Nonlinear pricing

Chapter 3. Models and Data Source

The demand function

- When a uniform price is used, the major purpose of data analysis is ultimately to predict the total quantity D(p) that would be sold at each price p

- Elasticity

$$\bar{\eta}(p) = -\frac{p}{\bar{D}(p)}\frac{d\bar{D}}{dp}(p)$$

The demand profile

- Nonlinear price schedule charges a possibly different price for each increment in the customer's purchase size

- N(p, q) - (Demand Profile) the number of customers purchasing the q –th unit at the marginal price p

- Elasticity

$$\eta(p,q) \equiv -\frac{p}{N}\frac{\partial N}{\partial p}$$

The demand profile

- The total demand in response to a uniform price p

$$\bar{D}(p) = N(p,\delta)\delta + N(p,2\delta)\delta + \cdots, \\ = \sum_{k=1}^{\infty} N(p,k\delta)\delta\,,$$

$$\bar{D}(p) = \int_0^{\infty} N(p,q)\,dq$$

  - δ is the size of an increment and the index k indicates the k -th increment purchased

- Total demand

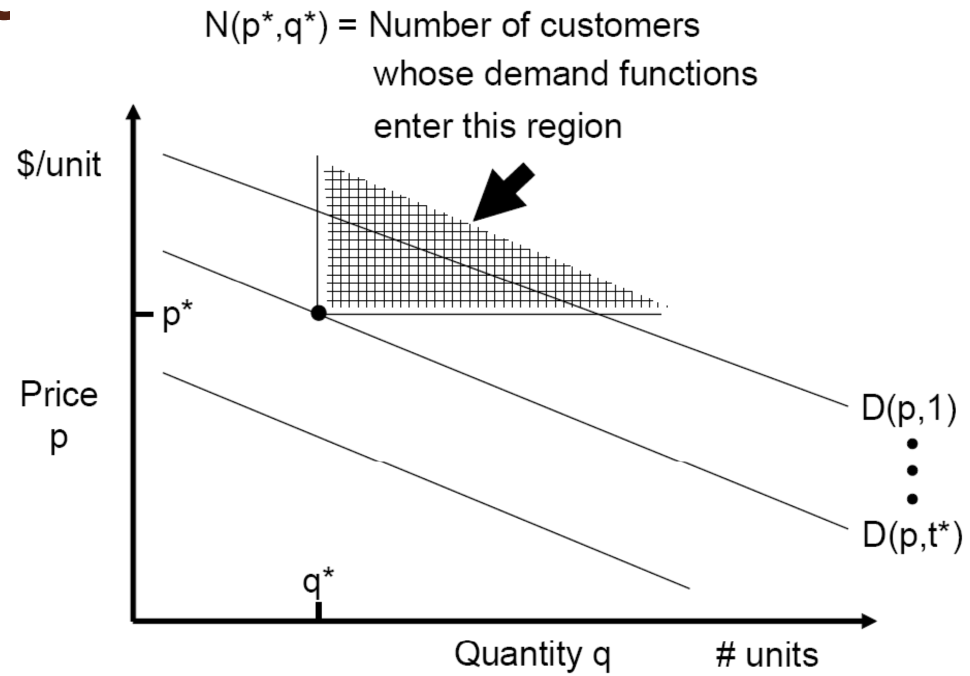
$$\sum_{k=1}^{\infty} N(p(k\delta),k\delta)\delta \quad \text{or} \quad \int_0^{\infty} N(p(q),q)\,dq$$

Interpretations of the demand profile

1) For each price p the demand profile specifies the number or fraction N(p, q) of customers purchasing at least q units.
   - If p is fixed and q is increased then a graph of N(p, q) depicts the declining number of customers purchasing each successive q -th unit. This is the same as the number of customers purchasing at least q units, so for each fixed price p the demand profile N(p, q) is the right-cumulative distribution function of the customers' purchase sizes

2) The demand profile specifies for each q -th unit the number or fraction N(p, q) of customers willing to pay the price p for that unit.
   - U(q, t) - Benefit function,
   - $v(q,t) \equiv \partial U(q,t)/\partial q$ - Marginal benefit
   - The demand profile N(p, q) measures the estimated number of customers whose types t are such that $v(q,t) \geq p$, indicating that they are willing to pay at least p for the q -th unit.

