V. Pressure Drop Calculation in Two Phase Flow

The mixture momentum equation is,

$$\frac{\partial p}{\partial z} = \underbrace{\frac{1}{g_c}\left[\frac{\partial G}{\partial t} + \frac{\partial}{\partial z}\left(\frac{G^2 A}{\langle \rho' \rangle}\right)\right]}_{\text{acceleration}} + \underbrace{\frac{g}{g_c}\langle \bar{\rho} \rangle \sin\Theta}_{\text{gravity}} + \underbrace{\frac{\tau_w P_w}{A}}_{\text{friction}} \qquad (1.5.1)$$

$$\therefore \quad \Delta p = \Delta p_{accel} + \Delta p_{grav} + \Delta p_{fric} + \Delta p_{local} \qquad (1.5.2)$$

(1) acceleration pressure drop (Δp_{accel})

$$-\Delta p_{accel} = \frac{1}{g_c}\int_0^{L_H} \frac{\partial}{\partial z}\left(\frac{G^2}{\langle \rho' \rangle}\right) dz = \underbrace{\int_0^{\lambda} \frac{d}{dz}\left(\frac{G^2}{\rho_f}\right) dz}_{0} + \int_{\lambda_0}^{L_H} \frac{d}{dz}\left(\frac{G^2}{\langle \rho_{2\phi} \rangle}\right) dz \qquad (1.5.3)$$

In homogeneous flow,

$$\langle \rho' \rangle = \langle \rho_H \rangle = 1/(v_f + \langle x \rangle v_{fg}) \qquad (1.5.4)$$

$$\therefore \quad \Delta p_{accel} = \frac{G^2}{g_c}\left[\frac{1}{\langle \rho(L_H) \rangle} - \frac{1}{\langle \rho(\lambda_0) \rangle}\right] = \frac{v_{fg}}{g_c} G^2 \langle x_e(L_H) \rangle \qquad (1.5.5)$$

In separate flow,

$$\frac{1}{\langle \rho' \rangle} = \left[\frac{(1-\langle x \rangle)^2}{\rho_l (1-\langle \alpha \rangle)} + \frac{\langle x \rangle^2}{\rho_v \langle \alpha \rangle}\right]$$

(2) gravitational pressure drop ($-\Delta p_{grav}$)

$$\Delta p_{grav} = \frac{g}{g_c}\int_0^{L_H} \langle \bar{\rho} \rangle dz \qquad (1.5.6)$$

i) single phase region

$$\Delta p^{1\phi}_{grav} = \frac{g}{g_c}\int_0^{\lambda_0} \langle \rho_H \rangle dz = \frac{g}{g_c}\rho_f \lambda_0 \qquad (1.5.7)$$

ii) two phase region

$$-\Delta p^{2\phi}_{grav} = \frac{g}{g_c}\int_{\lambda_0}^{L_H} \langle \rho' \rangle dz \qquad (1.5.8)$$

Heare, the axial density profile can be obtained from the energy balance by using the void quality relation and the definition of mixture density.

(3) frictional pressure drop ($-p_{fric}$)

$$-p^{2\phi}_{fric} > -p^{1\phi}_{fric} \qquad \text{where} \quad -p^{1\phi}_{fric} = K\frac{G^2}{2g_c\rho_l}$$

$$= -p^{1\phi}_{fric}\,\Phi^2_{lo} \qquad (1.5.9)$$

Here Φ^2_{lo} is called as the two phase multiplier.

$$\therefore \quad -p^{2\phi}_{fric} = K\frac{G^2}{2g_c\rho_l}\Phi^2_{lo}(\langle x\rangle, p, \ldots\ldots) \qquad (1.5.10)$$

Thus, $\Phi^2_{lo} = (dp/dz)_{2\phi}/(dp/dz)_{1\phi}$ (1.5.11)

To determine Φ^2_{lo}, we can write $-p_{2\phi}$ for two phase flow

$$-p^{2\phi}_{fric} = K\frac{G^2}{2g_c\langle\rho_{2\phi}\rangle} \qquad (1.5.12)$$

1) For homogeneous flow, $\langle\rho_{2\phi}\rangle = \langle\rho_H\rangle$

$$\therefore \quad K\frac{G^2}{2g_c\rho_l}\left(\frac{\rho_l}{\langle\rho_H\rangle}\right)$$

By comparing Eqs.(1.5.10) and (1.5.12),

$$\Phi^2_{lo} = \rho_l/\langle\rho_H\rangle = \left(1 + \frac{v_{fg}}{v_l}\langle x\rangle\right) \quad \text{from Eq.(1.5.1)} \qquad (1.5.13)$$

