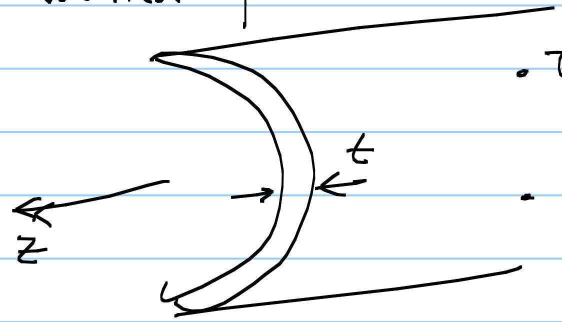
Lecture 3.4 :

Torsion of Thin-walled Open-sectioned Bar

노트 제목

- Thin-walled open



- Thin-walled
  $t \ll$ other dimension
- open : simply-connected cross-section

- Uniform / nonuniform torsion

$\boxed{\alpha = \text{const}}$ (uniform)

$\boxed{\alpha = f(z) \text{ vary along } z \text{ axis}}$ (nonuniform)

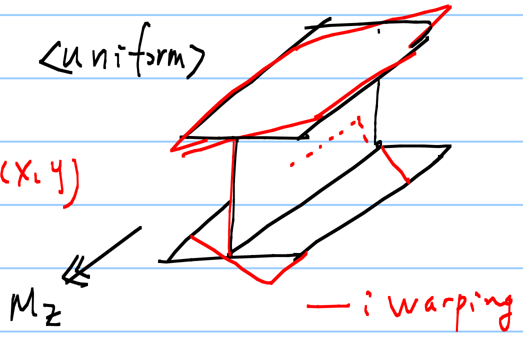
(where $M_t = C_t \alpha$

$\alpha = \dfrac{d\theta}{dz}$ (twist rate) $\sim \kappa$ in bending)

