Heat and Mass Transfer

이 윤 우

서울대학교 화학생물공학부

18

STEADY STATE HEAT CONDUCTION

Overview

Steady-State Heat Conduction in a

(1) Flat Wall

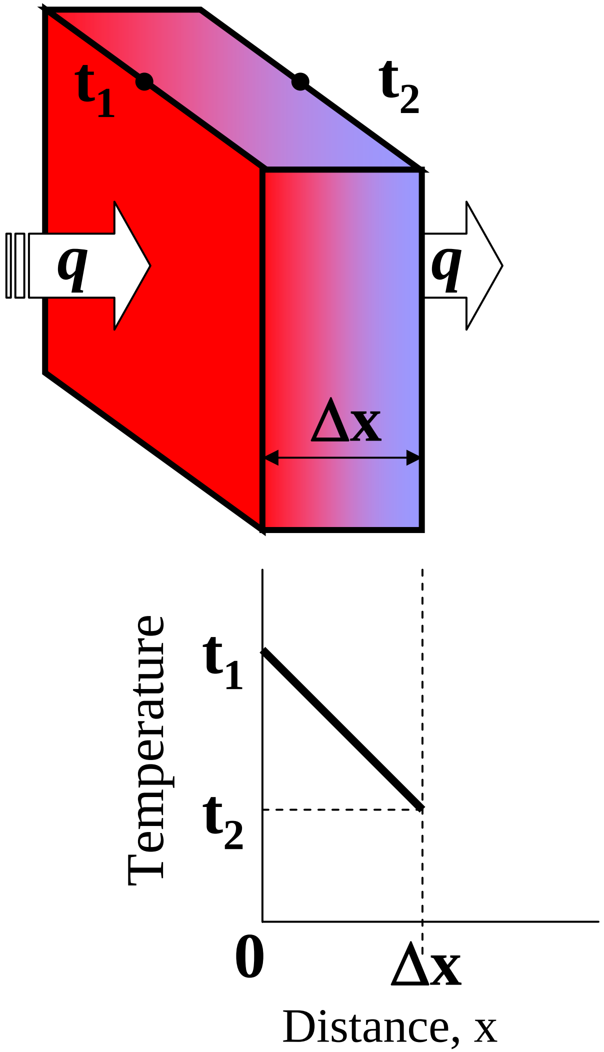
(2) Multilayer Flat Wall

(3) Hollow Cylinder

(4) Multilayer Cylinder

(5) Thermal Contact Resistance

Heat Conduction in a Flat Wall



Fourier conduction equation

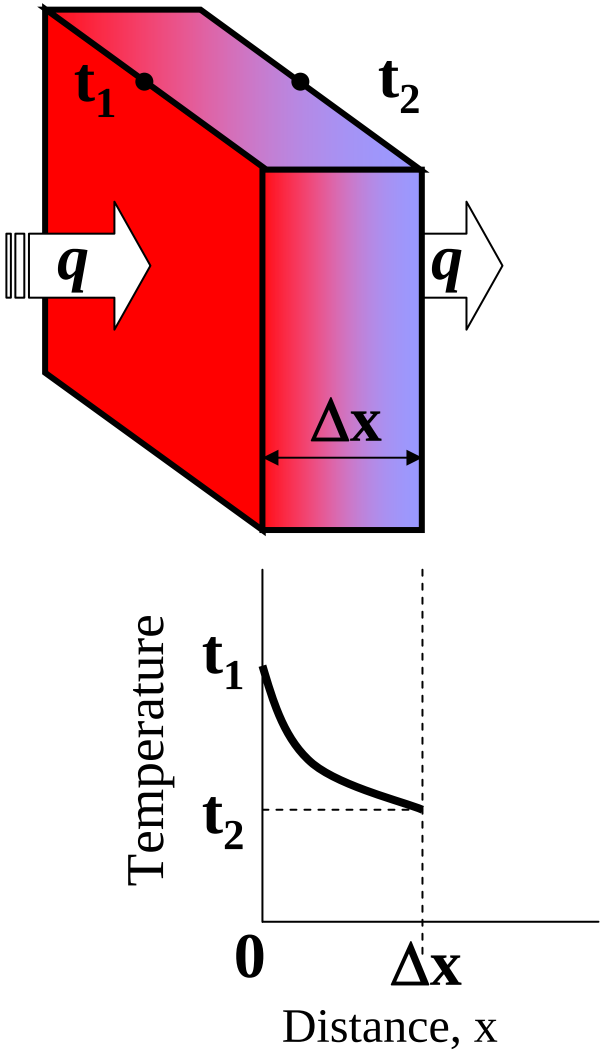
$$q = -kA\frac{dt}{dx}$$

▶ If heat is flowing normal to the principal surfaces, the area term *A* is constant.

▶ If *k* is assumed to be constant, *q* at any cross section is proportional to *dt/dx*.

▶ If energy is neither generated nor accumulated in the wall, *q* is identical at all cross sections and so is *dt/dx*.

Heat Conduction in a Flat Wall



Fourier conduction equation

$$q = -kA\frac{dt}{dx}$$

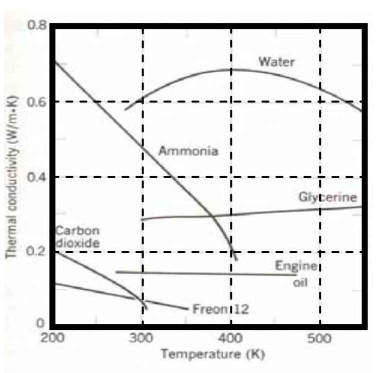
- If *k* varies with temp, *dt/dx* ≠ constant.
- If *q* and *A* = constant, and *k* increases with decreasing temp(Al_2O_3), the temperature gradient must diminish in the direction of decreasing temperature. Thus the curve representing temperature in steady-state flow for this system is concave upward.

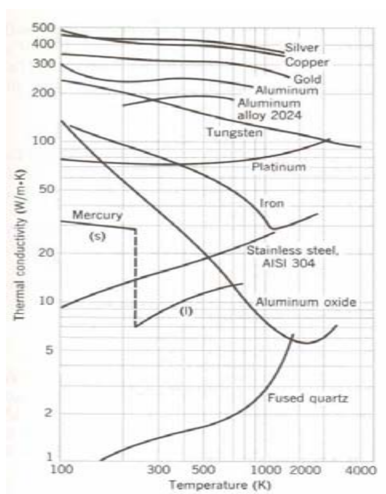
Influence of the temperature dependence of k on dt/dx

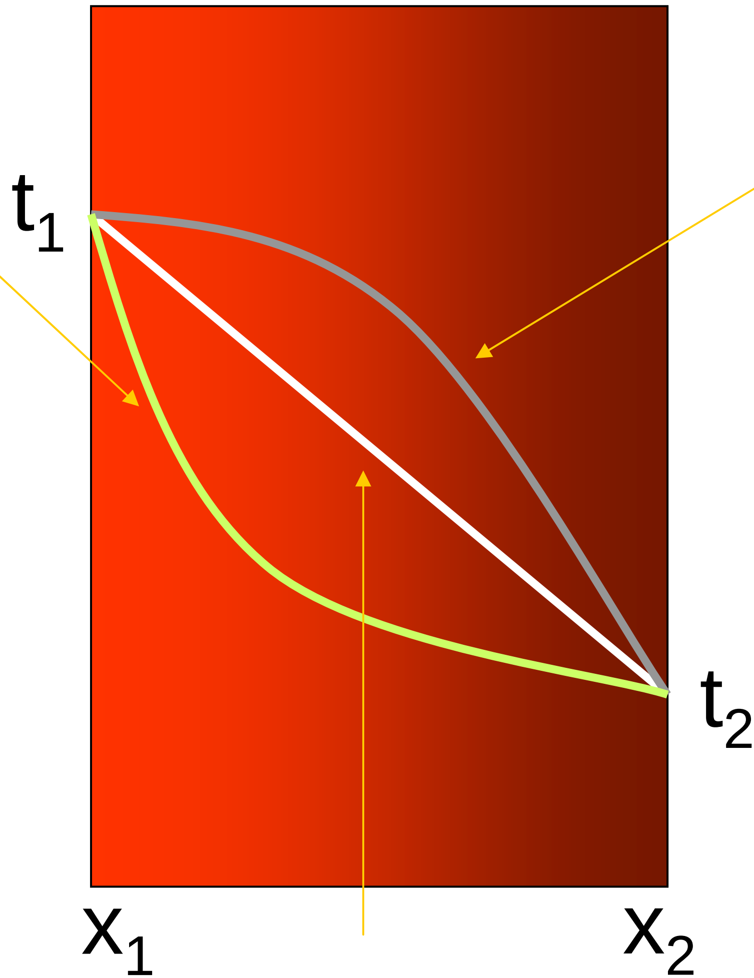
in a flat wall during heat transport by conduction

