Heat and Mass Transfer

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SOME DESIGN EQUATIONS FOR CONVECTION HEAT TRANSFER

How to obtain equations for predicting heat-transfer coefficients

1. Combination of momentum, energy, and continuity equations for laminar flow

2. von Karman integral method for turbulent flow

3. The analogy between heat and momentum transfer for turbulent flow

4. The dimensional analysis

Heat transfer coefficients

Flow parallel to a flat plate

Laminar flow

(Re < 5 x 10^5)

$$Nu_x = 0.332\,(\mathrm{Re}_x)^{1/2}\,\mathrm{Pr}^{1/3}$$

$$Nu_m = 0.664\,(\mathrm{Re}_L)^{1/2}\,\mathrm{Pr}^{1/3}$$

(22-19)

Turbulent flow

(Re < 5 x 10^5)

$$Nu_x = 0.0292\left(\mathrm{Re}_x\right)^{4/5}\mathrm{Pr}^{1/3} \quad (23\text{-}50)$$

$$Nu_m = 0.0365\left(\mathrm{Re}_L\right)^{4/5}\mathrm{Pr}^{1/3} \quad (23\text{-}51)$$

Flow in a pipe

Laminar flow

(Re<2100)

$$Nu = 1.86\,\mathrm{Re}^{1/3}\,\mathrm{Pr}^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \quad (22\text{-}42)$$

Turbulent flow

(Re>2100)

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} \quad (23\text{-}32)$$

Dimensional Analysis

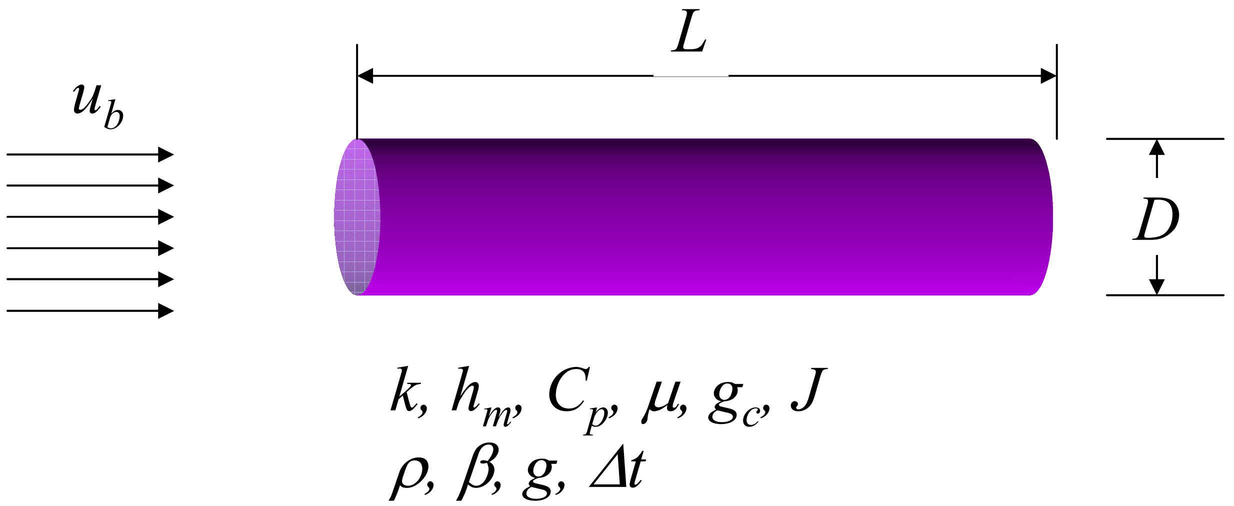
- can be used for both type flow (laminar/turbulent)

Turbulent flow: $$Nu = f(\mathrm{Re}, \mathrm{Pr}, \frac{D}{L}, \frac{\mu}{\mu_s}, \cdots)$$

Laminar flow: $$Nu = f(Ra, \mathrm{Pr}, \frac{D}{L}, \frac{\mu}{\mu_s}, \cdots)$$

Example 24-1 Dimensional Analysis

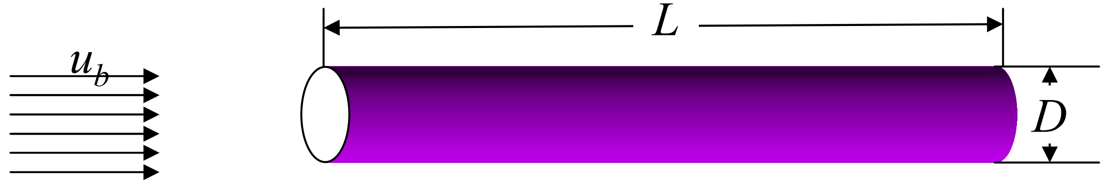
Heat-transfer coefficient in natural convection in a heated pipe



6 Fundamental dimensions: Length [L], mass [M], time [θ] temperature [T], heat [H], force [F]

- Heat & force can be expressed in terms of the other four dimensions
- Include gravitational constant, g_c & mechanical equivalent of heat, J

Example 24-1 Dimensional Analysis



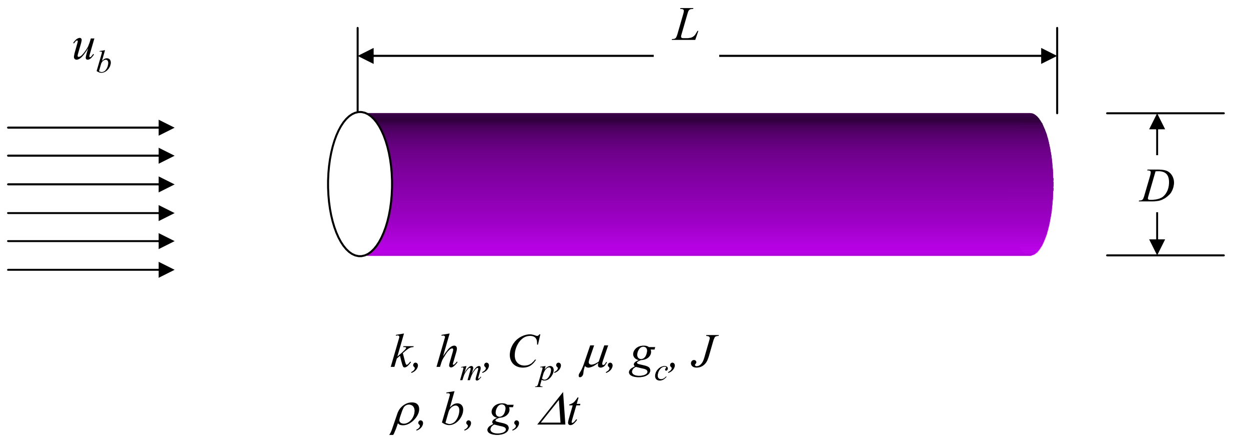
$k,\ h_m,\ C_p,\ \mu,\ g_c,\ J$
$\rho,\ b,\ g,\ \Delta t$

6 Fundamental dimensions:
Length [L], mass [M], time [θ], temperature [T], heat [H], force [F]

No.	Variable	Symbol	Dimension
1	Length of heated section	L	[L]
2	Fluid density	ρ	[M/L^3]
3	Fluid viscosity	μ	[M/Lθ]
4	Fluid thermal conductivity	k	[H/LθT]
5	Dimensional constant	g_c	[ML/Fθ^2]
6		J	[FL/H]
7	Mean heat-transfer coefficient	h_m	[H/θTL^2]
8	Temperature difference, $t_s - t_0$	Δt	[T]
9	Coefficient of thermal expansion	β	[T^{-1}]
10	Specific heat of fluid	C_p	[H/MT]
11	Gravitational acceleration	g	[L/θ^2]
12	Bulk velocity of fluid	u_b	[L/θ]
13	Diameter of pipe	D	[L]

Total number of variables is 13

Example 24-1 Dimensional Analysis



The total number of variables is 13, and we have chosen to express these in terms of six dimensions. As is often the case, the maximum number of variables which will not form a dimensionless group is equal to the number of fundamental dimensions.

According to Buckingham's theorem,
13 variables, 6 dimensions
Number of dimensionless groups: (13 – 6) = **7**
7 dimensionleess group=$\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7$

Example 24-1 Dimensional Analysis

According to Buckingham's theorem,

13 variables, 6 dimensions → Number of dimensionless groups: (13 – 6) = **7**

7 dimensionleess group = $\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7$

The variables which we choose to be common to all groups are first six in the table above.

$$\pi_1 = L^a \rho^b \mu^c k^d g_c^e J^f h_m^g$$

$$\pi_2 = L^a \rho^b \mu^c k^d g_c^e J^f (\Delta t)^g$$

$$\pi_3 = L^a \rho^b \mu^c k^d g_c^e J^f \beta^g$$

$$\pi_4 = L^a \rho^b \mu^c k^d g_c^e J^f C_p^g$$

$$\pi_5 = L^a \rho^b \mu^c k^d g_c^e J^f g^g$$

$$\pi_6 = L^a \rho^b \mu^c k^d g_c^e J^f u_b^g$$

$$\pi_7 = L^a \rho^b \mu^c k^d g_c^e J^f D^g$$

Each of the remaining seven variables will, in turn, be added to the first six, to give seven groups.

Example 24-1 Dimensional Analysis: the 1st group

$$\pi_1 = L^a \rho^b \mu^c k^d g_c^e J^f h_m^g$$

$$= L^a \left(ML^{-3}\right)^b \left(ML^{-1}\theta^{-1}\right)^c \left(HL^{-1}\theta^{-1}T^{-1}\right)^d \left(MLF^{-1}\theta^{-2}\right)^e \left(FLH^{-1}\right)^f \left(H\theta^{-1}T^{-1}L^{-2}\right)^g$$

L : $a-3b-c-d+e+f-2g=0$

M : $b+c+e=0$

Θ : $-c-d-2e-g=0$

T : $-d-g=0$

H : $d-f+g=0$

F : $-e+f=0$

$\therefore\ a=g,\ b=c=e=f=0,\ d=-g$

$$\boxed{\pi_1 = \left(\frac{h_m L}{k}\right)^g = \frac{h_m L}{k}} \longrightarrow \text{Nusselt number}$$

Example 24-1 Dimensional Analysis: the 2nd group

$$\pi_2 = L^a \rho^b \mu^c k^d g_c^e J^f (\Delta t)^g$$

$$= L^a \left(ML^{-3}\right)^b \left(ML^{-1}\theta^{-1}\right)^c \left(HL^{-1}\theta^{-1}T^{-1}\right)^d \left(MLF^{-1}\theta^{-2}\right)^e \left(FLH^{-1}\right)^f \left(T\right)^g$$

$L: a-3b-c-d+e+f=0$

$M: b+c+e=0$

$\theta: -c-d-2e=0$

$T: -d+g=0$

$H: d-f=0$

$F: -e+f=0$

$\therefore a=b=2g,\ c=-3g,\ d=e=f=g=1$

$$\pi_2 = \frac{L^2 \rho^2 k g_c J \Delta t}{\mu}$$

Example 24-1 Dimensional Analysis

How to make seven groups

$$\pi_2 = \boxed{L^a \rho^b \mu^c k^d g_c^e J^f} \Delta t \qquad \pi_2 = \frac{L^2 \rho^2 k g_c J \Delta t}{\mu^3} \rightarrow \text{Grashof number}$$

$$\pi_3 = L^a \rho^b \mu^c k^d g_c^e J^f \beta \qquad \pi_3 = \frac{\mu^3 \beta}{L^2 \rho^2 k g_c J}$$

$$\pi_4 = L^a \rho^b \mu^c k^d g_c^e J^f C_p \qquad \pi_4 = \frac{C_p \mu}{k} \rightarrow \text{Prandtl number}$$

$$\pi_5 = L^a \rho^b \mu^c k^d g_c^e J^f g \qquad \pi_5 = \frac{L^3 \rho^2 g}{\mu^2}$$

$$\pi_6 = L^a \rho^b \mu^c k^d g_c^e J^f u_b \qquad \pi_6 = \frac{L \rho u_b}{\mu} \rightarrow \text{Reynolds number}$$

$$\pi_7 = L^a \rho^b \mu^c k^d g_c^e J^f D \qquad \pi_7 = D/L$$

(24-1)

Example 24-1 Dimensional Analysis

The seven groups obtained in Example 24-1 can be used to correlate the results of experiments in natural convection. One common way of doing this is to write an equation of the form

$$\pi_1 = function(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7)$$
$$\rightarrow \pi_1 = \alpha \pi_2^{\beta} \pi_3^{\gamma} \pi_4^{\delta} \pi_5^{\varepsilon} \pi_6^{\zeta} \pi_7^{\eta} \qquad (24\text{-}1)$$

By experimental, $\beta, \gamma, \delta, \varepsilon, \zeta$ and η are to be determined

Example 24-1 Dimensional Analysis

✓The constants represented by the Greek letters can be determined by a statistical analysis of the experimental results, often the least-squares method.