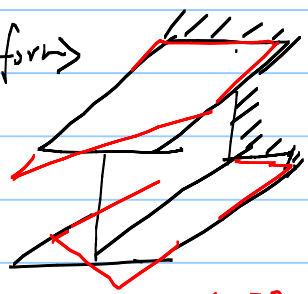
<uniform>

const

$u_z = \alpha\, \phi(x, y)$



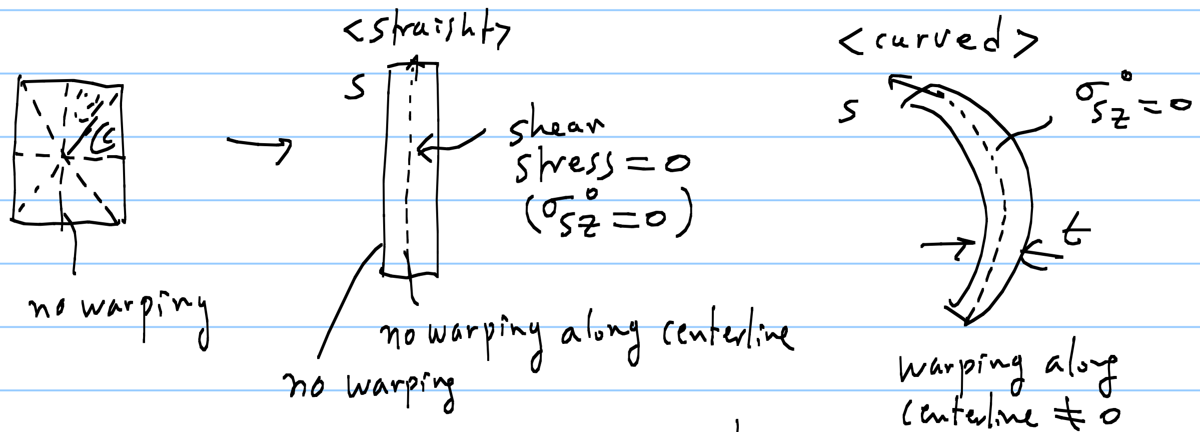
<non-uniform>



$u_z = \alpha(z)\, \phi(x, y)$

<Uniform Torsion in thin-walled open-sectioned bar>

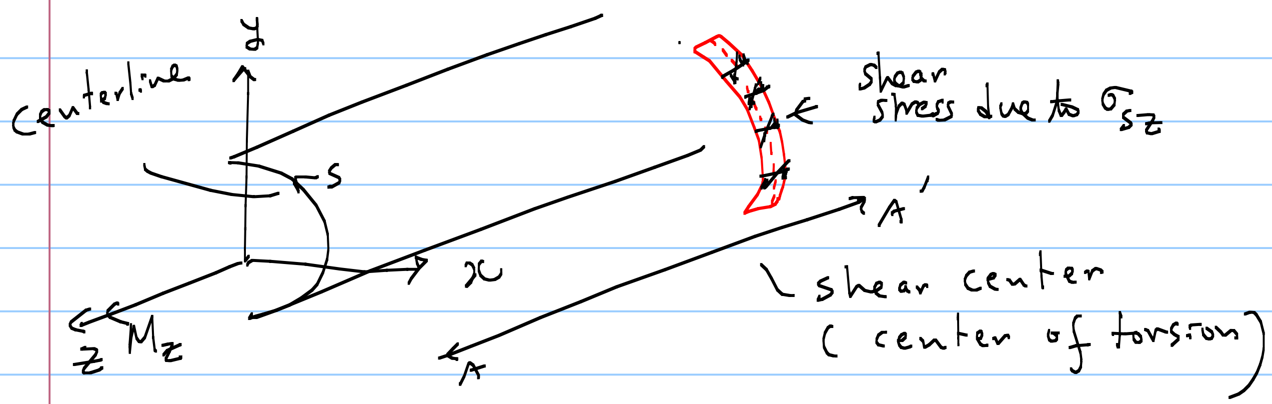
Recall warping in Rectangular section



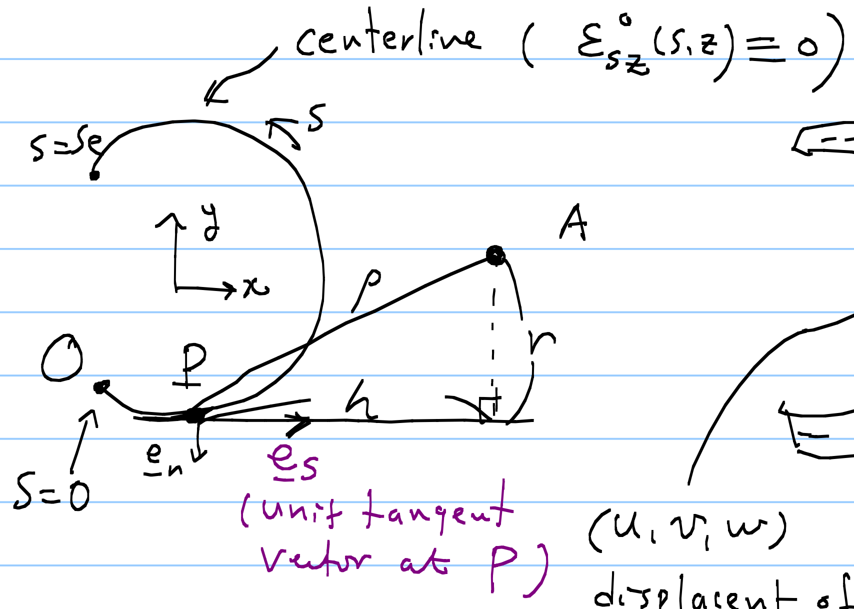
$\Rightarrow$ warping $\simeq 0$ everywhere
(However, shear stress cannot be zero)

