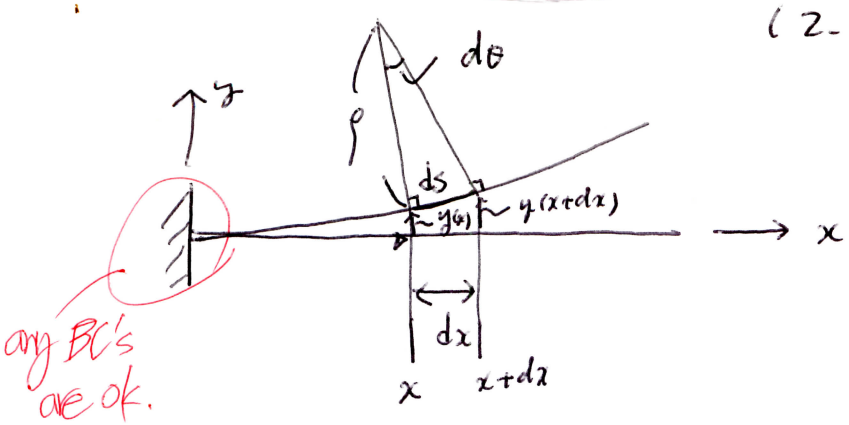
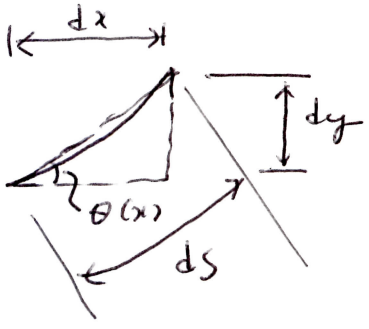
ppt & upload

Exact expression for curvature

(2-2절 보충자료)





$ds = \rho \cdot d\theta$; $\frac{1}{\rho}$ (= curvature) $\equiv \frac{d\theta}{ds}$ (by definition)

$\tan\theta(x) = \frac{dy}{dx}$; $\theta(x) = \tan^{-1}\left(\frac{dy}{dx}\right)$

$$\frac{1}{\rho} = \kappa = \frac{d\theta}{ds} = \frac{d}{dx}\left\{\tan^{-1}\left(\frac{dy}{dx}\right)\right\} \cdot \frac{dx}{ds}$$

chain rule

Note: ① $ds = \sqrt{dx^2+dy^2} = dx\sqrt{1+(y')^2}$

$$\therefore \frac{dx}{ds} = \frac{1}{\sqrt{1+(y')^2}}$$

② $$\frac{d}{dx}\left\{\tan^{-1}(y')\right\} = \frac{y''}{1+(y')^2}$$

$$\therefore \frac{1}{\rho} = \kappa = \frac{y''}{\left\{1+(y')^2\right\}^{3/2}}$$

(for small deflection assumption, $y' \ll 1$

$$\frac{1}{\rho} = \kappa \doteq y'' = \frac{d}{dx}(y')$$ / note: strain angle = curvature



Review "Column and Frame Stability" material (ppt file)

Chapter 2 Columns 기둥

2-1 Introduction

① "Simple bar-spring models" versus "deformable systems"

(chapter 1 의 내용)

↓

rigid body 거동

(no internal forces)

↓

Equilibrium equation

= algebraic eq.

(not differential eq.)

(chapter 2 의 거동)

↓

Internal forces will develop

↓

Internal forces

= a function of the derivatives of displacement variables

(ex: $M = EIy''$)

↓

Equilibrium eq.

= differential eq.

Differential calculus 필요

(주로 linear ordinary differential equations of 2nd order)

② The approach to stability analysis in this chapter

↳ the bifurcation approach

= the eigenvalue analysis approach

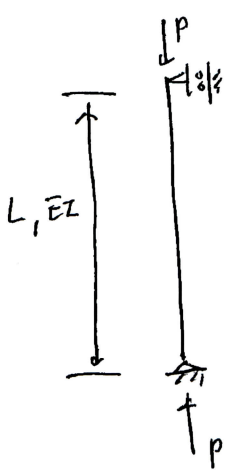
(Recall "methods of analyses in stability" in the section 1-3)

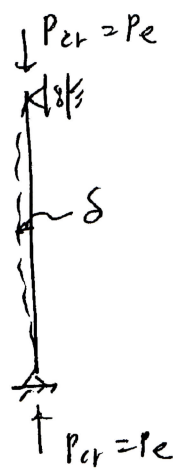
③ Euler column의 stability

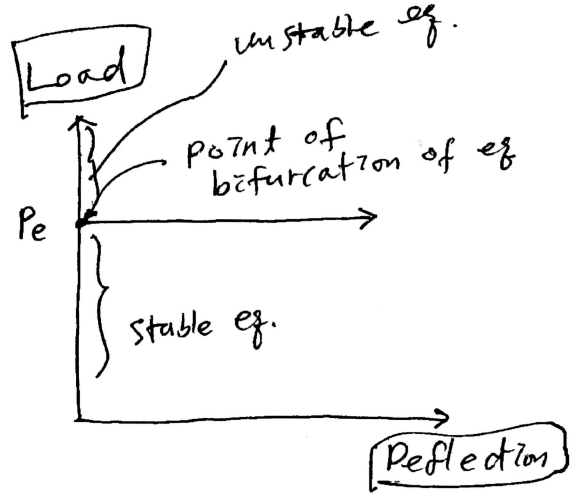
(= Perfect column) ← 지난 시간의 ppt 자료 참조

Review the concepts

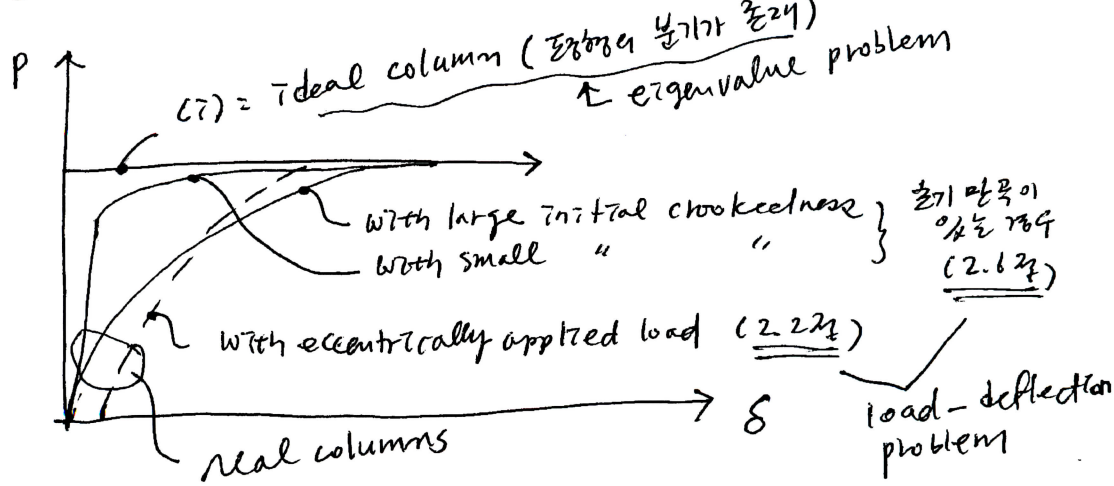
i) stable equilibrium
ii) unstable "
iii) Euler buckling load = $P_{cr} = \frac{\pi^2 EI}{L^2}$ $(= P_e)$
iv) Neutral equilibrium
v) stability surface
vi) bifurcation of equilibrium





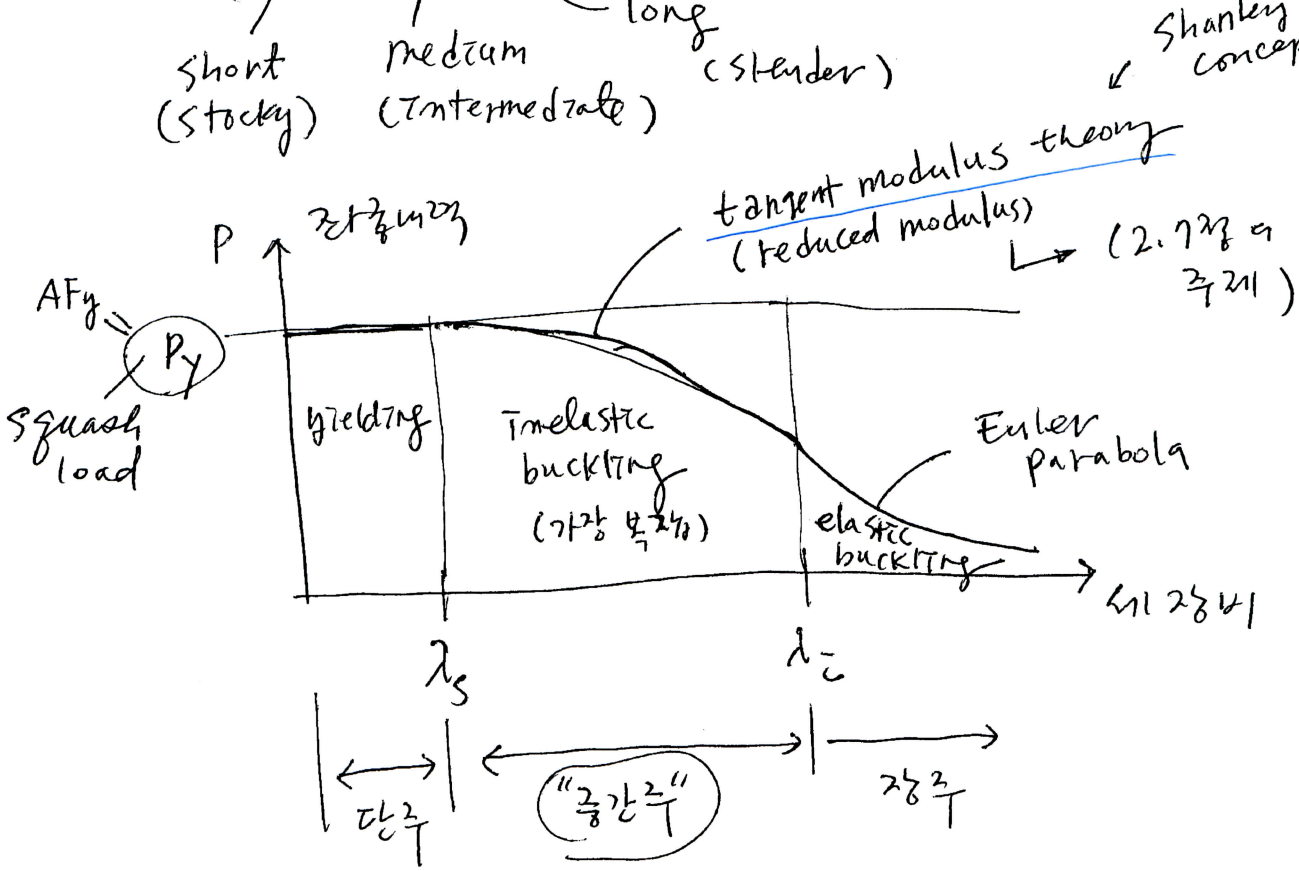


④ Load – deflection curves of ideal and real columns.