k decreasing with increasing T

ex: copper, tungsten, Al_2O3, $NH_3(l)$

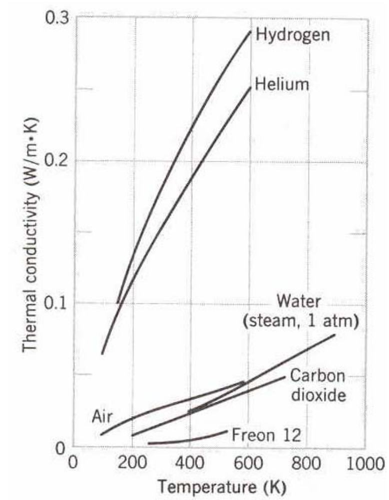






k increasing with increasing T

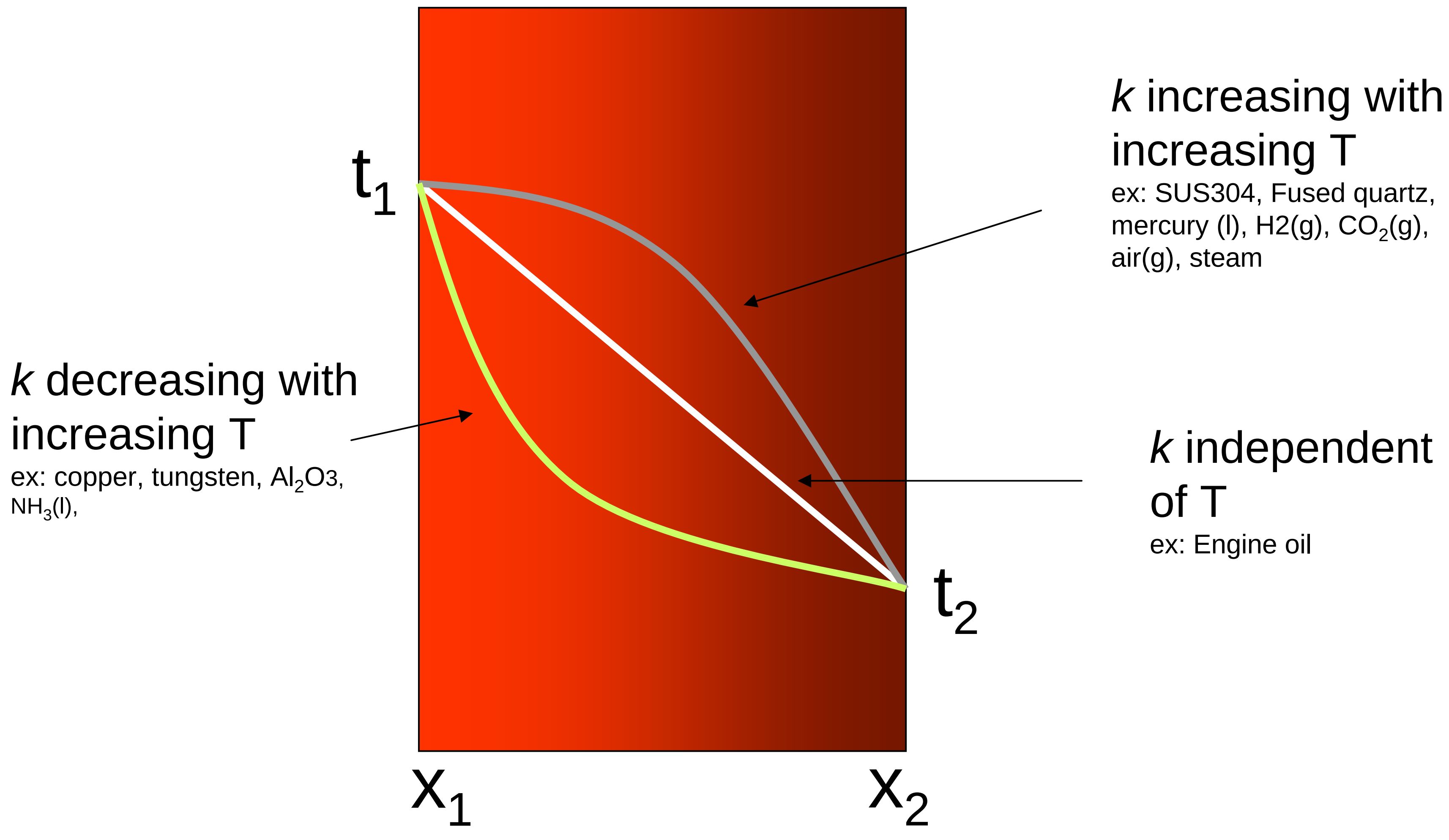
ex: SUS304, Fused quartz, mercury (l), $H_2(g)$, $CO_2(g)$, air(g), steam



k independent of T

ex: Engine oil, gylcerine

Influence of the temperature dependence of k on dt/dx
in a flat wall during heat transport by conduction



Supercritical Fluid Process Lab

Temperature Profile in a flat wall

Fourier conduction equation

$$q = -kA\frac{dt}{dx} \tag{18-1}$$

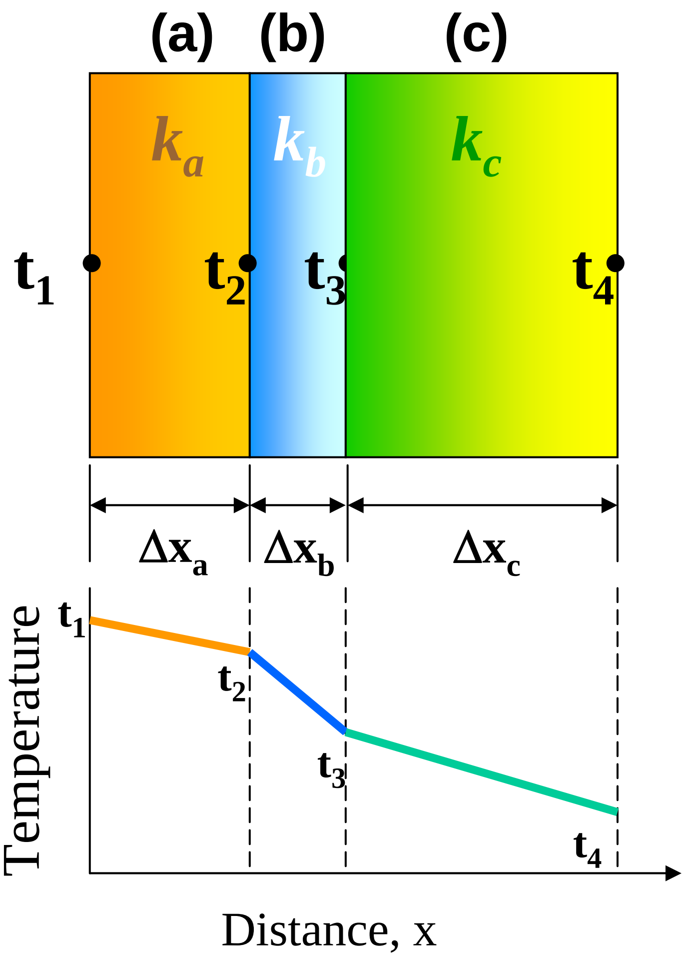
The integration of Eq (18-1) is readily performed when q, k, and A are constant,
And this gives

$$q = \frac{kA(t_1 - t_2)}{\Delta x} \tag{18-2}$$

The equation can also be integrated and solved for the temperature at any point x.

