Chapter 8. Strength of rock and rock masses

1. Definitions

(1) Intact rock:	 The unfractured blocks which occur between structural discontinuities in a typical rock mass
	- Ranging from a few millimeters to several meters in size
	- Generally elastic and isotropic
(2) Joints	- A particular type of geological discontinuity
(3) Strength	- The maximum stress level which can be carried by a specimen

2. Strength of intact rock

- (1) Hoek and Brown's attempt:
 - The failure criterion of good agreement with rock strength values (50 mmφ, core orientation ⊥ discontinuity surface)
 - Mathematically simple expression
 - Possibly of extension to deal with the failure of jointed rock mass



Figure 6.20 The Hoek–Brown empirical failure criterion.

Compare with Mohr-Coulomb failure criterion?

Compare with Mohr-Coulomb failure criterion?



2. Strength of intact rock

(2) Hoek and Brown failure criterion

$$\sigma_1' = \sigma_3' + \sigma_c \left(m_i \frac{\sigma_3'}{\sigma_c} + 1 \right)^{\frac{1}{2}}$$



 σ_1 ': major principal effective stress at failure

 σ_3 ': minor principal effective stress at failure

 σ_c : uniaxial compressive strength of the intact rock (50 mm $\phi \times$ 100 mmH lab test result)

m_i: material constant for the intact rock

For smaller diameter (d in mm) cores,

$$\sigma_{\rm c} = \frac{\sigma_{\rm cd}}{\left(\frac{50}{\rm d}\right)^{0.18}}$$

Ex) For 35 mm ϕ , σ_{cd} = 90 MPa was obtained.

$$\sigma_{\rm c} = \frac{90 \,\text{MPa}}{\left(\frac{50}{35}\right)^{0.18}} = 90 \,\text{MPa} \times 0.938 = 84 \,\text{MPa}$$

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$$\sigma_{\rm c} = \frac{\sigma_{\rm cd}}{\left(50/d\right)^{0.18}}$$

$$\frac{\sigma_{\rm c}}{\sigma_{\rm cd}} = \frac{1}{\left(50/d\right)^{0.18}}$$

2. Strength of intact rock

- (3) $\sigma_{c}~$ and $~m_{i}~$ obtained from the results of triaxial tests
 - Conventional triaxial cell











a or oystem









Typical Triaxial Cell Assembly





Description	Strength characteristics	Strength testing considerations	Theoretical
Intact rock	Brittle, elastic and gener- ally isotropic behaviour	Triaxial testing of core specimens relatively simple and inexpensive and results are usually reliable	Behaviour of elastic iso- tropic rock is adequately understood for most prac- ical applications
Intact rock with a single inclined discontinuity	Highly anisotropic, de- pending on shear strength and inclination of discon- tinuity	Triaxial tests difficult and expensive. Direct shear tests preferred. Careful interpretation of results required	Behaviour of discontinui- tics adequately understood for most practical applica- tions
Massive rock with a few sets of discontinuities	Anisotropic, depend-ing on number, orientation and shear strength of disconti- nuities	Laboratory testing very difficult because of sample disturbance and equipment size limitations	Behaviour of complex block interaction in sparse- ly jointed rock masses poorly understood
Heavily jointed rock masses	Reasonably isotropic, highly dilatant at low stress levels with particle break- age at high stress levels	Triaxial testing of repres- entative samples extremely difficult because of sample disturbance	Behaviour of interlocking angular pieces poorly un- derstood
Compacted rock- fill or weakly cemented con- glomerates	Reasonably isotropic, less dilatant and lower strength than in situ rock due to de- struction of fabric	Triaxial testing simple but expensive due to large equipment required to ac- commodate samples	Behaviour reasonably well understood from soil me- chanics studies on granular materials
Loose waste rock or gravel	Poor compaction and grading allow particle movement resulting in mobility and low strength	Triaxial or direct shear testing simple but expen- sive due to large size of equipment	Behaviour of loosely com- pacted waste rock and gravel adequately under- stood for most applications

Table 8.1: S	Summary of	f rock mass	characteristics,	testing methods	and	theoretical	considerations.
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2. Strength of intact rock

