양자역학의 기초

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Nothing to Do?

"It seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles. … The future truths of Physical Science are to be looked for in the sixth place of decimals." – Albert A. Michelson in 1894

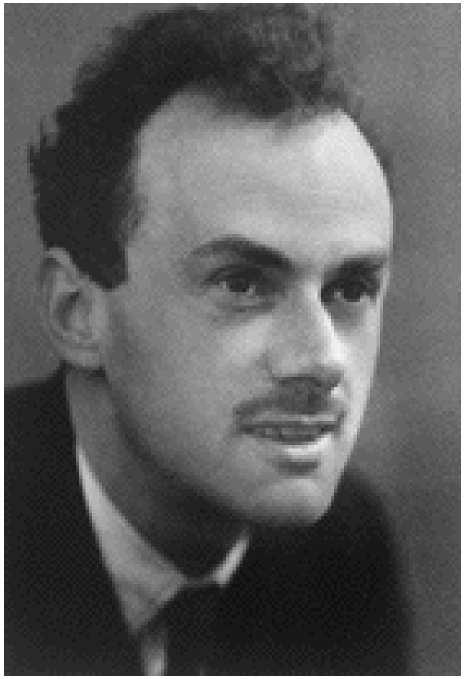
The 20th Century

"It was a marvelous time to be alive."

– Albert Einstein

Quantum Mechanics

“양자역학은 물리의 대부분과 화학의 전부를 설명.”
– Paul A. M. Dirac (1902-1984)



Do We Need to Know?

As electrical or electronic engineers, do we need to know QM?

No and Yes!

Is QM Hard to Learn?

Is the Quantum Mechanics difficult to understand?

No (in general) and yes in some details and subtle fundamental points.

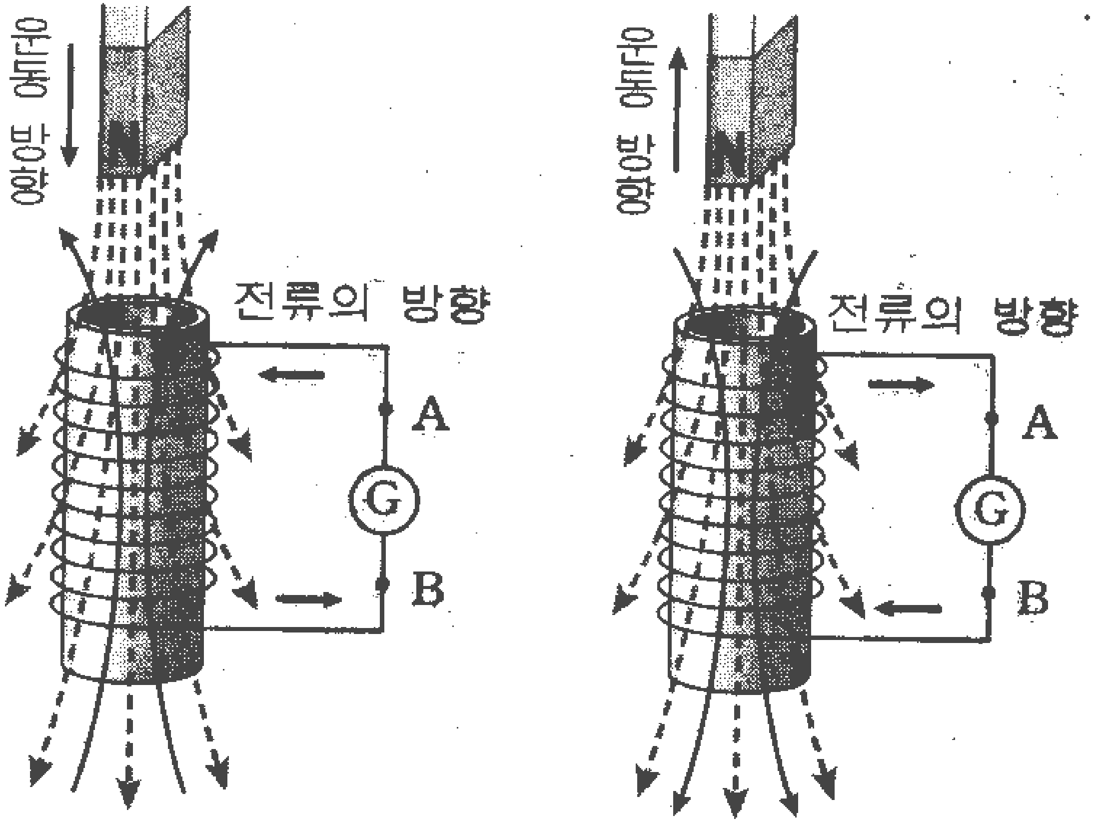
Basically, science is a simplified modeling to understand *Nature* and *Universe*.

Ch. 0. Electromagnetic Waves

James Clerk Maxwell.

James Clerk Maxwell (1831–1879)

Faraday의 법칙

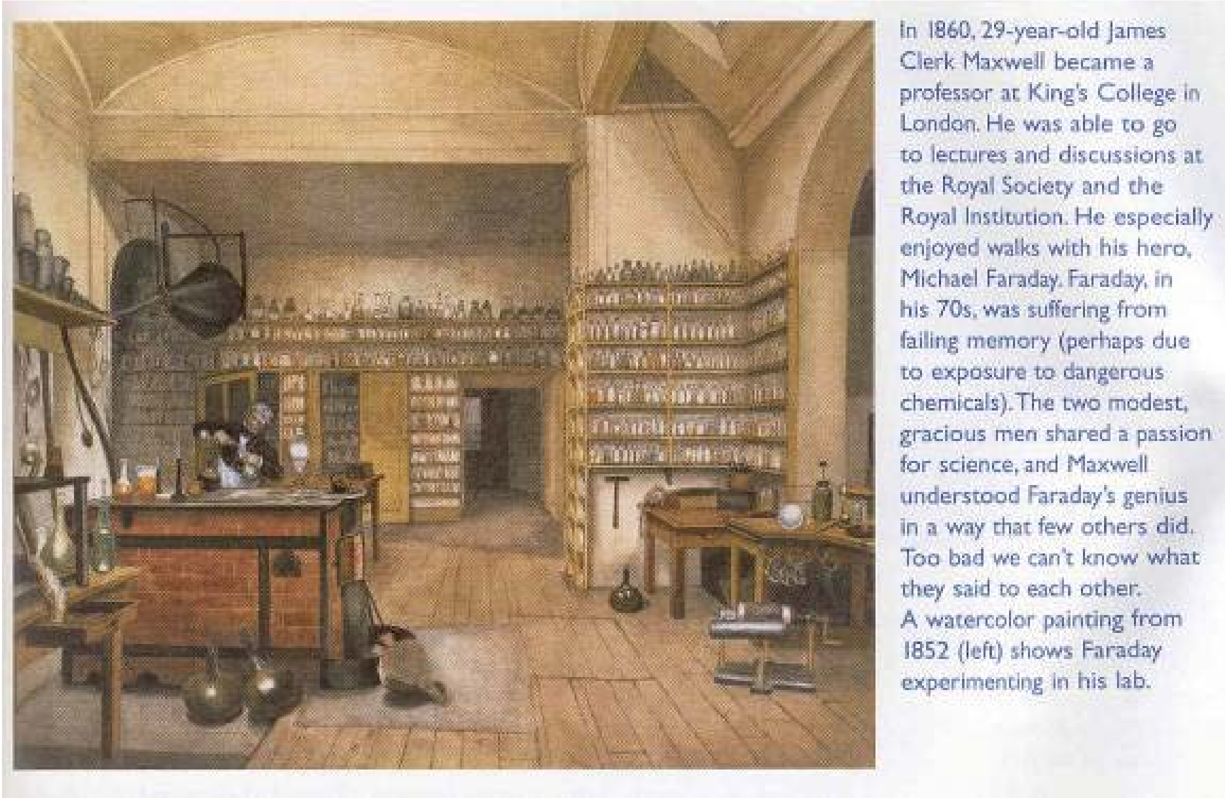


자계(자기장)의 변화 ⇒ 전류 유도

변압기

발전기

Faraday and Maxwell



In 1860, 29-year-old James Clerk Maxwell became a professor at King's College in London. He was able to go to lectures and discussions at the Royal Society and the Royal Institution. He especially enjoyed walks with his hero, Michael Faraday. Faraday, in his 70s, was suffering from failing memory (perhaps due to exposure to dangerous chemicals). The two modest, gracious men shared a passion for science, and Maxwell understood Faraday's genius in a way that few others did. Too bad we can't know what they said to each other.

