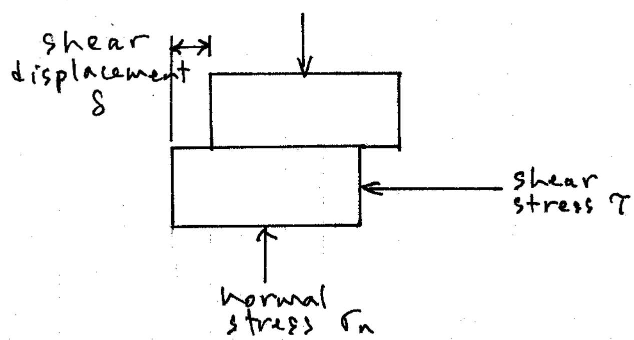
Chapter 5. Shear strength of discontinuities.

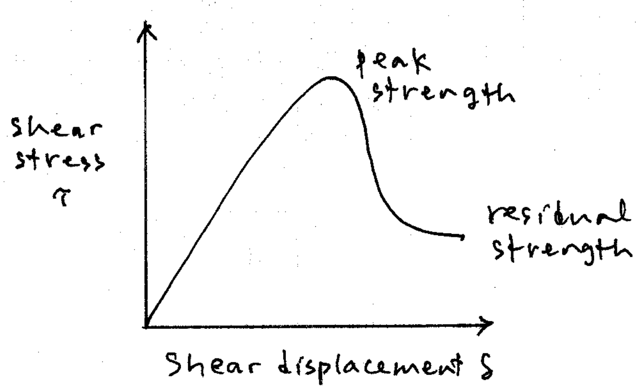
⊛ Hard rock at shallow depth. → discrete blocks
(bedding planes, joints, faults, etc).

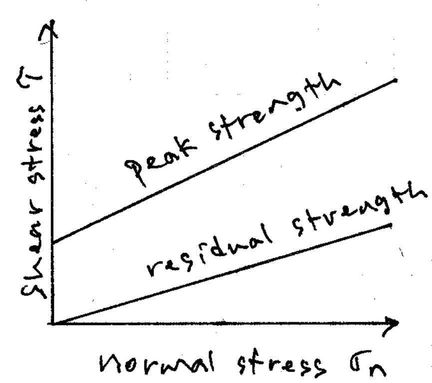
In tact rock failure (X) → sliding and rotation of rock blocks (O)
Behavior of rock mass.

1. Shear strength of planar surfaces

(1) Typical behavior







Peak shear strength

$$\tau_p = c + \sigma_n \tan\phi \quad \text{(Mohr-Coulomb Eq.)}$$

$$\begin{cases} c = \text{cohesive strength of the cemented surface} \\ \phi = \text{angle of friction.} \end{cases}$$

Residual strength

$$\tau_r = \sigma_n \tan\phi_r$$

ϕ_r = residual angle of friction.

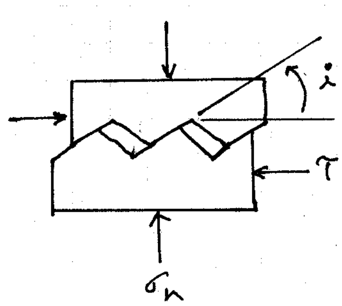
* cohesion – from the concept in soil mechanics (adhesion)
– In rock mechanics, it applies when cemented surfaces are sheared.
– usually, it is used for surface roughness. (intercept on the τ axis @ $\sigma_n=0$).

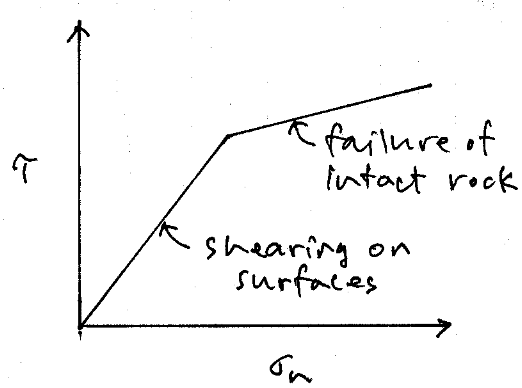
* Basic friction angle ϕ_b ($\simeq \phi_r$: residual friction angle).
– measured by testing sawn or ground rock surfaces.
– $\tau = \sigma_n \tan\phi_b$.

2. Shear strength of rough surfaces.

(1) Surface roughness increases the shear strength of the surface. Extremely important for the stability of underground openings.

(2) Saw-tooth specimen.





– Involves "dilation (an increase in volume)"
– $\tau = \sigma_n \tan(\phi_b + i)$ (Patton, 1966).

$\begin{cases} \phi_b : \text{basic friction angle of the surface} \\ i : \text{angle of the saw-tooth face.} \end{cases}$

– Valid at low normal stress.
At high normal stress, intact rock material will break off.

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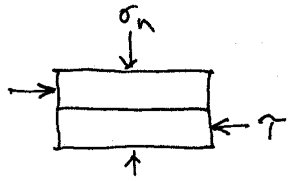
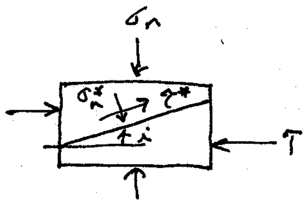


$$\tau = \sigma_n \tan\phi$$



$$\tau^* = \sigma_n^* \tan\phi \rightarrow \frac{\tau^*}{\sigma_n^*} = \tan\phi$$

$$\begin{cases} \tau^* = \tau\cos i - \sigma_n \sin i \\ \sigma_n^* = \sigma_n \cos i + \tau \sin i \end{cases}$$

$$\therefore \frac{\tau^*}{\sigma_n^*} = \frac{\tau\cos i - \sigma_n \sin i}{\sigma_n \cos i + \tau \sin i} = \tan\phi$$

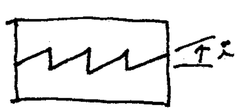
$$\frac{\frac{\tau}{\sigma_n} - \tan i}{1 + \frac{\tau}{\sigma_n}\tan i} = \tan\phi$$

$$\left(\frac{\tau}{\sigma_n} - \tan i\right) = \tan\phi \left(1 + \frac{\tau}{\sigma_n}\tan i\right)$$

$$\frac{\tau}{\sigma_n}(1 - \tan\phi \cdot \tan i) = \tan\phi + \tan i$$

$$\therefore \frac{\tau}{\sigma_n} = \frac{\tan i + \tan\phi}{1 - \tan i \cdot \tan\phi} = \tan(\phi + i)$$

$$\therefore \tau = \sigma_n \tan(\phi + i).$$



(3) Behavior of natural rock joints.

$$\tau = \sigma_n \tan\left[\phi_b + JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right)\right]$$

$\begin{cases} JRC: \text{rock roughness coefficient} \\ JCS: \text{joint wall compressive strength.} \end{cases}$