⑤ 단주 / 중간주 / 장주의 개념 ← ppt 과목 참조

Short (Stocky) — Medium (Intermediate) — long (Slender)



2.2 Classical column theory

"압축재" 기둥?

① column ← compression member 중 휨 (2차의 곡선)

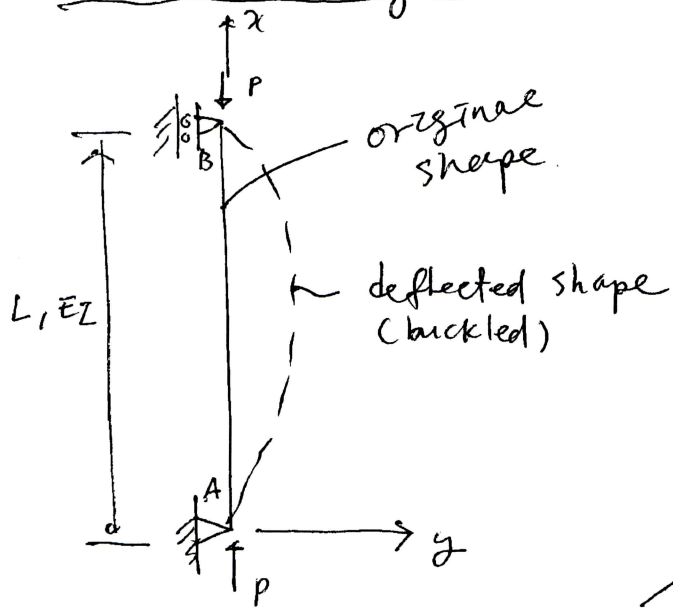
beam-column ← flexural-compression member 중 휨 (3차의 곡선)

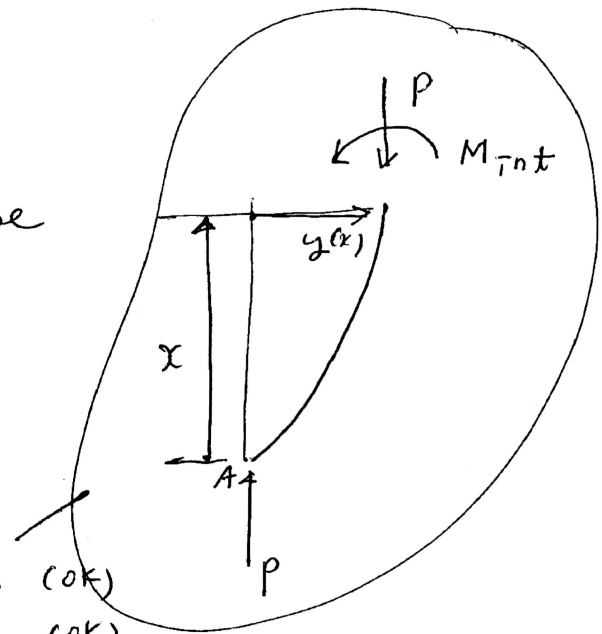
휨-압축재

② 기본가정 6가지 (geometry, kinematics, material)

↳ 교과서 p.48 참조.

Euler buckling load (1744년 Euler 연구)





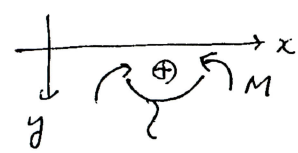
(1) – $\Sigma X = 0$ (OK)

(2) – $\Sigma Y = 0$ (OK)

(3) – $\Sigma M = 0$

↺ +

$\Sigma M_{about\ x} = M_{int} - P\,y(x) = 0$



"–" 부호

$M = \ominus EI\,y''$

미소처짐을 가정 (실제방향이 중요) → $-EIy'' - Py = 0$

$y'' + k^2 y = 0$ where $k^2 = \frac{P}{EI}$ (or $k = \sqrt{\frac{P}{EI}}$)

The general solution,

$y = A\sin kx + B\cos kx$

↳ 미지수 3개 ; A, B, k

↳ 가능한 경계조건 2개 : $y(0) = 0$, $y(L) = 0$

$y(0) = B = 0$

$y(L) = A\sin kL = 0$ [$A = 0$ → trivial

$kL = n\pi$ → non-trivial

$$k = \sqrt{\frac{P}{EI}} = \frac{n\pi}{L}$$

$$P = \frac{n^2\pi^2 EI}{L^2}$$

the smallest buckling load.

∴ The critical load $P_{cr} = \frac{\pi^2 EI}{L^2} = P_e$ (Euler load.)
($n = 1$)

∴ The buckled shape,

$$y = A \sin kx$$

$$= A \sin\left(\frac{\pi}{L}x\right)$$

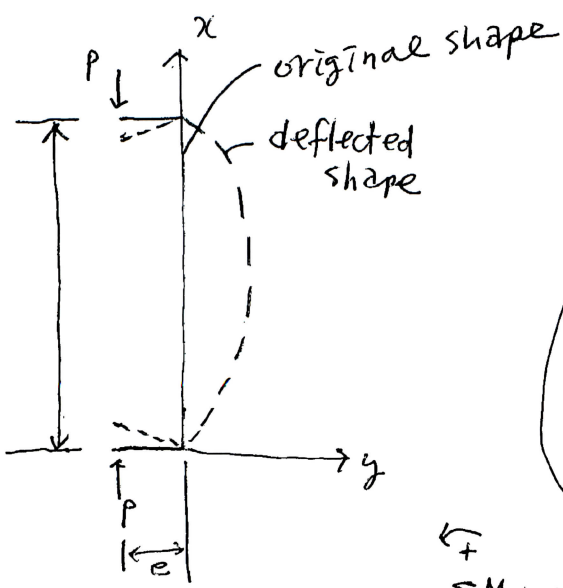
Still Indeterminate

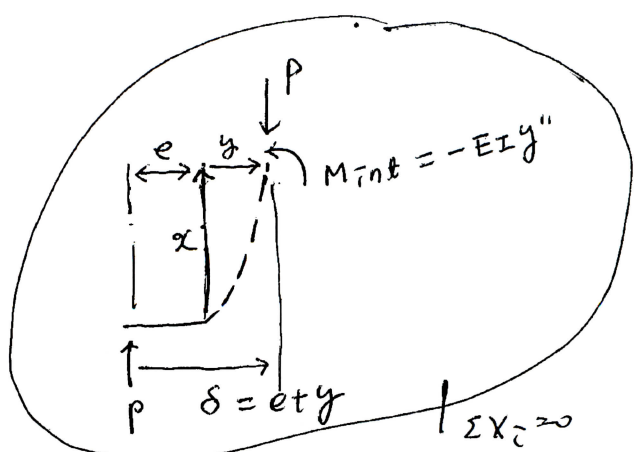
① 자체로도 중요하지만, 바로다음주제를 학습할 때 반드시 알아야 하는 중요한 reference load의 역할

② 이식의 물리적 의미: 지난 시간의 PPT 자료 참조

Eccentrically loaded column

↳ real column의 경우.





$\Sigma X_i = 0$

$\Sigma Y_i = 0$

$\Sigma M_i = 0$

$+\circlearrowleft \Sigma M_{about\ x} = -EIy'' - P(e+y) = 0$

or $EIy'' + Py = -Pe$ ↳ 2계 비제차 상미방 (고속의 물체가 아님!)

Using $\frac{P}{EI} = k^2$,

$$y'' + k^2 y = -k^2 e$$

The general solution is $y = y_c + y_p$:

$$\begin{cases} y_c = A\sin kx + B\cos kx \\ y_p = e \quad (\text{method of undetermined coefficients}) \end{cases}$$

미상 이 방법 사용

$$\therefore y = A\sin kx + B\cos kx + e$$

B/C's 에서 ; $y(0) = 0$, $y(L) = 0$

$$B = e \qquad A = \left(\frac{1-\cos kL}{\sin kL}\right)e$$

$$= \left[\left(\frac{1-\cos kL}{\sin kL}\right)\sin kx + \cos kx - 1\right] \times e$$

i) $M(x) = -EIy''$ ← (2.2.37)식

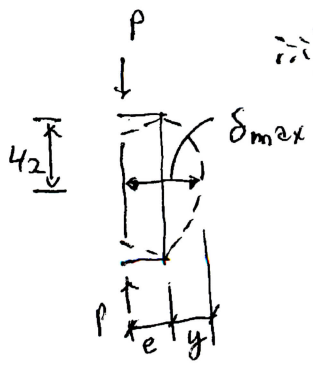
$M_{max} \doteq M(x = L/2)$ ← (2.2.39)식

$= EIk^2 e \sec\left(\frac{kL}{2}\right)$ ← (2.2.43)식

ii) $y_{max} = y(x = L/2)$

$= \left(\sec\frac{kL}{2} - 1\right) \times e$ ← (2.2.42)식

"삼각함수 공식!"



iii) The total deflection measured from the line of application of P $\equiv \delta_{max}$

$$\delta_{max} = y_{max} + e = \left(\sec\frac{kL}{2}\right) \times e$$

A_F = amplification factor (for deflection)

(cf. secant formula)

or $A_F = \sec\frac{kL}{2} = \sec\left[\frac{L}{2}\cdot\sqrt{\frac{P}{EI}}\right]$

$= \sec\left[\frac{\pi}{2}\sqrt{\frac{P}{\left(\frac{\pi^2 EI}{L^2}\right)}}\right] = \sec\left[\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right]$

Euler load

" Importance reference load in stability theory. "

iv) Similarly,

$M_{max} = EI(k^2)e\sec\left(\frac{kL}{2}\right)$ ← (2.2.43) 식 ($\frac{P}{EI}$)