A watercolor painting from 1852 (left) shows Faraday experimenting in his lab.

J. Hakim, The Story of Science – Newton at the Center, Smithsonian Books, Washington DC, USA, 2005

Hertz and Marconi

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Heinrich Hertz (1857–1894)

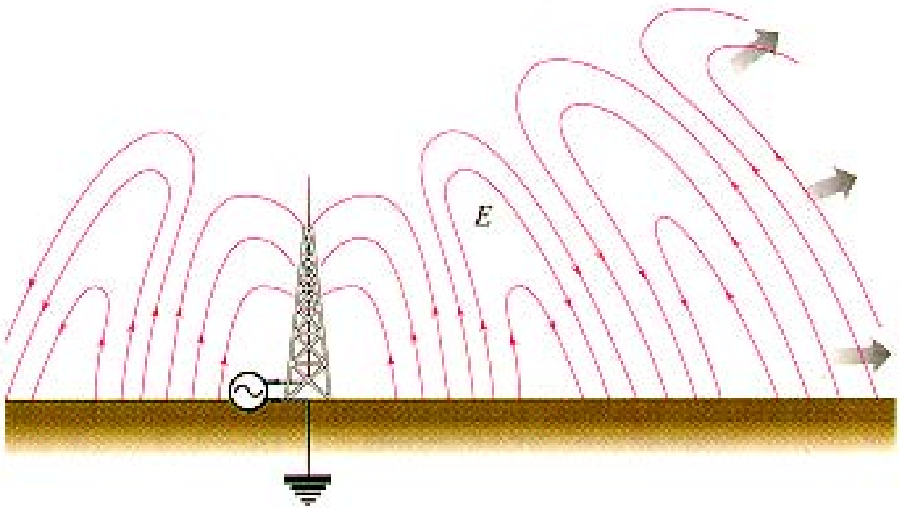
Guglielmo Marconi (1874–1937)

Tragically, Hertz was as unfortunate with his health as he was fortunate with his talent. The first sign of trouble was a series of toothaches in 1888, which led to removal of all his teeth in 1889. By 1892, he was suffering from pains in his nose and throat, and was often depressed. His doctors could give him no satisfactory diagnosis. Several operations failed to provide permanent relief. By December 1893, he knew he would not recover, and in a letter he asked his parents *"not to mourn... rather you must be a little proud and consider that I am among the especially elect destined to live for only a short while and yet to live enough. I did not choose this fate, but since it has overtaken me, I must be content; and if the choice had been left to me, perhaps I should have chosen it myself."*

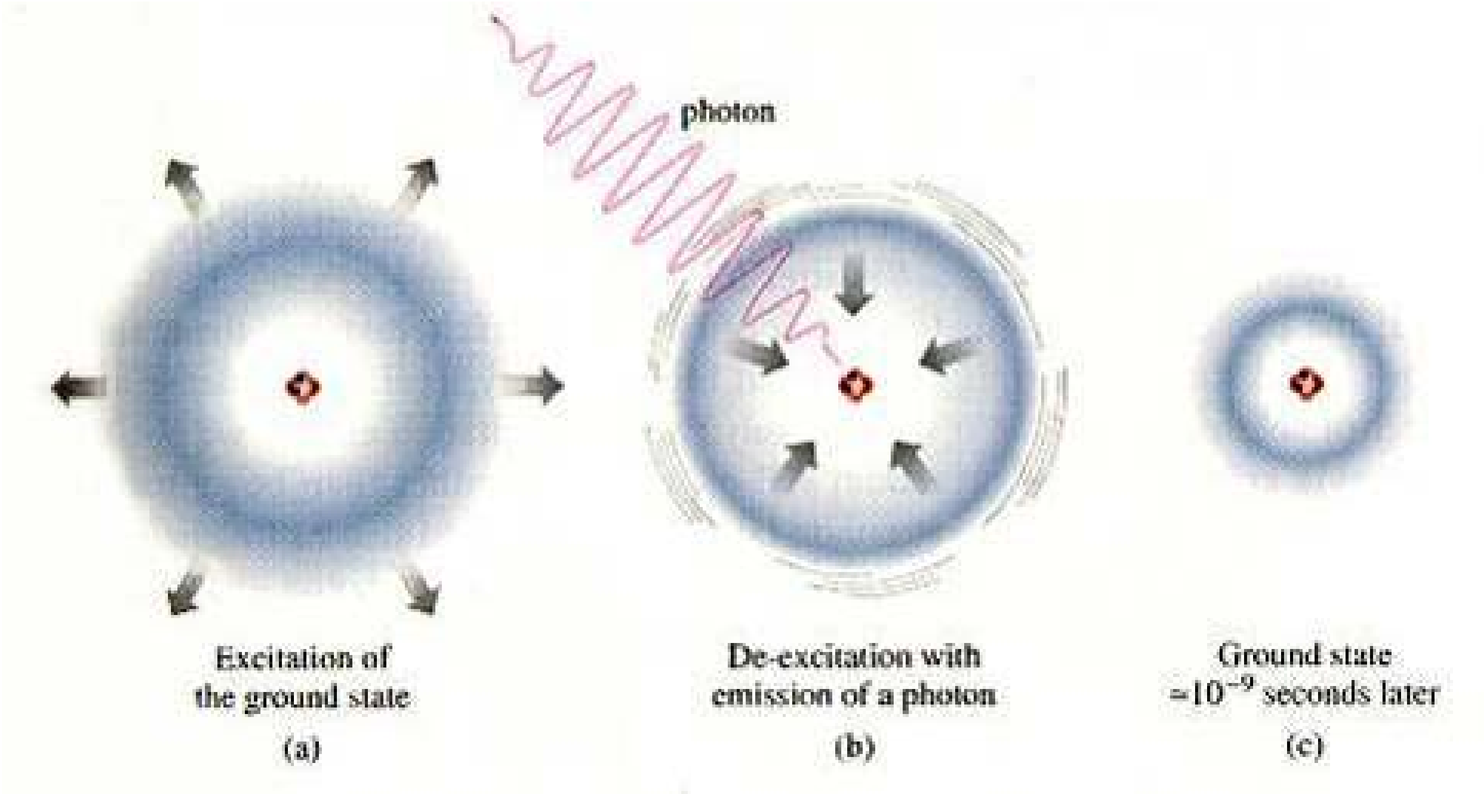
Hertz died of blood poisoning on New Year's Day, 1894; he was thirty-six years old.

