2. One Dimensional Two Phase Flow Model

<u>*Mass Continuity*</u>

$$\begin{pmatrix} \textit{Rate of Creation} \\ \textit{of Mass} \end{pmatrix} = \begin{pmatrix} \textit{Mass Outflow} \\ \textit{Rate} \end{pmatrix} \quad \begin{pmatrix} \textit{Mass Inflow} \\ \textit{Rate} \end{pmatrix} + \begin{pmatrix} \textit{MassStorage} \\ \textit{Rate} \end{pmatrix}$$

$$(ROC) \qquad O \qquad I \quad + \quad S$$

$$A\frac{\partial \rho}{\partial t} + \frac{\partial w}{\partial z} = 0 \;, \qquad \text{where} \quad w = \rho u A$$

For each phase,

$$\frac{\partial}{\partial t}(A\,\rho_k \alpha_k) + \frac{\partial}{\partial z}(A\,\alpha_k \rho_k u_k) = \Gamma_k \,, \qquad \sum_k \Gamma_k = 0$$

<u>*Momentum Equation*</u>

$$\begin{pmatrix} \textit{ROC of} \\ \textit{Momentum} \end{pmatrix} = \begin{pmatrix} \textit{Momentum} \\ \textit{Outflow Rate} \end{pmatrix} \quad \begin{pmatrix} \textit{Momentum} \\ \textit{Inflow Rate} \end{pmatrix} + \begin{pmatrix} \textit{Momentum} \\ \textit{Storage Rate} \end{pmatrix}$$

$$= \begin{pmatrix} \textit{Sum of the Forces} \\ \textit{acting on the C.V} \end{pmatrix} \left(\sum F\right)$$

$$\therefore \quad \sum F = \quad F_{body} + \quad F_{sh} + \quad F_{wall} + \quad F_{sur} + \quad F_{head}$$

For vapor phase,

$$A_g dp \quad dF_g \quad S \quad A_g dz \rho_g g \sin\Theta = \quad w_g du_g + dw_g u_g \quad dw_g u_l + \frac{\partial w_g}{\partial t} A_g \qquad (1.2.13)$$

For liquid phase,

$$A_l dp \quad dF_l + S \quad A_l dz \rho_l g \sin\Theta = \quad w_l du_l + \frac{\partial w_l}{\partial t} A_l \qquad (1.2.14)$$

Adding Eqs. (1.2.13) & (1.2.14) and $dw_g = \quad dw_l$

$$A dp \quad dF_g \quad dF_l \quad g \sin\Theta\, dz (A_l \rho_l + A_g \rho_g)$$

$$= \quad d(w_l u_l + w_g u_g) + \frac{\partial}{\partial t}(w_g A_g + w_l A_l) \qquad (1.2.15)$$

Here, interfacial shear stress, $S = \tau_i P_i \quad z$

where $\quad \tau_i = (C_f)_i \frac{\rho_g}{2g_c}(u_g \quad u_l)^2 \qquad$ with no entrainment

$$(C_f)_i = 0.005[1 + 75(1 - \langle\alpha\rangle)] \quad : \text{interfacial friction factor}$$

And $dw_l = -dw_g = -\delta w'$

$$= \frac{q'' P_H}{h_{fg}} \quad \text{: evaporation amount per unit axial length}$$

★ See Lahey's Table 5.II

Energy Equation

$$\begin{pmatrix}\text{Rate of Creation}\\ \text{of Energy}\end{pmatrix} = \begin{pmatrix}\text{Energy}\\ \text{Outflow Rate}\end{pmatrix} - \begin{pmatrix}\text{Energy}\\ \text{Inflow Rate}\end{pmatrix} + \begin{pmatrix}\text{Energy}\\ \text{Storage Rate}\end{pmatrix}$$

$$w(\delta Q - \delta W) = w\,dh + d\left[\frac{w_l u_l^2}{2} + \frac{w_g u_g^2}{2}\right] + wg\sin\Theta\,dz \qquad (1.2.16)$$

