

2009 fall

# Phase Transformation of Materials

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# Contents for previous class

- **Binary System** mixture/ solution / compound

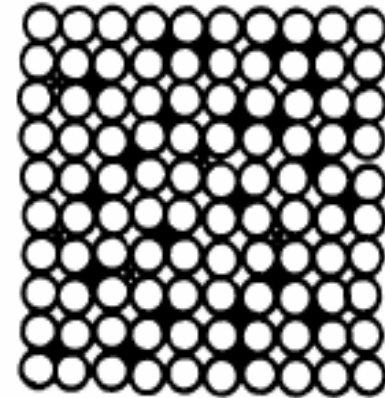
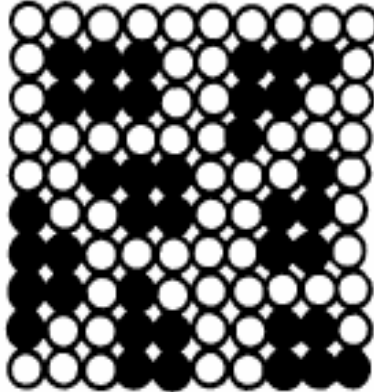
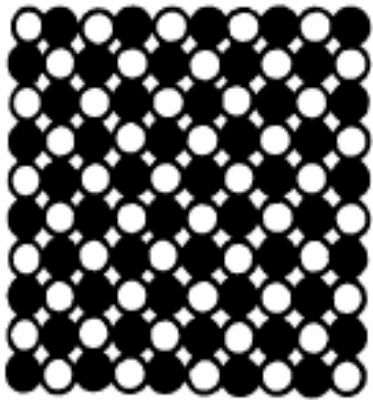
**Ideal solution** ( $\Delta H_{\text{mix}} = 0$ ) **Random distribution**

**Regular solution**  $\Delta H_{\text{mix}} = P_{AB}\epsilon$  where  $\epsilon = \epsilon_{AB} - \frac{1}{2}(\epsilon_{AA} + \epsilon_{BB})$   $\epsilon \approx 0$

$\Delta H_{\text{mix}} > 0$  or  $\Delta H_{\text{mix}} < 0$

**Real solution**

**Ordered structure**



(a)  $\epsilon < 0, \Delta H_{\text{mix}} < 0$

**Ordered alloys**

$P_{AB} \uparrow \longrightarrow$  내부  $E \downarrow$

(b)  $\epsilon > 0, \Delta H_{\text{mix}} > 0$

**Clustering**

$P_{AA}, P_{BB} \uparrow$

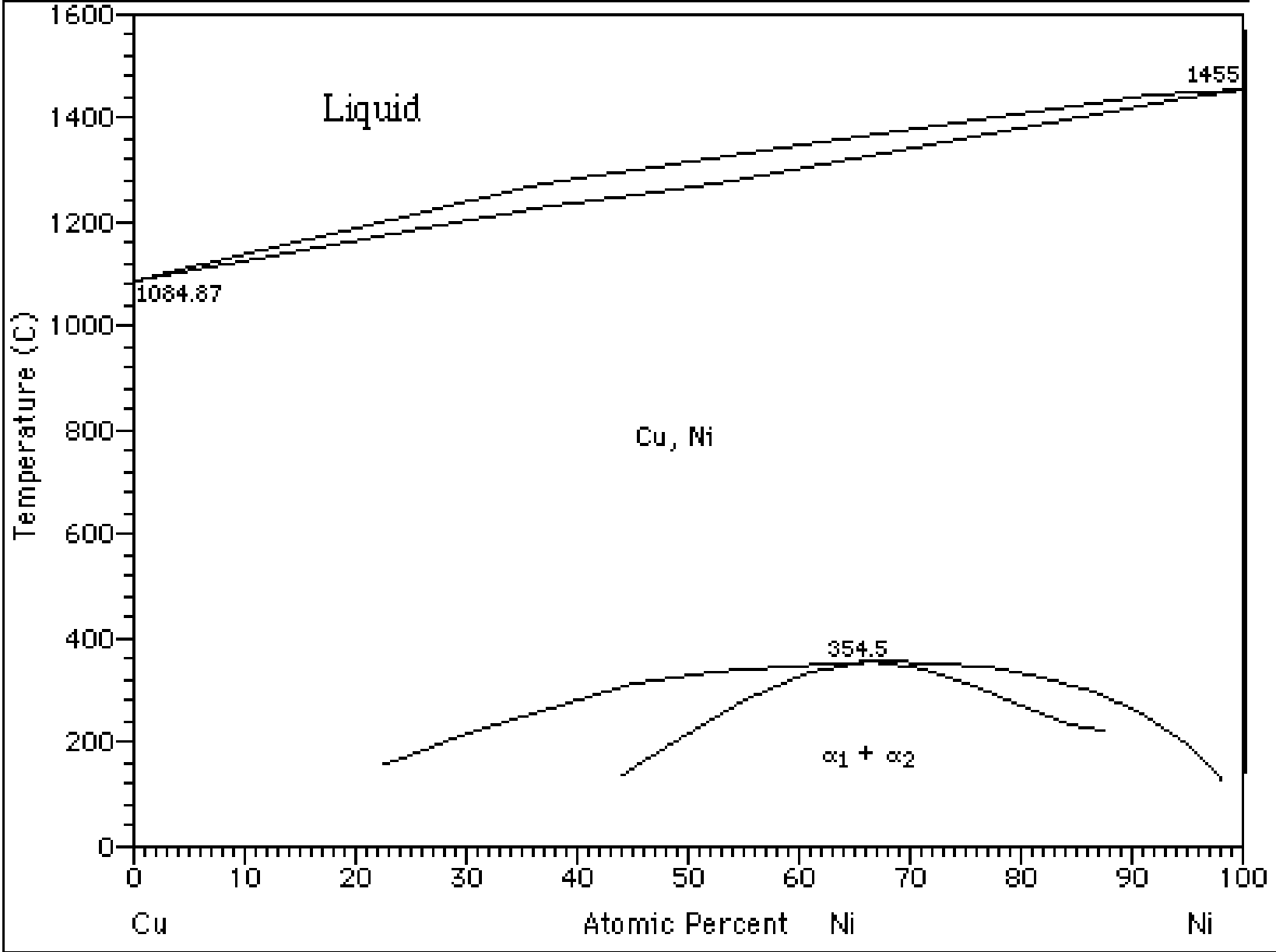
(c) *when the size difference is large*

**strain effect**

**Interstitial solution**

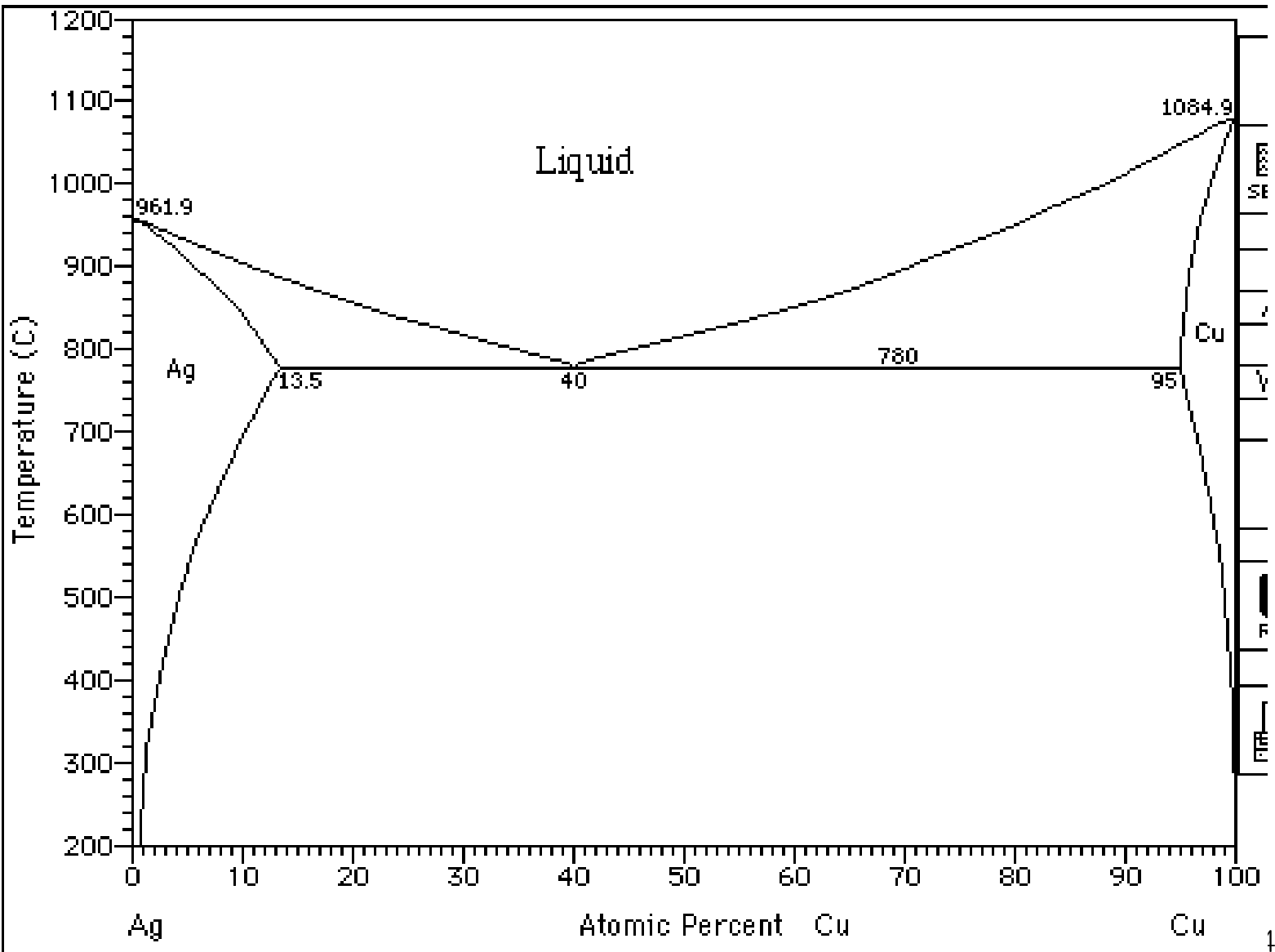
# Contents for previous class

$\epsilon > 0, \Delta H_{\text{mix}} > 0 / \Delta H_{\text{mix}} \sim +26 \text{ kJ/mol}$



