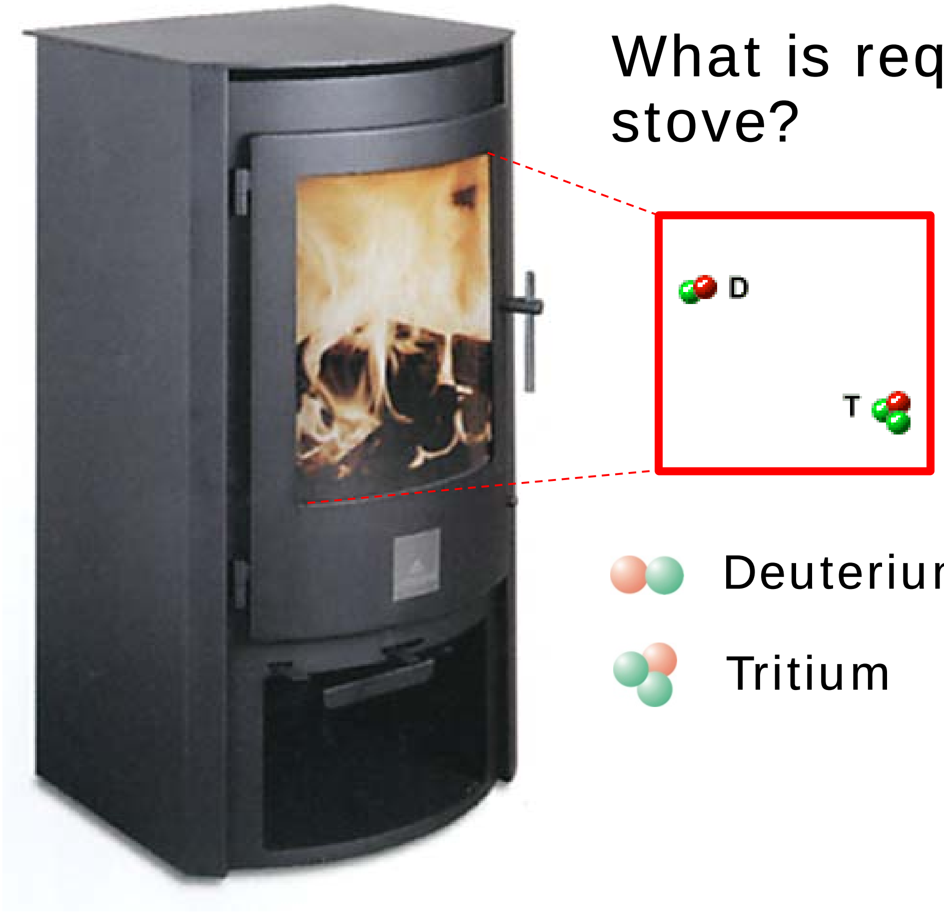
What is requirement of a fusion reactor?

Discussion

Fusion Reactor



Fusion Reactor Energetics



Deuterium

Tritium

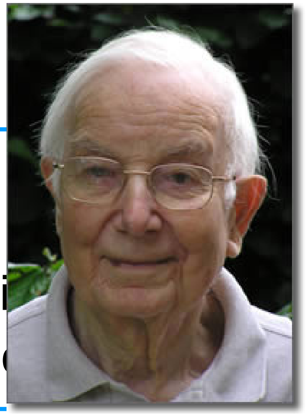
What is required to light a fire in a stove?

- Fuel: D, T
- Amount/density: n
- Heat insulation: τ
- Ignition temperature: T

n … ?

Cri… ed

(fusi… duct)



J. D. Lawson

Why nτT?