$$\boxed{t = -\frac{q}{kA}x + t_1} \tag{18-3}$$

Conduction in a multi-layer flat wall



$$q = k_a A \frac{t_1 - t_2}{\Delta x_a} = k_b A \frac{t_2 - t_3}{\Delta x_b} = k_c A \frac{t_3 - t_4}{\Delta x_c} \qquad (18\text{-}4)$$

$$t_1 - \cancel{t_2} = q \frac{\Delta x_a}{k_a A}$$

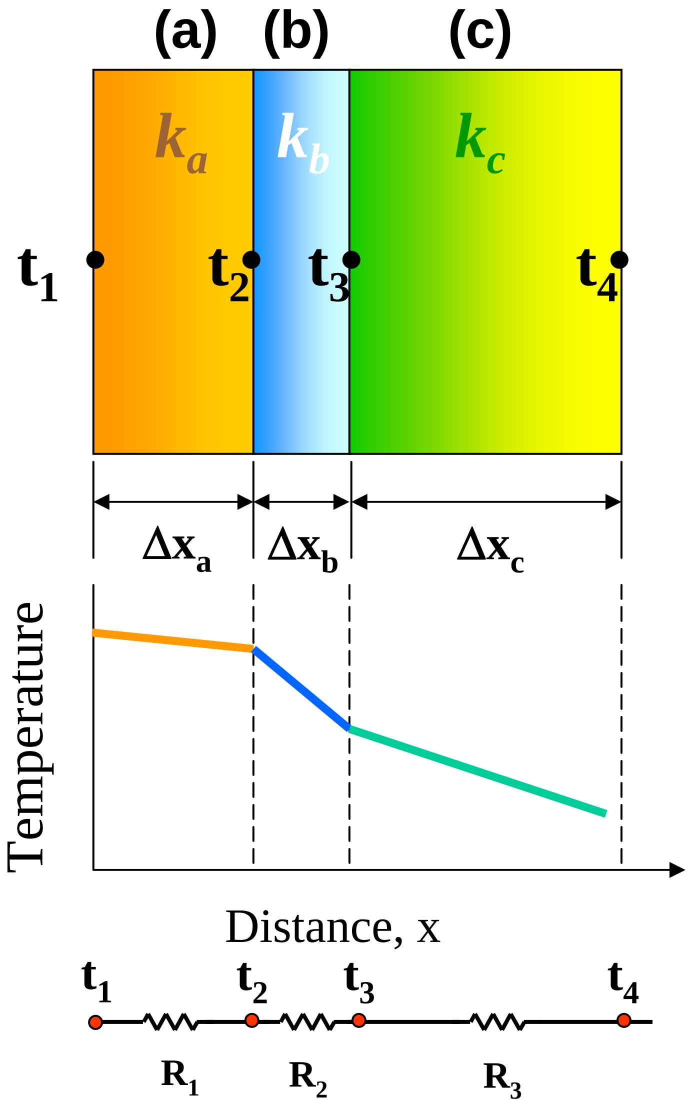
$$\cancel{t_2} - \cancel{t_3} = q \frac{\Delta x_b}{k_b A}$$

$$+ \quad \cancel{t_3} - t_4 = q \frac{\Delta x_c}{k_c A}$$

$$t_1 - t_4 = q \left(\frac{\Delta x_a}{k_a A} + \frac{\Delta x_b}{k_b A} + \frac{\Delta x_c}{k_c A} \right)$$

$$\boxed{q = \frac{t_1 - t_4}{\dfrac{\Delta x_a}{k_a A} + \dfrac{\Delta x_b}{k_b A} + \dfrac{\Delta x_c}{k_c A}}} \qquad (18\text{-}5)$$

Analogy to Ohm's law for electric conduction



Driving force
(Temp Difference)

Heat flow

$$q = \frac{t_1 - t_4}{\dfrac{\Delta x_a}{k_a A} + \dfrac{\Delta x_b}{k_b A} + \dfrac{\Delta x_c}{k_c A}}$$

Thermal resistances

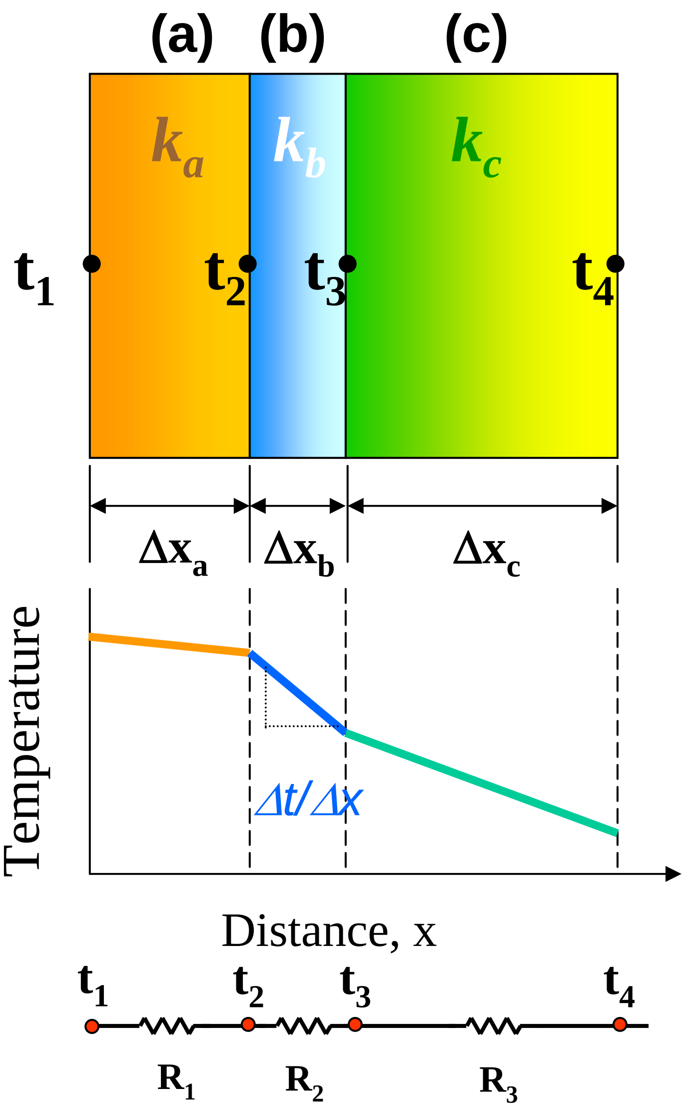
Driving force
(Voltage Difference)

Electric flow
(Current)

$$I = \frac{V_1 - V_4}{R_1 + R_2 + R_3}$$

Electric resistances

Overall Thermal resistances



$$q = \frac{t_1 - t_4}{\dfrac{\Delta x_a}{k_a A} + \dfrac{\Delta x_b}{k_b A} + \dfrac{\Delta x_c}{k_c A}}$$

Individual resistance

Overall resistance

$$\frac{q}{A} = k_a \frac{t_1 - t_2}{\Delta x_a} = k_b \frac{t_2 - t_3}{\Delta x_b} = k_c \frac{t_3 - t_4}{\Delta x_c}$$

Because *q/A* is the same for all layers, it follows that *k(Δt/Δx)* is the same for all layers; thus *Δt/Δx* is inversely proportional to the thermal conductivity (열전도도가 크면 온도구배가 작다).

Example 18-1

A cold-storage room has walls constructed of a 4-in layer of corkboard contained between double wooden walls, each 1/2 in thick. **(1)** Find the rate of heat loss in But/(h)(ft^2) if the wall surface temperature is 10°F inside room and 70 °F outside the room. In addition, **(2)** find the temperature at the interface between the outer wall and the corkboard.

Although thermal conductivity is a function of temperature, it is often assumed to be constant at the arithmetic average temperature of the layer involved. The conductivities of many materials have not been measured over a temperature range; in addition, other factors such as the density (in the case of corkboard) and the presence of impurities (e.g., moisture) can have an effect on the conductivity. Data limitations such as these often have a direct effect on the accuracy of the solution and guide the engineer in determining the degree of simplification he can use in his calculations.

Example 18-1

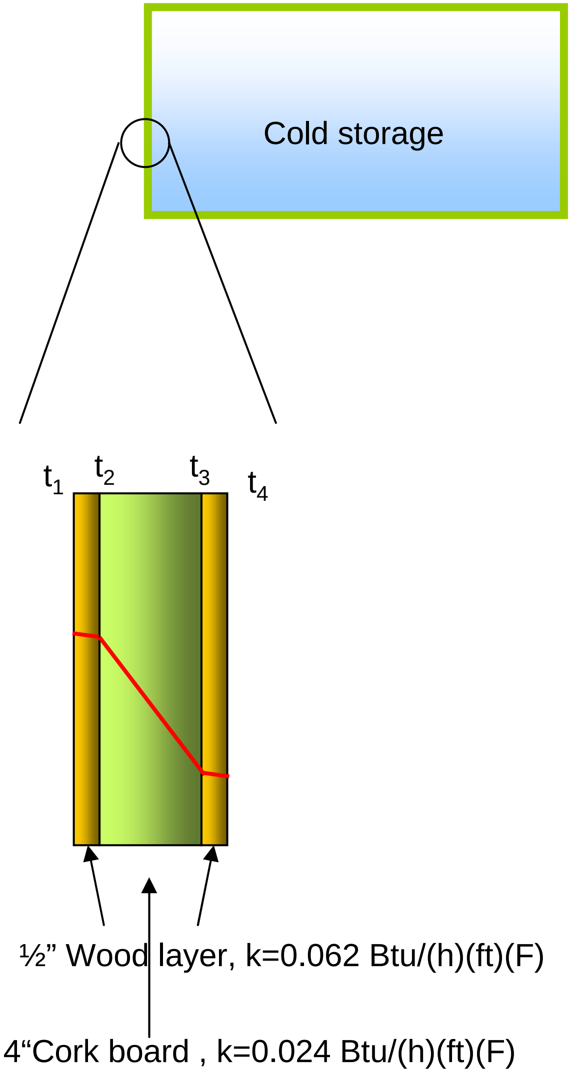
Thermal conductivity of corkboard ?

