8. Flexural Loading:
The Deflection of a beam

8.1 Introduction

- A beam design is frequently not complete until the amount of deflection has been determined for a specified load.
- The deflection of a beam depends on the stiffness of the material, the dimension of the beam, and the applied loads and supports.
- Four methods for calculating beam deflections owing to flexural stresses:

  (1) integration method

  (2) singularity function method

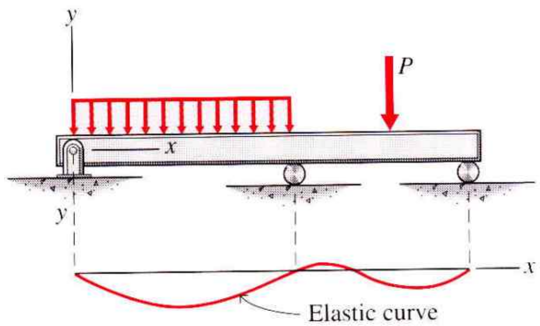
  (3) superposition method

  (4) energy method

8.2 The differential equation of the elastic curve

- Elastic curve: a curve defined by the centroidal axis of a beam

$$Slope = \frac{dv}{dx} = \tan\theta \approx \theta$$



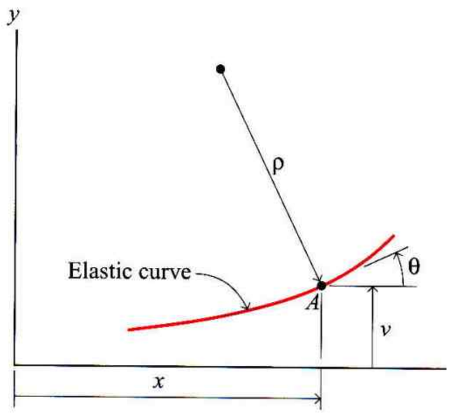
- Curvature: the reciprocal of a radius

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{\left[1+\left(dv/dx\right)^2\right]^{3/2}} \approx \frac{d^2v}{dx^2} = \frac{d\theta}{dx}$$

Meanwhile,

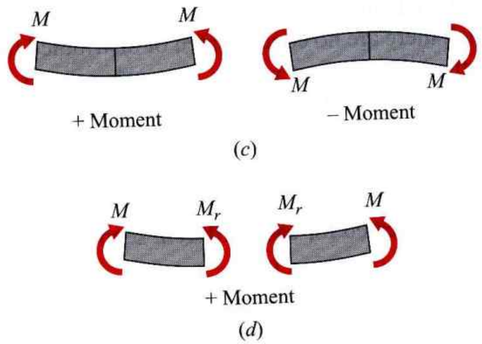
$$\sigma_x = E\varepsilon_x = E\left(\frac{-y}{\rho}\right) \quad and \quad \sigma_x = \frac{-M_r y}{I}$$

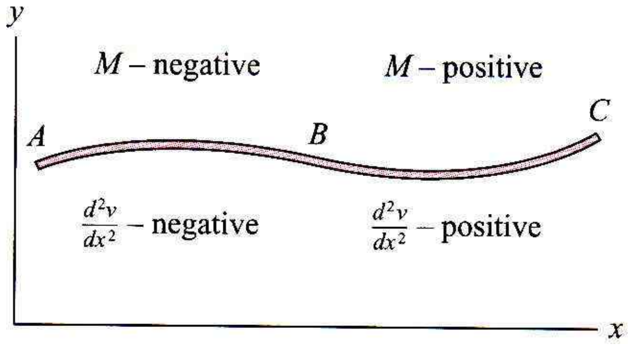
$$\rightarrow \frac{1}{\rho} = \frac{M_r}{EI} = \frac{d^2v}{dx^2} \quad \therefore M_r = EI\frac{d^2v}{dx^2}$$



8.2 The differential equation of the elastic curve

- Sign convention





- From deflection to load intensity

$$\text{Deflection} = v$$

$$\text{Slope} = \frac{dv}{dx}$$

$$\text{Moment}: M_r = EI\frac{d^2v}{dx^2}$$

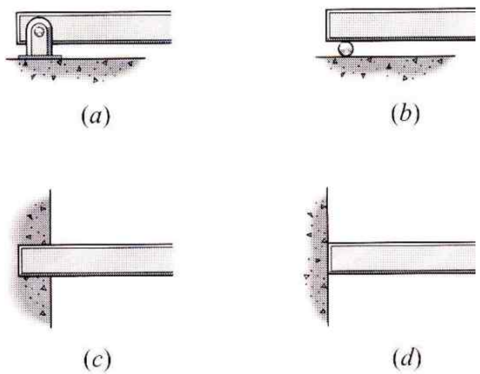
$$\text{Shear}: V = \frac{dM_r}{dx} = EI\frac{d^3v}{dx^3}$$

$$\text{Load intensity}: w = \frac{dV}{dx} = EI\frac{d^4v}{dx^4}$$

8.3 Deflection by integration

- The constants of integration can be evaluated from boundary or matching conditions