https://www.euro-fusion.org/glossary/lawson-criteria/

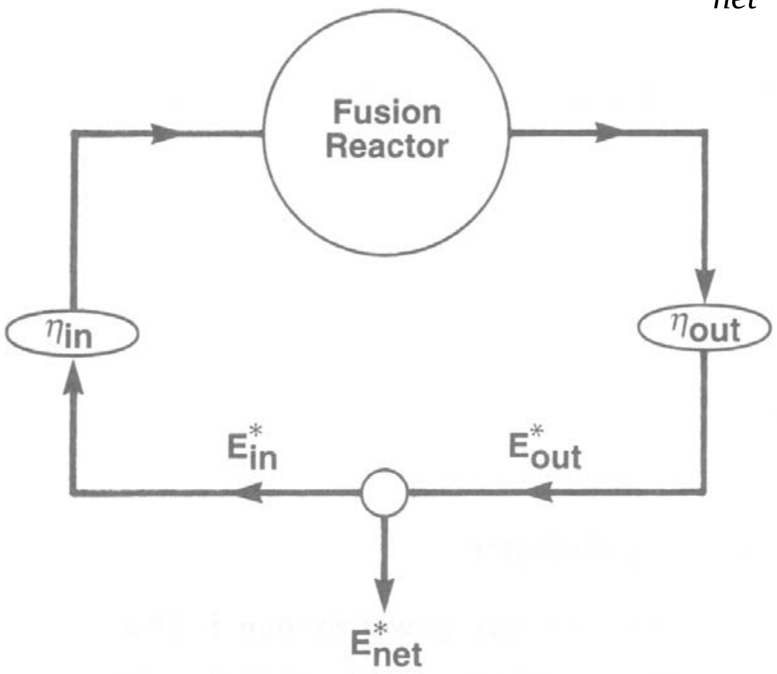
Fusion Reactor Energetics

- **Fundamental requirement of a fusion reactor system**

The overall net energy should be larger than the total energy externally supplied to sustain fusion reactions and associated processes subtracted from the total recovered energy

$$E_{net}^* = E_{out}^* - E_{in}^* > 0$$

*: referring to the entire reaction volume



Considering the time variations of power (Particularly for pulsed systems)

$$\int_0^{\tau_b} \left(\frac{dE^*}{dt} \right)_{net} dt = \int_0^{\tau_b} \left(\frac{dE^*}{dt} \right)_{out} dt - \int_0^{\tau_b} \left(\frac{dE^*}{dt} \right)_{in} dt > 0$$

τ_b: burning time

Fusion Reactor Energetics

- **Fusion Plasma Energy Balance**

$$\int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

$$E^*_{out} = E^*_{aux} + E^*_{fu}$$

$$E^*_{in} = E^*_{n} + E^*_{rad} + \int_o^{\tau_b}\frac{E^*_{th}}{\boxed{\tau^*_E}}dt$$

$$E^*_{aux} = \eta_{in}E^*_{in}$$

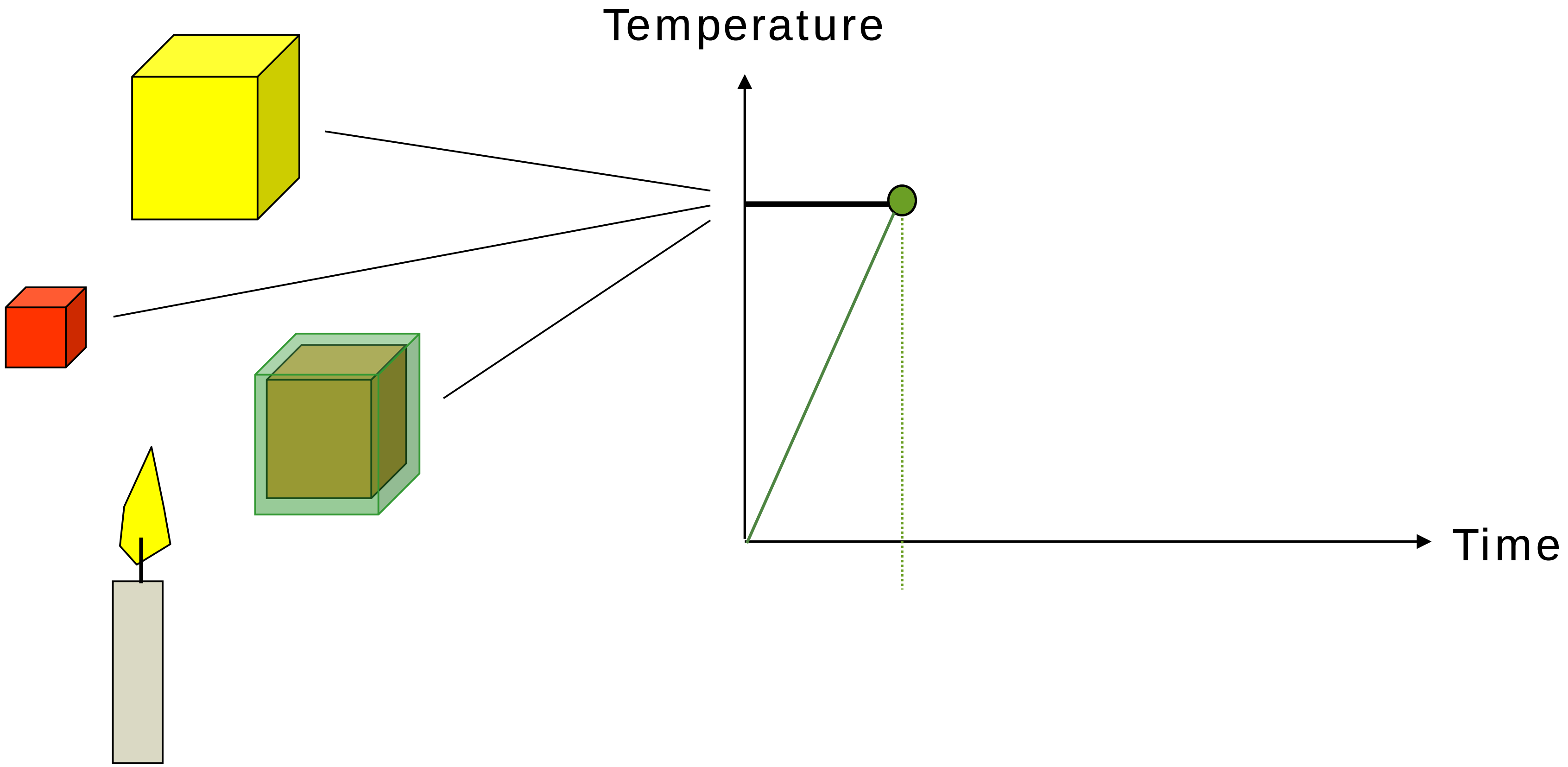
$$\frac{E^*_{alpha}}{E^*_{fu}} = f_c \qquad E^*_n + E^*_{alpha} = E^*_{fu} \qquad E^*_n = (1-f_c)E^*_{fu}$$

