457.309.02 Hydraulics and Laboratory

.10 Turbulent flow in rough pipe(2)

Prepared by Jin Hwan Hwang
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Today's objectives

- Leaning how to determine the friction factors
- Study on some empirical parameters for friction factors.

6. Pipe friction factors

- Since there is no exact solution for the pipe friction factors, determin ation of friction depends extensively on the experimental works.
- Friction factor, must depends on the shear stress, mean velocity, pip e diameter, mean roughness height, fluid density and viscosity.
- Dimensional analysis

$$\phi\left(\tau_0, V, d, e, \rho, \mu\right) = 0$$

- We have 6 variables which includes density (mass), therefore, 3 rec urring variables. That means we can come up with three non-dimens ional number describe this system.

$$\rho: \ \text{Mass}, \quad V: \ \text{Time}, \quad d: \text{Length}$$

6. Pipe friction factors

- Then $\pi_1 \Rightarrow \phi_1(\tau_0)$

$$\pi_2 \Rightarrow \phi_2(e)$$

$$\pi_3 \Rightarrow \phi_3(\mu)$$

$$M^0L^0T^0 = \phi_1(\rho, V, d, \tau_0) = \left[ML^{-3}\right]^a \left[LT^{-1}\right]^b \left[L\right]^c \left[ML^{-1}T^{-2}\right]^{-1}$$

$$a - 1 = 0,\ -3a + b + c + 1 = 0,\ -b + 2 = 0$$

$$a = 1,\ b = 2,\ c = 0$$

$$\pi_1 = \phi_1\left(\frac{\rho V^2}{\tau_0}\right) = \phi_1\left(\frac{\tau_0}{\rho V^2}\right)$$

- In the similar way,

$$\pi_2 = \phi_2\left(\frac{Vd\rho}{\mu}\right),\quad \phi_3 = f_3\left(\frac{e}{d}\right)$$

6. Pipe friction factors

- Therefore

$$\frac{\tau_0}{\rho V^2} = \phi\left(\frac{Vd\rho}{\mu}, \frac{e}{d}\right) = \phi\left(\mathrm{Re}, \frac{e}{d}\right)$$

$$\tau_0 = \rho V^2 \phi\left(\mathrm{Re}, \frac{e}{d}\right) \qquad \left(remember,\ \tau_0 = \frac{f\rho V^2}{8}\right)$$

