Risk Management and Decision Analysis

Seokho Chi*

*Professor, Department of Civil and Environmental Engineering, Seoul National University

Recall

Risk Determinants & Components

- Risk Determinants: Lack of Time, Information, Control
- Risk Components: Event, Probability, Impact

Project Risk Management Process

① Recognize need for risk management

② Identify types & sources of risks

③ Measure & prioritize risks

④ Adjust & control risks

PART I

MODELING UNCERTAINTY

- Theory of Probability

Approaches to Probability

Frequentist(빈도학파)

- A measure of the likelihood that a particular event will happen
- Based on the assumption the observation of some activity can be repeated
  - Roll a die: Frequency of cell “4” is 1/6

Subjectivist(주관주의학파)

- Based on the assumption that observation cannot be duplicated
- Probability is the “degree of personal belief” based on whatever information is available
  - Predicting the chance of hurricane: 30%
  - Predicting the chance that you will earn “A+” in this course: 60%

Terminology

We can model uncertainty in decision problems by using probability.

Terminology for probability theory

- Experiment: process of observation or measurement; e.g., coin flip
- Outcome: result obtained through an experiment; e.g., coin shows tails
- Sample space: set of all possible outcomes of an experiment; e.g., sample space for coin flip: S={H, T}
- Sample spaces can be finite or infinite.

Sample Space

Example: Finite Sample Space

- Roll two dice, each with numbers 1~6. Sample space:

$$S_1 = \{(x, y): x \in \{1,2, \dots, 6\} \land y \in \{1,2, \dots, 6\}\}$$

- Alternative sample space for this experiment – sum of the dice:

$$S_2 = \{x + y: x \in \{1,2, \dots, 6\} \land y \in \{1,2, \dots, 6\}\}$$

$$S_2 = \{z: z \in \{2,3, \dots, 12\}\}$$

Example: Infinite Sample Space

- Flip a coin until heads appears for the first time:

$$S_3 = \{H, TH, TTH, TTTH, TTTTH, \dots\}$$

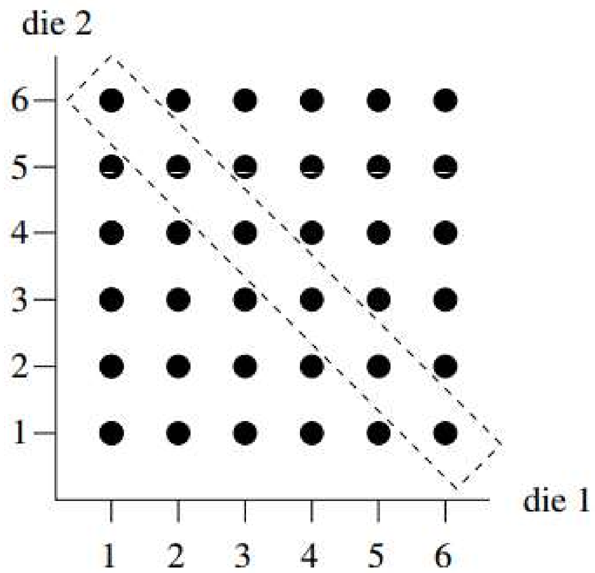
Events

Often we are not interested in individual outcomes, but in events. An event is subset of a sample space.

Example

- With respect to S_1, describe the event B of rolling a total of 7 with the two dice.

$$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$



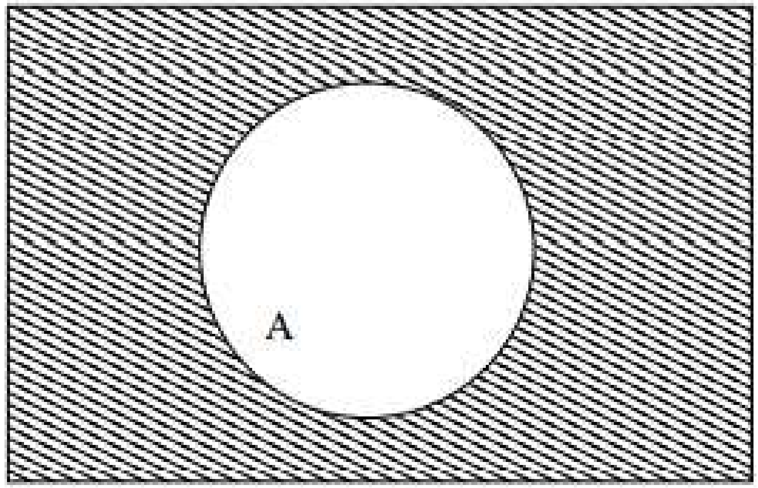
Events

Often we are interested in combinations of two or more events. This can be represented using set theoretic operations. Assume a sample space S and two events A and B:

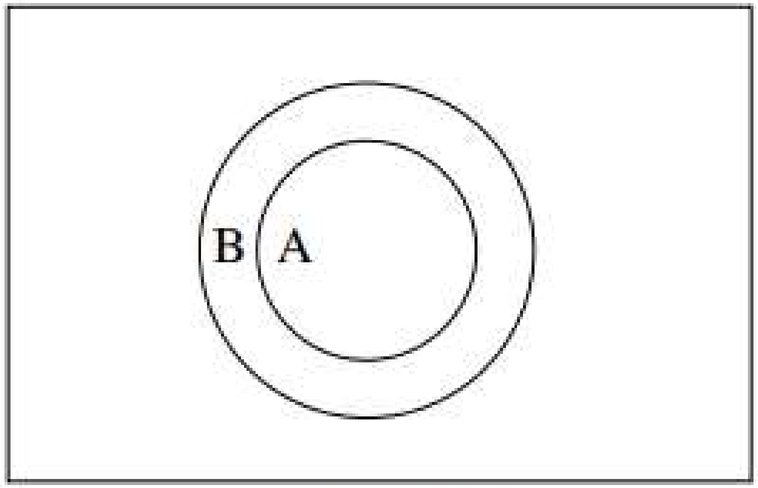
- complement $\bar{A}$ (also A'): all elements of S that are not in A;
- subset $A \subseteq B$: all elements of A are also elements of B;
- union $A \cup B$: all elements of S that are in A or B;
- intersection $A \cap B$: all elements of S that are in A and B.

These operations can be represented graphically using *Venn diagrams*.

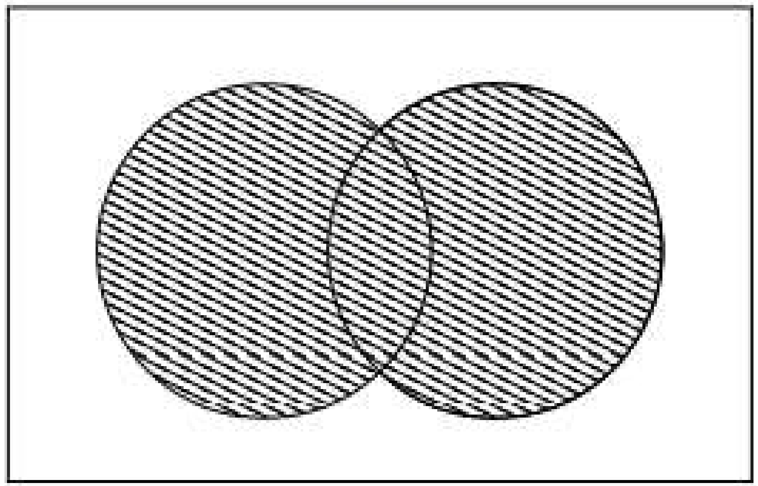
Venn Diagrams



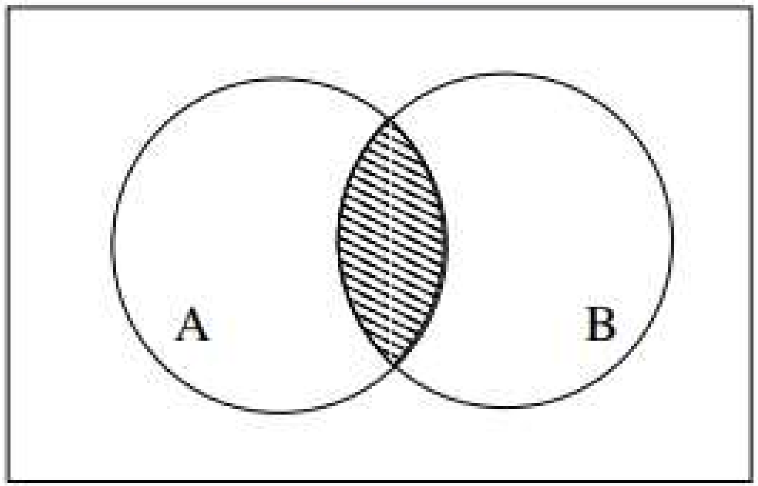
$\bar{A}$



$A \subseteq B$



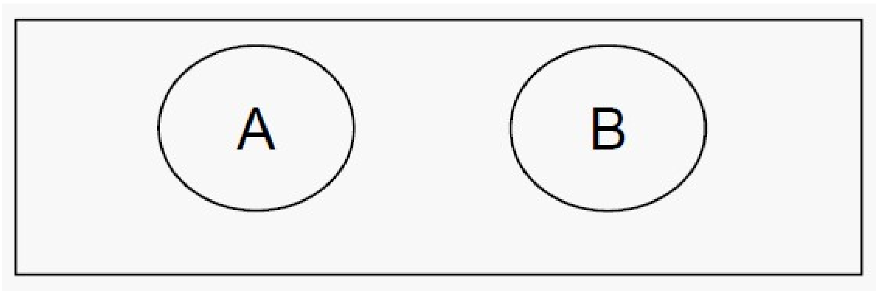
$A \cup B$



$A \cap B$

Basic Law of Probability

- Probability must lie between 0 and 1.
  - Sure occurrence: 1 (chance of rain in Seoul this year)
  - Non-occurrence: 0 (chance that the sun will disappear tomorrow)
- Probability must add up.
  - If A&B are mutually exclusive (If only one of several event can occur at one time): P(A or B) = P(A) + P(B)
  - Otherwise, P(A or B) = P(A) + P(B) - P(A and B)



- Total probability must equal 1: P(S)=1.

Basic Law of Probability

Theorem: Probability of an Event

- If A is an event in a sample space S and O_1, O_2, …, O_n are the individual outcomes comprising A, then $P(A) = \sum_{i=1}^{n} P(O_i)$

Example

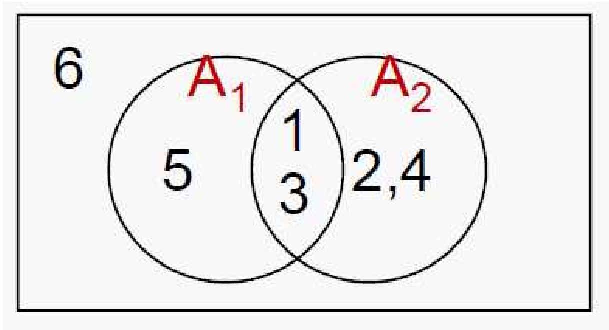
- 연속으로 세 개의 알파벳을 뽑는다고 할 때, 모음이 나올 확률은 얼마인가?

$$S = \{(x, y, z): x \in \{a, b, \dots, z\} \wedge y \in \{a, b, \dots, z\} \wedge z \in \{a, b, \dots, z\}$$
$$N = 26^3, \mathrm{n} = 5^3$$
$$P(A) = \frac{5^3}{26^3} \approx 0.00711$$

Exercise #1: Basic Law of Probability

Event “A”: Casting a single die

- A = [1, 2, 3, 4, 5, 6]
- Two outcomes
  - A_1 = odd outcome = [1, 3, 5]
  - A_2 = less than 5 = [1, 2, 3, 4]
  - Are they mutually exclusive?
- $P(A_1)$, $P(A_2)$ = ?
- $P(A_1$ and $A_2)$ = ?
- $P(A_1$ or $A_2)$ = ?



If A_2 = [2, 4, 6], What is the differences?

Basic Law of Probability

Here, we have some definitions,

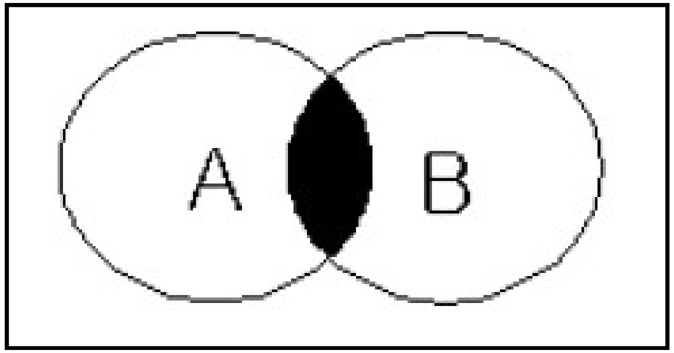
- **Marginal probability(주변확률, 한계확률)**: probability at the initial condition (boundary state) → P(A), P(B)
- **Union probability(합확률)**: P(A or B)
- **Joint probability(결합확률)**: P(A and B) → intersection
  - a probability that measures the likelihood that two or more events will happen at the same time
- **Conditional probability (|)**: "|" means "given that another event has already occurred"
  - P(B|A) → probability of event "B" after it is known that some other event "A" has ALREADY occurred

Basic Law of Probability

- **Law of Joint probability**
  - $P(A \text{ and } B) = P(A) \times P(B|A) = P(B \text{ and } A) = P(B) \times P(A|B)$

Marginal probability

- **Conditional probability**
  - $P(B|A) = P(A \text{ and } B)/P(A)$



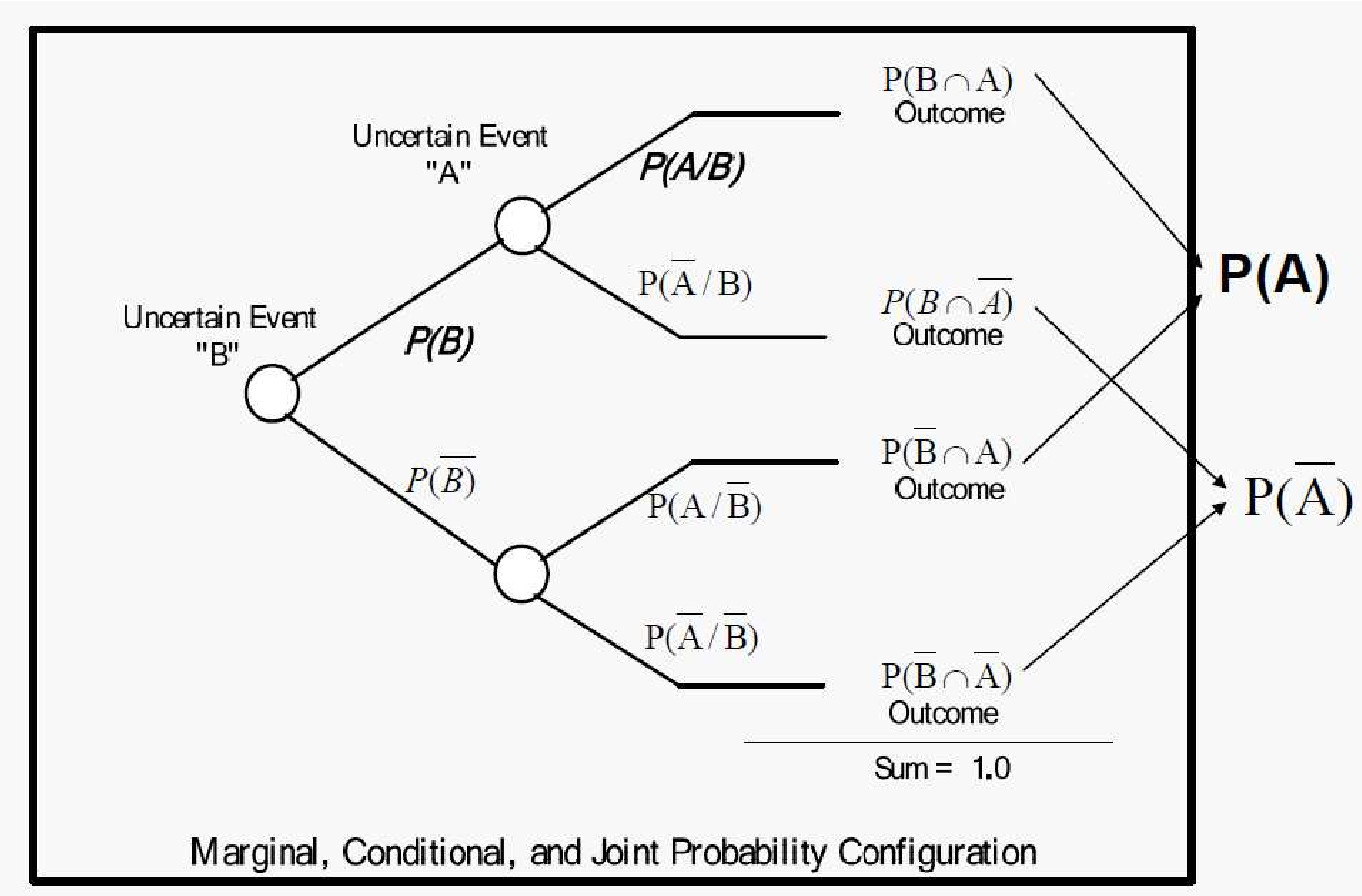
- **Complementary**
  - "A" Complement: $\overline{A}$
  - $P(\overline{A}) = 1 - P(A)$

- **Total probability**
  - $P(A) = P(A \cap B) + P(A \cap \overline{B})$

$$= P(B) \times P(A|B) + P(\overline{B}) \times P(A|\overline{B})$$

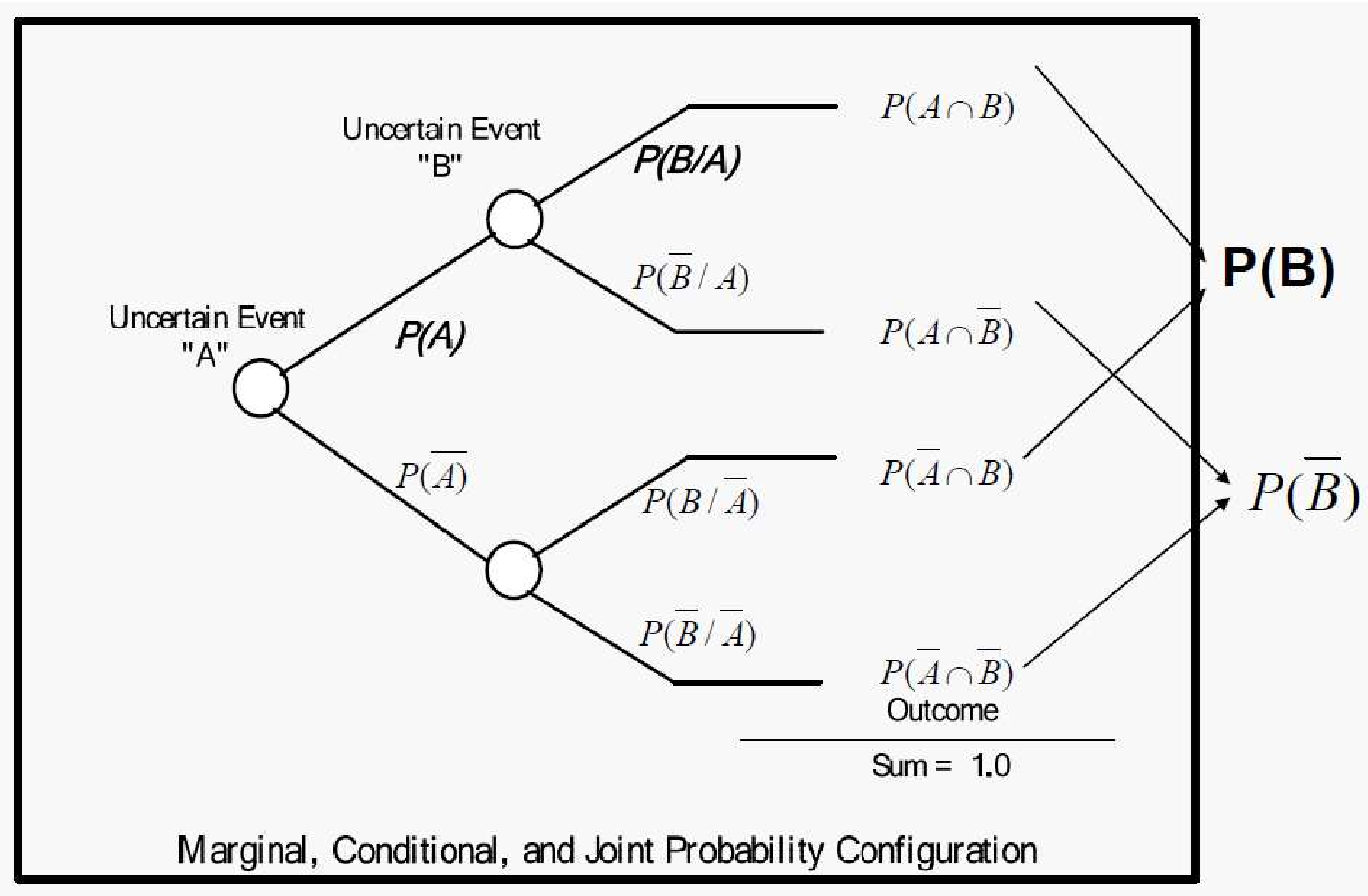
Basic Law of Probability

- **Probability Tree**



Basic Law of Probability

- **Probability Tree: We can FLIP it!!!**



Exercise #2: Conditional Probability

- A manufacturer knows that the probability of an order being ready on time is 0.80, and the probability of an order being ready on time and being delivered on time is 0.72. What is the probability of an order being delivered on time, given that it is ready on time?