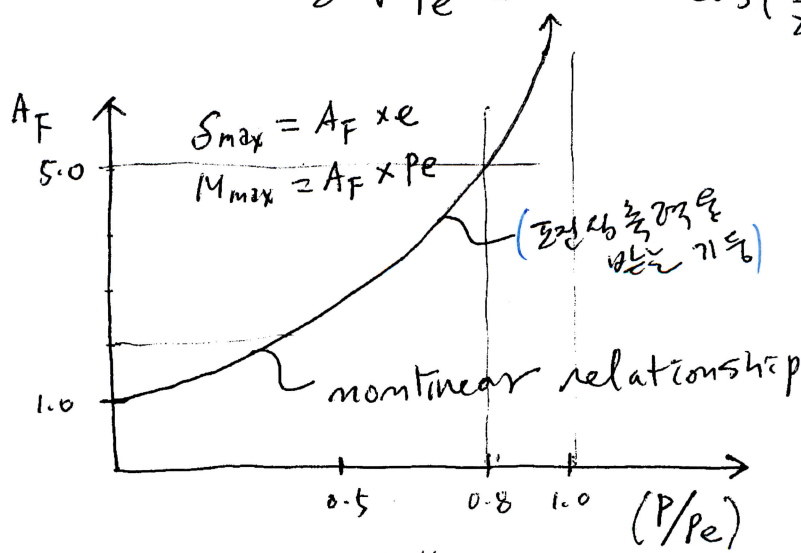
$= (P\cdot e)\times\sec\left(\frac{kL}{2}\right) = (P\cdot e)\times\sec\left[\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right]$

original geometry에 대해 작용하는 moment

A_F (for moment) : 처짐을 고려하여 동일하게 moment

A_F (for both deflection and moment)

$= \sec\left[\frac{\pi}{2}\cdot\sqrt{\frac{P}{P_e}}\right] = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)}$

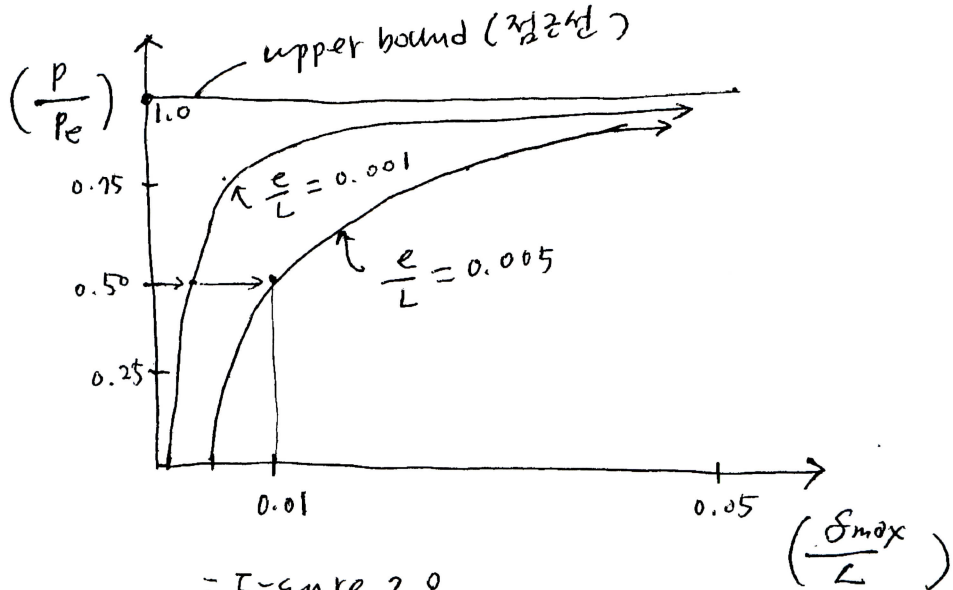


∴ Figure 2.7 참조

$$\delta_{max} = e \times \sec\left[\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right]$$

$$\left(\frac{\delta_{max}}{L}\right) = \left(\frac{e}{L}\right) \times \sec\left[\frac{\pi}{2}\sqrt{\left(\frac{P}{P_e}\right)}\right]$$

$\frac{e}{L}$: $\frac{1}{1000}$ (=0.001), $\frac{1}{200}$ (=0.005)



: Figure 2.8

The "Secant" formula

↑ to calculate the max. compressive stress in an eccentrically loaded "elastic" column.

재료역학 level과의 차이점?

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{I_z}c = (Pe) \times \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)$$

$I_z = Ar^2$

기하학적 비선형 반영

$$= \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)\right] \quad \cdots\cdots (2.2.52)$$

재료비선형까지 반영

용어 ① 편심축하중을 받는 압축재의 최대응력 산정 (trivial!)

② 실제 기둥의 imperfection factor의 calibration에 따른 목적 (다음 쪽 참조)

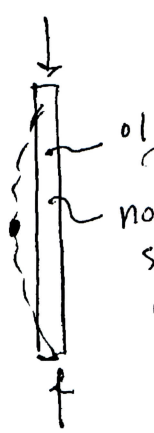
If the first yield of the material is used as the criterion for failure, the corresponding critical load (P_{cr}) can be calculated from Eq (2.2.52) by setting $\sigma_{max} = \sigma_y$.

$$\sigma_y = \frac{P_{cr}}{A}\left[1+\left(\frac{ec}{r^2}\right)\times \sec\left[\frac{\pi}{2}\sqrt{\frac{P_{cr}}{P_e}}\right]\right] \quad \cdots (*)$$

상기 공식(실험)의 의의

"nominally straight한" column의 좌굴하중을 불완전도를 (실험상) 알고 계산하는 것

The eccentricity factor (ec/r^2) is treated as an imperfection factor for an axially loaded column and is determined by calibration so that the formula will best fit the given experimental data.



이 시편의 항복 (σ_y등) 탄성 특성은 재료의 제특 가능 (known임)

nominally straight column (short column 가정)

실제로 초기상의 굽힘이 있으므로 (따라서 하중보다 extreme fiber의 yielding이 발생)

$P = P_{cr}$ ($= P_y$)

Compressive extreme fiber yielding의 상황

(*) 식에서

$$\sigma_y \times \frac{A}{P_{cr}} - 1 = \frac{ec}{r^2}\times \sec\left[\frac{\pi}{2}\sqrt{\frac{P_{cr}}{P_e}}\right]$$

$$\therefore e = \frac{\left(\sigma_y \cdot \frac{A}{P_{cr}} - 1\right)}{\sec\left[\frac{\pi}{2}\sqrt{\frac{P_{cr}}{P_e}}\right]} \times \frac{r^2}{c}$$

"calibrated" eccentricity

우변의 σ_y, 탄성특성 P_{cr} 등은 모두 실험적으로 계측된 자료에 근거

$$\frac{EIy''}{[1+(y')^2]^{3/2}} + Py = 0 \quad \cdots (2.2.5b)$$

↑ 탄성장굴 대변형 해석문제

Linear vs. Nonlinear Theory : 차이점

① (Wang (1953), Chajes (1974) , pp. 22-28 ← 「elliptical integral of the 1st kind 를 이용하여 풀이 가능」

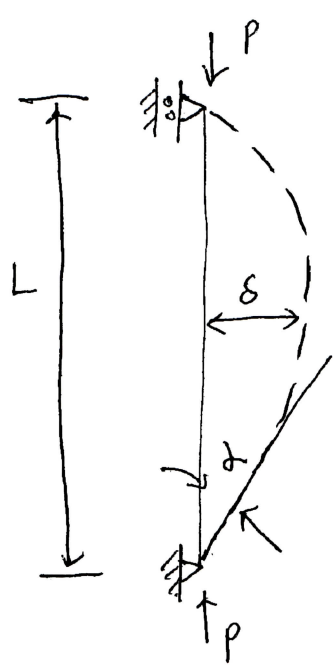
② See Figure 2.10 comments

(P/Pe versus δ/L)

$$\frac{\delta}{L} = \frac{2\sin\left(\frac{\alpha}{2}\right)}{\pi\sqrt{\frac{P}{Pe}}}$$

↙

← Gives P/Pe vs. ($\frac{\delta}{L}$ and α) relationship up to large deflection.



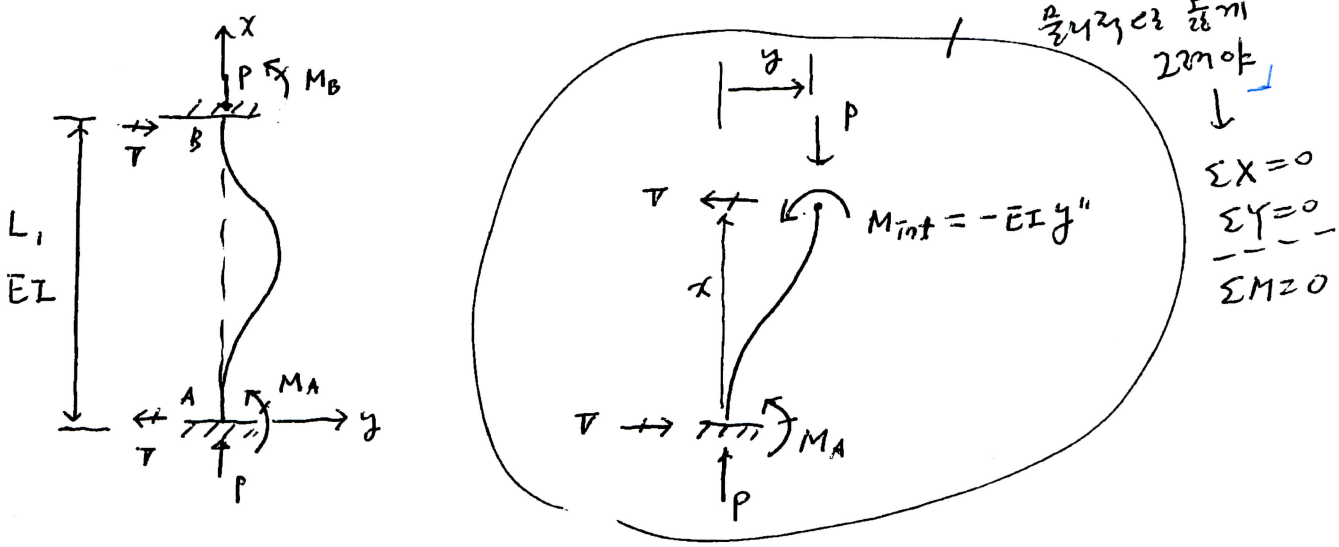
2.3 End-restrained columns

↳ the concept of effective length (여러 연결)

↳ Visualize as the distance between the two inflection points (real or imaginary) of the buckled shape

↳ See figures 2.11, 2.12, 2.13, etc.