(1) Homogeneous Model

◎ $u_g = u_l = u$

◎ thermodynamic equilibrium between phases

◎ use suitably defined f for two phase flow

★ valid for

1. bubbly and wispy annular flow pattern
2. high velocity flow in large channel

$$\langle \rho_H \rangle = 1/(v_f + \langle x \rangle v_{fg}) \qquad (1.2.17)$$

$$\langle h \rangle = h_f + \langle x \rangle h_{fg} \qquad (1.2.18)$$

$$G = \langle \rho_H \rangle \langle j \rangle, \quad \text{and } \langle j \rangle = u \text{ for homogeneous flow} \qquad (1.2.19)$$

For constant flow area,

1) <u>mass continuity</u>

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial z} = 0 \qquad (1.2.20)$$

2) <u>momentum equation</u>

$$\frac{\langle \rho_H \rangle}{g_c}\frac{Du}{Dt} = -\frac{\partial p}{\partial z} - \frac{g}{g_c}\langle \rho_H \rangle \sin\Theta - \frac{\tau_w P_f}{A} \qquad (1.2.21)$$

3) <u>energy equation</u>

$$\langle \rho_H \rangle \frac{D\langle e \rangle}{Dt} = q''\left(\frac{P_H}{A}\right) \quad \text{where } \langle e \rangle = \left(\langle h \rangle + \frac{u^2}{2g_c J} + \frac{g}{g_c}\frac{z\sin\Theta}{J}\right) \qquad (1.2.22)$$

★ thermal energy equation : Eq. (1.2.21) × u

$$\langle \rho_H \rangle \frac{D\langle h \rangle}{Dt} = \dot{q}'' \left(\frac{P_H}{A} \right) + u \left(\frac{\tau_w P_f}{JA} \right) \qquad (1.2.23)$$

I) quality propagation equation

From Eqs. (1.2.18) and (1.2.19), for constant system pressure, if we neglect Δp

compared to the system pressure,

$$\frac{D\langle x \rangle}{Dt} = \Omega \langle x \rangle - \Omega \frac{v_f}{v_{fg}} \qquad (1.2.24)$$

where Ω : characteristic frequency of phase change

(즉 speed at which phase change take place)

$$= \dot{q}'' \frac{P_H}{A} \frac{v_{fg}}{h_{fg}}$$

(2) Separate Flow Model

◎ $u_l \neq u_g$

◇ thermodynamic equilibrium between phases

◇ ϕ^2, α

★ valid for annular flow

For constant flow area

1) continuity equation

$$\frac{\partial \langle \bar{\rho} \rangle}{\partial t} + \frac{\partial G}{\partial z} = 0 \qquad (1.2.25)$$

where $\langle \bar{\rho} \rangle = \rho_l (1 - \langle \alpha \rangle) + \rho_g \langle \alpha \rangle$

$G = \rho_l (1 - \langle \alpha \rangle) u_l + \rho_g \langle \alpha \rangle u_g$

2) momentum equation

◇ liquid phase

$$-(1 - \langle \alpha \rangle) \frac{\partial p}{\partial z} - \frac{g}{g_c} \rho_l (1 - \langle \alpha \rangle) \sin\Theta - \frac{\tau_w P_f}{A} + \frac{\tau_i P_i}{A}$$

$$= \frac{1}{g_c} \left[\frac{\partial}{\partial t} [\rho_l (1 - \langle \alpha \rangle) u_l] + \frac{\partial}{\partial z} [\rho_l A (1 - \langle \alpha \rangle) u_l^2] \right] + \frac{\delta w'}{g_c A} u_i \qquad (1.2.26)$$