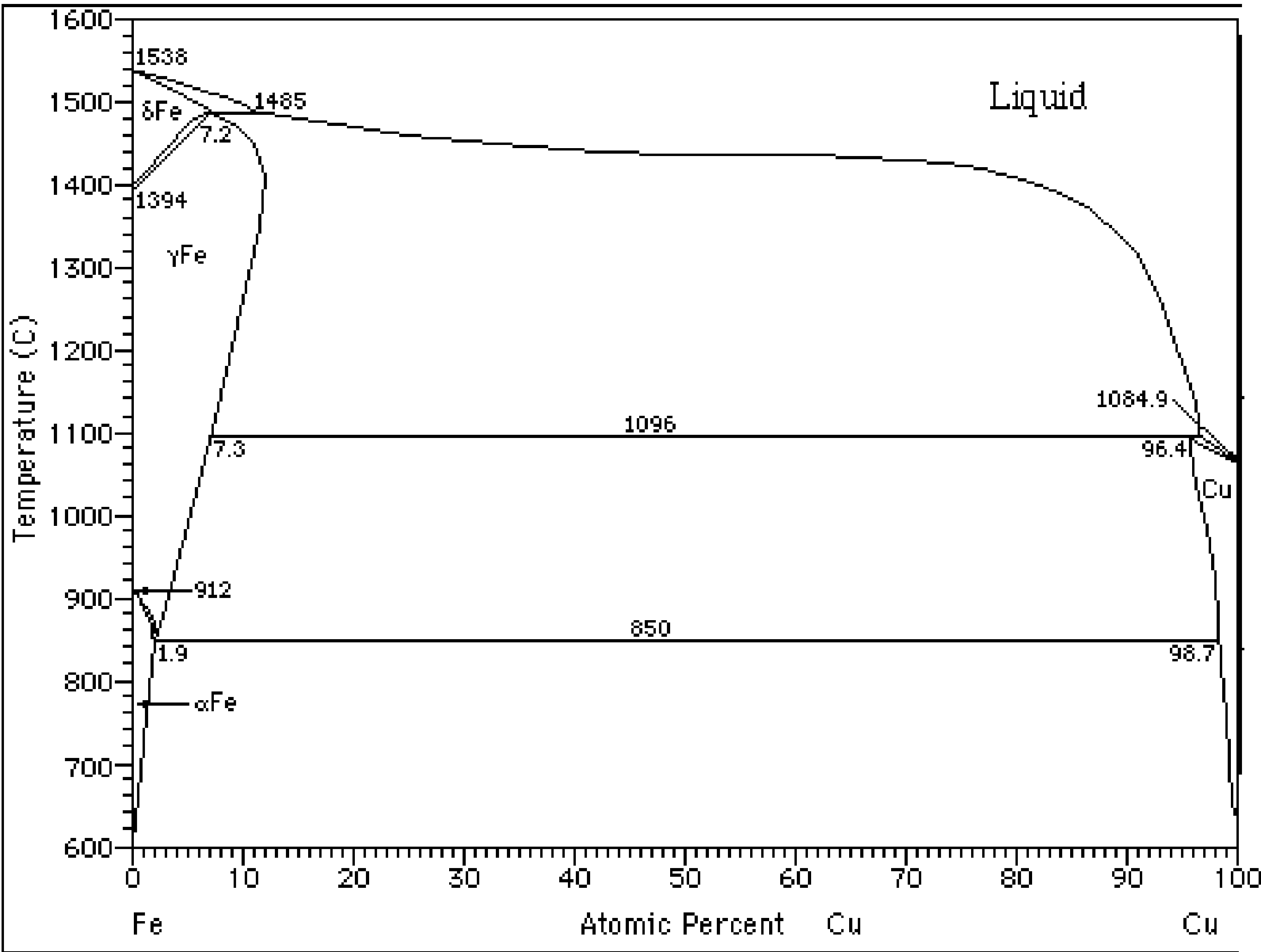
# Contents for previous class

$\epsilon > 0, \Delta H_{\text{mix}} > 0 / \Delta H_{\text{mix}} \sim +5 \text{ kJ/mol}$



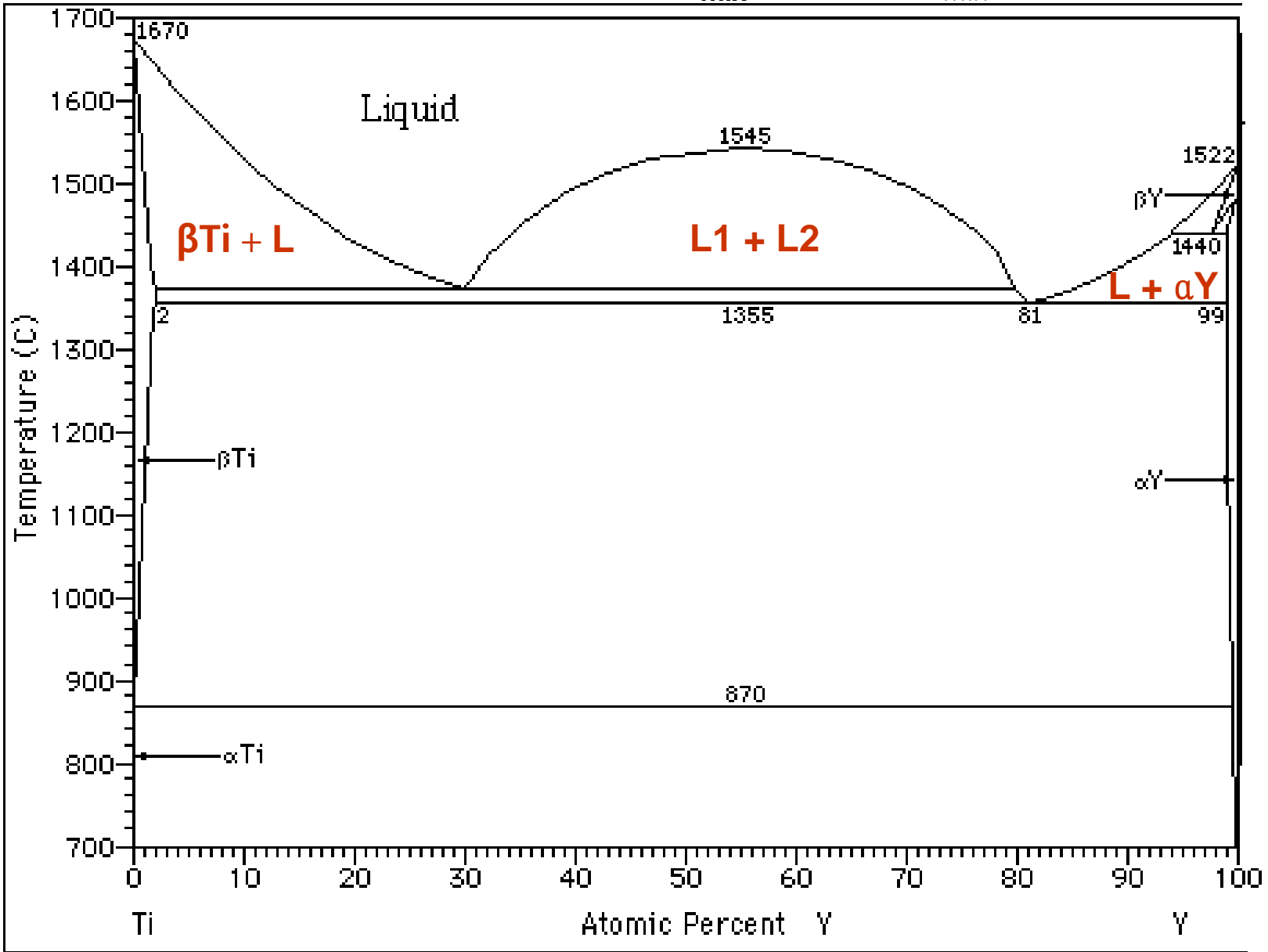
# Contents for previous class

$\epsilon \gg 0, \Delta H_{\text{mix}} \gg 0 / \Delta H_{\text{mix}} \sim +60 \text{ kJ/mol}$



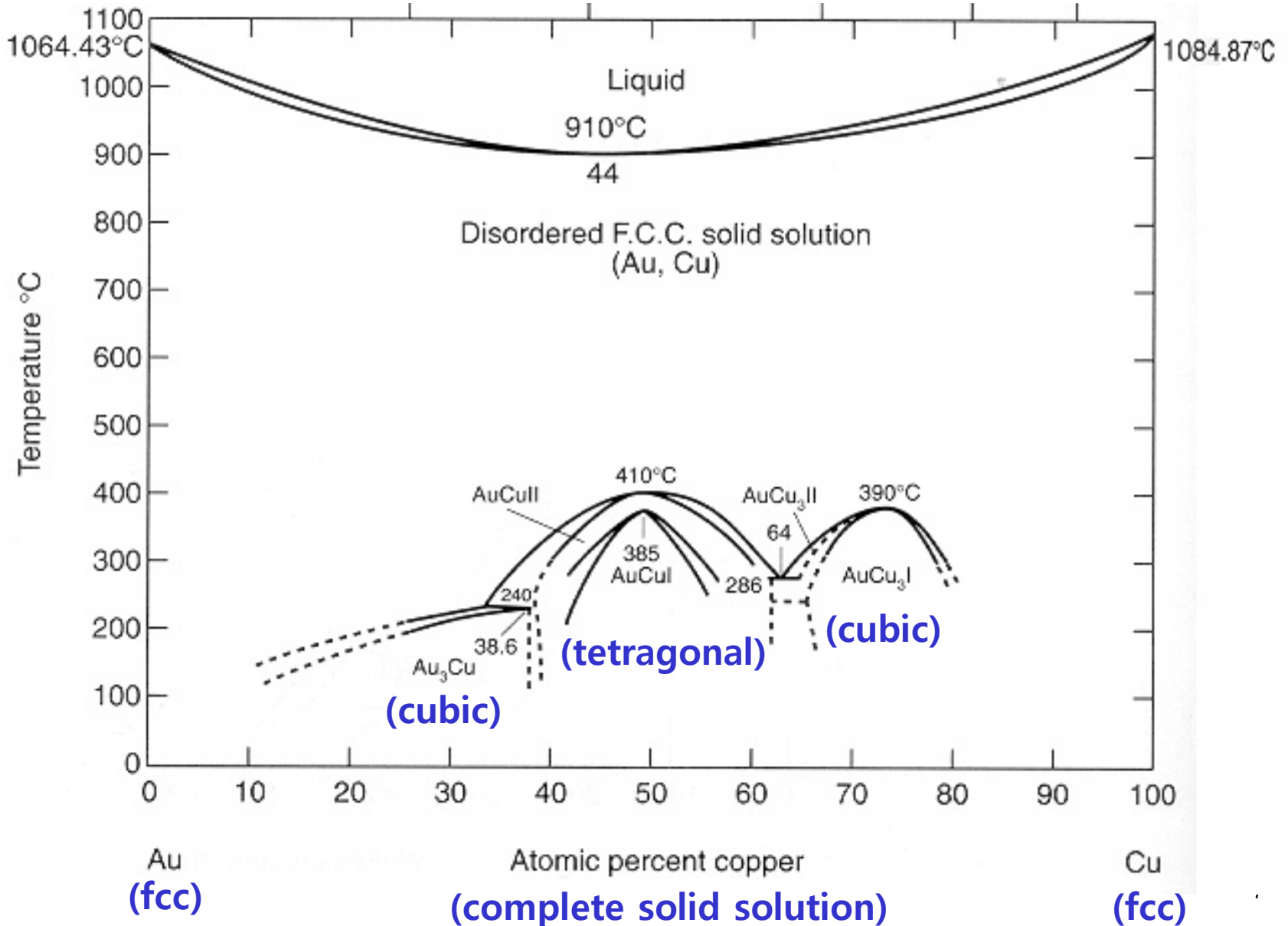
# Contents for previous class

$\epsilon \gg 0, \Delta H_{\text{mix}} \gg 0 / \Delta H_{\text{mix}} \sim +58 \text{ kJ/mol}$