Interpretations of the demand profile



- The number of customers purchasing at least q* units at the price p* , and the number willing to pay at least p* for the q* -th unit, are the same
  - both measured as the number of customers whose demand functions intersect the shaded region of price-quantity pairs (p, q) ≥ (p*; q*)

Interpretations of the demand profile

- Sizes q for each uniform price p

$$N(p,q) = \#\left\{t \mid D(p,t) \geq q\right\}$$

$D(p,t)$ - demand function of customers of type t

- Distribution of marginal valuations for each unit q

$$N(p,q) = \#\left\{t \mid v(q,t) \geq p\right\}$$

Estimation of the demand profile

1. Direct measurement of the demand profile

- Firm has used several uniform prices in the past.
- Firm has had the foresight to record for each price the distribution of customers' purchase sizes over a standard billing period
- Demand profile

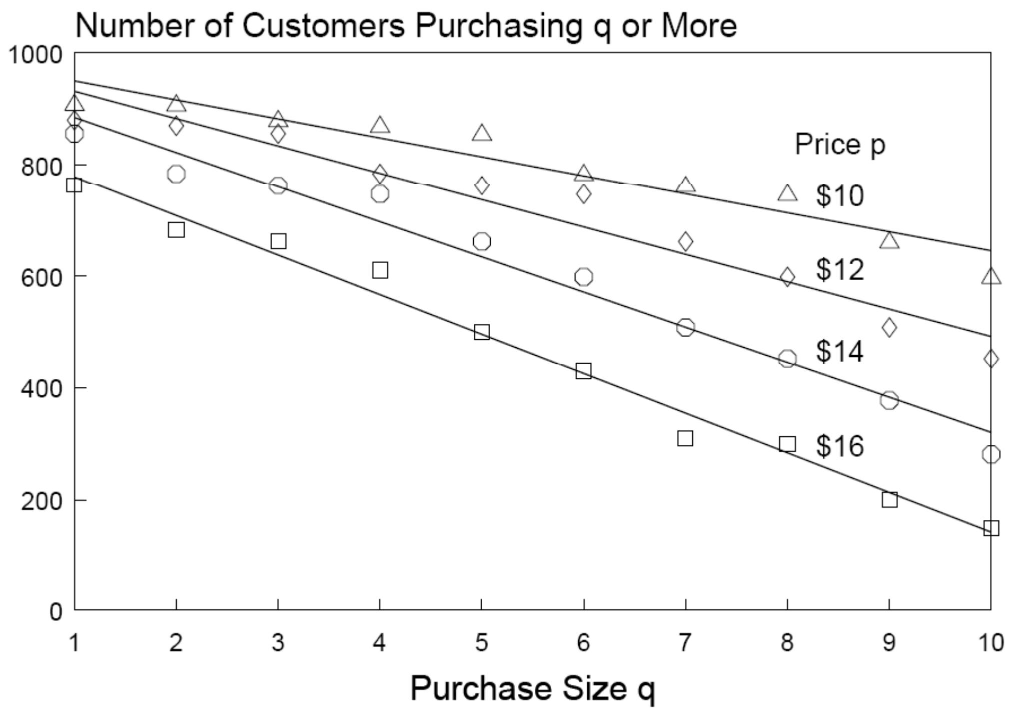
$$N(p_j, q_k) = \sum_{\ell \geq k} n_{j\ell}$$

  - j - index that distinguishes the different prices (cents per minute per mile in the case of a telephone company)
  - k - distinguishes several volume bands of total usage per billing period

- This estimate is insufficient because it does not cover all the possible prices and quantities that might be relevant for rate design.

Estimation of the demand profile

1. Direct measurement of the demand profile



• Useful step is to compare the two estimates of the demand profile based on its two interpretations

$$N(p_j, q_k) = \sum_{h \geq j} n_{hk}$$

Estimation of the demand profile

2. Indirect measurement of the demand profile

- Indirect approximation of the demand profile derived from estimation of customers' demand functions or benefit functions
- Example
  - Customer's benefit function

$$U(q,t) = t_1 q - \tfrac{1}{2} t_2 q^2$$

(Type parameters T = (t_1,t_2))

  - Marginal benefit function

$$v(q,t) = t_1 - t_2 q$$

  - $p = v(q,t)$ → q = $\frac{1}{t_2}[t_1 - p]$

  - Predicted demand function

$$D(p,t) = \frac{1}{t_2}[t_1 - p]$$

Estimation of the demand profile

2. Indirect measurement of the demand profile

- Converting this model into an Estimate of the demand profile requires an auxiliary datum, which is the distribution of types in the population of potential customers

$$N(p,q) = \int_{T(p,q)} f(t)\,dt\,, \qquad \text{or} \qquad N(p,q) = \int_{T(p,q)} dF(t)$$