Cork board ρ=10 lb/ft^3, k=0.024 Btu/(h)(ft^2)(^{o}F/ft) @30^{o}C

Thermal conductivity of wood ?

Wood Balsa (벽오동) ρ=7-8 lb/ft^3, k=0.025 Btu/(h)(ft^2)(^{o}F/ft) @30^{o}C
Wood Oak (참나무) ρ=51.5 lb/ft^3, k=0.12 Btu/(h)(ft^2)(^{o}F/ft) @15^{o}C
Wood Maple (단풍나무) ρ=44.7 lb/ft^3, k=0.11 Btu/(h)(ft^2)(^{o}F/ft) @50^{o}C
Wood Pine (소나무) ρ=34.0 lb/ft^3, k=0.087 Btu/(h)(ft^2)(^{o}F/ft) @15^{o}C
Wood Teak (티크나무) ρ=40.0 lb/ft^3, k=0.10 Btu/(h)(ft^2)(^{o}F/ft) @15^{o}C
White fir (전나무) ρ=28.1 lb/ft^3, k=0.062 Btu/(h)(ft^2)(^{o}F/ft) @60^{o}C
(전나무의 가격이 싸고 벽의 건축자재로 널리 사용됨)

Table A-14 Thermal conductivities of some building materials (Page 805)



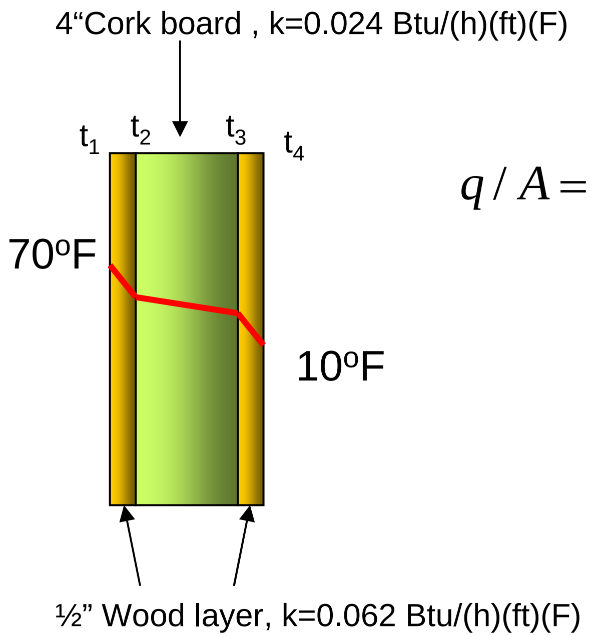
$$q/A = \frac{t_1 - t_4}{\dfrac{\Delta x_a}{k_a} + \dfrac{\Delta x_b}{k_b} + \dfrac{\Delta x_a}{k_a}}$$

$$\text{Thermal resistance wood layer} = \frac{\Delta x_a}{k_a} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{12}\right)}{(0.062)} = 0.67\,\frac{(h)(^\circ F)(ft^2)}{Btu}$$

$\Delta t/\Delta x$

$$\text{Thermal resistance cork board} = \frac{\Delta x_b}{k_b} = \frac{(4)\left(\frac{1}{12}\right)}{(0.024)} = 13.9\,\frac{(h)(^\circ F)(ft^2)}{Btu}$$

(1) the rate of heat loss: $$q/A = \frac{70-10}{0.67+13.9+0.67} = 3.9\ Btu/(h)(ft^2)$$



$$q/A = \frac{t_1 - t_4}{\frac{\Delta x_a}{k_a} + \frac{\Delta x_b}{k_b} + \frac{\Delta x_a}{k_a}} = \frac{t_1 - t_2}{\frac{\Delta x_a}{k_a}} = \frac{t_2 - t_3}{\frac{\Delta x_b}{k_b}} = \frac{t_3 - t_4}{\frac{\Delta x_a}{k_a}} = 3.9\ Btu/(h)(ft^2)$$

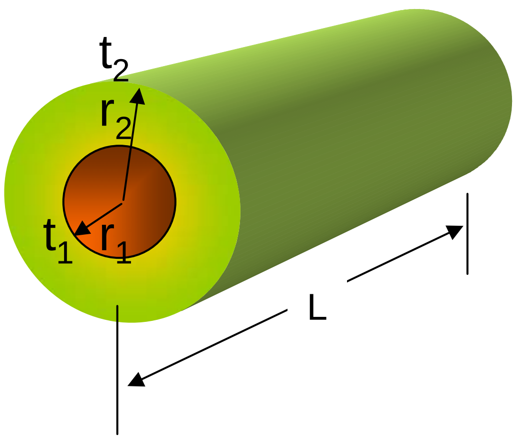
The temperature at the interface between the outer wooden wall and the corkboard can be determined by rearranging the equation for the individual temperature drops which led to Eq. (18-5).

$$q/A = \frac{t_1 - t_4}{\frac{\Delta x_a}{k_a} + \frac{\Delta x_b}{k_b} + \frac{\Delta x_a}{k_a}} = 3.9 = \frac{70 - t_2}{0.67} = \frac{t_3 - 10}{0.67}$$

$t_2 = 67.4^oF$

$t_3 = 12.6^oF$

Heat Conduction in the walls of a Hollow cylinder



The Fourier Equation in cylindrical Coordinates $$q = -kA\frac{dt}{dr} \qquad (18\text{-}6)$$

The area normal to heat flow $A = 2\pi rL$

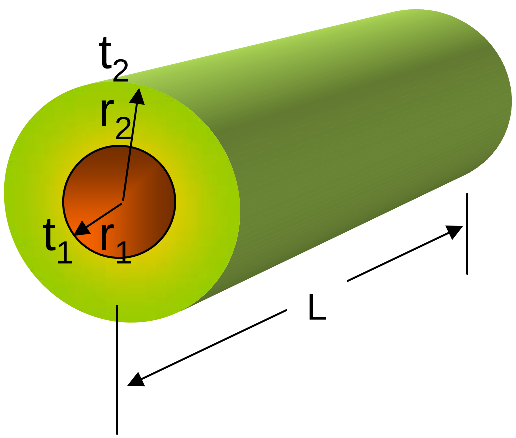
Integration Eq (18-6) yields

$$q\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi Lk\int_{t_1}^{t_2} dt \qquad (18\text{-}7)$$

$$q = \frac{2\pi Lk(t_1 - t_2)}{\ln(r_2/r_1)} \qquad (18\text{-}8)$$

$$q = \frac{2\pi Lk(r_2 - r_1)(t_1 - t_2)}{\ln(r_2/r_1)(r_2 - r_1)} = k\frac{(A_2 - A_1)(t_1 - t_2)}{\ln(A_2/A_1)(r_2 - r_1)} = k\frac{(A_2 - A_1)\Delta t}{\ln(A_2/A_1)\Delta r} \qquad \textbf{(18-10)}$$

Heat Conduction in the walls of a Hollow cylinder



$$q = k\frac{(A_2 - A_1)\Delta t}{\ln(A_2 / A_1)\Delta r} = kA_{lm}\frac{\Delta t}{\Delta r} \qquad (18\text{-}10)$$

$$A_{lm} = \frac{(A_2 - A_1)}{\ln(A_2 / A_1)}$$

The log mean area

Similar to

$$q = \frac{kA(t_1 - t_2)}{\Delta x}$$

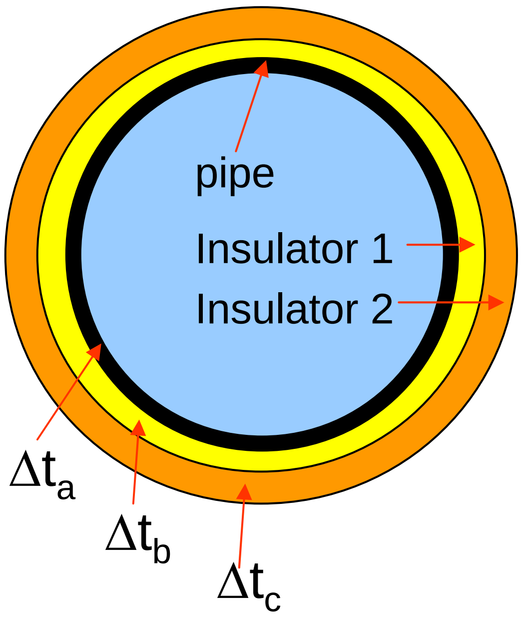
in flat wall

$$\Delta t = t_1 - t_2$$

$$\Delta r = r_2 - r_1$$

In most engineering applications (e.g., pipe), r_2/r_1<<2. In these circumstance the arithmetic mean area may be used in Eq. (18-10), with a consequent error in *q* less than 4%.