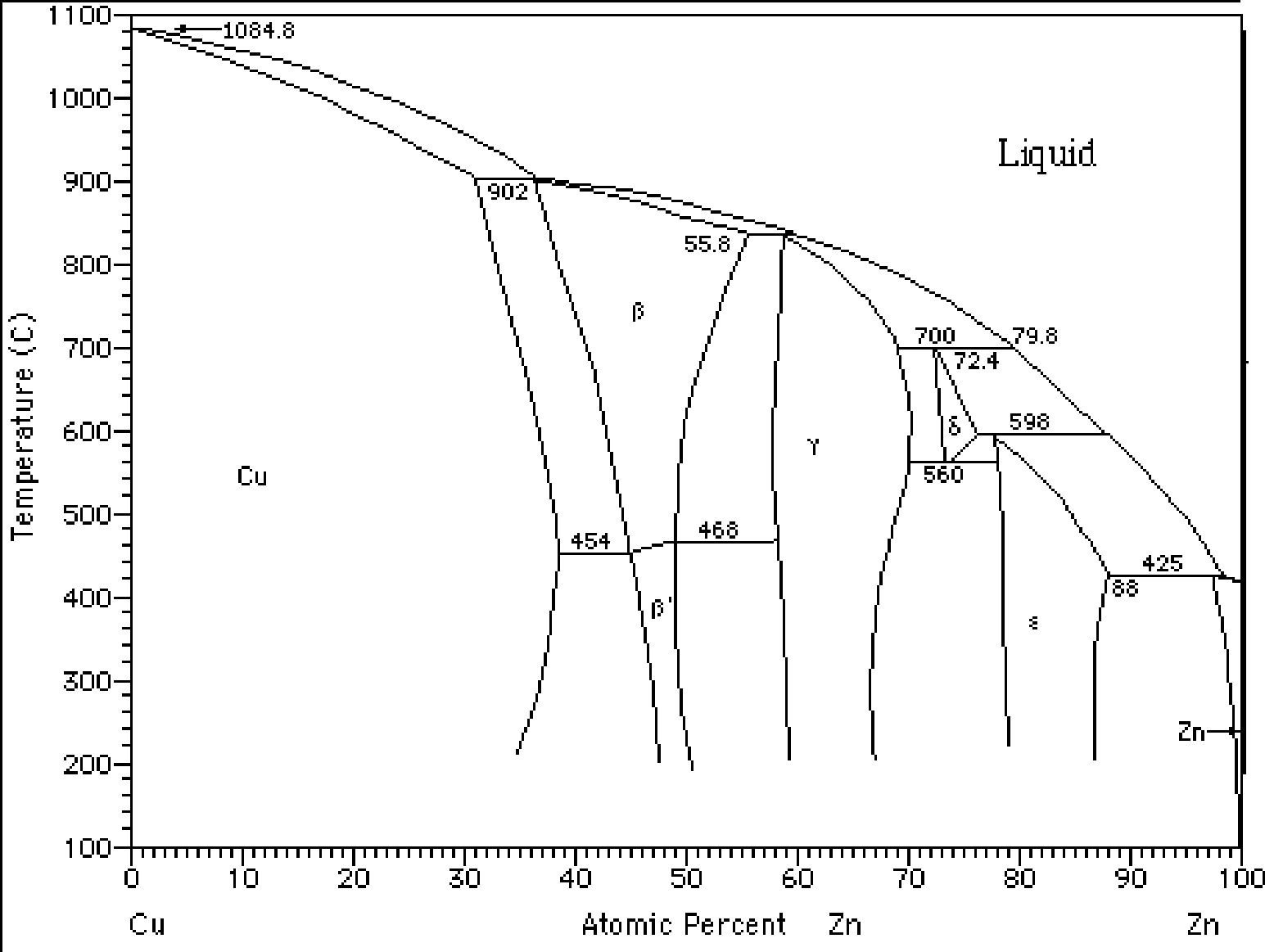
# Contents for previous class

$$\epsilon < 0, \Delta H_{\text{mix}} < 0 / \Delta H_{\text{mix}} \sim -20 \text{ kJ/mol}$$



# Contents for previous class

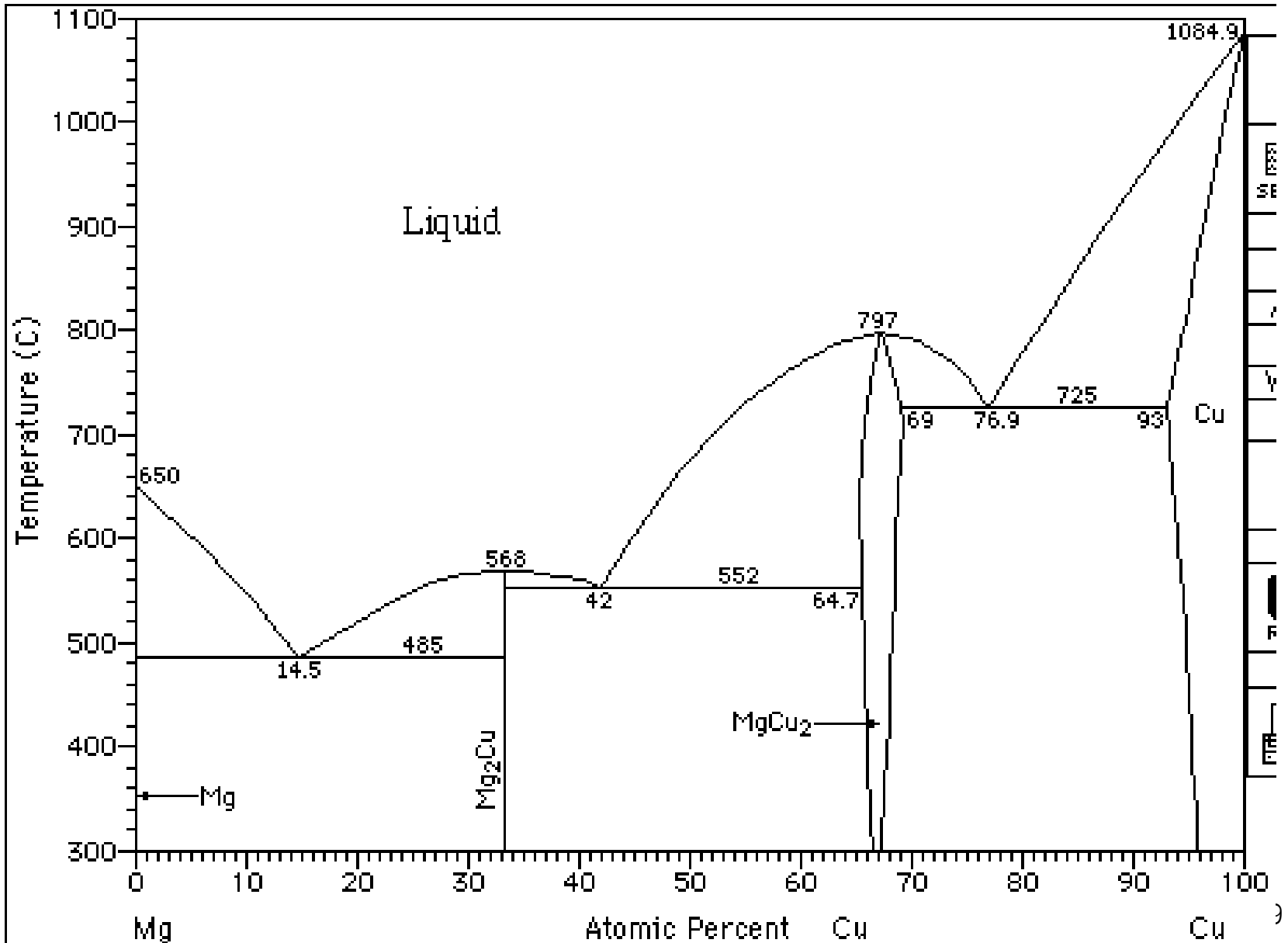
$\epsilon < 0, \Delta H_{\text{mix}} < 0 / \Delta H_{\text{mix}} \sim -21 \text{ kJ/mol}$