- Boundary condition: a set of values for x and v, or x and dv/dx at a specific location of a beam



ex.) $v = 0$ at a pin or roller support
$v = 0$ and $dv/dx = 0$ at cantilever beams

- Matching condition: the equality of deflection or slope at junctions of two intervals of load conditions

ex.) At $x = L/3$, v from the left equals v from the right equation

8.3 Deflection by integration

- Procedure of calculating a beam deflection by the double integration method

1) Select an interval(s), and place a set of coordinate axes at one end of the interval
   ex.) $0 \le x \le L/3$ and $L/3 \le x \le L$

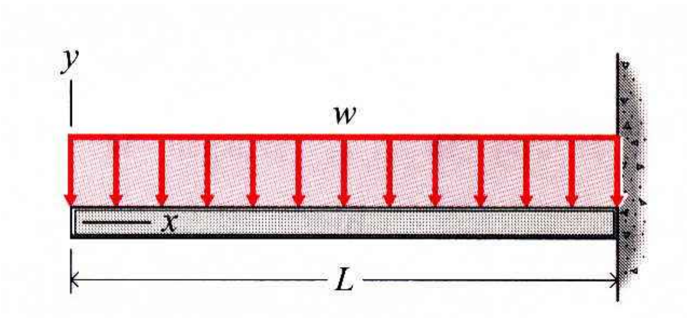
2) List the available boundary and matching conditions: Two condition are required to evaluate two constants.
3) Express the bending moment for each interval.
4) Solve the differential equation and evaluate all constants of integration.

- Example Problem 8-1

- The equations of the elastic curve

- The deflection at the left end

- The slope at the left end



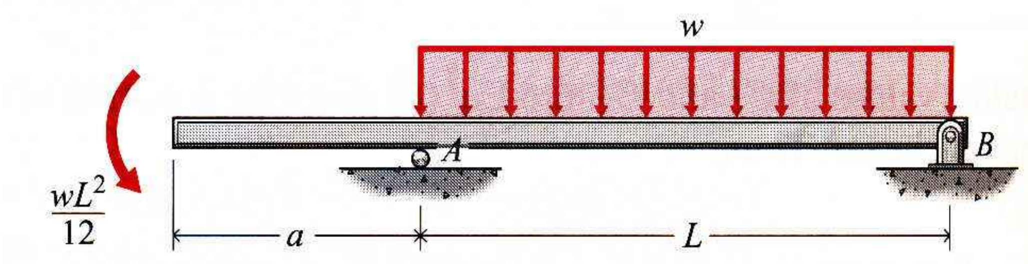
- Example Problem 8-2

- The equations of the elastic curve for the interval between the supports

- The deflection midway between the supports

- The point of max. deflection between the supports

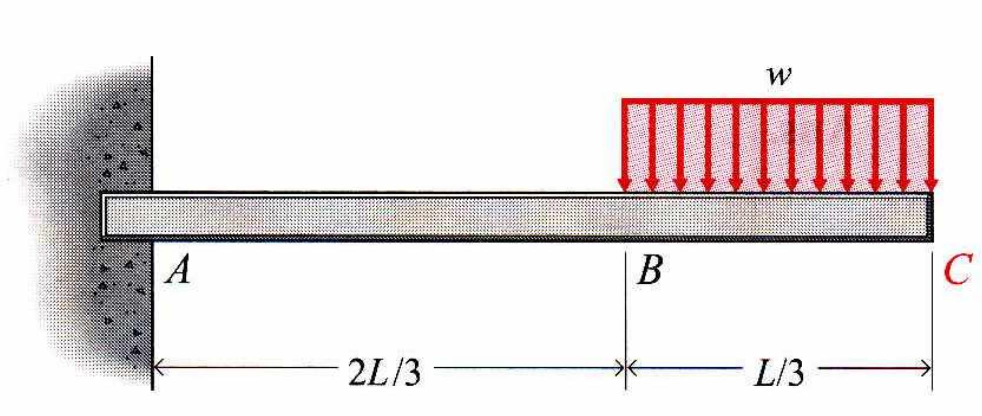
- The max. deflection in the interval between the supports



8.3 Deflection by integration

- Example Problem 8-3

- The deflection at the right end



• Example Problem 8-4

- The deflection at the left end