  R: order is ready on time; D: order is delivered on time.

$$P(R) = 0.80$$

$$P(R \cap D) = 0.72$$

$$P(D|R) = \frac{P(R \cap D)}{P(R)} = \frac{0.72}{0.80} = 0.90$$

Exercise #3: Marginal Probability

- In an experiment on human memory, participants have to memorize a set of words (B1), numbers (B2), and pictures (B3). These occur in the experiment with the probabilities; $P(B_1) = 0.5, P(B_2) = 0.4, P(B_3) = 0.1$.
- Then participants have to recall the items (where A is the recall event). The results show that $P(A|B_1) = 0.4, P(A|B_2) = 0.2, P(A|B_3) = 0.1$. Compute $P(A)$, the probability of recalling an item.
  - By the theorem of total probability:

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).

Eye Color	Hair Color				
	Black	**Brunette**	**Blond**	**Red**	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).
  - These are the joint probability: P(Eye Color, Hair Color)

Eye Color	Hair Color				
	Black	Brunette	Blond	Red	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).
  - Example: P(Eye Color=Brown, Hair Color=Brunette)=0.20

	Hair Color				
Eye Color	**Black**	**Brunette**	**Blond**	**Red**	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).
  - These are the marginal probability: P(Eye Color)

Eye Color	Hair Color				
	Black	Brunette	Blond	Red	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).
  - Example: $P(Eye\ Color = Brown) = \sum_{Hair\ Color} P(Eye\ Color = Brown, Hair\ Color)$
    $= 0.12 + 0.20 + 0.01 + 0.04 = 0.37$

Eye Color	Hair Color				
	Black	Brunette	Blond	Red	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).
  - These are the marginal probability: P(Hair Color)

Eye Color	Hair Color				
	Black	Brunette	Blond	Red	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).
  - Example: $P(Hair\ Color = Brunette) = \sum_{Eye\ Color} P(Hair\ Color = Brunette, Eye\ Color)$

    $= 0.14 + 0.20 + 0.14 = 0.48$

	Hair Color				
Eye Color	Black	Brunette	Blond	Red	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).

To obtain the conditional probability $P(Eye\ Color|Hair\ Color = Brunette)$, we do two things:

① Reduction: we consider only the probabilities in the brunette column;

	Hair Color				
Eye Color	Black	Brunette	Blond	Red	
Blue	0.03	0.14	0.16	0.03	0.36
Brown	0.12	0.20	0.01	0.04	0.37
Hazel/Green	0.03	0.14	0.04	0.05	0.27
	0.18	0.48	0.21	0.12	

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).

To obtain the conditional probability $P(Eye\ Color|Hair\ Color = Brunette)$, we do two things:

① Reduction: we consider only the probabilities in the brunette column;

	Hair Color				
Eye Color	Black	Brunette	Blond	Red	
Blue		0.14			
Brown		0.20			
Hazel/Green		0.14			
		0.48			

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).

To obtain the conditional probability $P(Eye\ Color|Hair\ Color = Brunette)$, we do two things:

② Normalization: we divide by the marginal P(Brunette), since all the probability mass is now concentrated here.

Eye Color	Hair Color				
	Black	Brunette	Blond	Red	
Blue		0.14/0.48			
Brown		0.20/0.48			
Hazel/Green		0.14/0.48			
		0.48			

Exercise #4: Joint, Marginal, & Conditional Prob.

- Proportions for a sample of a certain university students (N=592).

To obtain the conditional probability $P(Eye\ Color|Hair\ Color = Brunette)$, we do two things:

Example: $P(Eye\ Color = Brown|Hair\ Color = Brunette) = \frac{0.20}{0.48} = 0.417$

Eye Color	Hair Color				
	Black	Brunette	Blond	Red	
Blue		0.14/0.48			
Brown		0.20/0.48			
Hazel/Green		0.14/0.48			
		0.48			

Exercise #5: Joint, Marginal, & Conditional Prob.

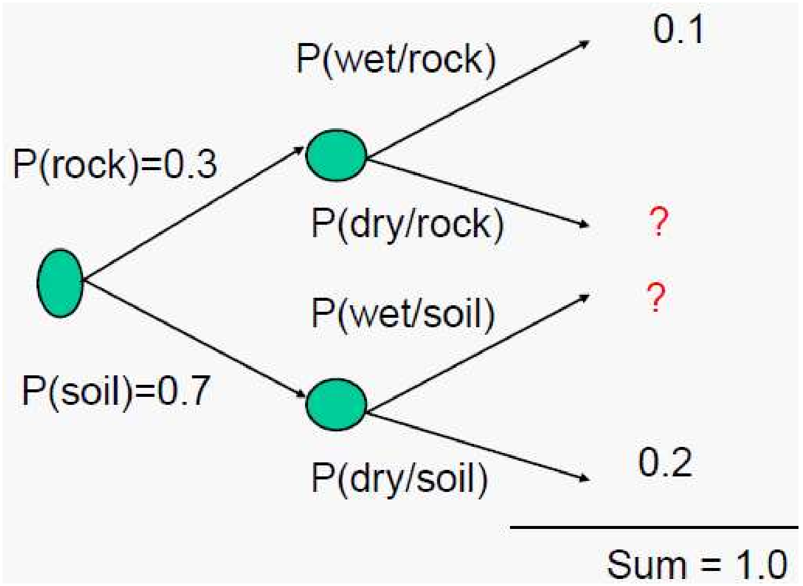
- A geotechnical database gives the following probabilities for certain subsurface conditions;
  - Mutually exclusive condition: “rock” & “soil”, “wet” & “dry”
  - P(rock) = 0.3
  - P(common soil) = 0.7
  - P(wet and rock) = 0.1
  - P(dry and common soil) = 0.2

1) Find the marginal probability of “wet” condition: P(wet)
2) Find the marginal probability of “dry” condition: P(dry)
3) Find the union probability of “wet or rock” conditions: $P(\text{wet} \cup \text{rock})$

Exercise #5: Joint, Marginal, & Conditional Prob.

Conditions

- P(rock) = 0.3, P(common soil) = 0.7
- P(wet ∩ rock) = 0.1
- P(dry ∩ common soil) = 0.2

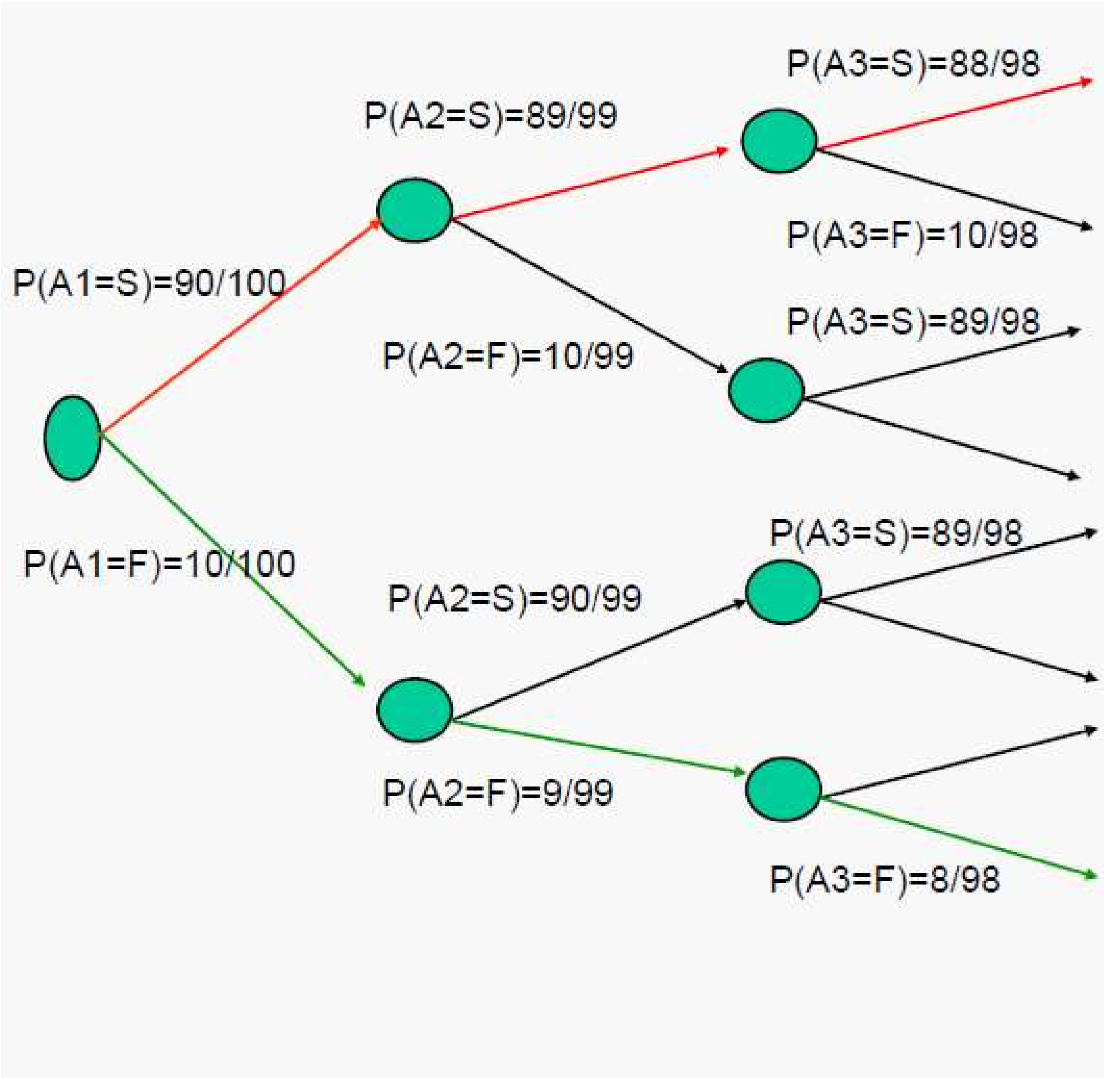


Answers

Exercise #6: Joint, Marginal, & Conditional Prob.

- Suppose that three strength tests are to be performed on the sample of 100 concrete cylinders. If there are 10 defective cylinders out of 100, what is the probability that all three tests will succeed? What is the probability that all three tests will fail? What is the probability that two tests will succeed?

Exercise #6: Joint, Marginal, & Conditional Prob.



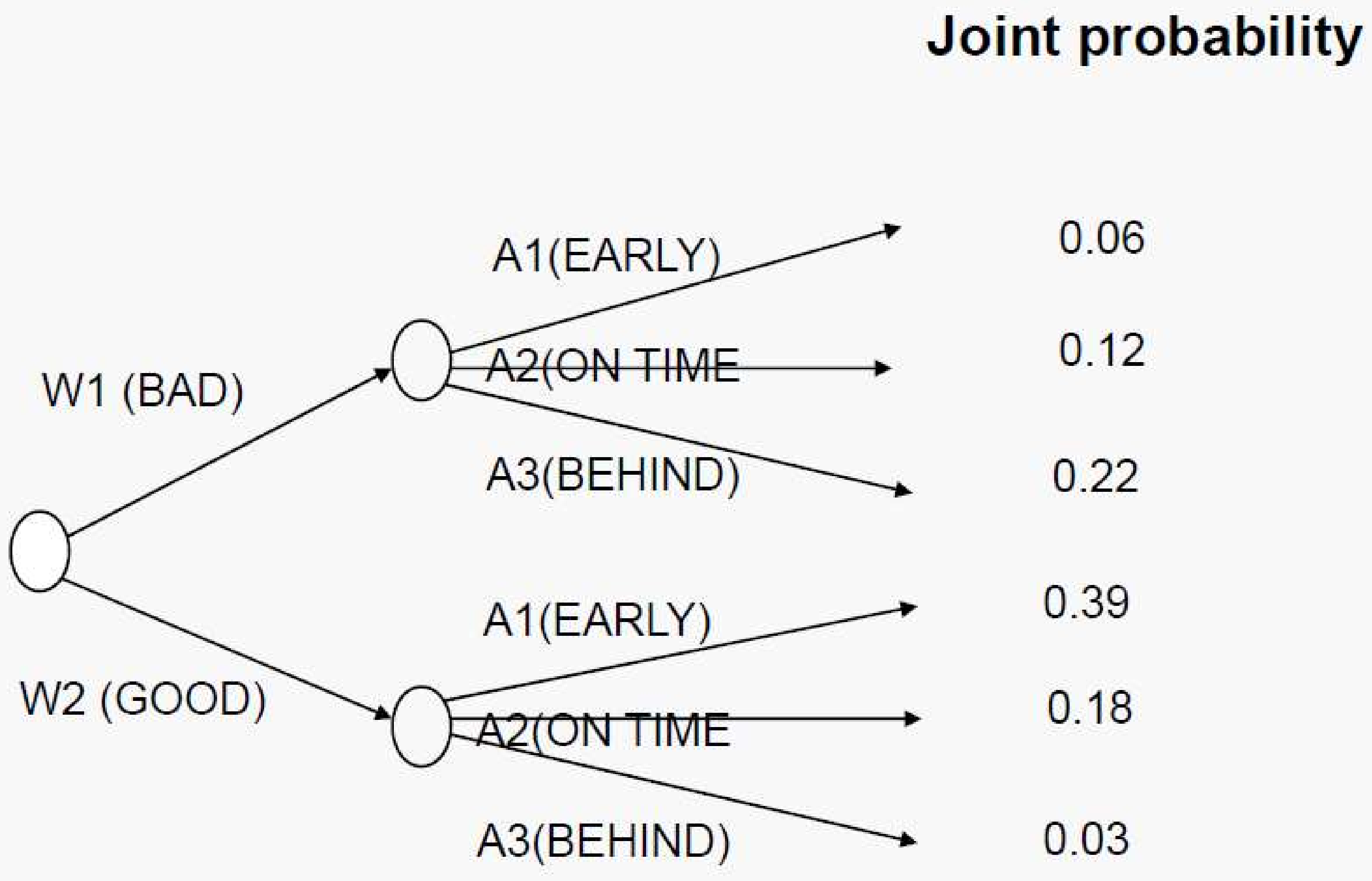
Exercise #7: Joint, Marginal, & Conditional Prob.

- On the construction project, there is either a good weather or a bad weather condition. Depending on which weather condition controls the project, time schedule may be either early, on time, and late. We define the following events:
  - W1: bad weather
  - W2: good weather
  - A1: ahead of schedule
  - A2: on time
  - A3: behind of schedule
- The joint probabilities are:
  - W1 & A1 = 0.06; W1 & A2 = 0.12; W1 & A3 = 0.22
  - W2 & A1 = 0.39; W2 & A2 = 0.18; W2 & A3 = 0.03

Exercise #7: Joint, Marginal, & Conditional Prob.

1. What are the **three conditional probabilities given a bad weather condition**?
2. **Given** you finish the project ahead of schedule, what is the **probability you had a good weather condition**?

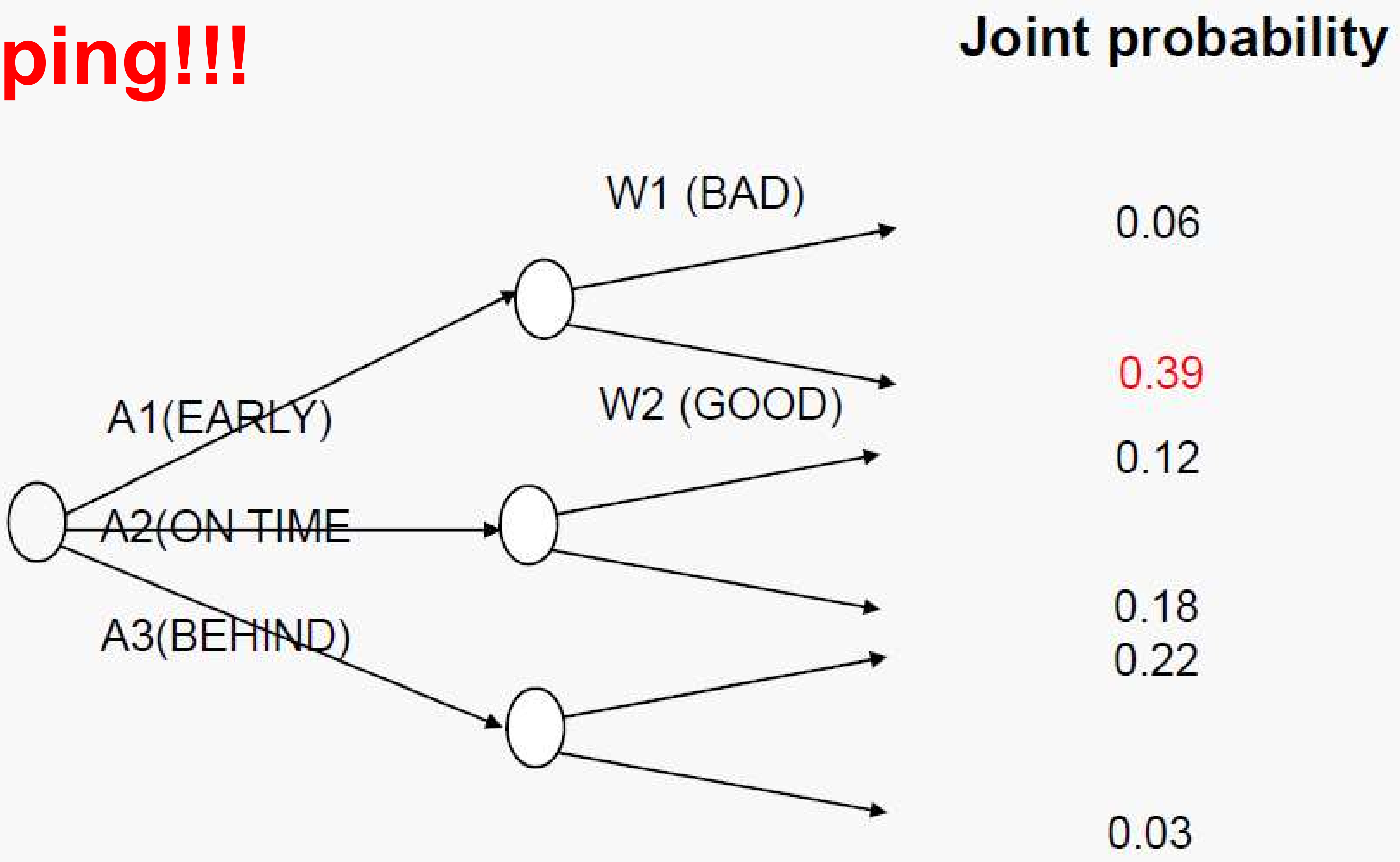
Exercise #7: Joint, Marginal, & Conditional Prob.



- P(W1) = 0.06 + 0.12 + 0.22 = 0.40
- P(A1/W1) = 0.06/0.40 = **0.15**

Exercise #7: Joint, Marginal, & Conditional Prob.

Flipping!!!



- P(A1) = 0.06 + 0.39 = 0.45
- P(W2/A1) = 0.39/0.45 = **0.87…by the same way!**

Exercise #8: $P(A|B)$ vs. $P(B|A)$

Disease Symptoms (from Lindley 2006)

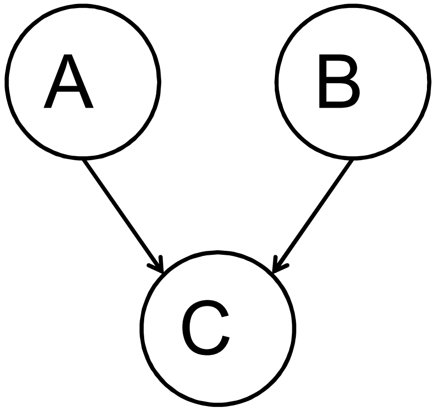
- Doctors studying a disease "D" noticed that 90% of patients with the disease exhibited a symptom "S".
- Later, another doctor sees a patient and notices that she exhibits symptom "S".
- As a result, the doctor concludes that there is a 90% chance that the new patient has the disease "D".

While $P(S|D) = 0.9$, $P(D|S)$ might be very different!!

Source: Lindley, Dennis V. (2006), *Understanding Uncertainty*. Wiley, Hoboken, NJ, US.