--- From 'Great Physicists' by W. H. Cropper

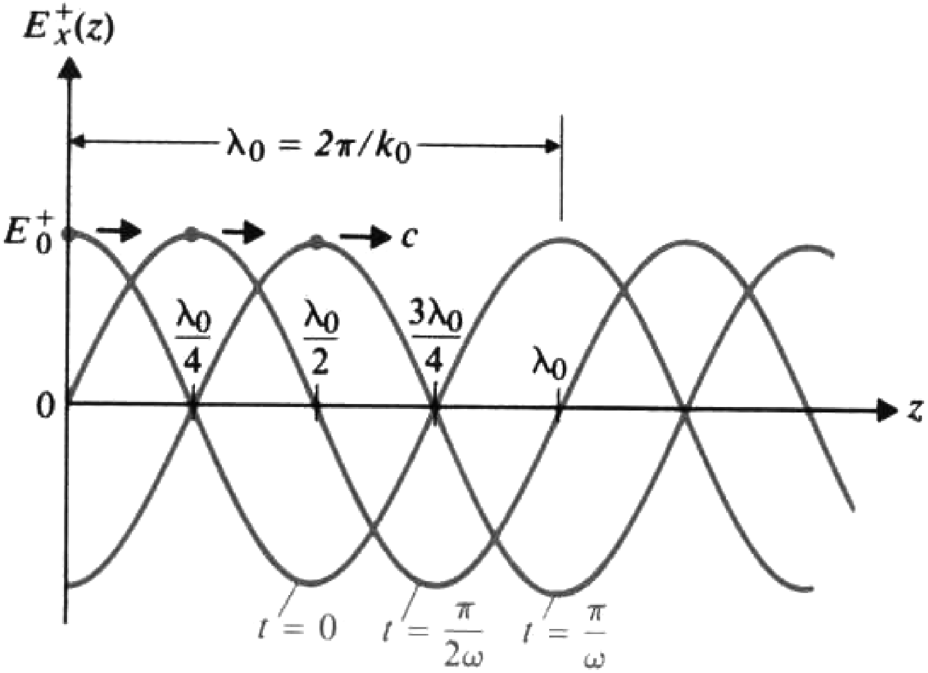
전자파의 발생 예 (안테나)



빛의 발생의 예

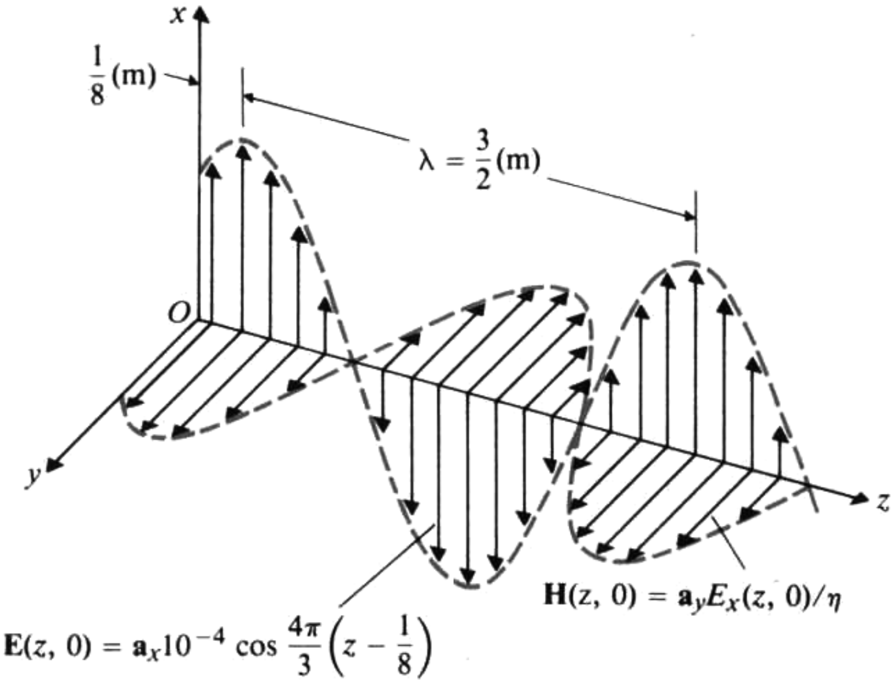

photon
Excitation of the ground state
(a)
De-excitation with emission of a photon
(b)
Ground state ≈10⁻⁹ seconds later
(c)

전자파



Wave traveling in positive *z* direction

$E_x^+ = E_0^+ \cos(wt-k_0 z)$, for several values of *t*.



E and H fields

of a uniform plane wave at a fixed *t*.