2.3.1 Both ends fixed



$$+\circlearrowright \quad \Sigma M_{\text{about } x} = -M_{int} + Py + Tx - M_A = 0$$

$(-EIy'')$

$$y'' + k^2 y = -\frac{T}{EI}x + \frac{M_A}{EI}$$

✓ $$y = \underbrace{A\sin kx + B\cos kx}_{y_h} + \underbrace{(-T\cdot x/P + M_A/P)}_{y_p}$$

✓ 경계조건: $y(0) = 0$, $y'(0) = 0$, $y(L) = 0$, $y'(L) = 0$

$y(0)=0$, $y'(0)=0$ → $B = -M_A/P$, $A = T/Pk$

나머지 2개의 조건을 대입 ; (2.3.9) (2.3.10)

"HW# 아래 식 확인"

In matrix form of the two equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V \\ M_A \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

("Buckling이 발생하려면 V, M_A 모두가 zero가 아닌 값을 가져야 함")

$$\text{Det}\left|\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right| = 0 \text{ 이어야}$$

$kL \sin kL + 2\cos kL - 2 = 0$

$\sin\frac{kL}{2}\left(\frac{kL}{2}\cos\frac{kL}{2} - \sin\frac{kL}{2}\right) = 0$

} By using the trigonometrical identity (삼각함수)

$\sin\frac{kL}{2} = 0 \rightarrow P_{cr} = \frac{(4\pi^2)EI}{L^2}$ ← symmetric buckling (critical) (40)

or,

$\left(\frac{kL}{2}\cos\frac{kL}{2} - \sin\frac{kL}{2}\right) = 0 \rightarrow \tan\left(\frac{kL}{2}\right) = \frac{kL}{2} \rightarrow P_{cr} = \frac{80.8EI}{L^2}$

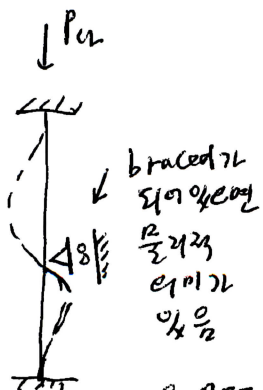
trial-error method 적용

↑ anti-symm buckling



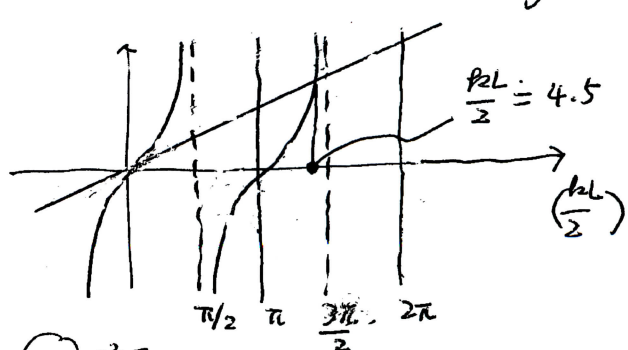
$P_{cr} = \frac{(4\pi^2)EI}{L^2}$

: symmetric



$P_{cr} = \frac{80.8EI}{L^2} = \frac{(8.20\pi^2)EI}{L^2}$

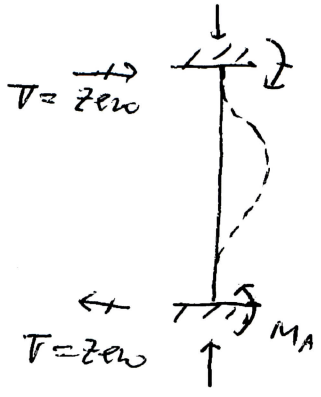
: anti-symmetric



Notes :

(i) buckled shape for symmetric buckling

→ $k = 2\pi/L$



$M_B = M_A$ 이고 $V = 0$ 이다

} 이 2가지 경계조건을 (2.3.5)식에 대입

$$y = \frac{M_A}{P}\left(1 - \cos\frac{2\pi}{L}x\right)$$

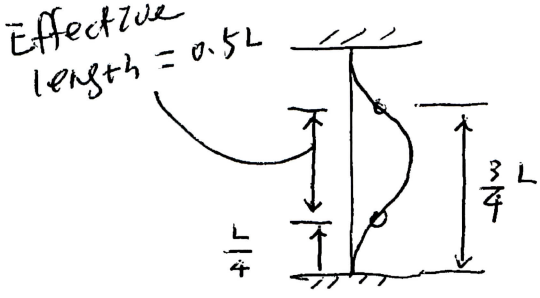
변곡점 위치

$$M = -EIy'' = \left(EI\,\frac{M_A}{P}\,\frac{4\pi^2}{L^2}\right) \times \cos\left(\frac{2\pi}{L}x\right) = 0$$

non-zero term × zero

$\frac{2\pi}{L}x = \frac{\pi}{2}, \frac{3}{2}\pi, \cdots$

or $x = \frac{L}{4}, \frac{3}{4}L$



(ii) Effective length factor K based on P_{cr}

$$P_{cr} = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{(kL)^2} \rightarrow K = \cancel{2.0}\ 0.5 \quad \text{(in this case)}$$

In general,

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2}, \quad K = \sqrt{\frac{\pi^2 EI/L^2}{P_{cr}}} = \sqrt{\frac{P_e}{P_{cr}}}$$

2.3.2 One end fixed and one end free

2.3.3 One end hinged and one end fixed)

↑ 내용 (follow-up 중)

"고려서 2.3.4 ~ 2.3.6절 보충 (upload 은)"

2.3 End-Restrained Columns ← 다양한 단부지지조건에서 K값은 얼마인가?

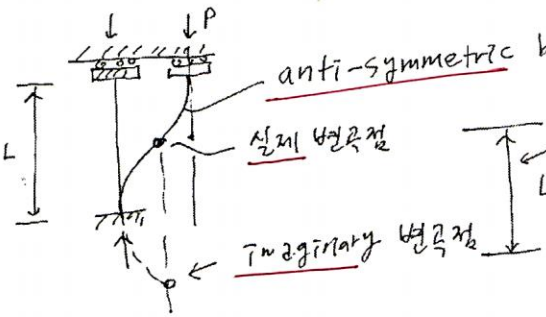
2.3.4 One End Fixed and One End Guided

↳ Big Implication



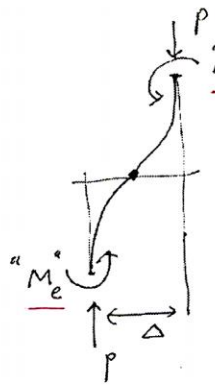
이 논리로 충분!

Note: ①



② Guided end: 「수평반력 생기지 않음, 따라서 고정단에서도 수평반력이 생기지 않음 ($\because \Sigma X_i = 0$)
↳ 따라서 beam shear = zero」

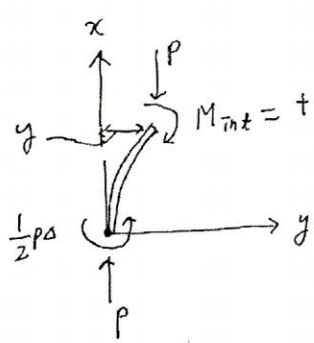
③ 역"Euler" 라출 모드:



$+\circlearrowleft \Sigma M_{about\ (End)}$ Lower

$= 2 \times M_e - P \cdot \Delta = 0$

$\therefore M_e = \frac{1}{2} P\Delta$



∴ Free-body

$\Sigma X = 0$; ok.

$\Sigma Y = 0$; ok.

$+\circlearrowleft \Sigma M_{about\ x} = EIy'' + Py - \frac{1}{2}P\Delta = 0$

$y'' + k^2 y = \frac{P\Delta}{2EI}$ --- (53)

$y = \underbrace{A \sin kx + B \cos kx}_{y_c} + \underbrace{\frac{\Delta}{2}}_{y_p}$ --- (54)

B/Cs available; $y(0) = 0 \rightarrow B = -\frac{\Delta}{2}$

$y'(0) = 0 \rightarrow A = 0$

$\Big) \; y = \frac{\Delta}{2}(1 - \cos kx)$

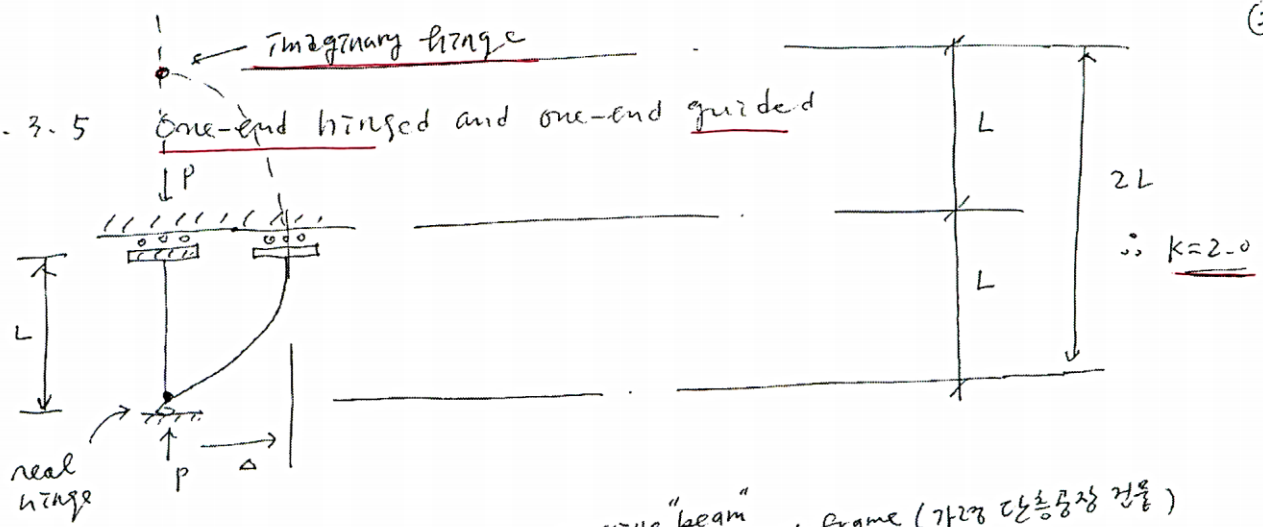
$y(x=L) = \Delta \rightarrow \Delta = \frac{\Delta}{2}(1 - \cos kL) \rightarrow \cos kL = -1$

$kL = \pi, 3\pi \cdots$

$\sqrt{\frac{P_{cr}}{EI}} \times L = \pi \quad \therefore P_{cr} = \frac{\pi^2 EI}{L^2}$