✓Groups which have no effect on h_m can easily be located because the exponents on these groups will be very small.

✓Two groups may have the same exponent, indicating that the variables they contain may be combined in a single group. For example, in Ex 24-1, it is found that the combined groups, π_2, π_3, π_5, give a single dimensional group, $L^3\rho^2 g\beta\Delta t/\mu_2$, known as the *Grashof number*, which together with π_1, the *Nusselt number*, and π_4, the *Prandtl number*.

✓The group π_6 and π_7 are meaningless in this analysis because $u_b=0$ and $D=\infty$.

Dimensional Analysis

✓The correlating equation is

$$\pi_1 = \alpha(\pi_2\pi_3\pi_5)^\beta(\pi_4)^\delta \qquad (24\text{-}2)$$

$$\frac{h_m L}{k} = 0.478\left(\frac{L^3\rho^2 g\Delta t}{\mu^2 T}\right)^{0.25} \qquad (22\text{-}25)$$

✓The *Prandtl number* is approximately constant at unity for most gases, so the absence of that group would be expected.

✓The *Grashof number* is in Eq. (22-25) differs from the product $\pi_2\ \pi_3\ \pi_5$ only in that for an ideal gas β equals 1/T, where T is the absolute temperature.

Dimensional Analysis

$$\pi_1 = \frac{Lh_m}{k} : \text{Nusselt number}$$

$$\pi_4 = \frac{C_p \mu}{k} : \text{Prandtl number}$$

$$\pi_2 \times \pi_3 \times \pi_5 = \frac{L^3 \rho^2 \beta g \Delta t}{\mu^2} : \text{Granshof Number}$$

$$\pi_6 \times \pi_7 = \frac{D \rho u_b}{\mu} : \text{Reynolds Number}$$

$$\begin{aligned} Nu_m &= \alpha \left(\pi_2 \pi_3 \pi_5 \right)^{\beta} \pi_4^{\delta} \pi_6^{\zeta} \pi_7^{\eta} \\ &= \alpha \mathrm{Gr}^a \mathrm{Pr}^b \mathrm{Re}^c \end{aligned}$$

In natural convection, $Nu_m = 0.478 Gr^{0.25}$

In forced convection, $Nu_m = \alpha \mathrm{Re}^{\beta} \mathrm{Pr}^{\gamma}$

Dimensional Analysis

Most equations : $\pi_1 = \alpha \pi_2^{\beta} \pi_3^{\gamma} \cdots$

$\mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}$ for tubulent flow in circular pipe

$\mathrm{Nu} = 0.53\mathrm{Gr}^{1/4}\mathrm{Pr}^{1/4}$ for natural convection from horizontal cylinder

In heat transfer - coefficients : $\pi_{11} = \alpha + \beta \pi_2^{\gamma} \pi_3^{\delta}$

$$\mathrm{Nu} = 2.0 + 0.6\left(\frac{\mathrm{D}u_0\rho}{\mu}\right)^{1/2}\left(\frac{\mathrm{C_p}\mu}{\mathrm{k}}\right)^{1/3}$$ for sphere

$$\mathrm{Nu} = 7.0 + 0.025\left(\frac{\mathrm{D}u_b\rho}{\mu}\right)^{0.8}\left(\frac{\mathrm{C_p}\mu}{\mathrm{k}}\right)^{0.8}$$ for liquid metal in circular pipe

Summary of Equations used for Predicting convective heat-transfer coefficients

Fundamental derivations

Empirical correlations of dimensionless groups

Equations

Predicting convective heat-transfer coefficients

Heat Exchanger Design

- Fundamental derivations
- Empirical correlations of dimensionless groups

$$\pi_1 = \alpha \pi_2^{\beta} \pi_3^{\gamma}$$

$$Nu_x = (\mathrm{Re}_x)^{1/2} \left(\int_0^{\infty} \exp\left(-\frac{\mathrm{Pr}}{2} \int_0^{\eta} f(\eta)\, d\eta \right) d\eta \right)^{-1}$$

Predicting Convective Heat-transfer Coefficients: $\mathbf{Nu_x} \rightarrow \mathbf{h_x}$

$$Nu_m = 0.478 Gr^{0.25}$$

$$Nu = 0.023 \,\mathrm{Re}^{0.8}\, \mathrm{Pr}^{1/3}$$

$$Nu_x = 0.332 (\mathrm{Re}_x)^{1/2}\, \mathrm{Pr}^{1/3}$$

Calculation of overall heat-transfer Coefficients

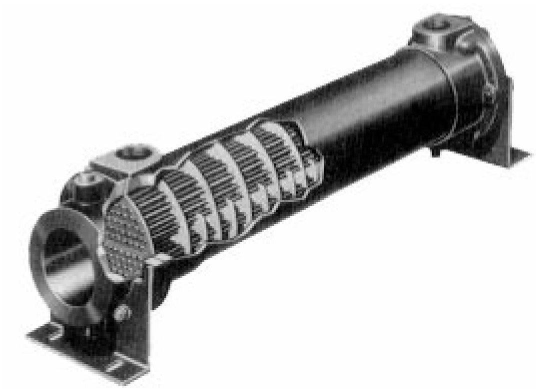
$$U_o = \frac{1}{\dfrac{A_o}{h_i A_i} + \dfrac{A_o}{h_{di} A_i} + \dfrac{\Delta r_b}{k_b}\dfrac{A_o}{A_{b,lm}} + \dfrac{\Delta r_c}{k_c}\dfrac{A_o}{A_{c,lm}} + \dfrac{1}{h_o}}$$

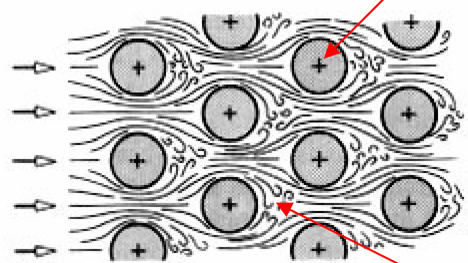
Calculation of heat transfer area

$$q = U_0 A \Delta t_{lm} \qquad A = \frac{q}{U_0 A} \qquad A = \pi D L$$

Shell and Tube Heat Exchanger

$$h_i = 0.023\left(\frac{k}{D}\right)\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}$$





$$h_o = 0.482\left(\frac{k}{D}\right)\left(\frac{D\rho u_{max}}{\mu}\right)^{0.556}$$

$$U_o = \frac{1}{\dfrac{A_o}{h_i A_i} + \dfrac{A_o}{h_{di} A_i} + \dfrac{\Delta r_b}{k_b}\dfrac{A_o}{A_{b,lm}} + \dfrac{\Delta r_c}{k_c}\dfrac{A_o}{A_{c,lm}} + \dfrac{1}{h_o}}$$

$$q = U_0 A \Delta t_{lm} \qquad A = \frac{q}{U_0 A} \qquad A = \pi D L$$

Summary of Equations used for Predicting convective heat-transfer coefficients

- ☺ Convection in circular pipe (Laminar/turbulent)
- ☺ Convection in non-circular conduits
- ☺ Convection normal to a cylinder
- ☺ Convection normal to a bank of circular tubes
- ☺ Convection from spheres
- ☺ Convective heat transfer between a fluid and a packed bed
- ☺ Convection from a plane surface
- ☺ Heat transfer to liquid metals
- ☺ Heat transfer to non-newtonian fluids
- ☺ Effect of surface roughness on heat transfer coefficient

Convection in circular pipe (Laminar flow)

Parabolic flow, uniform heat flux: overall temperature difference is constant

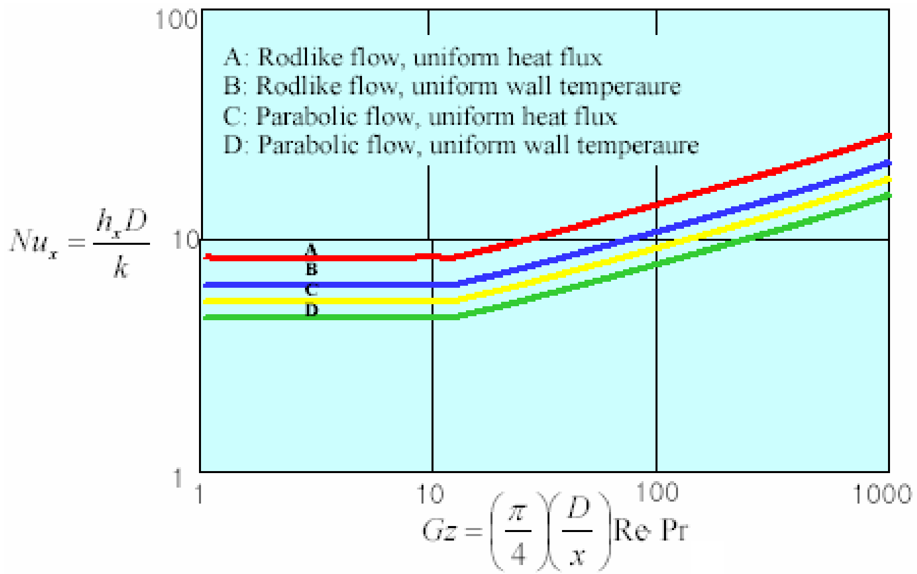
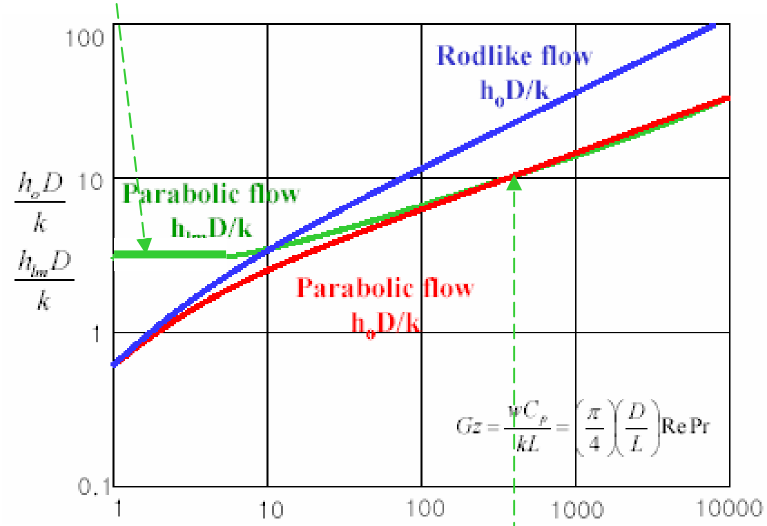


Fig. 22-3: convection in circular pipe

$$Gz < 5, \quad Nu_m \sim 3.6$$

Low flow rate
Long pipe



$$Gz > 10, \quad Nu_m = 1.62\left(\frac{4wC_p}{\pi k L}\right)^{1/3}$$

Fig. 22-4: convection in circular pipe with uniform wall temperature

Convection in circular pipe (Laminar flow)

For the case of constant wall temperature: condenser

$$Graetz\ Number = Gz_x = \frac{wC_p}{kx} = \frac{heat\ transfer\ by\ conduction}{heat\ transfer\ by\ convection}$$

$$Gz = \mathrm{Re} \cdot \mathrm{Pr} \cdot \frac{D}{L} = Pe \cdot \frac{D}{L}$$

t_s=*constant*

$$Gz < 5, \quad Nu_m \sim 3.6$$ (a low flow rate or a long pipe)

$$Gz > 10, \quad Nu_m = 1.62\left(\frac{4wC_p}{\pi kL}\right)^{1/3}$$

Convection in circular pipe (Laminar flow)

For the case of constant wall temperature : condenser

$$Gz > 10, \quad Nu_m = 1.62\left(\frac{4wC_p}{\pi kL}\right)^{1/3} \tag{22-40}$$

Account for the effect of temperature on viscosity

Sieder and Tate

$$w = \rho u \frac{\pi D^2}{4}$$

Arithmetic average temperature

$Gz>10$

$$\boxed{Nu_m = 1.86\,\mathrm{Re}^{1/3}\,\mathrm{Pr}^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}} \tag{22-42}$$

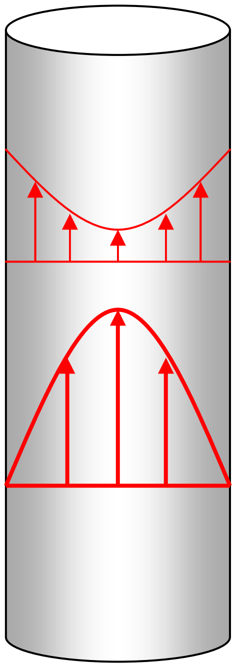
Wall temperature

Convection in vertical tube (Laminar flow)

Natural-convection flow effects are usually significant only when the forced convection flow is laminar and the temperature-difference driving force is large.

