MULTIDIMENSIONAL PRICING

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Multidimensional formulation

- Multidimensional product

  single product + several quality attributes
- Multiple products of the sort (§13)

- Assignment form
- Profile form

Assignment form

- m quality attributes

  $(q_1, \ldots, q_m)$ of magnitudes of m quality attributes
- Postal context (regular, airmail and express)

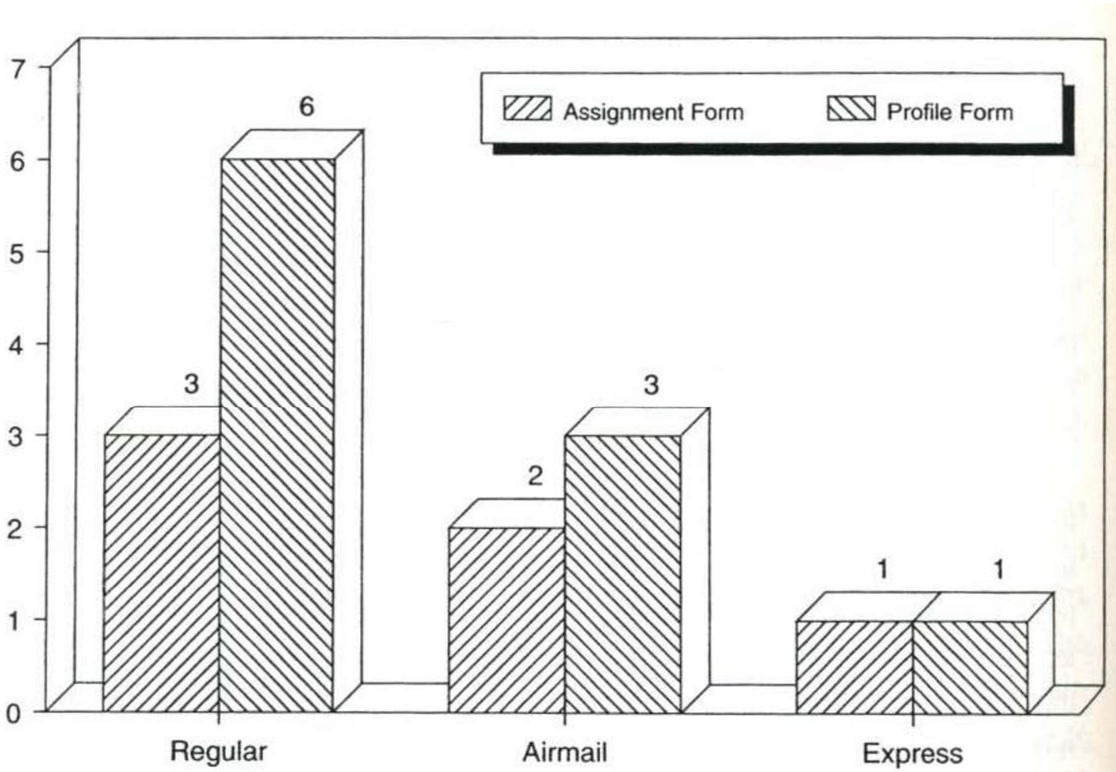
  m=1 with three possible magnitudes of the single dimension indicating speed of delivery
- n-lists (n=m+1)

  $(q_1, \ldots, q_m, q_{m+1})$

  q_{m+1} is the number of items assigned the m-list $(q_1, \ldots, q_m)$ of quality magnitudes

Profile form

- three, two and one letters by regular, airmail and express delivery



- §11 on capacity pricing

Profile form

- n-lists q^i

$$D(\hat{q}) = \sum_{i \in I(\hat{q})} q_n^i \qquad I(\hat{q}) = \{\, i \mid q_1^i \geq \hat{q}_1, \ldots, q_m^i \geq \hat{q}_m \,\}$$