f_c: alpha particle fraction of fusion product energy

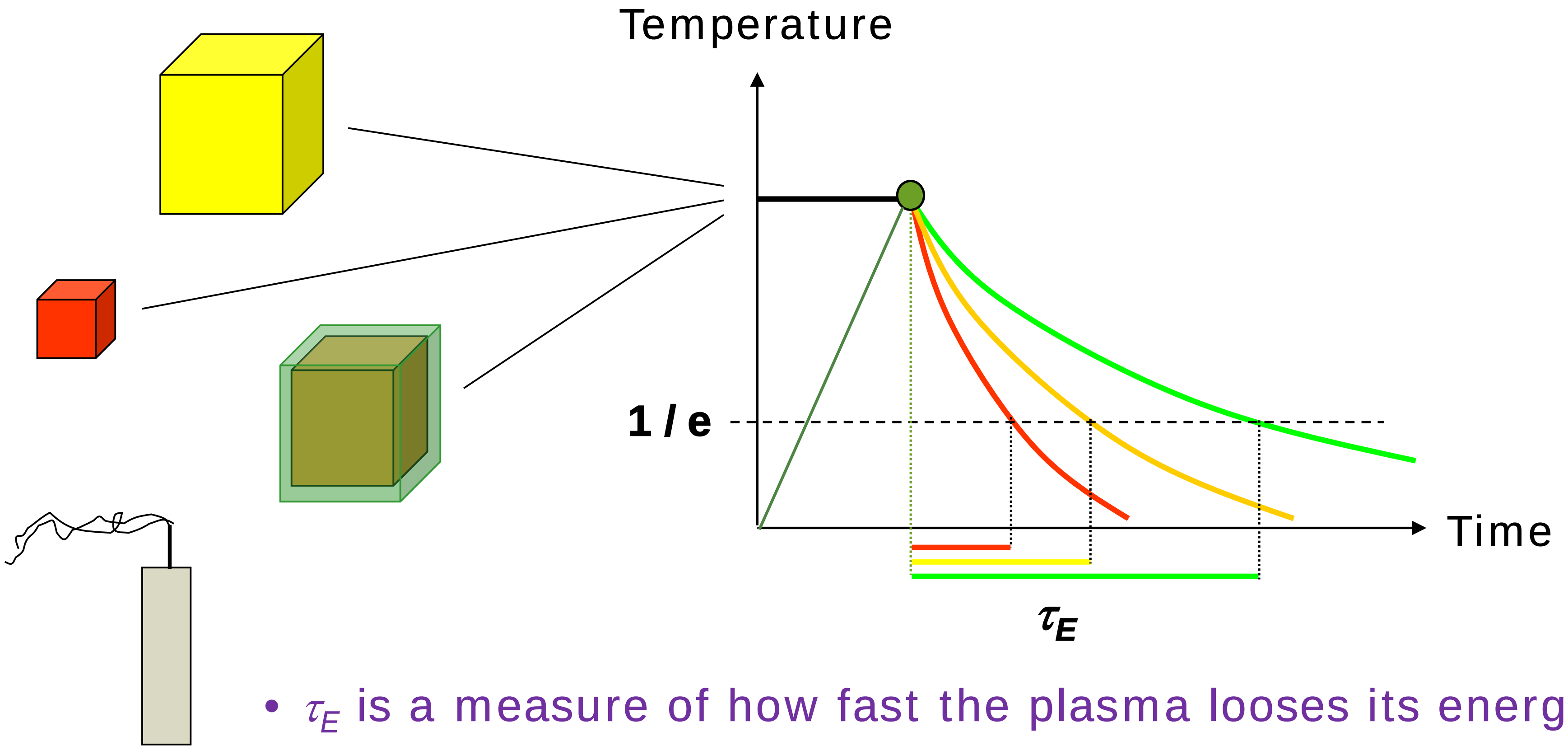
$$Q_p = \frac{E^*_{fu}}{E^*_{aux}} = \frac{E^*_{fu}}{\eta_{in}E^*_{in}}$$

