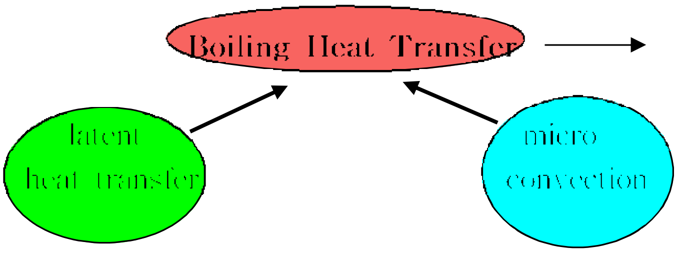
Chapter 2 Bubble Dynamics



- o Bubble Dynamics
- o Pool Boiling
- o Forced Convection Boiling

★ Description of Heat Transfer in the Isolated Bubble

1) dynamic bubble growth coupled with heat transfer

superheated layer

heat transfer though the bubble

1) free convection of liquid surrounding to bubble

2) heat transfer from super heated layer and vaporization of microlayer

micro convection

1) Step (1) : rapid vaporization of microlayer

2) Step (2) : temperature rise due to poor heat transfer through vapor

3) Step (3) : surface quenching by cold liquid during bubble departure

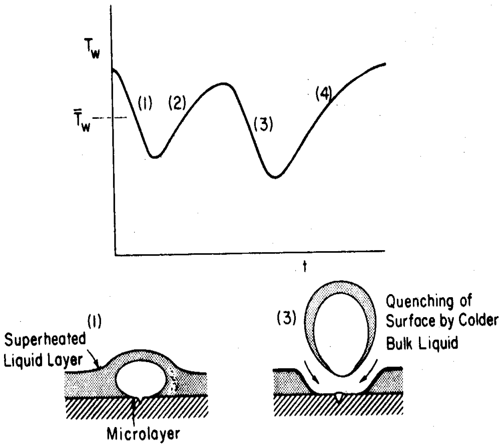


Fig. 2.1 Wall temperature change around bubble

microconvection or micropumping

4) Step (4) : temperature rise due to re-establish of thermal boundary

1. Basic Definitions

1) Surface tension (σ) [lbf/ft]

Energy per unit area to maintain an interface

2) Gibbs free energy function

$g = h - Ts$

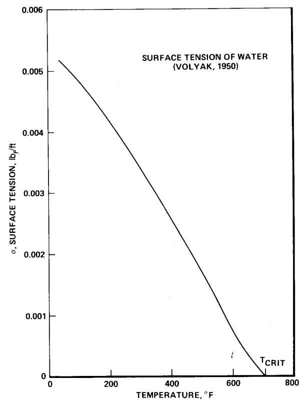


Fig. 2.2 σ vs T_{sat}

3 Contact angle (β)

= Angle between the solid surface and gas-liquid interface

4 *Equilibrium*

1) thermal equilibrium : $T_v = T_l$

2) mechanical equilibrium : $p_v = p_l$

3) phase equilibrium : $g_v = g_l$

★ Bubble equilibrium

- the condition of mechanical equilibrium for the curved surface

$$p_v = p_l + 2\frac{\sigma}{r}$$

- $T_v = T_l$
- $g_v = g_l$

Assume

$$T_v = T_l \approx T_{sat}(p_v)$$

Since $T_l = T_v$ and the vapor must be saturated,

$$T_l > T_{sat}(p_l)$$

Thus, the liquid is ***superheated***

2. Nucleation

- Homogeneous Nucleation
- Surface(Cavity) Nucleation

(1) *Homogeneous(or bulk) Nucleation*

: The supperheated liquid in meta stable suffers form an abrupt phase change due to the fluctuation of superheat.