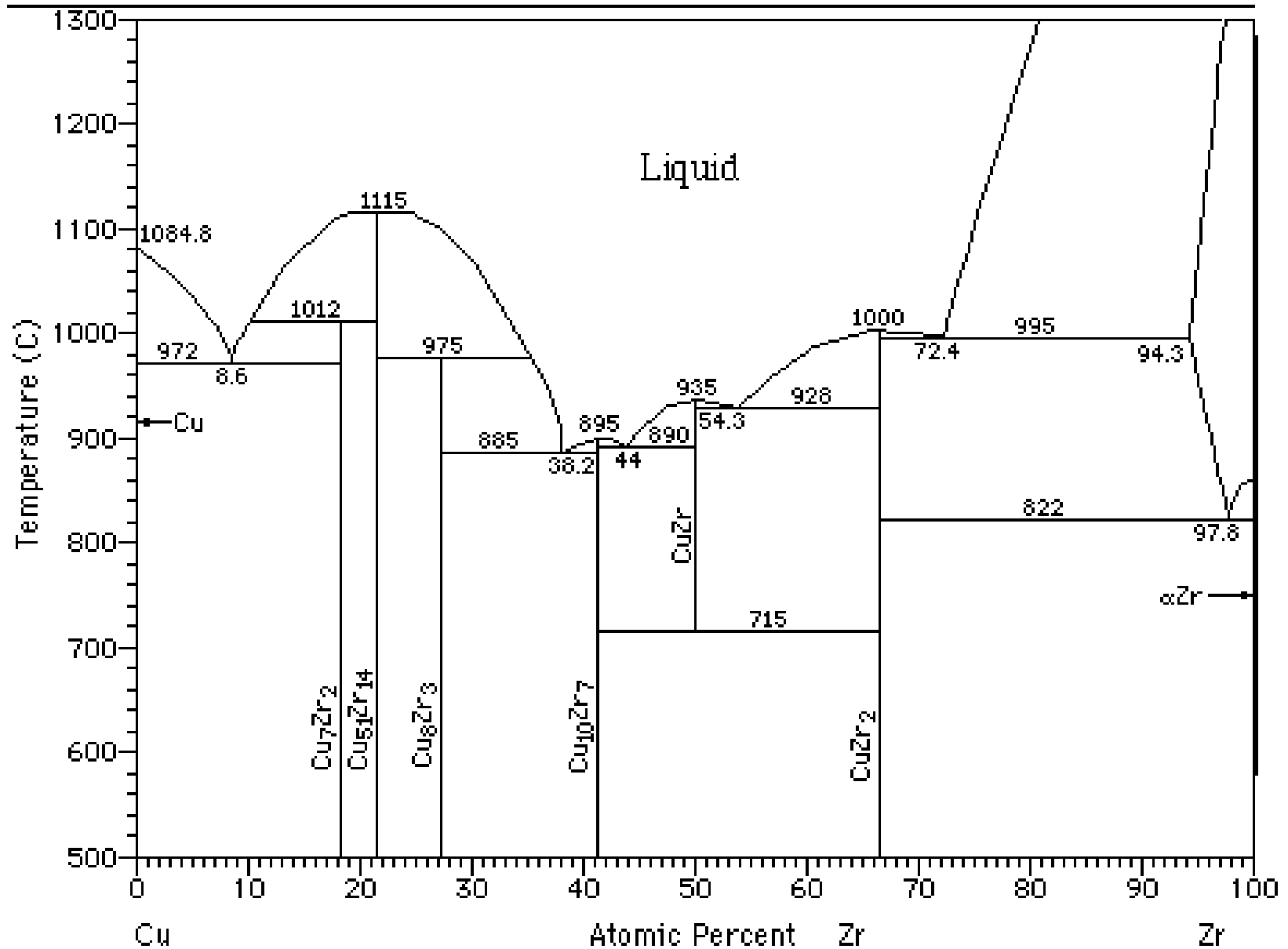
# Contents for previous class

$$\epsilon < 0, \Delta H_{\text{mix}} < 0 / \Delta H_{\text{mix}} \sim -38 \text{ kJ/mol}$$



# Contents for previous class

$$\varepsilon \ll 0, \Delta H_{\text{mix}} \ll 0 / \Delta H_{\text{mix}} \sim -142 \text{ kJ/mol}$$



# Contents for today's class

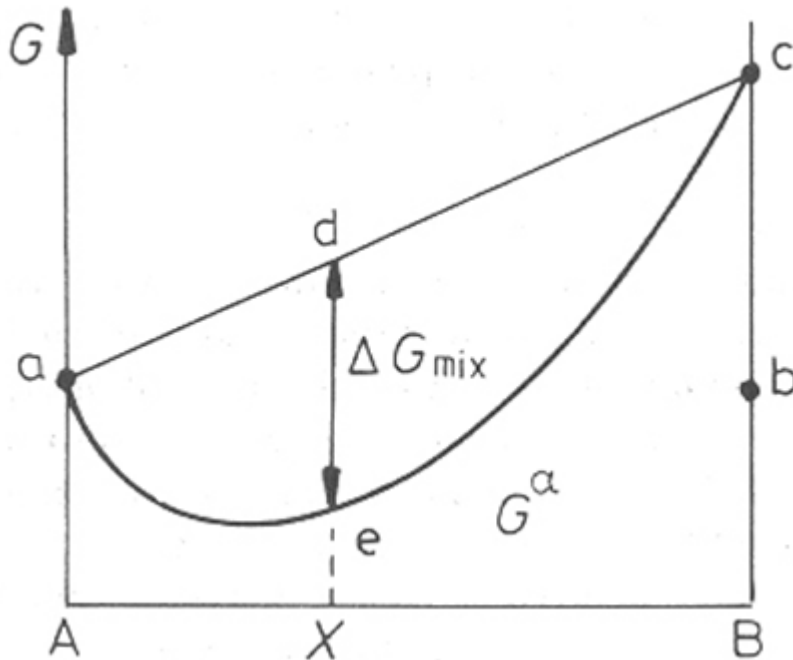
- **Equilibrium in Heterogeneous Systems**
- **Binary phase diagrams**
  - 1) **Simple Phase Diagrams**
  - 2) **Systems with miscibility gap**
  - 4) **Simple Eutectic Systems**
  - 3) **Ordered Alloys**
  - 5) **Phase diagrams containing intermediate phases**

# 1.4

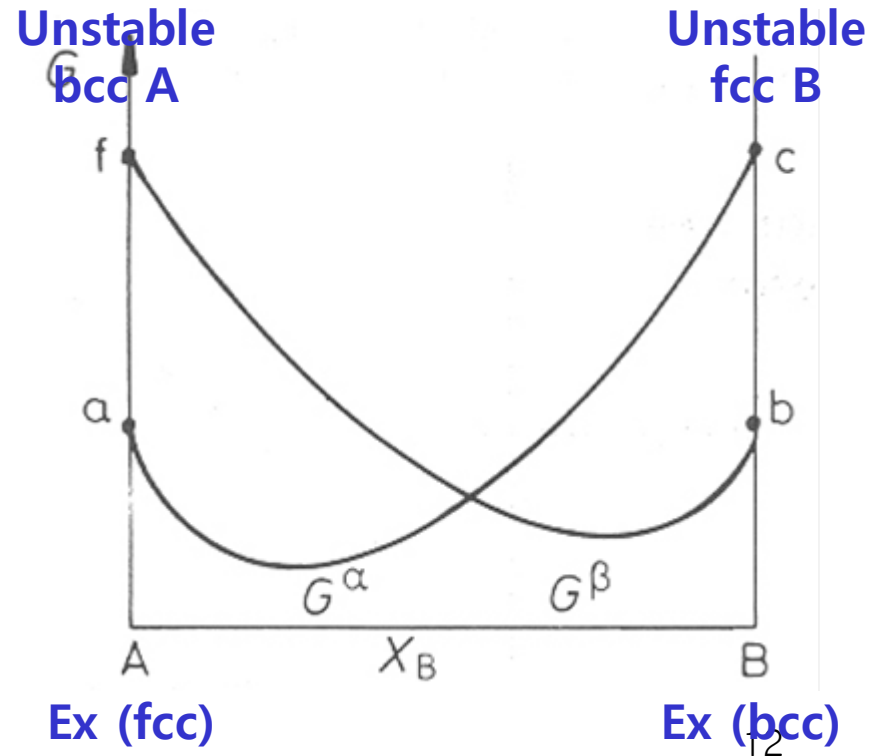
## Equilibrium in Heterogeneous Systems

We have dealt with the case where the components A and B have the same crystal structure.

$$G = X_A G_A + X_B G_B + \Omega X_A X_B + RT(X_A \ln X_A + X_B \ln X_B)$$



What would happen when the components A and B have a different crystal structure?  
 → **heterogeneous system**



## 1.4

# Equilibrium in Heterogeneous Systems

If  $G^\alpha(X_B^\alpha)$  and  $G^\beta(X_B^\beta)$  are given,  
 what would be  $G(\alpha + \beta)$  at  $X_B^o = ?$

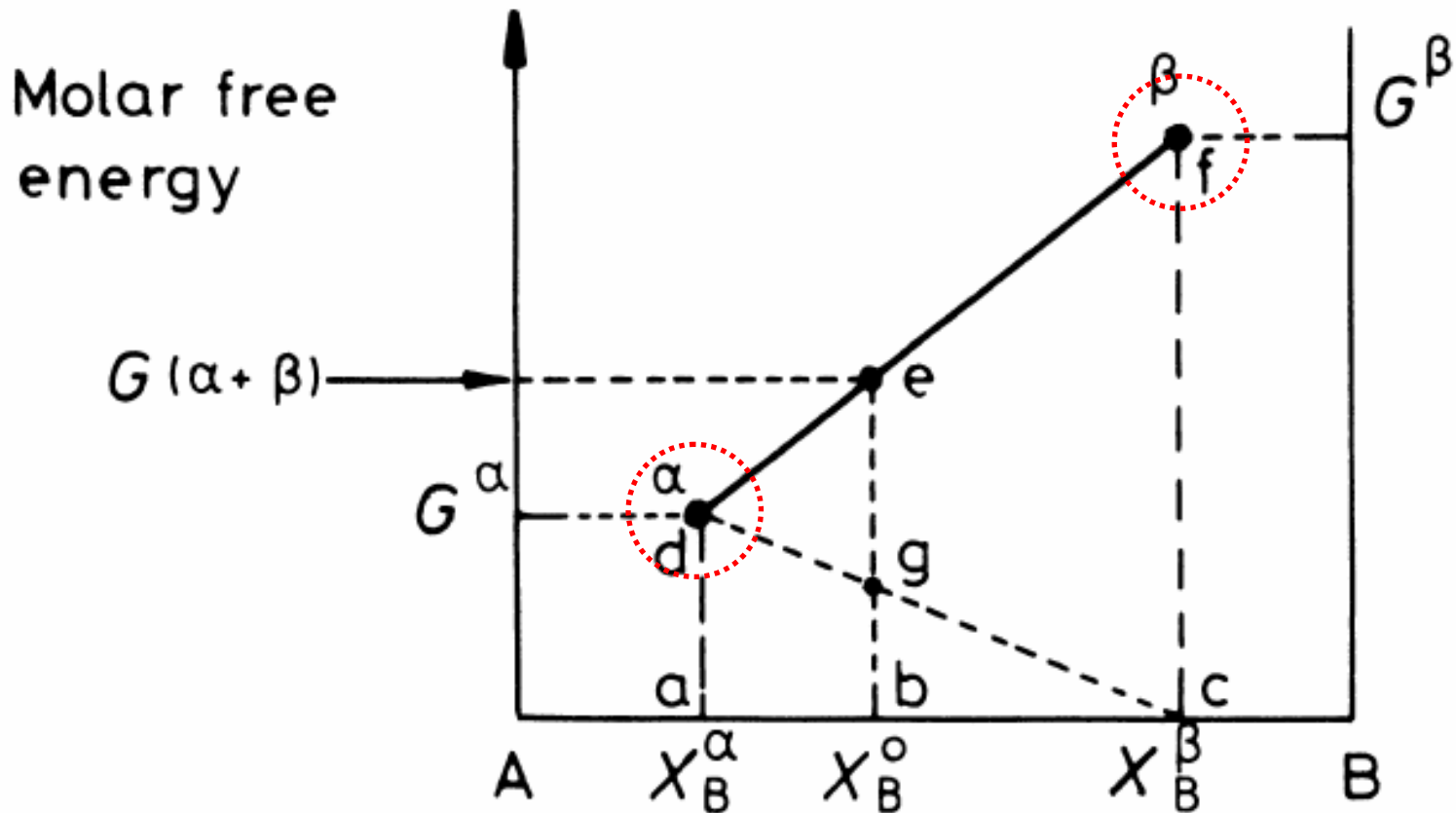
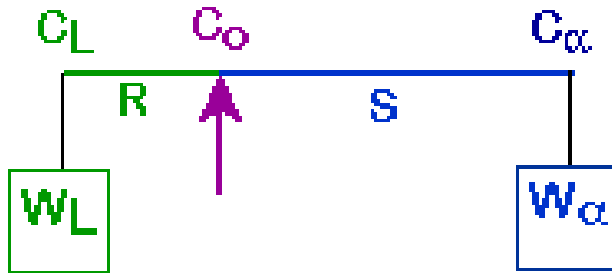