파장과 주파수

주기 T

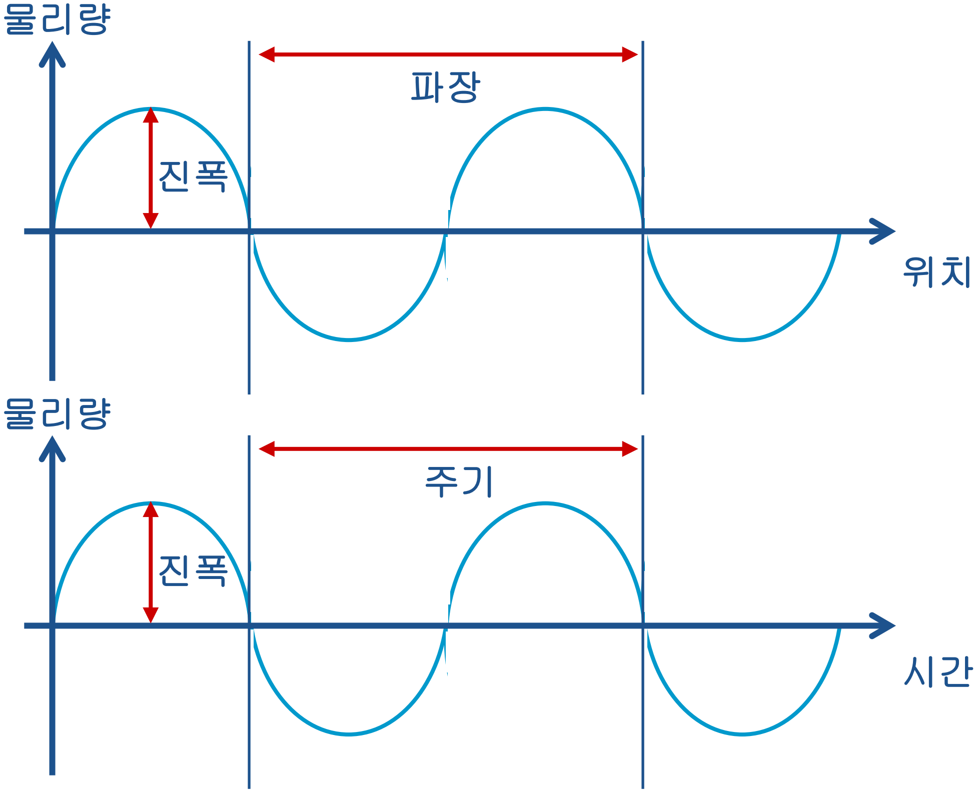
주파수 $f \quad (\nu)$

파장 λ

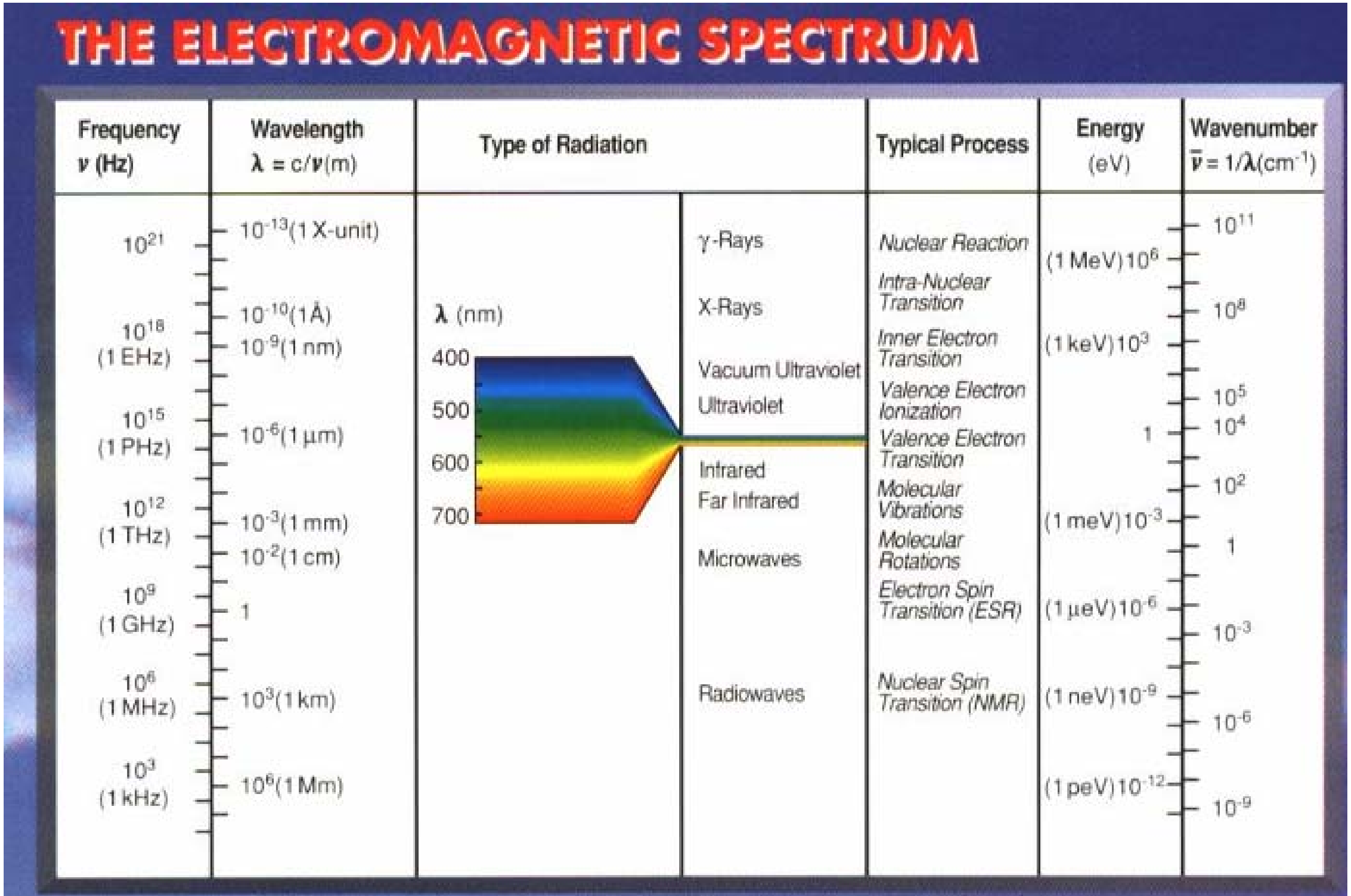
진공에서의 빛의 속력 c

$$f = \frac{1}{T}$$

$$c = \frac{\lambda}{T} = \lambda f$$



전자파의 스펙트럼

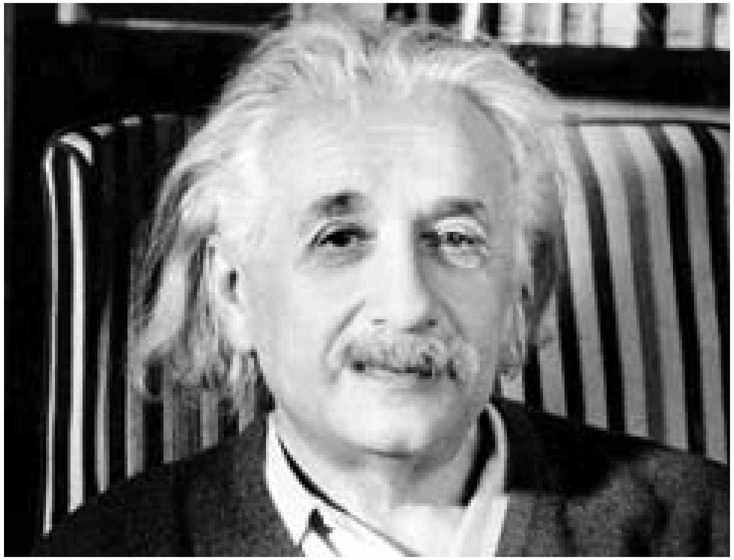


1 eV = 1.602177×10^{-19} J; h = 6.626075×10^{-34} J·s (Planck Constant); c = 2.997925×10^{8} m·s^{-1} (light velocity)

1Å = 10^{-10}m = 10^{-8}cm~2.997925×10^{18} s^{-1} (light frequency)~1.239852×10^{4} eV (energy of light quantum)

Ch. 1. Relativity

The King



Albert Einstein (1879–1955)

1905

Young Einstein at the Swiss Patent Office in Berne

Annalen der Physik, vol. 17 (1905)

“one of the most remarkable volumes in the whole scientific literature. It contains three papers by Einstein,
each dealing with a different subject and each today acknowledged to be a masterpiece.”
– Max Born

Abraham Pais, whose biography of Einstein is the best of the many written, leaves us with this glimpse of Einstein about three months before he died in 1955. He had been ill and unable to work in his office at the institute. Pais visited him at home and ...

"went upstairs and knocked at the door of [his] study. There was a gentle "Come." As I entered, he was seated in his armchair, a blanket over his knees, a pad on the blanket. he was working. He put his pad aside at once and greeted me. We spent a pleasant half hour or so; I do not recall what was discussed. Then I told him I should not stay any longer. We shook hands, and I said goodbye. I walked to the door of the study, not more than four or five steps away. I turned around as I opened the door. I saw him in his chair, his pad on his lap, a pencil in his hand, oblivious to his surroundings. He was back at work.

– *From "Great Physicists"*
by W. H. Cropper

Electromagnetic Wave

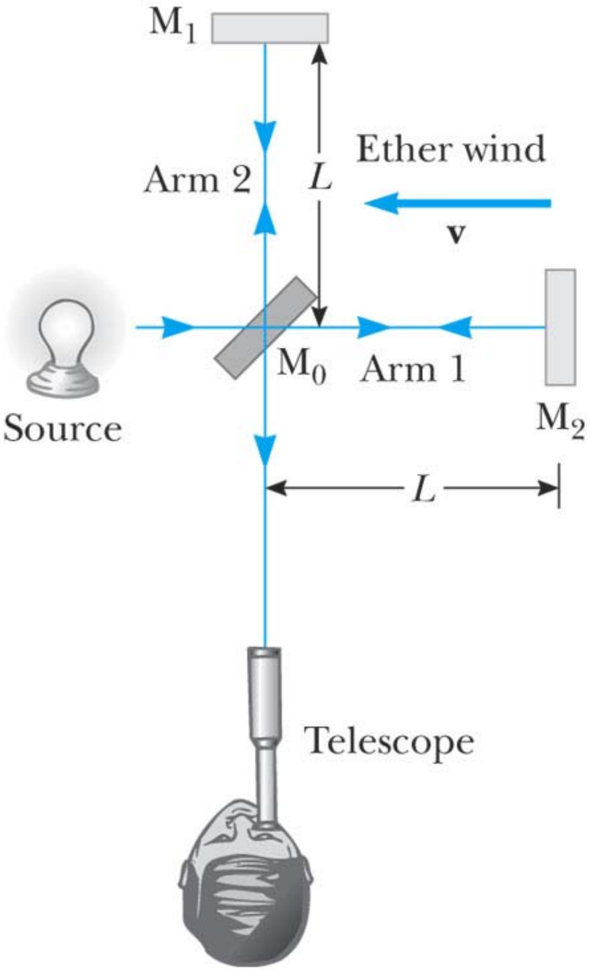
Wave equation

$$\nabla^2 \mathbf{E}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}(\mathbf{r},t)}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \approx 3\times 10^8\ \mathrm{m/s}$$

with respect to what?