Plasma Q-value (fusion multiplication factor): measure for how efficiently an energy input to the plasma is converted into fusion energy

Energy Confinement Time



Energy Confinement Time



- τ_E is a measure of how fast the plasma looses its energy.
- The loss rate is smallest, τ_E largest
  if the fusion plasma is big and well insulated.

Fusion Reactor Energetics

$$\int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

$$E^*_{aux} + E^*_{fu} - E^*_{n} - E^*_{rad} - \int_o^{\tau_b}\frac{E^*_{th}}{\tau^*_E} dt > 0 \longrightarrow \left(\frac{1}{Q_p} + f_c\right)E^*_{fu} - E^*_{rad} - \int_o^{\tau_b}\frac{E^*_{th}}{\tau^*_E} dt > 0$$

Describe reactor criteria in terms of Q_p

Fusion Reactor Energetics

- **Ignition**

  Energy viability of the fusion plasma:

  actual self-sustaining engineering reactor condition with no heating power

$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p \rightarrow \infty$$

$$\left(\frac{1}{Q_p} + f_c \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt > 0$$

$$f_c E^*_{fu} > E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt$$

Fusion Reactor Energetics

$$f_c E_{fu}^* > E_{rad}^* + \int_o^{\tau_b} \frac{E_{th}^*}{\tau_E^*} dt$$

$$f_{c,dt} \int_V d^3r \int_o^{\tau_b} R_{dt}(\vec{r},t) Q_{dt} dt > \int_V d^3r \left[\int_o^{\tau_b} (P_{br} + P_{cyc}^{net}) dt + \int_o^{\tau_b} \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} dt \right]$$

$$P_{br} = A_{br} n_i n_e Z^2 \sqrt{kT_e} \qquad A_{br} \approx 1.6 \times 10^{-38} \left[\frac{m^3 J}{\sqrt{eV} s} \right]$$

$$P_{cyc}^{net} = A_{cyc} n_e B^2 k T_e \psi \qquad A_{cyc} \approx 6.3 \times 10^{-20} \left[J eV^{-1} T^{-2} s^{-1} \right]$$

$$\int_V d^3r \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} = \frac{E_{th}^*(t)}{\tau_{E^*}(t)}$$

Volume integrated

Fusion Reactor Energetics

- In a homogeneous plasma, local D-T fusion ignition condition:
  Charged particle self-heating power > loss powers
  (radiation + plasma transport)

$$f_{c,dt}P_{dt}(n_i,T_i) > P_{br}(n_i,n_e,T_e) + P_{cyc}^{net}(n_e,T_e) + \frac{3}{2}\frac{(n_iT_i + n_eT_e)}{\tau_{E^*}}$$