$$\text{Finally} \quad : \quad f = \phi\left(\mathrm{Re}, \frac{e}{d}\right)$$

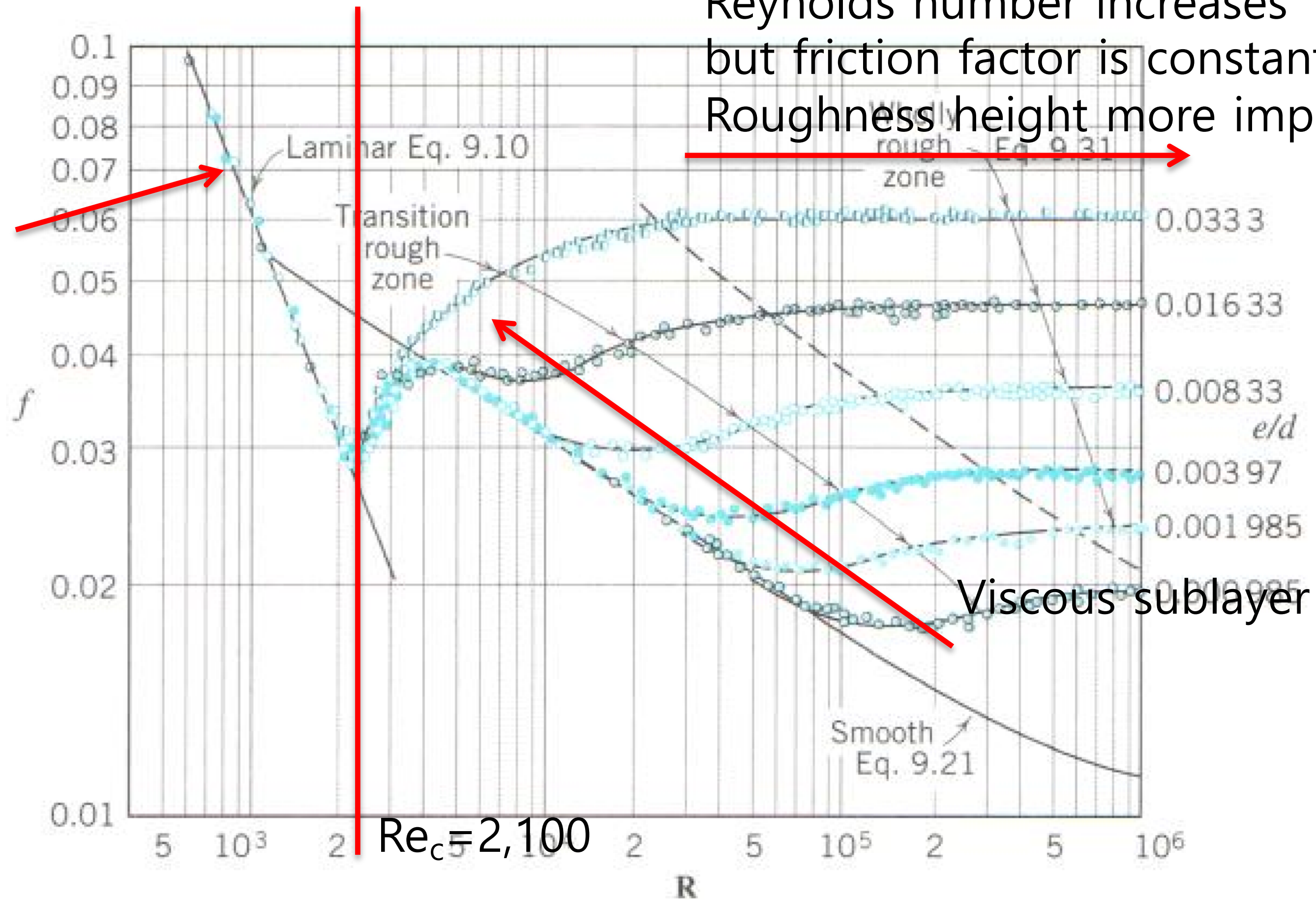
If dynamically and geometrically two systems are same , then their friction factors are same (similarity).

6. Pipe friction factors

- Blasius-Stanton diagram

$$f = \frac{64}{Re}$$

Reynolds number increases but friction factor is constant. Roughness height more important



Re_c=2,100

Viscous sublayer decreases

Example problem

- Water at 100°F flows in a 3 inch pipe at a Reynolds number of 80,000. If the pipe is lined with uniform sand grains 0.006 inches in diameter, how much head loss is to be expected in 1,000 ft of the pipe? How much head loss would be expected if the pipe were smooth?

$$\frac{e}{d} = \frac{0.006}{3} = 0.002 \text{ and } \text{Re}=80,000$$

$$f \cong 0.021 \quad \text{(Use figure)}$$

- To get velocity,

$$V = \frac{\text{Re} \cdot \nu}{d} = \frac{80,000 \times 0.739 \times 10^{-5}}{3/12} = 2.36\, ft/\sec$$

$$h_L = f \frac{l}{d} \frac{V^2}{2g_n} \cong 0.021 \frac{1000}{3/12} \frac{2.36^2}{2 \times 32.2} = 7.3\, ft$$