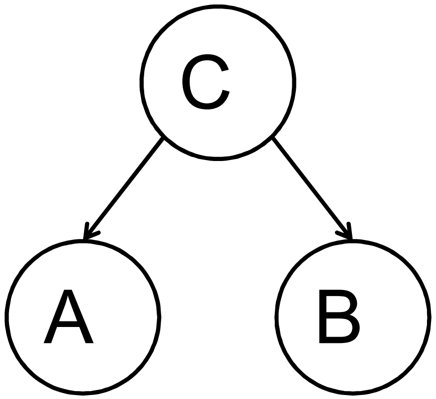
More Probability Relations…

- **Independent Probability**
  - P(B|A) = P(B)
  - P(A and B) = P(A) x P(B|A) = P(A) x P(B)



A, B are independent

- **Conditional Independence**
  - P(A|B,C) = P(A|C)
  - P(A,B | C) = P(A|C) P(B|C)



A, B are conditionally independent given C

Independence

- **Definition: Independent Events**
  - Two events A and B are independent if:

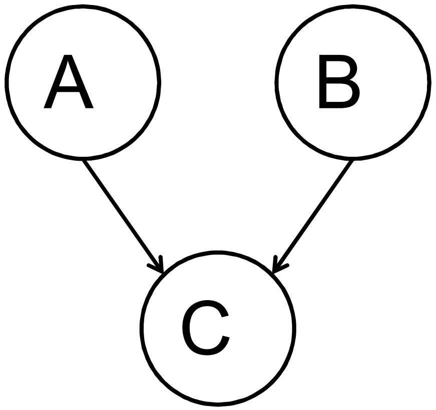
$$P(A \cap B) = P(A) \times P(B)$$

  - Intuition: two events are independent if knowing whether one event occurred does not change the probability of the other
  - Note that the following are equivalent:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$



A, B are independent

Exercise #9: Independence

- A coin is flipped three times. Each of the eight outcomes is equally likely.
  - A: heads occurs on each of the first two flips
  - B: tails occurs on the third flip
  - C: exactly two tails occur in the three flips
- Show that A and B are independent, B and C dependent.

$A = \{HHH, HHT\}$ $\quad P(A) = \frac{2}{8} = \frac{1}{4}$

$B = \{HHT, HTT, THT, TTT\}$ $\quad P(B) = \frac{4}{8} = \frac{1}{2}$

$C = \{HTT, THT, TTH\}$ $\quad P(C) = \frac{3}{8}$

$A \cap B = \{HHT\}$ $\quad P(A \cap B) = \frac{1}{8}$

$B \cap C = \{HTT, THT\}$ $\quad P(B \cap C) = \frac{2}{8} = \frac{1}{4}$

$\boldsymbol{P(A)P(B) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = P(A \cap B)}$**, hence A and B are independent.**

$\boldsymbol{P(B)P(C) = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16} \neq P(B \cap C)}$**, hence B and C are dependent.**

Exercise #10: Independence

- A simple example of two attributes that are independent: the suit and value of cards in a standard deck: there are 4 suits {♦, ♠, ♣, ♥} and 13 values of each suit {2, · · · , 10, J, Q, K, A}, for a total of 52 cards.
- Consider a randomly dealt card:
  - Marginal probability it's a heart:

$$P(suit = \heartsuit) = \frac{13}{52} = \frac{1}{4}$$

  - Conditional probability it's a heart given that it's a queen:

$$P(suit = \heartsuit | value = Q) = \frac{1}{4}$$

In general, $P(suit|value) = P(suit)$, hence suit and value are independent

Exercise #11: Independence

- We can verify independence by cross-multiplying marginal probabilities too. For every suit s ∈ {♦, ♠, ♣, ♥} and value v ∈ {2, · · · , 10, J, Q, K, A}:
  - $P(suit = s, value = v) = \frac{1}{52}$ (in a well-shuffled deck)
  - $P(suit = s) = \frac{13}{52} = \frac{1}{4}$
  - $P(value = v) = \frac{4}{52} = \frac{1}{13}$
  - $P(suit = s) \times P(value = v) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$
- Independence comes up when we construct mathematical descriptions of our beliefs about more than one attribute: to describe what we believe about combinations of attributes, we often assume independence and simply multiply the separate beliefs about individual attributes to specify the joint beliefs.

Conditional Independence

- **Definition: Conditionally Independent Events**
  - Two events A and B are conditionally independent given event C iff:

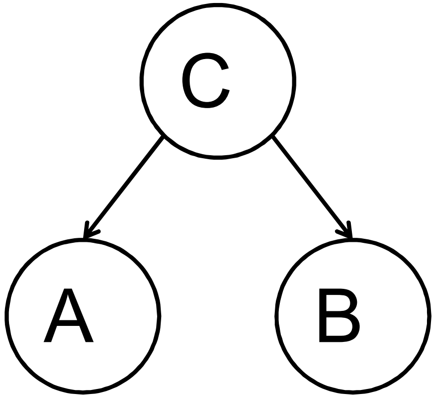
$$P(A, B|C) = P(A|C) \times P(B|C)$$

  - Intuition: Once we know whether C occurred, knowing about A or B doesn't change the probability of the other.
  - Note that the following are equivalent:

$$P(A, B|C) = P(A|C) \times P(B|C)$$

$$P(A|B, C) = P(A|C)$$

$$P(B|A, C) = P(B|C)$$



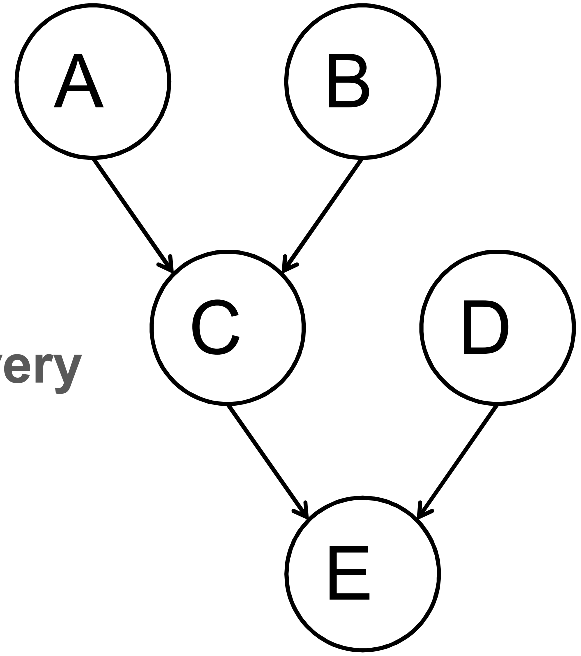
A, B are conditionally independent given C

Exercise #12: Conditional Independence

- In a noisy room, I whisper the same number $n \in \{1, \ldots, 10\}$ to two people A and B on two separate occasions. A and B imperfectly (and independently) draw a conclusion about what number I whispered. Let the numbers A and B think they heard be n_a and n_b, respectively.
- Are n_a and n_b independent (a.k.a. marginally independent)?
  - No.
  - E.g., we'd expect $P(n_a = 1 | n_b = 1) > P(n_a = 1)$.
- Are n_a and n_b conditionally independent given n?
  - Yes: if you know the number that I actually whispered, the two variables are no longer correlated.
  - E.g., $P(n_a = 1 | n_b = 1, n = 2) \ = P(n_a = 1 | n = 2)$

Assignment #3-1. Conditional Probability

- A: Weather
- B: Labor Skill
- C: Productivity
- D: Resource Delivery
- E: Schedule



EVENT	PROBABILITY
A1	0.7
A2	0.3
B1	0.4
B2	0.6
D1	0.4
D2	0.6
C1/A1, B1 (conditional probability of C1 given A1 & B1)	0.9
C1/A1, B2	0.8
C1/A2, B1	0.6
C1/A2, B2	0.5
C2/A1, B1	0.1
C2/A1, B2	0.2
C2/A2, B1	0.4
C2/A2, B2	0.5
E1/C1, D1	0.5
E1/C1, D2	0.4
E1/C2, D1	0.3
E1/C2, D2	0.2
E2/C1, D1	0.5
E2/C1, D2	0.6
E2/C2, D1	0.7
E2/C2, D2	0.8

아래 P(C1)와 P(E2, C2, D1)를 계산하시오

1. **P(C=C1): probability of productivity "high"**
2. **P(C=C2 and D=D1 and E=E2): Joint Prob.**

Assignment #3-2. Soldier's Problem

- **A soldier is taking an attack in which he is allowed three shots at a target airplane. The probability of his first shot hitting the plane is 0.1, that of his second shot is 0.3, and that of his third shot is 0.5. The probability of the plane crashing after one hit is 0.1; after two hits, the probability of crashing is 0.5, and the plane will crash for sure if hit three times. The attack is over when the soldier has fired all three shots or when the plane crashed.**
  1) What is the probability of the soldier shooting down the plane?
  2) What are the marginal probabilities of 0 hits, 1 hit, 2 hits, and 3 hits?
  3) What is the mean number of shots required to shoot down the plane?

PART II

MODELING UNCERTAINTY

- Theory of Probability II

Bayes' Theorem

- Based on the symmetry of the definition of conditional probability, we can predict the **posterior probability** based on the **prior information**

$$P(A \cap B) = P(A) \times P(B|A) = P(B \cap A) = P(B) \times P(A|B)$$

By the symmetry, $P(A) \times P(B|A) = P(B) \times P(A|B)$

$$\therefore P(A|B) = \frac{P(A) \times P(B|A)}{P(B)}$$

⇨ **Probability of "A" given the occurrence of "B"**

where, $P(B) = P(B \cap A) + P(B \cap \bar{A}) = P(A) \times P(B|A) + P(\bar{A}) \times P(B|\bar{A})$

Bayes' Theorem

- **P(A): Prior Probability (Base Rate)**

→ This is assigned before any empirical data is obtained.

- **P$(A|B)$: Posterior Probability**

→ This can be estimated based on **prior probability** and **given new data** on conditional events from historical data.

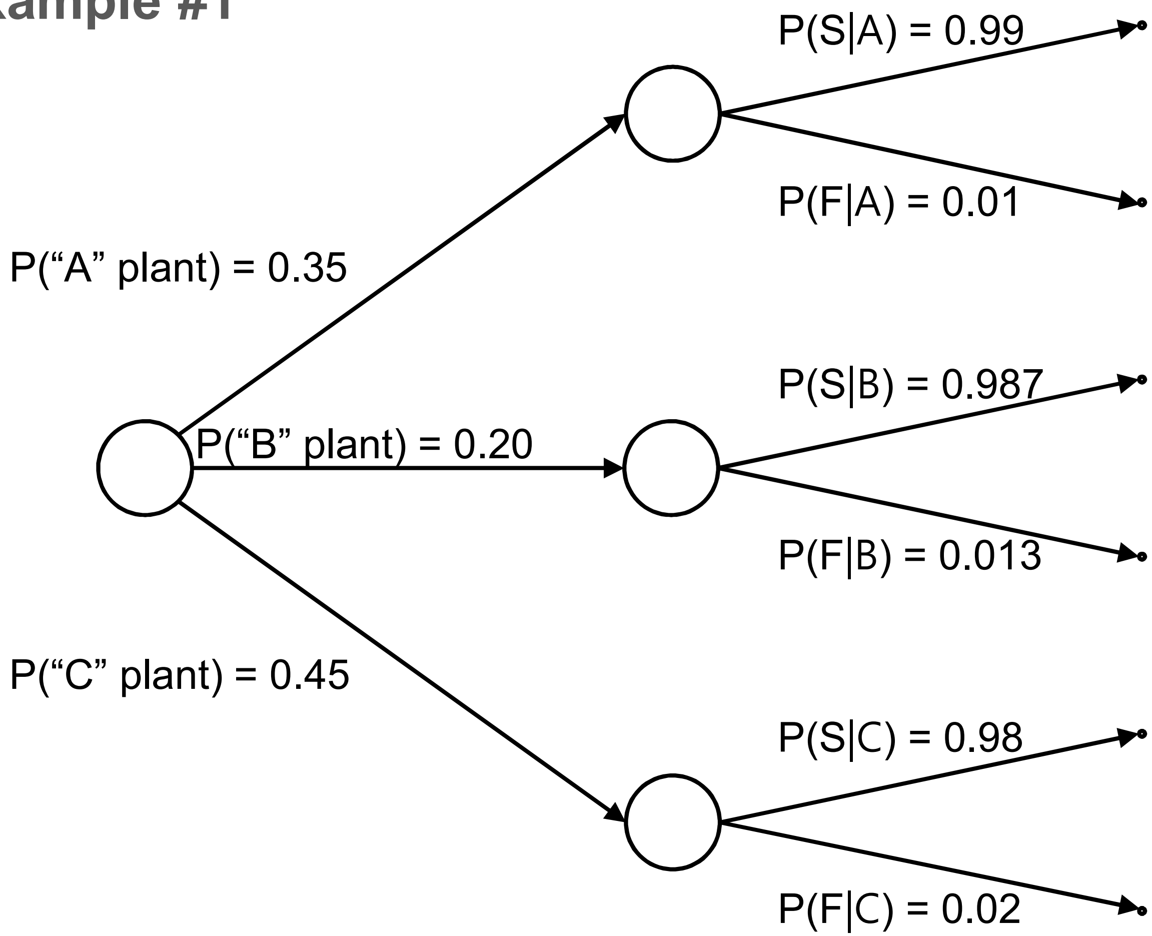
Bayes' Theorem

Example #1

- A construction company operates 3 batch plants (A, B, C) for producing concrete used in a highway project. Each plant is producing 35%, 20%, and 45% of the total quantity, respectively. According to the past historical data, the percentages of defective concretes from each plant are 1%, 1.3%, and 2%, respectively. If you test the samples randomly to confirm the quality of concrete produced,
  - What is the marginal probability that the sample is defective?
  - What are the conditional probabilities that any defective concrete was supposed to be produced from plant A, B, or C?

Bayes' Theorem

Example #1



Joint Probability

$P(A \cap S) = 0.35 \times 0.99 = 0.3465$

$P(A \cap F) = 0.35 \times 0.01 = 0.0035$

$P(B \cap S) = 0.20 \times 0.987 = 0.1974$

$P(B \cap F) = 0.20 \times 0.013 = 0.0026$

$P(C \cap S) = 0.45 \times 0.98 = 0.4410$

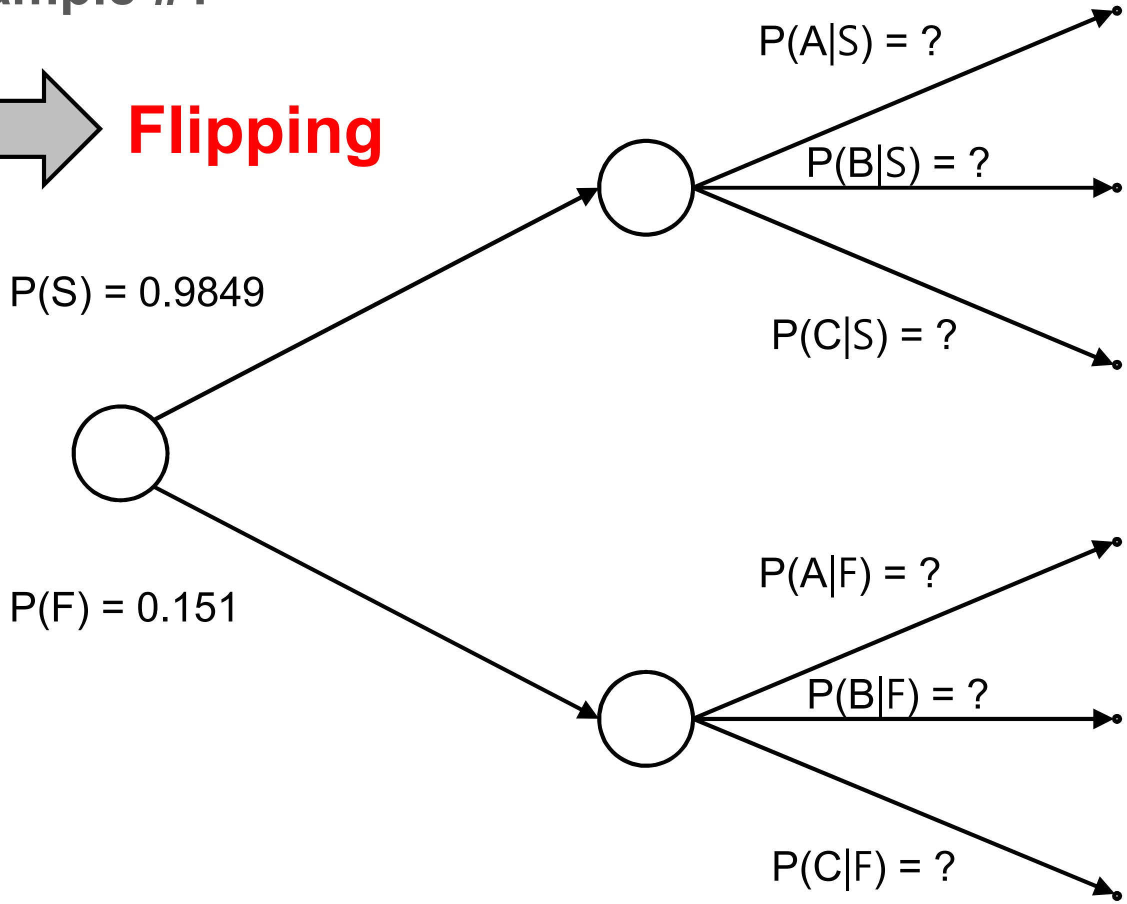
$P(C \cap F) = 0.45 \times 0.02 = 0.0090$

- P(Fail) = 0.0035 + 0.0026 + 0.0090 = 0.0151 = 1.51%
- P(Success) = 1 – 0.0151 = 0.9849 = 98.49%

Bayes' Theorem

Example #1

Flipping



Joint Probability

$P(A \cap S) = 0.3465$

$P(B \cap S) = 0.1974$

$P(C \cap S) = 0.4410$

$P(A \cap F) = 0.0035$

$P(B \cap F) = 0.0026$

$P(C \cap F) = 0.0090$

- P(A | F) = 0.0035 / 0.0151 = **0.23**
- P(B | F) = 0.0026 / 0.0151 = **0.17**
- P(C | F) = 0.0090 / 0.0151 = **0.60** → **High Probability**

Bayes' Theorem

Example #1

- P(A | F) = 0.0035 / 0.0151 = **0.23**

$$P(A|F) = \frac{P(F|A) \times P(A)}{P(F)} = \frac{0.01 \times 0.35}{0.0151} = \frac{0.0035}{0.0151} = 0.23$$

Prior Probability (ASSIGNED BASE RATE, "A" plant)

Given conditional probability from historical data
(1% defective rate)

Posterior Probability (Estimated/Predicted rate)

Bayes’ Theorem

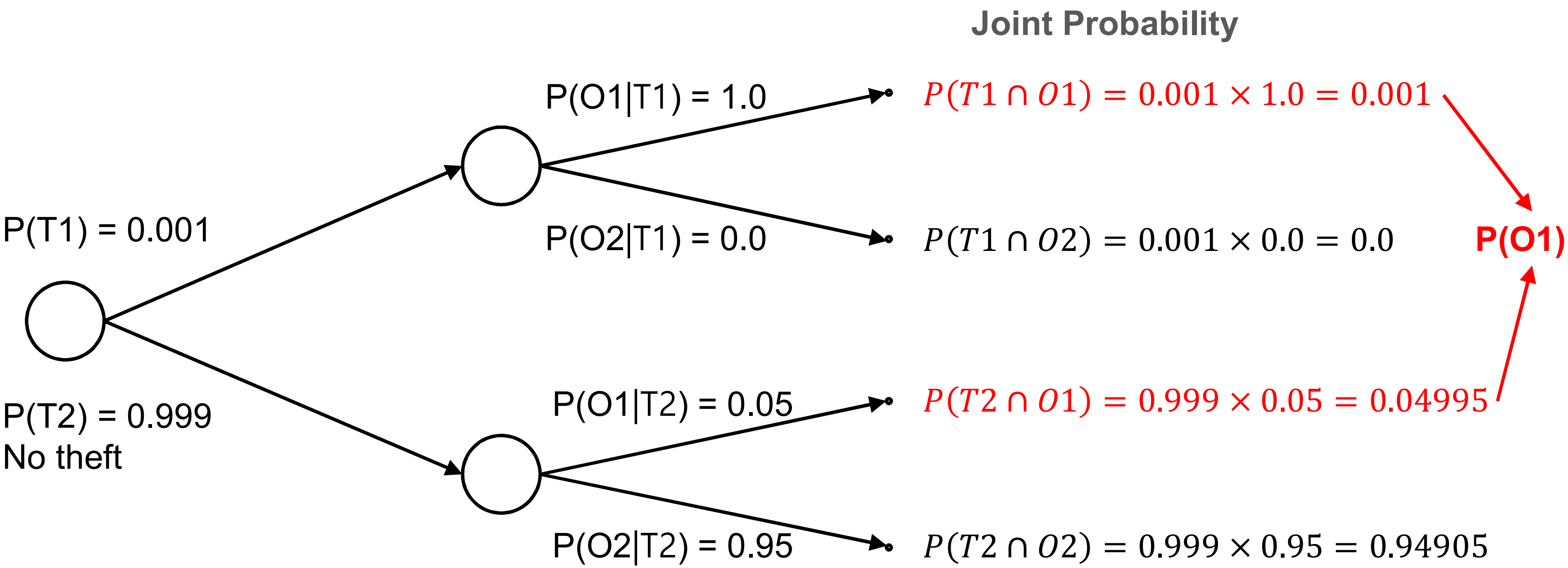
Example #2

- Mr. Smith was living in an apartment in a certain city.
- One day, the door was open when coming back to his apartment.
- He thought the probability of theft was highly probable.
- Based on the past history in this city, if the probability of “door open” given “no-theft” and “door open” given “theft” are supposed to be 5% and 100% respectively.
- Also, the base rate of theft probability in this city is 0.1%,
  - What is the probability of “theft” when the “door was open” in this case?

