III. Flow Regime Analysis

- As the shape or distribution of bubbles changes according to the break of the thermodynamic and local hydrodynamic equilibrium, the mixing of liquid and bubble has its unique form and thermal hydraulics characteristics.

  ➡ **Flow Patten** or **Flow Regime**

- When a given flow regime exit or where transitions occur,

  → different correlation for τ_w, $\langle\alpha\rangle$ at each region

  ∴ need appropriate analysis for each flow regime and couple these analysis at flow regime transition points.

- Observation of flow pattern

  1 flash and cine photography : reflection and refraction

  2 X radiography

  3 hot wire or electrical and optical probe

1. Flow pattern in cocurrent flow

◆ In vertical flow

1 bubbly flow : discrete small and spherical bubbles in a continuous liquid

2 slug flow : bubble coalescence having approximate diameter of pipe. <u>periodic</u>

3 churn flow : breakdown of large bubbles in slug flow, <u>chaotic</u>

4 annular flow : separate flow, <u>entrainment of droplets</u>

◆ In horizontal tube

1 bubbly flow : bubbles travel in the upper half of the pipe.

2 plug flow : similar to slug flow in the vertical flow

3 stratified flow : at very low v_f and v_g

4 wavy flow : as the vapor velocity increases, the interface disturbed.

5 slug flow

6 annular flow

◆ In helical coiled tube

(1) Bubbly flow

: Actually, no developed bubble flow due to coalesce and break of bubbles

But, for $\langle\alpha\rangle < 30\%$,

1. ideal bubbly regime : no interaction between bubbles
2. churn turbulent bubbly regime

   : exist a lot of interaction due to distortion and wake effects

※ the effect of **virtual mass** in momentum equation

caused by the relative acceleration of vapor phase with respect to liquid

As bubbles are accelerated, they increase kinetic energy of surrounding liquid. Thus this inviscid effect is modelled as an increase in the mass of the accelerating bubble by adding some liquid displaced.

∴ the virtual mass of bubble = true mass + additional hydrodynamic mass

and additional force on bubble = additonal mass × its relative acceleration

➡ **virtual mass force ;** $\pm\frac{\rho_l \xi \langle\alpha\rangle}{g_c} A_{x-s}\left[\frac{D_v(\langle u_v\rangle - \langle u_l\rangle)}{D_t}\right]$

- quantify the mom. transfer between phase
- important 2 phase cirtical flow
- improve numerical stability

In the mixture momentum equation, both terms are cancelled.

ξ is dependent geometric configulation and interaction between bubbles.

= 1/2 for spherical, noninteracting bubble

★ Note that the virtual mass force is not a viscous drag term(interfacial shear term) due to relative slip between phases, which is given as,

$$\frac{\tau_i P_i}{A_{x-s}} = \frac{3}{4}\left(\frac{C_d}{\langle D_b\rangle}\right)\rho_l\,[\langle u_v\rangle_v - \langle u_l\rangle_l]\,|\langle u_v\rangle_v - \langle u_l\rangle_l|$$

where $\left(\frac{C_d}{\langle D_b\rangle}\right)$ is constant for idela bubblely flow

$$= 1.4\,(1-\langle\alpha\rangle)^3\ m^{-1}$$

Kinematics of bubbly flow by Zuber and Hench

$$\langle u_r\rangle = U_T(1-\langle\alpha\rangle)^m \qquad \text{where } U_T\text{: bubble rise velocity} \qquad (13.1)$$

Recall the define

$$\langle u_r\rangle = \langle u_v\rangle - \langle u_l\rangle = \frac{\langle j_v\rangle}{\langle\alpha\rangle} - \frac{\langle j_l\rangle}{(1-\langle\alpha\rangle)} \qquad (13.2)$$

Fig. 1.8 shows m vs bubble size.

The wake interaction will be strong as the bubble become larger.