(4) Tables can be used for σ_c and m_i instead of triaxial tests

Grade*	Term	Uniaxial comp. strength (MPa)	Point load index (MPa)	Field estimate of strength	Examples**
R6	Extreme- ly strong	> 250	>10	Rock material only chipped under repeated hammer blows, rings when struck	Fresh basalt, chert, diabase, gneiss, granite, quartzite
R5 .	Very strong	100-250	4-10	Requires many blows of a geolog- ical hammer to break intact rock specimens	Amphibolite, sandstone, basalt, gabbro, gneiss, granodiorite, limestone, marble, rhyolite, tuff
R4	Strong	50-100	2-4	Hand held specimens broken by a single blow of geological hammer	Limestone, marble, phyllite, sandstone, schist, shale
R3	Medium strong	25-50	1-2	Firm blow with geological pick indents rock to 5 mm, knife just scrapes surface	Claystone, coal, concrete, schist, shale, siltstone
R2	Weak	5-25	***	Knife cuts material but too hard to shape into triaxial specimens	Chalk, rocksalt, potash
RI	Very weak	1-5	***	Material crumbles under firm blows of geological pick, can be shaped with knife	Highly weathered or altered rock
R0	Extreme-	0.25-1	***	Indented by thumbnail	Clay gouge

* Grade according to ISRM (1981).

**All rock types exhibit a broad range of uniaxial compressive strengths which reflect the heterogeneity in composition and anisotropy in structure. Strong rocks are characterised by well interlocked crystal fabric and few voids.

***Rocks with a uniaxial compressive strength below 25 MPa are likely to yield highly ambiguous results under point load testing.

Rock	Class Group Texture Course Medium Fine							
type		-	Course	Medium	Fine	Very fine		
	Clastic		Conglomerate (22)	Sandstone 19 Greyw	Siltstone 9 vacke>	Claystone 4		
SEDIMENTARY		Organic		← Chalk → 7 ← Coal →				
	Non-Clastic	Carbonate	Breccia (20)	Sparitic Limestone (10)	Micritic Limestone 8			
		Chemical		Gypstone 16	Anhydrire 13	· ·		
HIC	Non Foliated		Marble 9	Homfels (19)	Quartzite 24			
MORP	Slightly	foliated	Migmatite (30)	Amphibolite 31	Mylonites (6)			
META	Foliated*		Gneiss 33	Schists (10)	Phyllites (10)	Slate 9		
	Light		Granite 33		Rhyolite (16)	Obsidian (19)		
	· · ·	-	Granodiorite (30)		(17)			
EOUS		Dark			Andesite 19			
ION	D			Dolerite (19)	Basalt (17)	· ·		
			Norite 22					
	Extrusive py	roclastic type	Agglomerate (20)	Breccia (18)	Tuff (15)			

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*These values are for intact rock specimens tested normal to foliation. The value of m_i will be significantly different if failure occurs along a foliation plane (Hoek, 1983).

3. Strength of jointed rock masses

(1) General form of the Hoek-Brown criterion

$$\sigma_1' = \sigma_3' + \sigma_c \left(m_b \frac{\sigma_3'}{\sigma_c} + s \right)^a \qquad \qquad \frac{\sigma_1'}{\sigma_c} = \frac{\sigma_3'}{\sigma_c} + \left(m_b \frac{\sigma_3'}{\sigma_c} + s \right)^a$$

$$\sigma_1' = \sigma_3' + \sigma_c \left(m_i \frac{\sigma_3'}{\sigma_c} + 1 \right)^{\frac{1}{2}}$$

For intact rock,

s, a = constants (characteristics of the rock mass) σ_1 ', σ_3 ': axial and confining effective principal stresses σ_c : uniaxial compressive strength of intact rock piece m_b : value of the constant m for the rock mass

(2) For most rocks of good to reasonable quality (tightly interlocking angular rock pieces): a = 0.5

$$\sigma_1' = \sigma_3' + \sigma_c \sqrt{m_b \frac{\sigma_3'}{\sigma_c} + s}$$

(3) For poor quality rock mass (no tensile strength or cohesion): s = 0

$$\sigma_1' = \sigma_3' + \sigma_c \left(m_b \frac{\sigma_3'}{\sigma_c} \right)^a$$

(4) Above equations are of no practical value unless m_b , s and a can be estimated in some way.

