2019 Spring

# "Phase Equilibria in Materials"

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- **Binary System** mixture/ solution / compound
- Gibbs Free Energy in Binary System

$$G_1 = X_A G_A + X_B G_B \quad J/mol$$
  $G_2 = G_1 + \Delta G_{mix} \quad J/mol$ 

**Ideal solution (\Delta H\_{mix} = 0)**  $\Delta G^{mix} = RT(X_A \ln X_A + X_B \ln X_B)$ 

$$G = X_A G_A + X_B G_B + RT(X_A \ln X_A + X_B \ln X_B)$$

**Regular solution**  $\Delta H_{mix} = P_{AB}\varepsilon$  where  $\varepsilon = \varepsilon_{AB} - \frac{1}{2}(\varepsilon_{AA} + \varepsilon_{BB})$  $G = X_A G_A + X_B G_B + \Omega X_A X_B + RT(X_A \ln X_A + X_B \ln X_B)$ 

- Chemical potential and Activity

$$\mu_{\mathsf{A}} = \left(\frac{\partial \mathsf{G'}}{\partial \mathsf{n}_{\mathsf{A}}}\right)_{\mathsf{T}, \mathsf{P}, \mathsf{I}}$$

$$\mu_{A} = G_{A} + RT \ln a_{A} \quad \ln \left(\frac{a_{A}}{X_{A}}\right) = \frac{\Omega}{RT} (1 - X_{A})^{2}$$

 $\mu$ 는 조성에 의해 결정되기 때문에  $dn_A$ 가 매우 작아서 조성변화 없어야

- $\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$
- Chemical equilibrium → Gibbs phase rule 2

#### **Regular Solutions**



## At $T_c$ the term $d^2(\Delta G_m)/d(X_A)^2$ will be zero.

Since

then

$$\frac{d^{2}(\Delta G_{m})}{d(X_{A})^{2}} = -2NC + NkT_{c}\left(\frac{1}{X_{A}} + \frac{1}{1 - X_{A}}\right) = 0$$
$$2C = \frac{kT_{c}}{X_{A}(1 - X_{A})} \text{ or } T_{c} = \frac{2CX_{A}(1 - X_{A})}{k}$$

The term  $T_c$  will be a maximum when  $X_A = (1 - X_A) = 0.5$ . It follows that

$$T_c = \frac{C}{2k}.$$
(101)

A high value of the critical temperature is associated with a high positive value for  $C = z[H_{AB} - \frac{1}{2}(H_{AA} + H_{BB})]).$ 

The stronger the attraction between similar atoms, the higher  $T_c$ . In those binary phase diagrams with a miscibility gap in the solid state the gap has not the symmetrical form shown in Fig. 21. This is primarily because the initial simplifying assumption that the energy is the sum of interaction between pairs of atoms is never absolutely valid. The systems Pd-Ir\*, Pt-Ir\*\* and Pt-Au\*\*\* all have miscibility gaps in the solid state with varying degrees of asymmetry. Most binary phase diagrams with a positive value of  $\Delta H_m$  do not show a miscibility gap with a closure at temperature  $T_c$  since melting occurs before  $T_c$  is reached (for example the Ag-Cu system).



#### Activity, a : effective concentration for mass action



#### **Activity-composition curves for solutions**



• For a dilute solution of B in A  $(X_B \rightarrow 0)$ 

 $\gamma_{B} = \frac{\mathbf{a}_{B}}{X_{B}} \cong \text{constant} \quad (\text{Henry's Law})$  $\gamma_{A} = \frac{\mathbf{a}_{A}}{X_{A}} \cong 1 \qquad (\text{Rault's Law})$ 7

# The Gibbs Phase Rule

**Degree of freedom** (number of variables that can be varied independently)

#### = the number of variables – the number of constraints

- Number of phases : p, number of components : c,
- # of controllable variable : composition (c-1)p, temperature : p, pressure : p
- # of restrictions :

(p-1)c from chemical equilibrium $\mu_i^a = \mu_i^\beta = \mu_i^\gamma = \dots = \mu_i^p$ p-1 from thermal equilibrium $T^a = T^\beta = T^\gamma = \dots = T^p$ p-1 from mechanical equilibrium $P^a = P^\beta = P^\gamma = \dots = P^p$ 

- Number of variable can be controlled with maintaining equilibrium

$$f = (c-1)p + p + p - (p-1)c - (p-1) - (p-1) = c - p + 2$$
$$f = c - p + 2$$

- If pressure is constant : f = (c-1)p + p - (p-1)c - (p-1) = c - p + 1

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# **Q1: What is "Real Solution"?**



\* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.