Natural convection

Forced convection

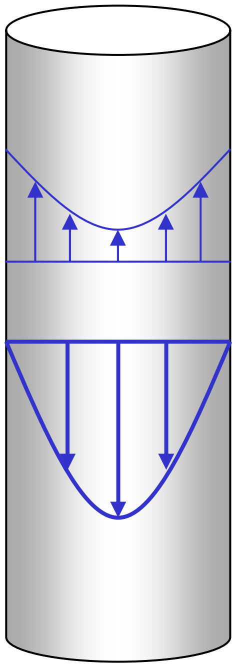


$Nu > Nu_x$

$Nu < Nu_x$

$$Nu_x = 1.62\,\mathrm{Re}^{1/3}\,\mathrm{Pr}^{1/3}\left(\frac{D}{L}\right)^{1/3}$$

(22-40)



Natural convection

Forced convection

AMC(Assisting Mixed Convection)

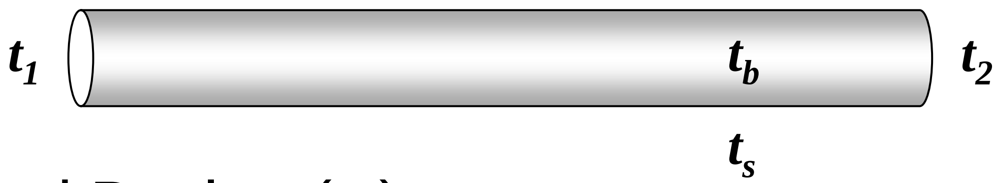
Higher heat-transfer coefficients

OMC(Opposing Mixed Convection)

Lower heat-transfer coefficients

Convection in circular pipe (Turbulent flow)

Pr > 0.7; L/D > 60



Dittus and Boelter (t_b)

$$t_b = \frac{t_1 + t_2}{2}$$

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.3}$$: cooling of the fluid

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.4}$$: heating of the fluid

Temp effect on viscosity in the laminar sub-layer.

Colburn (t_f)

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.3}$$

$$t_f = \frac{\frac{t_1 + t_s}{2} + \frac{t_2 + t_s}{2}}{2}$$

Sieder & Tate (t_b)

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

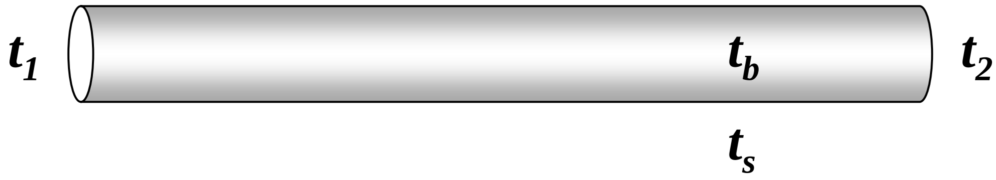
For $Pr > 10^4$

$$t_b = \frac{t_1 + t_2}{2}$$

t_s

Convection in circular pipe (Turbulent)

Pr > 0.7; L/D > 60



$$t_b = \frac{t_1 + t_2}{2}$$

$$t_f = \frac{\frac{t_1 + t_s}{2} + \frac{t_2 + t_s}{2}}{2}$$

Dittus and Boelter (t_b) (predicting within <13% for 651 data points)

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.3 \text{ or } 0.4} \qquad \textbf{(24-4)}$$

Least Square Correlation (t_b) (predicting within 10.2% for 651 data points)

$$\frac{h}{u_b\rho C_p} = \exp\left[-3.796 - 0.205\ln\mathrm{Re} - 0.505\ln\mathrm{Pr} - 0.0225(\ln\mathrm{Pr})^2\right] \qquad \textbf{(24-7)}$$

Convection in circular pipe (Turbulent flow)

Dittus and Boelter (t_b)

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.3}$$: cooling of the fluid

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.4}$$: heating of the fluid

Temp effect on viscosity in the laminar sub-layer.

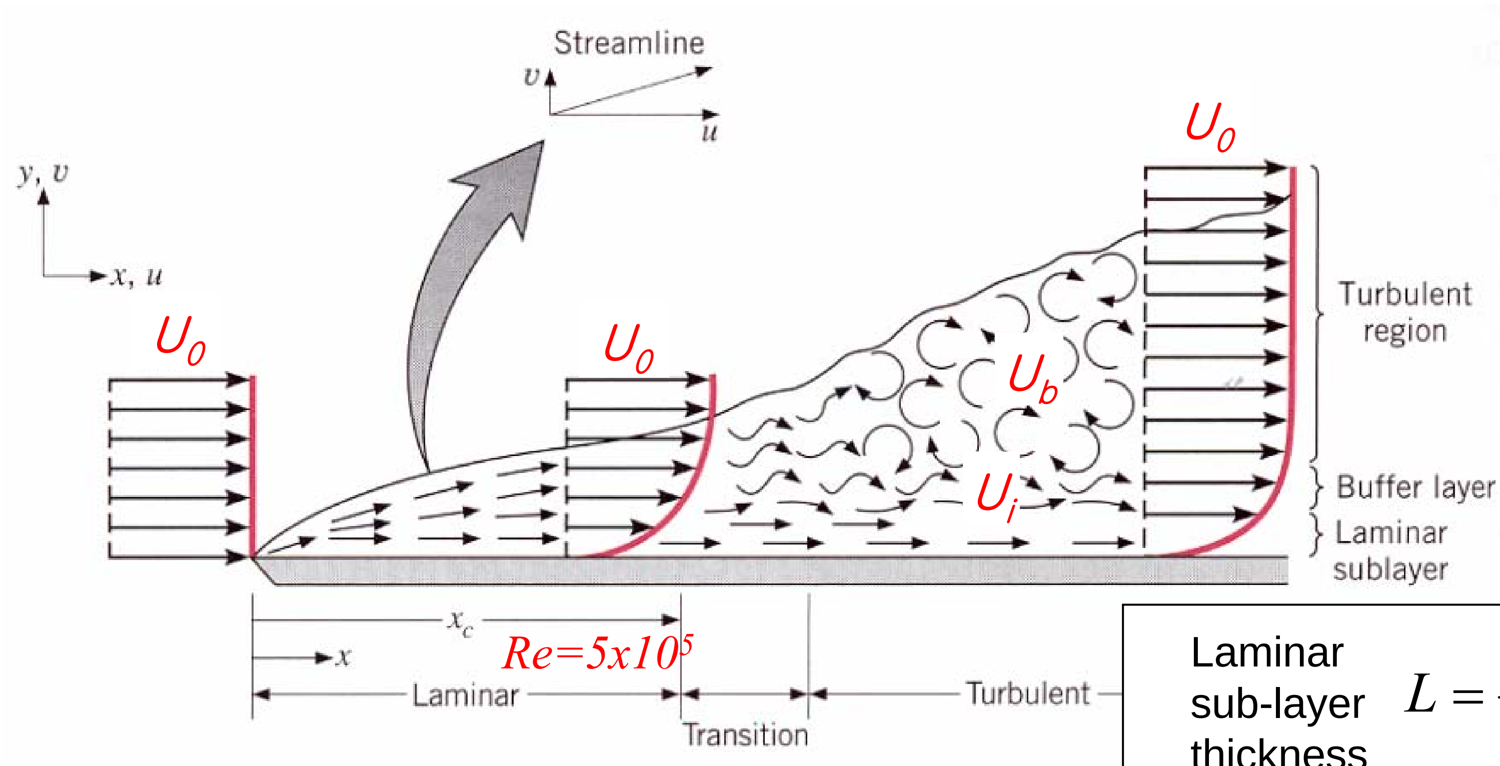
If two turbulent systems of the same fluid at the same bulk temperature are compared in heating and cooling experiment, the system being heated will have a higher temperature in the laminar sublayer than the system being cooled.

Consequently, if the fluid is a liquid, the sublayer of the heated system will be thinner, and the heat-transfer coefficient will therefore be higher than for the system being cooled. Since the Prandtl number is greater than 1 for most liquids, raising the exponent the above equation has the effect of raising *h*.

For most gases, the Prandtl number is near unity and approximately independent of temperature, so that the value of the exponent on Pr is of minor consequence. An understanding of this lack of effect for gases may be gained from the fact that viscosity and thermal conductivity increase at almost the same rate with temperature, so that the thermal resistance of the sublayer is approximately constant.

Heat Transfer with turbulent flow

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.3\ \text{or}\ 0.4}$$



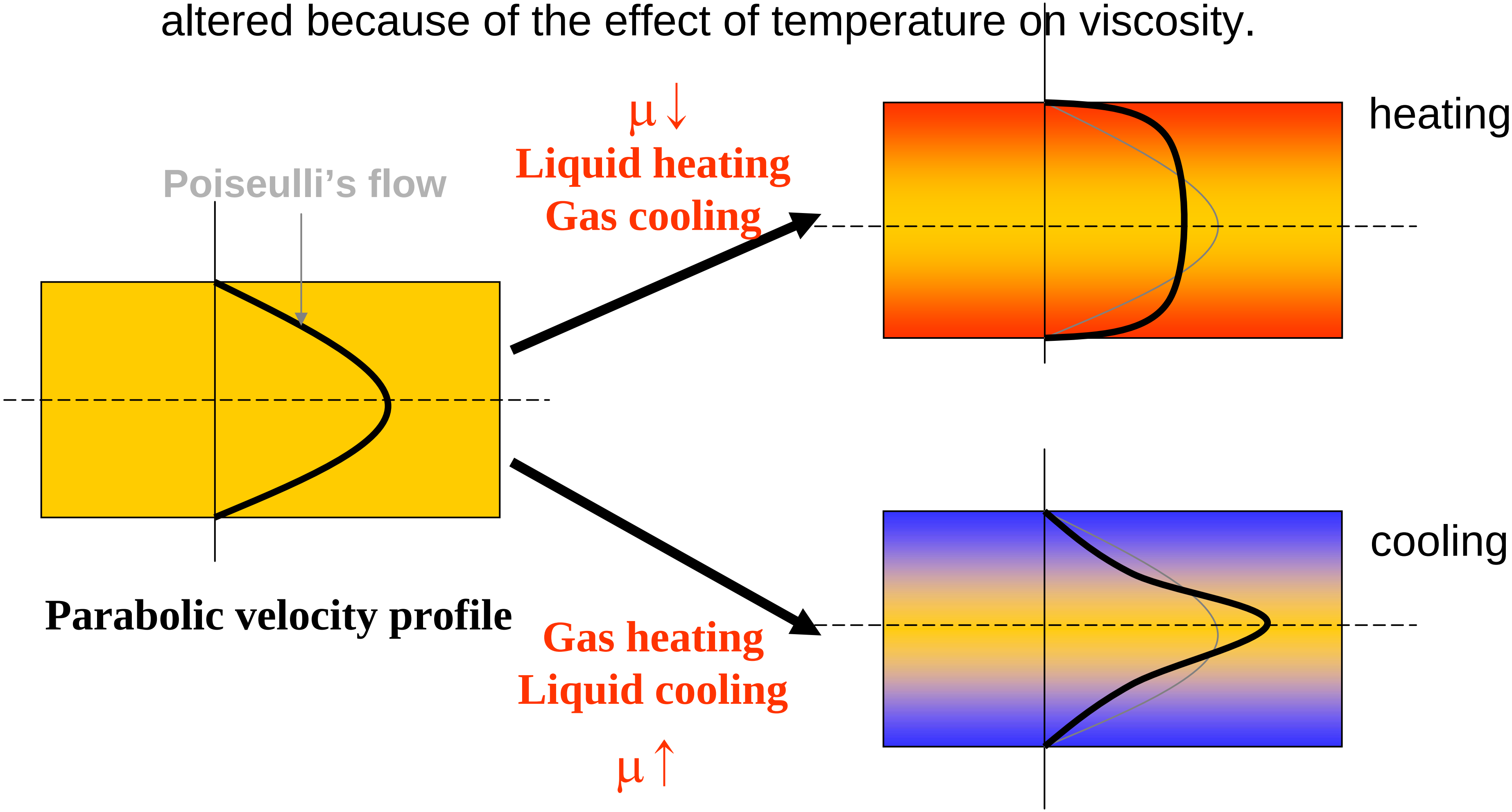
Laminar sub-layer thickness $\quad L = \dfrac{\mu u_i}{\tau_s g_c}$