$$E_{th.j} = \frac{3}{2}n_jT_j\,, \quad j = i,e$$

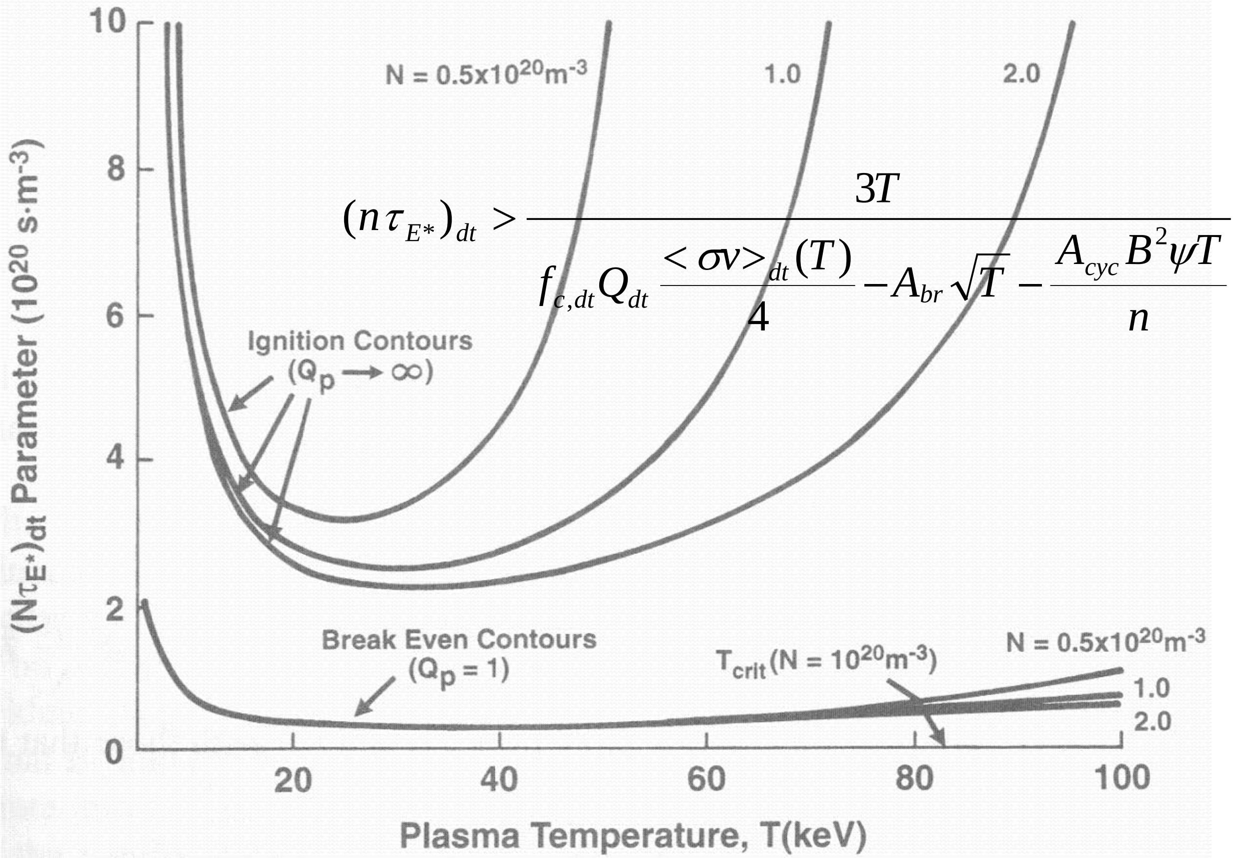
$$f_{c,dt}P_{dt}\tau_{E*} > (P_{br} + P_{cyc}^{net})\tau_{E*} + 3nT$$ ⟵ $n_i = n_e = n$, $T_i = T_e = T$

$$(n\tau_{E*})_{dt} > \frac{3T}{f_{c,dt}Q_{dt}\frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc}B^2\psi T}{n}}$$

Plot?

- complex interrelation between the plasma density
  and its temperature as required for ignition

Fusion Reactor Energetics



$$(n\tau_{E*})_{dt} > \frac{3T}{f_{c,dt} Q_{dt} \frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc} B^2 \psi T}{n}}$$

- $n = 10^{20}$ m^{-3}: T ~ 30 keV, $n\tau_{E*}$ ~ 2.7x10^{20} m^{-3}s, τ_{E*} ~ 2.7 s

