Homework of chap. 6

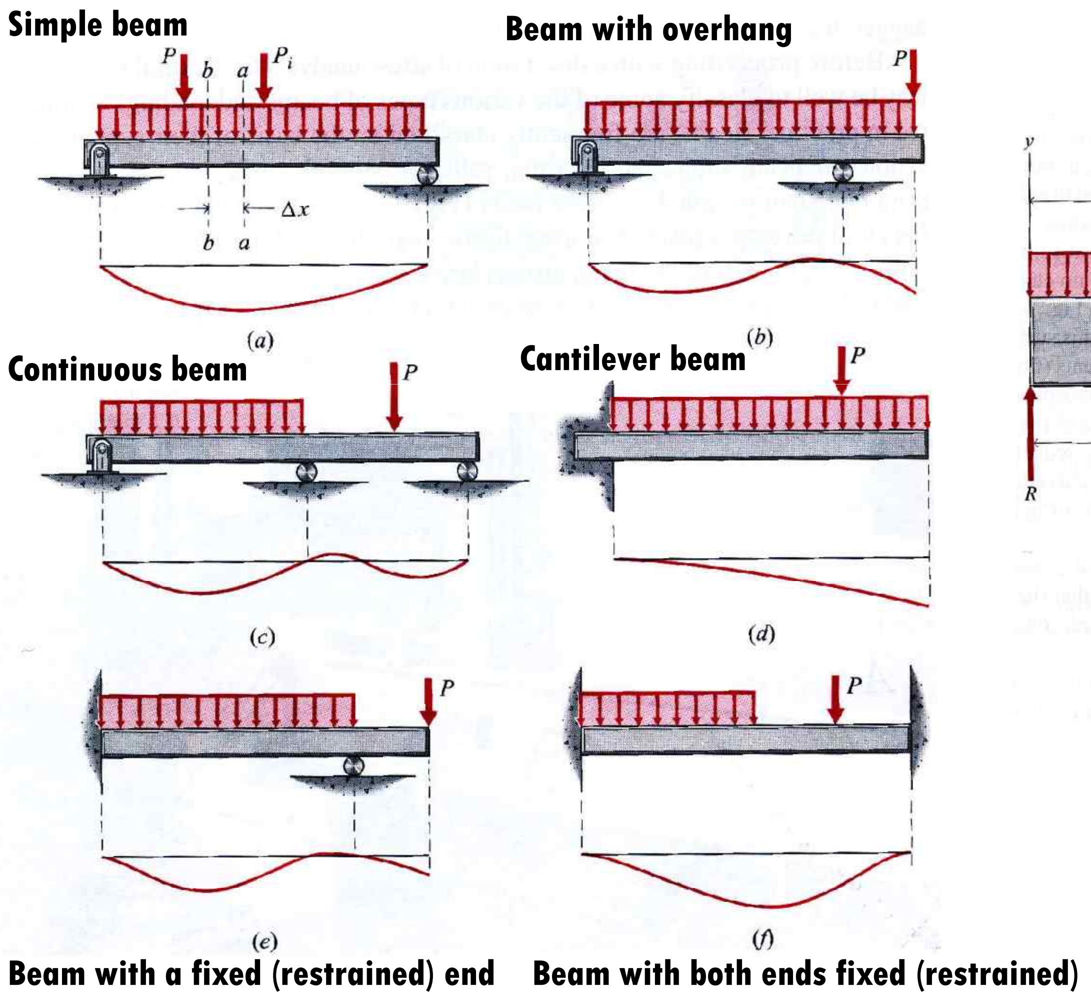
Problem 6-1, 15, 33, 49, 62, 81, 98, 111, 132, 147