◇ vapor phase

$$-\langle \alpha \rangle \frac{\partial p}{\partial z} - \frac{g}{g_c} \rho_g \langle \alpha \rangle \sin\Theta - \frac{\tau_i P_i}{A}$$

$$= \frac{1}{g_c} \left[\frac{\partial}{\partial t} [\rho_g \langle \alpha \rangle u_g] + \frac{\partial}{\partial z} [\rho_g A \langle \alpha \rangle u_g^2] \right] - \frac{\delta w'}{g_c A} u_i \qquad (1.2.27)$$

◎ two phase mixture

$$-\frac{\partial p}{\partial z} - \frac{g}{g_c}\langle\bar{\rho}\rangle\sin\Theta - \frac{\tau_w P_f}{A} = \frac{1}{g_c}\left[\frac{\partial G}{\partial t} + \frac{\partial}{\partial z}\left(\frac{G^2}{\langle\rho'\rangle}\right)\right] \qquad (1.2.28)$$

where $\frac{1}{\langle\rho'\rangle} = \left[\frac{(1-\langle x\rangle)^2}{\rho_l(1-\langle\alpha\rangle)} + \frac{\langle x\rangle^2}{\rho_g\langle\alpha\rangle}\right]$: momentum density

3) Energy Equation

$$\rho_l(1-\langle\alpha\rangle)\frac{De_l}{Dt} + \rho_g\langle\alpha\rangle\frac{De_g}{Dt} = \frac{q''P_H}{A} - \frac{\delta w'}{A}(e_g - e_l) \qquad (1.2.29)$$

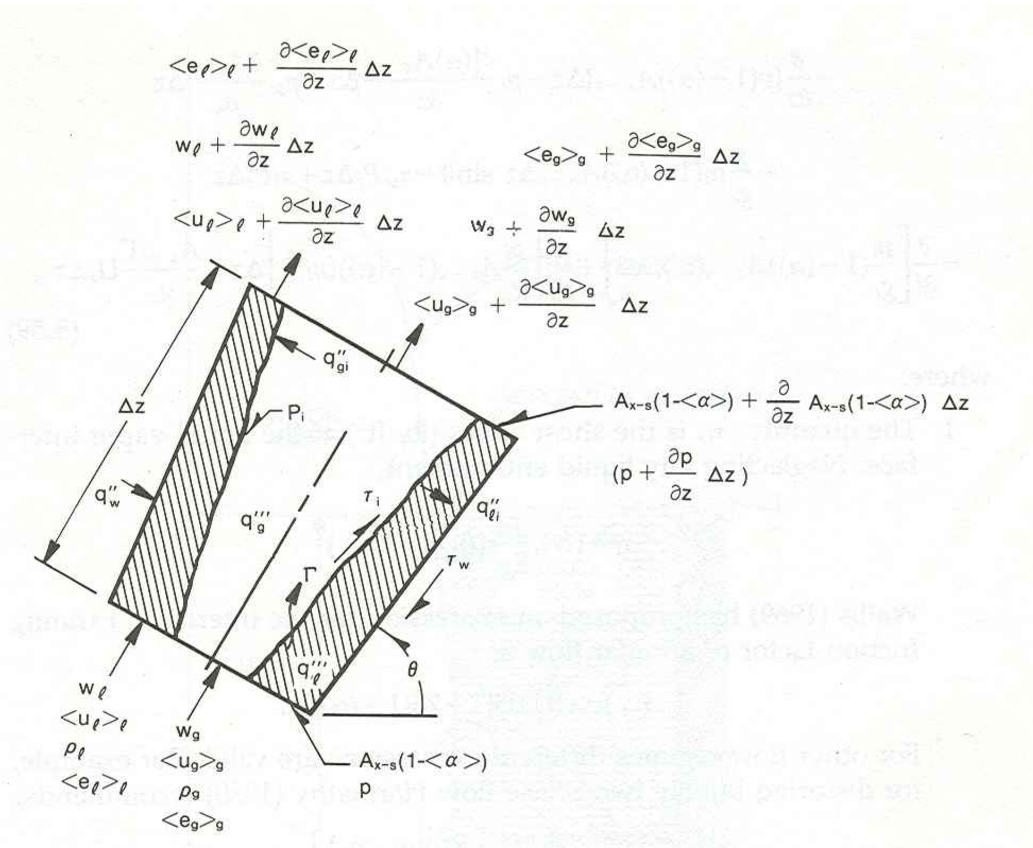

Fig.1.2 Control volume for separated flow(each phase) [Lahey's Fig.5.3]