- Hoek & Brown (1988) suggested a method to estimate them from RMR (\geq 25)
- Not satisfactory for all rock qualities

Empirical failure criterion $\sigma_1 = \sigma_3 + \sqrt{m\sigma_e\sigma_3 + s\sigma_e^2}$ $\sigma_1 = major principal stress;$ $\sigma_3 = minor principal stress;$ $\sigma_e = uniaxial compressive strength of intact rock, and m, s = empirical constants$	Carbonate rocks with well developed crystal cleavage Dolomite, limestone and marble	Lithified argillaceous rocks Mudstone, siltstone, shale and slate (normal to cleavage)	Arenaceous rocks with strong crystals and poorly developed crystal cleavage Sandstone and quartzite	Fine grained polyminerallic igneous crystalline rocks Andesite, dolerite, diabase and rhyolite	Coarse grained polyminerallic igneous and metamorphic crystalline rocks
Intact rock samples					
Laboratory size specimens free from joints RMR = 100 Q rating 500	m = 7.0 s = 1.0	m = 10.0 s = 1.0	m = 15.0 s = 1.0	m = 17.0 s = 1.0	m = 2 s = 1
Very good quality rock mass					
Tightly interlocking undisturbed rock with unweathered joints at 1 to 3 m RMR = 85 Q rating 100	m = 3.5 s = 0.1	m = 5.0 s = 0.1	m = 7.5 s = 0.1	m = 8.5 s = 0.1	m = 1 s = 0
Good quality rock mass					+
Fresh to slightly weathered rock, slightly disturbed with joints at 1 to 3 m RMR = 65 Q rating 10	m = 0.7 s = 0.004	m = 1.0 s = 0.004	m = 1.5 s = 0.004	m = 1.7 s = 0.004	m = 2 s = 0
Fair quality rock mass					
Several sets of moderately weathered joints spaced at 0.3 to 1 m RMR = 44 Q rating 1	m = 0.14 s = 0.0001	m = 0.20 s = 0.0001	m = 0.30 s = 0.0001	m = 0.34 s = 0.0001	m = (s = (
Poor quality rock mass					
Numerous weathered joints at 30 to 500 mm with some gouge. Clean compacted waste rock RMR = 23 Q rating 0.1	m = 0.04 s = 0.00001	m = 0.05 s = 0.00001	m = 0.08 s = 0.00001	m = 0.09 s = 0.00001	m = 0 s = 0
Very poor quality rock mass					
Numerous heavily weathered joints spaced < 50 mm with gouge. Waste rock with fines RMR = 3 Q rating 0.01	m = 0.007 s = 0	m = 0.010 s = 0	m = 0.015 s = 0	m = 0.017 $s = 0$	m = 0 s = 0

Notation: RMR - rock mass rating from the Geomechanics Classification; Q - quality of rock mass from the Q-System.

GENERA $\sigma_1' = majo$ $\sigma_3' = mino$ $\sigma_c = uniax$ piece m_b , s and the co	LISED HOEK-BROWN CRITERION $\sigma_1' = \sigma_3' + \sigma_c \left(m_b \frac{\sigma_3'}{\sigma_c} + s \right)^d$ or principal effective stress at failure or principal effective stress at failure tial compressive strength of <i>intact</i> are constants which depend on a composition, structure and surface inditions of the rock mass	SURFACE CONDITION	VERY GOOD Very rough, unweathered surfaces'	GOOD Rough, slightly weathered, iron stained surfaces	FAIR Smooth, moderately weathered or altered surfaces	POOR Slickensided, highly weathered surfaces with compact coatings or fillings containing angular rock fragments	VERY POOR Slickensided, highly weathered surfaces with soft clay coatings or fillings
	BLOCKY -very well interlocked undisturbed rock mass consisting of cubical blocks formed by three orthogonal discontinuity sets	m,∕m, s E_ V GSI	0.60 0.190 0.5 75,000 0.2 85	0.40 0.062 0.5 40,000 0.2 75	0.26 0.015 0.5 20,000 0.25 62	0.16 0.003 0.5 9,000 0.25 48	0.08 0.0004 0.5 3,000 0.25 34
	VERY BLOCKY-interlocked, partially disturbed rock mass with multifaceted angular blocks formed by four or more discontinuity sets	m,/m, s E GSI	0.40 0.062 0.5 40,000 0.2 75	0.29 0.021 0.5 24,000 0.25 65	0.16 0.003 0.5 9,000 0.25 48	0.11 0.001 0.5 5,000 0.25 38	0.07 0 0.53 2,500 0.3 25
	BLOCKY/SEAMY-folded and faulted with many intersecting discontinuities forming angular blocks	m,∕m, s a E, v GSI	0.24 0.012 0.5 18,000 0.25 60	0.17 0.004 0.5 10,000 0.25 50	0.12 0.001 0.5 6,000 0.25 40	0.08 0 0.5 3,000 0.3 30	0.06 0 2,000 0.3 20
	CRUSHED-poorly interlocked, heavily broken rock mass with a mixture of angular and rounded blocks	m,/m s a E_ GSI	0.17 0.004 0.5 10,000 0.25 50	0.12 0.001 0.5 6,000 0.25 40	0.08 0.5 3,000 0.3 30	0.06 0 0.55 2,000 0.3 20	0.04 0 0.60 1,000 0.3 10