## **Real solution:** sufficient disorder + lowest internal E

- 2) In systems where there is a size difference between the atom e.g. interstitial solid solutions,
- $\rightarrow \Delta E = \Delta H_{mix} + elastic strain$
- → quasi- chemical model ~ underestimate ∆E due to no consideration of elastic strain field



> New mathematical models are needed to describe these solutions.

# **Q2: Short range order in solid solution?**

**1.3 Binary Solutions** 

# **Ordered phase** $\epsilon < 0$ , $\Delta H_{mix} < 0$

SRO (Short Range Ordering) or LRO (Long Range Ordering)

- $\Omega < 0 \Rightarrow$  contain short-range order (SRO)
  - $\Delta \Omega = N_a z \epsilon$  **SRO parameter = s \_ degree of ordering**

$$s = \frac{P_{AB} - P_{AB}(random)}{P_{AB}(max) - P_{AB}(random)}$$



Fig. 1.19 (a) Random A-B solution with a total of 100 atoms and  $X_A = X_B = 0.5$ ,  $P_{AB} \sim 100$ , S=0. (b) Same alloy with short-range order  $P_{AB} = 132$ ,  $P_{AB}(max) \sim 200$ , S=(132-100)/(200-100)=0.32. 14



![](_page_15_Figure_0.jpeg)

# **Ordered phase: "Long range order (LRO)"**

(1)superlattice, 2)intermediate phase, 3)intermetallic compound)

 $\Delta H_{mix}^{S} < 0$ : Solid solution  $\rightarrow$  ordered phase

 $\Delta H_{mix}^{S} \ll 0$ : **Compound : AB, A<sub>2</sub>B...** 

G<sub>E</sub>

В

![](_page_17_Figure_2.jpeg)

 $\Delta H_{mix}^S \ll 0$ 

**Small composition range** → G ↓

![](_page_17_Figure_5.jpeg)

Fig. 1.23 Free energy curves for intermed pound with a very narrow stability range, ( stability range.

![](_page_17_Figure_7.jpeg)

liquid

α

 $X_{\rm B}$ 

Α

B

# **Q3: Superlattice**

\* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.

High temp.  $\longrightarrow$  Entropy effect  $\uparrow \longrightarrow$  Solution stability  $\uparrow$ X<sub>B</sub> \_\_\_\_ X<sub>B</sub> — **†** +0  $\Delta H_{mix}$  $\Delta \boldsymbol{G}_{mix}$  $= \Delta H_{mix} - T \Delta S_{mix}$  $\Delta H_{\rm mix}$ ·T & Smix -T & Smix  $\Delta G_{mix}$  $\Delta G_{mix}$ B В (b)  $\Omega < 0$ , low T  $\Omega < 0$ , high T (a)  $\Delta H_{\rm mix}$ +  $\Delta H_{\rm mix}$  $\Delta G_{mix}$ X<sub>B</sub> 0  $\cap$  $X_{\rm B}$  $\Delta G_{\rm mix}$  $-T_{\Delta}S_{mix}$  $-T \triangle S_{mix}$ В В A A (c)  $\Omega > 0$ , high T (d),  $\Omega > 0 \text{ low } T$ 20 **1.3 Binary Solutions** 

# **Ordered phase** $\epsilon < 0, \Delta H_{mix} < 0$

- \* In solutions with compositions that are close to a simple ratio of A:B atoms another type of order can be found.
- \* This is known as long-range order (LRO) CuAu, Cu<sub>3</sub>Au and many other intermetallics show LRO.

![](_page_20_Picture_4.jpeg)

(The atom sites are no longer equivalent but can be labelled as A-sites and B-sites.) \* A superlattice forms in materials with LRO

![](_page_20_Figure_6.jpeg)

# Superlattice formation: order-disorder transformation

 $-\varepsilon < 0$ ,  $\Delta H_{mix} < 0$ 

- between dissimilar atoms than between similar atoms
- Large electrochemical factor: tendency for the solute atoms to avoid each other and to associate with the solvent atoms
- Size factor just within the favorable limit: lead to atomic rearrangement

so as to relieve the lattice distortion imposed by the solute atoms

![](_page_21_Picture_6.jpeg)