■ Interfacial exchange for two fluid model

1) interfacial mass transfer

$$\Gamma = \frac{\delta w}{A} = a_i m \qquad (1.2.30)$$

where a_i : local interfacial area concentration (IAC)

the probability of the existence of interfacial area at local position in the two phase flow

m : averaged mass transfer rate per unit area

2) interfacial energy transfer

$$h_{ki}\Gamma_k + \frac{q_{ki}}{L_s} = a_i[m_{ki}h_{ki} + q_{ki}] \qquad (1.2.31)$$

(3) Drift Flux Model

[Ref] G.B. Wallis, "One dimensional Two phase Flow" McGraw Hill

★ Two fluid model : numerical complication and uncertainties in specification of interfacial transport

In drift flux model, concern the relative motion between phases rather than each phase motion

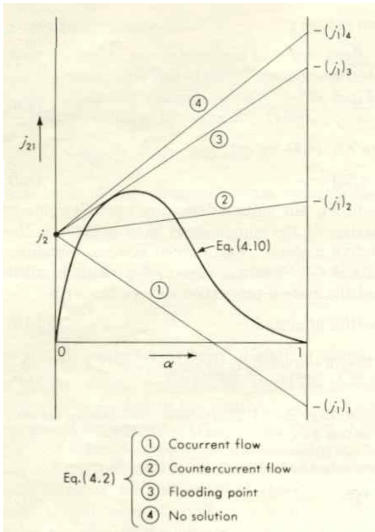

Fig. 4.1 Graphical solution of Eqs. (4.2) and (4.10).

Fig.1.2 Graphical solution of j_{gf}

o mixture center of mass velocity

o drift velocity : volumetric flux of each phase relative to a surface moving at average velocity, j

The drift flux is defined in many ways as,

V_{gj} (*local drift flux*) $= v_g - j$,

$j_{gf} = \alpha(v_g - j) = \alpha V_{gj}$ in Wallis

$\overline{\langle V_{gj} \rangle} = \dfrac{\langle \alpha(v_g - j)\rangle}{\langle \alpha \rangle}$ in Lahey & Zuber

$= \langle\langle V_{gj} \rangle\rangle$ in Ishii

Some relations :

$$j_{gf} = u_r\alpha(1 - \alpha) \qquad (1.2.32)$$

$$j_g = \alpha j + j_{gf}$$

Thus, all properties (α, ρ_m, etc) are sum of the homogeneous flow value and a correction factor which is a function of drift flux or additional term.

$$\alpha = \frac{j_g}{j}\left(1 - \frac{j_{gs}}{j_g}\right), \qquad \rho_m = \frac{j_f\rho_f + j_g\rho_g}{j} + (\rho_f - \rho_g)\frac{j_{gs}}{j}$$

$\therefore$ If the flow is homogeneous, $j_{gs} = 0$

or $\qquad j_g = \frac{\alpha}{1-\alpha} j_f + \frac{1}{1-\alpha} j_{gf}$

Then, for a constant α, a plot of j_g vs j_f will be a straight line of slope $\frac{\alpha}{1-\alpha}$

■ Modeling

- o area averaging
- o no information on change of variables normal to flow direction
- o need empirical correlations or simplified models for the transfer of momentum and energy between the wall and the fluid.

