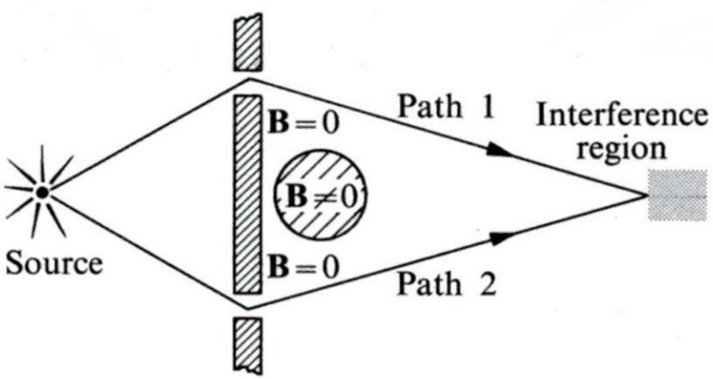
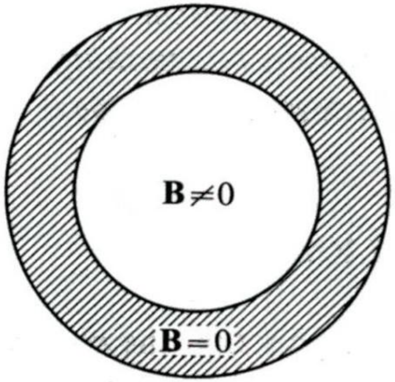
Aharonov-Bohm Effect

$$\frac{1}{2m}\left(-i\hbar\nabla - \frac{e\mathbf{A}}{c}\right)^2\psi + V\psi = E\psi$$

$$\psi = \psi_1^{(0)} \exp\left[\frac{ie}{\hbar c}\int_{\text{Path 1}}^{\mathbf{s}(\mathbf{x})} \mathbf{A}(\mathbf{x}')\cdot d\mathbf{s}'\right] + \psi_2^{(0)} \exp\left[\frac{ie}{\hbar c}\int_{\text{Path 2}}^{\mathbf{s}(\mathbf{x})} \mathbf{A}(\mathbf{x}')\cdot d\mathbf{s}'\right]$$

$$\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\left[\frac{e}{\hbar c}\oint \mathbf{A}\cdot d\mathbf{s}\right] = \begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\left[\frac{e}{\hbar c}\int \mathbf{B}\cdot\hat{n}\, dS\right] = \begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{e\Phi}{\hbar c}$$





Atom-Radiation Interaction

$$\hat{H} = \frac{1}{2m}\left[\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\mathbf{r},t)\right]^2 + V(\mathbf{r})$$

$$\hat{H} = \hat{H}^{(0)} + \hat{H}' + \hat{H}''$$

$$\hat{H}^{(0)} \equiv \frac{\hat{p}^2}{2m} + V; \qquad \hat{H}' = -\frac{e}{2mc}[\hat{\mathbf{p}}\cdot\mathbf{A} + \mathbf{A}\cdot\hat{\mathbf{p}}]$$

$$H'' = \frac{e^2}{2mc^2}A^2$$

$$\langle n'|H'|n\rangle = -\frac{e}{2mc}\int \psi_{n'}^*(\hat{\mathbf{p}}\cdot\mathbf{A} + \mathbf{A}\cdot\hat{\mathbf{p}})\psi_n\, d\mathbf{r}$$

$$\hat{\mathbf{p}}\cdot\mathbf{A}\psi_n = \frac{\hbar}{i}\nabla\cdot(\mathbf{A}\psi_n) = \frac{\hbar}{i}[\psi_n(\nabla\cdot\mathbf{A}) + (\mathbf{A}\cdot\nabla)\psi_n]$$

$$\mathbf{A}(\mathbf{r},t) = \mathbf{a}A_0\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

$$\langle n'|H'|n\rangle = -\frac{e}{mc}\int \psi_{n'}^*\mathbf{A}\cdot\hat{\mathbf{p}}\psi_n\, d\mathbf{r}$$

$$\langle n'|H'|n\rangle = -\frac{e}{mc}\int \psi_{n'}^*\hat{\mathbf{p}}\cdot\mathbf{A}\psi_n\, d\mathbf{r}$$

Atom-Radiation Interaction

$$\langle n'|\hat{H}'|n\rangle \rightarrow -\frac{e}{mc}\langle n';\hbar\omega|\hat{\mathbf{p}}|n\rangle \qquad (emission)$$

$$\langle n'|\hat{H}'|n\rangle \rightarrow -\frac{e}{mc}\langle n'|\hat{\mathbf{p}}|\hbar\omega;n\rangle \qquad (absorption)$$

$$\mathbf{A}(\mathbf{r},t) = \mathbf{a}A_0\cos(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