Heat Conduction in Multi-layer cylinder

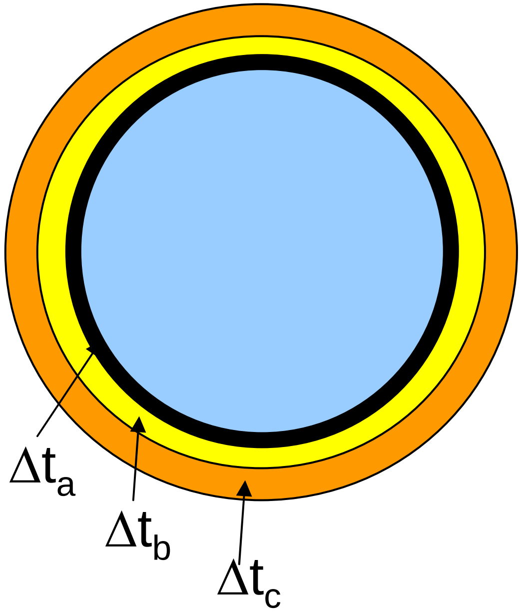


Consider the case of three concentric hollow cylinders, e.g., a pipe with two layers of insulation around it. The thickness the three layers will be designated Δr_a, Δr_b, and Δr_c, and the temperature drops over the individual layer Δt_a, Δt_b, and Δt_c .

The total heat-transfer rate, which will be the same for all the cylinders, can be written

$$q = \left(kA_{lm} \frac{\Delta t}{\Delta r} \right)_a = \left(kA_{lm} \frac{\Delta t}{\Delta r} \right)_b = \left(kA_{lm} \frac{\Delta t}{\Delta r} \right)_c \qquad (18\text{-}11)$$

Heat Conduction in Multi-layer cylinder



The individual temperature drops may be found by rearranging (18-11).

$$\Delta t_a = q\left(\frac{\Delta r}{kA_{lm}}\right)_a$$

$$\Delta t_b = q\left(\frac{\Delta r}{kA_{lm}}\right)_b$$

$$+ \quad \Delta t_c = q\left(\frac{\Delta r}{kA_{lm}}\right)_c$$

Adding and rearranging these three equations gives

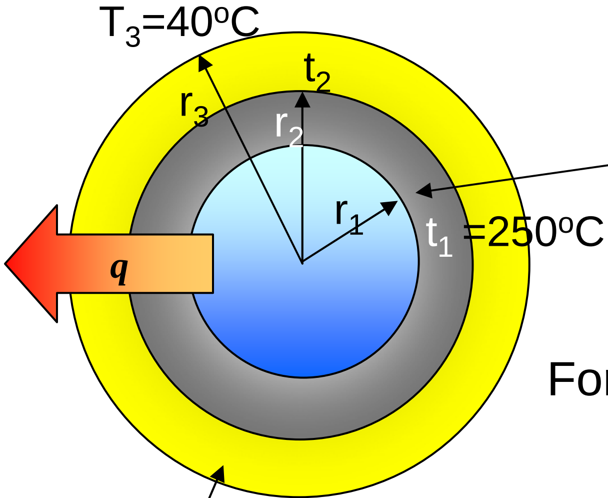
$$\boxed{q = \frac{\Delta t_{overall}}{\left(\frac{\Delta r}{kA_{lm}}\right)_a + \left(\frac{\Delta r}{kA_{lm}}\right)_b + \left(\frac{\Delta r}{kA_{lm}}\right)_c}} \qquad (18\text{-}12)$$

Individual resistance

Example 18-2

A 6-in, schedule-80 steel pipe is covered with a 0.1-m layer of 85 percent magnesia insulation. The temperature of the inner surface of the pipe is 250°C, and the temperature of the outer surface of the insulation is 40°C. **(1)** Calculate the rate of heat loss <u>per meter of pipe</u> and **(2)** the temperature at the interface between the pipe and the insulation.

The thermal conductivity of the steel pipe can be taken as 44.8W/m·K (Perry, p. 3-220), and that of the 85 percent magnesia as 0.066 (Perry, p. 3-221). The OD of the pipe is 0.1683 m, and the ID is 0.1463 m.



6” schedule 80 steel pipe
k=44.8 W/mK
ID=5.761”=0.1463m
OD=0.1683m
$\Delta r_{21}=r_2-r_1=0.011$m

$$q=\frac{\Delta t_{overall}}{\left(\frac{\Delta r}{kA_{lm}}\right)_a+\left(\frac{\Delta r}{kA_{lm}}\right)_b+\left(\frac{\Delta r}{kA_{lm}}\right)_c}$$

85% magnesia
k=0.066 W/mK
$\Delta r_{32}=r_3-r_2=0.1$m

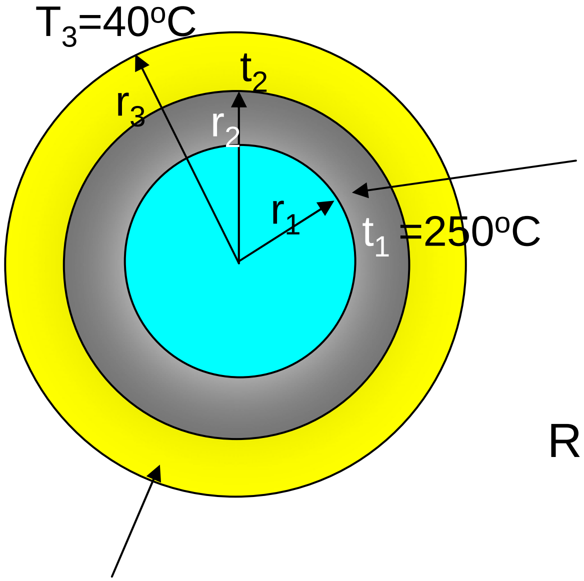
For the pipe
$$A_{lm}=\frac{\pi(0.1683-0.1463)}{\ln(0.1683/0.1463)}=0.49m^2$$

For the insulation
$$A_{lm}=\frac{\pi(0.3683-0.1683)}{\ln(0.3683/0.1683)}=0.80m^2$$
0.8023 m²

For the pipe
$$R_{pipe}=\frac{\Delta r_{21}}{kA_{lm}}=\frac{0.011}{(44.80)(0.49)}=0.00050\,K/W$$

For the insulation
$$R_{insulation}=\frac{\Delta r_{32}}{kA_{lm}}=\frac{0.1}{(0.066)(0.80)}=1.89\,K/W$$
1.8885 K/W

Rate of heat loss:
$$q=\frac{250-40}{0.0005+1.89}=111\,W$$
111.17 W



6" schedule 80 steel pipe
k=44.8 W/mK
ID=5.761"=0.1463m
OD=0.1683m
$\Delta r_{21}=r_2-r_1=0.011$m

85% magnesia
k=0.066 W/mK
$\Delta r_{32}=r_3-r_2=0.1$m

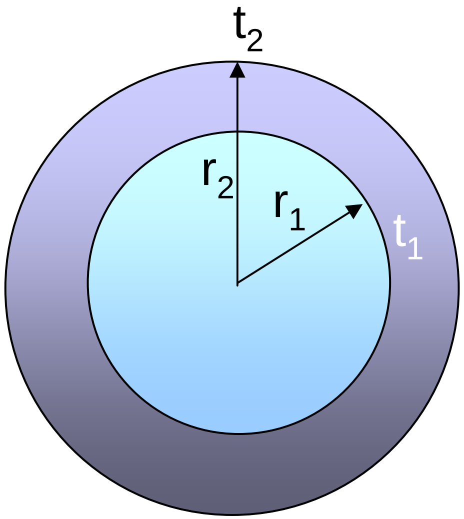
Rate of heat loss: $$q=\frac{250-40}{0.0005+1.89}=111\,W$$