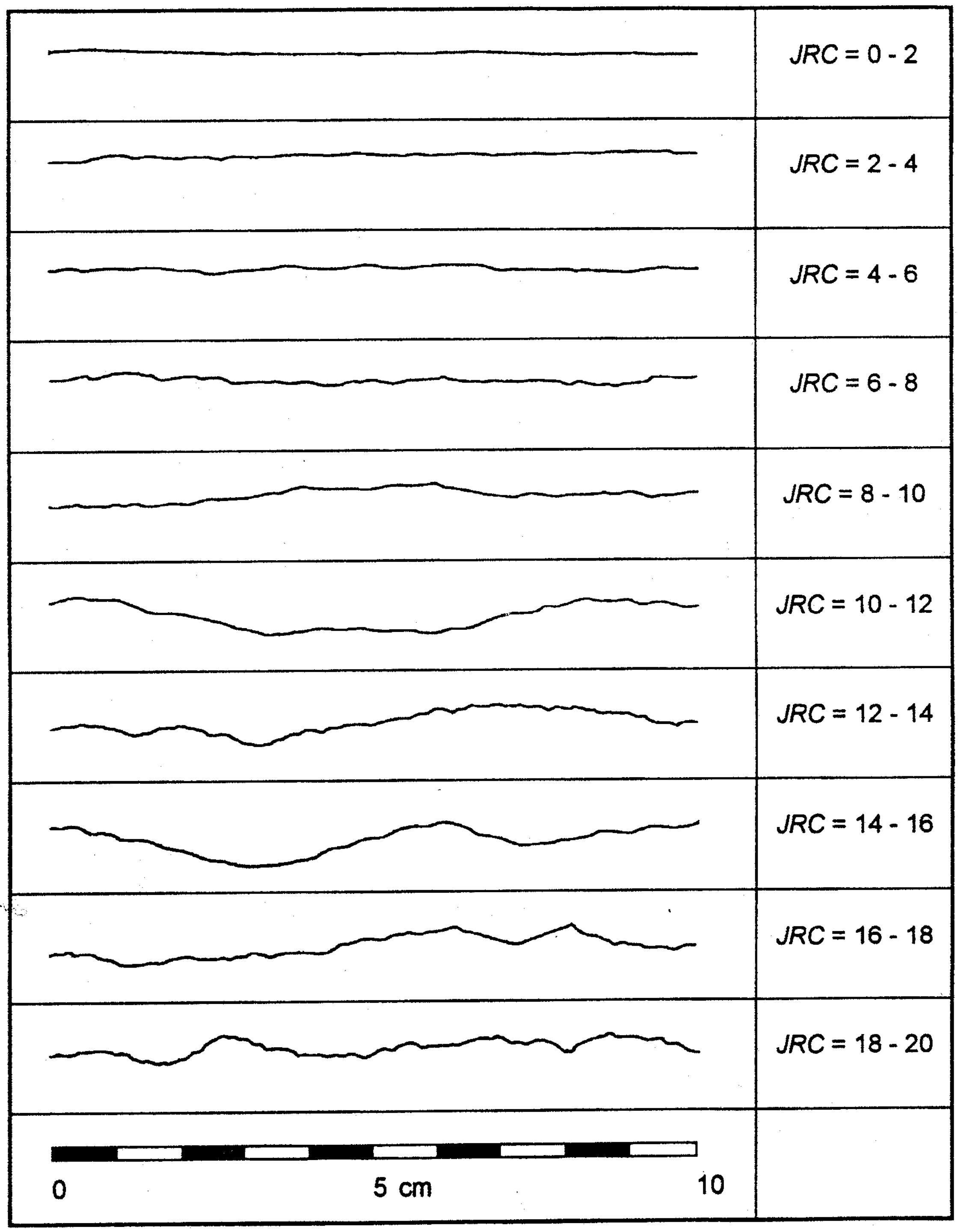


Figure 5.1: Roughness profiles and corresponding *JRC* values (After Barton and Choubey, 1977).

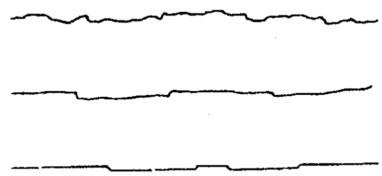
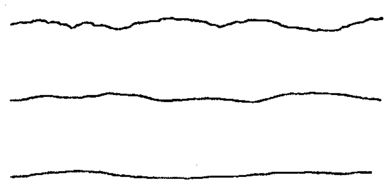
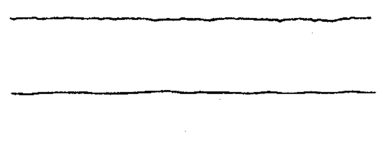
Description	Profile	J_r	*JRC* 200mm	*JRC* 1 m
Rough		4	20	11
Smooth		3	14	9
Slickensided	Stepped	2	11	8
Rough		3	14	9
Smooth		2	11	8
Slickensided	Undulating	1.5	7	6
Rough		1.5	2.5	2.3
Smooth		1.0	1.5	0.9
Slickensided	Planar	0.5	0.5	0.4

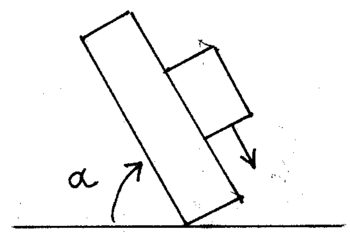
Figure 5.2: Relationship between J_r in the Q system and JRC for 200 mm and 1000 mm samples (After Barton, 1987).

① Field estimates of JRC

- Compare the appearance of a discontinuity surface with the profiles by Barton & Choubey (1977).

- Simple tilt test with a pair of matching discontinuity surfaces (Barton & Bandis, 1990).

$$JRC = \frac{\alpha - \phi_b}{\log_{10}\left[\frac{JCS}{\sigma_n}\right]}$$



Ex. $\alpha = 65°$, $\phi_b = 30°$, $JCS = 100\,MPa$, $\sigma_n = 0.001\,MPa$

$$JRC = \frac{65-30}{\log_{10}\left[\frac{100}{0.001}\right]} = 7$$

($\sigma_n = 0.001\,MPa$ for small samples.)

② Field estimates of JCS

- By ISRM suggested method (1978).
- By using the Schmidt rebound hammer. (Deere & Miller, 1966).