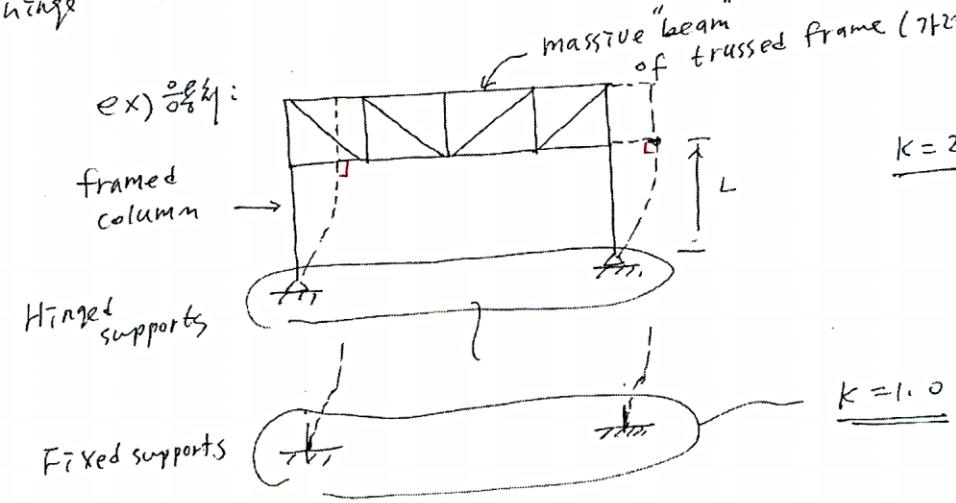
$= \frac{\pi^2 EI}{(1.0L)^2}$; $K = 1.0$

2.3.5 One-end hinged and one-end guided



∴ K = 2.0

ex) 응용례 :



K = 2.0

K = 1.0

2.3.6 AISC K factor and column curve

↳ See Table 2.1 (p. 74)

For an isolated column, regardless of the end conditions, the critical load can be expressed in the general form as

↙ framed column에도 응용가능

"Judgemental method"

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$ ← "Generalized" Euler formula

「design 초기에서 비교적 중요한 개념임」

$$\frac{P_{cr}}{P_y} = \frac{1}{A\sigma_y}\,\frac{\pi^2 EI}{(KL)^2} = \frac{\left(\frac{\pi^2 E}{\sigma_y}\right)}{\left(\frac{KL}{r}\right)^2} = \left(\frac{KL}{r}\right)^{-2} \times \left(\sqrt{\frac{\sigma_y}{\pi^2 E}}\right)^{-2}$$

$$= \left(\frac{KL}{r}\sqrt{\frac{\sigma_y}{\pi^2 E}}\right)^{-2}$$ ← λ_c = 무차원세장비 $= \left(\frac{KL}{r}\right)\sqrt{\frac{\sigma_y}{\pi^2 E}}$

$$= \lambda_c^{-2}$$

$$\therefore \boxed{P_{cr} = \lambda_c^{-2} \times P_y \;;\; \sigma_{cr} = \frac{1}{\lambda_c^2}\sigma_y}$$ ← 「일견 같은 하나 의미는 수식계산이기는 하나 P_{cr} 또는 σ_{cr}이 σ_y와 관계있는 것처럼 오해를 유발할 수 있는 표기법임」

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

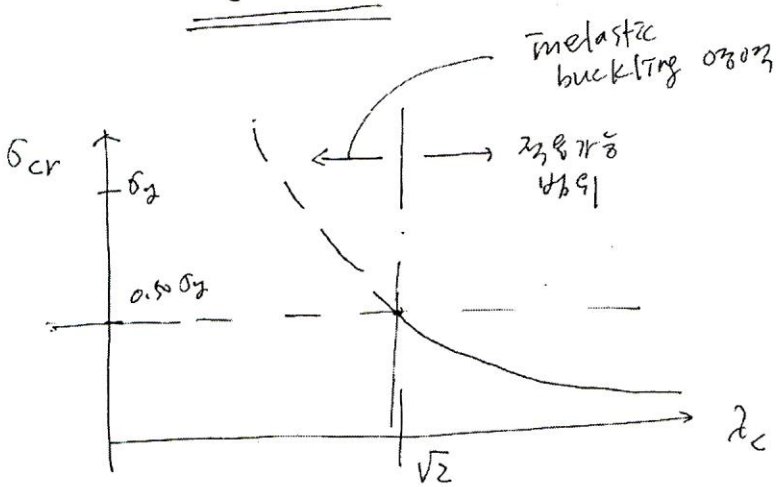
Note : $\frac{P_{cr}}{P_y} = \lambda_c^{-2}$ 의 적용범위 → "elastic buckling" 구간에서만 유효

(잔류응력의 영향 비교적 작을 때를 의미)

σ_r (residual stress) $\simeq 0.30\,\sigma_y$ (압연형강의 경우) 이지만 "real column"의 imperfection 등을 고려하여 보수적으로 $\sigma_r = 0.50\,\sigma_y$ 를 취하면

$$\sigma_{cr} = \frac{1}{\lambda_c^2}\sigma_y \le 0.50\,\sigma_y \; (= \sigma_y - \sigma_r)$$

$$\lambda_c \le \sqrt{2}$$



$$\lambda_c = \frac{kl}{r}\sqrt{\frac{\sigma_y}{\pi^2 E}} = \sqrt{2}$$

∴ SS400강재 (또는 A36 등 비슷) 의 경우

$\sigma_y = 2.4\ t/cm^2$

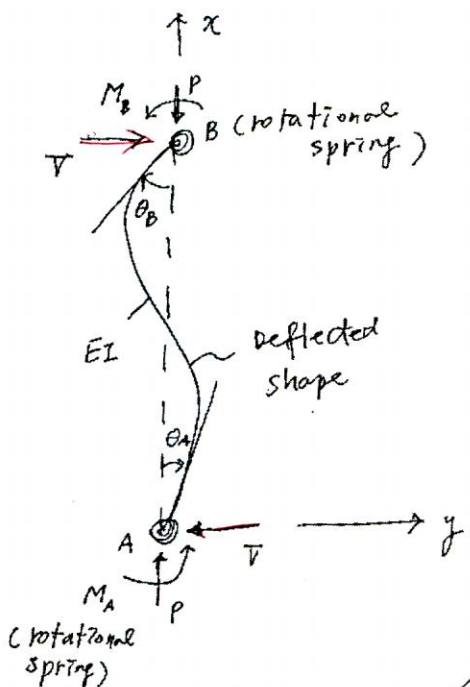
$E = 2100\ t/cm^2$ 을 대입하면

$$\left(\frac{kl}{r}\right)_p = \sqrt{2} \times \sqrt{\frac{\pi^2 \times 2100}{2.4}} \doteq 130$$

↑ 한계세장비

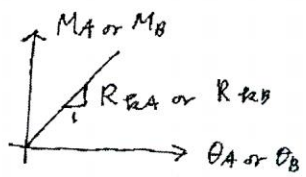
교재에 2.3.7절 보충

Elastically Restrained Ends ← 대부분의 Framed column의 경우 (neither fixed nor hinged)

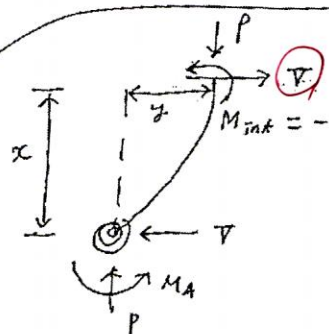


(겉보기 A, B의 방향 반대이나 θ 값은 같다)

Note: M_A, M_B, V는 절점의 돌기 단위는가?



: rotational spring behavior (linear spring)



$\Sigma X=0$;
$\Sigma Y=0$;

$\Sigma M=0$ →

$$\overset{\curvearrowleft +}{\Sigma} M_{about\ x} = -EIy'' - Py - Vx + M_A = 0 \quad ; \quad EIy'' + Py = -V \cdot x + M_A$$

$$y'' + k^2 y = -\frac{V}{EI}x + \frac{M_A}{EI} \quad \text{----- (84)}$$

The general solution is

$$y = A\sin kx + B\cos kx - \frac{V}{P}x + \frac{M_A}{P} \quad \text{--- (85)}$$

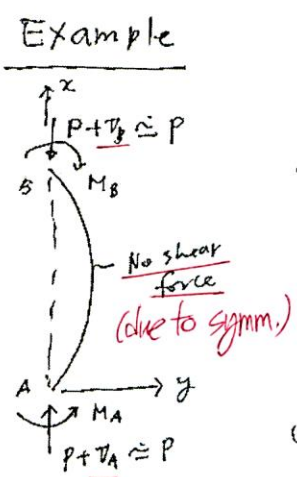
「Using the displacement and slope boundary conditions for the specific problem and recognizing that

$$M_A = R_{kA} \cdot \theta_A \quad ; \quad M_B = R_{kB} \cdot \theta_B \quad \text{--- (86, 87)}」$$

the Pcr can be obtained.

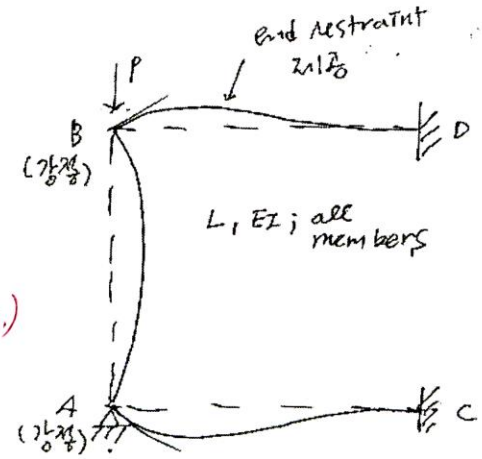
↳ Example 참조

Example

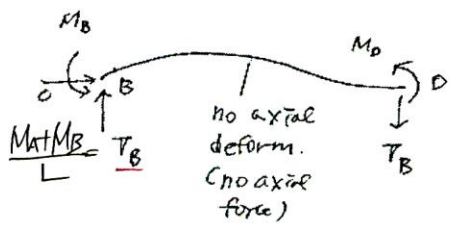


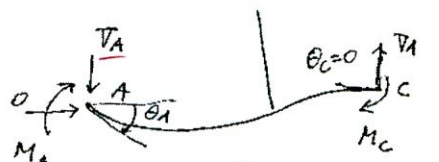
(c) Column deformation and member end forces

($P \gg V_A$ or V_B in general)

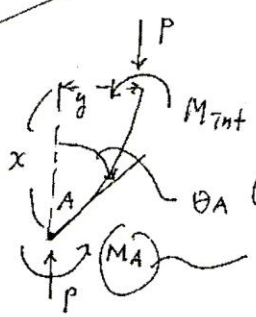


(a) Assumed buckling geometry (symmetrical mode)





(b) Beam deformation and member end forces



$\overleftarrow{+}$ $\Sigma M_{\text{about } A} = -EIy'' - py + M_A = 0$ or,

$$y'' + k^2 y = \frac{M_A}{EI} \quad \text{---- (88)}$$

↑ special form of Eqn (84) ; 대칭조건을 포함하므로 $M_A = M_B$ 이고 상호상대 회전은 column shear V가 zero인 경우로 생각가능

(88)의 일반해 : $y = A\sin kx + B\cos kx + \frac{M_A}{P}$ ------ (89)

가용한 경계조건 [$y(0)=0$, $y(L)=0$] / [대칭조건 $y'(x=\frac{L}{2})=0$]

} 3개중 2개가 Independent ; $y(0)=0$, $y'(\frac{L}{2})=0$ 이용.