Table 8.4: Estimation of constants $m_b / m_b s$, a, deformation modulus E and the Poisson's ratio v for the Generalised Hoek-Brown failure criterion based upon rock mass structure and discontinuity surface conditions. Note that the values given in this table are for an *undisturbed* rock mass.

Note 1: The in situ deformation modulus E_m is calculated from Equation 4.7 (page 47, Chapter 4). Units of E_m are MPa.

3. Strength of jointed rock masses



Mohr-Coulomb vs. Hoek-Brown Failure Criterion?



3. Strength of jointed rock masses

(6) Application to the Mohr-Coulomb failure criterion (Balmer, 1952)

$$\sigma_{n} = \sigma_{3} + \frac{\sigma_{1} - \sigma_{3}}{\partial \sigma_{1} / \partial \sigma_{3} + 1}$$
$$\tau = (\sigma_{1} - \sigma_{3}) \sqrt{\partial \sigma_{1} / \partial \sigma_{3}}$$

A set of (σ_n, τ) values $\rightarrow \sigma_n - \tau$ curve fitted (by linear regression analysis) \rightarrow c and ϕ determined from the plot



A simple spreadsheet (i.e. Excel) calculation can be carried out.

Input :	GSI =	62	sigci =	100	mi =	24		
Output:		sig3	sig1	ds1ds3	sign	tau	signtau	signsq
mb/mi =	= 0.26	0.10	14.48	22.47	0.71	2.91	2.07	0.51
mb =	= 6.18	0.20	16.55	19.89	0.98	3.49	3.41	0.96
S =	= 0.015	0.39	20.09	16.68	1.50	4.55	6.85	2.26
a =	= 0.5	0.78	25.87	13.31	2.53	6.39	16.20	6.42
Ε =	= 19953	1.56	34.91	10.26	4.52	9.48	42.90	20.46
phi =	= 48	3.13	48.70	7.78	8.32	14.48	120.44	69.18
coh =	= 3.4	6.25	69.56	5.88	15.45	22.31	344.80	238.78
sigcm =	= 18.0	12.5	101.20	4.48	28.68	34.26	982.51	822.60
				Sums =	62.70	97.88	1519.17	1161.16
Cell form	ulae:							
mb/mi =	= EXP((GSI	-100)/28)						
mb =	= mi*EXP((0	GSI-100)/28)						
S =	= IF(GSI>25	5 THEN EXP	((GSI-100)	/9) ELSE 0)				
a =	= IF(GSI>25	5 THEN 0.5 E	LSE (0.65	-GSI/200))				
E =	= 1000*10^((GSI-10)/40)	4					— F =
sig3 =	 sigci/2^n v 	where n starts	at 10 and	decreases b	y 1 for eac	ch subsequ	uent cell	\mathbf{L}_{m} –
sig1 =	 sig3+sigci 	*(((mb*sig3)/	sigci) + s)^	a				
ds1ds3 =	= IF(GSI>25	5 THEN 1+(n	nb*sigci)/(2	2*(sig1-sig3))	ELSE 1+((a*mb^a)*(sig3/sigci)^(a-1))
sign =	sig3+(sig1	-sig3)/(1+ds1	ds3)					
tau =	= (sign-sig3))*SQRT(ds1d	s3)					
signtau =	sign*tau		signsq =	sign^2				
phi =	= (ATAN((su	um(signtau)-(sum(sign)*	sum(tau))/8)/	(sum(sign/	sq)-((sum	(sign))^2)/8)))*180/PI()
n coh =	= (sum(tau)/	/8) - (sum(sig	n)/8)*TAN	(phi*PI()/180)				
sigcm =	= (2*coh*CC	OS(phi*PI()/18	30))/(1-SIN	(phi*PI()/180))			

* See a spreadsheet calculation

4. Use of rock mass classification for estimating GSI

(1) Bieniawski's 1976 RMR classification

Table 8.5: Part of Bieniawski's 1976 table defining the Geomechanics Classification or Rock Mass Rating (RMR) for jointed rock masses.