③ Influence of scale on JRC and JCS.

$$\tau = \sigma_n \tan\left[\phi_b + JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right)\right]$$

Three factors ① ϕ_b, ② JRC, ③ (JCS/σ_n)

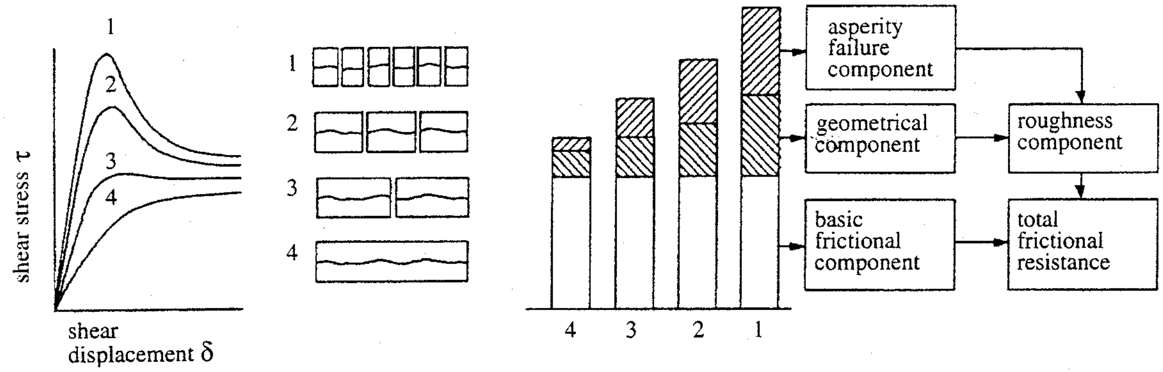


Figure 5.3: Influence of scale on the three components of the shear strength of a rough discontinuity. After Bandis (1990) and Barton and Bandis (1990).

Scale ↑ → effective roughness ↓ → shear strength ↓
→ weaknesses ↑ → strength ↓

$$JRC_n = JRC_0 \left[\frac{L_n}{L_0}\right]^{-0.02\,JRC_0}$$

$$JCS_n = JCS_0 \left[\frac{L_n}{L_0}\right]^{-0.03\,JCS_0}$$

$\begin{cases} JRC_0, JCS_0, L_0: \text{100mm lab. scale samples} \\ JRC_n, JCS_n, L_n: \text{in situ block sizes.} \end{cases}$

3. Shear strength of filled discontinuities.

Table 5.1: Shear strength of filled discontinuities and filling materials (After Barton, 1974).

Rock	Description	Peak c' (MPa)	Peak $\phi°$	Residual c' (MPa)	Residual $\phi°$
Basalt	Clayey basaltic breccia, wide variation from clay to basalt content	0.24	42		
Bentonite	Bentonite seam in chalk	0.015	7.5		
	Thin layers	0.09-0.12	12-17		
	Triaxial tests	0.06-0.1	9-13		
Bentonic shale	Triaxial tests	0-0.27	8.5-29		
	Direct shear tests			0.03	8.5
Clays	Over-consolidated, slips, joints and minor shears	0-0.18	12-18.5	0-0.003	10.5-16
Clay shale	Triaxial tests	0.06	32		
	Stratification surfaces			0	19-25
Coal measure rocks	Clay mylonite seams, 10 to 25 mm	0.012	16	0	11-11.5
Dolomite	Altered shale bed, ± 150 mm thick	0.04	14.5	0.02	17
Diorite, granodiorite and porphyry	Clay gouge (2% clay, PI = 17%)	0	26.5		
Granite	Clay filled faults	0-0.1	24-45		
	Sandy loam fault filling	0.05	40		
	Tectonic shear zone, schistose and broken granites, disintegrated rock and gouge	0.24	42		
Greywacke	1-2 mm clay in bedding planes			0	21
Limestone	6 mm clay layer			0	13
	10-20 mm clay fillings	0.1	13-14		
	<1 mm clay filling	0.05-0.2	17-21		
Limestone, marl and lignites	Interbedded lignite layers	0.08	38		
	Lignite/marl contact	0.1	10		
Limestone	Marlaceous joints, 20 mm thick	0	25	0	15-24
Lignite	Layer between lignite and clay	0.014-.03	15-17.5		
Montmorillonite	80 mm seams of bentonite (mont-	0.36	14	0.08	11
Bentonite clay	morillonite) clay in chalk	0.016-.02	7.5-11.5		
Schists, quartzites and siliceous schists	100-15- mm thick clay filling	0.03-0.08	32		
	Stratification with thin clay	0.61-0.74	41		
	Stratification with thick clay	0.38	31		
Slates	Finely laminated and altered	0.05	33		
Quartz / kaolin / pyrolusite	Remoulded triaxial tests	0.042-.09	36-38		

4. Influence of water pressure

(1) The surfaces of the discontinuities are forced apart. The normal stress σ_n is reduced.

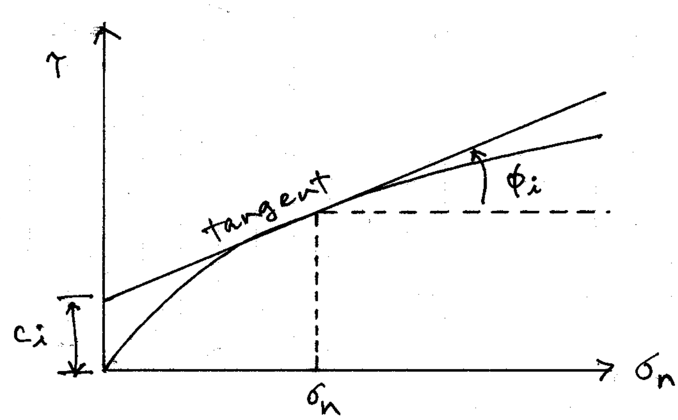
(2) Under steady state conditions,

$$\sigma_n' = (\sigma_n - u) \quad \begin{cases} u: \text{water pressure.} \\ \sigma_n': \text{effective normal stress.} \end{cases}$$

5. Instantaneous cohesion and friction.

(1) Mohr-Coulomb Eq. $[= f(c, \phi)]$

→ Non-linear relationship between σ and τ. $[\neq f(c, \phi)]$



Instantaneous cohesion c_i and instantaneous friction angle ϕ_i for a non-linear failure criterion.

$$\phi_i = \tan^{-1}\left(\frac{\partial \tau}{\partial \sigma_n}\right)$$

$$\frac{\partial \tau}{\partial \sigma_n} = \tan\left(JRC \log_{10}\frac{JCS}{\sigma_n} + \phi_b\right) - \frac{\pi JRC}{\ln 10}\left[\tan^2\left(JRC \log_{10}\frac{JCS}{\sigma_n} + \phi_b\right) + 1\right]$$

$$c_i = \tau - \sigma_n \tan\phi_i$$

Example. $\phi_b = 29°$. $JRC = 16.9$. $JCS = 96\,MPa$,

For practical meaning. ($\sigma_n \neq 0$).

$$\phi_b + JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right) \leq 70°$$

$$\Rightarrow 29 + 16.9 \log_{10}\left(\frac{96}{\sigma_n}\right) \leq 70°$$

$$\log_{10}\left(\frac{96}{\sigma_n}\right) \leq 2.43$$

$$\therefore \ \sigma_n \geq 0.36. \qquad (\leftarrow \sigma_{n,min})$$

$\sigma_{n,max} = ?$ $\Rightarrow$ $\sigma_n = JCS$.