Bayes’ Theorem

Example #2

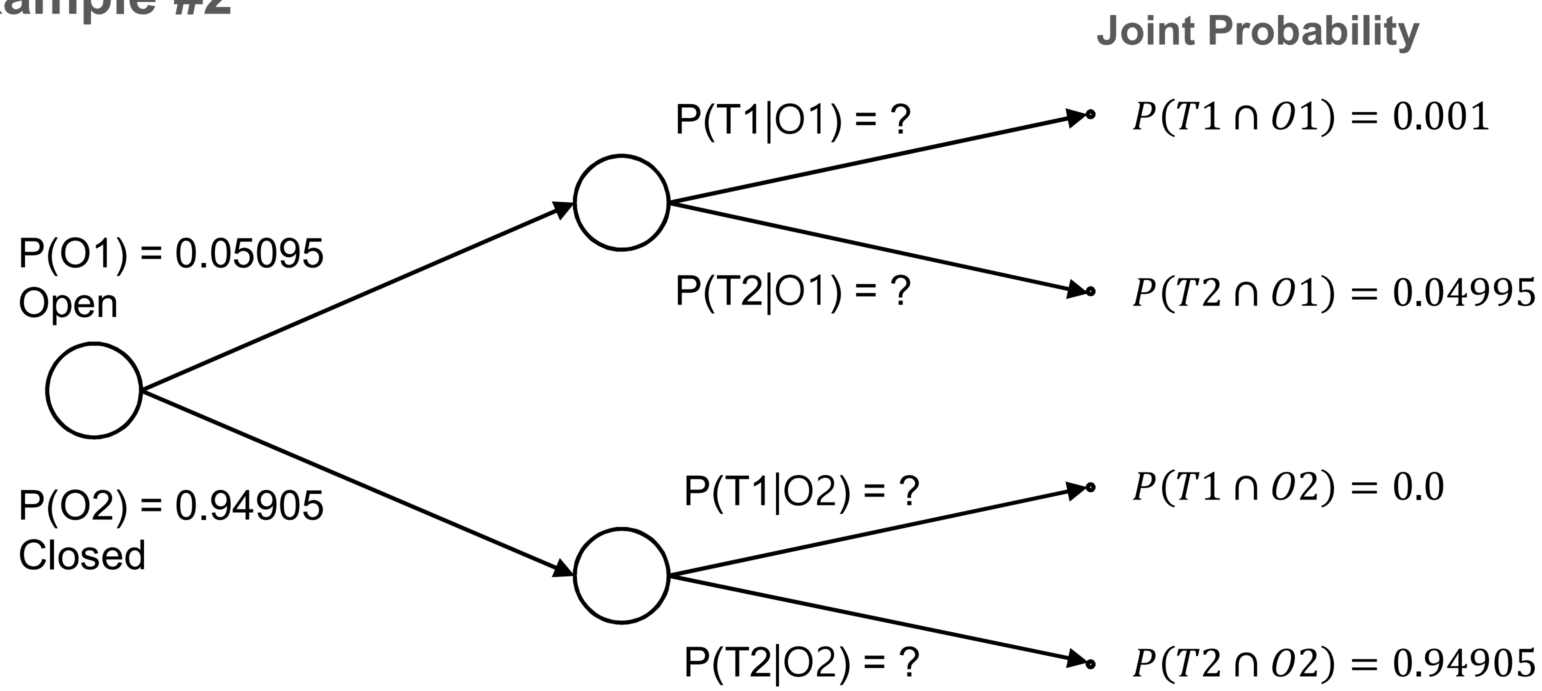
P(T1) = 0.001 (Base rate: theft)



By Flipping

Bayes' Theorem

Example #2



- $P(T1|O1) = [P(O1|T1) \times P(T1)]/P(O1) = 1 \times 0.001/0.05095 =$ **Only 1.96%**
- **Why? → This city is very safe!!**

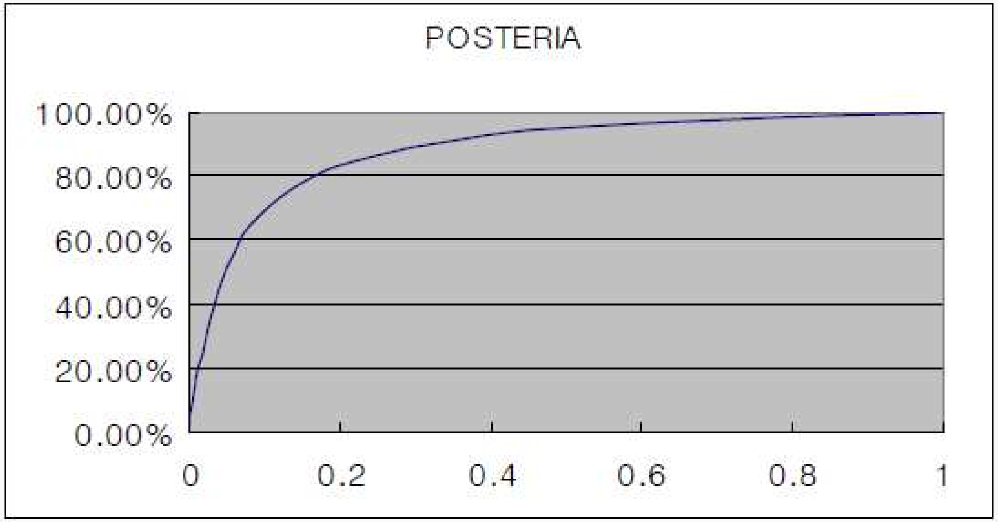
Posterior probability is highly sensitive to the prior probability.

Bayes' Theorem

Example #2

- **Sensitivity analysis on the prior probability**

PRIOR	POSTERIA
0.001	1.96%
0.01	16.81%
0.05	51.28%
0.1	68.97%
0.2	83.33%
0.4	93.02%
0.6	96.77%
0.8	98.77%
1	100.00%



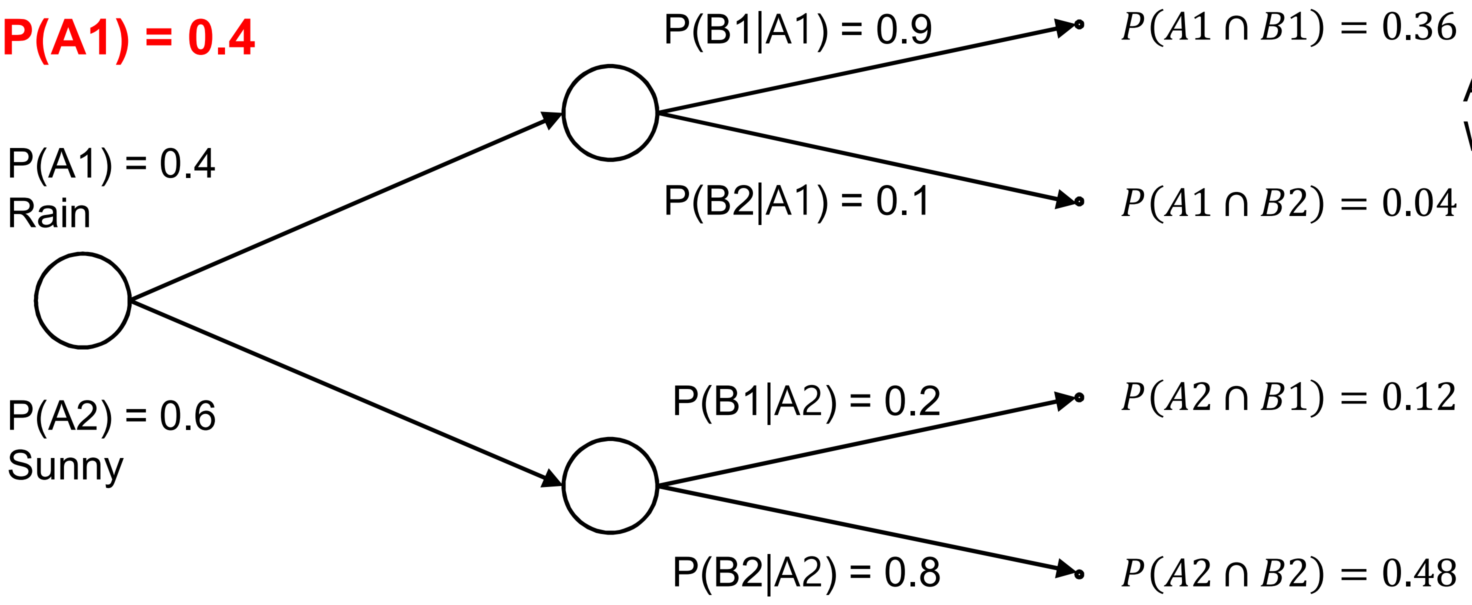
If this city is unsafe, the posterior probability will be highly anticipated.

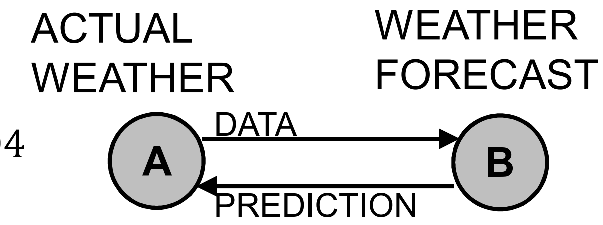
Bayes' Theorem

Example #3

- Past historical data gives (Information from previous data)
  - Under Actual rain, weather forecast was correct (rain) = 0.9
  - Under Actual sunny, weather forecast was incorrect (rain) = 0.2
- Probability of rainy day = 0.4 (base rate: prior probability)

P(A1) = 0.4





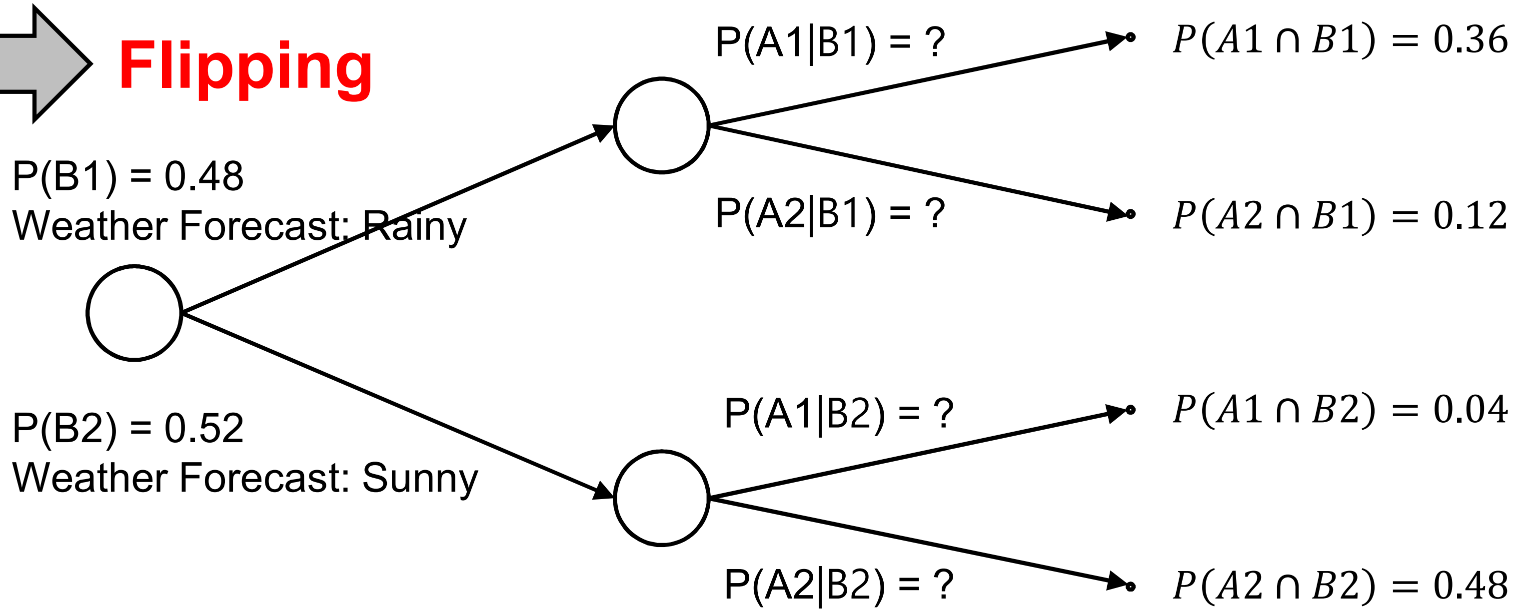
$$P(A1|B1) = \frac{P(B1|A1) \times P(A1)}{P(B1)}$$

Probability of rain (A1) given that the weather forecast is rainy (B1) tomorrow?

Bayes' Theorem

Example #3

Flipping



- $P(A1|B1) = [P(B1|A1) \times P(A1)]/P(B1) = 0.36/0.48 =$ **0.75 → Bayes' Theorem**
- $P(A2|B1) = 1 - 0.75 = 0.25$

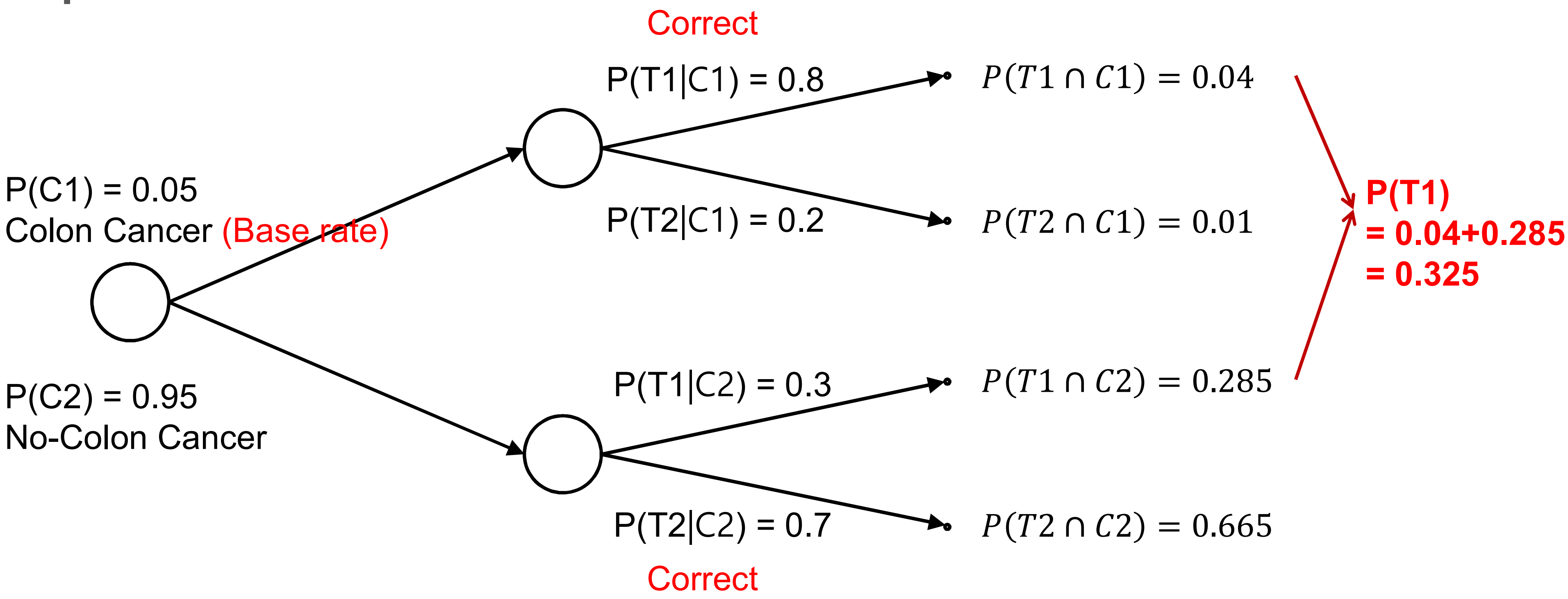
Bayes' Theorem

Example #4

- Frequently, people use tests to infer knowledge about something. The following example is the use of a blood test to see if a person has a colon cancer(대장암) or not. The test results reflect current knowledge of the colon cancer characteristics, and test accuracy may be a matter of concern. Suppose that a number of people (e.g. 1,000 persons) have taken a blood test with the following results;
  - Colon cancer (5%): positive blood test result (80%); negative blood test result (20%)
  - No colon cancer (95%): positive blood test result (30%); negative blood test result (70%)
- If a person has taken this test and the result turns out to be positive, what is the probability that he or she has a colon cancer? **Is it a reliable test?**

Bayes' Theorem

Example #4



By Bayes' Theorem

$$P(C1|T1) = \frac{P(T1|C1) \times P(C1)}{P(T1)} = \frac{0.8 \times 0.05}{0.325} = 0.123$$ **Only 12.3%!!**

Bayes' Theorem

Example #4

- If you can update the likelihood of test results from the improved test quality:
  - prior probability of colon cancer: 15%
  - positive blood test result given colon cancer: 85%
  - negative blood test result given no-colon cancer: 75%
- How do you **update the posterior probability** ?

Initial posterior Probability = 12.3% ➡ **Updated Posterior Probability = 24.4%**

Still, the reliability of this blood test seems to be very low!!

Bayes' Theorem

Example #5: Equipment Selection

- A construction company is considering the purchase of a new compactor. There is also the possibility of rebuilding the existing one or managing the old compactor without rebuilding.
- The compactors perform differently with two types of common material. When the material is good, both rebuilt and new compactor work much faster, but they will not perform adequately when the material is of poor quality.
- The payoffs are (thousands US$);

Material Type	New compactor	Rebuild	Old
Good	30	12	8
Bad	-15	3	6

- It is also possible to make a preliminary test at a cost of $600, but unfortunately, it is not infallible(결코 틀리지 않는).

Bayes' Theorem

Example #5: Equipment Selection

- The true soil quality can be determined only during the construction process. In the past, the test has been characterized by the following conditional probabilities;

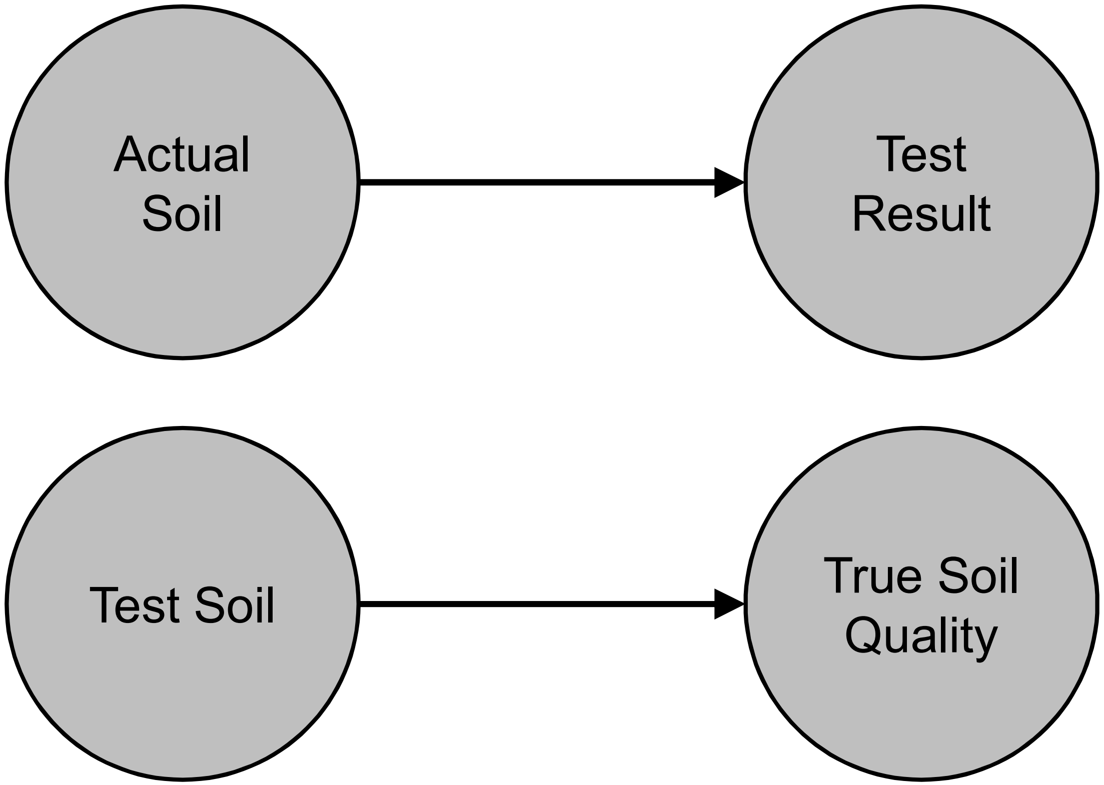
Actual Quality during construction	Test results before construction	
	Good soil	Bad soil
Good	0.8(correct)	0.2
Bad	0.3	0.7(correct)

- Past record shows that there is a 35% chance of getting good soil – base rate.