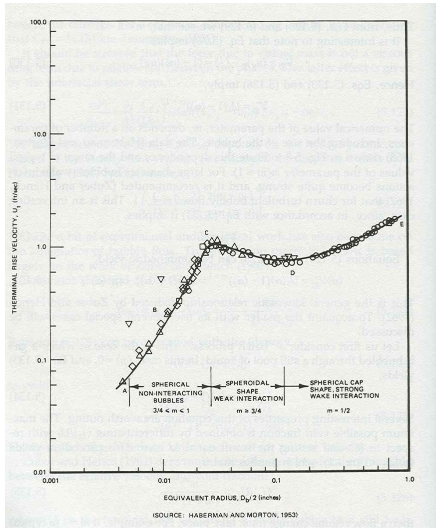


그림 1.8 Terminal rise velocity of air bubble vs bubble size[Lahey]

For churn turbulent flow, m = -1 (1.3.3)

Then $V_{gj}' = (1-\langle\alpha\rangle)u_r$

$$= U_t\,(1-\langle\alpha\rangle)^{m+1} \quad (1.3.1)$$

Thus from Eq. (1.3.3),

$$V_{gj}' = U_t$$

By multiplying Eqs. (1.3.1) and (1.3.2) by $\langle\alpha\rangle(1-\langle\alpha\rangle)$

$$U_t\langle\alpha\rangle(1-\langle\alpha\rangle)^{m+1} = \langle j_g\rangle(1-\langle\alpha\rangle)-\langle j_f\rangle\langle\alpha\rangle \quad (1.3.5)$$

$$= \langle\alpha\rangle V_{gj}' \quad \text{from Eq. (1.3.3)}$$

→ ***general kinetic relationship***

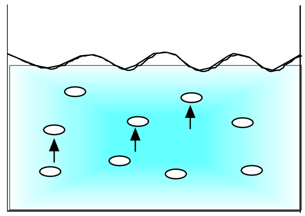
■ <u>Batch Process</u> : $\langle j_f\rangle = 0$

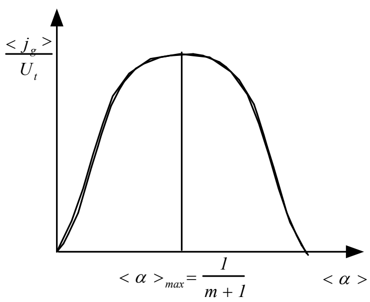
Eq. (1.3.5) will be

$$\frac{\langle j_g\rangle}{U_t} = \langle\alpha\rangle(1-\langle\alpha\rangle)^m$$

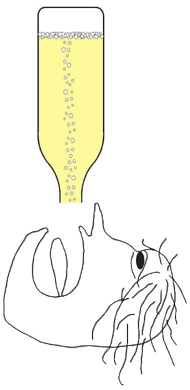
Thus, if $\frac{\langle j_g\rangle}{U_t} = \frac{m^m}{(m+1)^{m+1}}$, the flow regime changes

(Ex : $\frac{\langle j_g\rangle}{U_t} > \frac{1}{4}$, for m=1 (small bubble).)





■ Counter current flow : $\langle j_f\rangle = -\langle j_g\rangle$



Eq.(1.3.5) will be

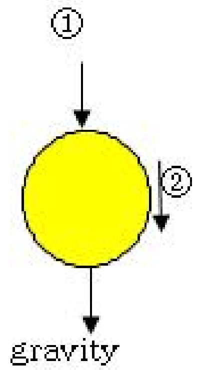
$$\langle j_g\rangle / U_t = \langle\alpha\rangle(1-\langle\alpha\rangle)^{m+1} \quad (1.3.6)$$

From Eqs. (1.3.1) and (1.3.2)

$$\langle u_g\rangle(1-\langle\alpha\rangle) = \frac{\langle j_g\rangle}{\langle\alpha\rangle} = -\langle u_f\rangle$$

$$\therefore \langle u_g\rangle > \langle u_f\rangle$$

Fractional analysis to obtain U_f



1. $Buoyancy\ force = \frac{1}{6}\pi D_b^3(\rho_f - \rho_g)\frac{g}{g_c}$

2. $Drag\ force = 3\pi U_f \mu_f D_b$

3. $Inertia\ force\ on\ bubble = \frac{\rho_f}{g_c} U_f^2\left(\frac{1}{4}\pi D_b^2\right)$

4. $Surface\ force = \pi D_b \sigma$

Assume the ratio between these forces is constant.

$$\frac{Inertia}{Buoyance} = \frac{\rho_f U_f^2}{D_b g(\rho_f - \rho_g)} = k_1^2$$

$$\frac{Inertia}{Surface} = \frac{\rho_f U_f^2 D_b}{g_c \sigma} = k_2^2 \qquad \therefore \quad \frac{\rho_f^2 U_f^4}{g g_c \sigma(\rho_f - \rho_g)} = k_1^2 k_2^2 = k_3^4$$