Thermal resistance of the insulation is so large

$$q=\frac{\Delta t_{overall}}{\left(\frac{\Delta r}{kA_{lm}}\right)_a+\left(\frac{\Delta r}{kA_{lm}}\right)_b+\left(\frac{\Delta r}{kA_{lm}}\right)_c}=\left(kA_{lm}\frac{\Delta t}{\Delta r}\right)_a=\left(kA_{lm}\frac{\Delta t}{\Delta r}\right)_b=\left(kA_{lm}\frac{\Delta t}{\Delta r}\right)_c$$

Interface temperature (t_2) = $250-\left(\frac{0.00050}{1.89+0.00050}\right)(210)$ = 249.94°C ////

Heat Conduction to the walls of a Hollow Sphere



The Fourier Equation in spherical Coordinates $$q = -kA\frac{dt}{dr}$$ (18-6)

The area normal to heat flow $A = 4\pi r^2$

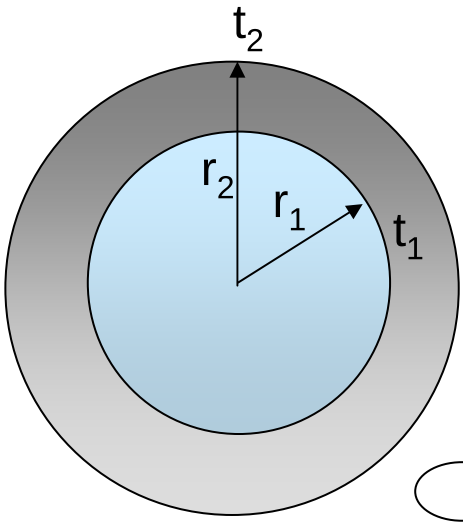
Integration Eq (18-6) yields

$$q\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi k\int_{t_1}^{t_2} dt \tag{18-13}$$

$$q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 4\pi k(t_1 - t_2) \tag{18-14}$$

$$q = 4\pi k(t_1 - t_2)\frac{r_1 r_2}{(r_2 - r_1)} = kA_{gm}\frac{\Delta t}{\Delta r} \tag{18-15, 16}$$

Heat Conduction to the walls of a Hollow Sphere



$$q = 4\pi k(t_1 - t_2)\frac{r_1 r_2}{(r_2 - r_1)} = kA_{gm}\frac{\Delta t}{\Delta r}$$

$$A_{gm} = \sqrt{A_1 A_2}$$: Geometric mean area

Similar to

$$q = \frac{kA(t_1 - t_2)}{\Delta x}$$

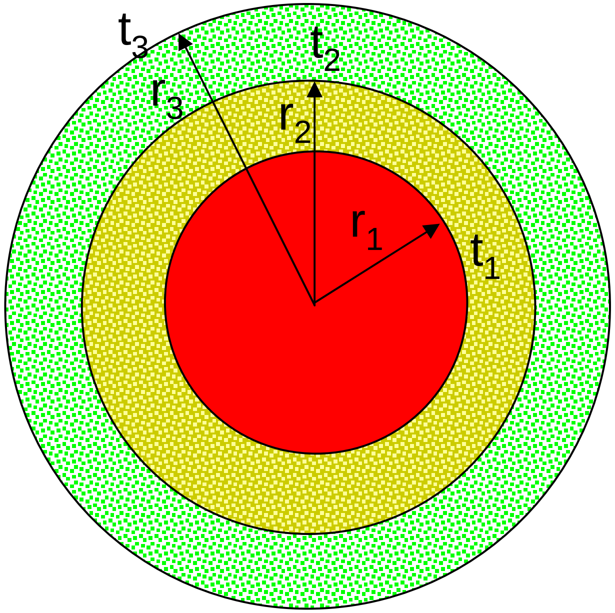
in flat wall

Integration of Eq. (18-13) to give the temperature t at any radial position r inside the wall of the sphere shows that

“the temperature is a linear function of 1/r.”

$$t = t_2 + \frac{q}{4\pi k}\left(\frac{1}{r} - \frac{1}{r_2}\right)$$

Heat Conduction of Three Concentric Spherical Layer



$$q = \frac{\Delta t_{overall}}{\left(\frac{\Delta r}{kA_{gm}}\right)_a + \left(\frac{\Delta r}{kA_{gm}}\right)_b + \left(\frac{\Delta r}{kA_{gm}}\right)_c}$$

Individual resistance

Heat Conduction

Multi-layer flat wall

$$q = \frac{\Delta t_{overall}}{\left(\frac{\Delta x}{kA}\right)_a + \left(\frac{\Delta x}{kA}\right)_b + \left(\frac{\Delta x}{kA}\right)_c}$$

$$A_{am} = \frac{A_1 + A_2}{2}$$

Multi-layer cylinder

$$q = \frac{\Delta t_{overall}}{\left(\frac{\Delta r}{kA_{lm}}\right)_a + \left(\frac{\Delta r}{kA_{lm}}\right)_b + \left(\frac{\Delta r}{kA_{lm}}\right)_c}$$

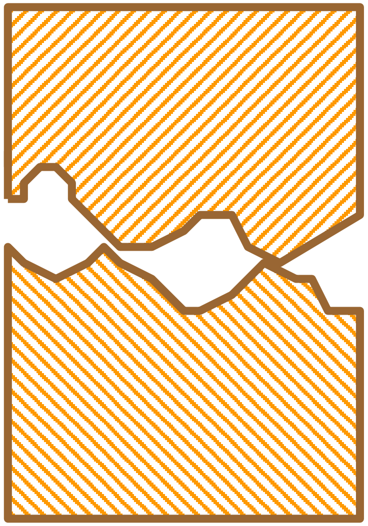
$$A_{lm} = \frac{(A_2 - A_1)}{\ln(A_2 / A_1)}$$

Walls of a Hollow Sphere

$$q = \frac{\Delta t_{overall}}{\left(\frac{\Delta r}{kA_{gm}}\right)_a + \left(\frac{\Delta r}{kA_{gm}}\right)_b + \left(\frac{\Delta r}{kA_{gm}}\right)_c}$$

$$A_{gm} = \sqrt{A_1 A_2}$$

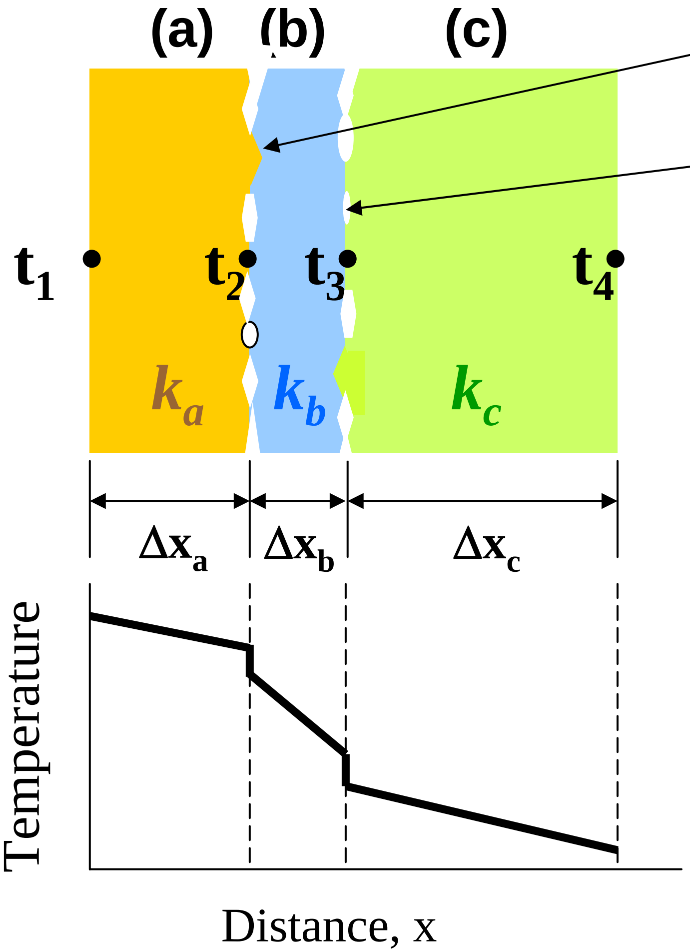
Thermal Contact Resistance



(1) Solid to solid conduction
(2) Conduction/convection/radiation through entrapped gas in the void spaces