β brass superlattice viewed as two inter-penetrating cubic lattices 22

# 1.3 Binary Solutions Five common ordered lattices

(a)L2<sub>0</sub>: (b) L1<sub>2</sub>: (c) L1<sub>0</sub>: CuZn/FeCo/NiAl/CoAl/ Cu<sub>3</sub>Au/Ni<sub>3</sub>Mn/Ni<sub>3</sub>Fe/Ni<sub>3</sub>Al/ CuAu/CoPt/FePt FeAl/AgMg/AuCd/NiZn Pt<sub>3</sub>Fe/Au<sub>3</sub>Cd/Co<sub>3</sub>V/TiZn<sub>3</sub>

![](_page_22_Picture_2.jpeg)

![](_page_22_Picture_3.jpeg)

![](_page_22_Picture_4.jpeg)

(b) Cu OAu

![](_page_22_Picture_5.jpeg)

(c) Cu OAu

![](_page_22_Picture_7.jpeg)

(d) Al OFe

(d) D0<sub>3</sub>:

Fe<sub>3</sub>Al/Cu<sub>3</sub>Sb/Mg<sub>3</sub>Li/Fe<sub>3</sub>Al/ Fe<sub>3</sub>Si/Fe<sub>3</sub>Be/Cu<sub>3</sub>Al (e)  $\bigcirc$  Cd  $\bigcirc$  Mg

(e) D0<sub>19</sub>:

23

Mg<sub>3</sub>Cd/Cd<sub>3</sub>Mg/Ti<sub>3</sub>Al/Ni<sub>3</sub>Sn/Ag<sub>3</sub>In/ Co<sub>3</sub>Mo/Co<sub>3</sub>W/Fe<sub>3</sub>Sn/Ni<sub>3</sub>In/Ti<sub>3</sub>Sn

# $\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix} \quad \text{Ordered phase} \qquad \varepsilon < 0, \ \Delta H_{mix} < 0$

Fig. 1.21. Part of the Cu-Au phase diagram showing the regions where the Cu<sub>3</sub>Au and CuAu superlattices are stable.

![](_page_23_Figure_2.jpeg)

- The entropy of mixing of structure with LRO is extremely small and the degree of order decrease with increasing temperature until above some <u>critical</u> <u>temperature</u> there is no LRO at all.
- This temperature is a maximum when the composition is the ideal required for the superlattice.
- The critical temperature for loss of LRO increases with increasing  $\Omega$  or  $\Delta H_{mix}$ , and in many systems the ordered phase is stable up to the melting point. <sub>2</sub>

![](_page_24_Figure_0.jpeg)

# **Q4: Order-disorder transition**

# **Order-disorder phase transformation**

- Not classical phase change=<u>~not depend on diffusion process</u>
- change of temperature allowed a continuous re-arrangement of atoms without changing the phase = "2<sup>nd</sup> order transition"
- boundary: ordered lattice & disordered lattice/phase rule could not applied there are cases in which an ordered phase of one composition exists in equilibrium with a disordered phase of a different composition.
- Simple composition of the type AB or AB<sub>3</sub> can the transformation

(i.e. at the temperature maximum) be considered diffusionless.

![](_page_26_Picture_6.jpeg)

**Intermediate Phase** 

 $\varepsilon < 0$ ,  $\Delta H_{mix} < 0 / \Delta H_{mix} \sim -21$  kJ/mol

![](_page_27_Figure_2.jpeg)

#### Cu-Zn Phase Diagram

#### **Eutectoid and Peritectic Reactions**

![](_page_28_Figure_2.jpeg)

<u>Eutectoid</u>: one solid phase transforms into two other solid phases upon cooling <u>Peritectic</u>: one solid and one liquid phase transform into another solid phase upon cooling

<u>Peritectoid</u>: two other solid phases transform into another solid phase upon cooling

![](_page_28_Figure_5.jpeg)

# **Q5: Intermediate phase vs Intermetallic compound**

## **Intermediate Phase**

![](_page_30_Figure_1.jpeg)

- \* Many intermetallic compounds have stoichiometric composition  $A_m B_n$  and a characteristic free energy curve as shown in Fig (a).
- \* In other structure, <u>fluctuations in composition can be tolerated</u> by some atoms occupying 'wrong' position or by atom sites being left vacant, and in these cases the curvature of the G curve is much less, Fig (b).

![](_page_31_Figure_0.jpeg)

#### **Intermediate Phase**

![](_page_32_Figure_1.jpeg)

## **Intermediate Phase**

ε << 0, ΔH<sub>mix</sub><< 0 / ΔH<sub>mix</sub>~ -142 kJ/mol

![](_page_33_Figure_2.jpeg)

# Q6: Main factors determining the structure of intermediate phase

1.3 Binary Solutions Intermediate Phase

\_"different crystal structure as either of the pure component"

## 3 main factors

determining the structure of Intermediate phase ?