Example problem

- If flow is in a smooth pipe, then we can apply Blasius power relation ship

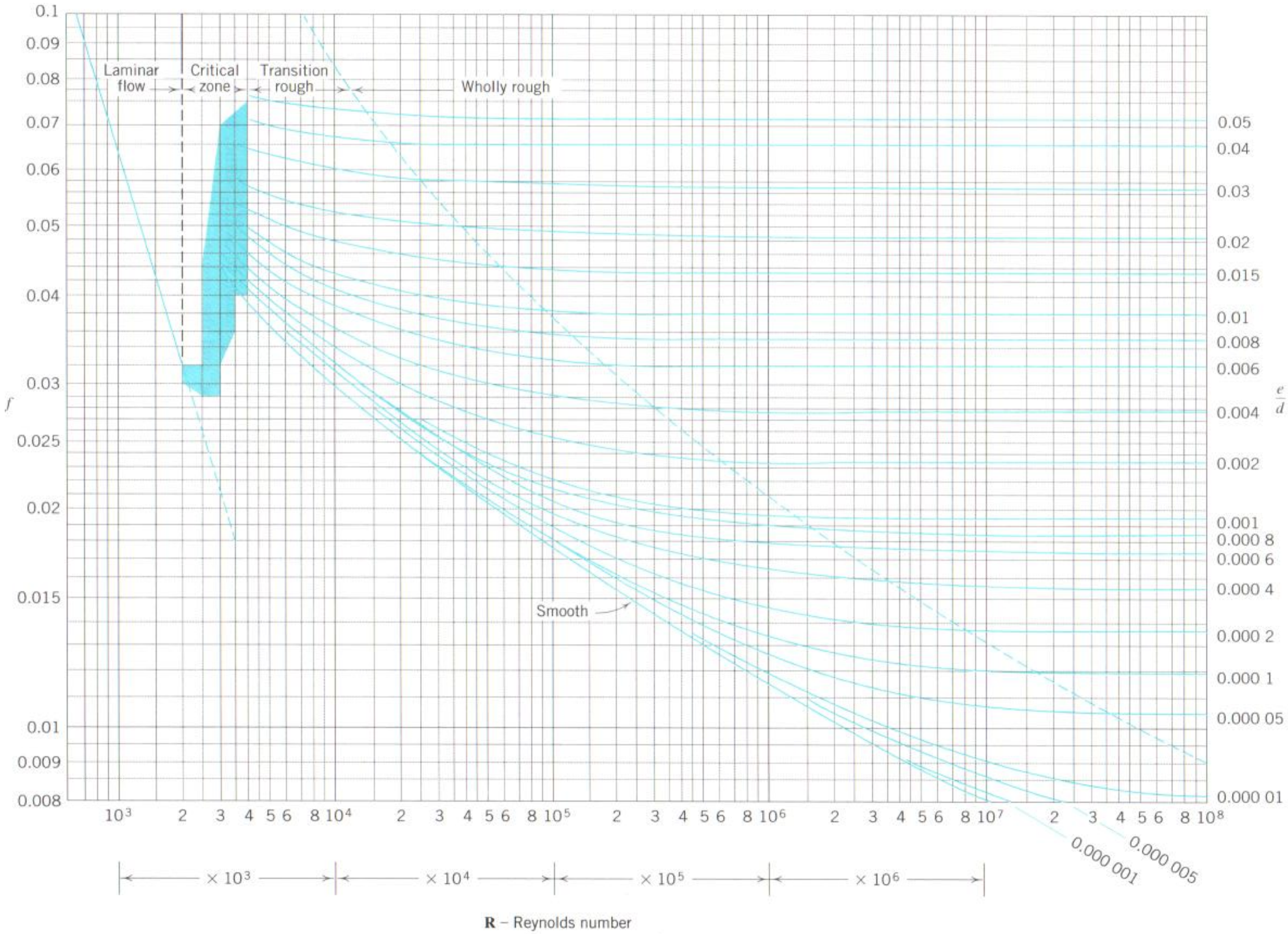
$$f = \frac{0.316}{\mathrm{Re}^{0.25}} = 0.0188$$