- Liquid superheated
- radiation attenuation
- neutron thermalization

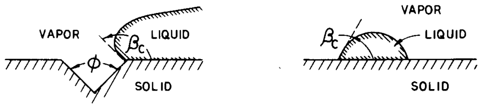


Fig. 2.3 Contact angle on nonwetting(left) and wetting(right) surface

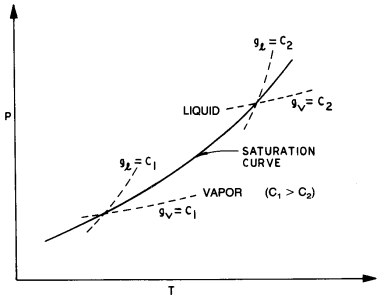
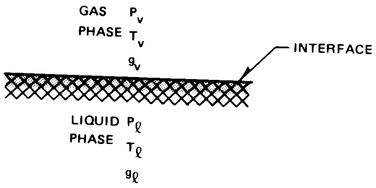
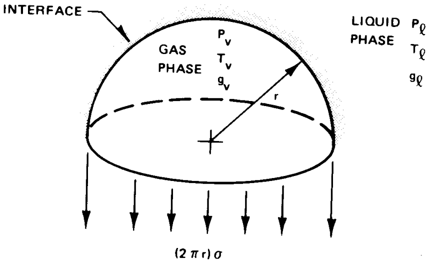


Fig 2.4 Saturation curve with Gibbs function



a. PLANE INTERFACE



b. CURVED INTERFACE (SPHERICAL)

Fig. 2.5 Curved and flat interfaces

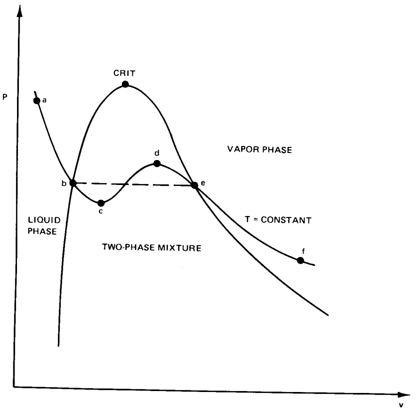


Fig. 2.6 P V Diagram

★ Metastability in the Two Phase

$\overline{bc}$ (metastable superheated liquid)

$\overline{de}$ (metastable subcooled vapor)

: important in BWR, with respect to to explaining bubble nucleation and transient accident phenomea

$\overline{cd}$ (completely unstable and cannot exist)

: $\frac{dp}{dv} > 0$

Energy required to form a bubble

= (surface tension) + (latent heat)

$$= \frac{1}{J}\int_0 \sigma\, dA + \int_0 \rho_g h_{fg}\, dV$$

$$= \frac{4\pi r_b^2}{J}\sigma + \frac{4}{3}\pi r_b^3 \rho_g h_{fg}$$

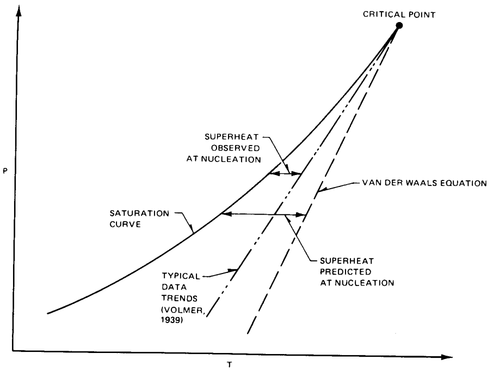


Fig. 2.7 Superheat required for homogeneous nucleation

Note Van de Waals equation overestimates the amount of superheat for homogeneous nucleation

(Ex)

	$T_{superheat}$	T_{sat}
air	320°C	100°C
benzene	225°C	80°C

But, in actual. ~ the order of 10°C

∵ Kinetic view by Cole

: exist a probability that sufficient number of molecules with greater than average energy to form a cluster with equilibrium radius

$$T - T_{sat} = \frac{T_{sat} v_{fg}}{h_{fg}} \frac{\rho_l}{\rho_l - \rho_v}\left[\frac{16\pi\sigma^3}{3kT\ln(nkT/\nu J)}\right]$$

where n is the number density of molecules

(2) *Surface(or cavity) Nucleation*

boiling at single spots(scratch or cavity) on the solid heating surface much lower wall temperature than the energy for homo. nucleation

● Clausius Clapeyron equation

: relate the vapor liquid pressure differential to liquid superheat

$$\left(\frac{dT}{dp}\right)_{sat} = \frac{T v_{lv}}{J h_{lv}} \qquad (2\text{-}1)$$

Mechanical equilibrium of a spherical bubble containing a noncondensible gas will be,

$$p_v + p_g - p_l = \frac{2\sigma}{r} \qquad (2\text{-}2)$$

Integrating the Clausius Clapeyron equation along the saturation curve,

$$T_v - T_{sat} = (p_v - p_l)\frac{T_{sat} v_{lv}}{J h_{lv}} \qquad (2\text{-}3)$$