Most liquid (Pr>1): T $\uparrow$, $\mu \downarrow$, $L \downarrow$ *and* $k \rightarrow h_x \uparrow$

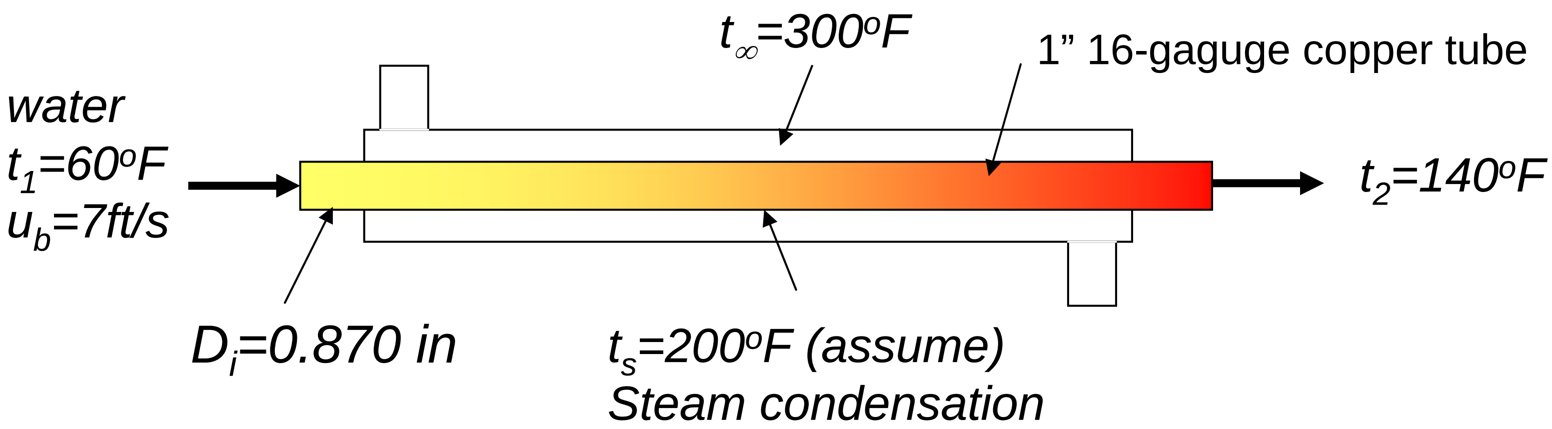
Most gas (Pr~1): T $\uparrow$, $\mu \uparrow$, $L \uparrow$ *but* $k \uparrow$, $h_x \rightarrow$

Influence of heating on velocity profile in laminar tube flow

If the fluid is heated or cooled, the velocity profile can be greatly altered because of the effect of temperature on viscosity.



Example 24-2: find *h* for the water



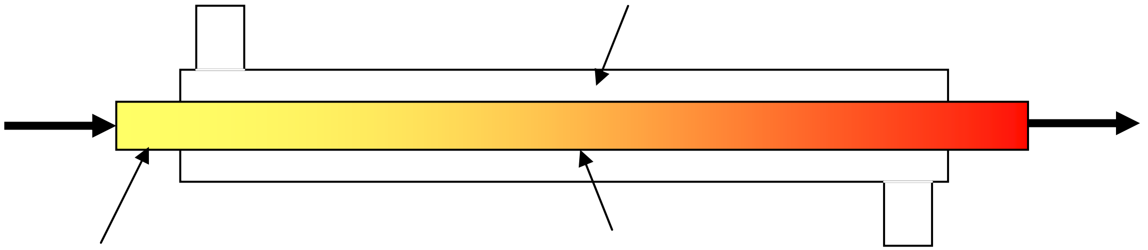
Arithmetic mean bulk temperature :

$$t_b = \frac{t_1 + t_2}{2} = \frac{60 + 140}{2} = 100^\circ F$$

Arithmetic mean of the wall and bulk-fluid temperatures

$$t_f = \frac{\frac{t_1 + t_s}{2} + \frac{t_2 + t_s}{2}}{2} = \frac{\frac{60 + 200}{2} + \frac{140 + 200}{2}}{2} = 150^\circ F$$

Example 24-2: find *h* for the water



	Arithmetic mean bulk temp	Arithmetic mean of the wall and bulk-fluid temperatures
	$at\ 100^\circ F \left(= \dfrac{60+140}{2} \right)$	$at\ 150^\circ F \left(= \dfrac{\frac{60+200}{2} + \frac{140+200}{2}}{2} \right)$
$\rho (lb/ft^3)$	62.0	61.2
$C_p (Btu/lb \cdot {}^\circ F)$	0.998	1.00
$\mu (lb/ft \cdot s)$	0.00458	0.000292
$k (Btu/h \cdot ft \cdot {}^\circ F)$	0.364	0.384

Example 24-2: find *h* for the water

100ºF

$$\mathrm{Re} = \frac{u_b D_i \rho}{\mu}$$

$$= \frac{(0.87/12)(7)(62.0)}{0.000458}$$

$$= 68{,}000$$

$$\mathrm{Pr} = \frac{C_p \mu}{k}$$

$$= \frac{(0.998)(0.000458)(3600)}{0.364}$$

$$= 4.53$$

150ºF

$$\mathrm{Re} = \frac{u_b D_i \rho}{\mu}$$

$$= \frac{(0.87/12)(7)(61.2)}{0.000292}$$

$$= 107{,}000$$

$$\mathrm{Pr} = \frac{C_p \mu}{k}$$

$$= \frac{(1.00)(0.000292)(3600)}{0.384}$$

$$= 2.73$$

Example 24-2: find *h* for the water

(a) Using the Dittus and Boelter equation (t_b)

heating

$$Nu = \frac{hD}{k} = 0.023\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.4} \qquad (24\text{-}4)$$

$$h = \frac{(0.023)(0.364)}{(0.870/12)}(68{,}600)^{0.8}(4.53)^{0.4}$$

$$= 1560\ Btu/(h)(ft^2)(^\circ F)$$

Example 24-2: find *h* for the water

(b) Using the Colburn equation (t_f)

$$\frac{h}{u_b \rho C_p}\left(\frac{C_p \mu}{k}\right)^{2/3} = 0.023\left(\frac{D u_b \rho}{\mu}\right)^{-0.2} \qquad (24\text{-}5)$$

$$h = \frac{(0.023)(7)(3600)(61.2)(1.0)}{(107{,}000)^{0.2}(2.73)^{2/3}}$$

$$= 1800\ Btu/(h)(ft^2)(^\circ F)$$

Example 24-2: find *h* for the water

(c) Using the Sieder and Tate equation (t_b)

$$\frac{h}{u_b \rho C_p}\left(\frac{C_p \mu}{k}\right)^{2/3}\left(\frac{\mu_s}{\mu}\right)^{0.14} = 0.023\left(\frac{D u_b \rho}{\mu}\right)^{-0.2} \qquad (24\text{-}6)$$

$$\frac{hD}{k} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.33}\left(\frac{\mu_s}{\mu}\right)^{0.14}$$

$$h = \frac{(0.023)(0.364)}{(0.870/12)}(68{,}600)^{0.80}(4.53)^{0.33}\left(\frac{0.000458}{0.000205}\right)^{0.14}$$

$$= 1580\ Btu/(h)(ft^2)(^\circ F)$$

Example 24-2: find *h* for the water

(d) Using the Empirical Method (@100^{o}F) (t_b)

$$\frac{h}{u_b \rho C_p} = \exp\left[-3.796 - 0.205 \ln Re - 0.505 \ln Pr - 0.0225 (\ln Pr)^2\right] \quad \textbf{(24-7)}$$

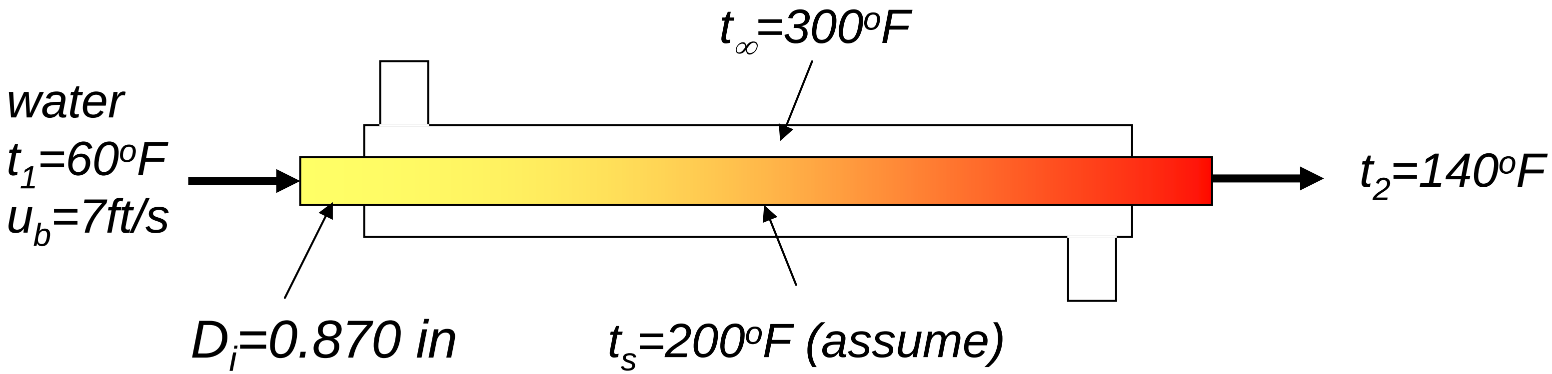
$$= \exp[-3.796 - 0.205 \ln 68{,}600 - 0.505 \ln 4.53 - 0.0225 (\ln 4.53)^2]$$

$$= \exp[-6.89]$$

$$h = (0.00105)(62.0)(7)(3600)(0.998)$$

$$= 1640 \quad Btu/(h)(ft^2)(^\circ F)$$

Example 24-2: find *h* for the water

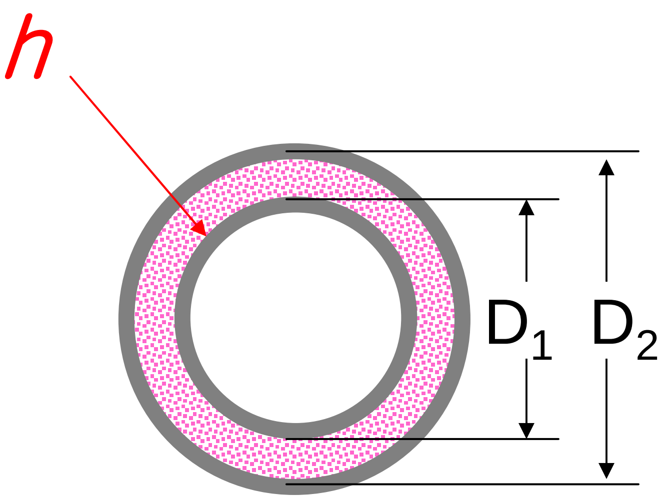


Equation	h (Btu/(h)(ft^2)(°F))
(a) the Dittus and Boelter equation (t_b)**(24-4)**	1560
(b) the Colburn equation (t_f)**(24-5)**	1800
(c) the Sieder and Tate equation (t_b)**(24-6)**	1580
(d) the Empirical Method (t_b)**(24-7)**	1640

Convection in Non-circular Conduits

Wiegand (heat transfer coefficient on the outer wall of inner pipe)

$$Nu = \frac{hD_{eq}}{k} = 0.023\left(\frac{D_{eq}u_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.4}\left(\frac{D_2}{D_1}\right)^{0.45} \quad \textbf{(24-8)}$$



Concentric pipes

$$D_{eq} = 4r_H \quad (14\text{-}4)$$

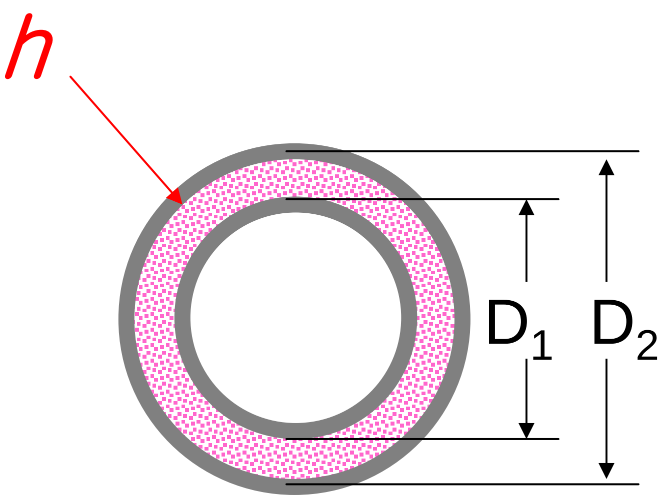
$$r_H = \frac{A}{l_p} = \frac{\pi\left(\frac{D_2^2}{4} - \frac{D_1^2}{4}\right)}{\pi(D_2 + D_1)} = \frac{D_2 - D_1}{4} \quad (14\text{-}5)$$

$$D_{eq} = D_2 - D_1$$

Convection in Non-circular Conduits

heat transfer coefficient on the inner wall of the outer pipe

$$\frac{h}{u_b \rho C_p}\left(\frac{C_p \mu}{k}\right)^{2/3}\left(\frac{\mu_s}{\mu}\right)^{0.14} = 0.023\left(\frac{D_{eq} u_b \rho}{\mu}\right)^{-0.2} \qquad \textbf{(24-6)}$$