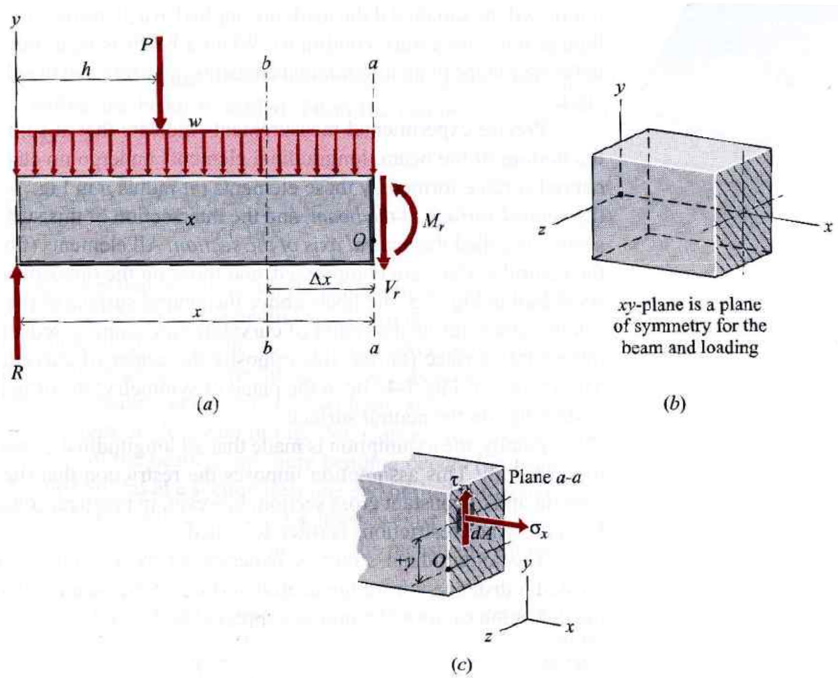
7. Stresses in Beams: Flexural Loading

7.1 Introduction

- Definition of terms
  - Beam: a member subjected to loads applied transverse to the long dimension of the member and which causes the member to bend.
  - Simple beam: a beam supported by a pin roller, or smooth surface at the ends
  - Simple beam with overhang: a beam which has either or both ends projecting beyond the supports
  - Continuous beam: a beam with more than two simple supports
  - Cantilever beam: a beam in which one end is built into a wall or other support so that the built-in end can neither move transversely nor rotate

7.1 Introduction

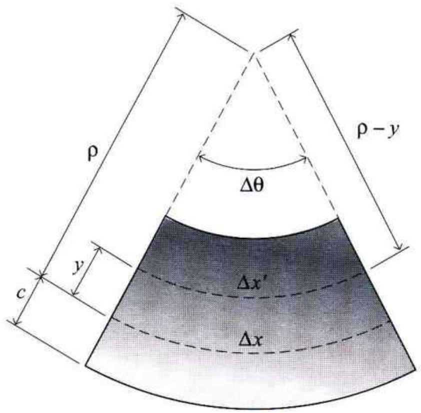




7.2 Flexural strains

- Neutral surface: a curved surface formed by the longitudinal elements undergoing no change in length
- Neutral axis: an intersection of the neutral surface with any cross section
- All elements on one side of the neutral surface are compressed while those on the opposite side are elongated.

- Longitudinal strain is directly proportional to the distance of the element from the neutral surface: valid for elastic or inelastic action.



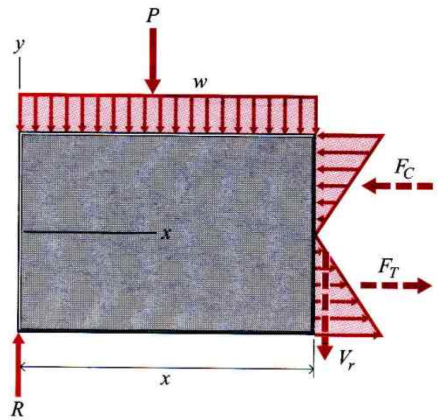
$$\varepsilon_x = \frac{\Delta x' - \Delta x}{\Delta x} = \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} = -\frac{1}{\rho}y$$

7.3 Flexural stresses

- The stress-strain diagram of a beam is assumed to be identical for compression and elongation throughout this book.
- The normal stress on the transverse cross section varies linearly with distance y from the neutral surface

$$\sigma_x = E\varepsilon_x = -\frac{E}{\rho}y$$

- For flexural loading and linearly elastic action, the neutral axis passes through the centroid of the cross section.