For solids whose thermal conductivities exceed that of the interfacial fluid, the contact resistance may be reduced by increasing the area of the contact spots. Such an increase may be effected by increasing the joint pressure and/or by reducing the roughness of the mating surfaces. The contact resistance may also reduced by selecting an interfacial fluid of large thermal conductivity. (e.g.; thermal grease such as silicon oil)

Thermal Contact Resistance



Protuberance at the contact point (spot)

Vacant at the contact point
Convection + radiation

$$q = \frac{t_1 - t_4}{\dfrac{\Delta x_a}{k_a A} + \dfrac{\Delta x_b}{k_b A} + \dfrac{\Delta x_c}{k_c A} + R_{TC}}$$

Thermal Contact Resistance

Two copper plates

Pressure	10^2 kN/m^2	10^4 kN/m^2
R_{TC}	1~10	0.1~1

Thermal Contact Resistance

Thermal Contact Resistance R_{TC}x10^4 (m^2·K/W)

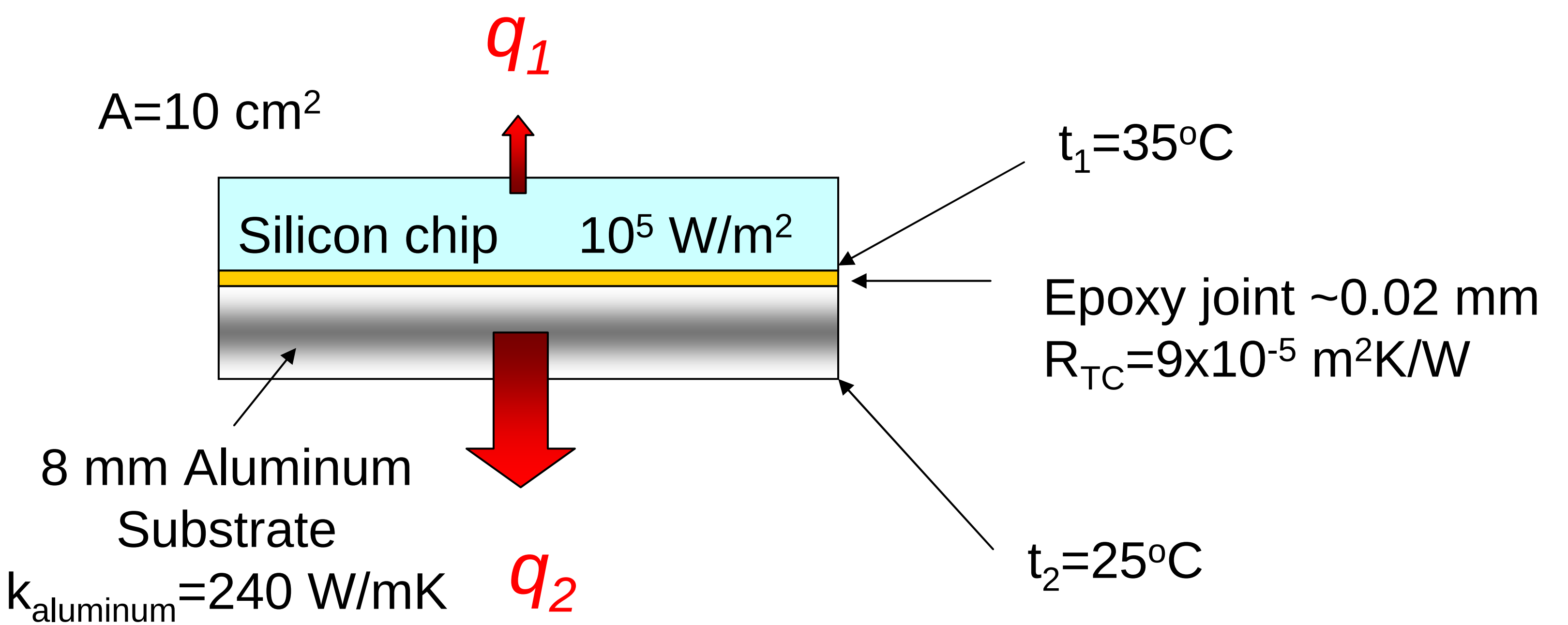
Contact pressure	10^2 kN/m^2	10^4 kN/m^2
SUS	6~25	0.7~4.0
Copper	1~10	0.1~0.5
Magnesium	1.5~3.5	0.2~0.4
Aluminum	1.5~5.0	0.2~0.4

Al-Al interface (10 μm roughness) under 10^5 kN/m^2
Thermal Contact Resistance R_{TC}x10^4 (m^2·K/W) with different interfacial fluids

Air	2.75
He	1.05
H_2	0.720
Silicone oil	0.525
Glycerine	0.265

Fried, E., “Thermal Conduction Contribution to Heat Transfer at Contacts”, in R.P. Tye, Ed., Thermal Conductivity, vol 2, Academic Press, London, 1969

Thermal Dissipation of the Chip



$$q_2 = \frac{35-25}{\underbrace{9\times10^{-5}}_{R_{TC}} + \frac{8\times10^{-3}}{\underbrace{240}_{k_{aluminum}}}} \underbrace{\times10^{-4}}_{\text{Cross-sectional area}} = 8.11\,W$$

Example 18-3

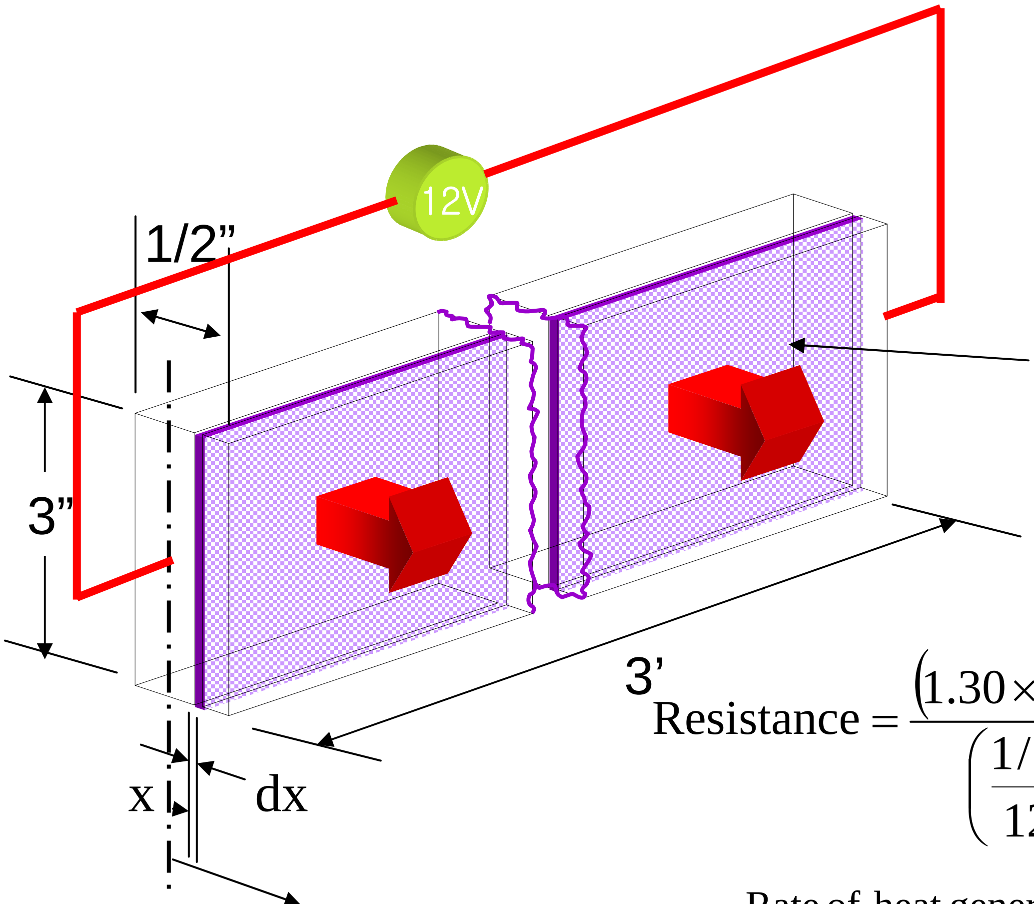
A heating element is constructed from carbon in the shape of a bar 3” wide, ½” thick, and 3 ft long. When a potential of 12 V is applied to the ends of the bar, its surface reaches a uniform temperature of 1400°F, as indicated by an optical pyrometer. What is the temperature at the center of the bar? The electrical resistivity of the bar is $1.30*10^{-4}(\Omega)(ft)$, and its thermal conductivity is 2.9 Btu/(h)(ft)(°F).