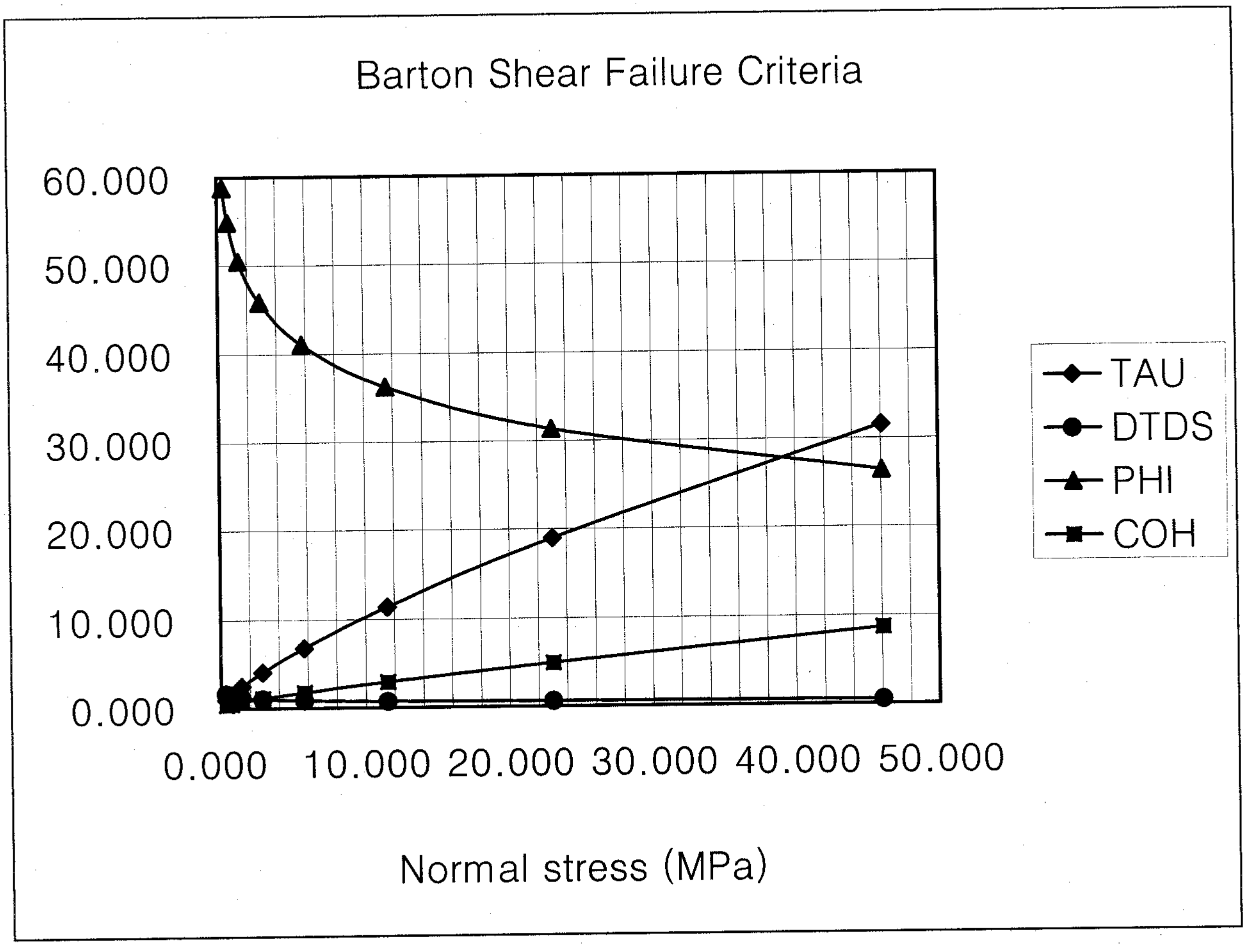
$$\sigma_n \geq \frac{JCS}{10^{\frac{70-\phi_b}{JRC}}}$$

Barton shear failure criterion

Input parameters

Basic friction angle (PHIB)–degrees	29.000
Joint roughness coefficient (JRC)	16.900
Joint compressive strength (JCS)	96.000
Minimum normal stress (SIGNMIN)	0.360

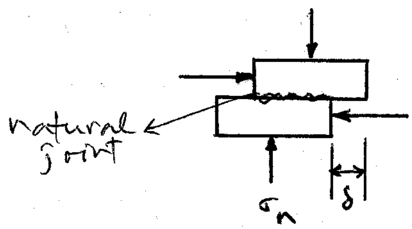
Normal stress (SIGN) Mpa	Shear Strength (TAU) Mpa	dTAU / dSIGN (DTDS)	Friction angle (PHI) degrees	Cohesive strength (COH) Mpa
0.360	0.989	1.652	58.818	0.394
0.720	1.538	1.423	54.910	0.513
1.440	2.477	1.213	50.494	0.730
2.880	4.073	1.030	45.845	1.107
5.759	6.779	0.872	41.072	1.760
11.518	11.344	0.733	36.223	2.907
23.036	18.973	0.609	31.326	4.953
46.073	31.533	0.496	26.395	8.667



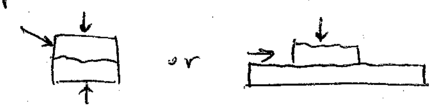
(5) Shear strength.

① Peak shear strength: $\tau_p = c + \sigma_n \tan\phi$
Residual shear strength: $\tau_r = \sigma_n \tan\phi_r$

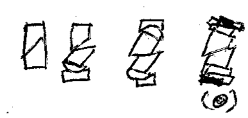
$\left\{ \begin{array}{l} c: \text{cohesive strength.} \\ \phi: \text{angle of friction} \\ \phi_r: \text{residual angle of friction} \end{array} \right.$



ⓐ To prevent rotation

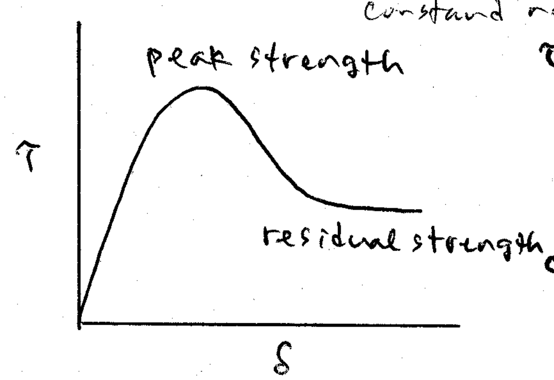


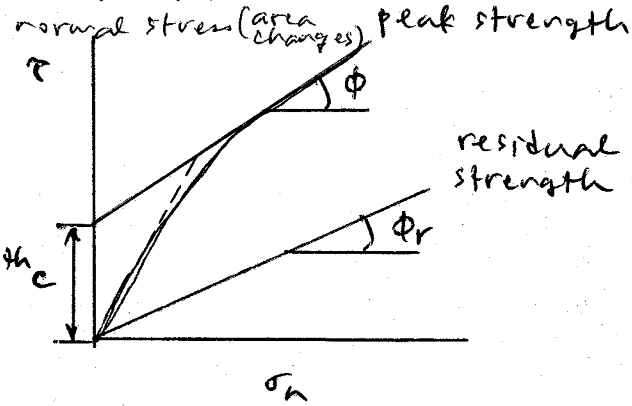
or Triaxial cell



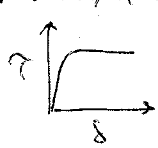
Drained / undrained

ⓑ Constant normal load
Constant normal stress (area changes)





Smooth / clean surfaces



- For soils, c is a result of the adhesion of the soil particles.
- For rocks, when cemented surfaces are sheared the true cohesion occurs.
- more related to surface roughness

② Basic friction angle ϕ_b ($\simeq \phi_r$):
Generally measured by testing sawn or ground rock surfaces.

$$\tau_r = \sigma_n \tan\phi_b.$$

Need to make corrections for surface roughness

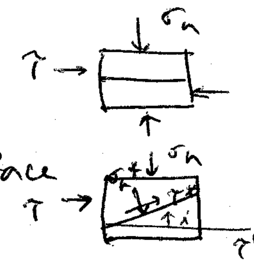
③ Rough surfaces : roughness increases the shear strength

Patton (1966)

$$\tau = \sigma_n \tan(\phi_b + i)$$

ϕ_b : basic friction angle of the surface

i : angle of the saw-tooth face.