Then, $$U_f = k_3\left[\frac{g g_c \sigma(\rho_f - \rho_g)}{\rho_f^2}\right]^{\frac{1}{4}}$$

For churn turbulent flow, $k_3 = 1.53$

$$\therefore \quad V_{gj}' = 1.53\left[\frac{g g_c \sigma(\rho_f - \rho_g)}{\rho_f^2}\right]^{\frac{1}{4}}$$

(2) Slug flow

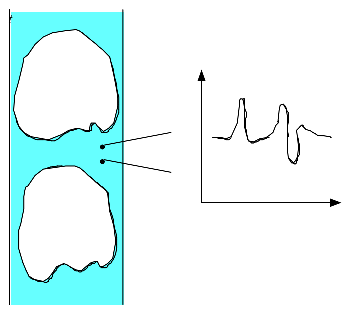
easily observable in adiabatic system not in diabatic system

$D_b / D_H > 0.5$

특성 : 1) bullet shaped Taylor bubble occupied most of channel area.

2) slug length ∝ liquid flow rate

3) liquid may downflow.



Concerns : $\begin{cases} \text{bubble rise velocity} \\ \text{effect of viscosity} \end{cases}$

Transition region between bubbly & slug flow

: $\langle\alpha\rangle = 0.1 \sim 0.3$

In case of churn turbulent flow, $U_f = V_{gj}'$

From fractional analysis, bubble rese velocity is

obtained from inertia ($\sim \rho_f U_f^2$) to bouyancy force ($\sim (\rho_f - \rho_g) g D_H$)

$$\therefore \quad U_l = k_1 \left[\frac{g D_b(\rho_l - \rho_g)}{\rho_l} \right]^{1/2} \qquad (4.3.7)$$

where $\quad k_1 = 0.345 \qquad$ for circular channel

$\qquad\qquad = 0.23 \qquad$ for noncircular channel

$\qquad$: inertia dominant case

★ viscous dominant case : $\qquad$ viscous force $\left(\sim \frac{\mu_l U_L}{D_b} \right) \sim \Delta\rho g D_b$

$$\therefore U_l = \frac{k g \Delta\rho D_b^2}{\mu_l} \qquad \textit{where } k=0.01$$

★ surface dominant: no bubble move

: depend on Bo (Bond 수 : $\frac{\Delta\rho g D_b^2}{4\sigma}$)

$\langle\alpha\rangle$ from drift flux model in slug flow

$$u_g = j + (u_g - j)$$

From Eq.(4.3.5), $j_g = \alpha j + \alpha(u_g - j)$

By cross sectional averaging, $\langle j_g \rangle = C_o \langle\alpha\rangle\langle j\rangle + V_{gj}\langle\alpha\rangle$

$$\therefore \langle\alpha\rangle = \frac{\langle j_g \rangle}{C_o \langle j \rangle + V_{gj}}$$

In slug flow $C_o \sim 1.2, \quad V_{gj} = U_l$

◆ ***Criterion of slug annular transition*** by Wallis

Define normalized superficial velocity as,

$$j_g^* = \frac{j_g \rho_g^{1/2}}{(g D_H \Delta\rho)^{1/2}} \quad : \quad \frac{\textit{mom. flux of each phase}}{\textit{buoyance force}}$$

$$j_l^* = \frac{j_l \rho_l^{1/2}}{(g D_H \Delta\rho)^{1/2}}$$

$$\therefore \quad j_g^* = 0.4 + 0.6 j_l^*$$

즉, if $\quad j_g^* > 0.4 + 0.6 j_l^*$, $\quad$ *annular flow*

※ Churn flow

similar characteristics to slug flow, but much more chaotic and frothy

Taylor bubble is distorted, its length increases and becomes narrow

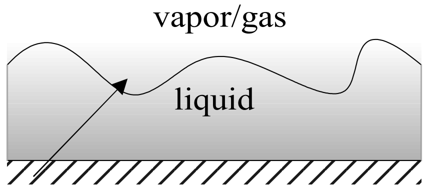
liquid falls downward where it accumulates and is lifted up by the gas again

$\therefore$ oscillatory motion of liquid.

➡ Characteristics of churn flow

(3) Annular Flow

- $\langle\alpha\rangle = 0.65 \sim 0.8$
- important in BWR (upper portion of fuel channel)
- ★ boiling heat transfer
- ★ interest in interface



What happens in interface ?