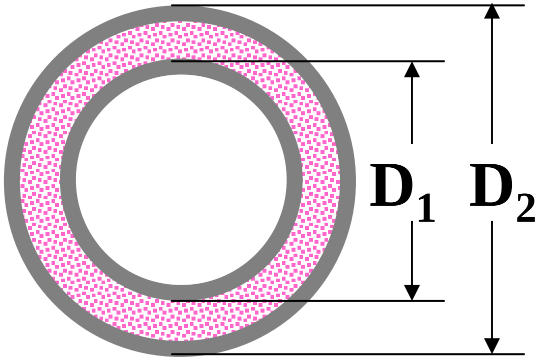
Concentric pipes

$$D_{eq} = 4r_H$$

$$r_H = \frac{A}{l_p} = \frac{\pi\left(\frac{D_2^2}{4} - \frac{D_1^2}{4}\right)}{\pi(D_2 + D_1)} = \frac{D_2 - D_1}{4}$$

$$D_{eq} = D_2 - D_1$$

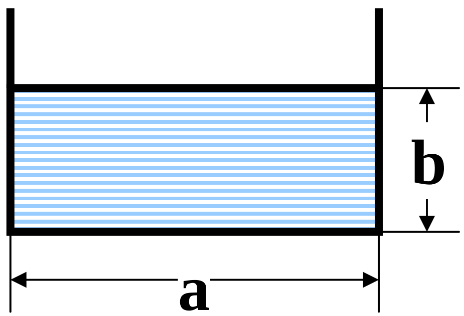
Convection in Non-circular Conduits (turbulent)



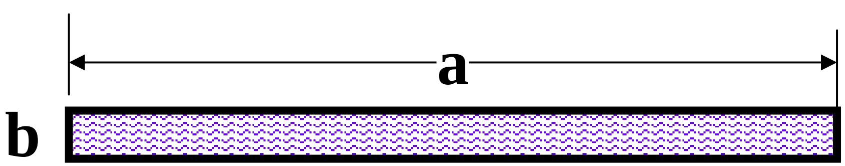
Concentric pipes

$$D_{eq} = 4r_H$$

$$r_H = \frac{A}{l_p} = \frac{\pi\left(\frac{D_2^2}{4} - \frac{D_1^2}{4}\right)}{\pi\left(D_2 + D_1\right)} = \frac{D_2 - D_1}{4}$$



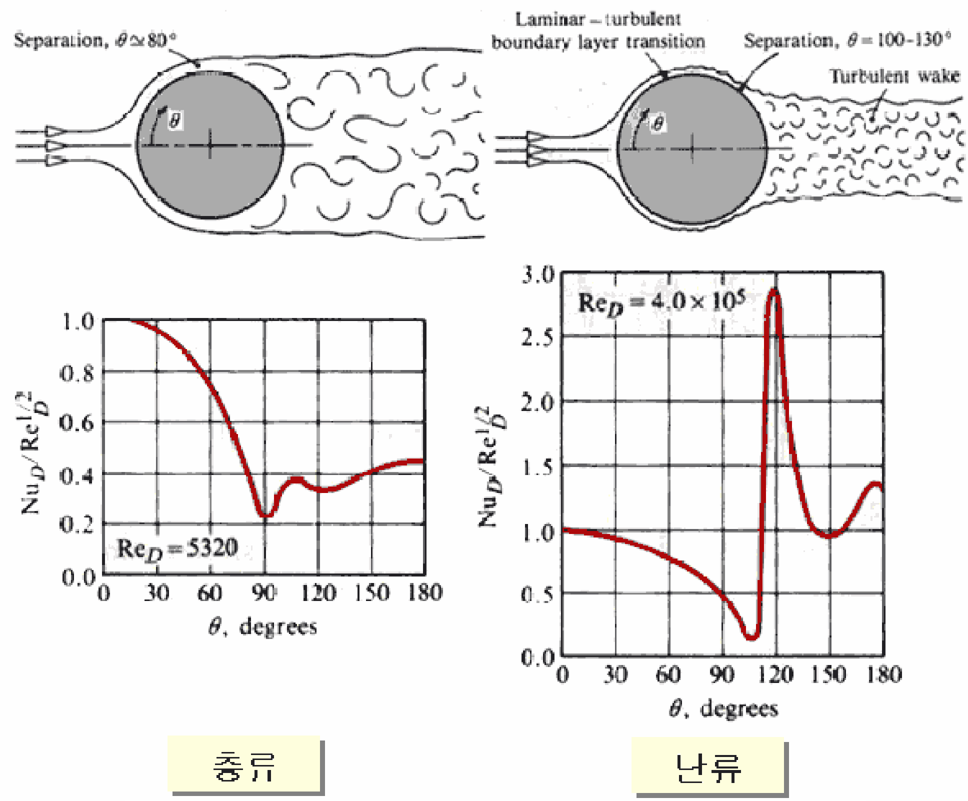
$$D_{eq} = \frac{4A}{l_p} = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$



b<<a

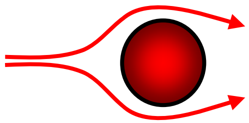
$$D_{eq} = \frac{4A}{l_p} = \frac{4ab}{2(a+b)} = \frac{2ab}{a+\cancel{b}} = 2b$$

실린더 주위에서의 열전달계수 분포



왼편 그림은 실린더 표면에서의 열전달계수를 원주방향에 따라 나타낸 것으로 θ=0°가 실린더의 맨 왼쪽을, θ=180°가 실린더의 맨 오른쪽을 나타낸다. 경계층의 박리(separation)가 θ~80°에서 일어나는 층류 경계층의 경우 박리 전에 열전달계수가 지속적으로 감소하다가 박리 이후 난동성분의 증가로 다소 증가하게 된다. 그러나 난류로의 천이가 일어나는 오른쪽 그림의 경우 천이로 인한 급속한 열전달계수 증가가 나타난다. 이는 평판 위를 지나는 유동의 경우에서도 나타났던 것이다. 그 이후 경계층의 박리에 의해 열전달계수는 다시 급속히 감소하게 된다.

Convection Normal to a Cylinder



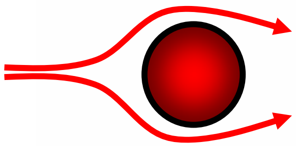
Average coefficients for the entire surface

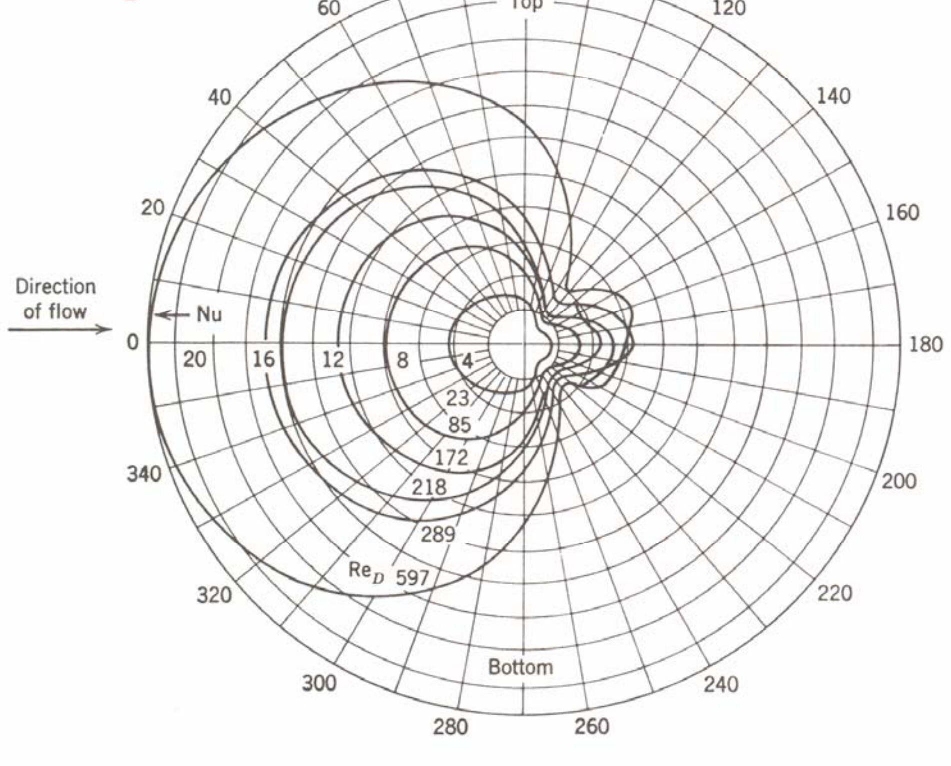
Gases: $$\frac{hD}{k} = b\left(\frac{Du_0\rho}{\mu}\right)^n$$ **(24-9)**

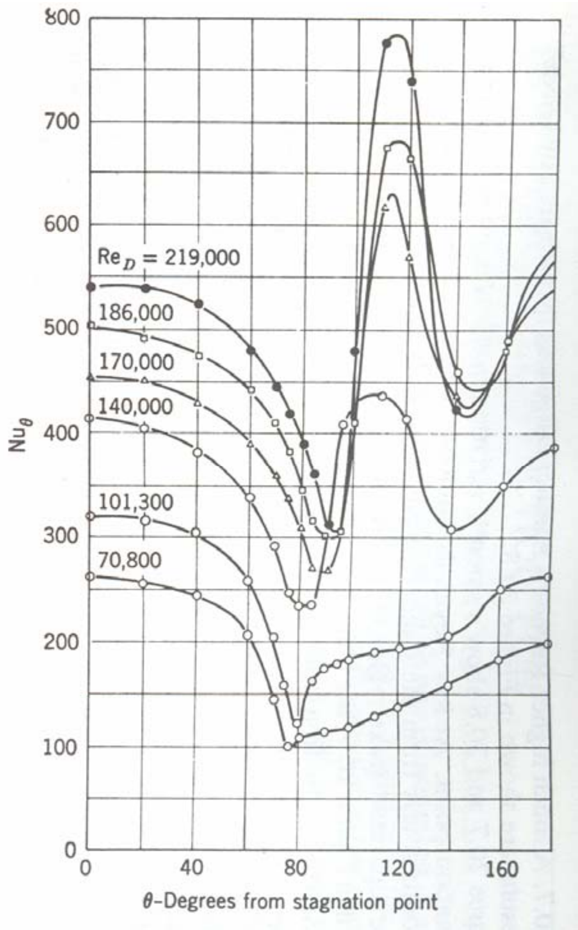
Liquids: $$\frac{hD}{k} = b\left(\frac{Du_0\rho}{\mu}\right)^n \left(1.1\,\mathrm{Pr}^{1/3}\right)$$

$Du_0\rho/\mu$	***n***	***b***
1-4	0.330	0.891
4-40	0.385	0.821
40-4000	0.466	0.615
4000-40,000	0.618	0.174
40,000-250,000	0.805	0.0239