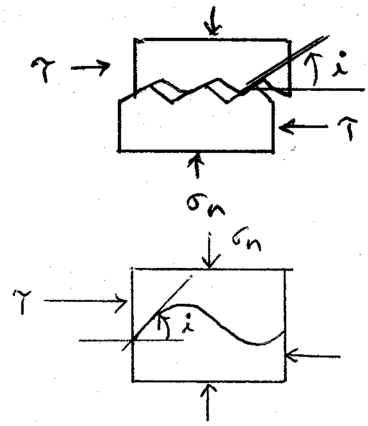
$$\frac{\tau}{\sigma_n} = \tan\phi_b$$

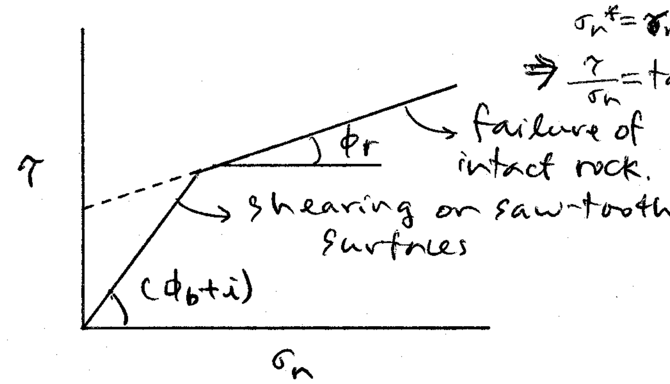
$$\frac{\tau^*}{\sigma_n^*} = \tan\phi_b$$

$$\tau^* = \tau\cos i - \sigma_n \sin i$$

$$\sigma_n^* = \sigma_n \cos i + \tau \sin i$$

$$\Rightarrow \frac{\tau}{\sigma_n} = \tan(\phi_b + i)$$





④ Barton's estimate of shear strength.

At higher normal stresses, the strength of the intact mat'l will be exceeded and the teeth will tend to break off, resulting in a shear strength behavior which is more closely related to the intact mat'l strength than to the frictional characteristics of the surfaces.

$$\tau = \sigma_n \tan\left[\phi_b + JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right)\right]$$

JRC : Joint roughness coefficient

JCS : Joint wall compressive strength.

Barton & Bandis (1982)

$$JRC_n = JRC_o \left(\frac{L_n}{L_o}\right)^{-0.02 JRC_o}$$

$$JCS_n = JCS_o \left(\frac{L_n}{L_o}\right)^{-0.03 JCS_o}$$

"o" : refer to 100 mm lab. scale sample

"n" : in situ block sizes

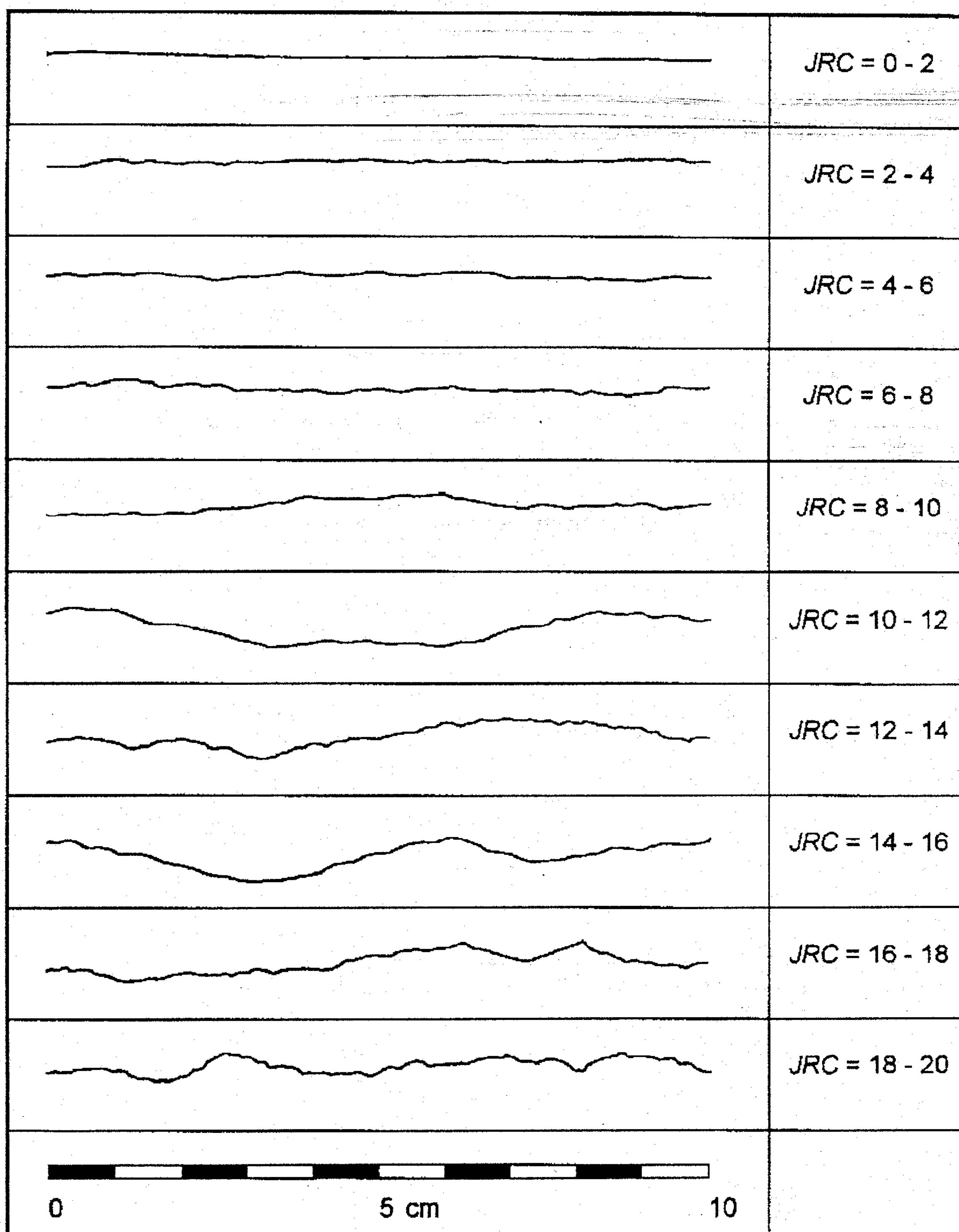
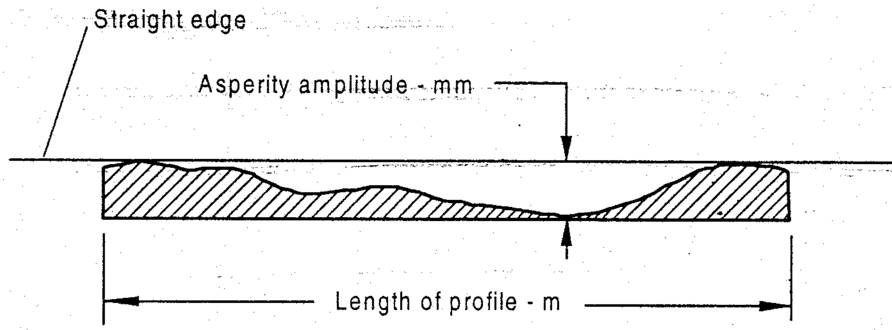