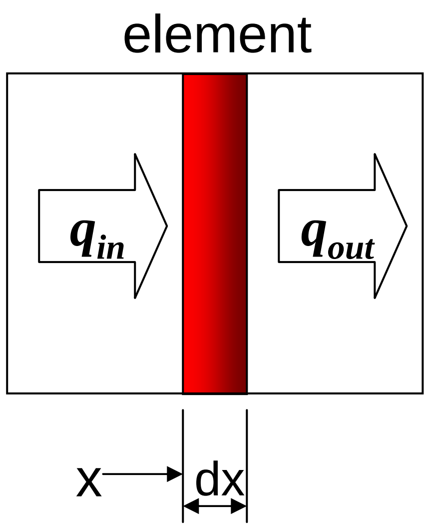
Only heat conduction normal to the largest forces of the bar will be considered, since heat leaves the bar principally through these faces. A differential equation is obtained by writing an energy balance on a differential segment *dx* of the bar, as shown in Fig. 18-4.

Energy Balance on the Element

@steady state

Heat flow in - heat flow out + rate of heat generation = 0





$$A = \left(\frac{3}{12}\right)(3)$$
$$= 0.75\, ft^2$$

$$\text{Resistance} = \frac{(1.30 \times 10^{-4}\,\Omega \cdot ft)(3\, ft)}{\left(\frac{1/2}{12}\, ft\right)\left(\frac{3}{12}\, ft\right)} = 0.0375\Omega$$

$$\text{Rate of heat generation in entire bar} = q_{in} = q_{out}$$

$$= \frac{V^2}{R} = \frac{(12)^2}{0.0375} = 3830W = 419000 Btu/(h)(ft^3)$$

Energy Balance on the Element

@steady state: **Heat flow in - heat flow out + rate of heat generation = 0**

Heat flow into element = $$-kA\frac{dt}{dx} \quad Btu/h \qquad (1)$$

Heat flow out of element = $$-kA\left(\frac{dt}{dx}+d\,\frac{dt}{dx}\right) \qquad -kA\left(\frac{dt}{dx}+\frac{\partial}{\partial x}\left(\frac{dt}{dx}\right)dx\right)$$

$$-kA\left(\frac{dt}{dx}+\frac{d^2t}{dx^2}dx\right) \quad Btu/h \qquad (2)$$

Rate of heat generation in the element = 419,000 Adx Btu/h (3)

Energy Balance: $$\boxed{-kA\frac{dt}{dx}+kA\left(\frac{dt}{dx}+\frac{d^2t}{dx^2}dx\right)+419{,}000\,Adx=0}$$

which reduces to

$$\frac{d^2t}{dx^2}=-\frac{419{,}000}{k}$$

$$\frac{d^2t}{dx^2} = -\frac{419{,}000}{k} = -\frac{419{,}000}{2.9} = -145{,}000^\circ F/ft^2$$

Integration with BCs: dt/dx=0 @x=0 (the bar is symmetrical)

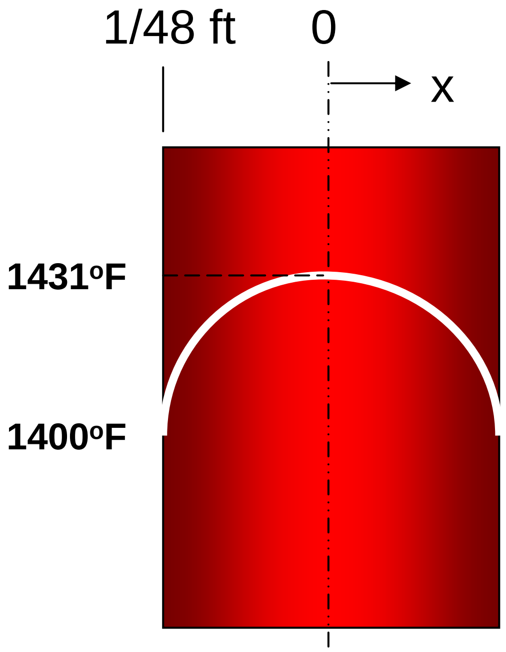
$$\frac{dt}{dx} = -145{,}000x + \overset{0}{\cancel{C_1}}$$

Second Integration with BCs: t=1400°F @x=1/48 ft

$$t = \frac{-145{,}000x^2}{2} + C_2$$

$$C_2 = 1400 + (145{,}000/2)(1/48)^2 = 1431$$

$$\boxed{t = -72{,}500x^2 + 1431}$$



Homework #1

PROBLEMS

18-1; 18-4; 18-5; 18-7; 18-8

Due date: Before quiz on September 21

Quiz: September 21 13:00-14:15 (302-509)

Closed book, Calculator

Differential element for mass balance

@steady state: Mass in - Mass out = Accumulate

In the x direction, output less input by mass flow is given by

$$[u_x\rho + d(u_x\rho)]dydz - u_x\rho dydz \quad (7\text{-}1)$$

The second term is the input in the x-direction through the face of area *dydz* located at a distance x from the plane x=0; the first term is the output through the parallel face located at x+dx. The express (7-1) reduces to

$$d(u_x\rho)dydz \quad (7\text{-}2)$$

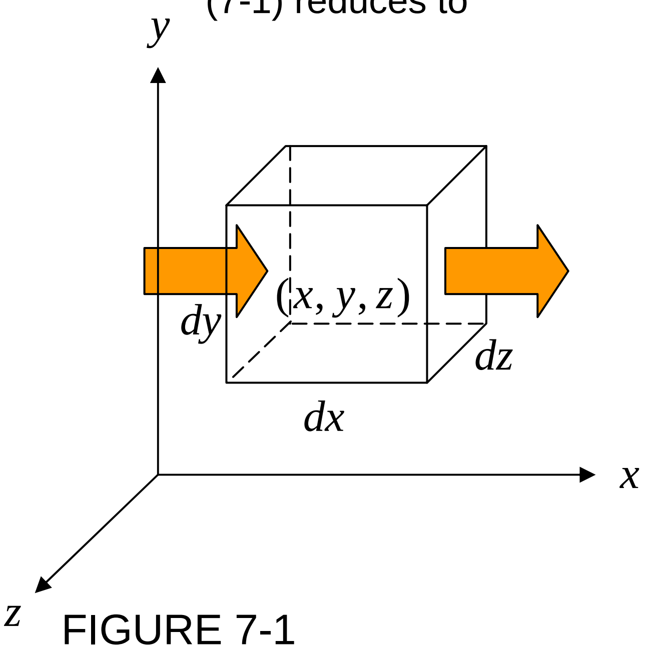


FIGURE 7-1
Differential element of mass balance

It will be convenient to express the differential change in $u_x\rho$ over the distance dx by

$$d(u_x\rho) = \frac{\partial(u_x\rho)}{\partial x}dx \quad (7\text{-}3)$$

As a result the output minus input in the x-direction is

$$\frac{\partial(u_x\rho)}{\partial x}dxdydz \quad (7\text{-}3)$$

Differential element for mass balance

Similar expression can be written for flow in the y and z directions.
The rate of accumulation in the element is

$$\frac{\partial \rho}{\partial \theta} dxdydz = 0 \qquad (7\text{-}5)$$

$$\frac{\partial (u_x \rho)}{\partial x} dxdydz + \frac{\partial (u_y \rho)}{\partial x} dydxdz + \frac{\partial (u_z \rho)}{\partial x} dzdxdy + \frac{\partial \rho}{\partial \theta} dxdydz = 0 \qquad (7\text{-}6)$$

$$\Downarrow$$

$$\frac{\partial (u_x \rho)}{\partial x} + \frac{\partial (u_y \rho)}{\partial x} + \frac{\partial (u_z \rho)}{\partial x} + \frac{\partial \rho}{\partial \theta} = 0 \qquad (7\text{-}7)$$

$$\Downarrow$$

$$\rho \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial x} + \frac{\partial u_z}{\partial x} \right) + u_x \frac{\partial \rho}{\partial x} + u_y \frac{\partial \rho}{\partial x} + u_z \frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial \theta} = 0 \qquad (7\text{-}8)$$