For two phase friction factor, $f_{2\phi}$,

1. $f_{2\phi} = f_{1\phi}$
2. $f_{2\phi}$ has the same Re dependence as $f_{1\phi}$

$$\frac{f_{2\phi}}{f_{1\phi}} = \frac{C_1/Re^n_{2\phi}}{C_1/Re^n_{1\phi}} = \left(\frac{\mu_{2\phi}}{\mu_{1\phi}}\right)^n \qquad (1.5.14)$$

★ appropriate correlations for two phase viscosity

a. McAdams et al. : $\frac{\mu_{2\phi}}{\mu_{1\phi}} = \left[1 + x\left(\frac{\mu_l}{\mu_g} - 1\right)\right]^{-1}$

b. Cichitti et al. : $\frac{\mu_{2\phi}}{\mu_{1\phi}} = \left[1 + x\left(\frac{\mu_g}{\mu_l} - 1\right)\right]$

c. Dukler et al. : $\frac{\mu_{2\phi}}{\mu_{1\phi}} = \left[1 + \beta\left(\frac{\mu_g}{\mu_l} - 1\right)\right]$

Thus, the modified Φ^2_{lo} considering the two phase viscosity effect is,

$$\Phi^2_{lo} = \left(1 + \frac{v_{fg}}{v_l}\langle x\rangle\right)\left[\frac{\mu_l}{\mu_g}\langle x\rangle + (1-\langle x\rangle)\right]^{-n} \qquad (1.5.15)$$

2) For separate flow.

By Chisholm, use momentum density against $\langle \rho_{2\phi} \rangle$ in Eq. (1.5.13)

$$\phi^2_{lo} = \frac{\rho_l}{\langle \rho' \rangle} = \left[\frac{(1-\langle x \rangle)^2}{(1-\langle \alpha \rangle)} + \frac{\rho_l}{\rho_g} \frac{\langle x \rangle^2}{\langle \alpha \rangle} \right] \qquad (1.5.16)$$

3) other ϕ^2_{lo}

1 Levy model for ideal annular flow

$$\phi^2_{lo} = \left[\frac{(1-\langle x \rangle)}{(1-\langle \alpha \rangle)} \right]^{2-n} \quad \text{where typical value of n is 0.25} \qquad (1.5.17)$$

2 Martinelli Nelson ϕ^2_{lo}

steam water data

$\phi^2_{lo} \propto \langle x \rangle$ at any given pressure

$$\phi^2_{lo} = \Omega(G, P)(1.2[(\rho_l/\rho_g)-1]\langle x \rangle^{0.824}) + 1.0 \qquad (1.5.18)$$

where $\Omega(G, p) = 1.36 + 0.0005p + 0.1(G/10^6) - 0.0007p(G/10^6)$, $\quad (G/10^6) < 0.7$

$= 1.26 - 0.0004p + 0.119(G/10^6) + 0.00028p(G/10^6)$, $\quad (G/10^6) > 0.7$

* negligible effects of G at a given quality
* not account for surface tension effect, important at high pressure

3 Baroczy correlation

correct for the influence of G on ϕ^2_{lo} for fluids other than water

$$\phi^2_{lo}(G) = \Omega\,\phi^2_{lo}(G_{ref})$$

$$= \left[1 + \frac{C}{X_{tt}} + \frac{1}{X^2_{tt}}\right](1-\langle x \rangle)^{2-n} \qquad (1.5.19)$$

where $C = -2.0 + (28 - 0.8G^{1/2}) \exp\left[-\frac{(\log_{10}\Lambda + 2.5)^2}{2.4 - G/10^4} \right]$

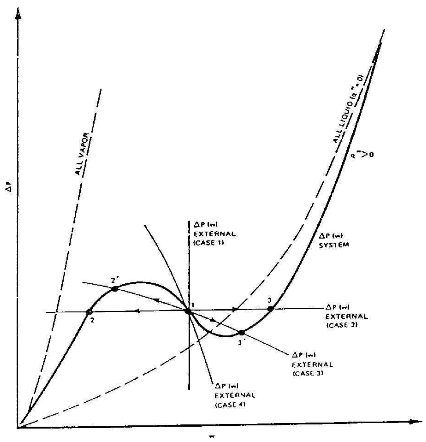
$$\Lambda = \frac{\rho_g}{\rho_l}\left(\frac{\mu_l}{\mu_g}\right)^{0.2}, \qquad n = 0.2$$

■ Models recommanded in 10CFR50 Appendix

- modified Baroczy correlation
- Thom correlation ($\geq$ 250 psia)
- Martinelli Nelson correlation ($<$ 250 psia)