$$N(p,q) \equiv N(p,(\hat{q},x)) = \#\{\, t \mid D(\hat{q},t) \geq x \,\}$$

x-th increment of quantity and each $\hat{q}_i$ -th increment of qualities

The load-duration curve

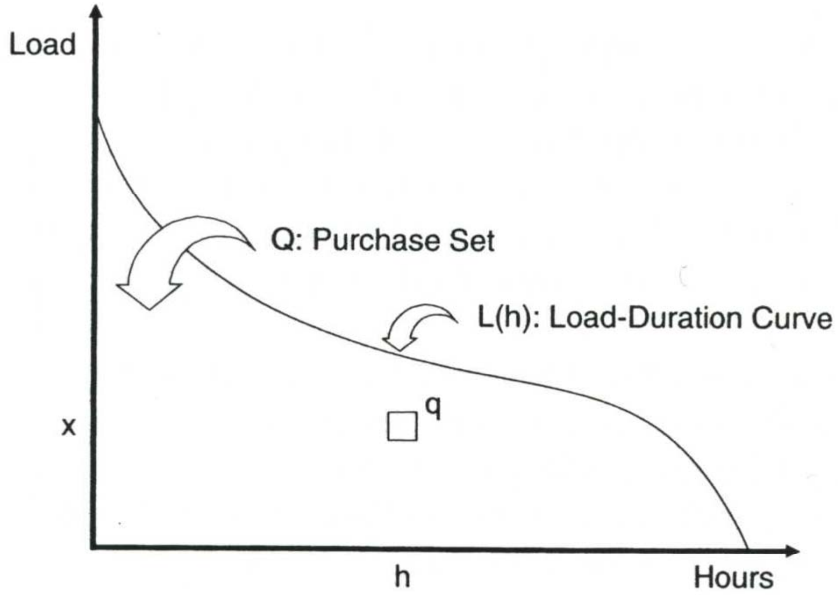
- Load level x [kW]
- H(x) : hours that the customer's load exceeds x

- m=1, n=2
- $\hat{q} \equiv \mathrm{x}$
- $D(\hat{q}) \equiv \mathrm{H(x)}$

- L(h) : the power, or load, demanded in the h-th ranked hour of the year

The load-duration curve

- m=1, n=2



- Time-of-use $P(h, L(h))$ $p(h, x) = \partial P(h, x)/\partial x$

- Wright tariff $\hat{P}(H(q), q)$ $\hat{p}(h, x) = \partial \hat{P}(h, x)/\partial h$

Design of multidimensional tariffs

- Purchase sets.

  Q : customer's purchase set

  $q=(q_1, \dots, q_n)$

  $dq=(dq_1, \dots, dq_n)$

  [q] : a particular kind of rectangular purchase set, consisting of all the small cubes below single point q

  [Q] : the smallest rectangular set that includes an arbitrary purchase set Q (§11)

Design of multidimensional tariffs

- The firm's cost

$$c(q) = \frac{\partial^n C([q])}{\partial q_1 \cdots \partial q_n} \qquad C(Q) = \int_Q c(q)\, dq$$

- A customer's benefits

$$v(q) = \frac{\partial^n U([q])}{\partial q_1 \cdots \partial q_n} \qquad U(Q) = \int_Q v(q)\, dq$$

- The optimal price schedule

$$p(q) = \frac{\partial^n P([q])}{\partial q_1 \cdots \partial q_n} \qquad P(Q) = \int_Q p(q)\, dq$$

Design of multidimensional tariffs

- The optimal price schedule
  - Purchase set

    $$Q(t) = \{ q \mid v(q,t) \geq p(q) \}$$

  - Maximized net benefit

    $$U(Q(t),t) - P(Q(t)) = \int_{Q(t)} [v(q,t) - p(q)]\, dq$$

  - Demand profile

    $$N(p(q),q) = \# \{ t \mid q \in Q(t) \}$$
    $$= \# \{ t \mid v(q,t) \geq p(q) \}$$

  - Firm's profit contribution

    $$\sum_t [P(Q(t)) - C(Q(t))] = \sum_t \int_{Q(t)} [p(q) - c(q)]\, dq$$
    $$= \int_0^\infty N(p(q),q) \cdot [p(q) - c(q)]\, dq$$