Local number for cross-flow about a circular cylinder

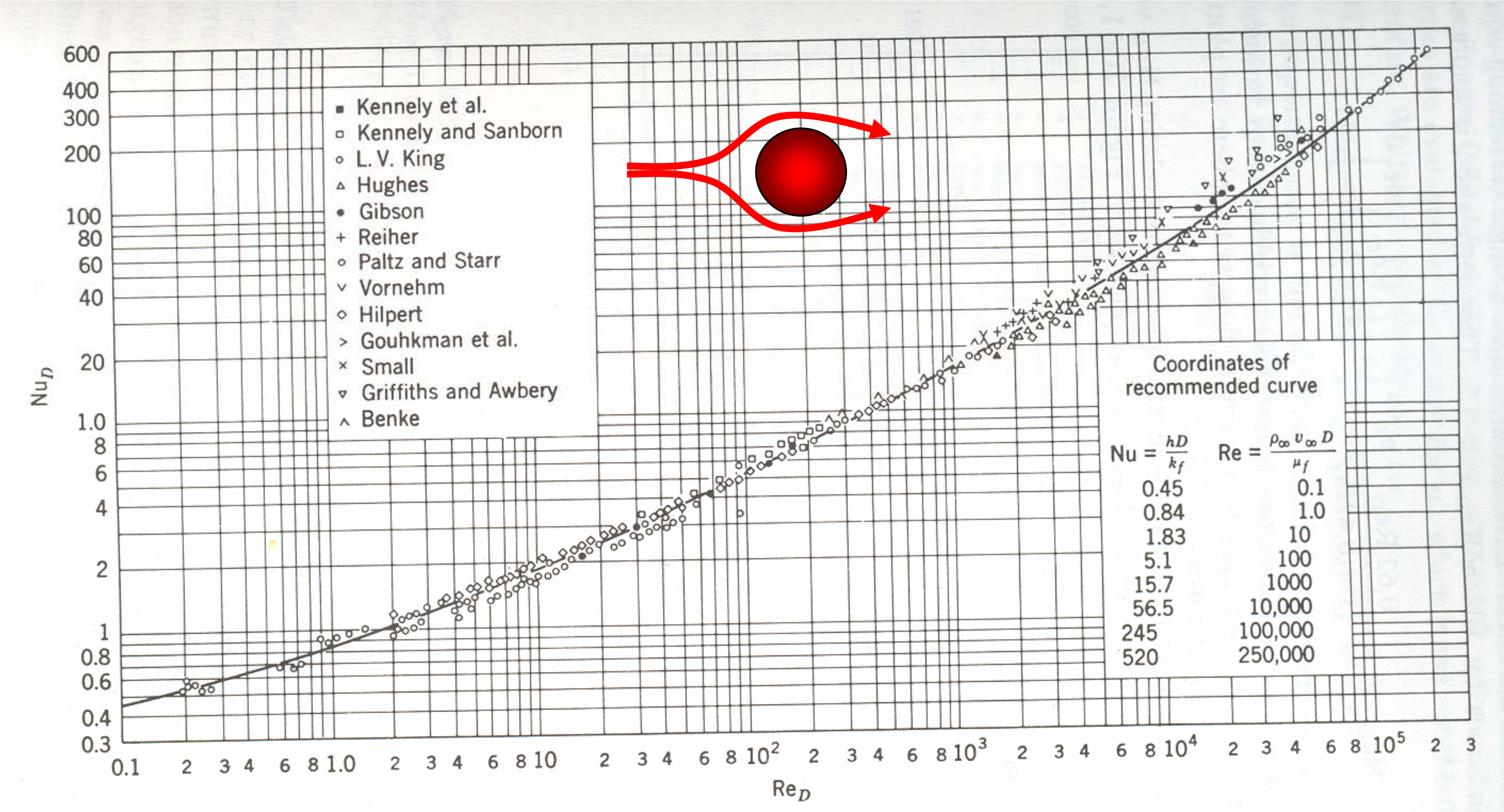






At low Reynolds numbers **At high Reynolds numbers**

Nu versus Re for flow normal to a single cylinder

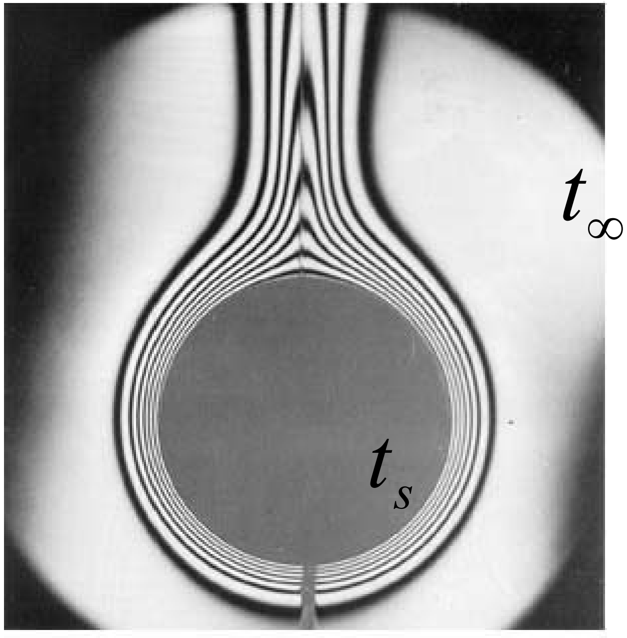


Natural convection from horizontal cylinder

A single horizontal cylinder

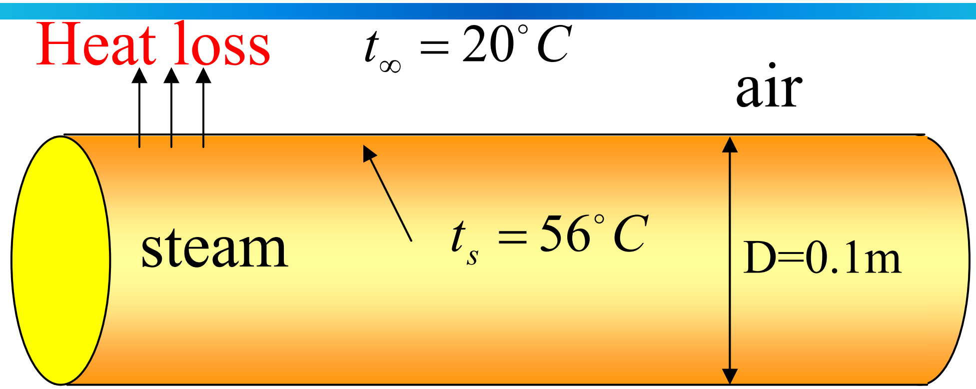
$10^3 < Gr \cdot Pr < 10^9$

$$\frac{hD}{k} = 0.53\left(\frac{D^3 \rho^2 g \beta \Delta t}{\mu^2}\right)^{1/4}\left(\frac{C_p \mu}{k}\right)^{1/4} \qquad \textbf{(24-10)}$$



$$t_f = \frac{t_s + t_\infty}{2}$$

Example 24-3: find *h* between the surface and air



$$t_f = \frac{t_s + t_\infty}{2} = \frac{56+20}{2} = 38^\circ C$$

$$k = 0.0266\ W/m \cdot K$$

$$\rho = 1.14\ kg/m^3$$

$$\beta = 0.00322\ K^{-1}$$

$$\mu = 1.92 \times 10^{-5}\ Pa \cdot s$$

$$C_p = 1000\ J/kg \cdot K$$

$$Gr = \frac{(0.1)^3(1.14)^2(9.8)(0.00322)(36)}{\left(1.92 \times 10^{-5}\right)^2}$$

$$= 4.00 \times 10^6$$

$$\Pr = \frac{(1000)(1.92 \times 10^{-5})}{0.0266}$$

$$= 0.722$$

$10^3 < \boldsymbol{PrGr} < 10^9$

(24-10)

$$\frac{hD}{k} = 0.53\left(\frac{D^3 \rho^2 g \beta \Delta t}{\mu^2}\right)^{1/4}\left(\frac{C_p \mu}{k}\right)^{1/4} \Rightarrow h = 5.81\ W/m^2 K$$

Convection Normal to a Bank of Circular Tubes

Grimison Equation (> 10 rows)

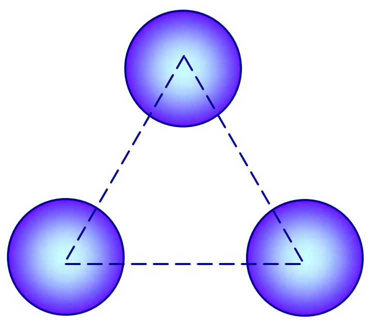
$$t_f = \frac{t_s + t_\infty}{2}$$

Gases: $$\frac{hD}{k} = b\left(\frac{DG_{max}}{\mu}\right)^n$$

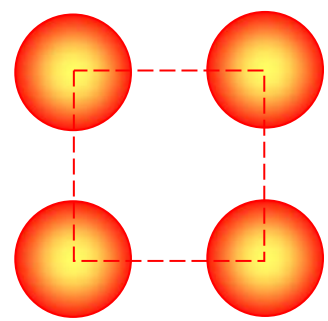
(24-11)

Liquids: $$\frac{hD}{k} = b(1.1)\left(\frac{DG_{max}}{\mu}\right)^n \mathrm{Pr}^{1/3}$$

$G_{max} = \rho U_{max}$ (velocity at the minimum cross section)



b=0.482
n=0.556



b=0.229
n=0.632

Convection Normal to a Bank of Circular Tubes

$$t_f = \frac{t_s + t_\infty}{2}$$

Grimison Equation (< 10 rows deep)

Gases: $$\frac{hD}{k} = b\left(\frac{DG_{max}}{\mu}\right)^n$$

(24-11)

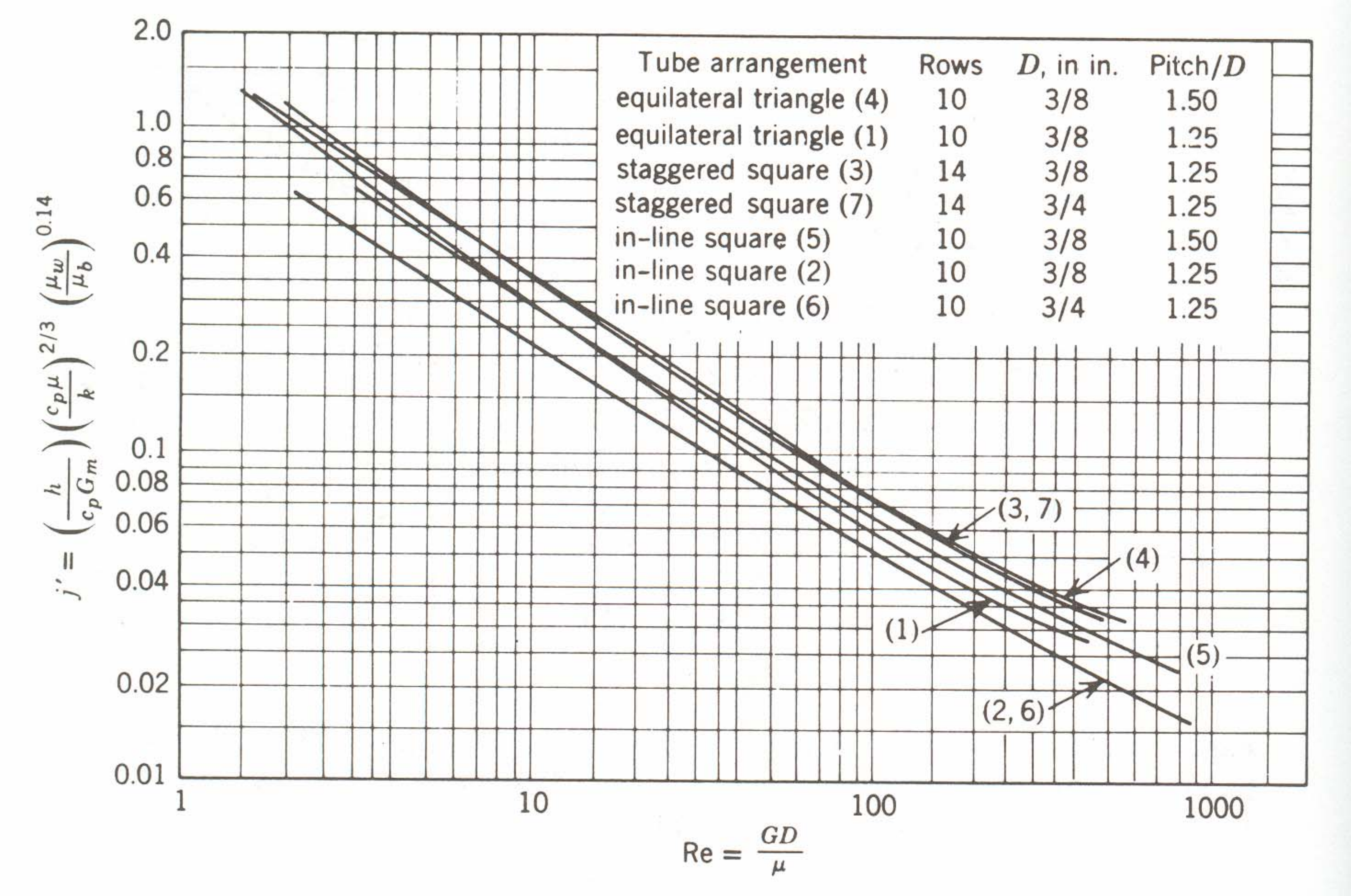
Liquids: $$\frac{hD}{k} = b(1.1)\left(\frac{DG_{max}}{\mu}\right)^n \mathrm{Pr}^{1/3}$$