Figure 4.2: Roughness profiles and corresponding JRC values (After Barton and Choubey 1977).



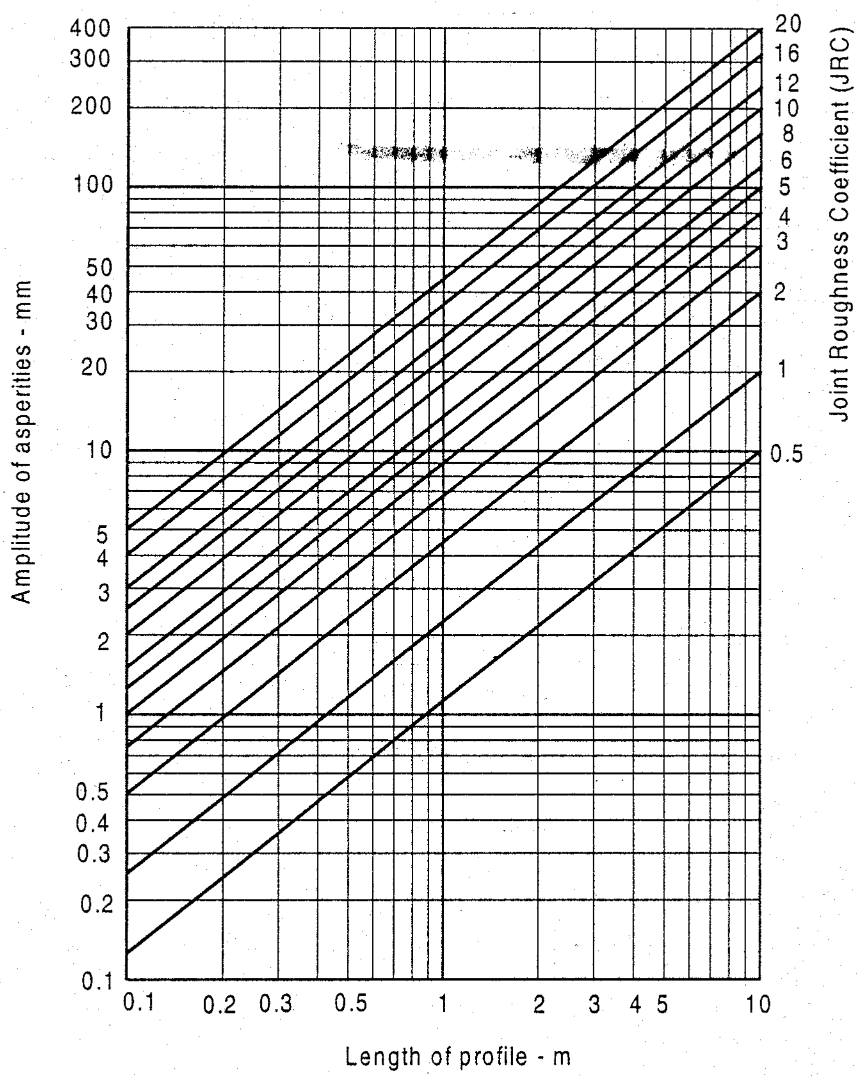


Figure 4.6: Alternative method for estimating *JRC* from measurements of surface roughness amplitude from a straight edge (Barton 1982).

4.6 Field estimates of JCS

Suggested methods for estimating the joint wall compressive strength were published by the ISRM (1978). The use of the Schmidt rebound hammer for estimating joint wall compressive strength was proposed by Deere and Miller (1966), as illustrated in Figure 4.7.