Design of multidimensional tariffs

- The optimal price schedule

$$N(p(q), q) \cdot [p(q) - c(q)]$$

$$N(p(q), q) + N_p(p(q), q) \cdot [p(q) - c(q)] = 0$$

$$\alpha N(p(q), q) + N_p(p(q), q) \cdot [p(q) - c(q)] = 0$$

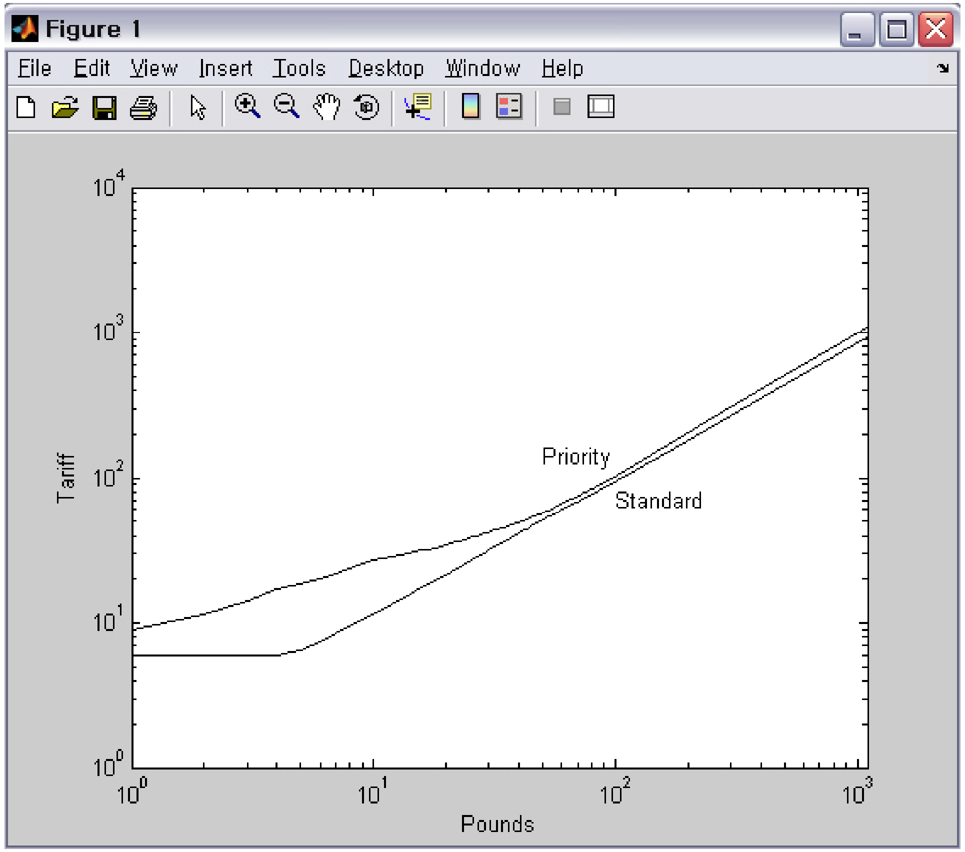
Example 9.1

	Increment q		N(p, q)											
Price p($)	Mode	Quantity :	1	2	3	4		Stamps	Letters					
1	Reg		100	60	30	15		205	81		100	60	30	15
	Air		50	40	25	9		124	49		50	40	25	9
	Exp		30	25	15	5		75	75		30	25	15	5
2	Reg		80	42	18	7		147	66		160	84	36	14
	Air		35	28	14	4		81	32		70	56	28	8
	Exp		20	17	11	1		49	49		40	34	22	2
3	Reg		55	29	12	3		99	57		165	87	36	9
	Air		18	14	9	1		42	23		54	42	27	3
	Exp		11	6	2	0		19	19		33	18	6	0
			Optimal Schedule								누적 Prices			
Prices($) :	Reg		3	3	2	1					3	3	2	1
	Air		2	2	2	1					5	5	4	2
	Exp		2	2	2	1					7	7	6	3
											각 구간의 Demand			
Demands($) :	Reg		55	29	18	15		117	31		20	1	4	6
	Air		35	28	14	9		86	33		15	11	3	4
	Exp		20	17	11	5		53	53		20	17	11	5
											각 구간의 Revenue			
Revenues($) :	Reg		165	87	36	15		303	77		60	3	8	6
	Air		70	56	28	9		163	150		75	55	12	8
	Exp		40	34	22	5		101	340		140	119	66	15
							Total :	567	567					

Example 9.2

- Federal Express' Rate Schedule

% q_th_Pound	Priority	Standard
1	9.00	6.00
2	2.50	0.00
3-4	2.75	0.00
5	1.75	0.50
6-10	1.65	1.00
11-50	0.75	1.00
51-100	0.90	0.85
101-300	1.02	0.86
301-500	0.99	0.84
501-1000	0.96	0.82
1000-	0.90	0.77