Michelson Interferometer



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Special Relativity

Two postulates:

- The Principle of Relativity

All the laws of physics have the same form in all inertial reference frames.

- The Constancy of the Speed of Light

The speed of light in vacuum has the same value in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

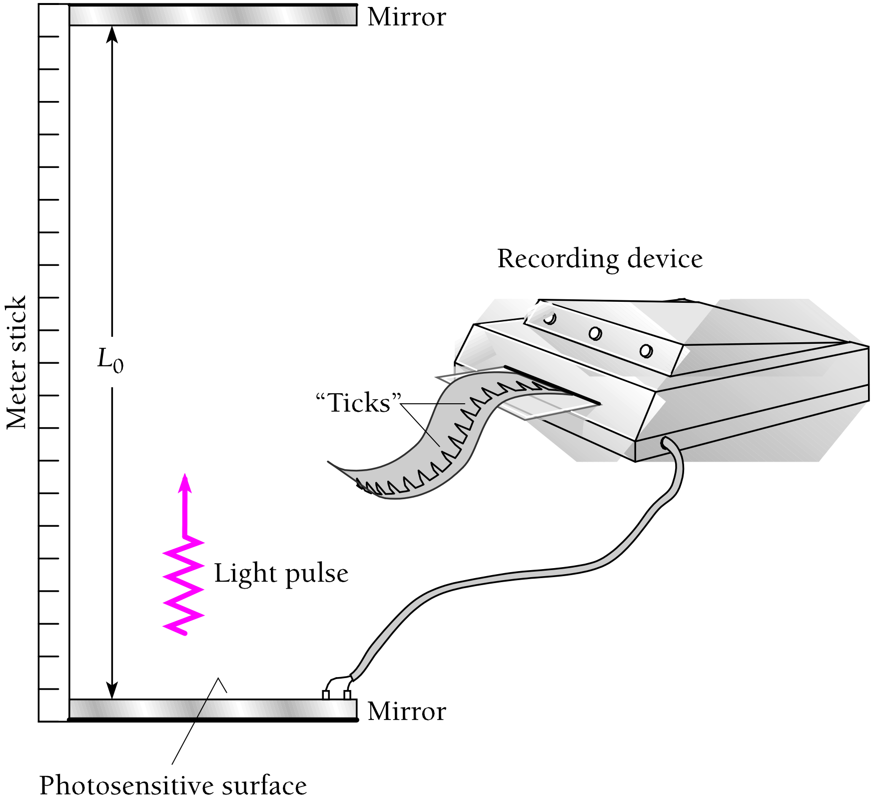
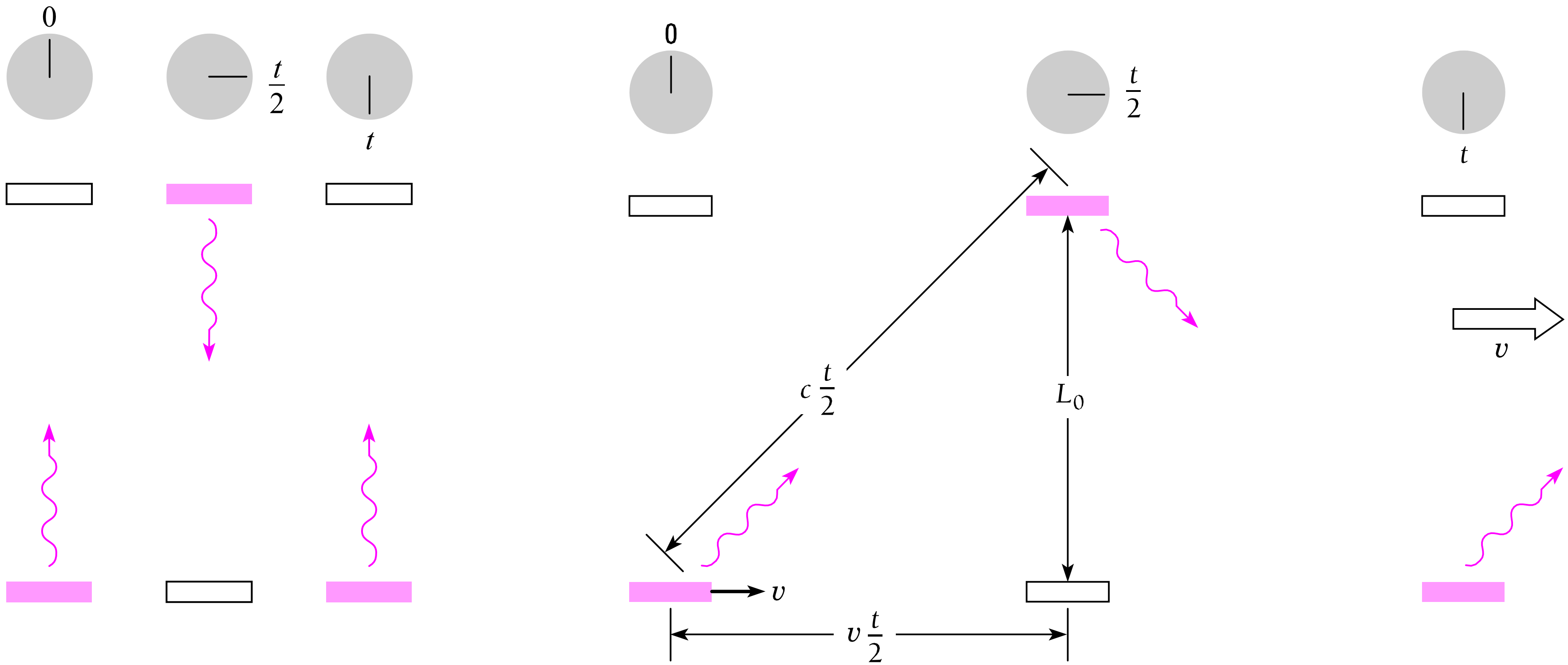


그림 1.3 간단한 시계. 각각의 똑딱거림은 아래 거울에서 위 거울까지 빛 펄스의 왕복시간에 해당한다.