Fusion Reactor Energetics

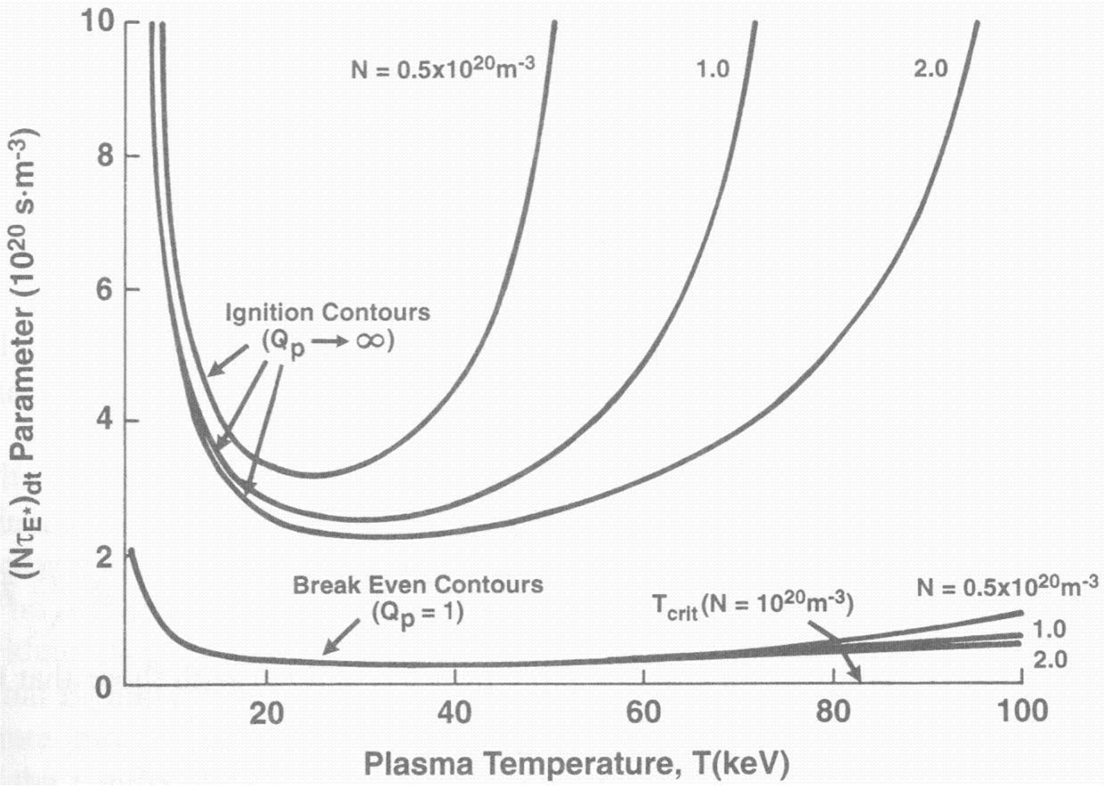
- **Break-even (scientific)**

  The total fusion energy production amounts to a magnitude equal to the effective plasma energy input.

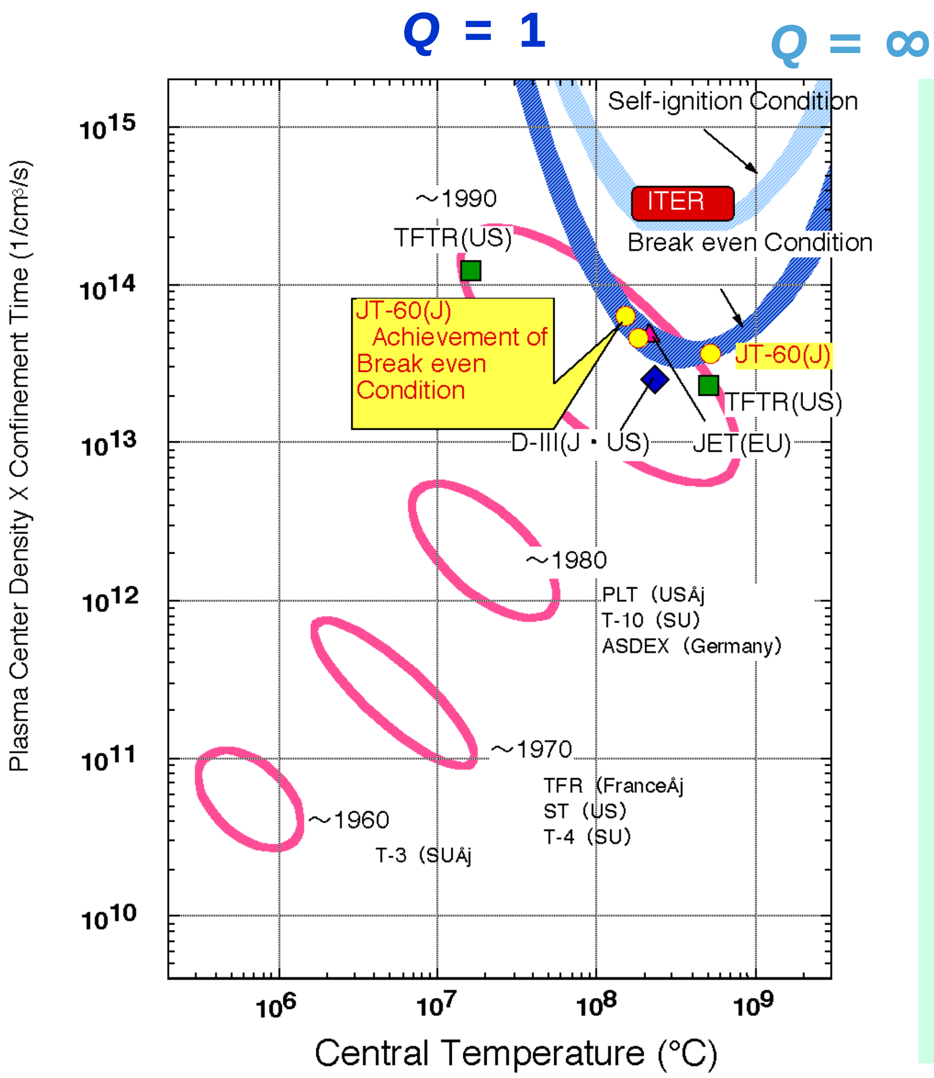
$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p = 1$$