Example 9.2

- Tariff

$$P(w \mid s) = A(s)w^b$$

- Marginal charge

$$P_w(w \mid s) = A(s)bw^{b-1}$$

  - w : weight, s : speed

- Suppose
  - valuations : v(s, w; t)
  - mean : M(s, w)
  - demand profile : $N(p; s, w) = e^{-p/M(s,w)}$
  - $M(s, w) = a(s)w^{b-1}$
  - marginal cost : c(s, w)=0

Example 9.2

- Ramsey pricing rule
  - α = 1/n : the result of competition among n firms using Cournot model (§12)

  - $p(s, w) = M(s, w)/n$

    $\log\ p(s, w) = \log[a(s)/n] + [b - 1] \cdot \log\ w$

Example 9.3

- Suppose
  - t : customers' types
  - demand function : D(p, h; t)=t-hp

    $h_* < h < 1$ and $h_* > 0$
  - valuation v(h, x; t)=[t-x]/h
- Demand profile $$N(p; h, x) = \#\{t \mid t \geq x + hp\} = \max\{0, 1 - [x + hp]\}$$
- Price schedule $$p(h, x) = \min\left\{[1 - x]/h, \frac{1}{1+\alpha}\left(c(h, x) + \alpha[1 - x]/h\right)\right\} = \frac{1}{1+\alpha}\left(c(h, x) + \alpha[1 - x]/h\right)$$

$$c(h, x) \leq [1 - x]/h$$

Example 9.4

- F : distribution function
- $\bar{F}$ (t)=1-F(t) : fraction of the population with type parameters exceeding t
- Benefit function

$$U(Q,t) = t \int_Q v(q)\, dq$$

- tv(q) : t's marginal valuation
- Suppose c(q)=0
- Demand profile

$$N(p,q) = \bar{F}(p/v(q))$$

Example 9.4

- Optimal marginal price schedule

$$p(q)/v(q) = \alpha \bar{F}(p(q)/v(q))/f(p(q)/v(q)) = k(\alpha) \leq 1$$

  - f : density of F

$$p(q) = k(\alpha)v(q)$$

An item-assignment formulation

- Customer's total benefit is the aggregate of the marginal service values → inaccurate
  - item-specific attributes

- t : customer's type
- a : item attribute
- G(a, t) : distribution for the attributes of the customer's items
- g(a, t) : associated density function
- u(s, a, t) : benefit function
- s : delivery delay
- $v(s, a, t) \equiv \partial u(s, a, t)/\partial s$: marginal value of delay

An item-assignment formulation

- $P(q(s), s)$: charge
- $p(q, s) \equiv \partial P(q, s)/\partial q$: marginal charge
- $s(a)$: delay assigned to an item with attribute a
- $a(s)$: attribute of items sent with delay s
- $G(a(s), t)$: distribution of items' delays
- $q(s)=g(a(s), t)a'(s)$

  $q(s(a))=g(a, t)/s'(a)$: corresponding density of delays

An item-assignment formulation

- Customers' optimal assignments
  - Net benefit

$$\int_A u(s(a),a,t)g(a,t)\,da - \int_S P(g(a(s),t)a'(s),s)\,ds$$
$$\equiv \int_A \{u(s(a),a,t)g(a,t) - P(g(a,t)/s'(a),s(a))s'(a)\}\,da$$

$$v(s(a),a,t) = \frac{d}{ds}p(q(s),s)$$

  - Two transversality conditions

$$u(s(a),a,t) \geq p(q(s),s)$$

$$u(s(a),a,t)q(s) \geq P(q(s),s)$$

An item-assignment formulation

- The firm's preferred assignments

$$\int_T \left\{ \int_S \left[P(q(s,t),s) - q(s,t)c(s) \right] ds \right\} dF(t)$$

$$\frac{dG(a(s,t),t)}{dt} \cdot F(t) \cdot \frac{d[p(q(s,t),s) - c(s)]}{ds}$$

$$V(s,a,t) \equiv F(t) \cdot [v(s,a,t) - c'(s)]$$

$$= F(t) \cdot \frac{d[p(q(s,t),s) - c(s)]}{ds}$$

$$\frac{G_t(a,t)}{G_a(a,t)} = \frac{V_t(s,a,t)}{V_a(s,a,t)}$$