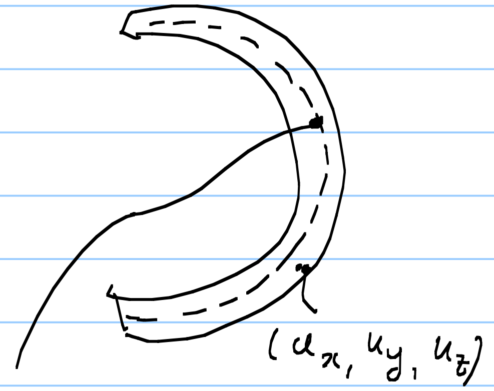
$\rightarrow$ may assume same warping over t

$\boxed{\text{Thus, we want to determine warping disp of the centerline.}}$



※ The section rotates about AA′ axis.





u : normal disp

v : tangential

w : axial

To determine w

① use $\varepsilon^{o}_{sz}(s,z) = \dfrac{\partial w(s,z)}{\partial s} + \dfrac{\partial v(s,z)}{\partial z} = 0 \quad (1)$

② v = In-plane tangential Disp along e_s

$$= \Big(\underbrace{\Theta(z)\, \underline{e}_z}_{\text{rotation angle}} \times \underbrace{\underline{r}_{AP}}_{\text{Distance from A to P}} \Big) \cdot \underline{e}_s$$

(valid only for small rotation ; velocity $= \underline{\omega} \times \underline{r}$)

$$= \Theta(z)\, \underline{e}_z \times \left(-h\, \underline{e}_s + r\, \underline{e}_n\right) \cdot \underline{e}_s$$

(n, z, s)

$$= \left(h\, \Theta\, \underline{e}_n + r\, \Theta\, \underline{e}_s\right) \cdot \underline{e}_s$$

$$= \Theta(z)\, r(s) \qquad (2)$$

(2) → (1)

$$\frac{\partial w}{\partial s} = -\frac{\partial v}{\partial z} = -\alpha\, r(s)$$

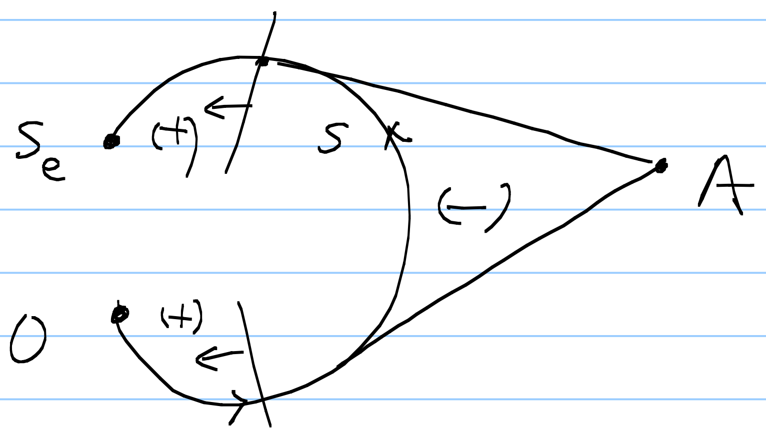
$$\therefore \boxed{w = w^{0} - \alpha \underbrace{\int_0^s r(s)\, ds}_{\text{denotes the warping displacent of the centerline of thin-walled open section}}}$$

5

Let $\Omega_s(s) = -\int_0^s r(s)\,ds$

Sign convention for $r(s)$

- if ρ rotates ccw as s increases, then $r(s) > 0$
- if ρ rotates cw as s increases then $r(s) < 0$



Sign for $r(s)$

To satisfy $\int_0^{s_e} w(s)\,ds = 0$ for torsional warping;

$$0 = \int_0^{s_e} \Big[w^0 + \alpha \underbrace{\int_0^{s} -r(s')\, ds'}_{\Omega_s(s)} \Big] ds$$