1) ideal annular flow
   - no entrainment
   - smooth and symmetric interface

2) wispy annular flow
   - at high G (> $1.0 \times 10^6\ lb/h\ ft^2$)
   - entrained liquid flowing in large agglomerate

3) spray annular flow
   - most frequent in BWR
   - entrained of relatively small liquid droplet
     - ★ droplet size

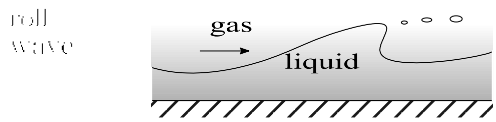
$$We_{crit} = \left[\frac{\rho_g(\langle u_g\rangle - \langle u_f\rangle)^2 D_d}{g_c\sigma}\right]_{crit} = 6.5 \sim 22$$

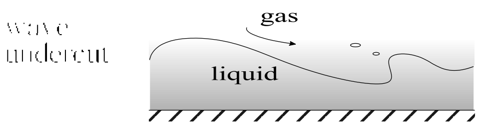
   - liquid entrainment (m_e'')
     - ★ Fully-developed entrainment fraction

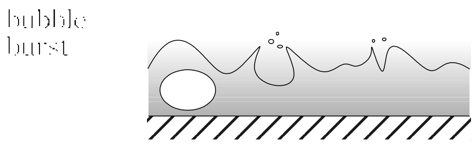
$E = W_{fe}/W_{f,T}$

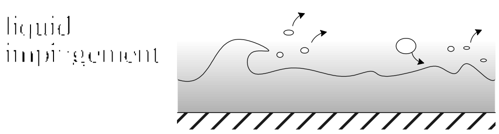
which is related as,

$$\pi_g \doteq \frac{\rho_g}{\rho_f}\left(1 + E\frac{G_f}{G_g}\right)\left(\frac{G_g\mu_g}{\rho_g\sigma}\right)^2\left(\frac{\mu_g}{\mu_f}\right)^{1.5}$$









▶ mechanism of liquid deposition

$$m_d'' = kC ,$$

where C: mean concentration of droplet (kg/ m^3)

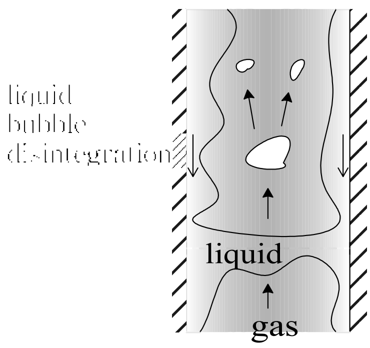
k: empirical mass transfer coefficient (m/s)

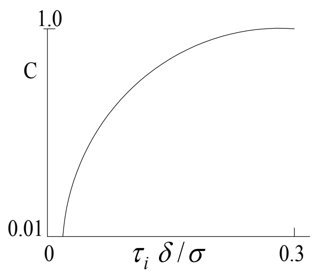
In "hydrodynamic equillibrium"

$$m_d'' \approx m_e''$$

By Whally and Hewitt ($k \propto \sigma$)

$$\frac{k}{\sqrt{\tau_i/\rho_g}} = 87\sqrt{\frac{\mu_f^2}{D\sigma\rho_f}}$$





■ modification of momentum equation accounting for the effect of the entrained liquid

$$G_{lf} = \frac{w_{lf}}{A_{x-s}}$$

$$\langle u_g \rangle = \frac{G\langle x \rangle}{\langle \alpha \rangle \rho_g} \quad (1)$$

$$\langle u_l \rangle_f = \frac{G(1-\langle x \rangle)(1-E)}{\langle \Lambda \rangle \rho_l} \quad (2)$$

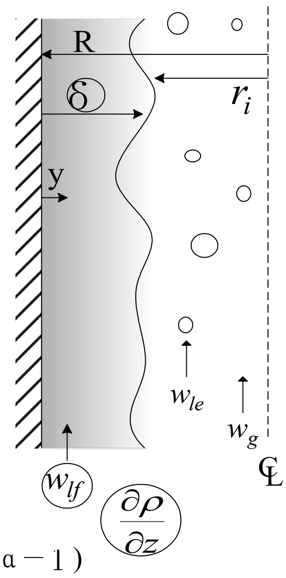
$$\langle u_l \rangle_e = \frac{G(1-\langle x \rangle)E}{(1-\langle \alpha \rangle-\langle \Lambda \rangle)\rho_l}, \quad (3)$$