A. Cross Section Averaged Kinematic Equation

$$\langle F \rangle = \frac{1}{A}\int_A F dA$$

For constant system pressure, the vapor phase continuity equation, will be

$$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial z}[\alpha(j + V_{gj})] = \frac{\Gamma}{\rho_g} \qquad (1.2.33)$$

Taking $\frac{1}{A}\int_A [Eq.(1.2.33)]dA$,

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \frac{\partial}{\partial z}[\langle \alpha j \rangle + \langle \alpha V_{gj} \rangle] = \frac{\langle \Gamma_g \rangle}{\rho_g}$$

$$\therefore \quad \frac{\partial \langle \alpha \rangle}{\partial t} + \frac{\partial}{\partial z}\left[C_o \langle \alpha \rangle \langle j \rangle + \frac{\langle \alpha V_{gj} \rangle}{\langle \alpha \rangle}\langle \alpha \rangle\right] = \frac{\langle \Gamma_g \rangle}{\rho_g} \qquad (1.2.34)$$

where $C_o = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}$

$$\therefore \quad \frac{\partial \langle \alpha \rangle}{\partial t} + C_o \langle j \rangle \frac{\partial \langle \alpha \rangle}{\partial z} + C_o \langle \alpha \rangle \frac{\partial \langle j \rangle}{\partial z} + \overline{V_{gj}}\frac{\partial \langle \alpha \rangle}{\partial z} = \frac{\langle \Gamma_g \rangle}{\rho_g} \qquad (1.2.35)$$

Since $\frac{\partial \langle j \rangle}{\partial z} = \Omega$, where $\Omega = v_{fg} \frac{\delta w'}{A_{x-s}} = \frac{q_w'' P_H v_{fg}}{A_{x-s} h_{fg}}$

$$\frac{\partial \langle \alpha \rangle}{\partial t} + [C_o \langle j \rangle + \overline{V_{gj}}] \frac{\partial \langle \alpha \rangle}{\partial z} = (\frac{\rho_f}{\Delta \rho} - C_o \langle \alpha \rangle)\Omega \quad (4.2.36)$$

Now, let's define C_K (kinematic velocity) = $C_o \langle j \rangle + \overline{V_{gj}}$

Then,

$$\frac{\partial \langle \alpha \rangle}{\partial t} + C_K \frac{\partial \langle \alpha \rangle}{\partial z} = (\frac{\rho_f}{\Delta \rho} - C_o \langle \alpha \rangle)\Omega \quad (4.2.37)$$

=> ***Void Propagation Equation***

B. Drift Flux parameters

1. C_o : concentration parameter

$$C_o = \frac{1}{\langle j \rangle \langle \alpha \rangle A} \int\int_A (j\alpha) dA \quad (4.2.38)$$

$$= \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}$$: global slip due to cross sectional averaging

From definition of average velocity of vapor phase,

$$v_g = \frac{\langle j_g \rangle}{\langle \alpha \rangle} \quad (4.2.39)$$

Averaging Eq.(4.2.32), $\langle j_{gj} \rangle = \langle j_g \rangle - \langle \alpha j \rangle$

$$\therefore \frac{\langle j_g \rangle}{\langle \alpha \rangle} = C_o \langle j \rangle + \overline{V_{gj}} \quad (4.2.40)$$

Or $\langle \alpha \rangle = \frac{\langle j_g \rangle}{[C_o \langle j \rangle + \overline{V_{gj}}]}$

Note that, from Eq.(4.1.18),

$$S = \frac{1 - C_o \langle \alpha \rangle}{1 - \langle \alpha \rangle} + \frac{V_{gj}(1 - \langle \alpha \rangle)}{(1 - C_o \langle \alpha \rangle) \langle j_f \rangle} \quad (4.2.41)$$

integral slip local slip

For homogeneous model ($V_{gj} = 0$), local slip will be vanished.

And,

$$\langle \alpha \rangle = \frac{\langle x \rangle}{C_o \left[\langle x \rangle + \frac{\rho_g}{\rho_f}(1 - \langle x \rangle) \right] + \frac{\rho_g V_{gj}}{G}} \quad (4.2.42)$$

=> Zuber-Findley Void Quality Correlation

Since $\langle j \rangle = \frac{Q_1 + Q_2}{A}$, $\langle j_g \rangle = \frac{Q_2}{A}$ and $\overline{V_g} = \frac{\langle j_g \rangle}{\langle \alpha \rangle}$.