  - **Find the appropriate decision based on Expected value criteria**
  - **Should the company perform a test?**

Bayes' Theorem

Example #5: Equipment Selection



From historical data

(quality of test)

P(T|A)

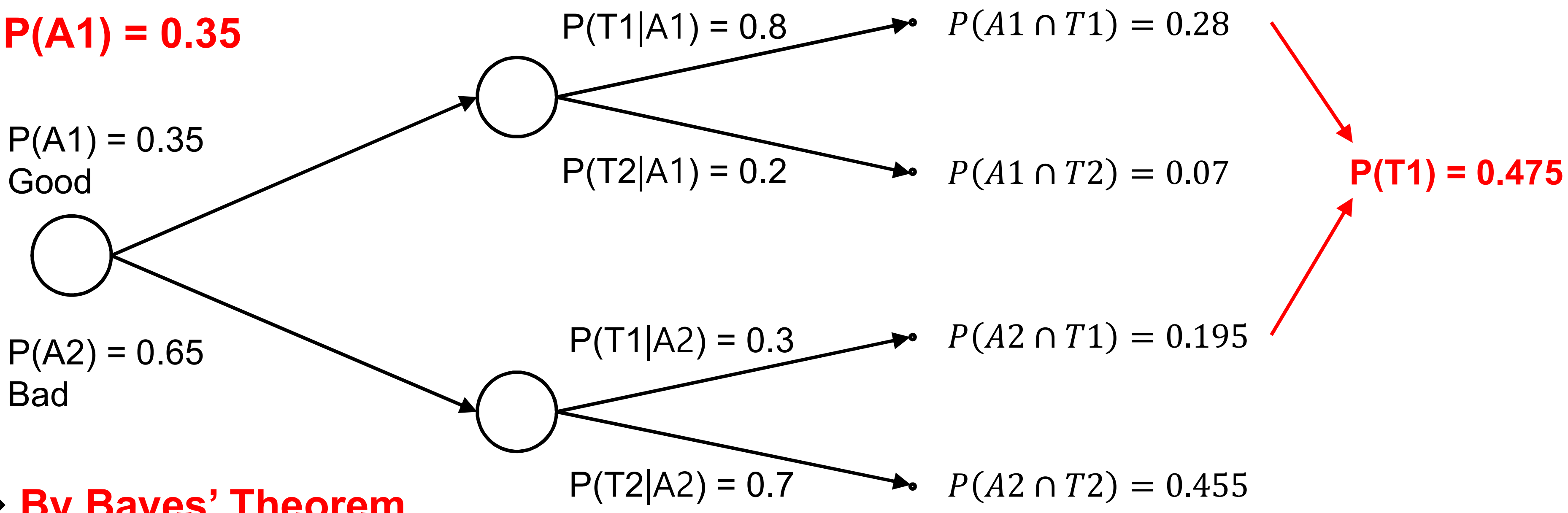
Prediction (Bayes' Theorem)

P(A|T)=P(T|A) x P(A) / P(T)

Bayes' Theorem

Example #5: Equipment Selection

P(A1) = 0.35



By Bayes' Theorem

$P(A_1/T_1) = P(T_1/A_1) \times P(A_1) / P(T_1) = 0.8 \times 0.35 / (0.35 \times 0.8 + 0.65 \times 0.3) =$ **0.589**

→ posterior probability that true soil will be good given the test result is good soil.

$P(A_2/T_1) = 1 - 0.59 =$ **0.411**

$P(A_1/T_2) = P(T_2/A_1) \times P(A_1) / P(T_2) = 0.2 \times 0.35 / (0.35 \times 0.2 + 0.65 \times 0.7) =$ **0.133**

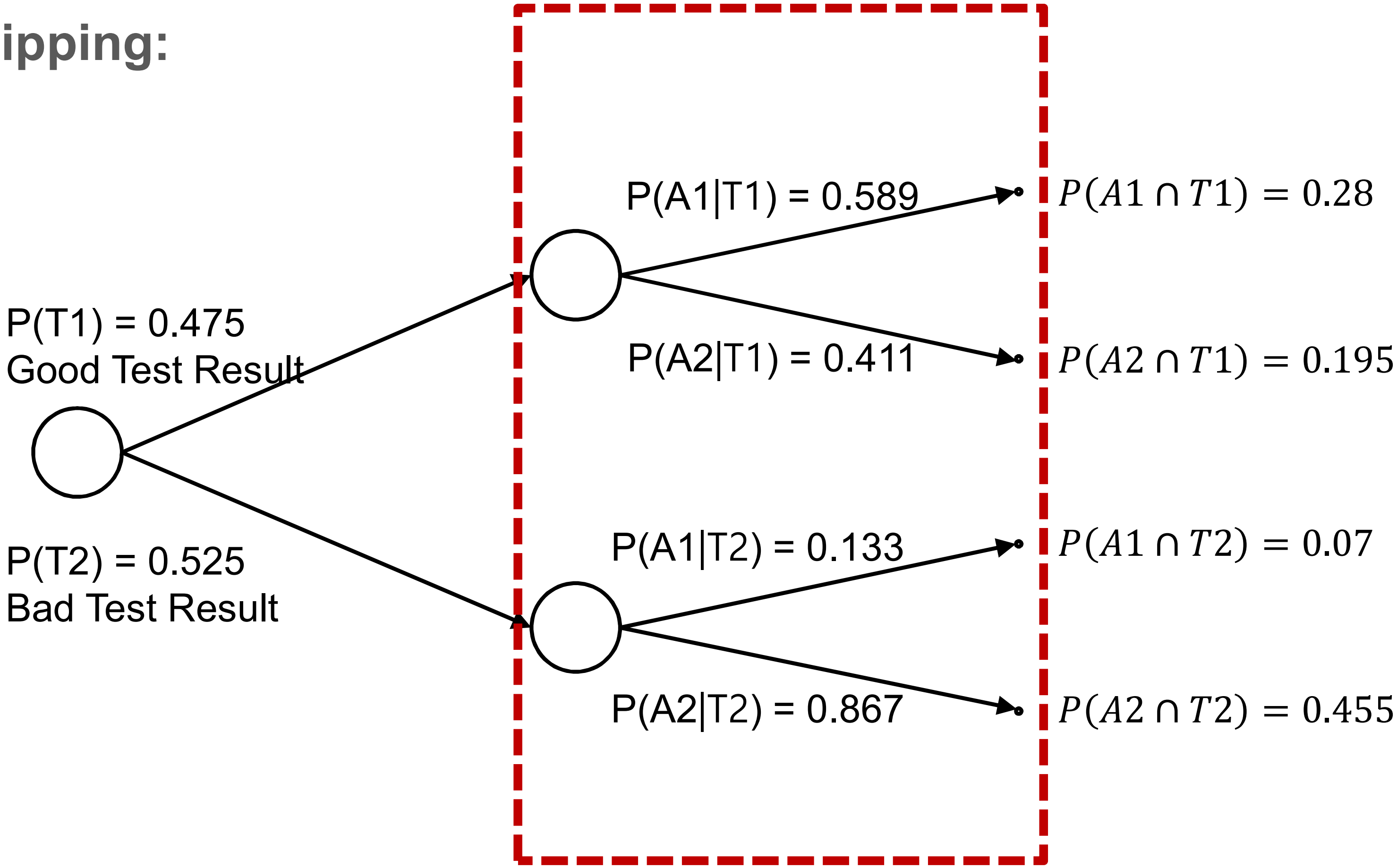
$P(A_2/T_2) = 1 - 0.13 =$ **0.867**

Bayes' Theorem

Example #5: Equipment Selection

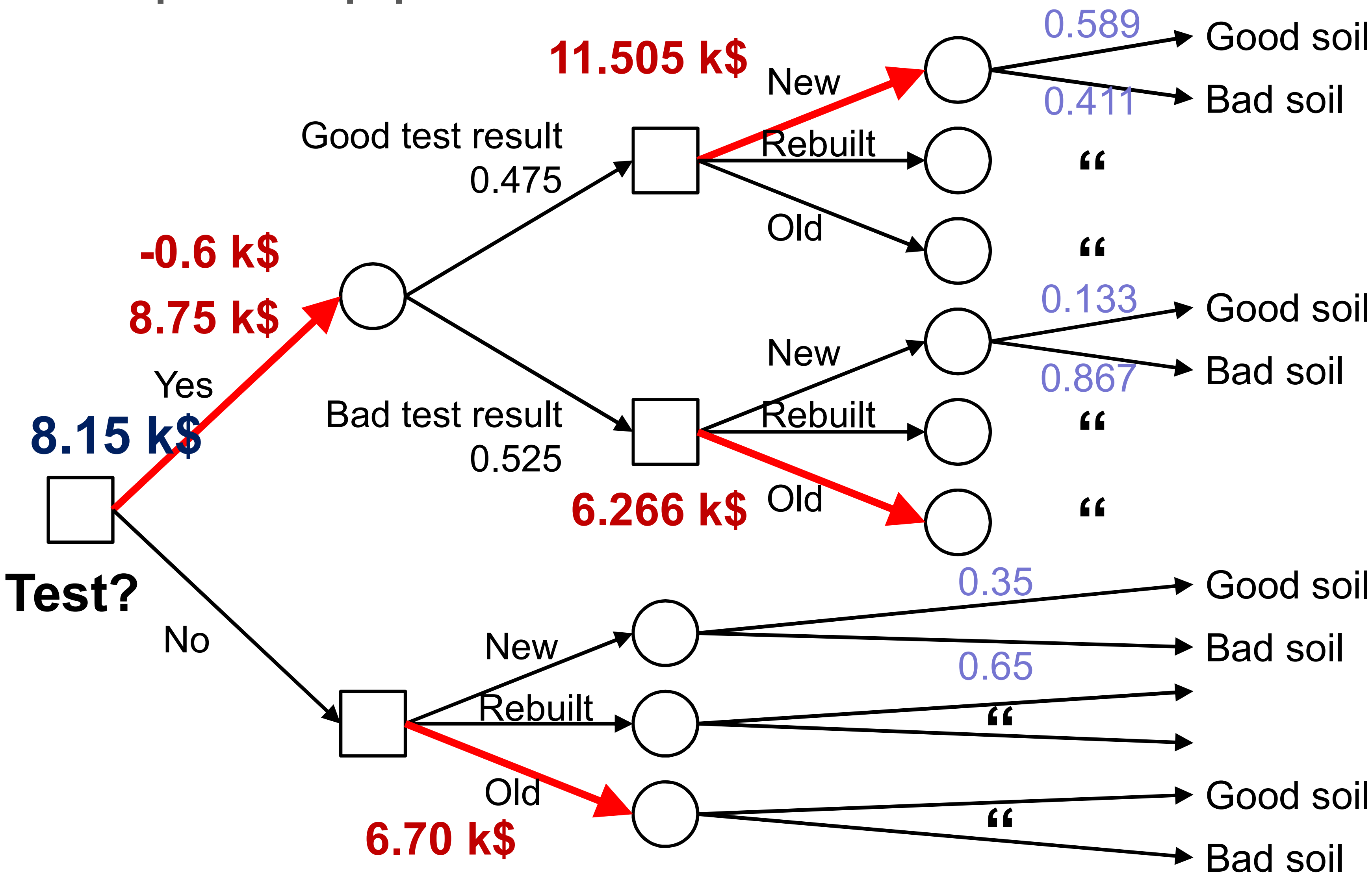
- **By Flipping:**

By Bayes' Theorem



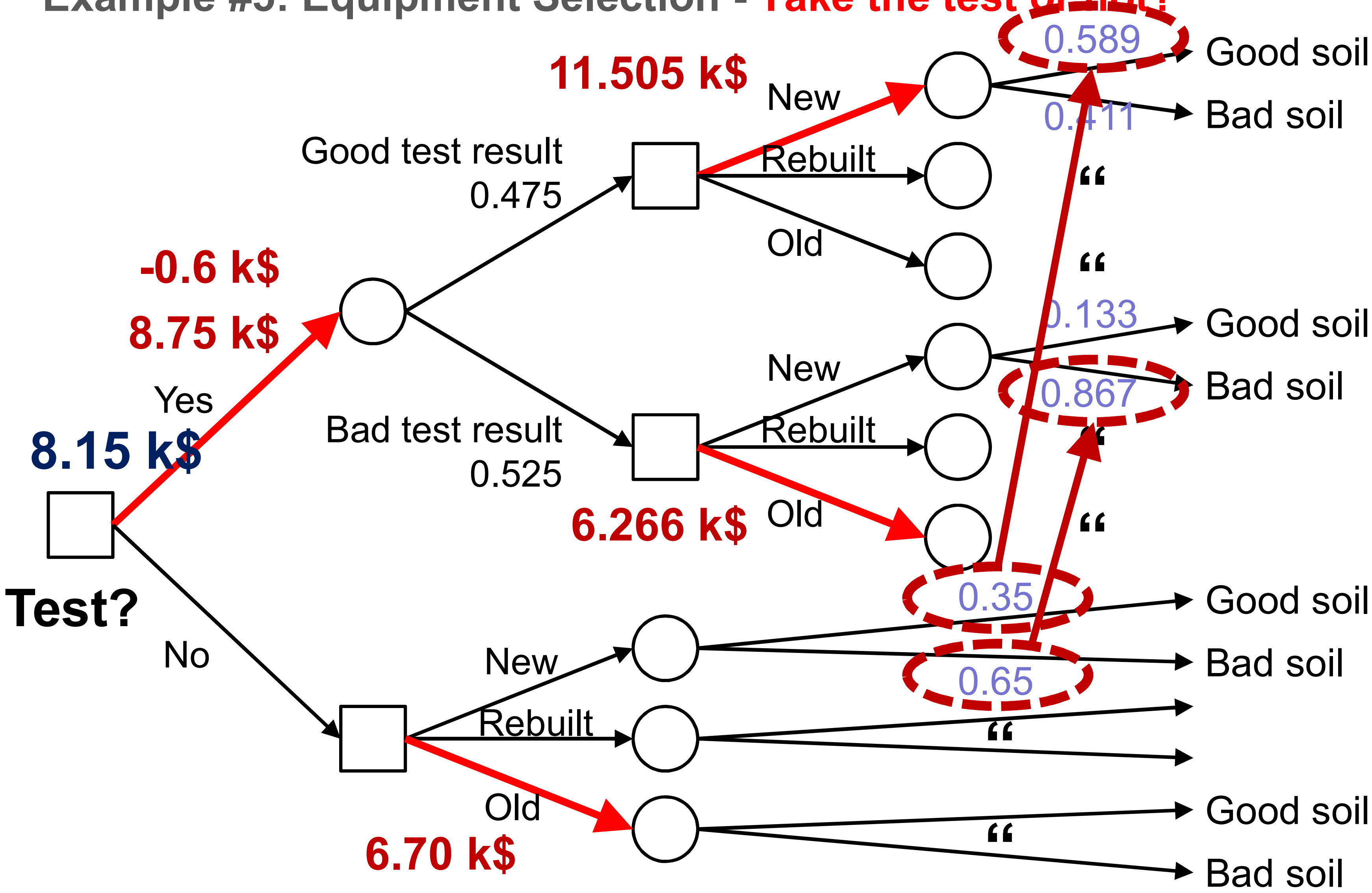
Bayes' Theorem

Example #5: Equipment Selection - Take the test or not?



Why? – more updated information

Example #5: Equipment Selection - Take the test or not?



Bayes' Theorem - Bayesian Updating

- Bayes' Theorem is useful in **updating** prior probability (i.e., P(A)) given new data on conditional events.

$$P(A|B) = \frac{P(A) \times P(B|A)}{P(B)}$$

where, $P(B) = P(B \cap A) + P(B \cap \bar{A}) = P(A) \times P(B|A) + P(\bar{A}) \times P(B|\bar{A})$

- So, you can update **P(A)** when given that $B\ or\ \overline{B}$ has occurred.

Bayes' Theorem - Bayesian Updating

- In soil test & equipment selection problem,
  - Initial prior probability of good soil P(A1) = 0.35
  - Initial prior probability of bad soil P(A2) = 0.65
- If we have data to predict the quality of soil based on the previous soil test, we can update the P(A1) or P(A2) given the results of soil test,
- That is;
  - P(A1|good test) = 0.589: accuracy increased from 0.35 to 0.589
  - P(A2|bad test) = 0.867: accuracy increased from 0.65 to 0.867

Bayes' Theorem - Bayesian Updating

- If we will collect more data to update the prior probabilities and their conditional states, i.e.;
  - Initial prior probability of good soil P(A1) = 0.40, bad soil P(A2) = 0.60
  - P(good test/A1) = 0.80 → 0.85
  - P(bad test/A2) = 0.70 → 0.75
- So, we can predict the quality of soil more accurately based on the updated data (soil test results) through Bayes' theorem.
- That is;
  - $P(A1|\text{good test}) = 0.694$: accuracy increased from 0.589 to 0.694
  - $P(A2|\text{bad test}) = 0.882$: accuracy increased from 0.867 to 0.882
- We can say it as "**Bayesian updating**" to update the uncertainties.
- This sequential updating process will continue indefinitely!

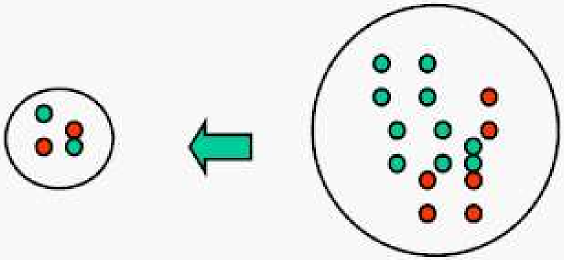
PART III

MODELING UNCERTAINTY

- Measurement of uncertain variables

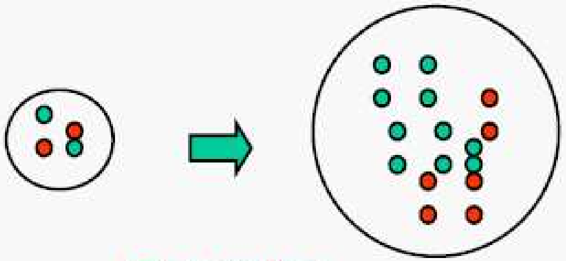
1. Probability vs. Statistics

- Probability
  - Deductive reasoning: use knowledge of a *population* to understand a *sample*



What will it be? Probability of failure

- Statistics
  - Inductive reasoning via inferential statistics: understand the *population* by studying a *sample*



Generalize

Types of Statistics

- ***Descriptive Statistics***

  - for organizing & summarizing data

- ***Inferential Statistics / Inductive Reasoning***

  - for drawing **conclusions** about a population based on knowledge of a sample (i.e., regression model)

Important Terms

Measure of central tendency:

- mean, median, mode
  - mean is sensitive to outliers
  - mean is analogous(유사한) to expected value [E(X)]

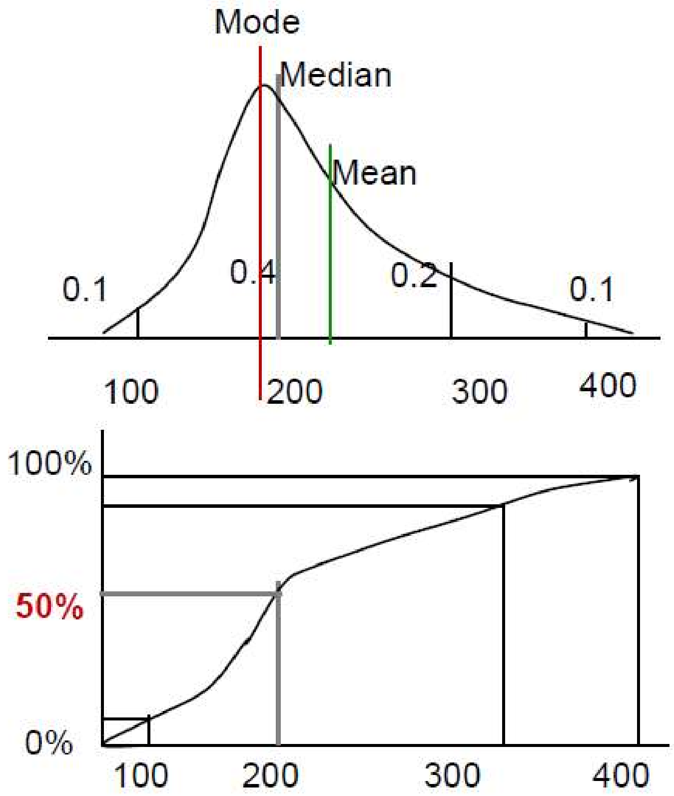
Measure of variability & dispersion(확산):

- range, deviation from mean(편차), standard deviation, variance(분산), coefficient of variance (COV), box plots w/ quartiles