By combining Eqs.(2-2) and (2-3), also with the thermal equilibrium

$$\boxed{T_l - T_{sat} = \left(\frac{2\sigma}{r} - p_g\right)\frac{T_{sat} v_{lv}}{J h_{lv}}} \qquad (★)$$

the ***superheat*** required for spherical vapor bubble of radius r to remain in equilibrium

★ If p_g is neglected the equilibrium bubble size will be,

$$\boxed{r^* = \frac{2\sigma}{J h_{lv}\rho_v}\,\frac{T_{sat}}{(T_l - T_{sat})}} \quad \text{for } v_v \gg v_l \qquad (2\text{-}4)$$

For an ideal gas,

$$v_{lv} \approx v_v = \frac{R T_{sat}}{p_v} \qquad (2\text{-}5)$$

Thus, Eq.(★) will be,

$$T_l - T_{sat} = \left(\frac{2\sigma}{r} - p_g\right)\frac{R T_{sat}^2}{J p_v h_{lv}} \qquad (2\text{-}6)$$

The presence of dissolved gas reduces the superheat required.

(3) Bubble growth process

If superheat > Eq.(★) → bubble grows

If superheat < Eq.(★) → bubble condense

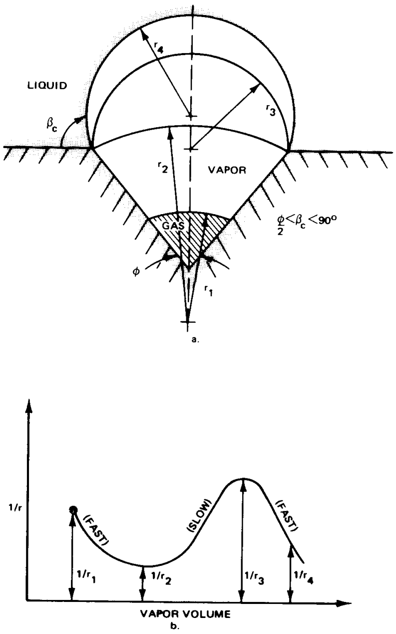


Fig. 2.8 기포 성장

3. Nucleation criterion for heated commercial surface

(1) Nucleation Criteria

In laminar sublayer. Fourier's law gives,

$$q'' = -k_\ell \frac{\partial T}{\partial y} \qquad (2.7)$$

should provide the superheat greater than that of Eq.(★)

By combining Eq. (2.7) and equation obtained by differentiating Eq.(★),

$$q'' = -k_\ell \frac{\partial T}{\partial y}\bigg|_{y=r_c} = \frac{2k_\ell \sigma}{r_c^2}\left(\frac{T_{sat} v_{fg}}{J h_{fg}}\right) \qquad (2.8)$$

Thus, the critical cavity radius to nucleate will be,

$$r_c = \left(\frac{2\sigma T_{sat} v_{fg} k_\ell}{J h_{fg} q''}\right)^{1/2} \qquad (2.9)$$

For a linear temperature gradient through liquid in the laminar sublayer. i.e.

$$T_\ell = T_w - \frac{q'' y}{k_\ell} \qquad (2.10)$$

Then,

$$T_\ell - T_{sat} = (T_w - T_{sat}) + \frac{q'' y}{k_\ell} \qquad (2.11)$$

From Eq.(★),

$$T_g - T_{sat} = \frac{2\sigma}{r_c}\frac{T_{sat} v_{fg}}{J h_{fg}} + \frac{q'' y}{k_\ell} \qquad (C1.23) \qquad (2.12)$$

If only one cavity size($r^* = r_a$) is present,

$$q'' = k_\ell \frac{(T_w - T_{sat})}{r_a} - \frac{2\sigma k_\ell}{r_a^2}\frac{T_{sat} v_{fg}}{J h_{fg}} \qquad (2.13)$$

But, if all cavity sizes are present, r^* will be eliminated as,

$$q'' = \frac{J h_{fg} k_\ell}{8\sigma T_{sat} v_{fg}}(T_w - T_{sat})^2 \qquad (2.14)$$