Atom-Radiation Interaction

$$\langle U \rangle = \frac{1}{4\pi}\langle \mathcal{B}^2 \rangle = \frac{1}{4\pi}\langle \mathcal{E}^2 \rangle$$

$$\boldsymbol{\mathcal{B}} = \boldsymbol{\nabla} \times \mathbf{A}$$

$$\mathbf{A} = \frac{A_0}{2}\mathbf{a}[e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}]$$

$$\mathbf{A}_{\pm} = c\left(\frac{2\pi\hbar}{\omega}\right)^{1/2}\mathbf{a}e^{\pm i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\hat{H}' = \hat{\mathbb{H}}_{\pm}(r)e^{\mp i\omega t}$$

$$\langle n'|\mathbb{H}_{\pm}|n\rangle = -\frac{e}{m}\left(\frac{2\pi\hbar}{\omega}\right)^{1/2}\langle n'|\mathbf{a}\cdot\hat{\mathbf{p}}e^{\pm i\mathbf{k}\cdot\mathbf{r}}|n\rangle$$

The Dipole Approximation

$$\langle n' | \mathbb{H}_{\pm} | n \rangle = -\frac{e}{m} \left(\frac{2\pi \hbar}{\omega} \right)^{1/2} \langle n' | \mathbf{a} \cdot \hat{\mathbf{p}} | n \rangle$$

$$\langle n' | \mathbf{p} | n \rangle = -\frac{im}{\hbar} (E_n{}^{(0)} - E_{n'}{}^{(0)}) \langle n' | \mathbf{r} | n \rangle$$

$$\langle n' | \mathbf{p} | n \rangle = -im\omega \langle n' | \mathbf{r} | n \rangle$$

$$\begin{aligned} \langle n' | \mathbb{H}_{+} | n, \mathbf{k} \rangle &= \langle n', \mathbf{k} | \mathbb{H}_{-} | n \rangle \\ &= ie(2\pi \hbar \omega)^{1/2} \langle n' | \mathbf{a} \cdot \mathbf{r} | n \rangle \end{aligned}$$

Quantum Mechanical Description of Vector Potential and its Results

$$\mathbf{A}(\mathbf{x}, t) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \sum_{\alpha} c\sqrt{\frac{\hbar}{2\omega}} [a_{\mathbf{k},\alpha}(0)\boldsymbol{\epsilon}^{(\alpha)} e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t} + a^{\dagger}_{\mathbf{k},\alpha}(0)\boldsymbol{\epsilon}^{(\alpha)} e^{-i\mathbf{k}\cdot\mathbf{x} + i\omega t}]$$

$$\begin{aligned}
&\langle B; n_{\mathbf{k},\alpha} - 1 \,|\, H_{\text{int}} \,|\, A; n_{\mathbf{k},\alpha} \rangle \\
&\quad = -\frac{e}{mc} \langle B; n_{\mathbf{k},\alpha} - 1 \,|\, \sum_i c\sqrt{\frac{\hbar}{2\omega V}} a_{\mathbf{k},\alpha}(0) e^{i\mathbf{k}\cdot\mathbf{x}_i - i\omega t} \mathbf{p}_i \cdot \boldsymbol{\epsilon}^{(\alpha)} \,|\, A; n_{\mathbf{k},\alpha} \rangle \\
&\quad = -\frac{e}{m}\sqrt{\frac{n_{\mathbf{k},\alpha}\hbar}{2\omega V}} \sum_i \langle B \,|\, e^{i\mathbf{k}\cdot\mathbf{x}_i} \mathbf{p}_i \cdot \boldsymbol{\epsilon}^{(\alpha)} \,|\, A \rangle e^{-i\omega t}
\end{aligned}$$

$$\langle B; n_{\mathbf{k},\alpha} + 1 \,|\, H_{\text{int}} \,|\, A; n_{\mathbf{k},\alpha} \rangle = -\frac{e}{m}\sqrt{\frac{(n_{\mathbf{k},\alpha} + 1)\hbar}{2\omega V}} \sum_i \langle B \,|\, e^{-i\mathbf{k}\cdot\mathbf{x}_i} \mathbf{p}_i \cdot \boldsymbol{\epsilon}^{(\alpha)} \,|\, A \rangle e^{i\omega t}$$

$$\text{absorption:} \quad c\sqrt{\frac{n_{\mathbf{k},\alpha}\hbar}{2\omega V}} \boldsymbol{\epsilon}^{(\alpha)} e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t},$$

$$\text{emission:} \quad c\sqrt{\frac{(n_{\mathbf{k},\alpha} + 1)\hbar}{2\omega V}} \boldsymbol{\epsilon}^{(\alpha)} e^{-i\mathbf{k}\cdot\mathbf{x} + i\omega t}$$

Perturbation Theory

$$c_m^{(1)}(t) = \frac{1}{i\hbar}\int_0^t dt' \langle m | H_I(t') | l \rangle e^{i(E_m - E_l)t'/\hbar}$$

$$c_m^{(2)}(t) = \frac{1}{i\hbar}\sum_n \int_0^t dt'' \langle m | H_I(t'') | n \rangle e^{i(E_m - E_n)t''/\hbar} c_n^{(1)}(t'')$$

$$= \frac{1}{(i\hbar)^2}\sum_n \int_0^t dt'' \int_0^{t''} dt' \langle m | H_I(t'') | n \rangle e^{i(E_m - E_n)t''/\hbar} \langle n | H_I(t') | l \rangle e^{i(E_n - E_l)t'/\hbar}$$