# 1) Relative atomic size

- Laves phase (size difference: 1.1~1.6 ex: MgCu<sub>2</sub>)
   <u>fill space most efficiently ~ stable</u>
- Interstitial compound: MX, M<sub>2</sub>X, MX<sub>2</sub>, M<sub>6</sub>X

M= Cubic or HCP ex: Zr, Ti, V, Cr, etc, X= H, B, C, and N

# 2) Relative valency electron

- electron phases ex\_α & β brass
   # of valency electrons per unit cell
  - $\rightarrow$  depending on compositional change

# 3) Electronegativity

- very different electronegativites  $\rightarrow$  ionic bond\_normal valency compounds ex Mg<sub>2</sub>Sn

![](_page_35_Figure_13.jpeg)

## MgCu<sub>2</sub> (A Laves phase)

"Clustering"? → Phase separation Q7: Metastable vs Stable miscibility gap \* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.

![](_page_37_Figure_1.jpeg)

![](_page_38_Picture_0.jpeg)

![](_page_38_Figure_2.jpeg)

![](_page_39_Figure_1.jpeg)

#### Microstructure of as-cast CoCrFeNiCu HEA

![](_page_40_Figure_1.jpeg)

![](_page_40_Figure_2.jpeg)

![](_page_40_Figure_3.jpeg)

#### Compositional analysis of as-cast CoCrFeNi/Cu HEA (dendrite)

![](_page_41_Figure_1.jpeg)

- ROI 1, 2 : 1.4 nm x 2 nm x 2 nm
- ROI 3 : 1.2 nm x 2 nm x 23 nm

(1D concentration profile)

1	at%	2	at%
Со	26.19	Со	0.33
Cr	24.15	Cr	0.46
Fe	24.59	Fe	0.39
Ni	19.59	Ni	5.00
Cu	4.74	Cu	93.56
3-1'	at%	3-2'	at%
3-1' Co	at% 25.29	3-2' Co	at% 2.01
3-1' Co Cr	at% 25.29 25.63	3-2' Co Cr	at% 2.01 3.35
3-1' Co Cr Fe	at% 25.29 25.63 23.63	3-2' Co Cr Fe	at% 2.01 3.35 2.56
3-1' Co Cr Fe Ni	at% 25.29 25.63 23.63 20.66	3-2' Co Cr Fe Ni	at% 2.01 3.35 2.56 6.90

- Dendrite 는 matrix (4.74 at%Cu) 와 2nd phase (93.56 at%Cu)로 구성됨
- Matrix 와 2nd phase 계면에서의 segregation 없음

![](_page_42_Figure_0.jpeg)

![](_page_43_Picture_0.jpeg)

![](_page_43_Figure_2.jpeg)

![](_page_44_Picture_0.jpeg)

## $\varepsilon >> 0$ , $\Delta H_{mix} >> 0 / \Delta H_{mix} \sim +60 \text{ kJ/mol}$

![](_page_44_Figure_2.jpeg)

![](_page_45_Picture_0.jpeg)

#### $\epsilon >> 0$ , $\Delta H_{mix} >> 0 / \Delta H_{mix} \sim +58 \text{ kJ/mol}$

![](_page_45_Figure_2.jpeg)

"Clustering"? → Phase separation Q8: Spinodal decomposition

# 5.5.5 Spinodal Decomposition

# Spinodal mode of transformation has no barrier to nucleation

: describing the transformation of a system of two or more components in a metastable phase into two stable phases

![](_page_47_Figure_3.jpeg)

Fig. 5.38 Alloys between the spinodal points are unstable and can decompose into two coherent phasees  $\alpha_1$  and  $\alpha_2$  without overcoming an activation energy barrier. Alloys between the coherent miscibility gaps and the spinodal are metastable and can decompose only after nucleation of the other phase.

#### How does it differ between inside and outside the inflection point of Gibbs free energy curve?

1) Within the spinodal  $\frac{d^2G}{dX^2} < 0$ 

![](_page_47_Picture_7.jpeg)

: phase separation by small fluctuations in composition/ "up-hill diffusion"

2) If the alloy lies outside the spinodal, small variation in composition leads to an increase in free energy and the alloy is therefore metastable.

> The free energy can only be decreased if nuclei are formed with a composition very different from the matrix.