Population vs. Sample

Mean, Median, Mode

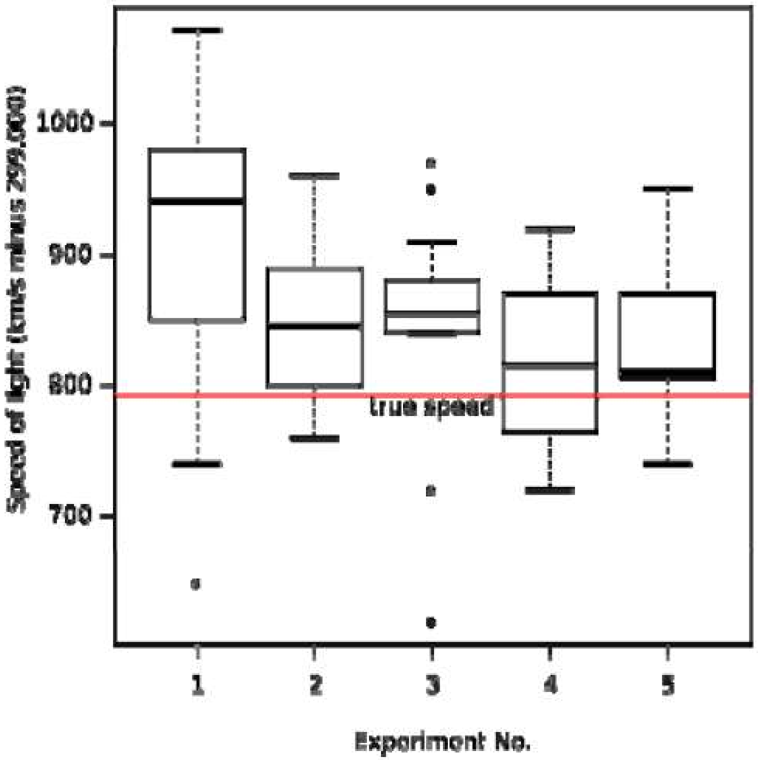
- Mean: Average value "E(x)"
- Median: 50% of cumulative curve (Not sensitive to outlier)
- Mode: Most frequently value (PEAK POINT)



In normal distribution, three values are all identical

Degree of Dispersion

EX. Box plot



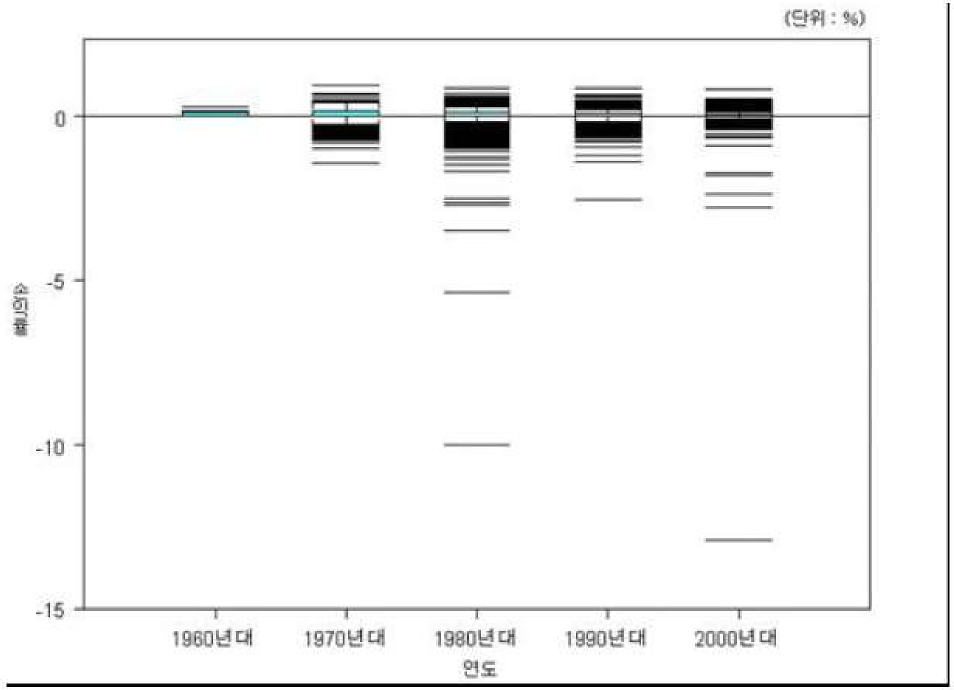


Fig. Profit rates with respect to time periods

Population and Sample

https://m.blog.naver.com/sw4r/221021838997

- Suppose that the number of samples from population are "N";

$$Population\ \ Mean = \frac{1}{N}\sum x_i = \overline{X}$$

$$Population\ \ Variance = \frac{1}{N-1}\sum (x_i - \overline{X})^2$$

Population is more spread than samples

- The net weight of the concrete samples randomly selected from the population: 85.4, 85.3, 84.9, and 85.0

$$\text{Population Mean} = \frac{1}{N}\Sigma x_i = \frac{85.4 + 85.3 + 84.9 + 85}{4} = 85.3$$

$$\text{Population Variance} = \frac{1}{N-1}\Sigma (x_i - \overline{X})^2 = \frac{(85.4-85.3)^2 + (85.3-85.3)^2 + (84.9-85.3)^2 + (85-85.3)^2}{4-1}$$

$$= \frac{0.26}{3} = 0.087$$

2. How to Measure Uncertain Input Data?

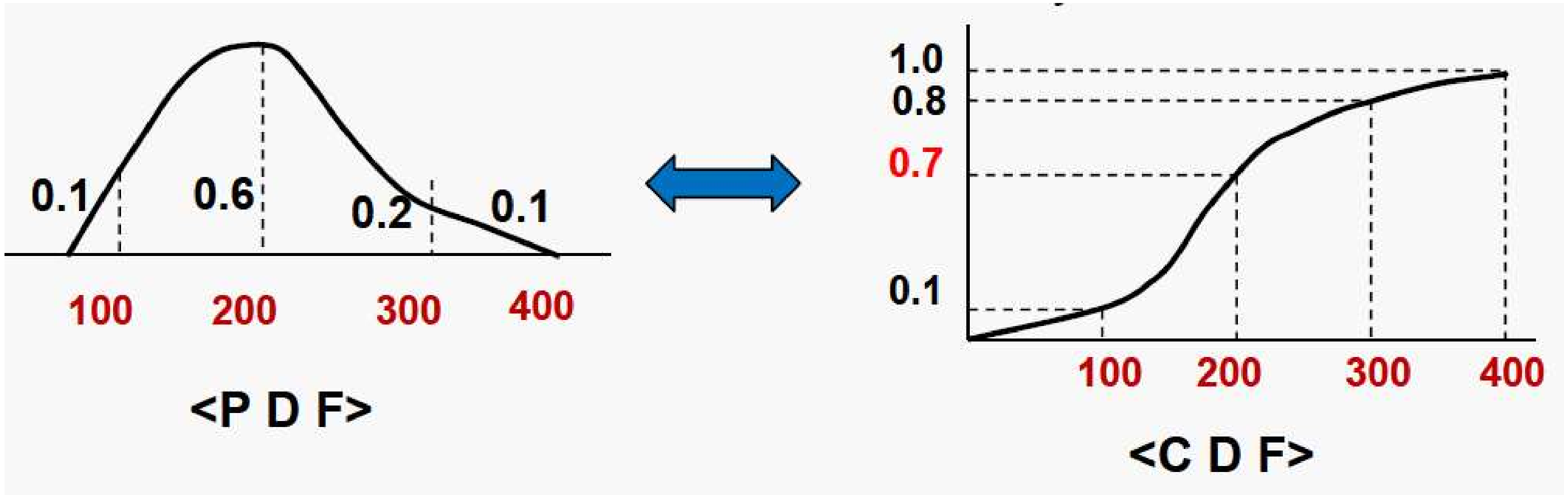
- Uncertain Inputs are essentially a mathematical description of the “frequency” and “severity” of the variability (e.g. histogram)
- Uncertainty can be often measured by;
  - $Mean = E(X)$ → measure of central tend.
  - $\text{Standard Deviation} = \sigma(X) = \sqrt{V(X)}$ → measure of dispersion
  - $\text{COV} = \frac{\sigma(X)}{E(X)}$ (coefficient of variance) → measure of spread
- There are two forms in displaying uncertainty
  - PDF (Probability Density Function)
  - CDF (Cumulative Distribution Function)

PDF and CDF

- **PDF(Probability Density Function)**
  - $y = f(x)$
- **CDF (Cumulative Distribution Function)**
  - $Y = \int f(x)dx$

    $Y(x_1) = \int_0^{x_1} f(x)dx$

    $Y(200) = 0.1 + 0.6 = 0.7$



Discrete and Continuous Distribution

- **Discrete:** Uncertainty can take only specific value

  (1,2,3,4, … → histogram)

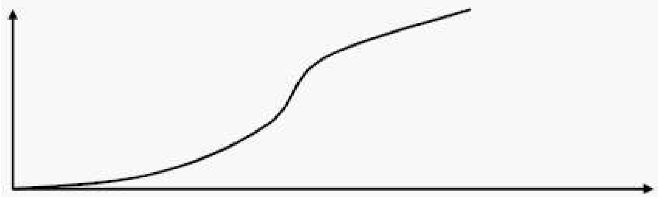
  - Ex) number of operations computer performs in any given second

$$E(X) = \sum_X x_i \times P(x_i)$$

$$V(X) = \sum_X (x_i - E(X))^2 \times P(x_i)$$

- **Continuous:** Uncertainty can take any value within the some range (15~20)

  - Ex) tomorrow temperature, measurements such as height



$$E(X) = \int x \times f(x)dx$$

$$V(X) = \int (x - E(X))^2 f(x)dx$$

Discrete and Continuous Distribution

Example

- Discrete casting die
  - E(X) = 1(1/6) + 2(1/6) + ……+ 6(1/6) = 3.5
  - V(X) = $(3.5-1)^2$ (1/6) + …….+ $(3.5-6)^2$ (1/6) = 2.92
  - σ(X) = $(2.92)^{0.5}$ = 1.71

- Continuous function
  - $f(x) = ae^{-ax}$
  - $E(X) = \int x \times f(x)dx = \int x \times ae^{-ax}dx = \frac{1}{a}$
  - $V(X) = \int (x - \frac{1}{a})^2 ae^{-ax}dx = \frac{1}{a^2}$

3. Concept of Moments

- We have considered only two values to estimate uncertainty (mean & variance)
- In general, we need concept of moments
  - First central moment: mean
  - Second: variance
  - Third: skewness (right or left?)
  - Fourth: flatness (uniform or central peak?)

n^{th} central moment

$$\mu_n = \int_x (x-\mu)^n f(x)dx - \text{continuous}$$

$$\mu_n = \sum_x (x_i - \mu)^n \times p(x_i) - \text{discrete}$$

- First central moment(mean value)

$$\mu_1 = \sum_x x_i \times p(x_i) - \text{discrete}$$

$$\mu_1 = \int_x xf(x)dx - \text{continuous}$$

- 2^{nd} moment

$$\mu_2 = \text{variance}$$

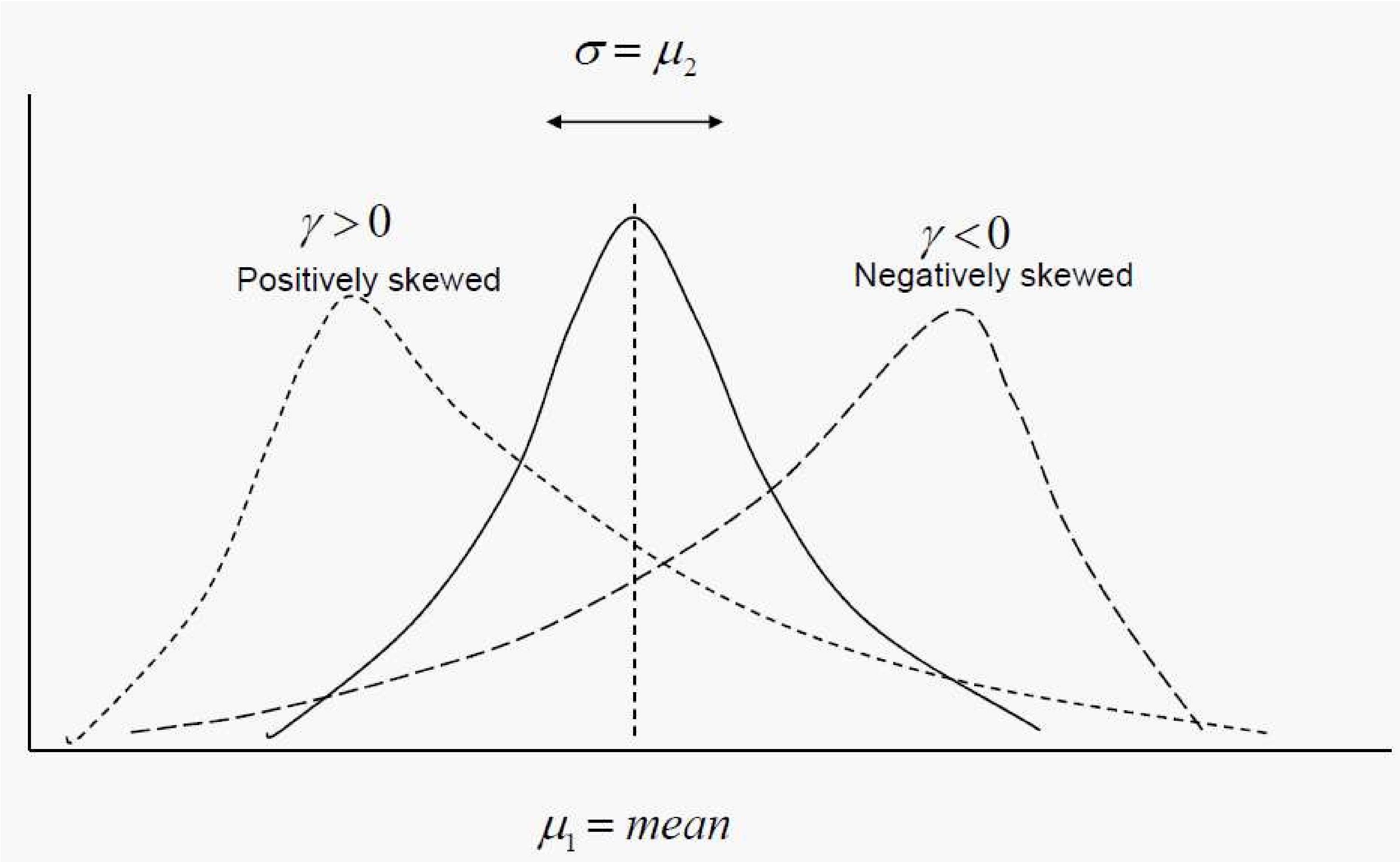
$$\sigma = \sqrt{\mu_2} \quad \text{(standard deviation)}$$

- 3^{rd} moment

$$\gamma = \frac{\mu_3}{\sigma} \quad \text{(skewness)}$$

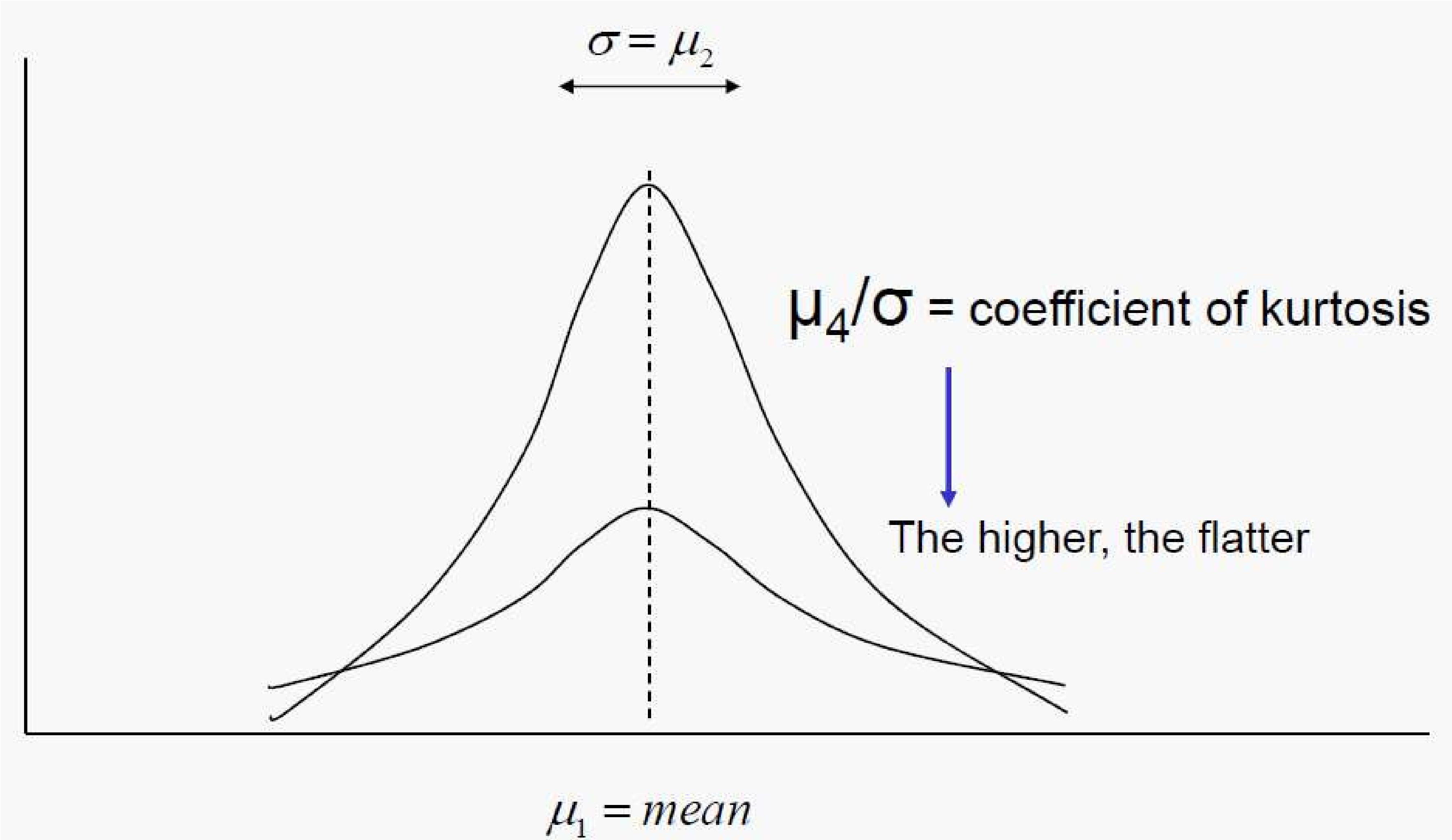
Charactering probability distribution

Skewness



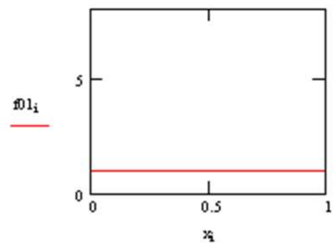
Charactering probability distribution

Flatness



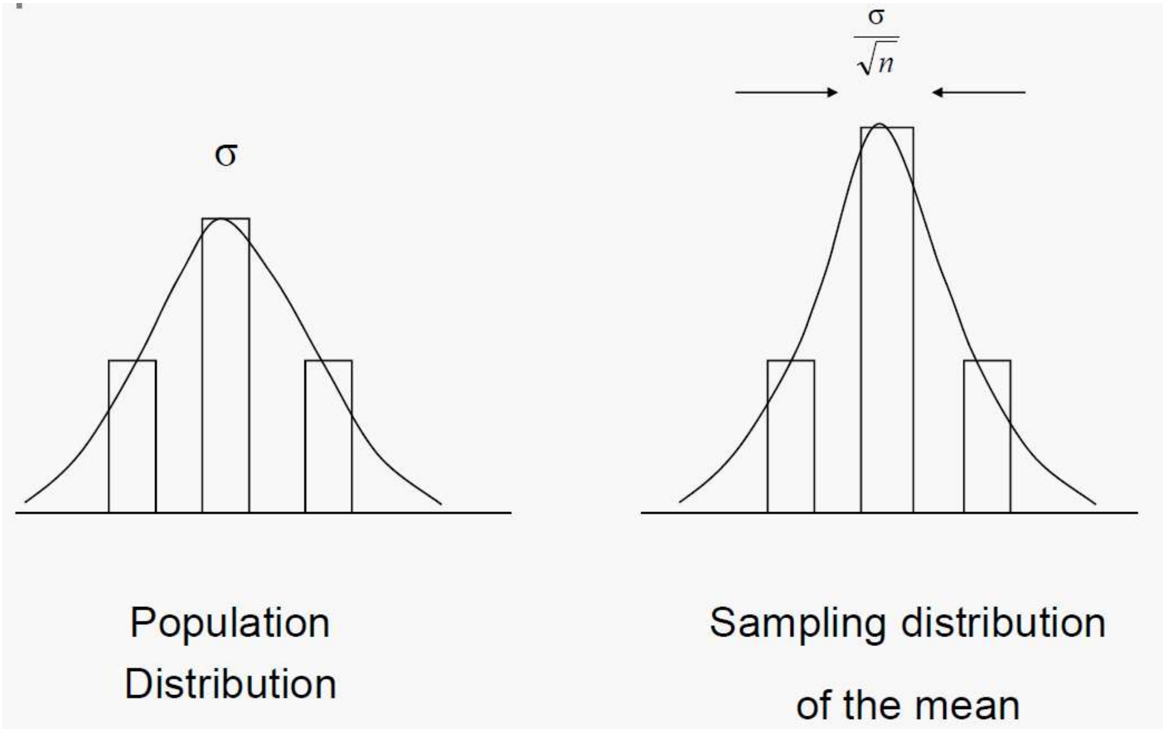
4. Central Limit Theorem (CLT) – predicting distribution

- For a population with a mean μ and a population standard deviation σ, the sampling distribution of the mean of all possible sample size generated from the population will be approximately normally distributed – with the mean of the sampling distribution equal to μ and the standard deviation equal to $\sigma/\sqrt{n}$ – assuming the sample size is sufficiently large
  - The sum of a large number of independent random variables will tend toward a normal density function irrespective of their original distributions
  - For large *n*, any reasonable distribution on the "x_i", distribution "*y*" will tend to become normal