Fig. 1.26 The molar free energy of a two-phase mixture ( $\alpha + \beta$ )

# Lever rule

A geometric interpretation:



moment equilibrium:

$$W_L R = W_\alpha S$$

↑  
1 - W<sub>α</sub>

solving gives Lever Rule

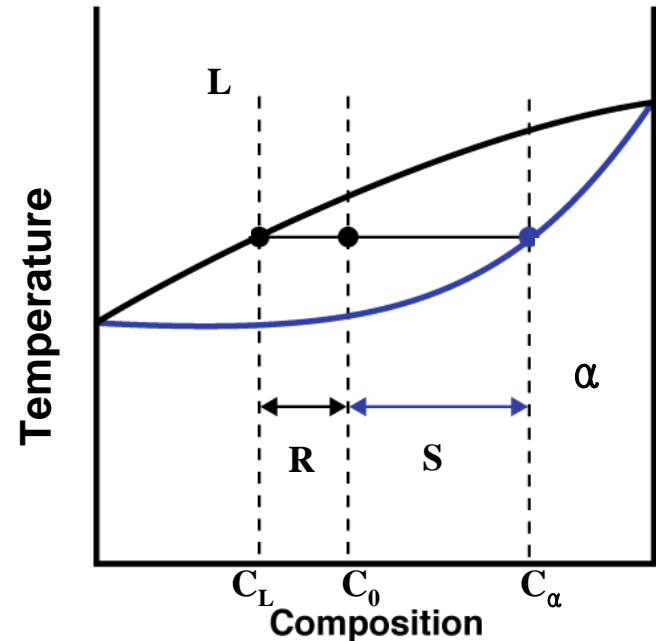
Sum of weight fractions:  $W_L + W_\alpha = 1$

Conservation of mass (Ni):  $C_0 = W_L C_L + W_\alpha C_\alpha$

Combine above equations:

$$W_L = \frac{C_\alpha - C_0}{C_\alpha - C_L} = \frac{S}{R + S}$$

$$W_\alpha = \frac{C_0 - C_L}{C_\alpha - C_L} = \frac{R}{R + S}$$



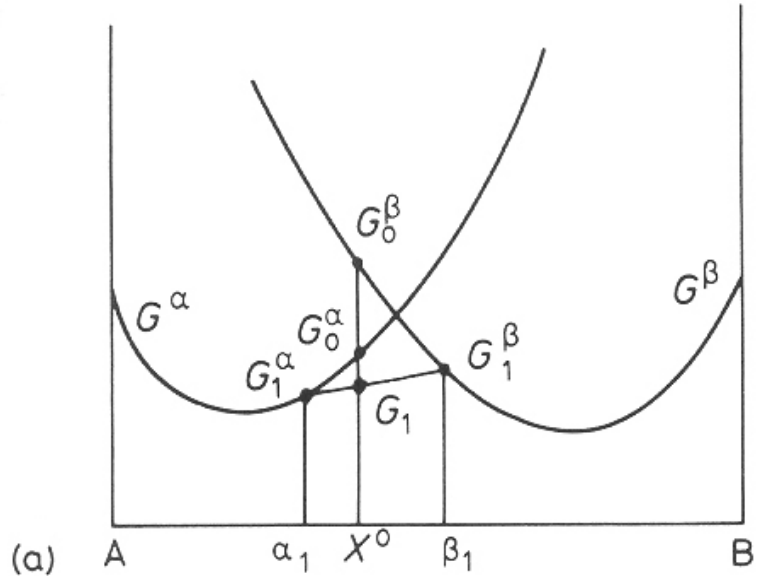
# 1.4

## Equilibrium in Heterogeneous Systems

In  $X^0$ ,  $G_0^\beta > G_0^\alpha > G_1$

A 및 B 원자 교환

→  $\alpha + \beta$  로 분리

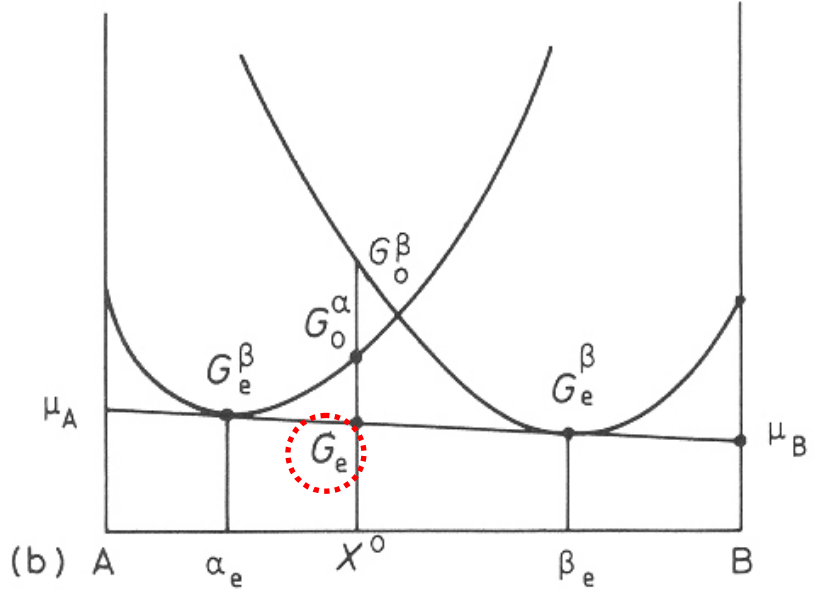


**Chemical Equilibrium ( $\mu$ , a)**  
 → multiphase and multicomponent  
 ( $\mu_i^\alpha = \mu_i^\beta = \mu_i^\gamma = \dots$ ), ( $a_i^\alpha = a_i^\beta = a_i^\gamma = \dots$ )

$$\mu_A^\alpha = \mu_A^\beta$$

$$\mu_B^\alpha = \mu_B^\beta$$

두상의 화학 포텐셜 일치



# Variation of activity with composition

The most stable state, with the lowest free energy, is usually defined as the state in which the pure component has unit activity of A in pure  $\alpha$ .

when  $X_A = 1 \rightarrow a_A^\alpha = 1$

when  $X_B = 1 \rightarrow a_B^\beta = 1$

when  $\alpha$  and  $\beta$  in equil.