Eq. (1.2.10) can be rewritten as,

$$\langle \alpha \rangle = \frac{Q_2 A \langle j_g \rangle}{C_o (Q_1 + Q_2)} = \frac{1}{C_o} \frac{Q_2}{Q_1 + Q_2}$$

1) Bankoff model

Neglecting the local slip with $V_{gj} = 0$, Eq. (1.2.10) will be

$$\frac{\langle \alpha \rangle}{\langle \beta \rangle} = \frac{1}{C_o} = \kappa, \qquad \text{where } \kappa = 0.71 + 0.0001\, p \text{ (psia) for water}$$

Here β, volumetric flow fraction, is defined as, $= \frac{\langle j_g \rangle}{\langle j \rangle}$

- However, V_{gj} is important for the flow of low G.
- applicable to determine the two phase level in steam generators

2) Dix model

: C_o is function of $\langle \beta \rangle$ which is dependent on pressure and quality

$$C_o = \langle \beta \rangle [1 + (1/\langle \beta \rangle - 1)^b] \qquad \text{where } b = (\rho_g/\rho_f)^{0.1}$$

If $C_o = 1$,

$$\langle \beta \rangle = \langle \alpha \rangle = \frac{\langle x \rangle}{\left[\langle x \rangle + \frac{\rho_g}{\rho_f}(1 - \langle x \rangle) \right]} \tag{1.2.13}$$

2) Vgj

From fractional analysis for bubbly slug flow, the important forces on a bubble are follows:

net bouyancy force : $F_B = \frac{1}{6}\pi D_b^3 (\rho_f - \rho_g)\frac{g}{g_c}$

inertia force : $F_I = \frac{\rho_f}{g_c} U_f^2 \left(\frac{1}{4}\pi D_b^2\right)$

surface tension force : $F_\sigma = \pi D_b \sigma$

In the churn turbulent flow, the most important force groups are

1) ratio F_I to F_B

2) ratio F_I to F_σ

Then, $\frac{\rho_f U_f^2}{D_b g (\rho_f - \rho_g)} = k_1^2$ and $\frac{\rho_f U_f^2 D_b}{g_c \sigma} = k_2^2$

The product of these dimensionless groups yields,

$$\frac{\rho_f^2 U_f^4}{g g_c \sigma (\rho_f - \rho_g)} = k_1^2 k_2^2 = k_3^4$$

Then, the terminal rise velocity will be,

$$U_t = k_3\left[\frac{g g_c \sigma (\rho_f - \rho_g)}{\rho_f^2}\right]^{1/4}$$

For churn turbulent flow, $V_{gj} = U_t$

- ▶ For bulk boiling (Lahey, 1977), $k_3 = 2.9$
- ▶ For churn turbulent flow (Zuber, 1967), $k_3 = 1.41$
- ▶ For slug flow (Zuber, 1969), $V_{gj} = 0.35\left[\frac{g \Delta\rho D}{\rho_f}\right]^{1/2}$
- ▶ For two phase pseudo jet flow (Zuber, 1969) $V_{gj} = constant\left[\frac{\sigma g \Delta\rho}{\rho_f^2}\right]^{1/4}$
- ▶ For the dispersed two phase system in vertical motion