NonNormal Distribution of X

4. Central Limit Theorem (CLT) – predicting distribution

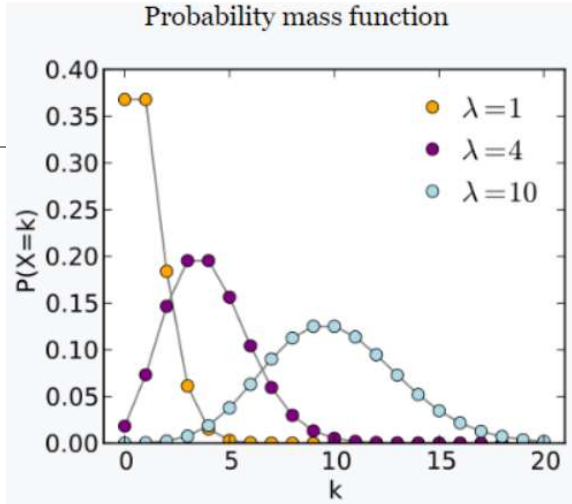


- Required conditions for CLT:
  - The RV(random variable)s are independent and identically distributed
  - The RVs are not identically distributed, but each RV has a small impact on the total
  - The RVs are not independent, but the correlations are small

5. Probability Distribution Models

- To provide reasonable representation of the observed data
- Find a standard distribution that provides close fit to the observed/subjective probability
- It will be easier for a decision maker to make calculation of probability and its value when we can model the shape of probability distribution
- Typical types of distributions
  - Normal, Lognormal, Binomial, Poisson, Uniform, Exponential, Triangular, Beta, …

Variables & Distributions

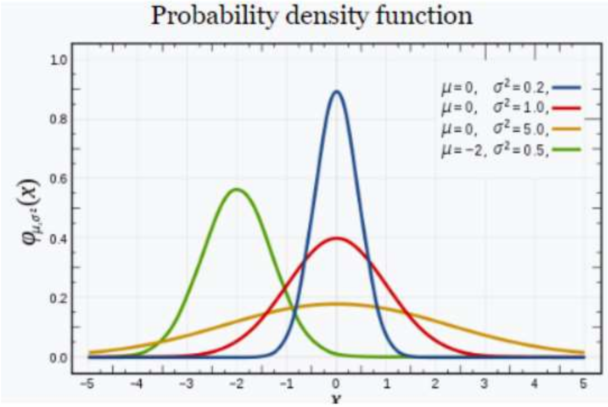


"Random" ***Variables***

- Discrete vs. Continuous

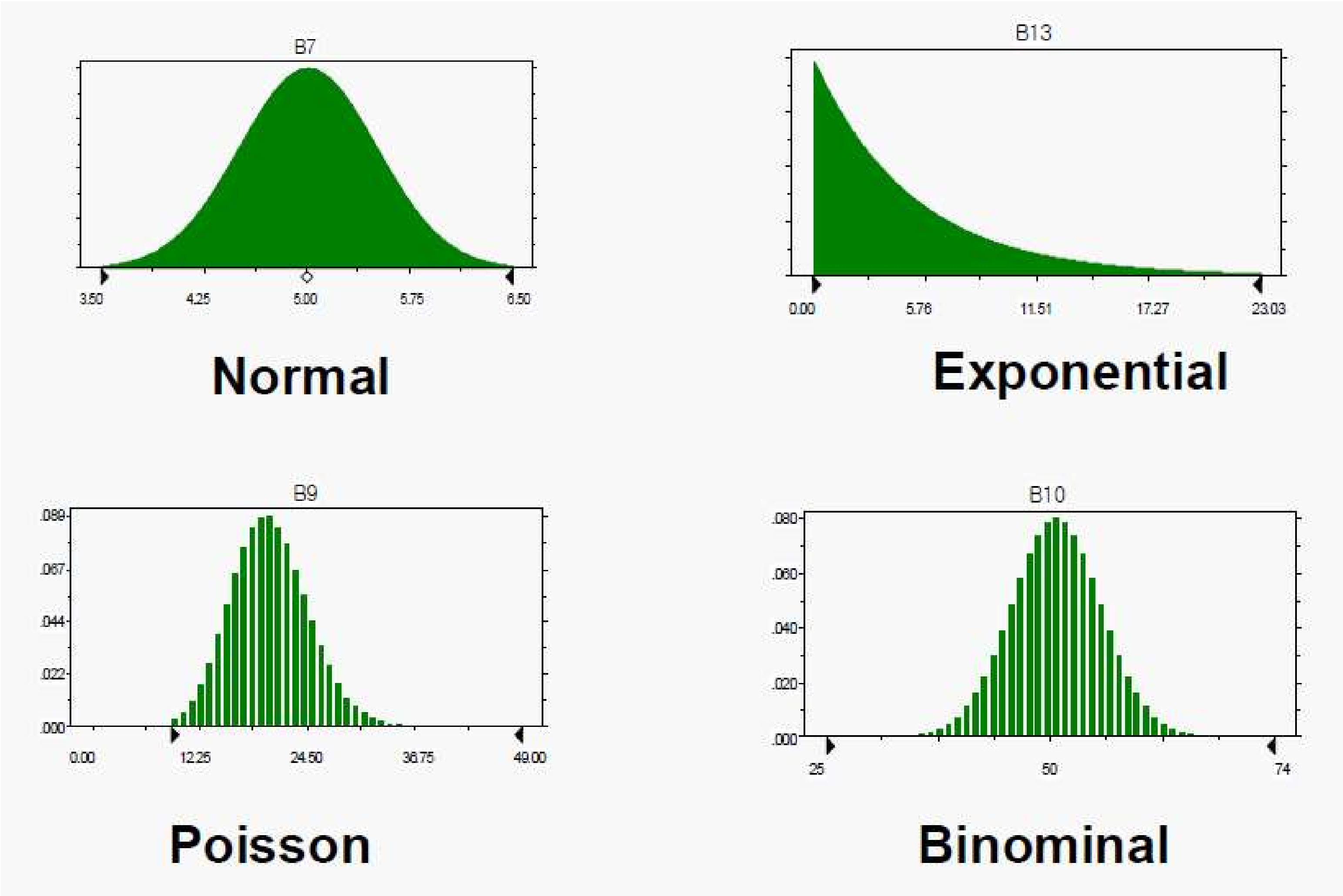
Distributions: how is the total probability distributed among all possible values?

- Probability Mass Function (PMF)
  - Discrete distribution for discrete variable (point masses)
- Probability Density Function (PDF)
  - Continuous distribution for continuous variable



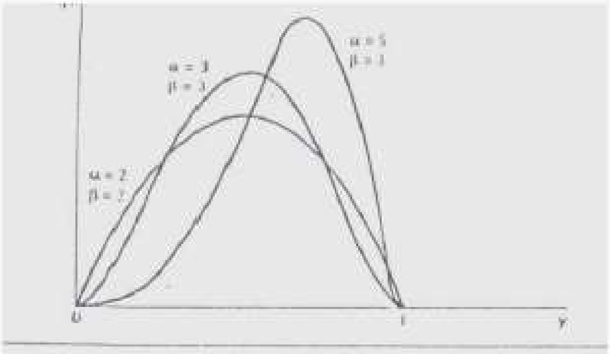
Variables & Distributions

Types of Distribution I

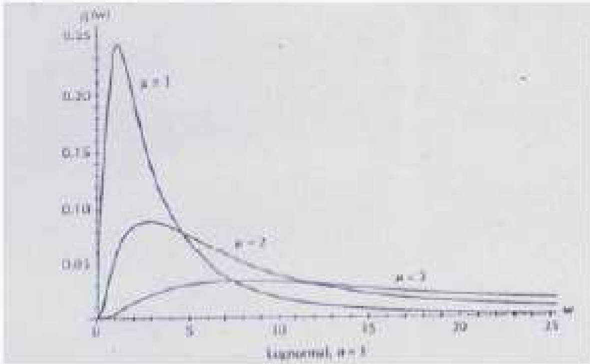


Variables & Distributions

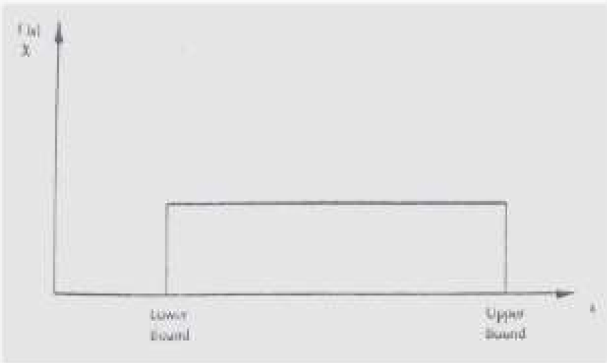
Types of Distribution II



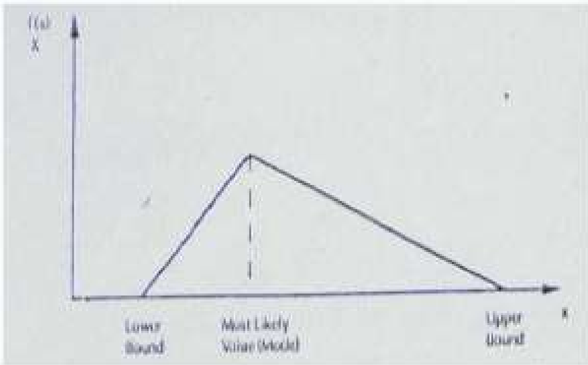
β



Lognormal

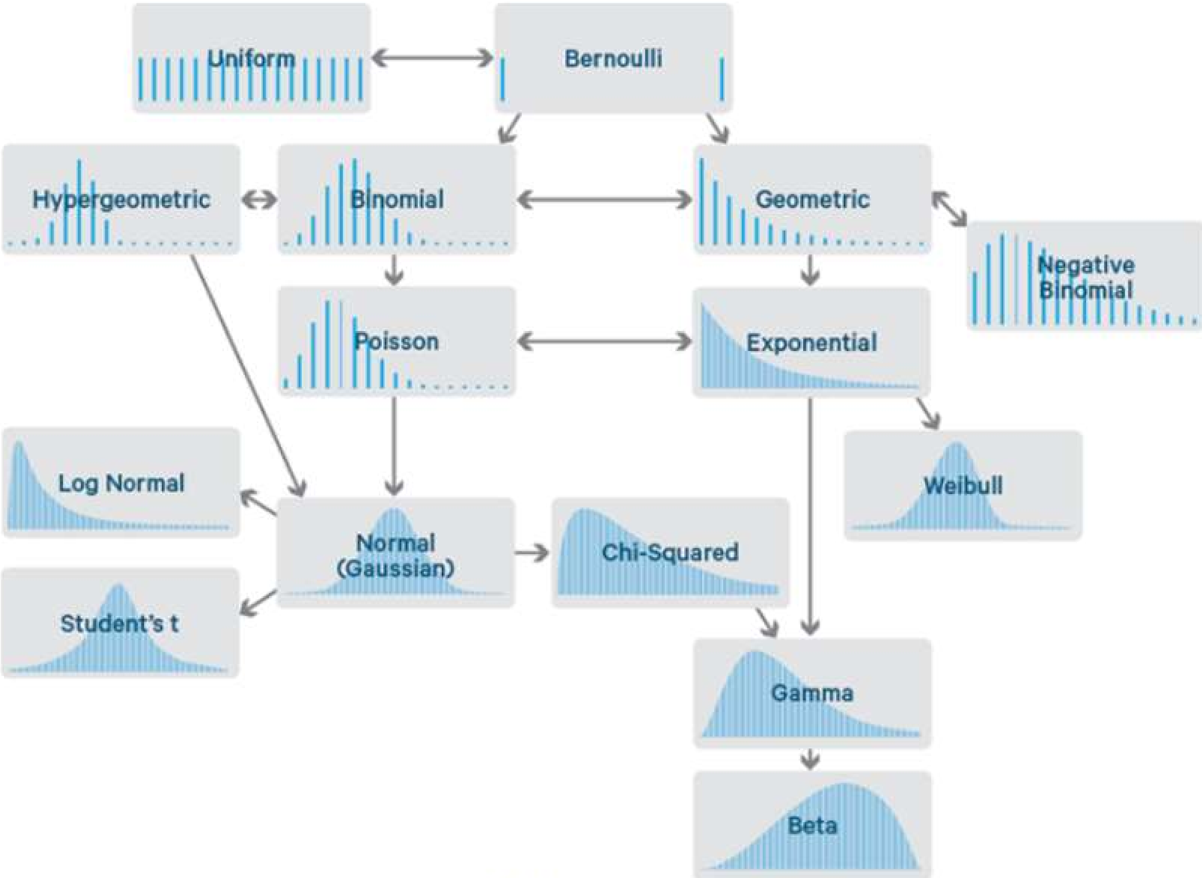


Uniform

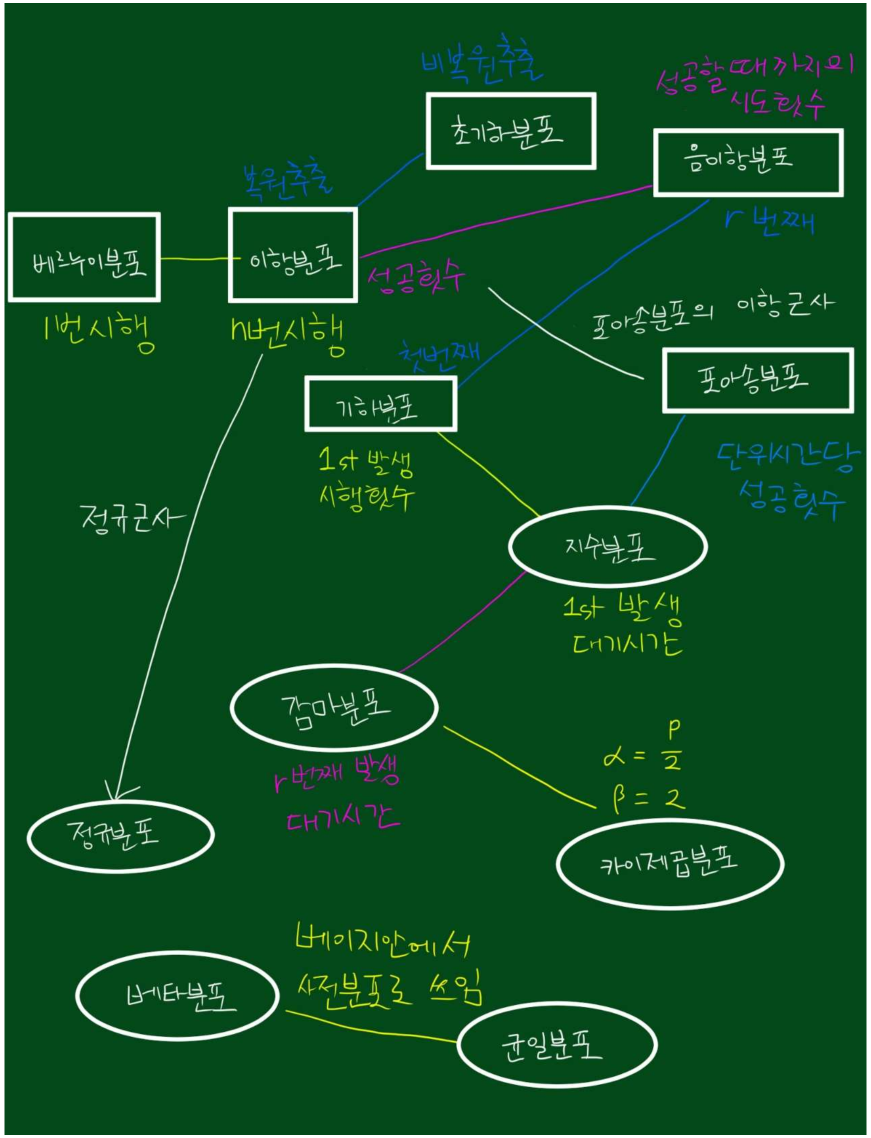


Triangle

Variables & Distributions



< 그림 1. 확률 분포 사이의 역학 관계도 >



Variables & Distributions

- Bernoulli: possible values = 1 or 0 (동전의 앞/뒤처럼 이벤트가 0 또는 1 밖에 일어나지 않는 분포)
- Binomial: possible outcomes = success or failure (동전을 N번 던졌을 때 P번만큼 앞면이 나올 확률에 대한 분포. 즉, 각각이 Bernoulli 분포를 갖는 이벤트에 대해 1 또는 0이 발생할 횟수에 대한 확률 분포)
- Poisson: for modeling the # of events that may occur during a given period (주어진 시간 동안 몇 번 발생. 1시간에 평균 10번의 전화통화가 온다고 할 때 한시간에 12번 전화통화가 올 확률. 시행 횟수가 크고 이벤트가 일어날 확률이 작은 Binomial 분포가 Poisson 분포에 수렴)
  - e.g., # of trucks arrived, rate of accidents, etc.

Variables & Distributions

- Uniform: flat (주사위처럼 모든 결과에 대한 확률이 동일한 분포)
- Normal: symmetrical, single model w/ tails (매우 많은 수의 동일 확률분포를 가진 샘플들의 산술평균은 그 샘플들이 어떤 분포를 따르든 결국 Gaussian 분포로 수렴)
  - Widely applicable due to *Central Limit Theorem*: Sample mean is Normally distributed for large samples drawn from non-Normal populations
- Geometric: 주사위를 굴렸을 때 한번에 6이 나올 확률, 두 번 만에 6이 나올 확률... 어떤 이벤트가 일어날 때까지의 횟수에 대한 확률, 얼마나 실패한 후에 성공할 것인가에 대한 분포 → 가장 첫번째에 이벤트가 발생할 확률이 가장 크다
- Exponential: sloped curve (Binomial의 연속버전이 Poisson, Geometric의 연속버전이 Exponential. 평균 5분만에 전화가 걸려온다고 할 때 다음 전화가 7분 후에 걸려올 확률. 일정시간 동안 발생하는 사건의 횟수가 Poisson 분포를 따른다면 다음 사건이 일어날 때 까지의 시간에 대한 분포는 Exponential 분포를 따름)
- Lognormal: 변수의 Log값이 Gaussian을 나타내는 분포. Gaussian을 Exponential한 함수
- Beta: distortable, skewable Normal-like, **thus very useful** (두 매개변수 알파와 베타에 따라 [0, 1] 구간에서 정의되는 연속확률분포)

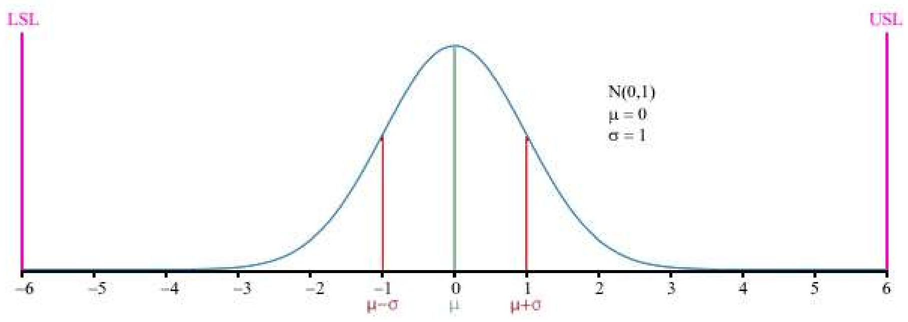
Characteristics of Probability Model

Normal Distribution

- The mean value is most likely
- Symmetrical about the mean
- Bell shape curve resulting from unbiased measurement error
- Lots of applications such as peoples' height & weight, production rate, inflation, project's profitability, quality control

- $\mu - \sigma < Y < \mu + \sigma$: 68%
- $\mu - 1.96\sigma < Y < \mu + 1.96\sigma$: 95%
- $\mu - 2.57\sigma < Y < \mu + 2.57\sigma$: 99%

6 sigma: $\mu - 6\sigma < Y < \mu + 6\sigma$: 99.99966%



Characteristics of Probability Model

Normal Distribution – Example

- A distilled water machine dispenses an amount of water that is Normally distributed with μ = 64 oz. and σ = 0.78 oz. per hour.
- What container size is needed to limit overflow to 0.5% of the time?

Sol.