  - f(t) – density function
  - F(t) – number of potential customers with type parameters not exceeding t
  - T(p, q) - set of types t for which $v(q,t) \geq p$ or $D(p,t) \geq q$

- Estimate the distribution of types in the population of potential customers
  - First distribution function F(t; β) that depends on a list β of parameters → demand profile N(p; q; β) depends on these coefficients.
  - Estimate the coefficients using data

Welfare considerations

1. Surplus measurement via parameterized models

- Producer's surplus

$$\text{Producer's Surplus} \equiv \int_a^b \left[P(q(t)) - C(q(t)) \right] \, dF(t)$$

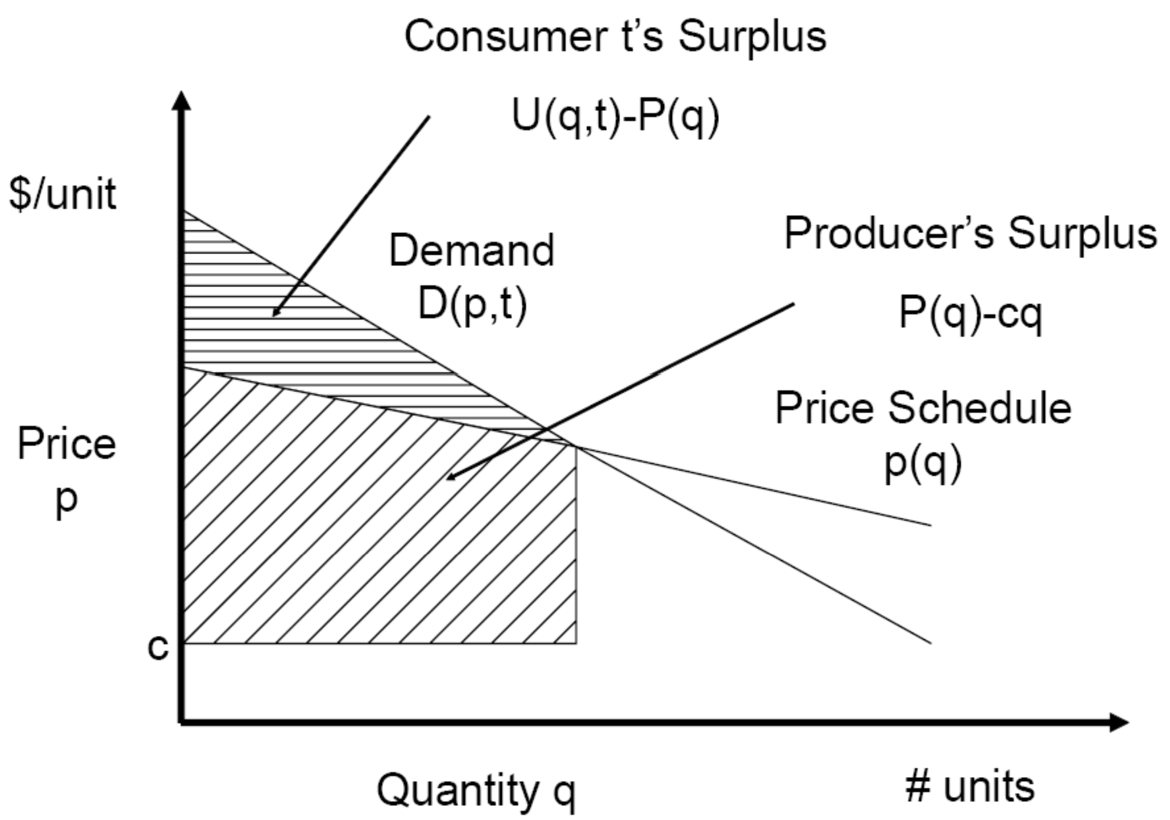
  - q(t) - purchase selected by type t
  - P(q(t)) - customer pays the tariff
  - C(q(t)) - cost incurred by the firm
  - a≤ t ≤b - range of type parameters in the population

- Consumer's surplus

$$\text{Consumers' Surplus} \equiv \int_a^b \left[U(q(t), t) - P(q(t)) \right] \, dF(t)$$