Time Dilation



$$t_0 = \frac{2L_0}{c}$$

proper time

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Length Contraction

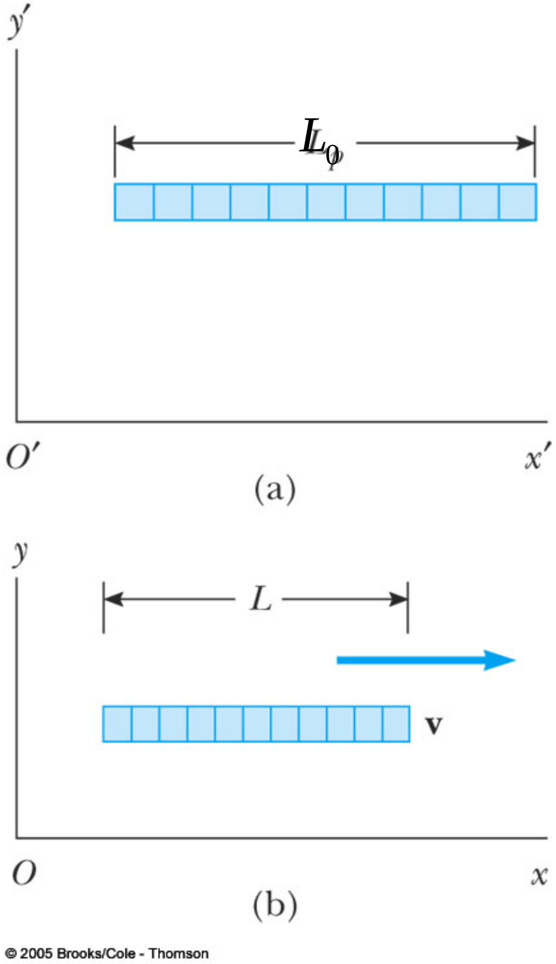


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Fig. 1-13, p. 19

L_0 proper length

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

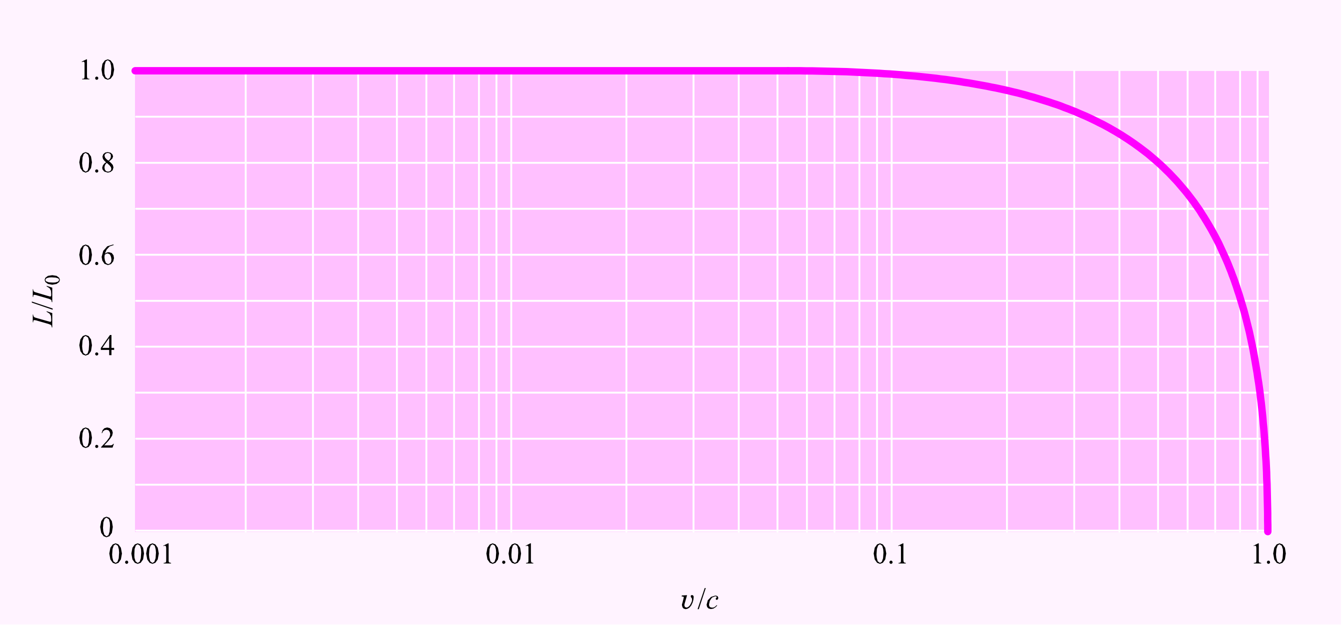
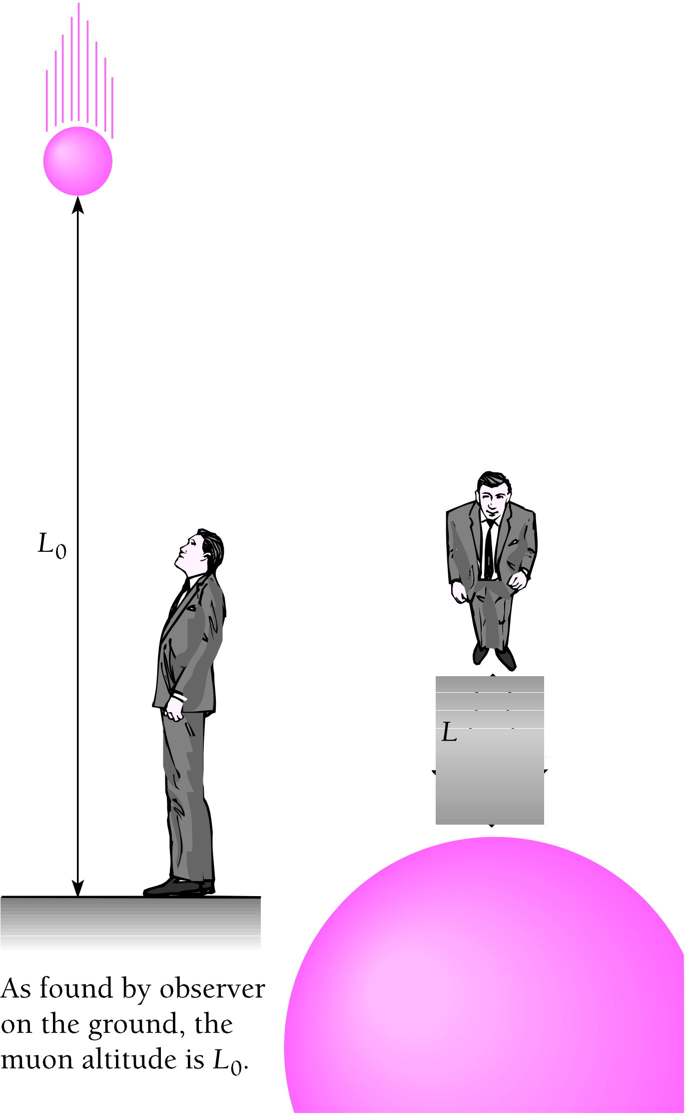
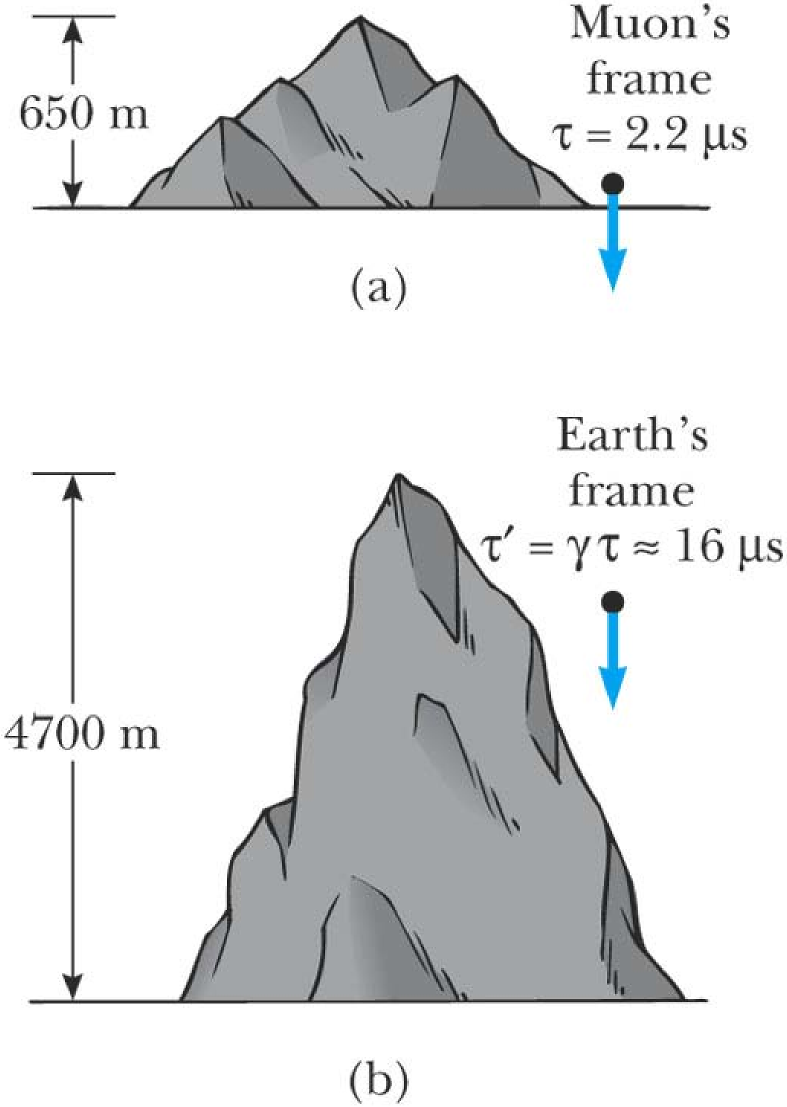


그림 1.10 상대론적 길이수축. 운동방향으로의 길이만이 영향을 받는다. 수평축은 로그 단위이다.