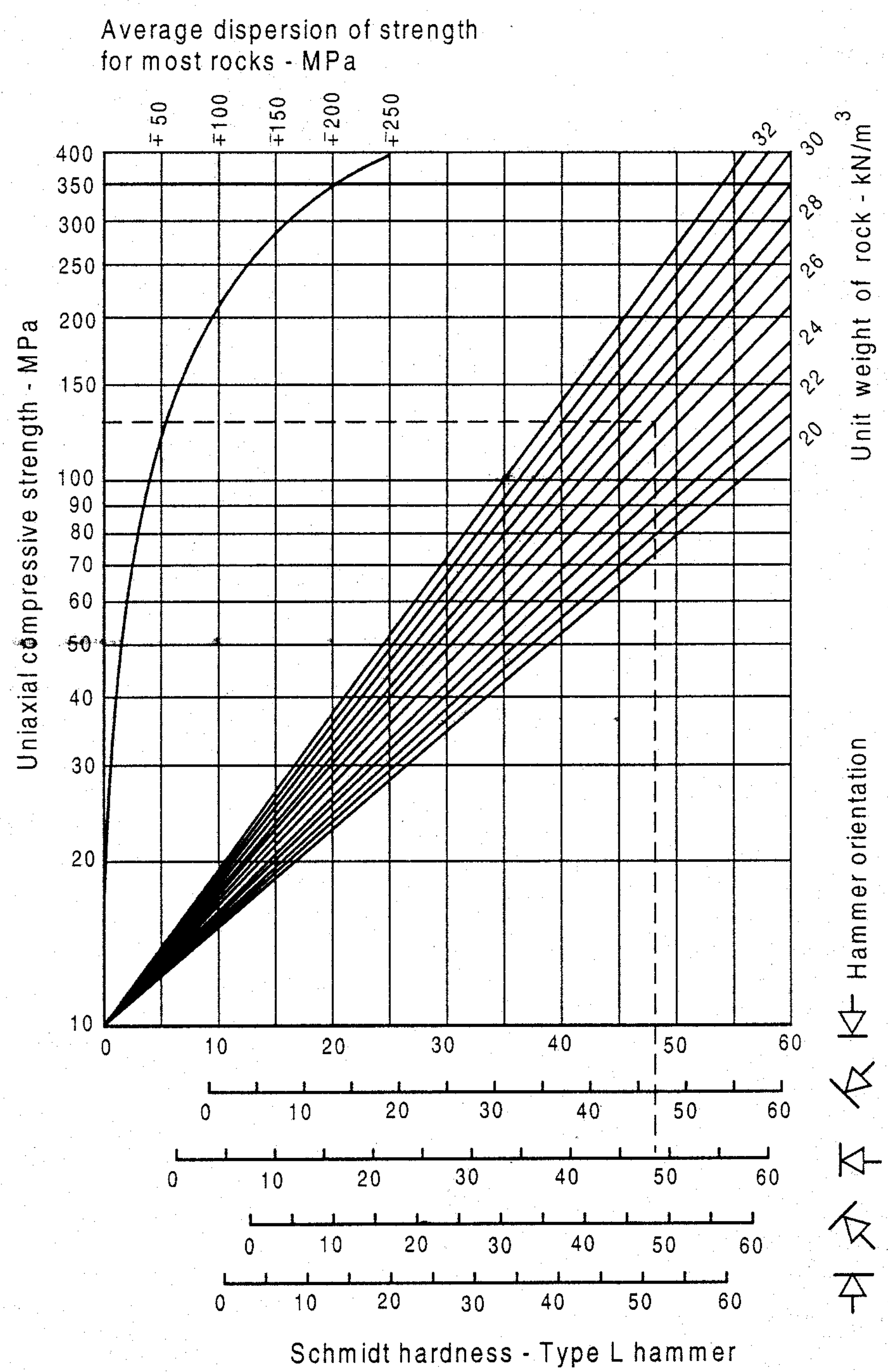
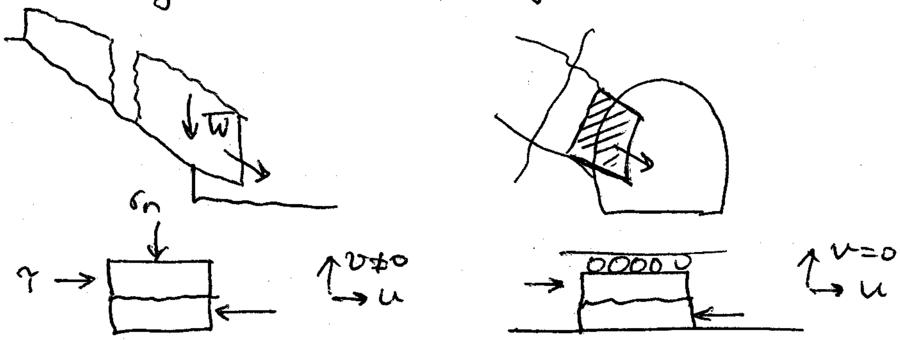


Figure 4.7: Estimate of joint wall compressive strength from Schmidt hardness.

⑤ Dilatancy and shear strength.



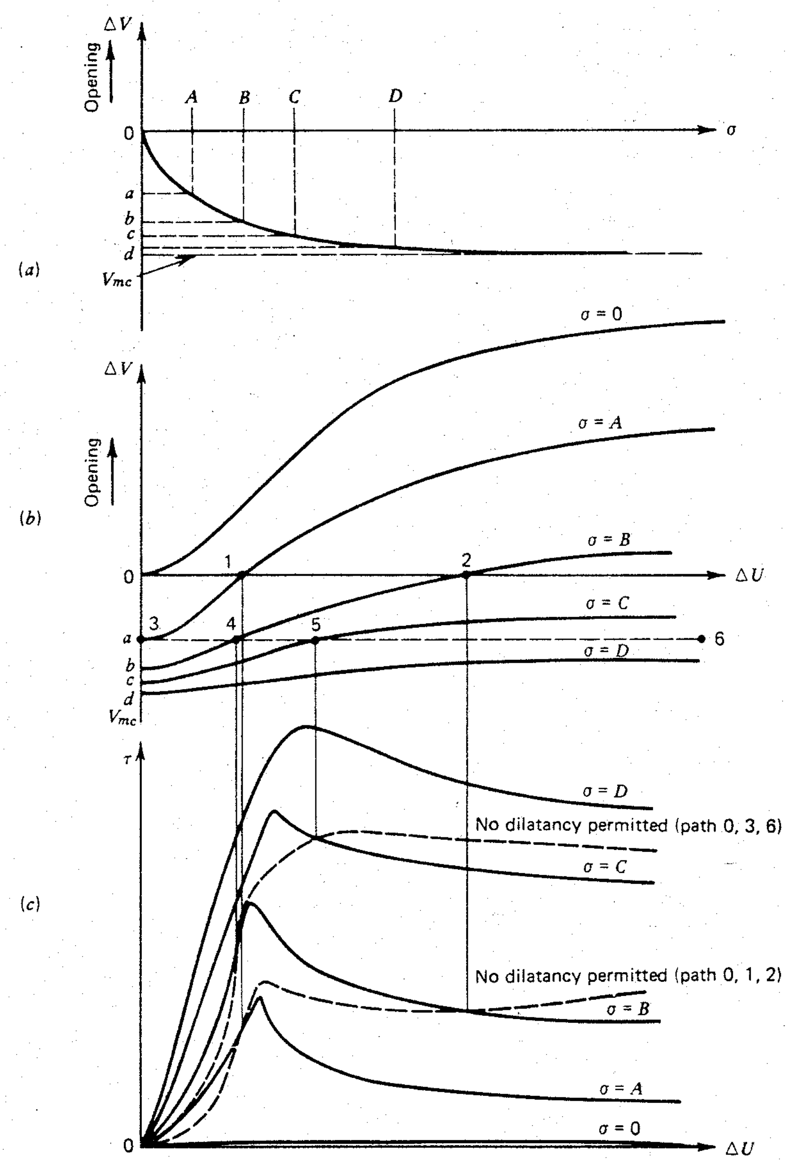


Figure 5.17 Coupling of the normal deformation, shear deformation, and dilatancy laws for rough joints and analysis of path dependency.