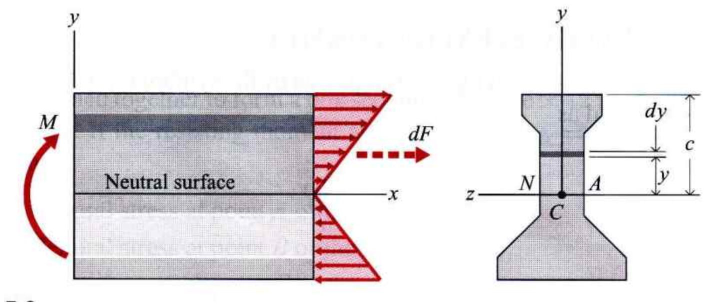
$$\sum F_x = \int_A dF = \int_A \sigma_x dA = \int_A -\frac{E}{\rho}y\, dA = -\frac{E}{\rho}\int_A y\, dA = 0$$

7.3 Flexural stresses

- The maximum normal stress on the cross section can be written as

$$\sigma_{\max} = -\frac{E}{\rho}c \quad \rightarrow \quad \sigma_x = \frac{y}{c}\sigma_{\max} = \frac{y}{c}\sigma_c$$

- The resisting moment on a cross section is as follow.



$$M_r = -\int_A y\sigma_x \, dA = -\frac{\sigma_c}{c}\int_A y^2 \, dA = -\frac{\sigma_x}{y}\int_A y^2 \, dA$$

$\int_A y^2 \, dA$: the second moment of area (I)

$$M_r = -\frac{\sigma_x}{y} I$$

7.4 The elastic flexure formula

- The elastic flexure formula:

$$\sigma_x = -\frac{M_r y}{I}$$

- The section modulus of a beam, S:

$$\sigma_{\max} = \frac{M_r c}{I} = \frac{M_r}{S} \quad \therefore S = I/c$$

- Case of nonsymmetric sections:

$$\sum M_y = 0$$

$$\sum M_y = \int_A z\sigma_x dA = \int_A z\frac{\sigma_c}{c} y\, dA = \frac{\sigma_c}{c}\int_A zy\, dA = \frac{\sigma_c}{c} I_{yz} = 0$$

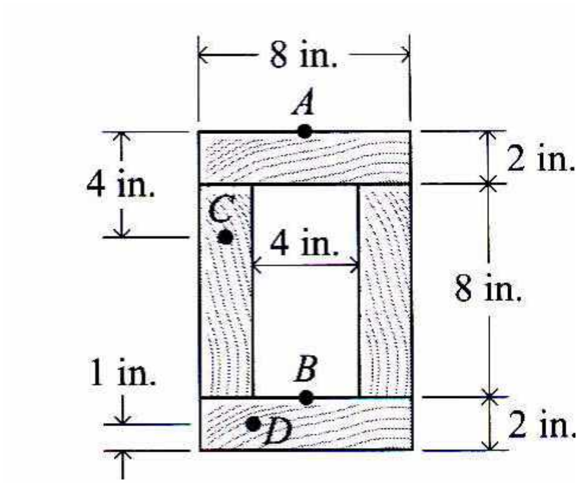
$I_{yz} =$ the mixed second moment of area

7.4 The elastic flexure formula

- Example Problem 7-1

A timber beam consists of four 2 x 8 in. planks fastened together to form a box section 8 in. wide x 12 in. deep. The resisting moment at the section is -200 kip ·in.

- The flexural stress at point A, B, C, and D of the cross section



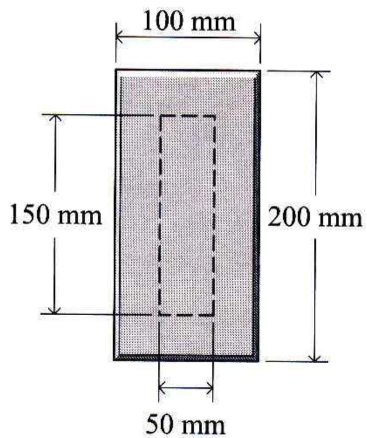
7.4 The elastic flexure formula

• Example Problem 7-2

The max. flexural stress at a certain section is 15 MPa.