↳ 대칭이므로 $y(0)=0$ 이면 $y(L)=0$ 일 것임

$y(0)=0 \rightarrow B = -\frac{M_A}{P}$

$y'(\frac{L}{2})=0 \rightarrow A = -\frac{M_A}{P}\tan(\frac{kL}{2})$

$$y = \frac{M_A}{P}\left(1 - \tan\frac{k_2 L}{2}\sin k_2 x - \cos k_2 x\right) \quad \cdots (94)$$

$$y' = \frac{k_2 M_A}{P}\left(-\tan\frac{k_2 L}{2}\cos k_2 x + \sin k_2 x\right) \quad \cdots (95)$$

「주어진 문제에서 만족되어야 하는 endpoint 조건은 절점 A에서의 변형에 관한 조건이다 : 즉 기둥 하부의 회전각이 (절점) 보 AC의 단부 회전각(θ_A)과 동일한 값이 되도록 조건을 부과하여야 한다.」

(95)식에서,

$$y'(x=0) = \frac{k_2 M_A}{P}\left(-\tan\frac{k_2 L}{2}\right) = \theta_A \quad \cdots (*)$$

보 AC의 단부 회전각 θ_A는 처짐각 방정식을 사용하여 고려된 부재로 (member end moment)로서 풀이할 수 있다 : ← "simple case"

$$M_A = 0 + \frac{2EI}{L}(2\theta_A + 0) \rightarrow \theta_A = \frac{L}{4EI} \times M_A \quad \cdots (**)$$

$$\left(M_C = 0 + \frac{2EI}{L}(0 + \theta_A) = \frac{1}{2}M_A\right)$$

→ ① 증명 REA에 상향

(*) = (**) 이어야 하므로

$$\frac{k_2 M_A}{P}\left(-\tan\frac{k_2 L}{2}\right) = \frac{L M_A}{4EI}$$

or $$M_A \times \left(\frac{L}{4EI} + \frac{k_2}{P}\tan\frac{k_2 L}{2}\right) = 0 \;;\; M_A\left(\frac{L}{4}\cdot\frac{P}{EI} + k_2\tan\frac{k_2 L}{2}\right) = 0 \;;$$

$$M_A\left(\frac{k_2^2 L}{4} + k_2\tan\frac{k_2 L}{2}\right) = 0 \;;$$

$$M_A \times \left(\frac{k_2 L}{4} + \tan\frac{k_2 L}{2}\right) = 0$$

$M_A \neq 0$ 이어야 하므로

$$\frac{k_2 L}{4} + \tan\frac{k_2 L}{2} = 0$$

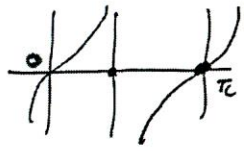
좌굴이 발생하면

삼각방정식 (transcendental equation)
↳ solution by graphical method or by trial and error.

↓

P_{cr}

Note: Bounding Solutions



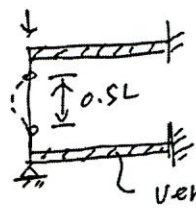
Note: ① 만일 보의 $EI \to \infty$ 라면 (massive beam),

→ One extreme

$\theta_A = 0$; $\frac{kM_A}{P}(-\tan\frac{kL}{2}) = 0$; $\tan\frac{kL}{2} = 0$

(**)式 $\frac{kL}{2} = \pi$; $\frac{\sqrt{\frac{P}{EI}}L}{2} = \pi$; $\sqrt{\frac{P}{EI}} = \frac{\pi}{(0.5L)}$

$\therefore P_{cr} = \frac{\pi^2 EI}{(0.5L)^2}$ →



② 만일 보의 $EI \to 0$ 라면 (very weak beam),

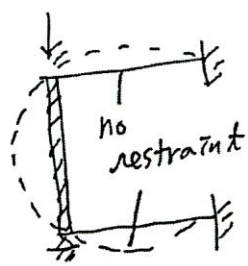
→ Other extreme

$\theta_A = \infty$; $\frac{kM_A}{P}(-\tan\frac{kL}{2}) = \infty$ very larger

very large

$\frac{kL}{2} = \frac{\pi}{2}$ or $\sqrt{\frac{P}{EI}} = \frac{\pi}{L}$

$\therefore P_{cr} = \frac{\pi^2 EI}{L^2}$ → Euler load!



Recall "Judgemental" Method.

$$0.5 \le k_{actual} \le 1.0$$

↳ 기둥이 sidesway가 구속된 경우의 유효좌굴길이 계수 상계 및 하계.

(Alignment chart의 범위 상기하여 볼 것)

Back to the problem,

$\frac{kL}{2} = x$; $\tan x = -\frac{x}{2}$

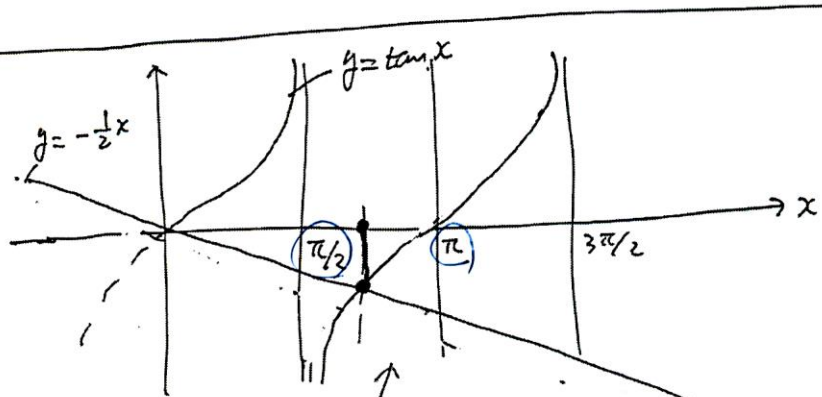
$\frac{\pi}{2} < x = \frac{kL}{2} < \pi$

$\frac{\pi}{3.14} < kL < \frac{2\pi}{6.28}$

$kL \doteq \frac{1}{2}(3.14 + 6.28)$

$= 4.71 \leftrightarrow 4.59$ (exact)

not a bad guess!



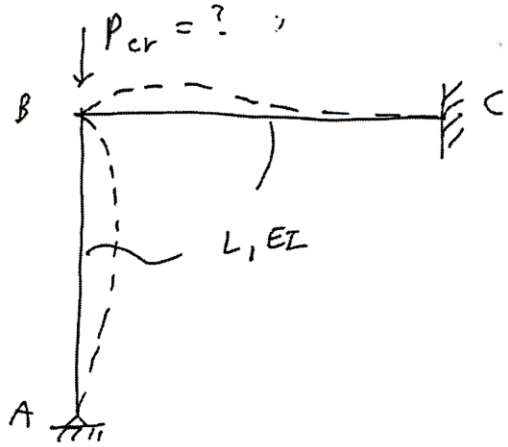
근을 구하는 algorithm? (가령 bi-section method)

$\therefore P_{cr} = \frac{21.03EI}{L^2}$

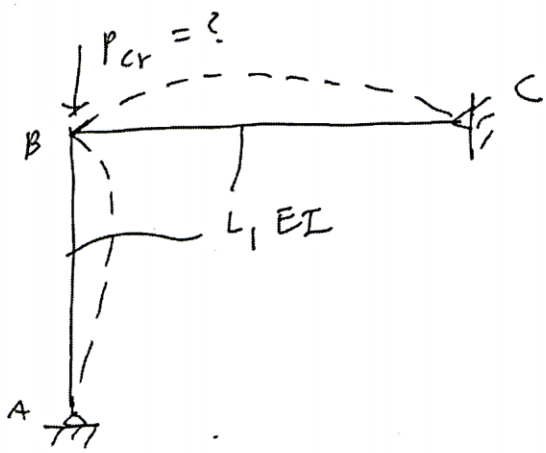
$= \frac{\pi^2 EI}{(0.69L)^2}$ "

HW #

(1)



(2)



* 휨좌굴 방정식에 의하여 trigonometric transcendental equations 은 graphical method 및 numerical trial and error 등의 방법을 적용하여 구할 것.

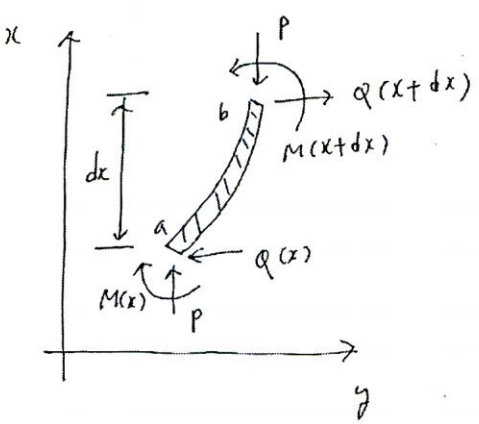
"2.4 ~ 2.6절 보충자료" (upload용)

2.4 4계 상미방 풀이에 대한 보충설명

[2계 상미방 (2.3절의 방법) Recall! → ① equilibrium consideration of a column segment of "finite" size
4계 상미방 (2.4절의 방법) → ② "geometric boundary conditions" enforced to find the critical load

↳ ① equilibrium consideration of an infinitesimal segment of the column.

② In most cases, mixed (or both "geometric" and "force-type") boundary conditions are needed to be specified to find the critical load



∴ Free-body & positive sign convention for internal forces M, Q, and P

$\overset{+\uparrow}{\Sigma} Y_i = P - P = 0$, OK.