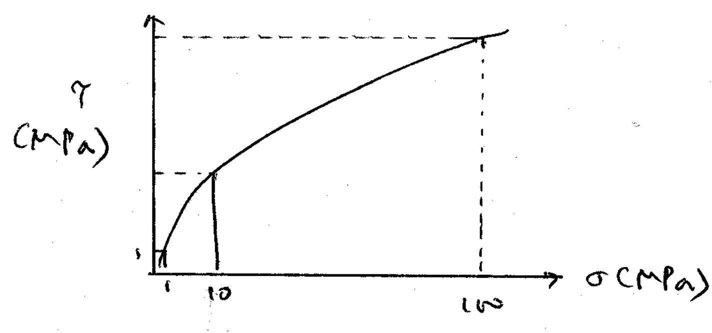
<u>Example</u>. $\phi_r = \phi_b = 25°$, JRC = 10, JCS = 100 MPa

$\sigma_n = 1, 10, 100$ MPa

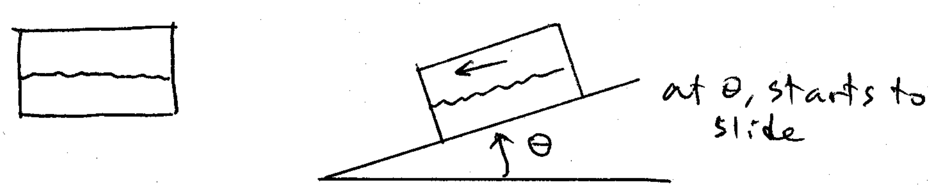
① $\sigma_n = 1$. $\tau = 1 \times \tan(25 + 10 \cdot \log 100) = \tan 45° = 1$ MPa

② $\sigma_n = 10$, $\tau = 10 \times \tan(25 + 10 \log 10) = 10 \tan 35° \doteqdot 7.0$ MPa

③ $\sigma_n = 100$ MPa, $\tau = 100 \times \tan(25 + 10 \log 1) = 100 \tan 25° \doteqdot 47$ MPa



<u>Example</u>



Assume $\phi_j = 25° = \phi_b$

Measured $\theta = 75°$

JCS = 100 MPa, $\sigma_n = 0.001$ MPa

From Barton's formula

$$\phi = \mathrm{JRC} \log\left(\frac{\mathrm{JCS}}{\sigma_n}\right) + \phi_j$$

$$\mathrm{JRC} = \frac{\phi_{meas.} - \phi_j}{\log(\mathrm{JCS}/\sigma_n)} = \frac{75 - 25}{\log(100/0.001)} = 10$$

4. Effect of water

(1) Water – chemical deterioration & pore water pressure. (weakening)

(2) Terzaghi's effective stress law : $\sigma' = \sigma - p_w$

$$\sigma_1 = b_u + \sigma_3 \tan^2(45+\tfrac{\phi}{2}) \quad : \text{Mohr-Coulomb failure crit.}$$

$$\Rightarrow \sigma_1' = b_u + \sigma_3' \tan^2(45+\tfrac{\phi}{2})$$

$$\sigma_1' - \sigma_3' = b_u + \sigma_3' [\tan^2(45+\tfrac{\phi}{2}) - 1]$$

$$\sigma_1 - \sigma_3 = b_u + (\sigma_3 - p_w)[\tan^2(45+\tfrac{\phi}{2}) - 1]$$

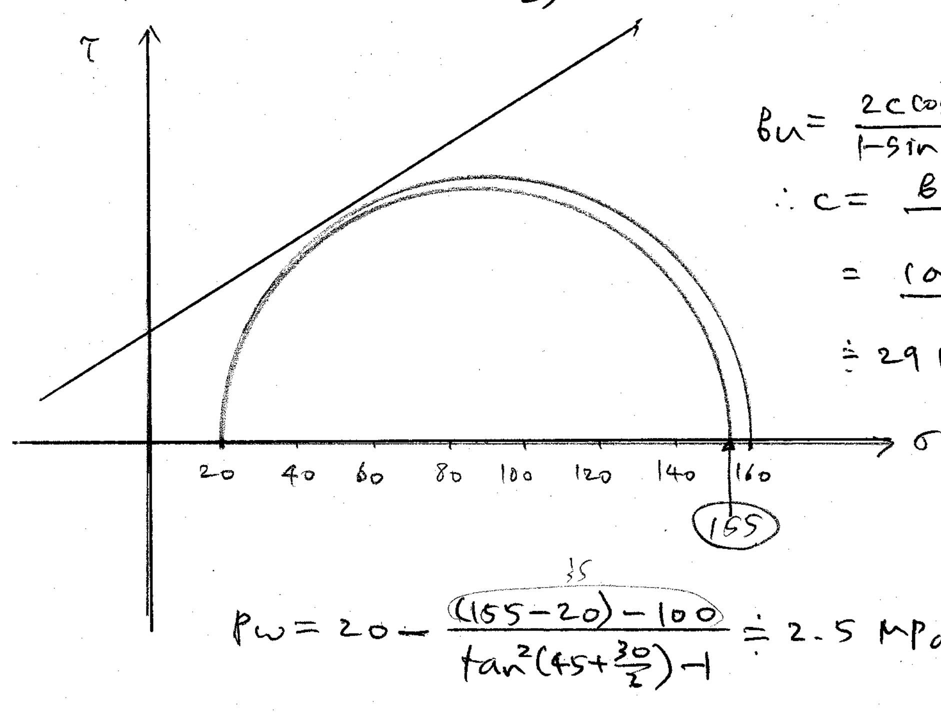
$$\therefore p_w = \sigma_3 - \frac{(\sigma_1-\sigma_3) - b_u}{\tan^2(45+\frac{\phi}{2}) - 1}$$

Example

(1) $\sigma_1 = 155\text{MPa}$, $\sigma_3 = 20\text{MPa}$, $b_u = 100\text{MPa}$, $\phi = 30°$ → Fail or not?

$$\sigma_1 = b_u + \sigma_3 \tan^2(45+\tfrac{\phi}{2}) = 100 + 20\tan^2(45+\tfrac{30}{2}) = 160\text{ MPa}$$

(won't fail)

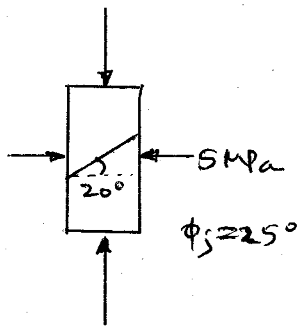


$$b_u = \frac{2c\cos\phi}{1-\sin\phi}$$

$$\therefore c = \frac{b_u(1-\sin\phi)}{2\cos\phi} = \frac{100(1-\sin 30°)}{2\cdot\cos 30°} \doteq 29\text{ MPa}$$

$$p_w = 20 - \frac{\overset{35}{(155-20)-100}}{\tan^2(45+\frac{30}{2})-1} \doteq 2.5\text{ MPa}$$

Example



$\phi_j = 25°$

$\tau = \sigma \tan\phi_j$

$$\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{(\sigma_1-\sigma_3)}{2}\cos 2\beta$$
$$= 10 + 5\cdot\cos 40°$$
$$\doteqdot 13.8 \text{ MPa}$$

$\therefore$ $S = 13.8 \cdot \tan 25° = 6.44$ MPa
= shear stress to cause slip.

$$\tau = \frac{\sigma_1-\sigma_3}{2}\sin 2\beta = 5\cdot\sin 40° = 3.21 \text{ MPa}$$

$\Rightarrow$ Does NOT slip.

Water pressure to cause a slip.

$\tau = (\sigma - p_w)\tan\phi_j$

$$p_w = \sigma - \frac{\tau}{\tan\phi_j} = 13.8 - \overset{6.88}{\frac{3.21}{\tan 25°}} \doteqdot 6.92 \text{ MPa}$$