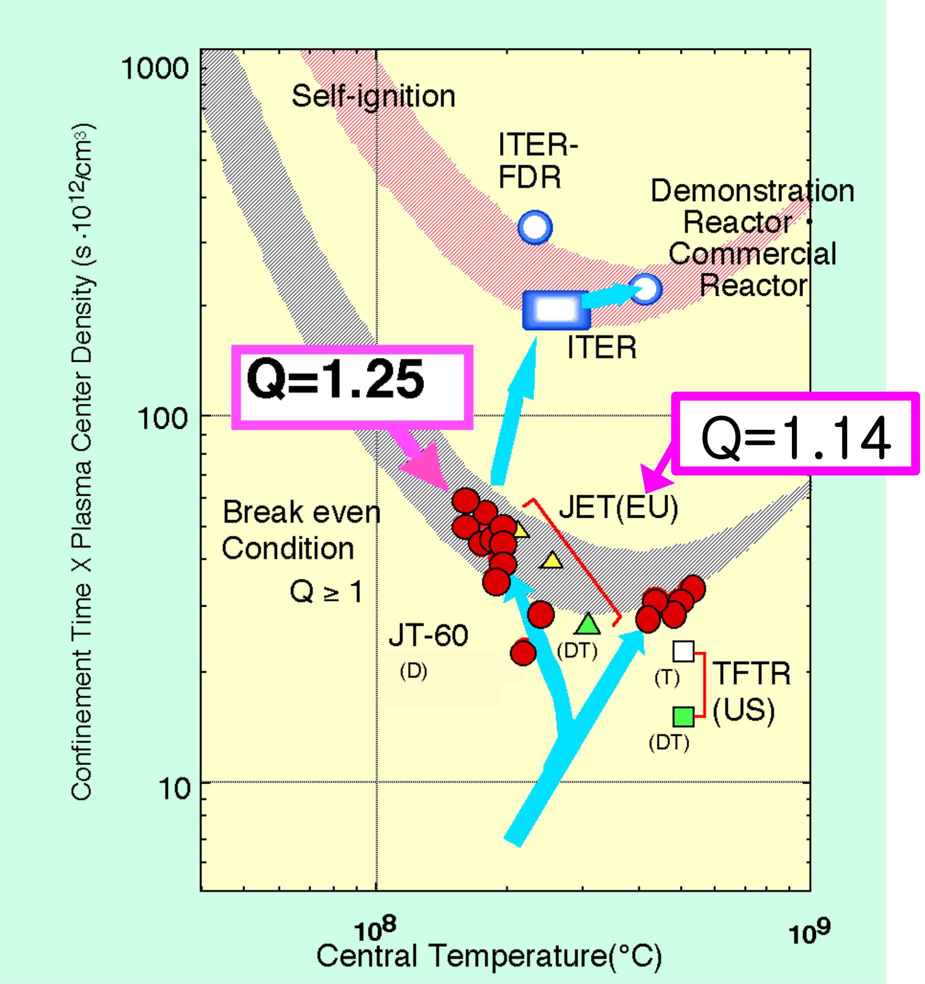
$$\left(\frac{1}{Q_p} + f_c\right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt > 0$$