$$0 = w^0 s_e + \alpha \int_0^{s_e} \Omega_s(s)\, ds$$

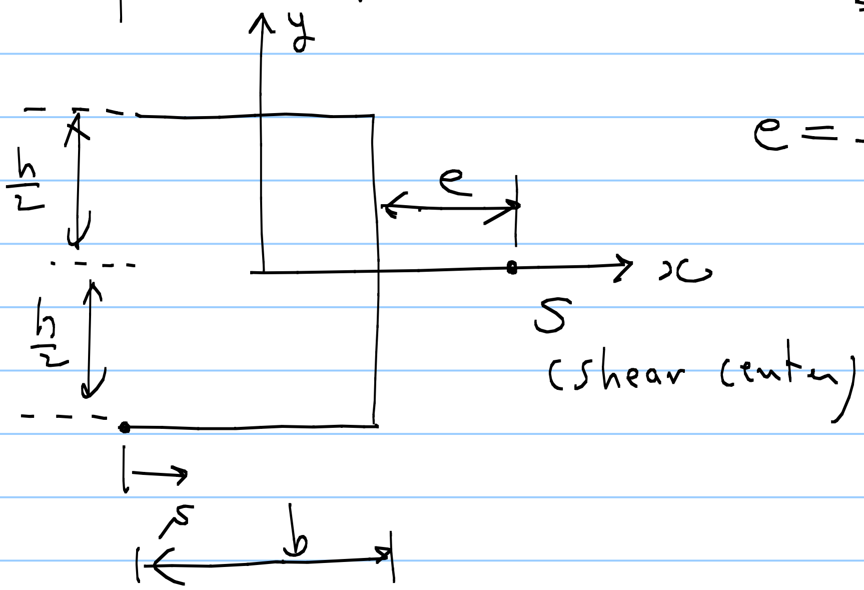
$$w^0 = -\frac{\alpha}{s_e} \int_0^{s_e} \Omega_s(s)\, ds \triangleq -\alpha \bar{\Omega}_s$$

$$\therefore w(s) = \alpha \left(\Omega_s - \bar{\Omega}_s \right)$$
$$\triangleq \alpha\, \omega(s)$$

$$\boxed{\omega_s(s) = \Omega_s - \bar{\Omega}_s}$$

called warping ftn
for thin-walled section.

Example 3-7-1 : Determine $\omega_s(s) = ?$



$$e = \frac{3b^2}{6b+h}$$

procedure — ① determine $\Omega_s = -\int_0^s r(s)\,ds$

② $\omega_s = \Omega_s - \bar{\Omega}_s$

③ Thus must know $r(s)$

(i) for $0 \le s \le b$

$r = h/2$, $\quad \Omega_s = -\int_0^s r\,ds$

$$= -\frac{h}{2}s$$

ii) for $b \le s \le b+h$

$r = -e$

$$\Omega_s = -\int_0^s r\,ds$$

$$= -\left[\int_0^b \underbrace{r}_{h/2}\,ds + \int_b^s \underbrace{r}_{(-e)}\,ds\right]$$

$$= -\left[\frac{bh}{2} - e(s-b)\right]$$

$$= \frac{bh}{2} + e(s-b)$$

iii) for $b+h \le s \le 2b+h$

$$r = \frac{h}{2}$$

$$\Omega_s = -\int_0^s r\,ds$$

$$= -\left[\int_0^b + \int_b^{b+h} \int_{b+h}^s r(s)\,ds\right]$$

$$= -\left[\int_0^b \frac{h}{2}\,ds + \int_b^{b+h} (-e)\,ds + \int_{b+h}^s \frac{h}{2}\,ds\right]$$

$$= -\left[\frac{h}{2}b - eh + (s-(b+h))\frac{h}{2}\right]$$

$$= he + \frac{h}{2}(h-s)$$

9

(V) $\bar{\Omega}_s = \frac{1}{2b+h}\int_0^{2b+h} \Omega_s(s)\, ds$

$= \frac{h}{2}(e-b)$

Finally

$\omega_s(s) = \Omega_s(s) - \bar{\Omega}_s$

warping function $= \begin{cases} \frac{h}{2}(b-e-s) & (0 \le s \le b) \\ e\left(-b-\frac{h}{2}+s\right) & b \le s \le b+h \\ \frac{h}{2}(b+e+h-s) & b+h \le s \le 2b+h \end{cases}$