As found by observer on the ground, the muon altitude is L_0.



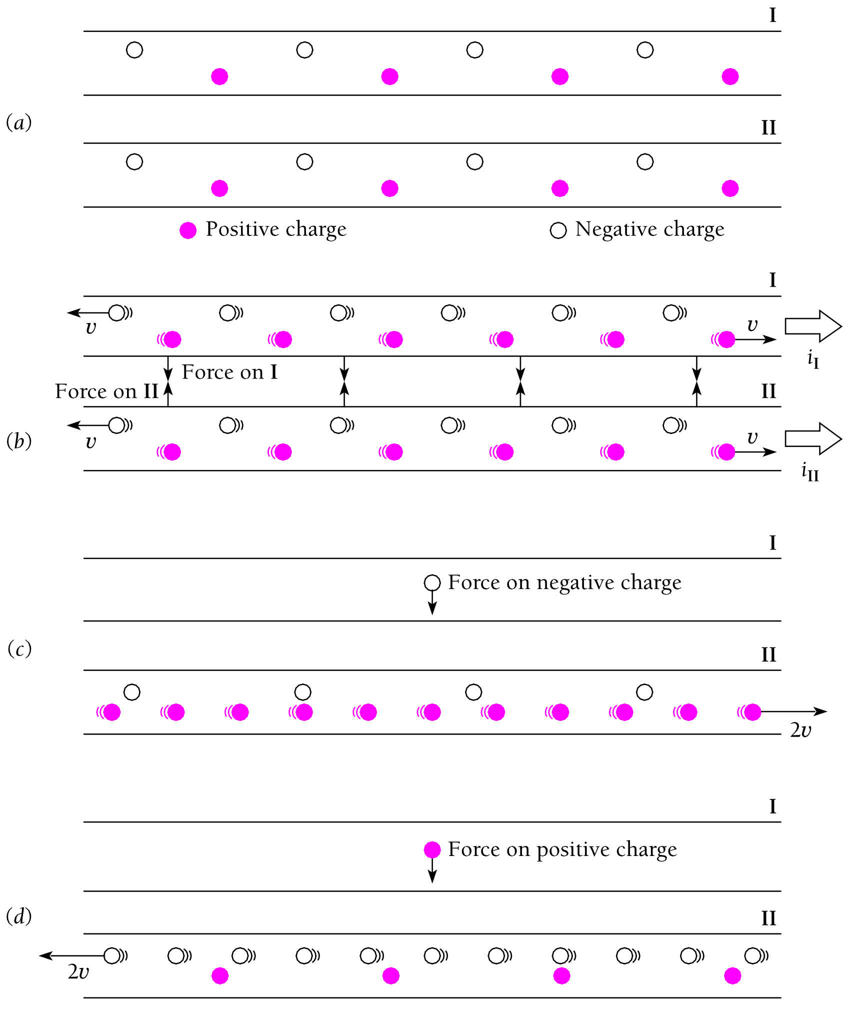


그림 1.12 평행한 전류 사이의 인력

(Einstein's) Causality

Transmission of signal cannot be faster than the speed of light in vacuum.

The Time Machine – H. G. Wells, 1895

→ Impossible

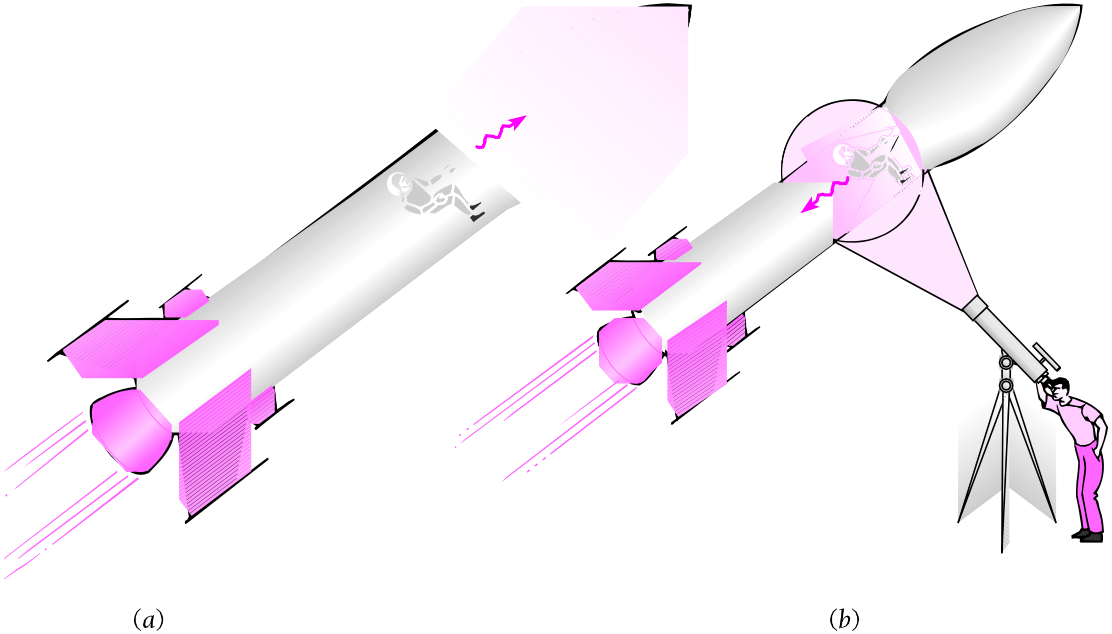
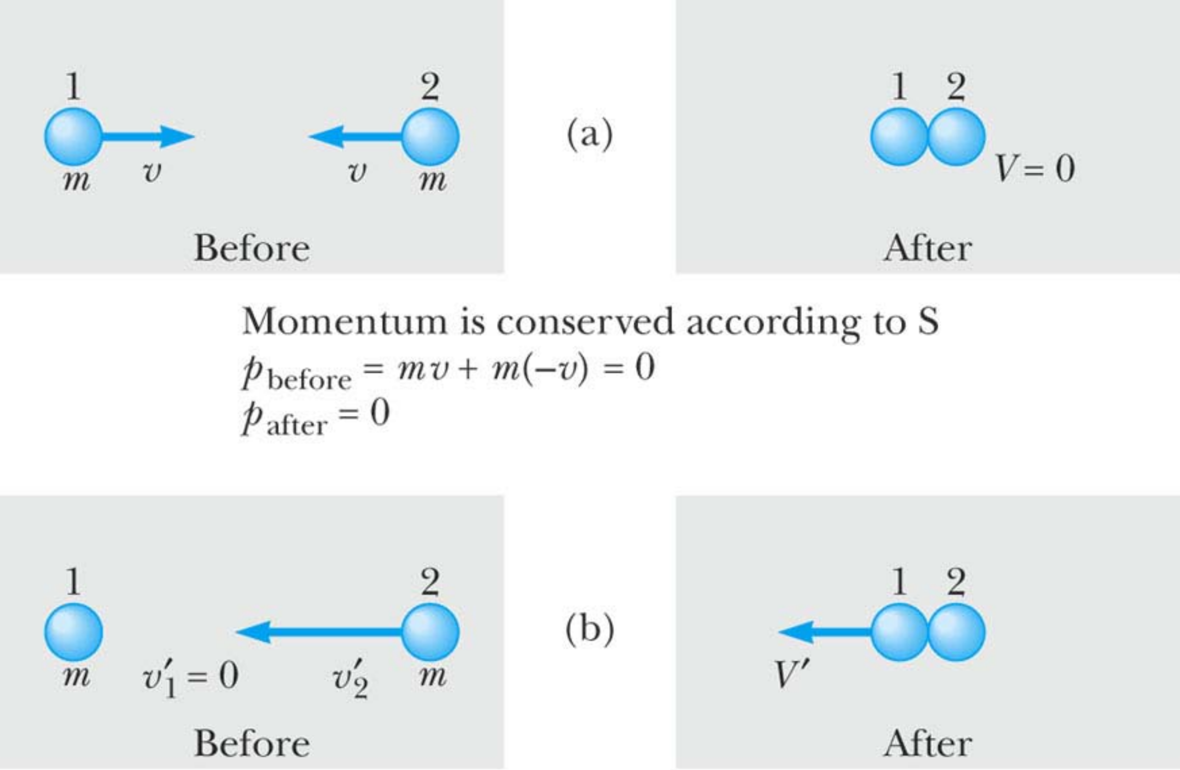