where $\langle \Lambda \rangle = \frac{A_{lf}}{A_{x-s}}$: volumetric flux of liquid flim

$$-\frac{\partial P}{\partial z} = \frac{g}{g_c}\langle \bar{\rho} \rangle \sin\Theta + \frac{\tau_w P_f}{A_{x-s}} + \frac{1}{g_c}\left[\frac{\partial G}{\partial t} + \underbrace{\frac{1}{A_{x-s}}\frac{\partial}{\partial z}\left\{A_{x-s}G^2\left(\frac{E^2(1-\langle x \rangle)^2}{(1-\langle \alpha \rangle-\langle \Lambda \rangle)\rho_l} + \frac{(1-E)^2(1-\langle x \rangle)^2}{\langle \Lambda \rangle \rho_l} + \frac{\langle x \rangle^2}{\langle \alpha \rangle \rho_g}\right)\right\}}_{\text{spatial acceleration term}}\right] \quad (4)$$

To obtain $\langle \Lambda \rangle$, assume "no slip" between vapor phase and entrained liquid. From Eqs. (1) & (3),

$$\langle \Lambda \rangle = 1 - \langle \alpha \rangle - \frac{\rho_g \langle \alpha \rangle (1-\langle x \rangle)E}{\rho_l \langle x \rangle}$$



■ Contitutive relation for τ_i, τ_w

From a simple force balance for idealized annular flow

$$(2\pi r\Delta z)\tau(r) = \pi r^2\left[p-\left(p+\frac{dP}{dz}\Delta z\right)\right] - \frac{g\Delta z}{g_c}\sin\Theta\int_0^r [\rho_g\alpha+\rho_l(1-\alpha)]2\pi r'\,dr'$$

$$\therefore \tau(r) = \frac{r}{2}\left[-\frac{dP}{dz}\right] - \frac{g}{g_c}\sin\frac{\Theta}{r}\int_0^r [\rho_g\alpha+\rho_l(1-\alpha)]2\pi r'\,dr'$$

For special case of no liquid entrainment and smooth interface ($\alpha = 1$)

$$\tau(R-\delta) \doteq \tau_i = \frac{(R-\delta)}{2}\left(-\frac{dP}{dz} - \frac{g}{g_c}\rho_g\sin\Theta\right) \quad (5)$$

And no voids in liquid film $\alpha = 0$, $R-\delta = R\sqrt{\langle \alpha \rangle}$

$$\tau(R) \doteq \tau_w = \frac{R}{2}\left(-\frac{dP}{dz}\right) - \frac{g}{g_c}\left(\frac{\sin\Theta}{R}\right)\left[\int_0^{R\sqrt{\alpha}}\rho_g r'\,dr' + \int_{R\sqrt{\alpha}}^{R}\rho_l r'\,dr'\right]$$

$$= \frac{R}{2}\left(-\frac{dP}{dz} - \frac{g}{g_c}\langle \bar{\rho} \rangle \sin\Theta\right) \quad (6)$$

If the pressure gradient has been assumed to be same in both phases, from Eqs. 5 and 6

$$\tau_w = \frac{R}{2}\left[\frac{2\tau_i}{(R-\delta)} - \frac{g}{g_c}(\rho_l - \rho_g)(1-\langle\alpha\rangle)\sin\Theta\right]$$

1) Engineering approach

let $R - r_i = \delta$, $\pi(R^2 - r_i^2) \approx R\delta 2\pi$ for small δ

$$\tau_w = f_w\frac{G^2}{2\rho_l}, \qquad \tau_i = f_i\frac{\rho_g(v_g - v_l)^2}{2}$$

$f_i \approx K f_w$ by Wallis, where $K = \begin{cases} \approx 1 & \text{for smooth surface} \\ 1 + 300\delta/D & \text{for wavy surface} \end{cases}$

2) Second Method

turbulence effect

account for entrainment

Calculation of W_l when δ, $\frac{dP}{dz}$ are known

1) calculate τ_i from $r_i = R - \delta$ and $\frac{dP}{dz}$

2) calculate $\tau(r)$ in liquid film from δ, τ_i, $\frac{dP}{dz}$

3) calculate $u(r)$ within liquid film

$$\tau = \mu_l\frac{du}{dy}$$

4) calculate of liquid film flow rate

$$W_l = \int_0^\delta 2\pi\, u(r)\rho_l\, dr$$

2. Flow pattern map and transients

- w_f vs w_g : [illegible]