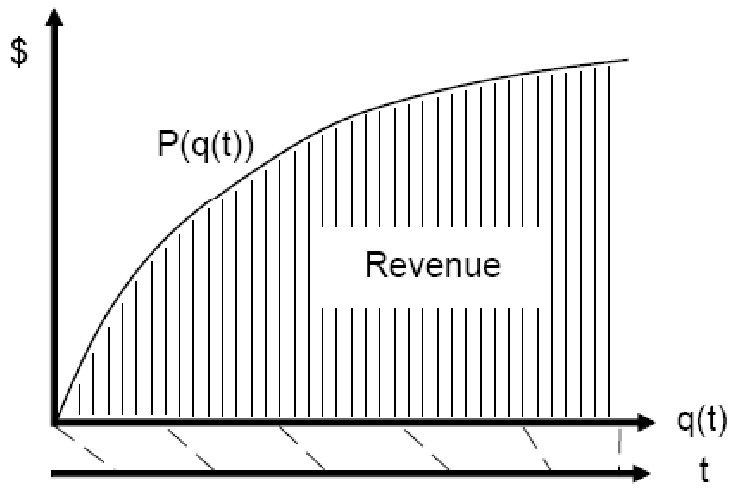
Welfare considerations

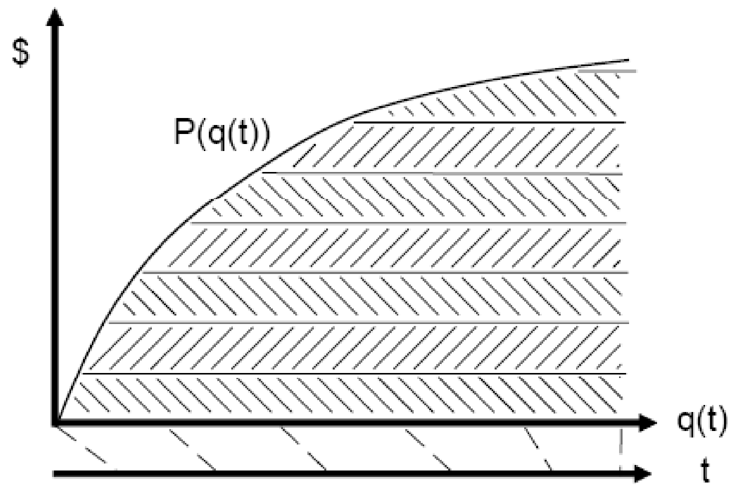
1. Surplus measurement via parameterized models



Welfare considerations

2. Surplus measurement via the demand profile





- The right diagram represents this revenue as the marginal price charged for each unit times the number of customers purchasing this unit, as represented by the corresponding horizontal slice of the area representing the revenue
- The number of q -th units purchased is the number of customers purchasing at least q units

Welfare considerations

2. Surplus measurement via the demand profile

- Interpreting the demand profile as the right-cumulative distribution function of reservation prices(p), the associated number or density of customers with the reservation price p for the q -th unit is therefore -∂N(p; q)/ ∂p

- Total consumer's surplus from purchases of q-th unit

$$\int_{p(q)}^{\infty} [p - p(q)][-\partial N(p, q)/\partial p]\, dp \equiv \int_{p(q)}^{\infty} [p - p(q)]\, d[-N(p, q)]$$
$$= \int_{p(q)}^{\infty} N(p, q)\, dp\,.$$

- Total consumer's surplus

$$\int_{0}^{\infty} \int_{p(q)}^{\infty} N(p, q)\, dp\, dq$$

Cautions and Caveats

- Customers' benefits and demand behaviors are assumed to be exogenous; that is, their behaviors are unaffected by the introduction of nonlinear pricing

  → appreciable quantity discounts for large purchases may induce some customers to alter end-uses and their investments in appliances and production technologies

- Customers' benefits are assumed to be denominated in money terms.

  → A more general approach allows income effects, risk aversion, impatience or discounting of delayed benefits, and other behavioral parameters

- The exposition of parameterized models assumes that the firm knows or can estimate the distribution of types in the population

  → this distribution is usually variable and at any one time the firm can usually only estimate the underlying probabilistic process by which customers' types are created or evolve over time

Cautions and Caveats

- Customers are assumed to obtain benefits directly from consumption of the product

  → In fact, many customers are other firms who are intermediaries in production or distribution

- The firm is assumed to be a monopoly

  → Estimates of the demand profile and customers' benefits are always conditioned on the prices prevailing for other products, especially those that are close substitutes or complements