- $\rightarrow$  nucleation and growth 48
  - : "down-hill diffusion"

![](_page_48_Figure_0.jpeg)

Fig. 5.39 & 5.40 schematic composition profiles at increasing times in (a) an alloy quenched into the spinodal region ( $X_0$  in Figure 5.38) and (b) an alloy outside the spinodal points ( $X_0$ ' in Figure 5.38) 49

# **Q9: Phase separation**

![](_page_50_Figure_0.jpeg)

#### Positive heat of mixing relation among constituent elements

Nucleation and growth ↔ Spinodal decomposition without any barrier to the nucleation process

#### \* Ti-Y-Al-Co system

![](_page_51_Figure_1.jpeg)

 $(Y_{56}Al_{24}Co_{20})_{25}(Ti_{56}Al_{24}Co_{20})_{75}$ 

 $(Y_{56}Al_{24}Co_{20})_{50}(Ti_{56}Al_{24}Co_{20})_{50}$ 

 $(Y_{56}Al_{24}Co_{20})_{65}(Ti_{56}Al_{24}Co_{20})_{35}$ 

#### \* La-Zr-Al-Cu-Ni system

![](_page_52_Figure_1.jpeg)

#### FIGURE 5.17

Schematic of the miscibility gap and the sequence of phase formation during cooling in the La–Zr–Al–Cu–Ni system. The positions of letters (a) to (d) in the diagram on the left correspond to the schematic microstructures (a) to (d) on the right. (Reprinted from Kündig, A.A. et al., *Acta Mater.*, 52, 2441, 2004. With permission.)

![](_page_53_Picture_0.jpeg)

# Phase separation in metallic glasses

![](_page_53_Picture_2.jpeg)

Q10: Microstructure determining parameters of phase separation in metallic glasses

(a) Composition

(b) Critical temperature, T<sub>c</sub>

(c) Asymmetry of the spinodal curve/decomposition range

(d) Glass-forming ability of the separated liquid

#### Synthesis of metallic glass composites using phase separation phenomenon

![](_page_55_Figure_1.jpeg)

# Possibility of two phase !!! Ti-Al-Co, Gd-Al-Co

(a) Composition section selected by rectangular plane intersection in quaternary Gd-Ti-Al-Co composition tetrahedron. (b) Gibbs free energy surface of liquid phase at 1000 K for the composition section given in (a). This Gibbs free energy surface shows two minima (arrows) implying that the phase separation can occur in that region.

![](_page_55_Figure_4.jpeg)

![](_page_56_Figure_0.jpeg)

![](_page_57_Figure_0.jpeg)

Chang et al., Acta Mater (2010)

#### (a) Composition

## Microstructure evolution (GdTiAlCu)

![](_page_58_Figure_2.jpeg)

![](_page_58_Figure_3.jpeg)

Chang et al., Acta Mater (2010)

![](_page_59_Figure_0.jpeg)

## (c) Asymmetry of spinodal curve / Decomposition range

![](_page_60_Picture_1.jpeg)

\* Symmetric spinodal curve / smaller decomposition range

![](_page_60_Picture_3.jpeg)

![](_page_60_Figure_4.jpeg)

#### \*Asymmetric spinodal curve / larger decomposition range

![](_page_60_Picture_6.jpeg)

![](_page_60_Figure_7.jpeg)

# \* Schematic drawings of the microstructures showing variation of microstructure depending on alloy composition and second phase separation mechanism.

![](_page_61_Figure_1.jpeg)

# **Contents for today's class**

- Binary System mixture/ solution / compound Ideal solution ( $\Delta H_{mix} = 0$ ) Random distribution **Regular solution**  $\Delta H_{mix} = P_{AB} \varepsilon$  where  $\varepsilon = \varepsilon_{AB} - \frac{1}{2} (\varepsilon_{AA} + \varepsilon_{BB})$   $\varepsilon \approx 0$  $\int \Delta H_{mix} > 0 \text{ or } \Delta H_{mix} < 0$ **Real solution Ordered structure** (c) when the size difference is large (a)  $\varepsilon < 0$ ,  $\Delta H_{mix} < 0$ (b)  $\epsilon > 0$ ,  $\Delta H_{mix} > 0$ strain effect Ordered alloys Clustering Interstitial solution P<sub>AA</sub>, P<sub>BB</sub> 1  $P_{AB} \uparrow \longrightarrow \text{Internal } E \downarrow$ 63

![](_page_63_Figure_0.jpeg)