☞ **Nucleation Criteria**

→ low bound of nucleation

Note that the number of active nucleating cavities increases as $\dot{q}$ increases.

$$\dot{q} \sim n^m$$

where m=1 at low $\dot{q}$ and decreases to around $\frac{1}{2}$ at high $\dot{q}$

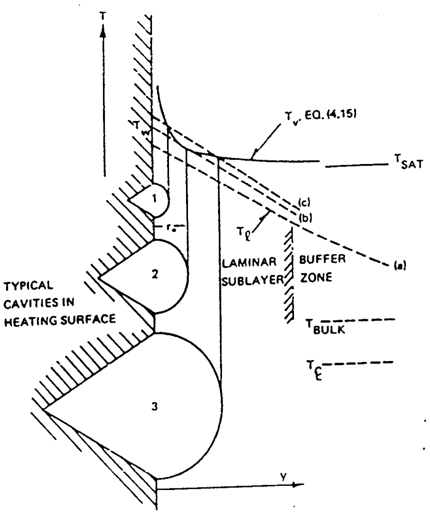


Fig. 2.9 강제대류하 상업가열면에서
기포 성장

(2) Boiling (or Ebullition) process

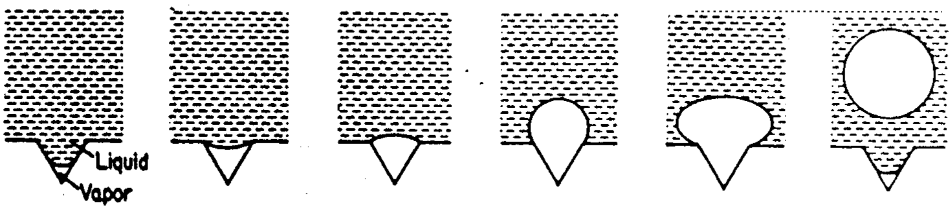
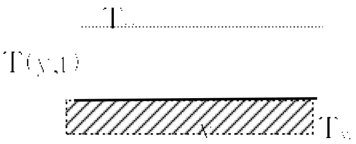


Fig 2.10 Boiling Process



※ Prediction of t_w

$$\frac{\partial T}{\partial t} = \left(\frac{k_l}{\rho_l c_{pl}}\right)\frac{\partial^2 T}{\partial y^2} \qquad (2\text{-}15)$$

I.C : $T(y,0) = T_l(\infty)$

B.C : $T(\infty,t) = T_l(\infty)$, $T(0,t) = T_w$

$$\rightarrow \quad \frac{T(y,t)-T_w}{T_l(\infty)-T_w} = erf\left(\frac{y}{2\sqrt{\alpha_l t}}\right) \qquad (2\text{-}16)$$

Thus, t_w can be obtained until reaching $T(y,t_w) = T^*$ of Eq.(★)

- *Forces exerted on a bubble*

1) surface force
2) buoyancy force
3) liquid inertia due to bubble growth
4) hydrodynamic drag force
5) force due to the liquid convection around the bubble

The size of bubble at departure from the surface can be obtained from the dynamic force balance.

$$\frac{\pi}{6}D_b^3\frac{g}{g_c}(\rho_l-\rho_v) = \pi D_b \sigma C_d(\beta_c). \qquad (2\text{-}17)$$

as,

$$D_b = \sqrt{\frac{6C_d\,\sigma\, g_c}{g(\rho_l-\rho_v)}} \qquad (2\text{-}18)$$

✣ For small cavity size, account only for buoyancy and liquid inertia force

By Fritz and Wark,

$$D_b = C_d\beta_c\sqrt{\frac{2\sigma g_c}{g(\rho_l-\rho_v)}} \qquad \text{where } C_d=0.0148$$

By Cole and Rohsenow

$$E_o^{1/2} = \begin{cases} 1.5\times 10^{-4}(J_a^*)^{5/4} & \textit{for water} \\ 1.65\times 10^{-4}(J_a^*)^{5/4} & \textit{for other fluids} \end{cases}$$

where

$$E_o = \frac{g(\rho_L - \rho_v) D_B^2}{\sigma}$$

$$J_a = \frac{\rho_L C_{pL} T_{sat}}{\rho_v h_{fg}} \quad \left(= \frac{sensible\ heat}{latent\ heat}\right)$$