$\overset{+\rightarrow}{\Sigma} X_i = Q(x+dx) - Q(x)$

$= dQ = \frac{dQ}{dx} \times dx = 0$

$\therefore \frac{dQ}{dx} = 0$ (or Q = constant) --- (2.4.2)

$\overset{+\circlearrowright}{\Sigma} M_{\text{about } b} = 0$ 에서

$Q = \frac{dM}{dx} - P\frac{dy}{dx}$ --- (2.4.1)

↳ 만약 $P = 0$ 이면 $Q = \frac{dM}{dx}$ (beam의 경우)

(2.4.1)식을 x로 미분 → $\frac{dQ}{dx} = \frac{d^2M}{dx^2} - P\cdot\frac{d^2y}{dx^2} = 0$ ← (2.4.2)식 대입

$\frac{d^2}{dx^2}(-EIy'') - P\cdot\frac{d^2y}{dx^2} = 0$

정리하면, $EIy'''' + Py'' = 0$

$y'''' + k^2 y'' = 0$ --- (2.4.6)

↳ ($P = 0$ 이면 beam의 지배방정식 $EI\cdot\frac{d^4y}{dx^4} = 0$ ↳ 재료역학의 4차 연속적분)

The general solution to Eq (2.4.6)

$y = e^{\lambda x}$ 라 하면

$\lambda^4 e^x + k^2 \lambda^2 e^x = 0$

$\lambda^2(\lambda^2 + k^2) = 0$; $\lambda = 0$ or $\lambda = \pm k i$

$\lambda = 0$: 중근 (중복도 2)

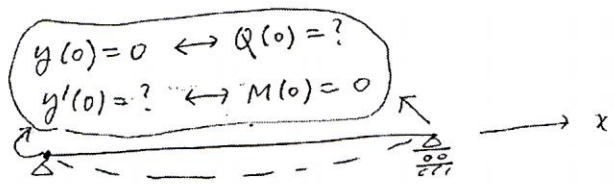
$\lambda = \pm ki$: 서로 basic의 관계

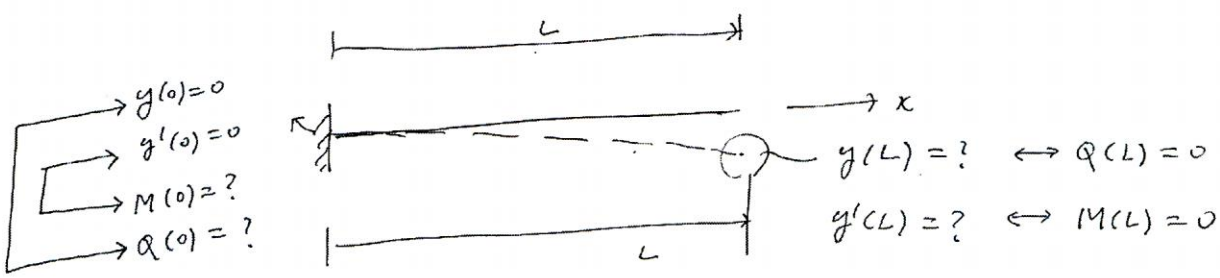
$$\therefore y = \underbrace{A \sin kx + B \cos kx}_{\lambda = \pm ki \text{ 에 대응하는 basis}} + \underbrace{D + Cx}_{\lambda = 0 \text{ 에 대응}} \quad (2.4.7)$$

중복도 2이므로

4개의 BCs 필요

Note :





Force-type BCs		Geometric BCs
Shear	⟷	deflection
Moment	⟷	slope
known	⟷	unknown
unknown	⟷	known

} 이 4가지 중 2개씩 택일.

↑ 처짐곡선의 2차 도함수

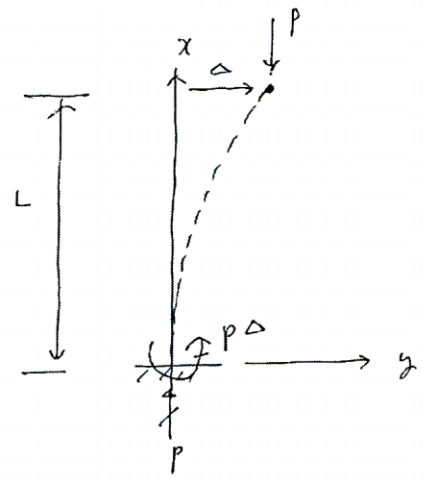
$M = -EI y''$

$Q = \frac{dM}{dx} = -EI y'''$

* 반력은 왜 생기는 것인가?

* Table 2.2 요약 참조 (P.85)

Fixed - Free Column



$$y = A\sin kx + B\cos kx + Cx + D \quad \cdots (2.4.1)$$

general solution

가용한 경계조건 :

(1) 고정단 (2개)

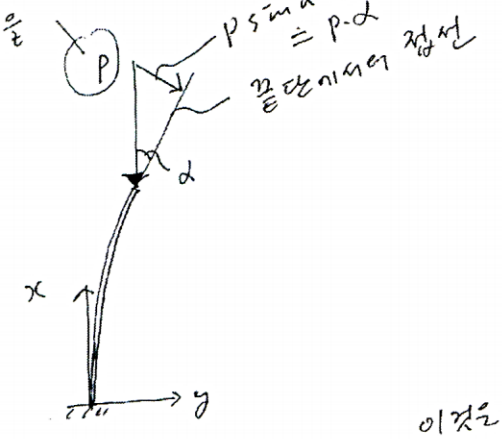
$y(0) = 0 \cdots (i)$ ← 모든 "기하학적 경계조건"

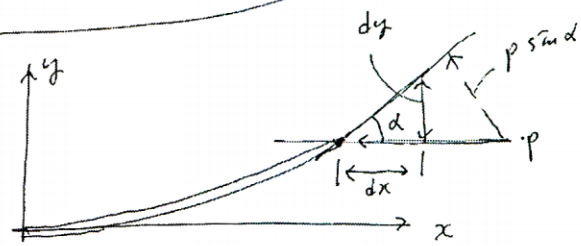
$y'(0) = 0 \cdots (ii)$

(2) 자유단 (2개) ← 모두 "힘 관련" 경계조건

$M = EI\,y'' = 0 \rightarrow y''(L) = 0 \cdots (iii)$

"좌굴 이후에도 연직하방을 유지"





$$\tan\alpha = \frac{dy}{dx} = y'$$

for small α,

$$\tan\alpha \simeq \alpha = \frac{dy}{dx}$$

미소처짐을 가정하면 처짐각은 곧 처짐곡선의 1차 도함수로서 곡선방향 일치를 내용에 상함

이것은 (+)양의 방향과 일치

따라서

$$Q(L) = -EI\,y'''(L) = P\sin\alpha$$
$$= P\alpha$$
$$= P\,y'(L)$$

or $y'''(L) + k^2 y'(L) = 0 \cdots (iv)$ ← Table 2.2의 세번째 case

Note :

$$y' = Ak\cos kx - Bk\sin kx + C$$
$$y'' = -Ak^2\sin kx - Bk^2\cos kx$$
$$y''' = -Ak^3\cos kx + Bk^3\sin kx$$

경계조건에서 계수조건을 대입하면,

$y(0)=0 \rightarrow B+D=0$ ---- (i)

$y'(0)=0 \rightarrow Ak+C=0$ --- (ii)

2차미분 후 계수조건에서,

$y''(L)=0 \rightarrow A\sin kL + B\cos kL = 0$ --- (iii)

$y'''(L)+k^2y'(L) = \left(-Ak^3\cos kL + Bk^3\sin kL\right) + k^2\left[Ak\cos kL - Bk\sin kL + C\right]=0$

$\therefore C=0$ --- (iv)

따라서 (iv) 번째 조건을 (ii) 에 대입하면 다음의 3가지 조건을 만족해야 함.

"관심있는 약간라즘" →
$$\begin{cases} B+D=0 & \checkmark \text{ ---- } (*) \\ Ak+0=0 \rightarrow A=0 & \\ 0\times\sin kL + B\cos kL=0 \rightarrow B\cos kL=0 & \text{-- } (**) \end{cases}$$

$$\begin{bmatrix} 1.0 & 1.0 \\ \cos kL & 0 \end{bmatrix}\begin{bmatrix} B \\ D \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\det|| = 0 \rightarrow \cos kL - 0 = 0 \rightarrow kL = \frac{\pi}{2}, \frac{3}{2}\pi \cdots$

$\therefore \sqrt{\frac{P}{EI}}L = \frac{\pi}{2}\ ;\ P_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 EI}{(2L)^2}\ ;\ K=2.0$

Note : ① Cramer's rule을 이용치 않을 경우

(**) 식에서 $B=0$ 이라면, (*) 식의 관계에서 $D=0$ 이 되어 trivial solution이 얻어진 것임. 따라서 $\cos kL=0$ 이 되어야 한다는 조건에서 구해질 수 있을 것임.

② 라플로는 $D=-B$ 이므로

$$y(x) = B\left[\cos\left(\frac{\pi}{2L}x\right)-1\right]$$

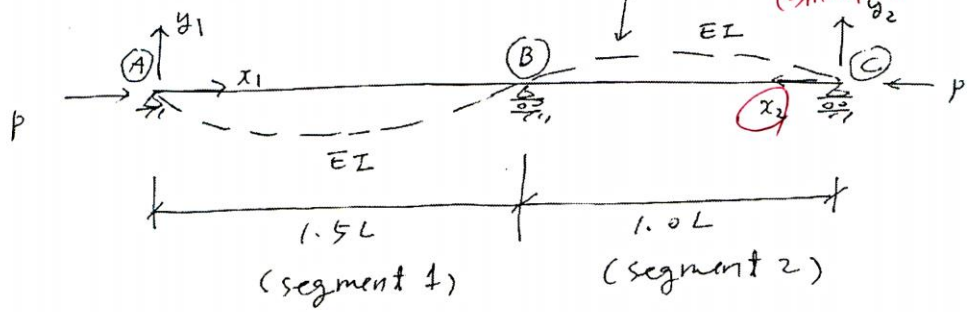


2.5 special members

2.5.1 two-axial-force column ← 앞에서 설명

2.5.2 Continuous member ↳ 해석 / follow-up 과제 HW#---

Reflected shape (smooth & continuous function)



For segments 1 & 2

$$y_1'''' + k^2 y'' = 0 \quad (k^2 = \frac{P}{EI})$$

Seg. 1

$$y_1 = A\sin kx_1 + B\cos kx_1 + C x_1 + D \quad \text{--- (30)}$$

Seg. 1

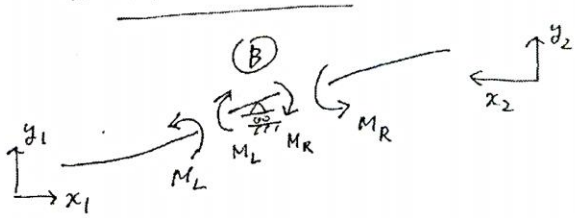
$$y_2 = E\sin kx_2 + F\cos kx_2 + G x_2 + H \quad \text{--- (31)}$$

경계 BCs 적용

경계 $\begin{cases} y_1(0) = 0 \\ y_1''(0) = 0 \\ y_1(\frac{3}{2}L) = 0 \end{cases}$ Ⓐ Ⓑ

경계 $\begin{cases} y_2(0) = 0 \\ y_2''(0) = 0 \\ y_2(L) = 0 \end{cases}$ Ⓒ Ⓑ

★ 적용 조건 : "모멘트 평형조건식"



$$M_L + M_R = 0 \quad \text{--- (8)}$$

$$+ EI\, y_1''(\tfrac{3}{2}L) + \{-EI\, y_2''(L)\} = 0$$

$$\therefore\ y_1''(\tfrac{3}{2}L) = y_2''(L) \quad \text{--- (8)}$$

↑ 모라서의 연속조건이라는 설명을 이해해야 함.