$$a_A^\alpha = a_A^\beta$$

$$a_B^\alpha = a_B^\beta$$

두 성분의 activity 일치

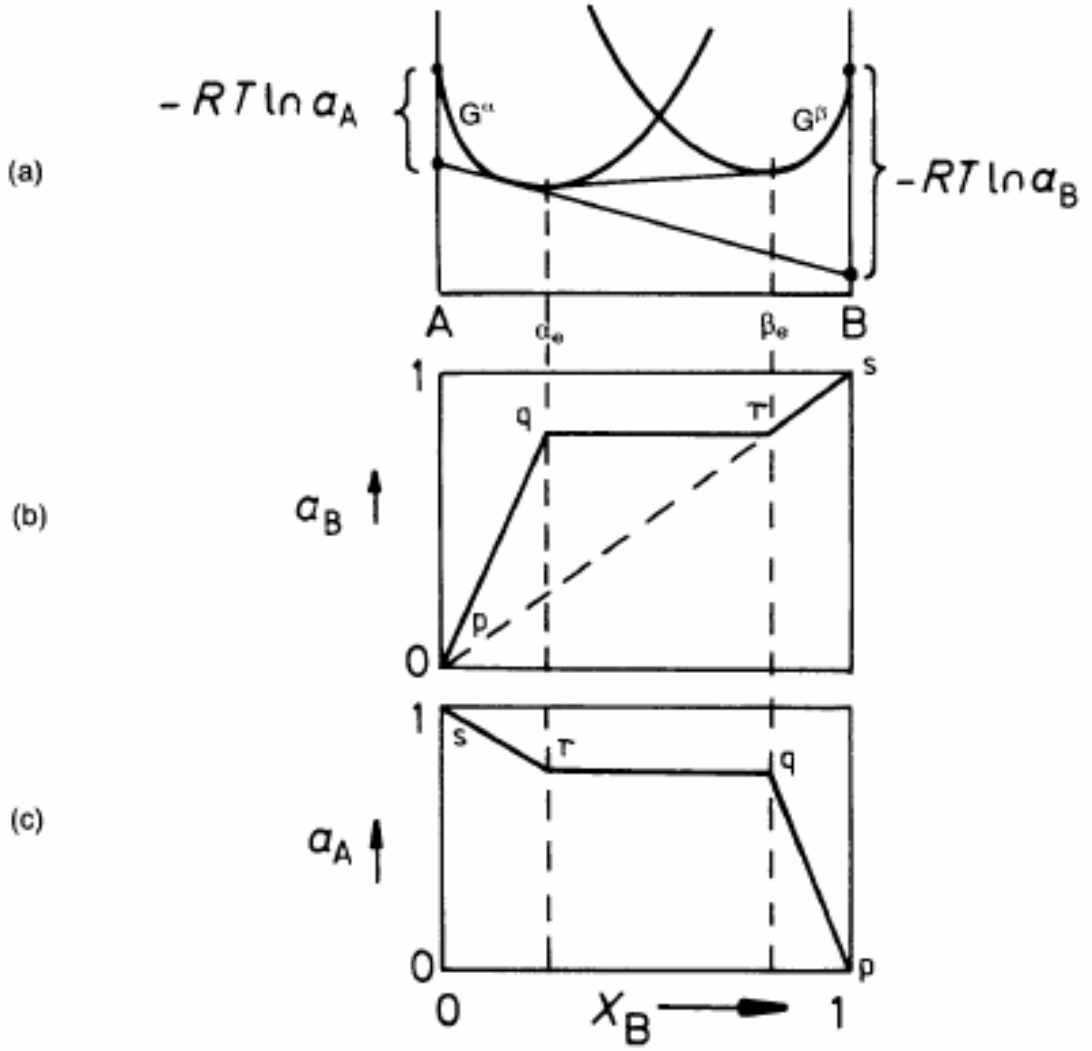
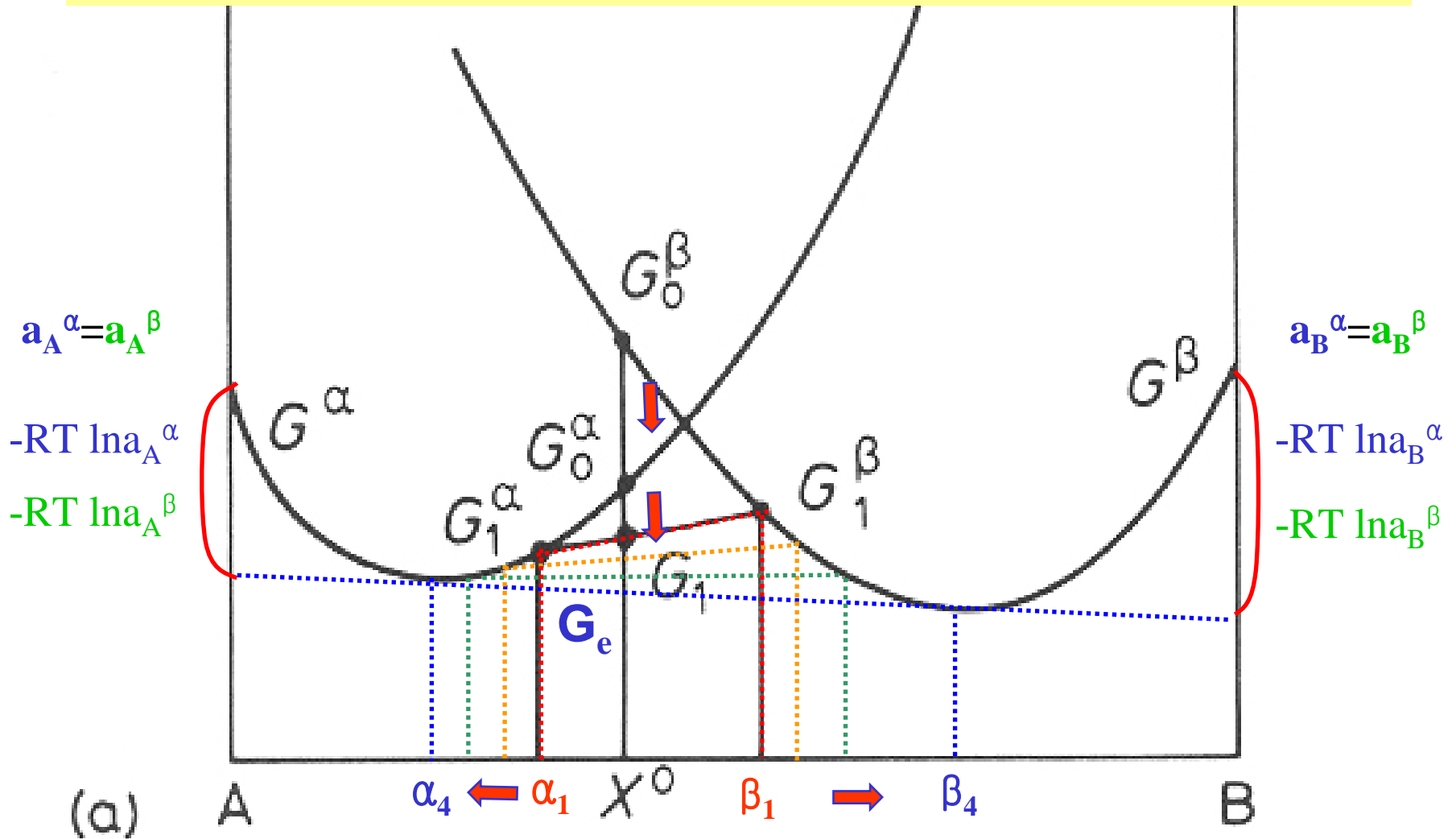


Fig. 1.28 The variation of  $a_A$  and  $a_B$  with composition for a binary system containing two ideal solutions,  $\alpha$  and  $\beta$



# Equilibrium in Heterogeneous Systems

In  $X^0$ ,  $G_0^\beta > G_0^\alpha > G_1 \rightarrow \alpha + \beta$  로 분리  $\rightarrow$  두상의 화학 포텐셜 일치



# 1.5 Binary phase diagrams

## 1) Simple Phase Diagrams

가정: (1) completely miscible in solid and liquid.

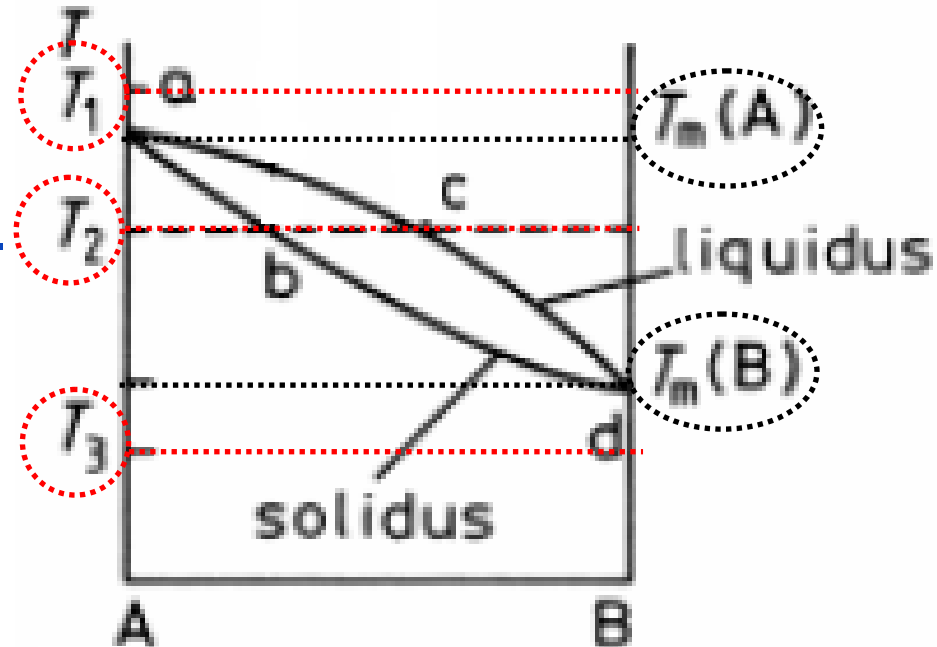
(2) Both are ideal soln.

$$\Delta H_{mix}^L = 0 \quad \Delta H_{mix}^S = 0$$

(3)  $T_m(A) > T_m(B)$

(4)  $T_1 > T_m(A) > T_2 > T_m(B) > T_3$

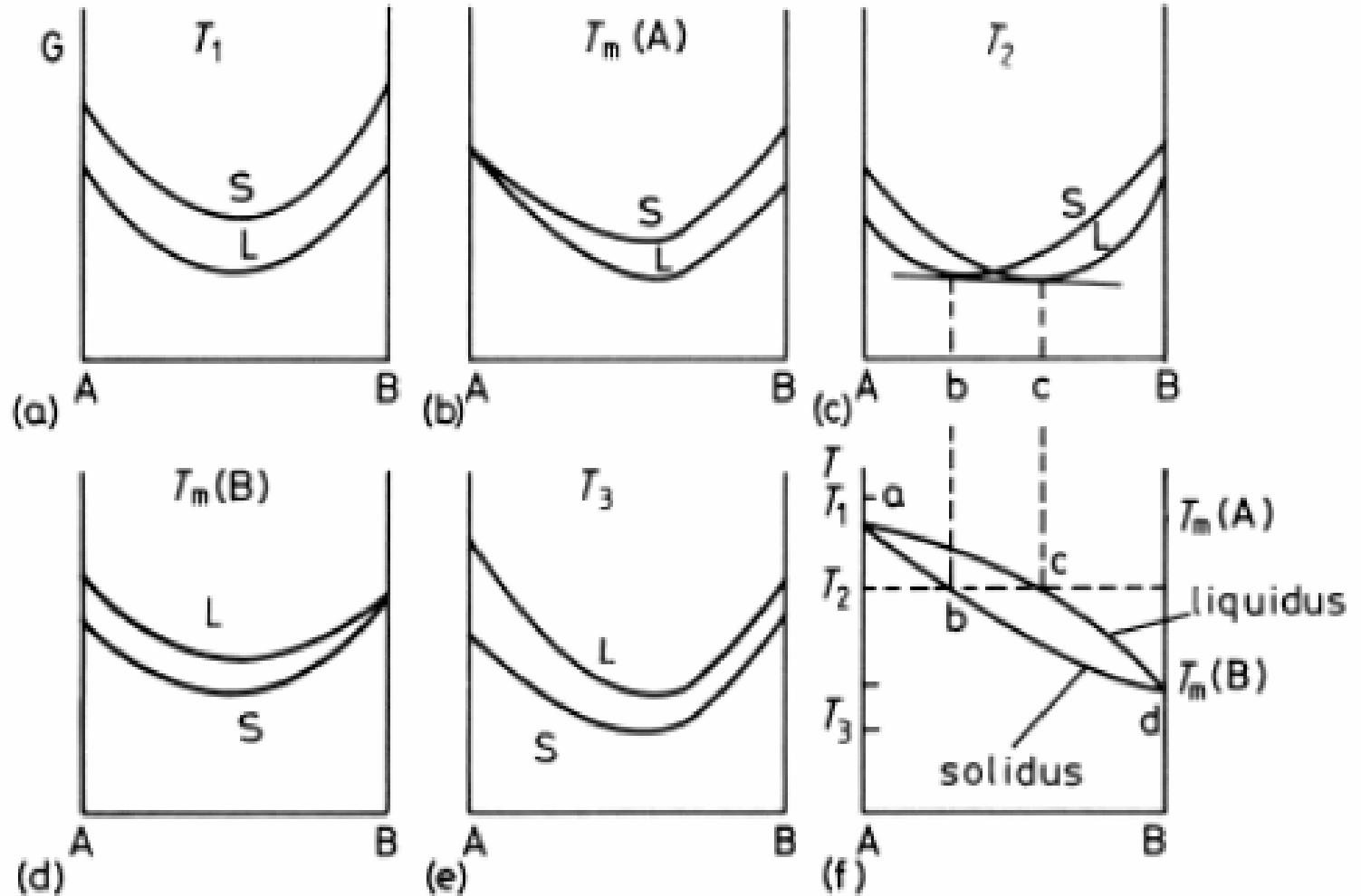
Draw  $G^L$  and  $G^S$  as a function of composition  $X_B$  at  $T_1$ ,  $T_m(A)$ ,  $T_2$ ,  $T_m(B)$ , and  $T_3$ .