- D_B decreases as the mass flow rate increases due to the increase of drag force of bubble on the active nucleation site (Levy, Koumoutsos)
- D_B increases as the wall heat flux (Unal)

Departure frequency

$$f_B = \frac{1}{t_c + t_d}$$

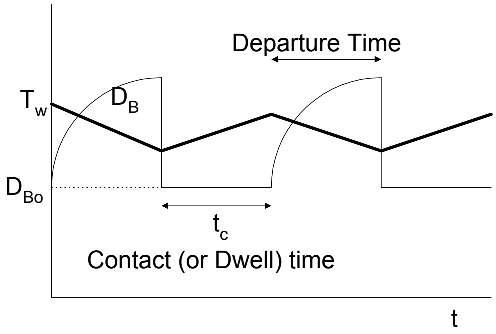


Fig. 2.11 Tw during bubble departue

Jacob and Linke let the average bubble departure velocity be,

$$U_B = \frac{D_B}{t_d} \qquad \therefore \quad D_B = U_B t_d$$

Then, $D_B f_B = \dfrac{t_d}{t_c + t_d} U_B \simeq f(p)$

Max Jacob assumed $t_c = t_d$, and by Peebles & Garber,

$$D_B f_B = \frac{1.18}{2}\left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2}\right]^{1/4}$$

$$\simeq 920 \text{ ft/hr}$$

→ nearly constant, because, before leaving the cavity the bubble grows faster in the beginning and then slow down its growth before ejecting

But, $t_c \neq t_d$, thus cannot be constant

(예 : $D_B f_B \simeq 1200$ ft/hr for Carbon tetrachloride by Westwater)

4. Bubble Dynamics

Continuity Equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho_l v_r) + \frac{\partial \rho_l}{\partial t} = 0 \qquad (2\text{-}19)$$

Momentum Equation

$$\frac{\partial p}{\partial r} = \frac{1}{g_c}\rho_l \frac{Dv_r}{Dt} \qquad (2\text{-}20)$$

For incompressible and inviscid liquid, $u(r) = u(R)\left(\frac{R}{r}\right)^2$ (2.21)

At the interface, if there is no mass transfer,

$$u(r) = \dot{R}\left(\frac{R}{r}\right)^2 \qquad (2.22)$$

Inserting the above equation into the momentum equation, we can obtain Rayleigh equation which governs the dynamic controlled bubble growth.

$$\boxed{\frac{\rho_l}{g_c}\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = \frac{h_{fg}(T_\infty - T_{sat})\rho_v}{T_{sat}} - \frac{2\sigma}{R}}$$

→ **_Rayleigh Equation_**

Energy Equation

$$\frac{\partial T}{\partial t} + u(r)\frac{\partial T}{\partial r} = \alpha_l \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right]$$

Note that $u(r) = \dot{R}\left(\frac{R}{r}\right)^2\left(1 - \frac{\rho_v}{\rho_l}\right)$

I.C : T(r,0) = T

B.C : T(R,t) = T ($T_l - T_v - T_{sat}$)

T(∞,t) = T

T

Bubble Growth Mechanism

(1) Isothermal Bubble Dynamices (dynamic controlled)

initial stage : $t \ll 1$

1. $r \gg r_{cri}$
2. $T_l - T_v \gg T_{sat}$
3. $P_v \gg P_{v,exit}$ ($= P_l + \frac{2\sigma}{r}$)

With assumption of $\lim \frac{R}{R_0} \to \infty$, where R_0: initial radius

$$R(t) = Kt + R_0$$

or $\boxed{R^{-} - t^{+}}$

(2) Isobaric Bubble Dynamics (thermally controlled)

$t^{+} \gg 1$

With assumption of $T_v = T_{sat}$

$$R(t) \propto \sqrt{t}$$

Introducing J_a

$$R(t) = 2\sqrt{\frac{1}{\pi}}\, J_a\sqrt{\alpha_l t} \quad \text{where,} \quad J_a = \frac{\rho_l C_{pl}(T_{\infty} - T_{sat})}{\rho_v h_{fg}}$$

By Mikic, Rohsenow and Graffith

$$R^{+} = \frac{2}{3}\left[(t^{+} + 1)^{\frac{3}{2}} - (t^{+})^{\frac{3}{2}} - 1 \right]$$