$$V_{gj} = v_\infty (1 - \alpha)^n$$

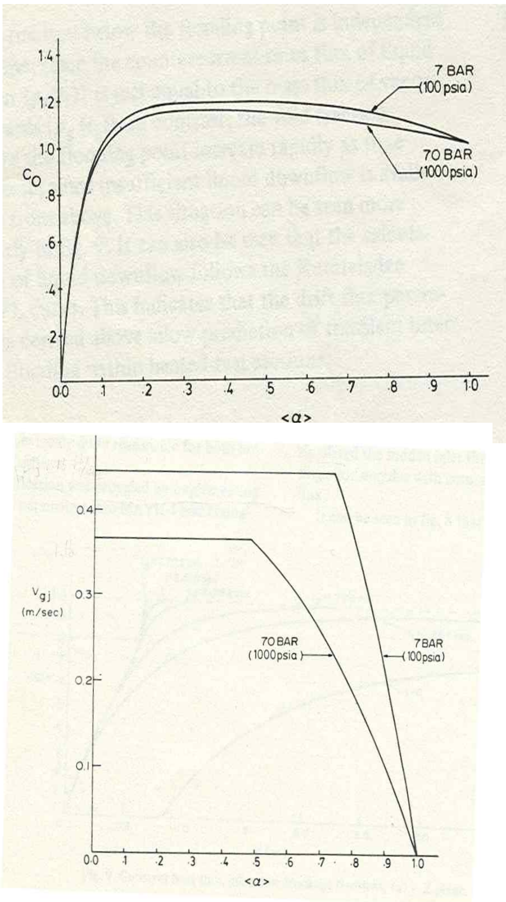

Fig.4.6 V_{gj} vs void fraction

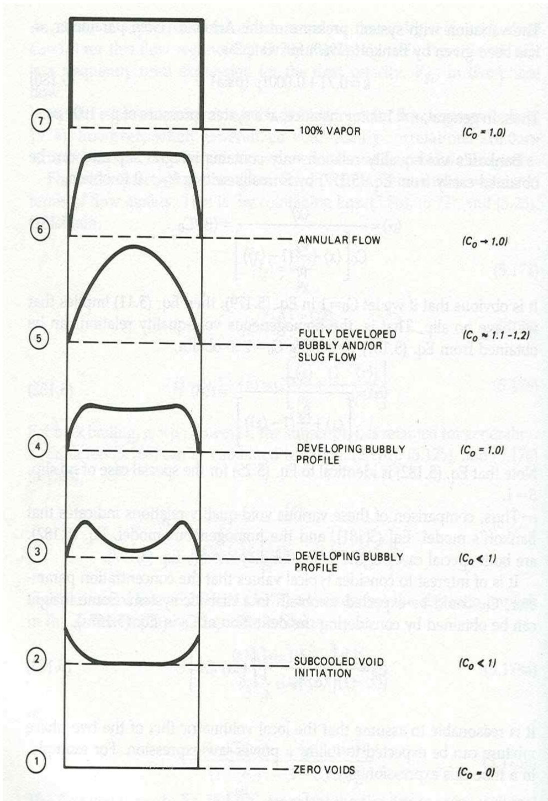

Fig.4.5 Void concentration profile and flow regime[Lahey]

(4) Numerical Stability in Two Phase Modeling

For two fluid model, generally

$$A\frac{\partial U}{\partial t} + B\frac{\partial U}{\partial z} = C \qquad (1.2.4)$$

where $U = (\alpha,\ p,\ u_l,\ u_v,\ h_l,\ h_g)^T$

Thus,

$$[A]\frac{\partial}{\partial t}\begin{bmatrix}\alpha \\ p \\ u_l \\ u_v \\ h_l \\ h_g\end{bmatrix} + [B]\frac{\partial U}{\partial z} = [S] \qquad \Rightarrow \qquad |A\lambda - B| = 0\ ,\quad \lambda_i = \frac{\partial}{\partial t}$$

a) If all six roots are real and distinct $\Rightarrow$ Hyperbolic

b) If we have some same roots $\Rightarrow$ Parabolic

c) If we have complex conjugate pair $\Rightarrow$ Elliptic

1) $\dfrac{\partial \phi}{\partial t} + v\dfrac{\partial \phi}{\partial z} = 0 \qquad (1.2.5)$

Assume a perturbation in ϕ of the form,

$$\delta\phi = \delta\phi_o e^{i(kz - \omega t)}\ , \quad \text{where k is real} \qquad (1.2.6)$$

In order for this system to be well posed, ω should be real.