- The head loss in the smooth pipe

$$h_L = f \frac{l}{d} \frac{V^2}{2g_n} = 0.0188 \frac{1000}{3/12} \frac{2.36^2}{2 \times 32.2} = 6.5\,ft$$

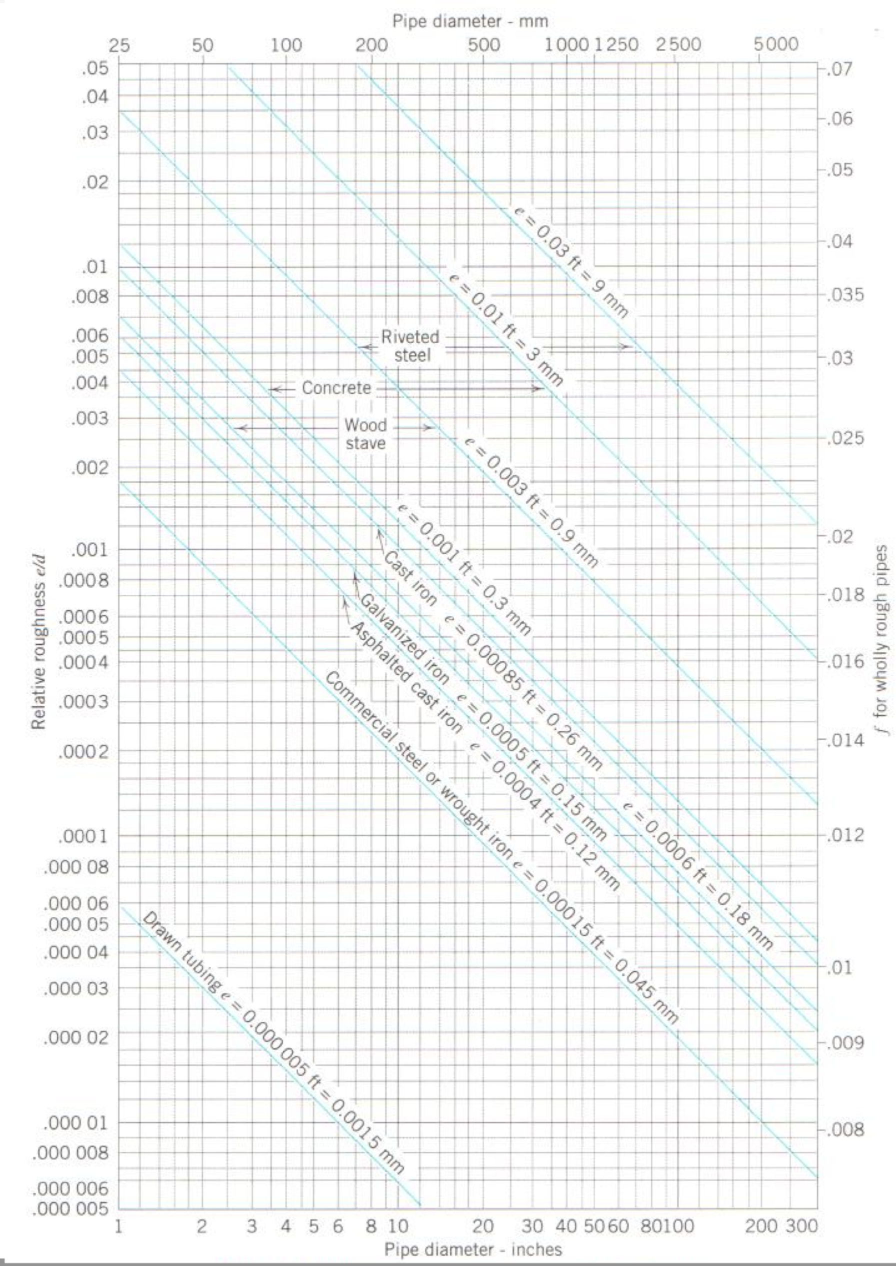
Moody Diagram

- As we discussed before, the commercial pipe does not follow Nikuradse's. In this case, we'd better use Moody diagram