The ratios given in Table 24-4

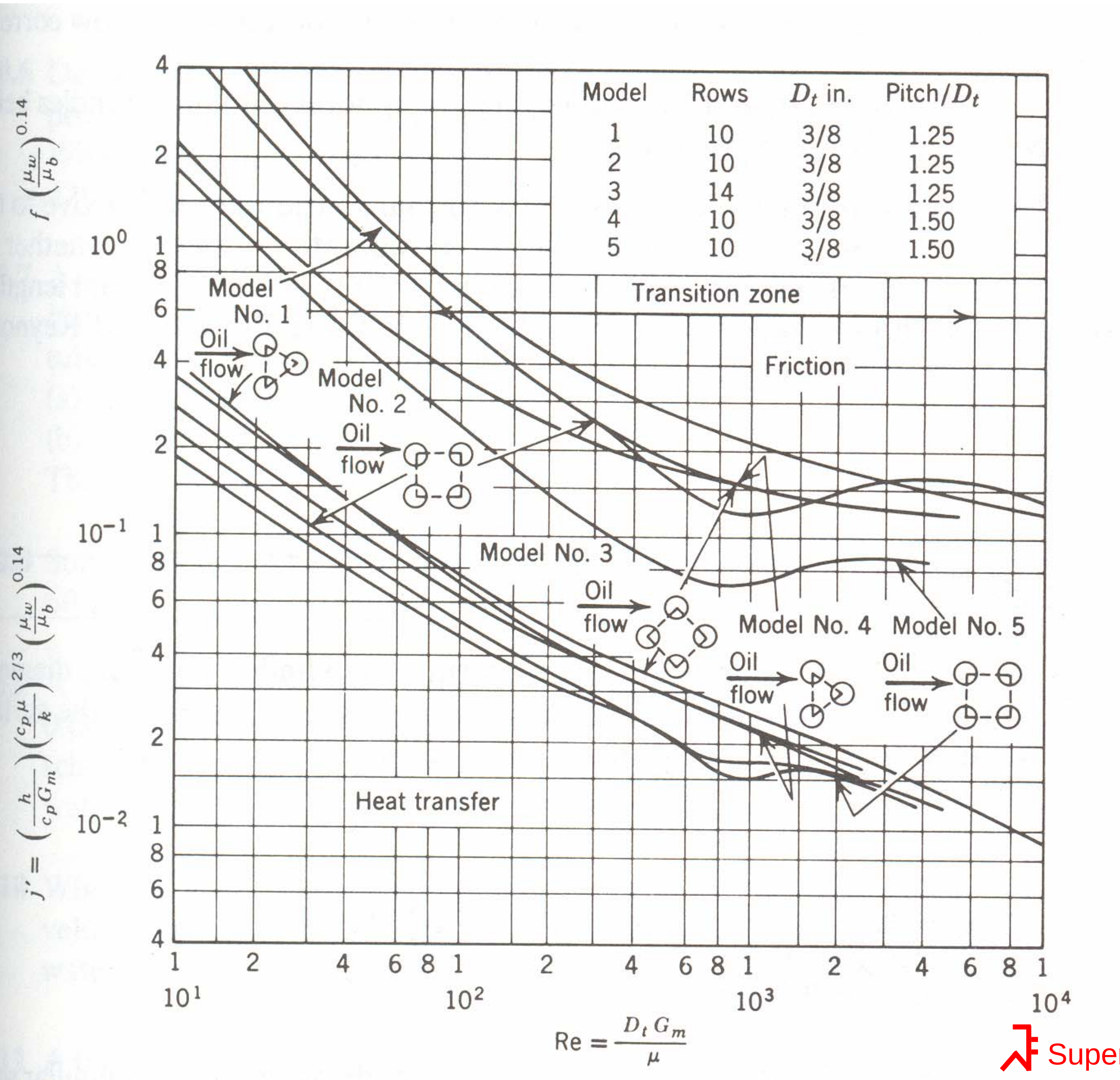
N	1	2	3	4	5	6	7	8	9	10
triangle	1	1.10	1.22	1.31	1.35	1.40	1.42	1.44	1.46	1.47
square	1	1.25	1.36	1.41	1.44	1.47	1.50	1.53	1.55	1.56

Convective heat transfer exchange

between liquids in laminar flow and tube bundles



Energy transfer and frictional loss for liquids in laminar and transition flow past tube bundles

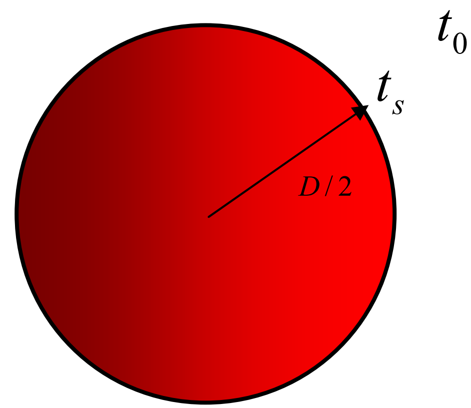


Convection from Spheres

Froessling Equation

$$Nu = \frac{hD}{k} = 2.0 + 0.6\left(\frac{C_p\mu}{k}\right)^{1/3}\left(\frac{Du_0\rho}{\mu}\right)^{1/2} \qquad \textbf{(24-12)}$$

Heat transfer by conduction in an infinite stagnant medium



Steady state conduction:

$$q = -k(4\pi r^2)\frac{dt}{dr} = const \qquad \textbf{(24-13)}$$

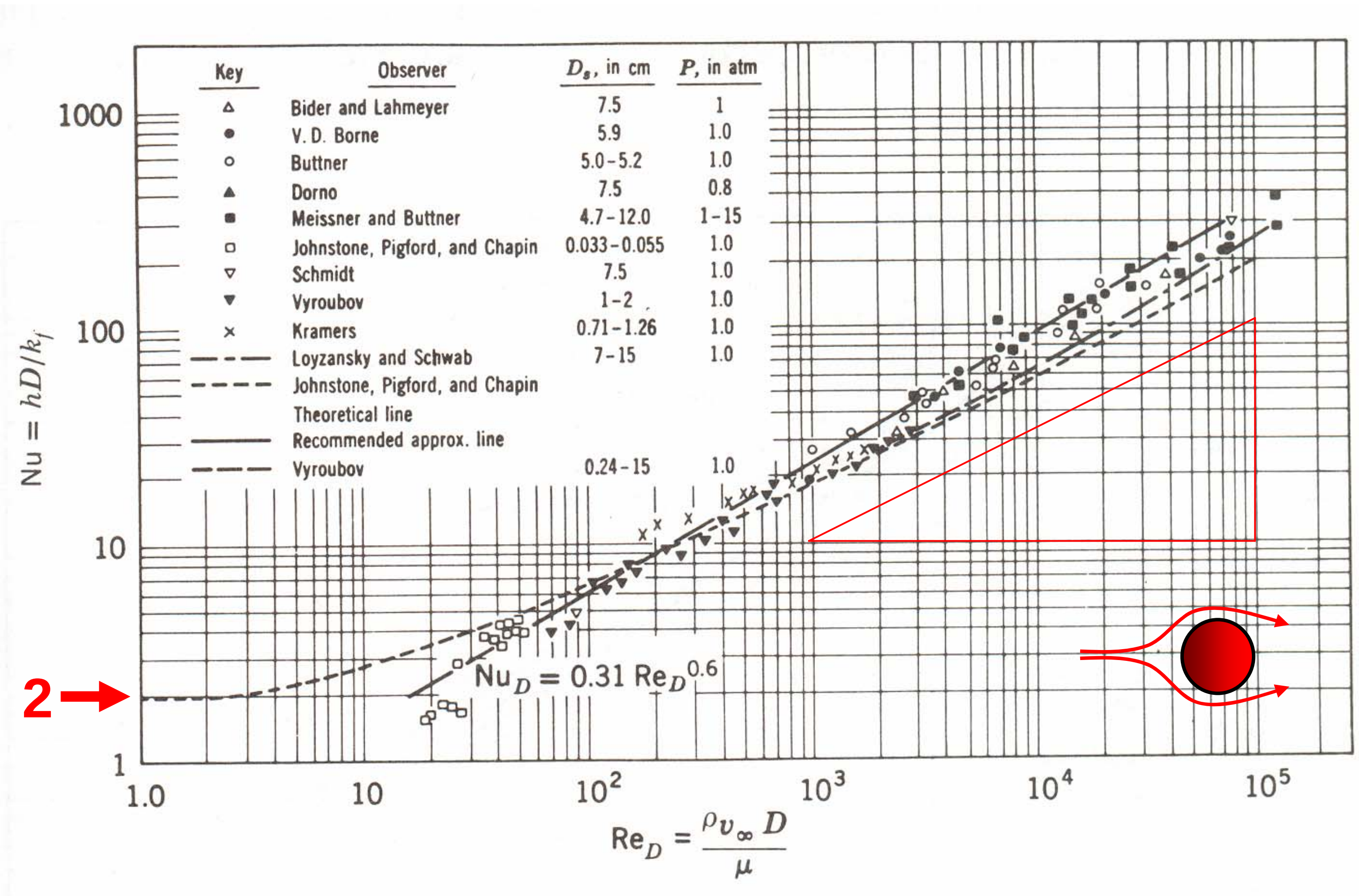
$$-\frac{q}{4\pi k}\int_{D/2}^{\infty}\frac{dr}{r^2} = \int_{t_s}^{t_0} dt \qquad \textbf{(24-14)}$$

$$-\frac{q}{4\pi k}\left(-\frac{1}{\infty} + \frac{1}{D/2}\right) = t_0 - t_s \qquad \textbf{(24-15)}$$

$$q = 2\pi Dk(t_s - t_0) = h(\pi D^2)(t_s - t_0) \qquad \textbf{(24-16) \& (24-17)}$$

$$\frac{hD}{k} = 2 \qquad \textbf{(24-18)}$$

Nu versus Re for air-flow past single sphere



Convective Heat Transfer between a Fluid and a Packed Bed

Bradshaw (1963) $400 < \boldsymbol{Re}_p < 10{,}000$

$$\boxed{j_H = \frac{h}{u_{bs}\rho C_p}\mathrm{Pr}^{2/3} = 2.50\left[\frac{Du_{bs}\rho}{\mu(1-\varepsilon)}\right]^{-1/2}} \qquad \textbf{(24-19)}$$

$$where \quad u_{bs} = \varepsilon u_b, \quad \varepsilon : \text{void fraction}$$

$$\mathrm{Re}_p = \frac{Du_{bs}\rho}{\mu(1-\varepsilon)} \qquad (14\text{-}20)$$

$$u_{bs} = \varepsilon u_b$$

U_b = Average interstitial velocity at any cross section in the bed
U_{bs} = Superficial velocity

Convection from a Plane Surface

Natural convection

Ideal gas heated by laminar Natural convection from vertical plate

$$Nu_m = 0.478 Gr^{1/4} \quad \textbf{(22-25)}$$

McAdams Eq. Laminar natural convection

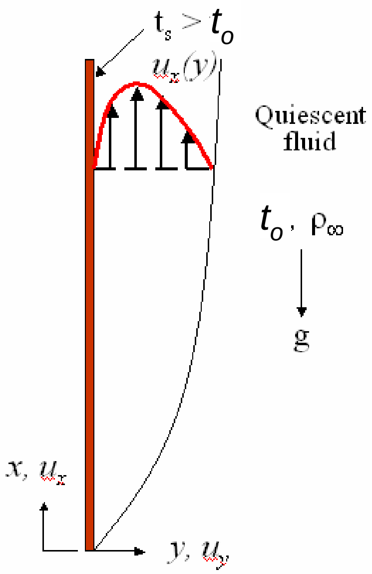
(24-20)

$$Nu_m = 0.59 Gr^{1/4} \Pr^{1/4} = 0.59 Ra^{1/4} \qquad 10^4 < Ra < 10^9$$

Combination of Laminar+Turbulent

$$Nu_m = 0.13 Gr^{1/3} \Pr^{1/3} \qquad Ra > 10^9$$

(24-21)



The fluid properties are evaluated at the arithmetic average temperature $(t_s+t_o)/2$

Determination of heat loss to the atmosphere by natural convection of air

The simplified forms of Eqs. (24-20) and (24-21) in which values of the physical properties of air at typical ambient conditions have already been substituted.

J.H. Perry (air, water, organic liq.)