I. w as the controlling parameters
II. w and x as the controlling parameters
III. dimensional analysis
IV. other analytical and empirical approach

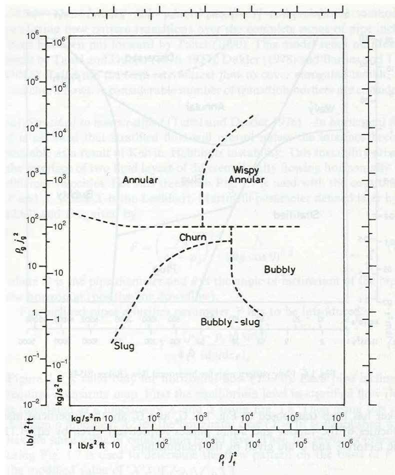


Fig. 4.9 Flow pattern map for vertical flow

Flow pattern map by Hewitt & Robert [Fig. 4.9]

Category II

$$\rho_f j_f^2 = \frac{[G(1-x)]^2}{\rho_f}, \qquad \rho_g j_g^2 = \frac{[Gx]^2}{\rho_g}$$

- cannot show the effects of physical properties of fluis or geometry effect

Thus, Baker's chart (horizontal tube)[Fig.4.10] used

G_g/λ vs. $G_f\lambda\psi/G_g$,

where $\lambda = \left[\left(\frac{\rho_g}{0.075}\right)\left(\frac{\rho_f}{62.3}\right)\right]^{1/2}$

$$\psi = \left(\frac{0.005}{\sigma}\right)\left[\frac{\mu}{2.12}\left(\frac{62.3}{\rho_f}\right)\right]^{1/3}$$

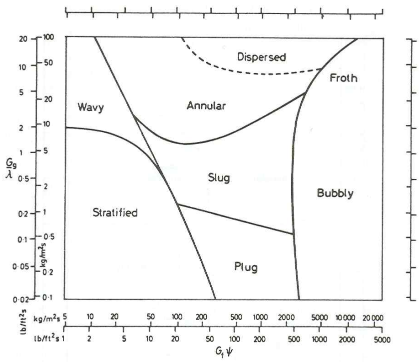


Fig. 4.10 Flow pattern map for horizontal flow

(1) bubbly flow - slug flow transition

$\alpha < 0.3$

By Taitel and Dukler

$$\frac{j_f}{j_g} = 2.34 - 1.07\frac{[g(\rho_f - \rho_g)\sigma]^{1/4}}{j_g \rho_f^{1/2}} \qquad (4.3.8)$$

(2) Slug flow - churn flow transition

slug flow: $\beta < 0.8 \sim 0.85$

upper limit of slug flow - flooding

∴ transition to churn flow by break-up of long vapor bubble

By Porteons

U'_f: Eq. (4.3.7)

$$\frac{j}{U'_f} = 0.3\left(\frac{\rho_f}{\rho_g}\right)^{0.5} \quad \text{for slug flow}$$

Then the upper limit of slug flow

$$j = 0.105\,\rho_g^{-0.5}[gD(\rho_f - \rho_g)]^{0.5}$$

(3) Churn flow - annular flow transition

low limit of annular flow - flow reversal

$$j_g^* = 0.9$$

By Zuber, $j_g^* = 4\left(\frac{\rho_L}{\rho_G}\right)^{0.5}[j_L^* + 0.35]$

By Taitel and Dukler

$$j_g = 3.09\,\rho_g^{-0.5}[g(\rho_l - \rho_g)\sigma]^{0.25} \qquad \text{for} \quad X_p \ll 1$$

$$j_g = 30.9\frac{[g(\rho_l - \rho_g)\sigma]^{0.25}}{\rho_g^{0.5}X_p} \qquad \text{for} \quad X_p \gg 1$$

★ β vs. Fr ($\beta \doteq \frac{Q_g}{Q_l + Q_g}$, $Fr = \frac{[(Q_l + Q_g)/A_p]^2}{gD_p}$)

Bubble - slug $\qquad \beta = 0.05(Fr)^{0.5}$

Slug - churn $\qquad = 0.12(Fr)^{0.15}$

Churn - annular $\qquad = 0.65(Fr)^{0.65}$