Moody Diagram

- The relative roughness should be determined by the following (if you don't know well the roughness (but you know material)



Example problem

- Water at 100°F flows in a 3 inch pipe at a Reynolds number of 80,000. This is a commercial pipe with an equivalent sand grain roughness of 0.006 in. What head loss is to be expected in 1,000 ft of this pipe?

$$\frac{e}{d} = \frac{0.006}{3} = 0.002 \text{ and } \text{Re}=80{,}000$$

$$f \cong 0.0255 \quad \text{(Use Moody diagram)}$$

$$V = \frac{\text{Re}\cdot \nu}{d} = \frac{80{,}000 \times 0.739 \times 10^{-5}}{3/12} = 2.36\, ft/\sec$$

$$h_L = f\frac{l}{d}\frac{V^2}{2g_n} \cong 0.0255 \frac{1000}{3/12}\frac{2.36^2}{2\times 32.2} = 8.8\, ft$$

7. Pipe friction in noncircular pipes

- How determines the friction factor and head loss in rectangular duct s? Or other conduits of noncircular form.
- Hydraulic Radius

$$R_h = \frac{P}{A} \qquad (P \text{ is wetted parameter, } A \text{ is area})$$

- First calculate the hydraulic radius and determine the equivalent dia meter of the circular pipe (assume it).

$$d = 4R_h \qquad \left(R_h = \frac{\pi R^2}{2\pi R} = \frac{R}{2} = \frac{d}{4} \right)$$

- Use this diameter for Moody diagram
- In turbulent flow, it seems work but in laminar flow not applicable.

Example problem

- Calculate the loss of head and the pressure drop when air at an abs olute pressure of 101.kPa and 15°C flows through 600 m of 450 mm by 300 mm smooth rectangular duct with a mean velocity of 3 m/s.

$$R_h = \frac{A}{P} = \frac{0.45m \times 0.30m}{2 \times 0.45m + 2 \times 0.30} = 0.090$$

$$\mathrm{Re} = \frac{Vd\rho}{\mu} = \frac{V(4R_h)\rho}{\mu} = \frac{3m/s \times (4 \times 0.090m) \times 1.225kg/m^3}{1.789 \times 10^{-5}} = 73{,}950$$

$$f \cong 0.019 \quad \text{(From the Moody diagram for smooth diagram)}$$

$$\mathrm{h}_L = f\frac{l}{d}\frac{V^2}{2g_n} = f\frac{l}{4R_h}\frac{V^2}{2g_n} = 14.5m$$

$$\Delta p = \gamma h_L = \rho g h_L = 174\,pa$$

8. Empirical Formulas

- Hazen-Williams
  - Permitting the capacity of ***pipes*** to convey water
- Manning
  - Application to ***open channel***, but also used for pipe flow.
  - For a given pipe

(U.S. Customary units) $V = \dfrac{1.49}{n} R_h^{2/3} S^{1/2}$

(SI units) $V = \dfrac{1}{n} R_h^{2/3} S^{1/2}$

$f = \dfrac{185 n^2}{d^{1/3}}$ *where* n is roughness coefficient for manning

8. Empirical Formulas

- Manning's
  - There is no Reynolds number effect so the formula must be used only in the ***wholly rough flow zone*** where its horizontal slope c an accurately match Darcy-Weisbach values provided the proper n-value is selected.
  - The relative roughness effect is correct in the sense that, for a gi ven roughness, a ***larger pipe will have a smaller factor***.
  - In general sense, because the formula is ***valid only for rough pi pes***, the rougher the pipe, the more likely the Manning formula w ill apply.