Inserting Eq. (1.1.6) into Eq.(1.1.5),

$$i\omega\,\delta\phi_o e^{i(kz-\omega t)} + ikv\,\delta\phi_o e^{i(kz-\omega t)} = 0$$

$$\therefore\ -i\omega + ikv = 0 \quad \text{or}\ \omega = kv$$

i.e The eigenvalue of this system, v, should be real.

For real v, stable when $C_v \left(= \frac{v\Delta t}{\Delta z}\right) \le 1$

For complex eigenvalue on a finite difference scheme [Handout 참조]

$$v = v_o(1 + i\varepsilon)$$

a) For $\varepsilon = 0$, stable when $C_v \le 1$

b) For $\varepsilon \ne 0$, may be unstable for some n even when $C_v \le 1$

2) Charecteristic Analysis and Time Discretization of the HEM Model

$$A\frac{\partial U}{\partial t} + B\frac{\partial U}{\partial z} = 0$$

where $U = [\rho,\ u,\ v,\ p]^T$, u : internal energy

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & \rho & 0 \\ 1 & 1/\beta^2 & 0 & 1/c^2 \end{bmatrix}, \quad B = \begin{bmatrix} v & 0 & \rho & 0 \\ 0 & \rho v & p & 0 \\ 0 & 0 & \rho v & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad 1/\beta^2 = \frac{\partial \rho}{\partial u}, \quad \frac{1}{c^2} = \frac{\partial \rho}{\partial p}$$

Now, for explicit time discretization and forward differencing, we can perturb the continuity and momentum equation

$$\rho_j^n = \rho_o + \delta\rho_j^n$$

$$v_j^n = v_o + \delta v_j^n$$

Assume $\delta\phi_j^n = \delta\phi^n e^{ijk\Delta z}$

Then, the linearized continuity and momentum equation will be

$$U^{t+\Delta t} = GU^t$$

where $U = [\delta\rho,\ \delta v]^T$ and $G = \begin{bmatrix} (1-\bar{v}) & \rho\frac{\Delta t}{\Delta z}iK' \\ -\frac{c^2}{\rho}\frac{\Delta t}{\Delta z}iK' & (1-\bar{v}) \end{bmatrix}$

$$\bar{v} = \frac{v_o \Delta t}{\Delta z}[1-e^{-ik\Delta z}], \qquad K' = 2\sin(k\Delta z/2)$$

For

$$|G - I\lambda| = 0, \qquad \lambda_{1,2} = \begin{bmatrix} 1-\bar{v}+iC_n \\ 1-\bar{v}-iC_n \end{bmatrix}$$

where

$$C_n = \frac{C\Delta t}{\Delta z}K'$$

For $v \ll C$, $\lambda_{1,2} > 1$, **unstable**

Thus, <u>the implicit solution can be recommanded.</u>

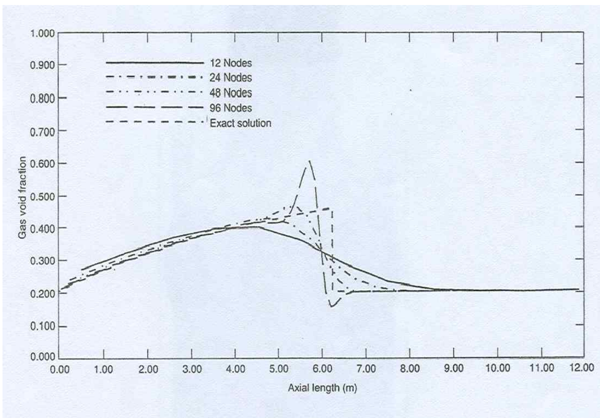

Fig. 1.7 Example of spatial convergence of void vs length