$$f_c E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt > 0$$



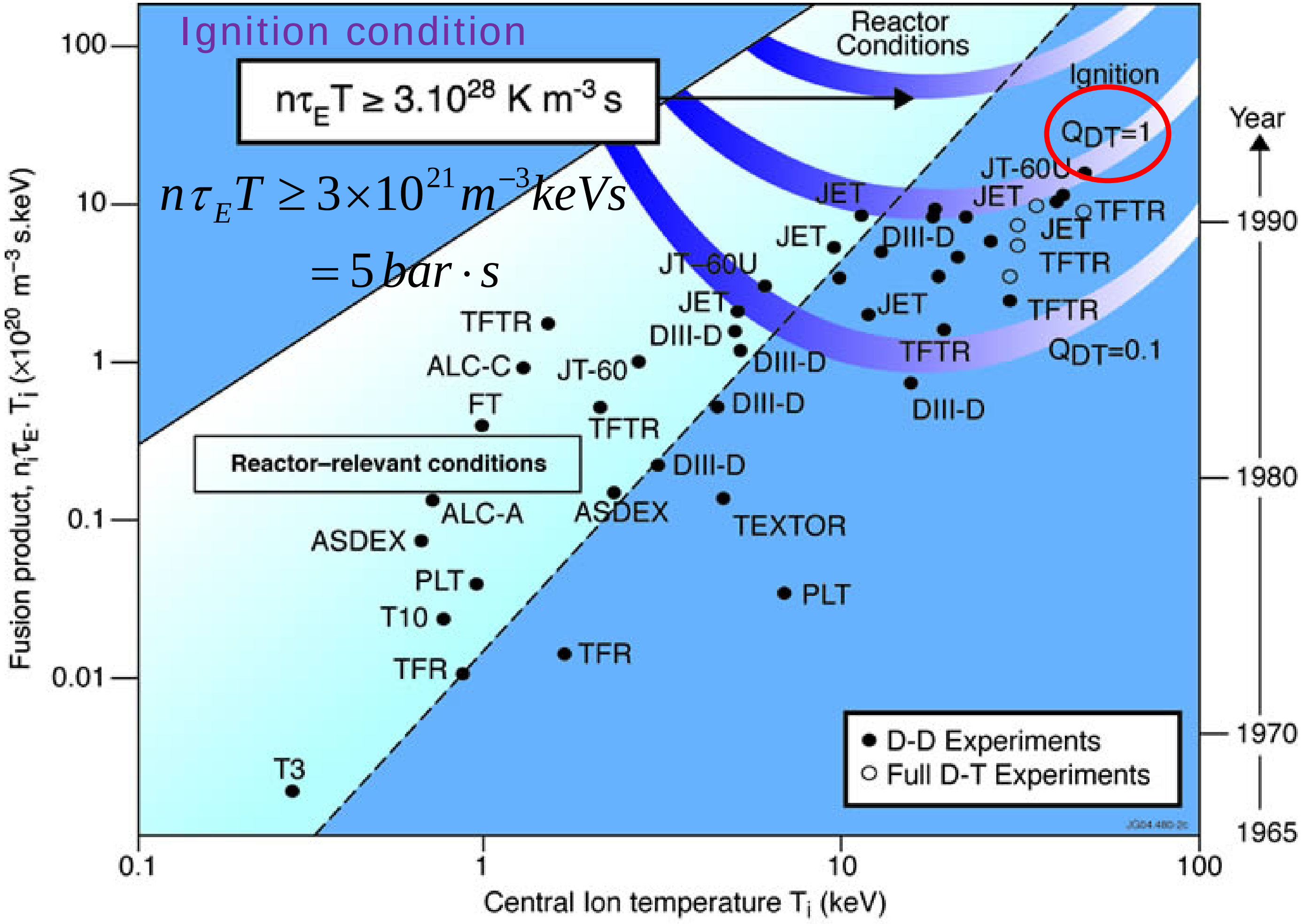
Status of the Tokamak Research





in D-D reactions

Status of the Tokamak Research



$$1\, atm = 1.01325\, bar$$

https://www.euro-fusion.org/2011/09/progress-in-fusion