- The resisting moment developed at the section

- The percentage decrease in the resisting moment if the dotted central portion of is removed



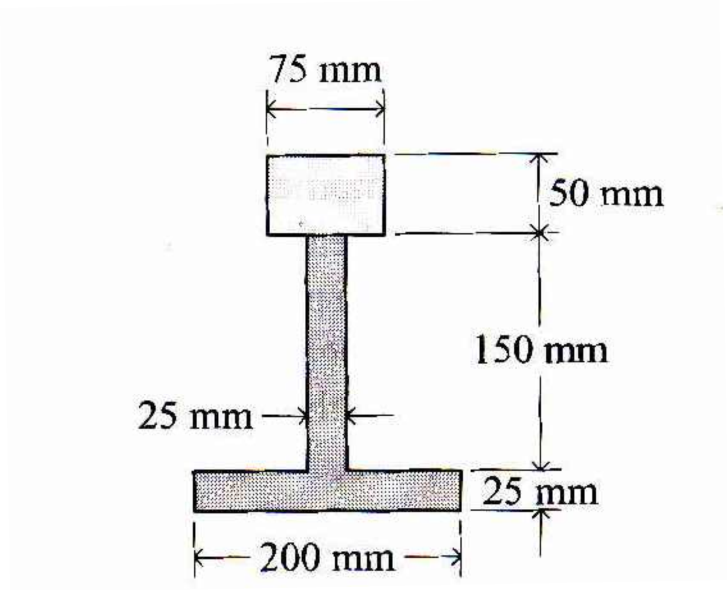
7.4 The elastic flexure formula

- Example Problem 7-3

On a section where the resisting moment is -75 kN·m,

- The max. tensile flexural stress

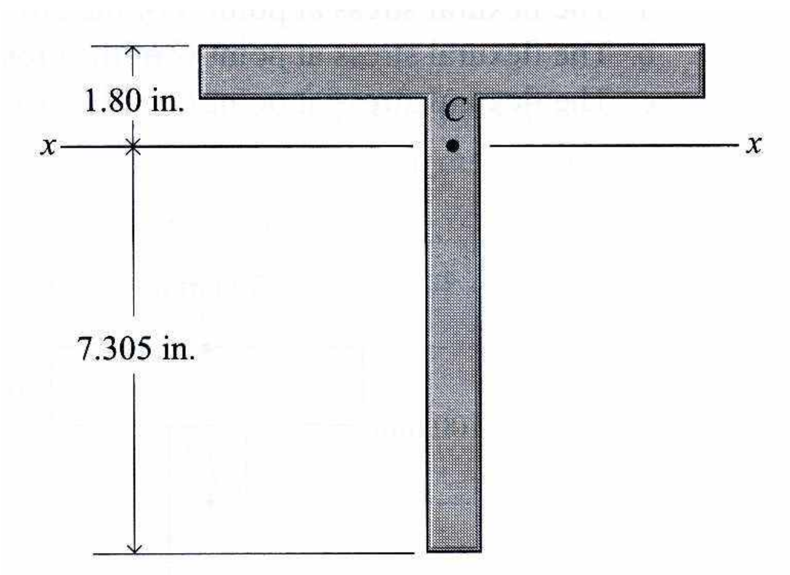
- The max. compressive flexural stress



7.4 The elastic flexure formula

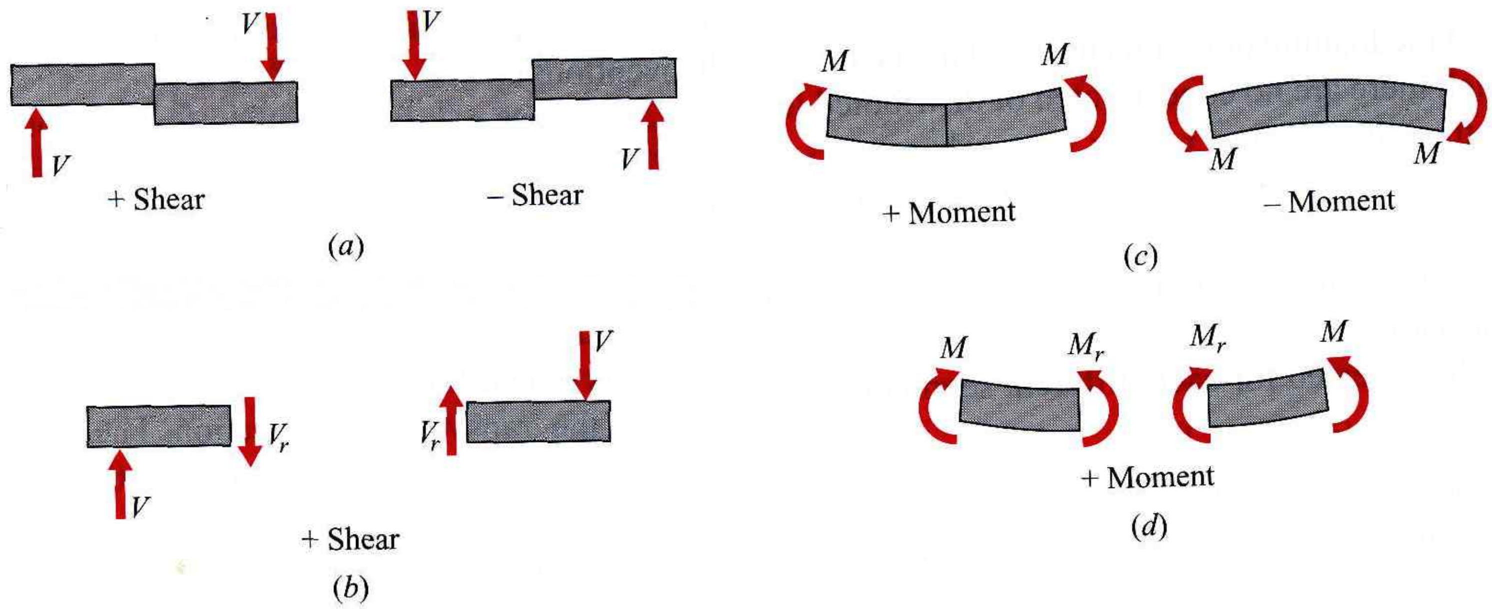
- Example Problem 7-4

Determine the largest positive bending moment that can be applied to a WT9x38 structural T-beam if the allowable flexural stresses are 20 ksi in tension and 25 ksi in compression.



7.5 Shear forces and bending moments in beams

- The resisting moment is also called bending moment.
- By definition, the shear at a section is positive when the portion of the beam to the left of the section tends to move upward with respect to the portion to the right of the section (clockwise rotation).
- By definition, the bending moment is positive at the sections for which the top of the beam is in compression and the bottom is in tension.



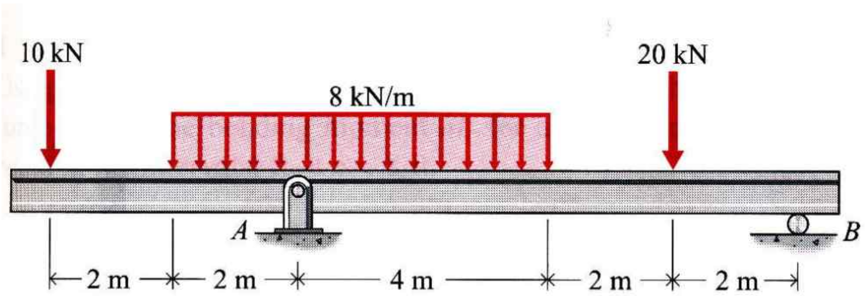
7.5 Shear forces and bending moments in beams