(D) local pressure loss (Δp_{local})

For abrupt change of flow area in single phase f



$$p_1 + \frac{G_1^2}{2g_c\rho_l} = p_2 + \frac{G_2^2}{2g_c\rho_l} + K\frac{G^2}{2g_c\rho_l}$$

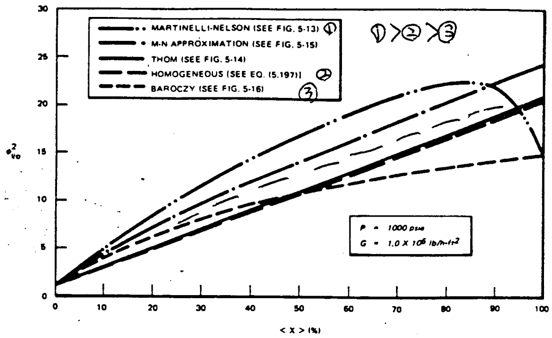


Fig. 5-18. A comparison of various ϕ_{lo}^2 correlations.

From momentum balance for uniform velocity profile,

$$K = \left(1 - \frac{A_1}{A_2}\right)^2 = (1-\sigma)^2 \qquad \text{where } \sigma(\textit{flow area ratio}) = \frac{A_1}{A_2}$$

Thus, $$-\Delta p_{exp} = [(1-\sigma^2) - K_{exp}]\frac{G_1^2}{2g_c\rho_l} \qquad (4.5.21)$$

$$-\Delta p_{coll} = [(1-\sigma^2) + \sigma^2 K_{coll}]\frac{G_1^2}{2g_c\rho_l} \qquad (4.5.22)$$

$(1-\sigma^2)\frac{G_1^2}{2g_c\rho_l}$ is representative the reversible pressure change which can be derived directly from the single phase Bernoulli equation without the head loss term.

$$\Delta p_{total} = \Delta p_{rev} + \Delta p_{irrev}$$

By Vennard,

- $K_{exp} = (1-\sigma)^2$
- K_{con}

σ	0	0.2	0.4	0.5	0.6	0.8	1.0
K_{con}	0.385	0.34	0.266	0.219	0.164	0.053	0

For two phase flow,

$$-\Delta p_{2\phi} = K\frac{G^2}{2g_c\langle\rho_{2\phi}\rangle} = K\frac{G^2}{2g_c\rho_l}\Phi \qquad (4.5.23)$$

$$-\Delta p_{exp} = \frac{G_1^2\sigma^2}{g_c\rho_l}\left([\frac{\rho_l}{\rho_g}x^2(\frac{1}{\alpha_1\sigma} - \frac{1}{\alpha_2})] + (1-x)^2[\frac{1}{\sigma(1-\alpha_1)} - \frac{1}{(1-\alpha_2)}]\right)$$

$$-\Delta p_{coll} = \frac{G_1^2}{2\sigma^2 g_c}\left[\left(\frac{1}{C} - 1\right)^2 + (1-\sigma^2)\right]\left[\frac{x}{\rho_g} + \frac{(1-x)}{\rho_l}\right]$$

where

α_1, α_2 : void fractions upstream and downstream

C : vena contracta area ratio

Thus, for sudden expansion with no phase change,

$$\phi^2_{lo} = \frac{\rho_l}{\langle \rho' \rangle} = \left[\frac{(1-\langle x \rangle)^2}{(1-\langle \alpha \rangle)} + \frac{\rho_l}{\rho_v}\frac{\langle x \rangle^2}{\langle \alpha \rangle} \right] \tag{1.5.26}$$

However, the experimental results for the grid type spacer showed,

$$\Phi = (1+\frac{v_{fg}}{v_f}\langle x \rangle)$$

Thus, $$-p_{spacer} = K_{sp}\frac{G^2}{2g_c\rho_l}(1+\frac{v_{fg}}{v_f}\langle x \rangle) \tag{1.5.27}$$

● pressure drop across the grid or spacer

1) By De Stordeur(1961)

$$p_{spacer} = \rho C_s V_s^2 s/2g_c A$$

where A, V_s, s are unrestricted area, velocity in the spacer region and projected area of the spacer.

C_s(Drag Coefficient) ~ 1.65 for strap type grids at Re ~10^5)

2) By Rehme(1973)

$$p_{spacer} = C_v(\rho V_B^2/2g_c)(s/A)^2$$

where C_v : modified drag coefficient = 9.5 and 6.5 for Re=10^4 and 10^5

V_B : average bundle fluid velocity