All surface: $h = 0.18(\Delta t)^{1/3} \quad Ra > 10^9$

Horizontal cylinder: $h = 0.50(\Delta t / D_0')^{1/4} \quad 10^3 < Ra < 10^9$

$[in]$

Vertical plate: $h = 0.28(\Delta t / L)^{1/4} \quad 10^3 < Ra < 10^9$

$[ft]$

Heated horizontal plates facing upward: $h = 0.38(\Delta t)^{1/4} \quad 10^5 < Ra < 10^7$

Heated horizontal plates facing downward: $h = 0.20(\Delta t)^{1/4} \quad 10^5 < Ra < 10^7$

Heat Transfer to Liquid Metals

(removing heat from nuclear reactors)

Liquid metal (same μ, high k) : Pr<<1, $\delta_{th}>>\delta$

For circular pipe, uniform heat flux, L/D>60, Pe>100

Lyon Equation

$$Nu = 7.0 + 0.025\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.8} \qquad \textbf{(24-27)}$$

For circular pipe, const wall temp, L/D>60, Pe>100

Seban and Shimazaki Equation

$$Nu = 5.0 + 0.025\left(\frac{Du_b\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.8}$$

Heat Transfer to Liquid Metals for Flat Plate

Grosh and Cess

$$u_0 \frac{\partial t}{\partial x} = \alpha \frac{\partial^2 t}{\partial y^2} \quad \Rightarrow \quad \frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial y^2}; \quad \left(\theta = \frac{x}{u_0} \right)$$

$$\frac{t_s - t}{t_s - t_0} = erf \frac{y}{\sqrt{4\alpha x / u_0}} = \frac{2}{\sqrt{\pi}} \frac{y}{\sqrt{4\alpha x / u_0}} = y \sqrt{\frac{u_0}{\pi \alpha x}}$$

$$h_x = k \left\{ \frac{d(t_s - t)/(t_s - t_0)}{dy} \right\} = k \sqrt{\frac{u_0}{\pi \alpha x}}$$

$$\frac{h_x x}{k} = 0.564 \left(\frac{x u_0 \rho}{\mu} \right)^{1/2} \left(\frac{C_p \mu}{k} \right)^{1/2}$$

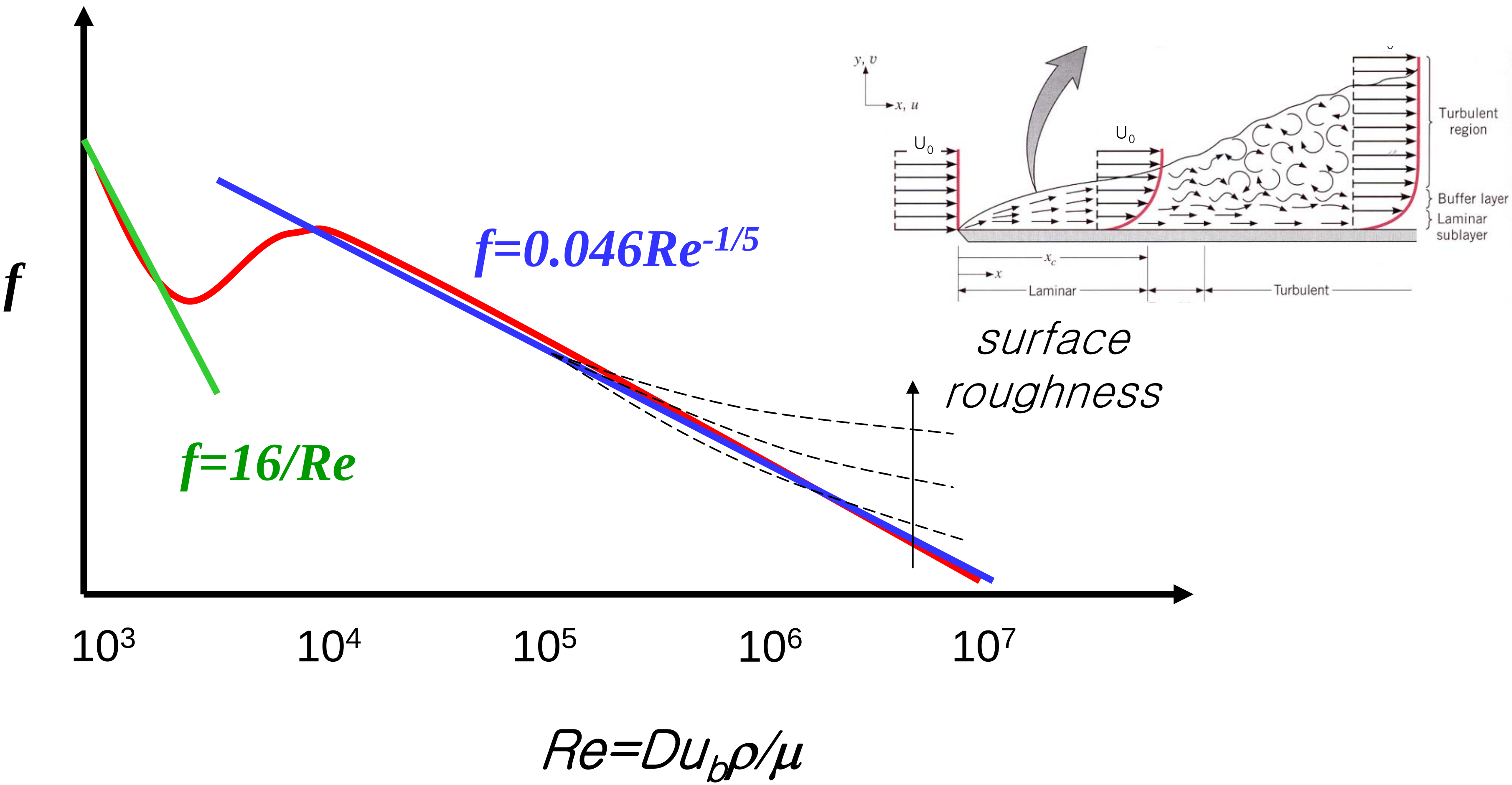
$$erf\ n = \frac{2}{\sqrt{\pi}} \left(n - \frac{n^3}{3 \cdot 1!} + \frac{n^5}{5 \cdot 2!} - \frac{n^7}{7 \cdot 3!} + \cdots \right)$$

$$\sim \frac{2n}{\sqrt{\pi}} \quad \text{when n is small}$$

$$\boxed{Nu_x = \sqrt{\frac{\mathrm{Re}_x \Pr}{\pi}}}$$

0.005<**Pr**<0.025

Effect of Surface Roughness on Heat-transfer Coefficient



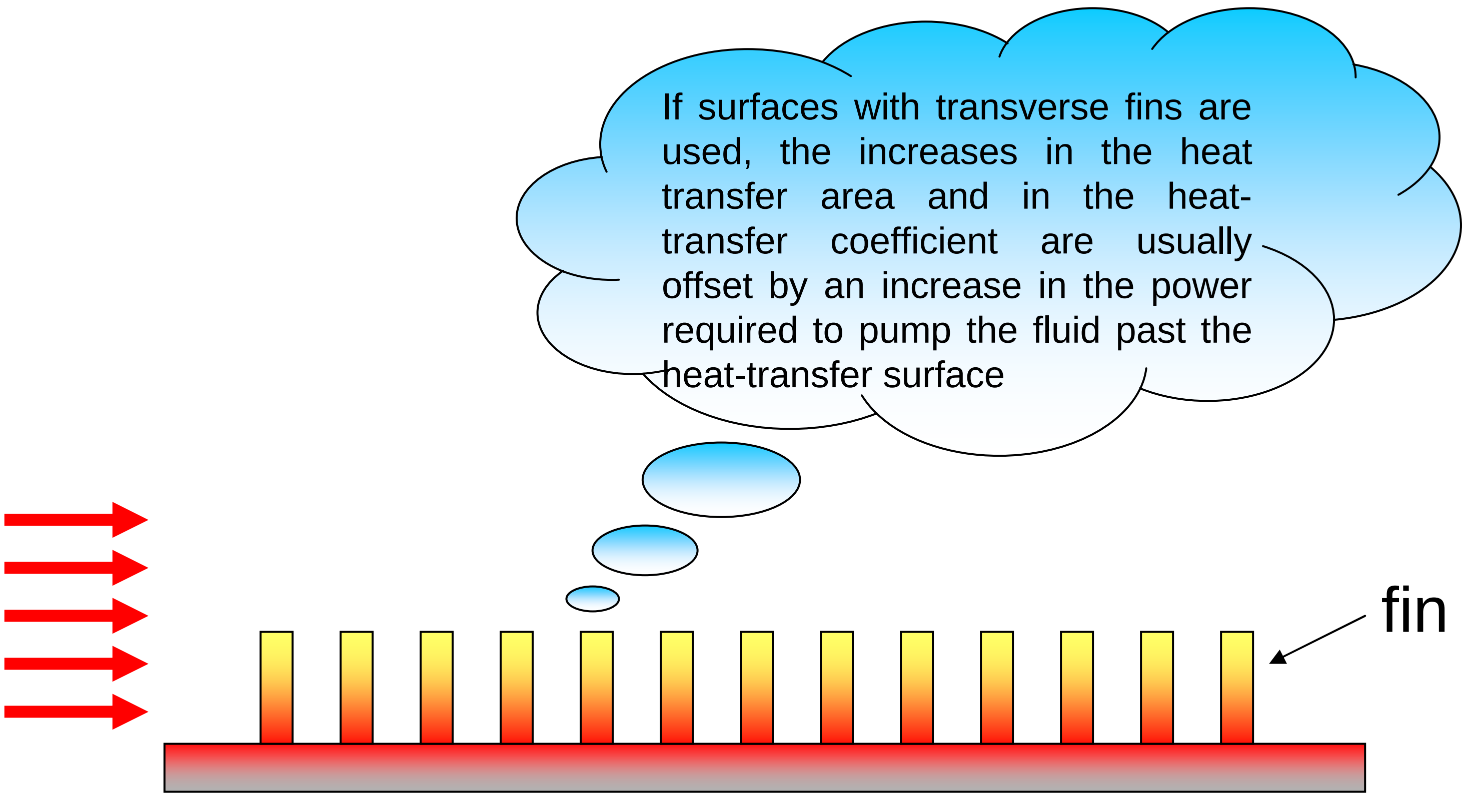
Protuberance < laminar sub-layer ⇨ No enhancement of *h*

Effect of Surface Roughness on Heat-transfer Coefficient

Protuberance > laminar sub-layer

→ Enhancement of *h*

→ Loss of energy due to friction

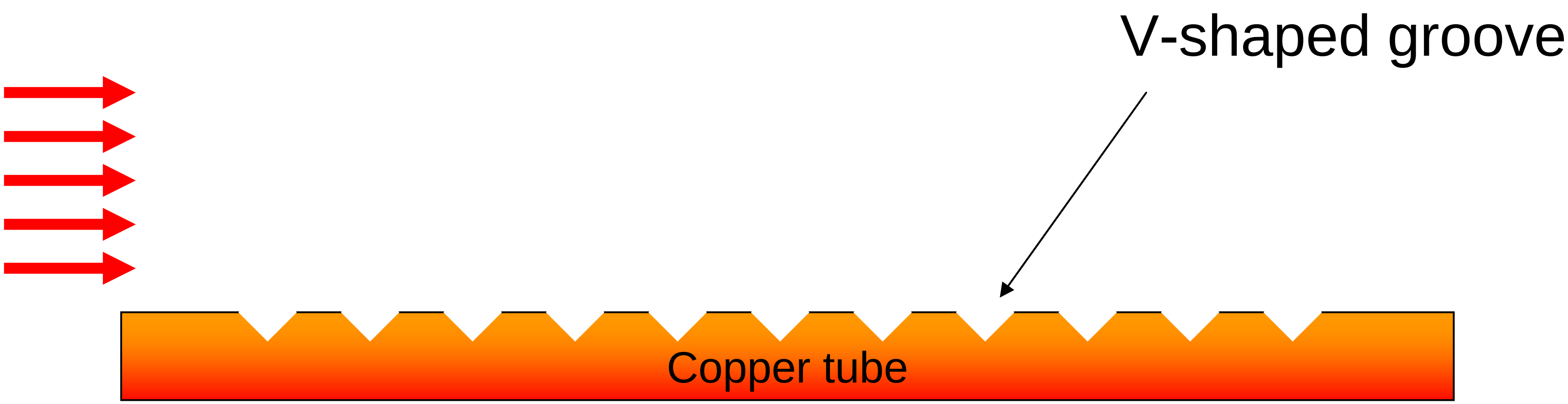


Effect of Surface Roughness on Heat-transfer Coefficient

Brouillette

- Heat transfer coefficient become double
- friction factor increase fourfold.

-Form drag is present !



Roughness ratio
= protuberance height/tubing diameter
=0.05

Homework

24-1

24-4

24-6

24-14