- Example Problem 7-6

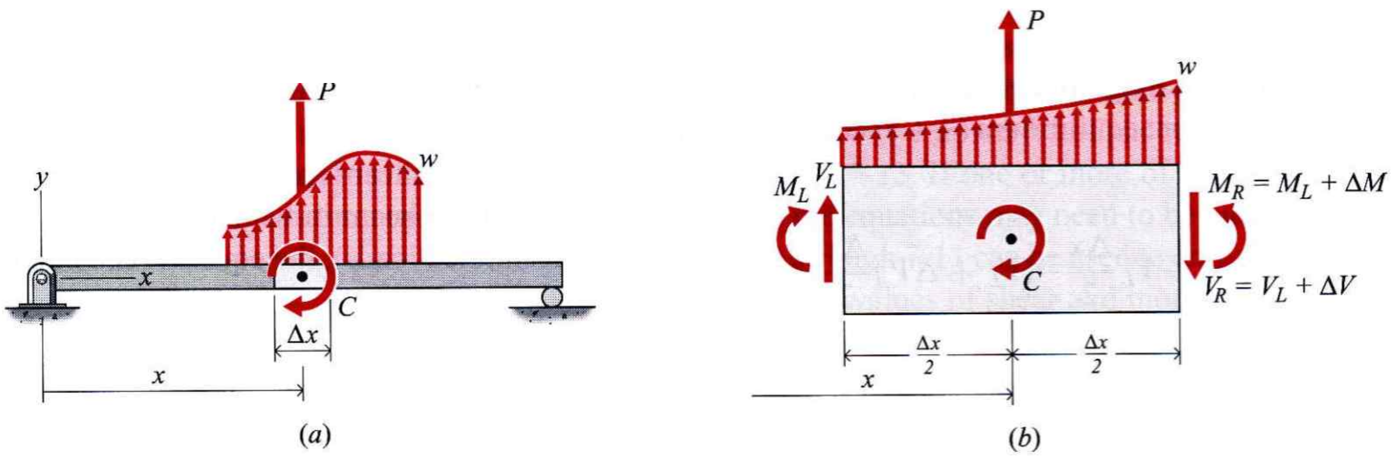
On a section 3 m to the right of A of an S152x19 steel beam whose second moment of area, section modulus, and depth are $9.2(10^{-6})$ m^4, $121(10^{-6})$ m^3, and 0.1524 m, respectively,

- The flexural stress at a point 25 mm below the top of the beam

- The max. flexural stress on the section



7.6 Load, shear force, and bending moment relationships



- Force equilibrium

$$V_L + w_{avg}\Delta x + P - (V_L + \Delta V) = 0 \quad \rightarrow \quad \Delta V = P + w_{avg}\Delta x$$

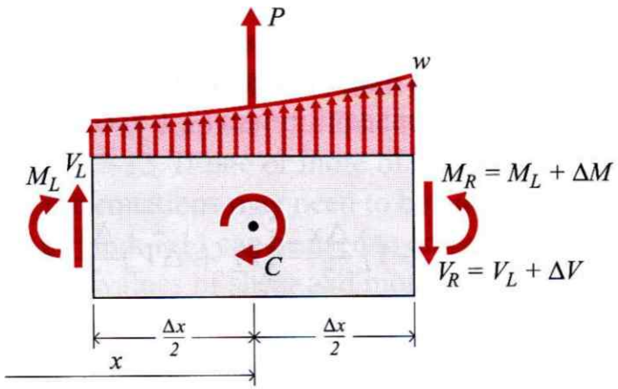
1) In any segment of a beam where there are no loads, the resisting shear force is constant: $\Delta V = 0 \quad or \quad V_L = V_R$
2) Across any concentrated load P ($\Delta x \rightarrow 0$), the shear force graph jumps by the amount of the concentrated load: $\Delta V = P \quad or \quad V_R = V_L + P$
3) The slope of the shear force graph is equal to the intensity of loading if the concentrated load P is zero: $\Delta V / \Delta x = w_{avg}\ , \quad dV/dx = w\left(\Delta x \rightarrow 0\right)$

7.6 Load, shear force, and bending moment relationships

4) The change in shear between sections at x_1 and x_2 is equal to the area under the load diagram between the two sections if P is zero:

$$V_2 - V_1 = \int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w\,dx$$

- Moment equilibrium



$$(M_L + \Delta M) - M_L - C - V_L \frac{\Delta x}{2} - (V_L + \Delta V)\frac{\Delta x}{2} + a\left(w_{avg}\Delta x\right) = 0$$

$$\rightarrow \Delta M = C + V_L \Delta x + \Delta V \frac{\Delta x}{2} - a\left(w_{avg}\Delta x\right)$$

1) Across any concentrated couple C ($\Delta x \rightarrow 0$), the bending moment graph jumps by the amount of the concentrated couple: $\Delta M = C \quad or \quad M_R = M_L + C$

2) The slope of the bending moment graph is equal to the value of the shear force as $\Delta x \rightarrow 0$ if C and P are both zero : $dM/dx = V$