그림 1.6 지구에 대해 빛 보다 더 빠르게 나는 우주선 안에서 손전등을 켰다. (a) 우주선 기준계에서는 손전등이 우주선 앞을 비춘다. (b) 지구 기준계에서는 불빛이 우주선 뒤를 향한다. 우주선과 지구에서 서로 다른 현상으로 보게 됨으로 상대성의 원리에 위배된다. 결론은 우주선이 지구에 대해(혹은 다른 어느 모든 것에 대해) 광속보다 더 빠르게 움직일 수 없다.

Momentum Conservation?



Momentum is conserved according to S

$p_{\text{before}} = mv + m(-v) = 0$

$p_{\text{after}} = 0$

Momentum is *not* conserved according to S′

$$p'_{\text{before}} = \frac{-2mv}{1 + v^2/c^2}$$

$p'_{\text{after}} = -2mv$

Momentum Conservation!

Momentum

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m\mathbf{v}$$

where

v is the velocity of the particle and

m is the proper mass

(in some books it is called rest mass and denoted by m_0.)

Newton’s Second Law – Einstein’s Version

Relativistic form of Newton’s second law

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left(\gamma m \mathbf{v}\right)$$

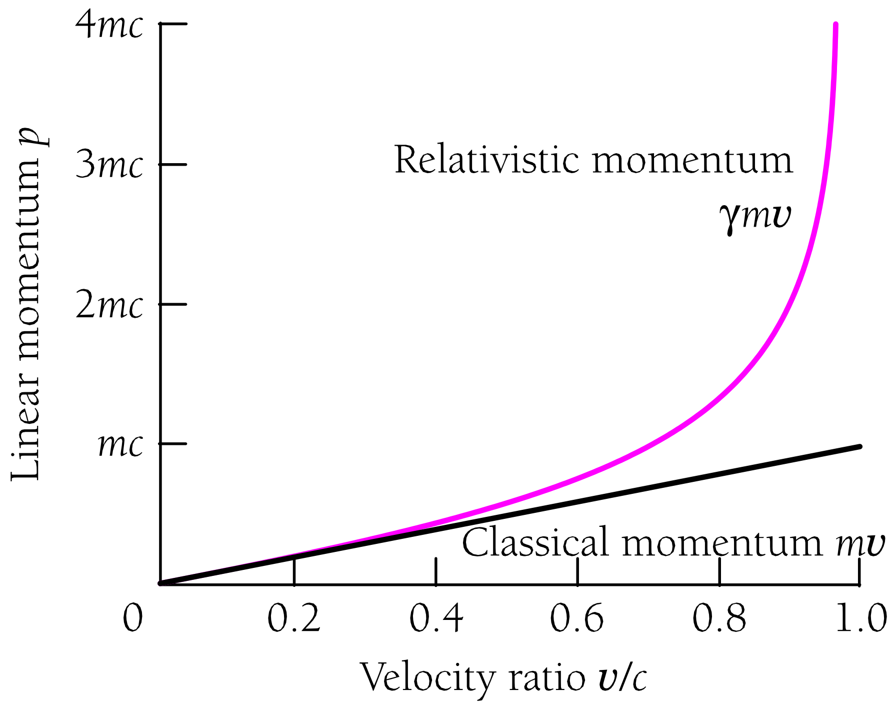


그림 1.14 관측자에 대해 속도 v로 움직이는 물체의 운동량.

Relativistic Energy

Work

$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} \frac{dp}{dt} dx = \int_{x_1}^{x_2} \frac{d\left(\gamma m v\right)}{dt} dx$$

$$= \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

Relativistic kinetic energy

$$K = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = \gamma mc^2 - mc^2$$

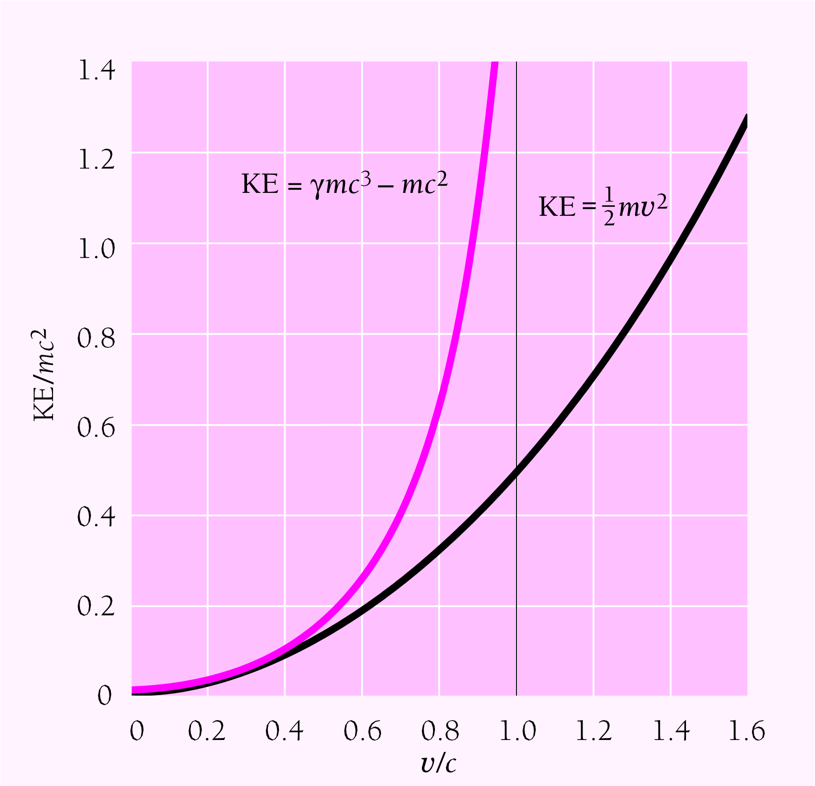


그림 1.16 어떤 움직이는 물체의 운동 에너지 KE와 정지질량 mc^2의 비율에 대한 고전적인 식과 상대론적인 식의 비교. 낮은 속력에서는 두 식이 같은 결과를 주지만 빛의 속력으로 접근하면서 두 식의 결과는 차이가 생긴다. 상대론적인 역학에 따르면 물체가 빛의 속력으로 움직이기 위해서는 무한 한 운동 에너지가 필요 하지만, 고전 역학에서는 운동 에너지가 정지 질량의 절반만 되면 그 물체는 빛의 속력으로 움직이는 것으로 된다.

Relativistic Energy

Total energy

$$E = \gamma mc^2 = K + mc^2$$

(in some books $E = mc^2 = \gamma m_o c^2$.)

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

For a photon, the proper mass (some call it rest mass) is zero, and hence,

$$E = pc$$

Mass-Energy Equivalence

Conservation of mass-energy