	PARAM	IETER			RANGE OF VALUES				
	Strength of	Point-load strength index	>8 MPa	4-8 MPa	2-4 MPa	1-2 MPa	For this ial comp preferred	low rang ressive t l	euniax- est is
1	intact rock material	Uniaxial compressive strength	>200 MPa	100-200 MPa	50-100 MPa	25-50 MPa	10-25 MPa	3-10 MPa	1-3 MPa
	R	ating	15	12	7	4	2	2 1	
	Drill core quality RQD		90%- 100%	75%- 90%	50%-75%	25%-50%	25%-50% <		
2	Rating		20	17	13	8		3	
	Spacin	g of joints	> 3 m	1-3 m	0.3-1 m	50-300 mm	< 50 mm		
3	R	ating	30	25	20	10	10		
4	Condition of joints		Very rough surfaces Not continuous No separation Hard joint wall contact	Slightly rough surfaces Separation < 1 mm Hard joint wall contact	Slightly rough surfaces Separation < 1 mm Soft joint wall contact	Slickensided surfaces or Gouge < 5 mm thick or Joints open 1-5 mm Continuous joints	es Soft gouge >5 m or k Joints open > Continuous j		m thick 5 mm bints
	R	ating	25	20	12	6		0	

 $\Sigma_{max} = 90 + 10$ (for groundwater condition)

 $\Sigma_{\rm min}$ = 8 + 10 = 18

RMR_{76'} < 18 (Inaccurate)

Bieniawski's 1989 RMR classification

Table 4.4: Rock Mass Rating System (After Bieniawski, 1989).

	I	Parameter			Range of values				
Streng		Point-load strength index	>10 MPa	4-10 MPa	2-4 MPa	1-2 MPa	For this uniaxial test is pr	low rang compre- eferred	ge - ssive
1 2 3	intact re materi	ock Uniaxial comp. al strength	c Uniaxial comp. >250 MPa 100-250 MPa 50-100 MPa strength	25-50 MPa	5-25 MPa	I-5 MPa	<1 MPa		
	1. Baller	Rating	15	12	7	4	2	1	0
	Drill	core Quality RQD	90%-100%	75%-90%	50%-75%	25%-50%	< 25%		
2	bell mile	Rating	20	• 17	13	8		3	
	Spacin	ng of discontinuities	> 2 m	0.6-2 . m	200-600 mm	60-200 mm < 0		< 60 mm	
3		Rating	20	15	10	8	5		
4	Condit	ion of discontinuities (See E)	Very rough surfaces Not continuous No separation Unweathered wall rock	Slightly rough surfaces Separation < 1 mm Slightly weathered walls	Slightly rough surfaces Separation < 1 mm Highly weathered walls	Slickensided surfaces or Gouge < 5 mm thick or Separation 1-5 mm Continuous	Soft gou thick Separatio Continue	ge >5 m or on > 5 m ous	າກ
		Rating	30	25	20	10	1	0	
		Inflow per 10 m tunnel length (l/m)	None	< 10	10-25	25-125		> 125	
5	Ground water	(Joint water press)/ (Major principal σ)	0	< 0.1	0.1,-0.2	0.2-0.5		> 0.5	
		General conditions	Completely dry	Damp	Wet	Dripping	1	Flowing	
		Rating	15	10	7	4		0	

 Σ_{max} = 85 + 15 (for groundwater condition)

 $\Sigma_{\rm min}$ = 8 + 15 = 23

RMR₈₉, < 23 (Inaccurate)

4. Use of rock mass classification for estimating GSI

(1) Bieniawski's 1976 RMR classification

For RMR_{76'} > 18, GSI = RMR_{76'} RMR_{76'} < 18, Use Q' value. RMR_{76'} cannot be used to estimate GSI)

(2) Bieniawski's 1989 RMR classification

For $RMR_{89'} > 23$, $GSI = RMR_{89'} - 5$ $RMR_{89'} < 23$, Use Q' value. $RMR_{89'}$ cannot be used to estimate GSI)

(3) Modified Barton, Lien and Lunde's Q' classification

J_w = 1, SRF = 1 (Dry, medium stress condition) (Q_{min}' = 0.0208, GSI ≈ 9) $Q' = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF} = \frac{RQD}{J_n} \times \frac{J_r}{J_a}$ GSI = 9 · log_e Q'+44

5. When to use the Hoek-brown failure criterion



Figure 8.3: Rock mass conditions under which the Hoek-Brown failure criterion can be applied.