3) The change in bending moment between sections at x_1 and x_2 is equal to the area under the shear force graph between the two sections if $C=P=0$:

$$M_2 - M_1 = \int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V\,dx$$

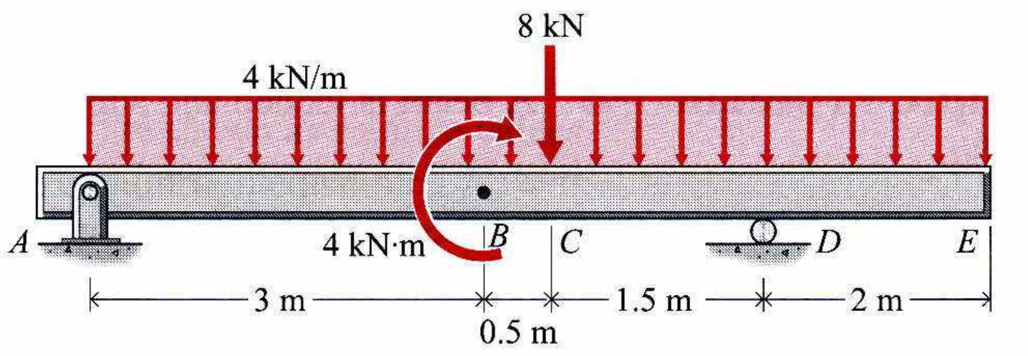
7.6 Load, shear force, and bending moment relationships

7.6.1 Shear and Bending Moment Diagrams

-Writing algebraic equations for the shear force and bending moment and constructing curves from the equations: accurate but relatively time consuming, divide the beam into intervals bounded by the abrupt changes in the loading

-Drawing the shear diagram from the load diagram and the bending moment diagram from the shear diagram by using previously introduced relationships: the max. or min. values are more easily obtained.

Example Problem 7-8



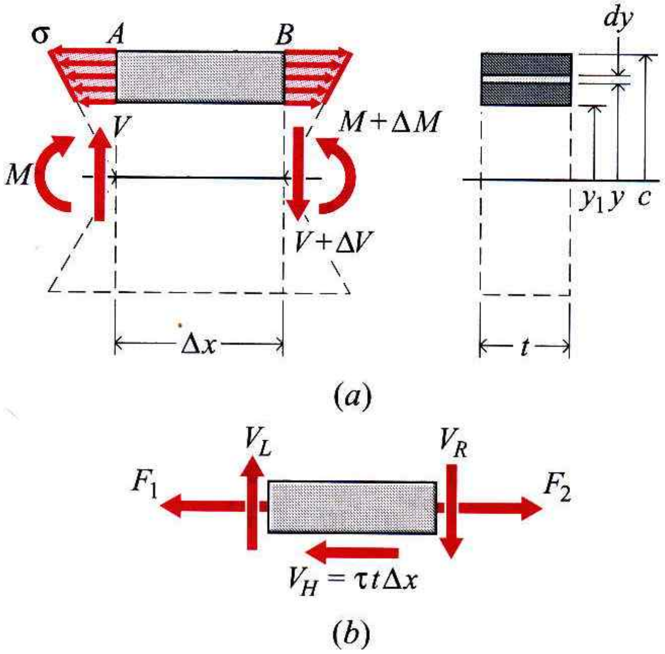
-Write equations for the shear and the bending moment for the interval CD.

- Draw complete shear and bending moment diagram.

- Determine the max. tensile and compressive flexural stresses in the beam made of S 457 x 104 whose second moment of area, section modulus, and depth are $358(10^{-6})$ m^4, $1690(10^{-6})$ m^3, and 0.4572 m, respectively.