Continuity 조건

↳ 지점 Ⓑ 에서

$y_1(\frac{3}{2}L) = y_2(L) = 0$ 이므로 — 이 조건은 이미 앞에서 사용되었음!

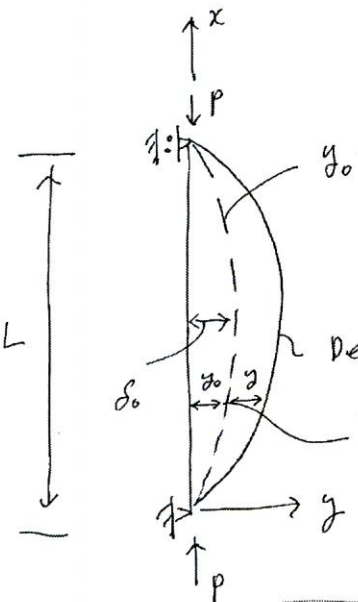
$$y_1'(\tfrac{3}{2}L) = y_2'(L) \times (-) \quad \text{--- (7)}$$

↳ minus 부호 적용

以下 생략!

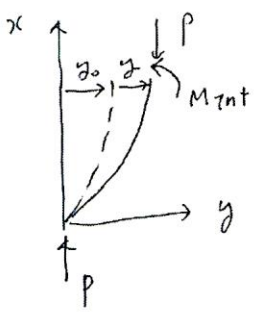
2.6 Initially "crooked" columns ← "Load-deflection problem 의 2"

In reality, all columns are imperfect. [geometric (initial out-of-straightness)
material (e.g., residual stress)
↳ 2.7 절의 주제



$y_0 = \delta_0 \sin\frac{\pi}{L}x$ (초기만곡)
↑ mid-height value

(즉 초기만곡 상태에서 유발되는 internal moment는 없음)



$\Sigma M = 0 \rightarrow M_{int} - P(y_0 + y) = 0$

$-EIy'' - P(y_0 + y) = 0$
↑ y_0 과 상관이 없음

$$EIy'' + Py = -Py_0 = -P \times \delta_0 \sin\left(\frac{\pi}{L}x\right)$$

$$y'' + k^2 y = -k^2 \delta_0 \sin\frac{\pi}{L}x \quad \text{--- (2.6.4)}$$

$$y = \underbrace{A\sin kx + B\cos kx}_{y_c} + \underbrace{C\sin\frac{\pi}{L}x + D\cos\frac{\pi x}{L}}_{y_p \leftarrow \text{"}\sin\frac{\pi}{L}x\text{"}} \quad \text{--- (2.6.6)}$$

특수해 y_p 의 계수 C 및 D 는 "미정계수법" 을 사용; y_p 를 (2.6.4) 식에 대입하여 결정

$$\left[C\left(k^2-\frac{\pi^2}{L^2}\right)+k^2\delta_0\right]\sin\frac{\pi x}{L}+\left[D\left(k^2-\frac{\pi^2}{L^2}\right)\right]\cos\frac{\pi x}{L}=0 \quad \cdots (2.6.7)$$

서로 orthogonal (or independent)

첫째 항등항 = zero,

$$\checkmark C=\frac{\delta_0(P/Pe)}{1-P/Pe} \quad \cdots (2.6.8)$$

둘째 항등항 = zero

$$\checkmark D=0 \quad \text{or} \quad \left(k^2-\frac{\pi^2}{L^2}=0\right)$$

Euler 좌굴하중에 대응되는 계수로서 0이 아님

따라서 $y=\underbrace{A\sin kx+B\cos kx}_{y_p}+\underbrace{\left[\frac{\delta_0(P/Pe)}{1-P/Pe}\right]\times\sin\frac{\pi}{L}x}_{y_p}$

A, B는 경계조건에서 산정

$y(0)=0 \rightarrow B=0$

$y(L)=0 \rightarrow A\sin kL+[\,\cdot\,]\times\underbrace{\sin\frac{\pi}{L}\times L}_{\text{zero}}=0$

$A\sin kL=0$

$A=0$ or $(\sin kL=0)$

역시 Euler 좌굴하중에 대응되는 계수

$$y=\left[\frac{\delta_0(P/Pe)}{1-P/Pe}\right)\sin\left(\frac{\pi}{L}x\right)$$

The total deflection (y_0+y)

$$\therefore\ y_{total}=\delta_0\sin\left(\frac{\pi}{L}x\right)+\frac{\delta_0(P/Pe)}{1-P/Pe}\sin\left(\frac{\pi}{L}x\right)$$

$$\frac{1-P/Pe+P/Pe}{1-P/Pe}$$

$$=\delta_0\sin\left(\frac{\pi}{L}x\right)\left[1+\frac{P/Pe}{1-P/Pe}\right]$$

A_F (증폭계수)

$$=\left(\frac{1}{1-P/Pe}\right)\times\delta_0\sin\left(\frac{\pi}{L}x\right)$$

Initial deflection

유사하게

M (the moment in the column, or the 2nd order moment)

= P × (y₀ + y)

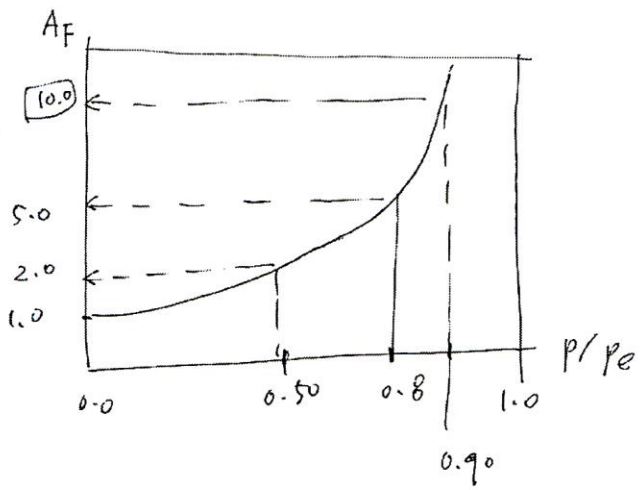
= P × $\left(\frac{1}{1-P/Pe}\right)$ × δ₀ sin $\left(\frac{\pi}{L}x\right)$ ← First-order moment

(deformed geometry-based)

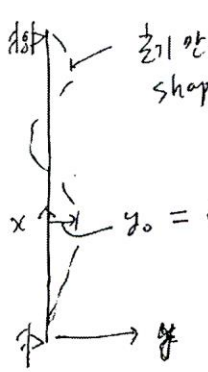
∴ $M_{2nd} = \left(\frac{1}{1-P/Pe}\right) \times M_{1st}$

→ A_F (증폭된 정도를 의미) = 「moment magnification factor」

처짐량



Note:



처짐량의 일련의 shape 인데

$y_0 = \delta_1 \sin\frac{\pi x}{L} + \delta_2 \sin\frac{2\pi x}{L} + \cdots$ (Fourier sine series expansion)

$y_{total} = \frac{\delta_1}{1-P/Pe}\sin\frac{\pi x}{L} + \frac{2^2\delta_2}{2^2-P/Pe}\sin\left(\frac{2\pi}{L}x\right) + \frac{3^2\delta_3}{3^2-P/Pe}\sin\left(\frac{3\pi}{L}x\right) + \cdots$

linear system (or differential eq.) 이므로

각각의 해를 중첩 가능.

$\begin{cases} a y_1'' + y_1 = x_1 \\ a y_2'' + y_2 = x_2 \end{cases}$

then, $a(y_1'' + y_2'') + (y_1 + y_2) = x_1 + x_2$ or $a(y_1+y_2)'' + (y_1+y_2) = (x_1+x_2)$

← output의 중첩 / input의 중첩

2.6.2 Perry-Robertson Formula → EuroCode에서 활용

↑ "편심축하중"을 받는 경우의 압축응력도값을 구하는 Secant formula와 흡사 (2.2.52식, P.58)

초기처짐을 갖는 기둥의 최대 수직응력도는 기둥 중앙부 높이에서 발생하므로,

$\delta_0 \sin(\frac{\pi}{L}x)$ 형태의 경우

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{I} \times c \left(\frac{1}{1-P/P_e}\right) P\delta_0 \times \sin\frac{\pi x}{L}\Big|_{x=L/2}$$

($\sin\frac{\pi x}{L}\Big|_{x=L/2} = 1.0$)

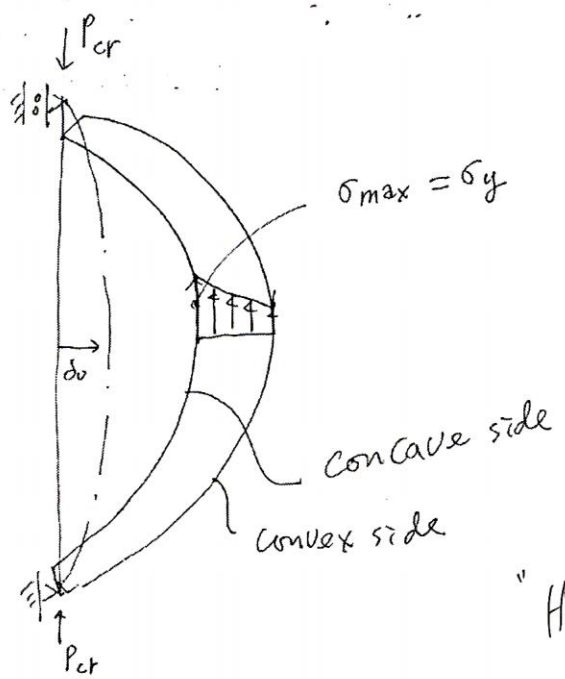
$$= \frac{P}{A} + \left(\frac{P\delta_0}{1-P/P_e}\right) \cdot \frac{c}{Ar^2}$$

$$\therefore \sigma_{max} = \frac{P}{A}\left[1 + \frac{\delta_0 c}{r^2}\,\frac{1}{(1-P/P_e)}\right] \quad \text{--- (2.6.27)}$$

★ $\sigma_{max} = \sigma_y$ 가 되면, fiber의 휨항복을 유발하는 "임계하중" P_{cr} 을 구할 수 있음

주어진 초기처짐에 대하여

구한하중과 동일한 값.



"HW제출"