# 1.5 Binary phase diagrams

## 1) Simple Phase Diagrams

- 가정: (1) completely miscible in solid and liquid.  
(2) Both are ideal soln.  
(3)  $T_m(A) > T_m(B)$   
(4)  $T_1 > T_m(A) > T_2 > T_m(B) > T_3$

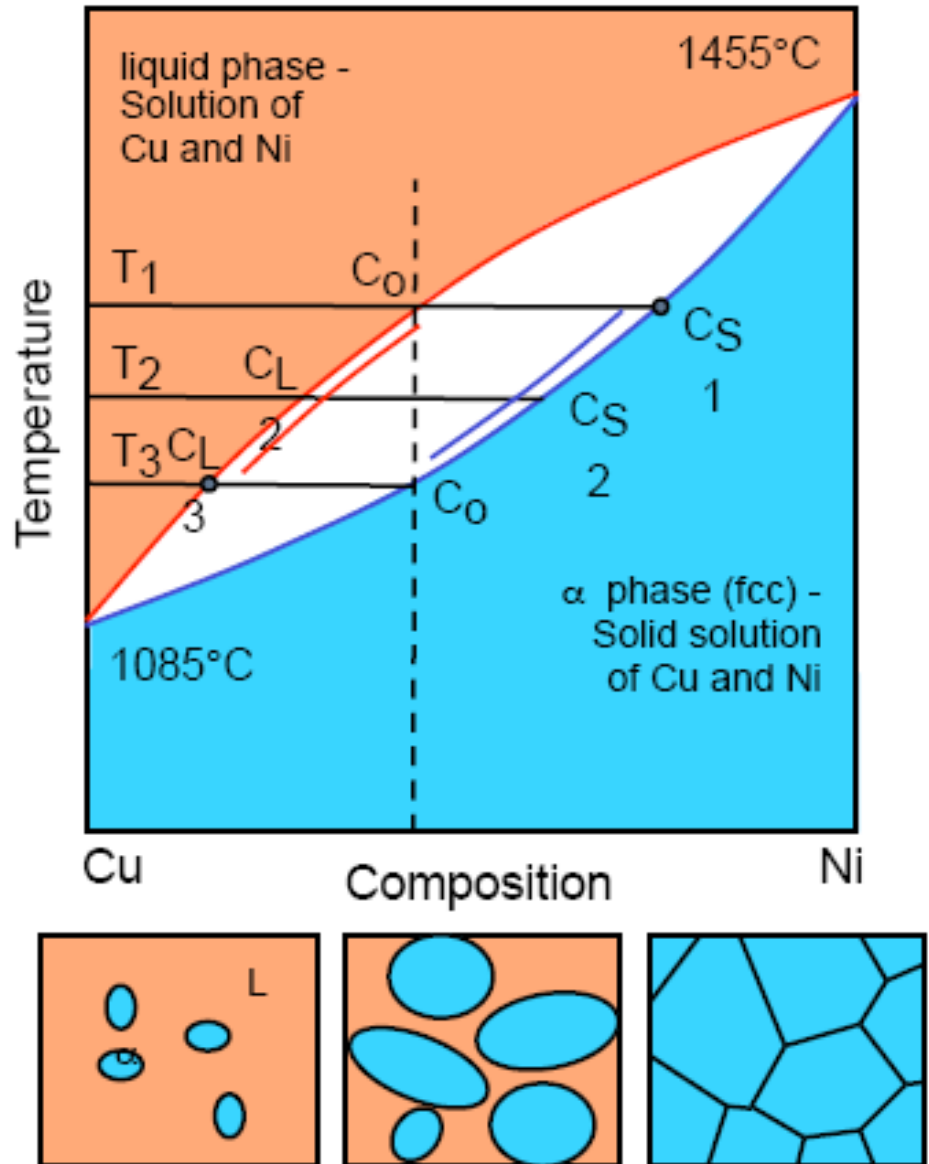


# 1.5 Binary phase diagrams

## 1) Simple Phase Diagrams

The simplest type of binary phase diagrams is the isomorphous system, in which the two constituents form a continuous solid solution over the entire composition range. An example is the Ni-Cu system.

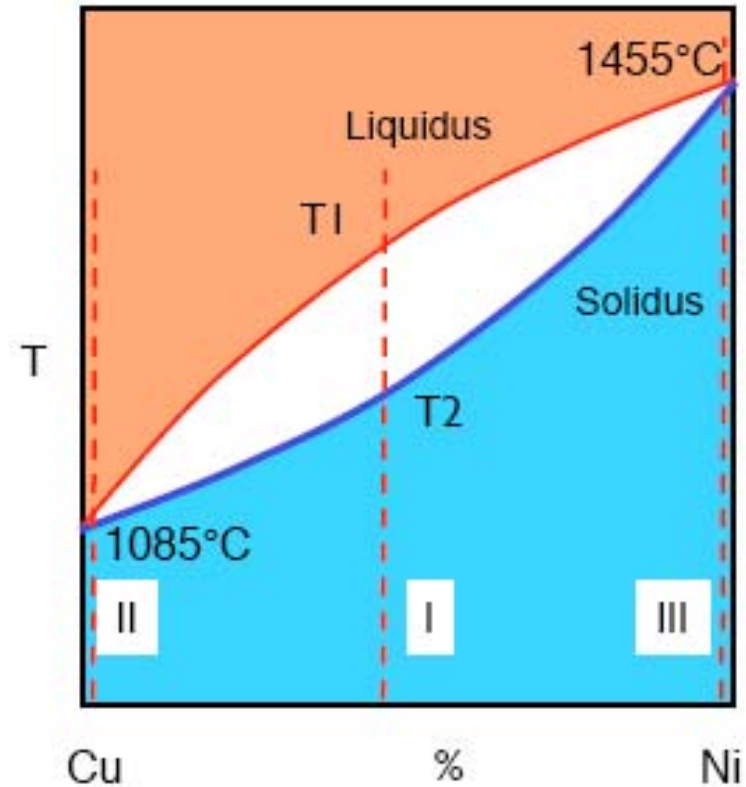
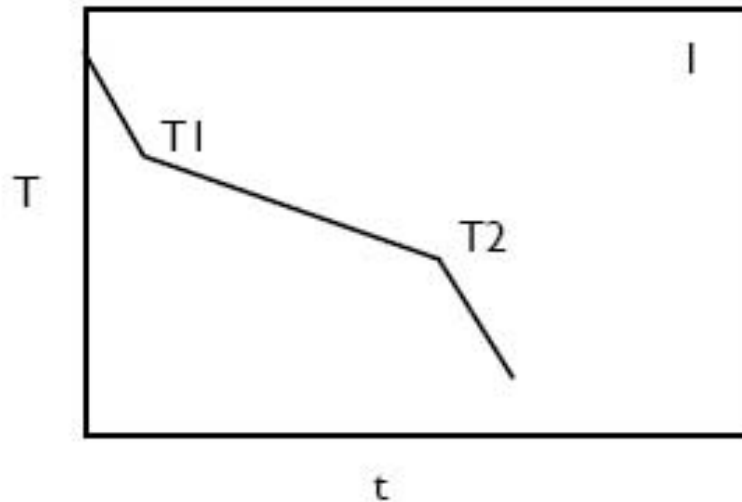
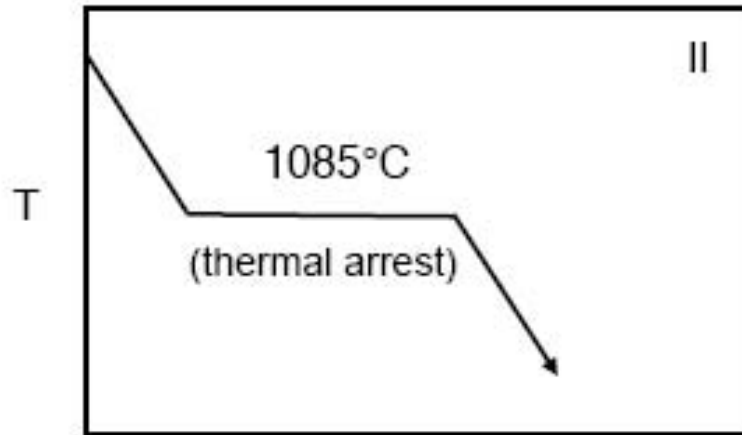
Solidification of alloy  $C_0$  starts on cooling at  $T_1$ . The first solid formed has a composition of  $C_{S1}$  and the liquid  $C_0$ . On further cooling the solid particles grow larger in size and change their composition to  $C_{S2}$  and then  $C_0$ , following the solidus whereas the liquid decrease in volume and changes its composition from  $C_0$  to  $C_{L3}$  following the liquidus. The solidification completes at  $T_3$ .



## 1.5 Binary phase diagrams

# Cooling Curves

## determination of Phase diagrams



## 1.5 Binary phase diagrams

# Example

At temperature  $T_1$ , alloy  $C_0$  is in the dual phase region, comprising the liquid phase and the  $\alpha$ -phase.