7.7 Shearing stresses in beams

- The elementary solution of Jourawski (1821-1891) is introduced: easy to apply but limited to elastic action.



$$F_1 = -\frac{M}{I}\int_A y\,dA = -\frac{M}{I}\int_{y_1}^{c} y(t\,dy)$$

$$F_2 = -\frac{M+\Delta M}{I}\int_{y_1}^{c} y(t\,dy)$$

$$V_H = F_2 - F_1 = -\frac{\Delta M}{I}\int_{y_1}^{c} y(t\,dy)$$

$$\tau_{avg} = \frac{V_H}{A_s} = -\frac{\Delta M}{It\Delta x}\int_{y_1}^{c} y(t\,dy)$$

$$\tau = \lim_{\Delta x \to 0}\frac{\Delta M}{\Delta x}\left(-\frac{1}{It}\right)\int_{y_1}^{c} t\,y\,dy = \frac{dM}{dx}\left(-\frac{1}{It}\right)\int_{y_1}^{c} t\,y\,dy$$

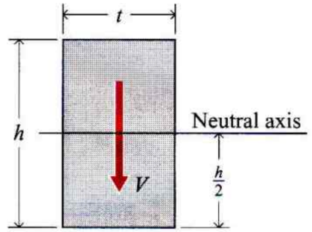
$$\tau = \frac{V_r Q}{It}$$

7.7 Shearing stresses in beams

- The variation of shearing stress on a transverse cross section of a beam

$$\tau = \frac{V_r Q}{It} = \frac{V}{It}\int_A y\,dA = \frac{V}{It}\int_{y_1}^{c} t\,y\,dy$$

$$= \frac{V}{I}\int_{y_1}^{h/2} y\,dy = \frac{V}{2I}\left[\left(\frac{h}{2}\right)^2 - y_1^2\right]$$



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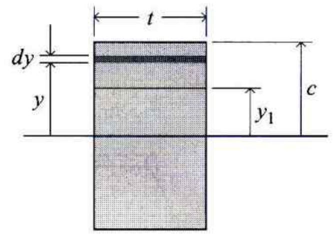
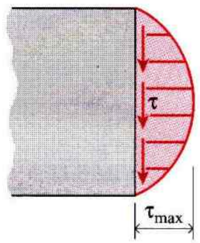


(b)



(c)

7.7 Shearing stresses in beams

- Example Problem 7-10

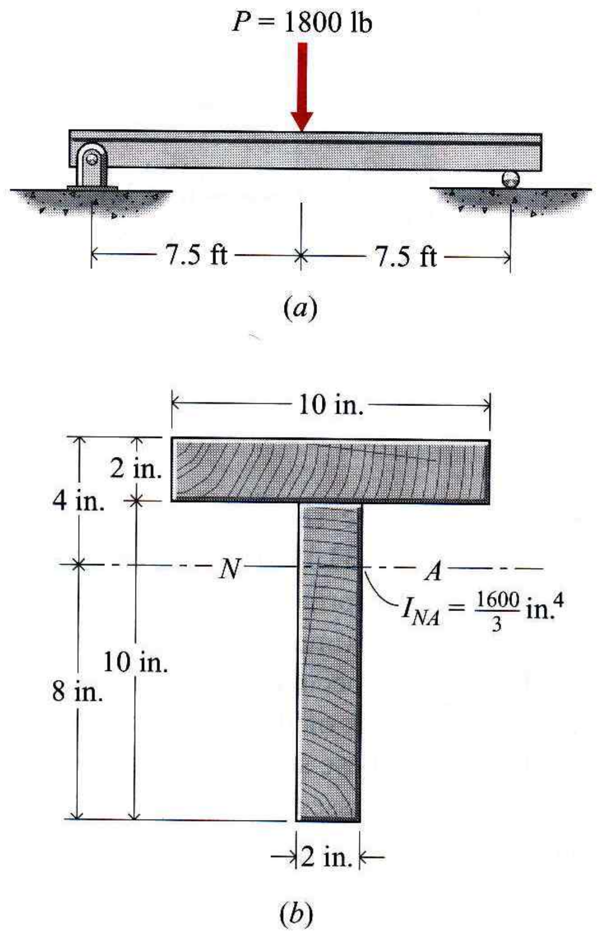
-The average shearing stress on a horizontal plane 4 in. above the bottom of the beam and 6 ft from the left support

-The max. transverse shearing stress

-The avg. shearing stress in the joint at a section 6 ft from the left support

-The force transmitted from the flange to the stem by the glue in a 12-in. length of the joint centered 6 ft from the left support

-The. max. tensile flexural stress



Homework of chap. 7

Problem 7-1, 18, 41, 64, 82