- Determine the compositions of the two phases;
- Determine the weight fractions of the two phases

Read from the tie line:

Liquid phase: Cu-30%Ni

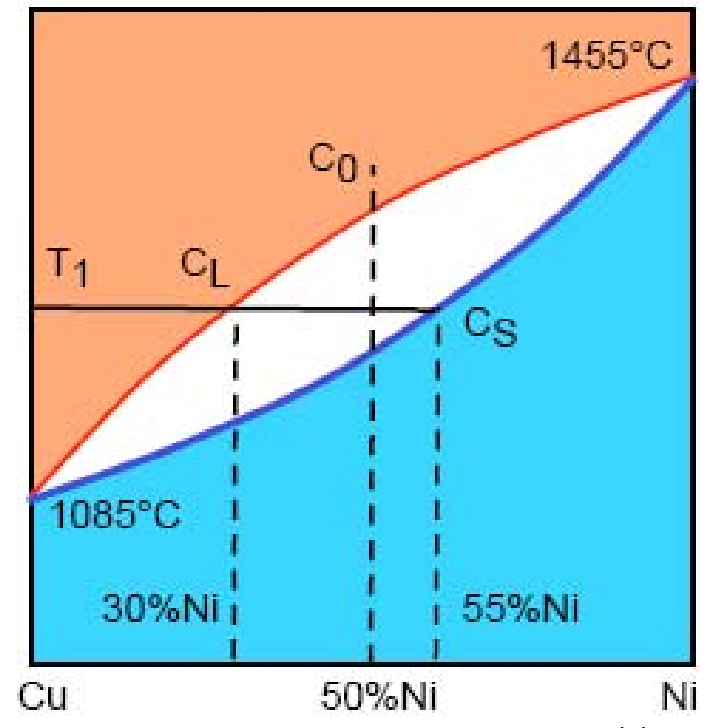
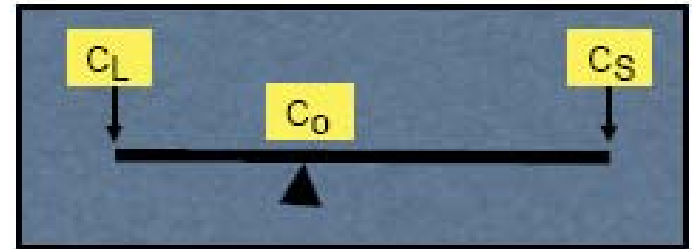
$\alpha$ -phase: Cu-55%Ni

$$W_L = \frac{C_S - C_0}{C_S - C_L} = \frac{55 - 50}{55 - 30} = 0.2 = 20\%$$

$$W_\alpha = \frac{C_0 - C_L}{C_S - C_L} = \frac{50 - 30}{55 - 30} = 0.8 = 80\%$$

or

$$W_\alpha = 1 - W_L = 1 - 0.2 = 0.8 = 80\%$$

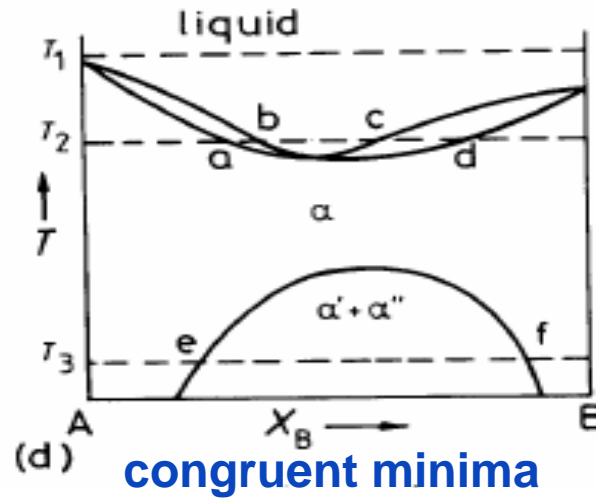
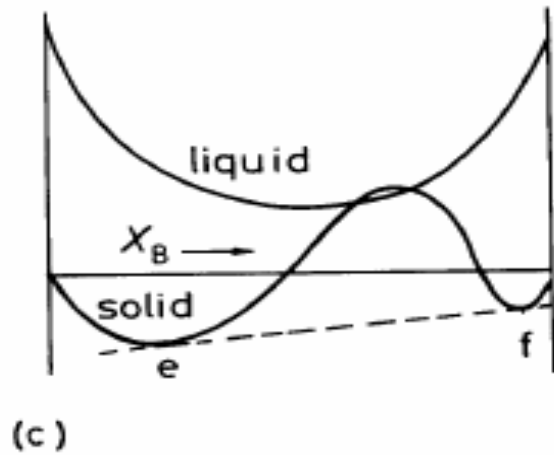
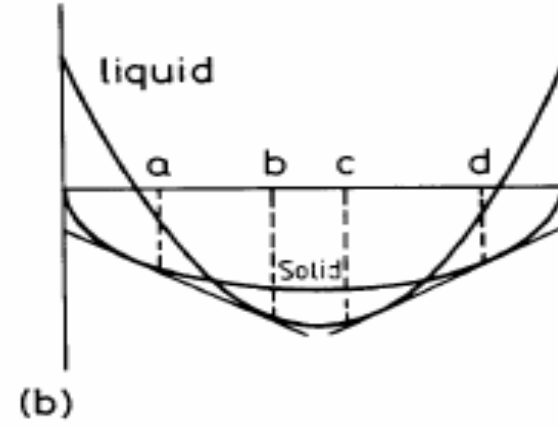
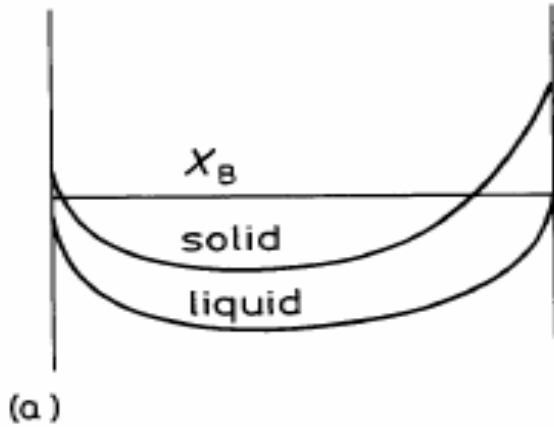


# 1.5 Binary phase diagrams

## 2) Systems with miscibility gap

$$\Delta H_{mix}^L = 0$$

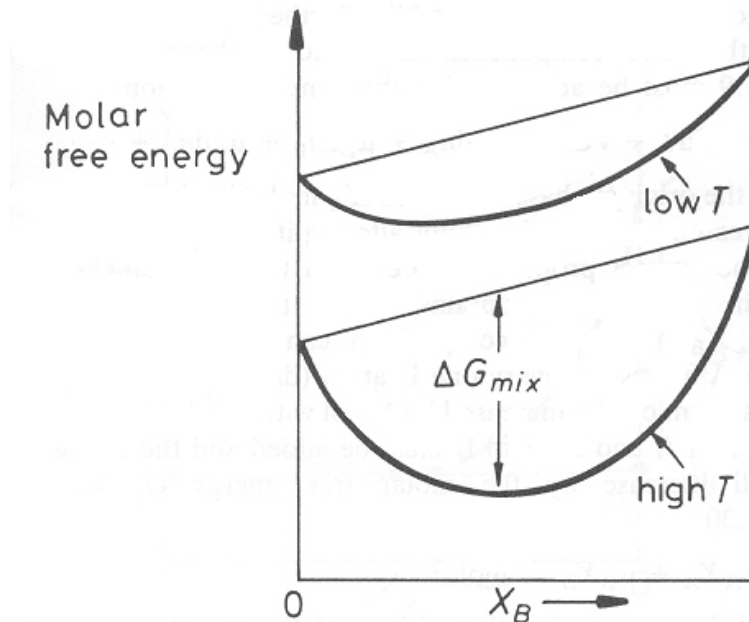
$$\Delta H_{mix}^S > 0$$



How to characterize  $G^S$  mathematically  
in the region of miscibility gap between e and f ?

# Ideal Solutions

$$G_2 = G_1 + \Delta G_{mix}$$



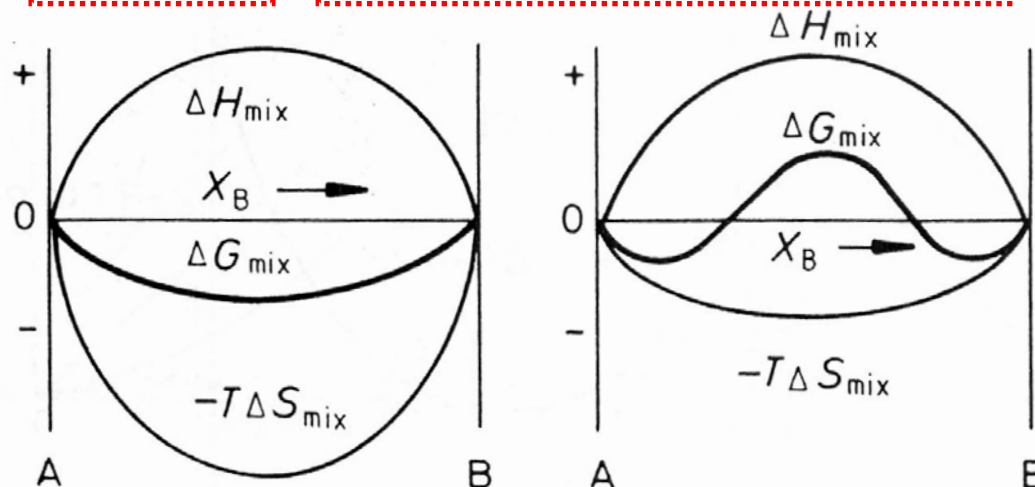
# Regular Solutions

$$G = X_A G_A + X_B G_B + \underbrace{\Omega X_A X_B}_{\Delta H_{mix}} + \underbrace{RT (X_A \ln X_A + X_B \ln X_B)}_{-T\Delta S_{mix}}$$

Reference state

Pure metal  $G_A^0 = G_B^0 = 0$

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$



(c)  $\Omega > 0$ , high  $T$

(d),  $\Omega > 0$  low  $T$



# 1.5 Binary phase diagrams

## 2) Systems with miscibility gap

$$\Delta H_{mix}^L = 0 \quad \Delta H_{mix}^S > 0$$

- When A and B atoms dislike each other,  $\Delta H_{mix} > 0$
- In this case, the free energy curve at low temperature has a region of negative curvature,  $\frac{d^2G}{dX_B^2} < 0$
- This results in a 'miscibility gap' of  $\alpha'$  and  $\alpha''$  in the phase diagram