- *c* = size of container
- $P(x \leq c) = 0.995$
- "z" value associated with [=0.995]: +2.58 from the statistic table
  - This is the number of σ from μ associated with 0.995
- Thus, *c* = 64 + 2.58 (0.78) = **66.01 oz.**

Characteristics of Probability Model

Lognormal Distribution

- Natural logarithm of the distribution is a normal distribution
  - If X is log-normally distributed with parameters μ and σ, then $Y = \ln(X)$ is normal with same mean and standard deviation
- Upper limit is unlimited, but values cannot fall behind zero
- Distribution is positively skewed, with most values near lower limit
- Applied to the situations where values are positively skewed, but cannot be negative such as real estate price, stock price, oil reservoir size, labor productivity

Characteristics of Probability Model

Triangle Distribution

- Lower bound and upper bound are fixed
- Most likely value in this range forms a triangle
- When you have limited data
- Applied when you know the minimum, maximum, and most likely values such as sales estimate, # of cars sold per week, durations of construction activity

Uniform Distribution

- Only you have Worst Scenario and Best Scenario
- All values in this range is equally likely to occur
- When you know the extreme range and all possible values are equally likely such as location of a leak on a pipeline, which high wind may approach to the building

Characteristics of Probability Model

Binomial Distribution

- For each trial, only 2 outcomes are possible, usually success and fail
- The trials are independent
- The probability is the same from trial to trial
- Describe the number of times an event occurs in a fixed number of trials
  - such as number of heads in 100 flips of a coin, likelihood of defective sheets in the production of 500 sheets.
- $P(x = r) = p^r \times (1 - p)^{n-r} \times \frac{n!}{r!(n-r)!}$

  where, *p* is probability of success in each trial, *r* is number of success, *n* is total number of trial

Characteristics of Probability Model

Binomial Distribution – Combinations(조합) / Permutations(순열)

- **# of Combinations**: *# of unordered sets* of size *r* from sample of *n* objects

$$\binom{n}{r} = \frac{n!}{r! \times (n-r)!}$$

  - Complication: are you sampling *with* or *without* replacement?

- **# of Permutations**: *# of sets with objects in different order with set size r from a sample of n objects*

$$P(n,r) = \frac{n!}{(n-r)!}$$

  - Always larger than # of Combinations

Characteristics of Probability Model

Binomial Distribution – Example

- In the project, a defective bolting occurs at random with probability 1/500. What is probability that the number of defects is 5 in a 500 bolting?

Sol.

$$P(x=5) = p^r \times (1-p)^{n-r} \times \frac{n!}{r!\,(n-r)!} = (\frac{1}{500})^5 (\frac{499}{500})^{495} \frac{500!}{5!\,495!} = 3\%$$

Characteristics of Probability Model

Poisson Distribution

- Describe the number of times an event occur in a given interval (usually time in Min, hour, day)
- Number of possible occurrences is not limited
- Occurrences are independent
- Applied such as number of telephone calls per minutes, number of trucks arrived per minutes, number of defects per 100 square yards of concrete slab
- $P(x = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$

  where, λ is average arrival rates (defects) per unit

Characteristics of Probability Model

Poisson Distribution – Example

- Past history reveals 3 flaws in producing a 20m steel sheet piles. What is the probability of 0 flaw in this type of pile?

Sol.

$$P(r = 0) = \frac{e^{-\lambda}\lambda^{r}}{r!} = \frac{e^{-3}3^{0}}{0!} = 5\%$$

Characteristics of Probability Model

Exponential Distribution

- When event (such as arrivals, defects, accidents) are purely random, the time between successive events are described by exponential distribution
- Distribution is not affected by previous events
- Applied such as time between incoming calls, time between truck arrival in earthwork

- $f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$
- $P(X \leq x) = 1 - e^{-\frac{x}{\mu}}$

  where, μ is average mean time $(= \frac{1}{\lambda})$

Characteristics of Probability Model

Exponential Distribution – Example

- The historical data indicates that it has been normally recorded 7 days of rains during May. Estimate the probability of no-rain during any week in May?

Sol.

- Mean time between rain $= \frac{31}{7} \times 24 = 106.28HR$
- $x = 7 \times 24 = 168HR$ (one week)
- Rain within a week: $P(X \leq 168) = 1 - e^{-\frac{168}{106.28}} = 0.794 = 79.4\%$
- $No - rain = 1 - 79.4\% = 20.6\%$

Characteristics of Probability Model

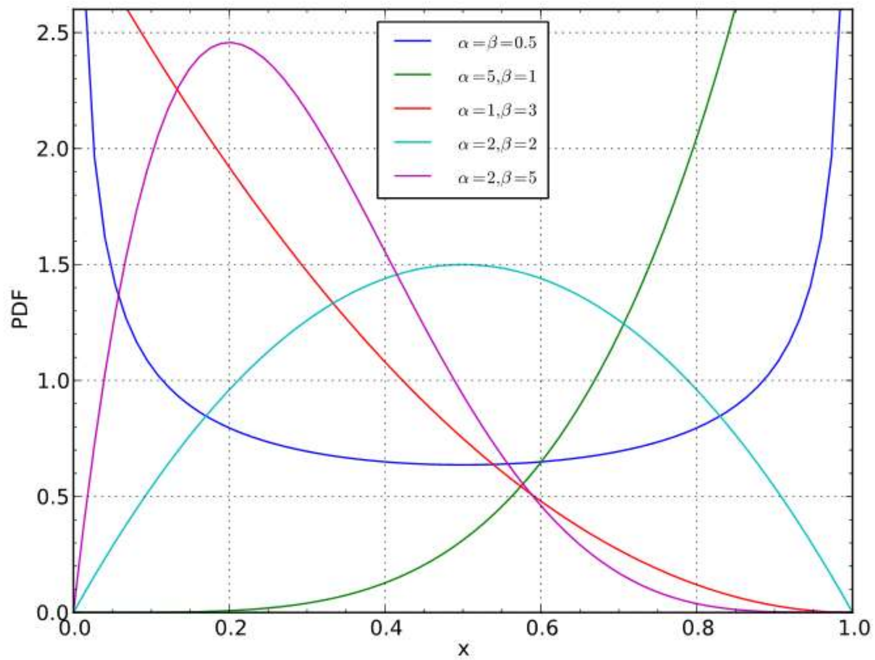
Beta Distribution

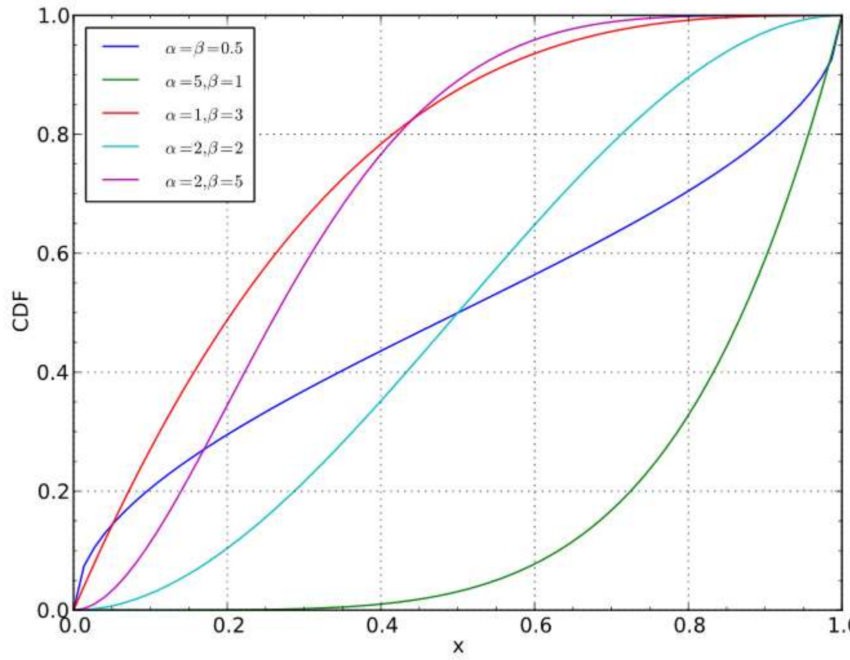
- Range is between 0 and a positive value
- Shape can be specified with two positive values, “α” and “β”
- Represent variability over a fixed range
- Describe empirical data such as representing the reliability of a company’s devices

Characteristics of Probability Model

Beta Distribution

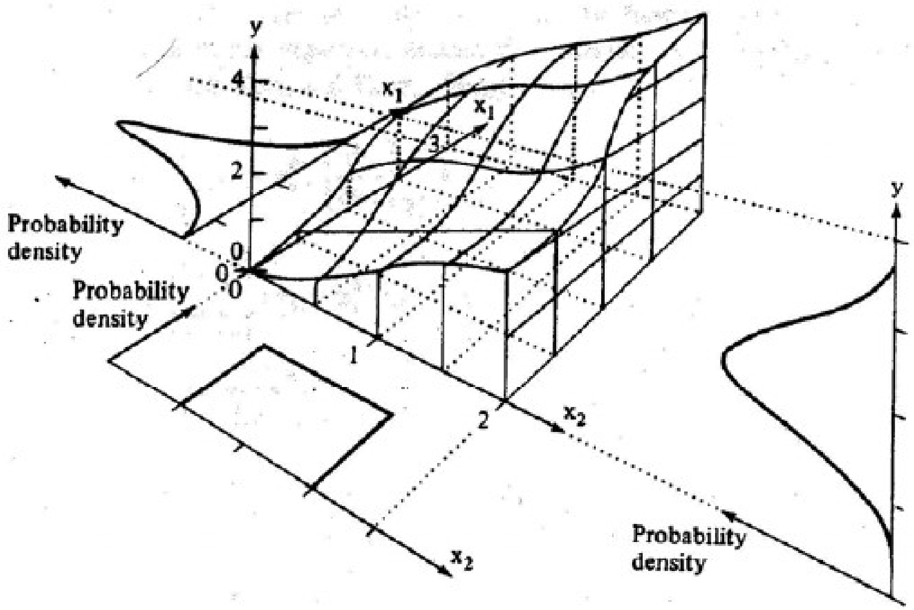
- PDF can be shaped with use of parameters α and β:
  - Large α: tight distribution; Small α: spread-out or flat distribution
  - $\beta < \frac{\alpha}{2}$: skewed to left; $\beta = \frac{\alpha}{2}$: symmetrical about 0.5; $\beta > \frac{\alpha}{2}$: skewed to right
- CDF provided in the figure (with various α and β)

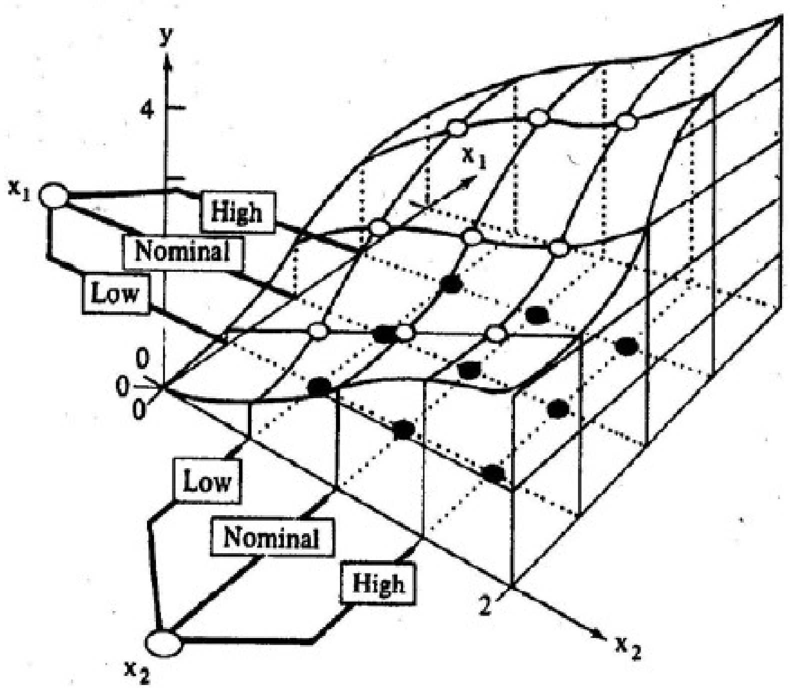




6. Discrete Approximation

- Most of uncertain quantities are continuous
- Often, we cannot know the exact density function
- We need to transform continuous into discrete trees attempting to apply them in risk analysis





6. Discrete Approximation

How to apply continuous PDF models to Decision Trees?

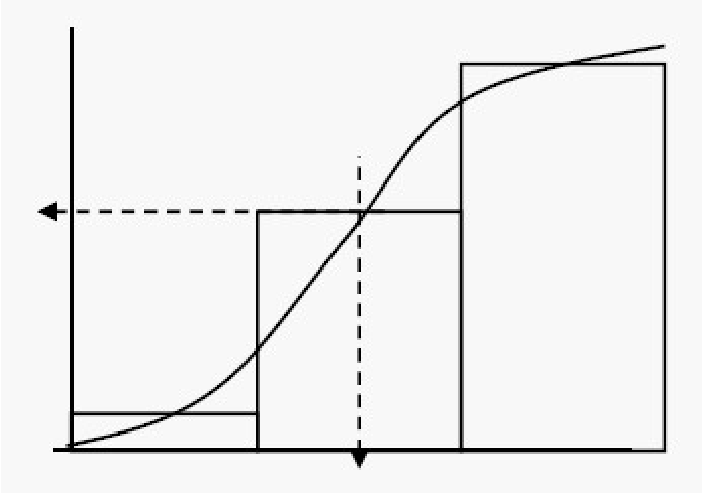
- Two alternative approaches:
  - Use Monte Carlo simulation instead of decision tree (with its discrete branches); MC simulation software easily handles continuous functions
  - Discretize the PDF in order to ascertain discrete tree branch probabilities

    But are you loosing valuable information?

6. Discrete Approximation

How to apply continuous PDF models to Decision Trees?

- How best to convert a continuous distribution into a discrete one? AND How to control the size of tree?
- Conventional Method
  - Middle value is chosen equal to the mean of continuous distribution
  - The other point is chosen to minimize the total area
  - ➢ Usually underestimating
- Discretize the distribution
  - Bracket median
  - Pearson-Tukey 3 points method



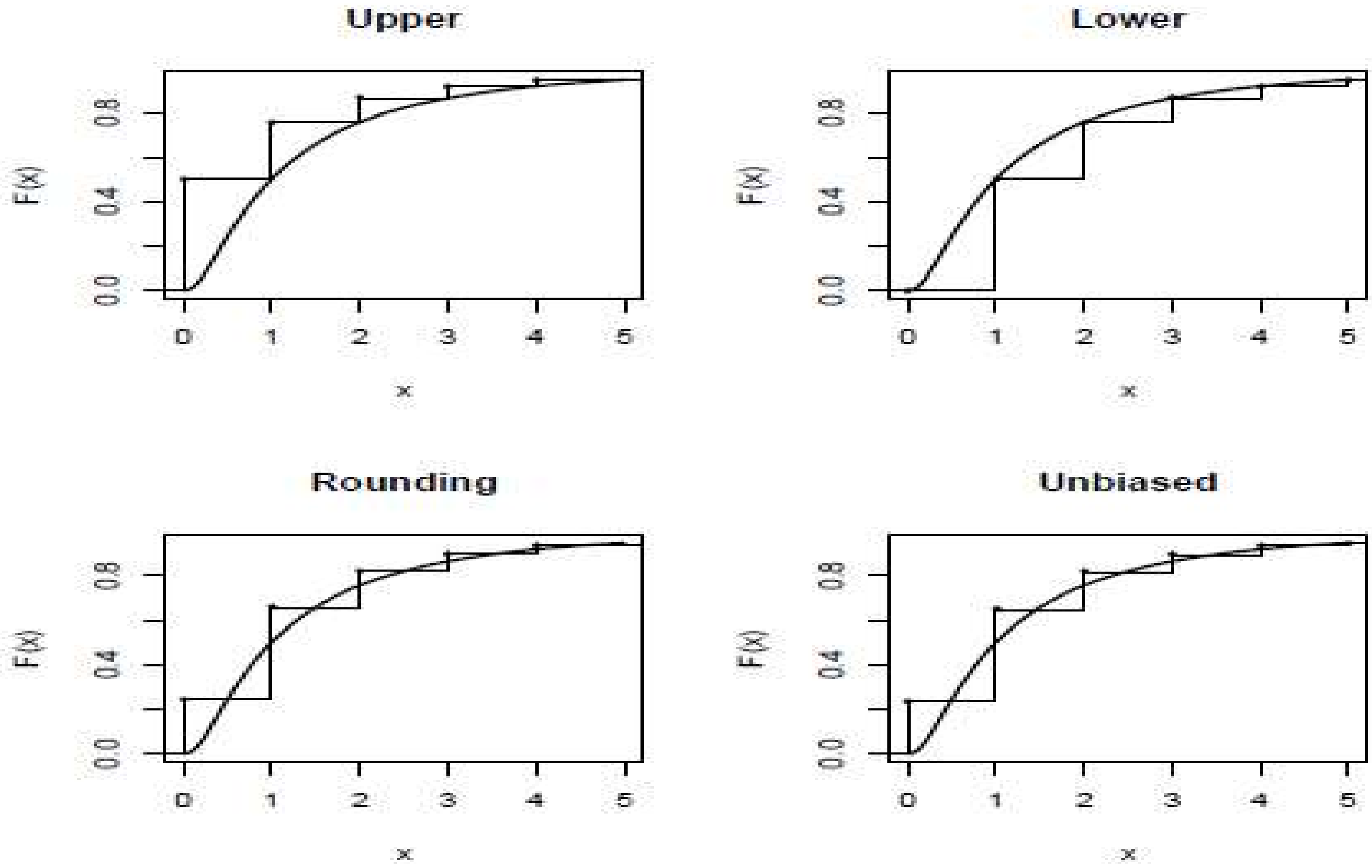
6. Discrete Approximation

Discretizing a PDF – Approach #1

1. Draw the CDF $F(x)$
2. Establish # of intervals for x and draw vertical lines so that intervals of x are approximately equal
3. For each interval, draw horizontal line such that areas between CDF and line (above & below) are equal
4. Determine the estimator of x for each interval of x from the intersection of the horizontal line and CDF
5. Cumulative probabilities associated with each estimator of x can be read off the vertical axis at the interval separations
6. Summarize discrete probabilities for each x estimator with a histogram

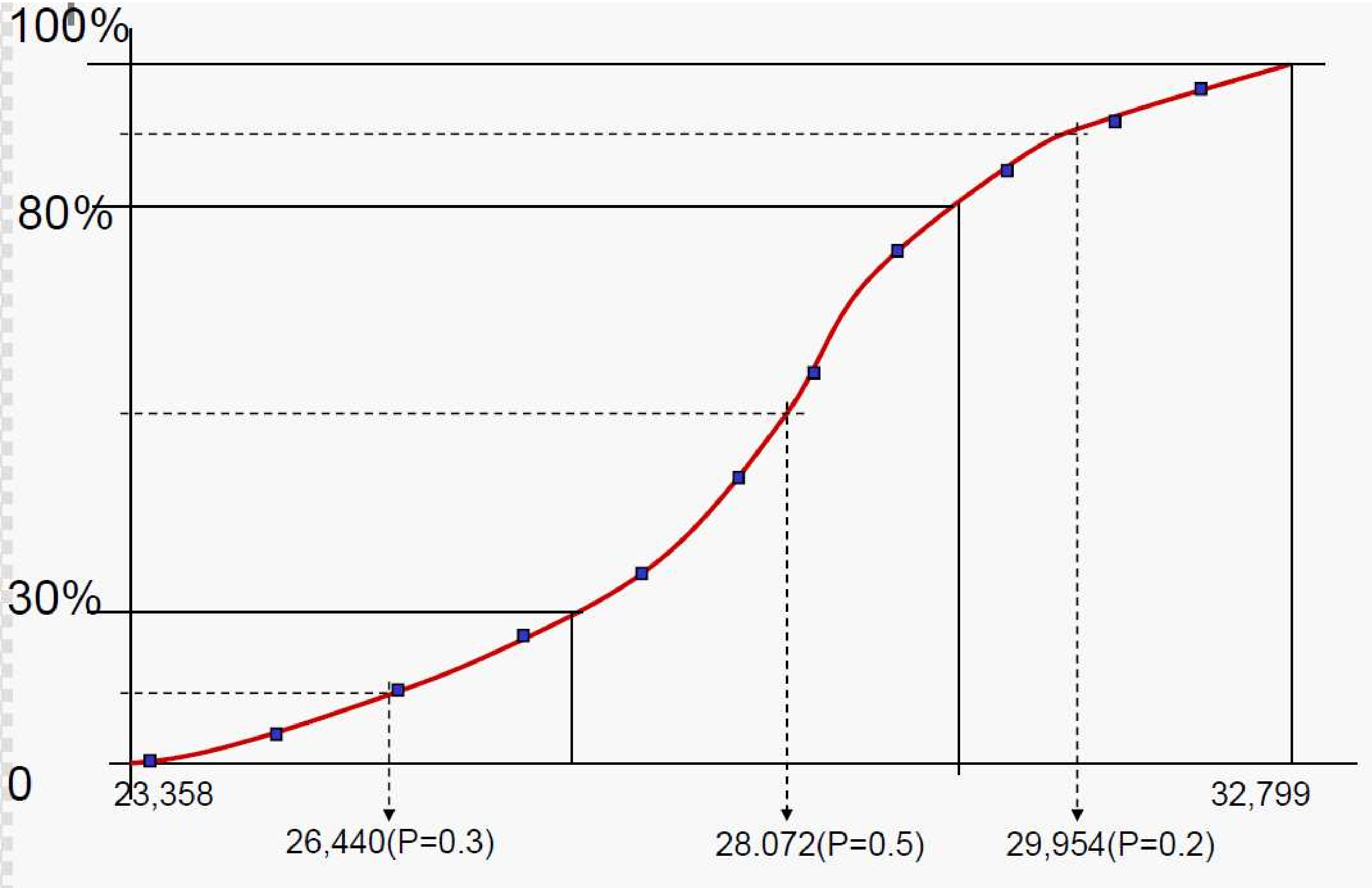
6. Discrete Approximation

Discretizing a PDF – Approach #1



6. Discrete Approximation

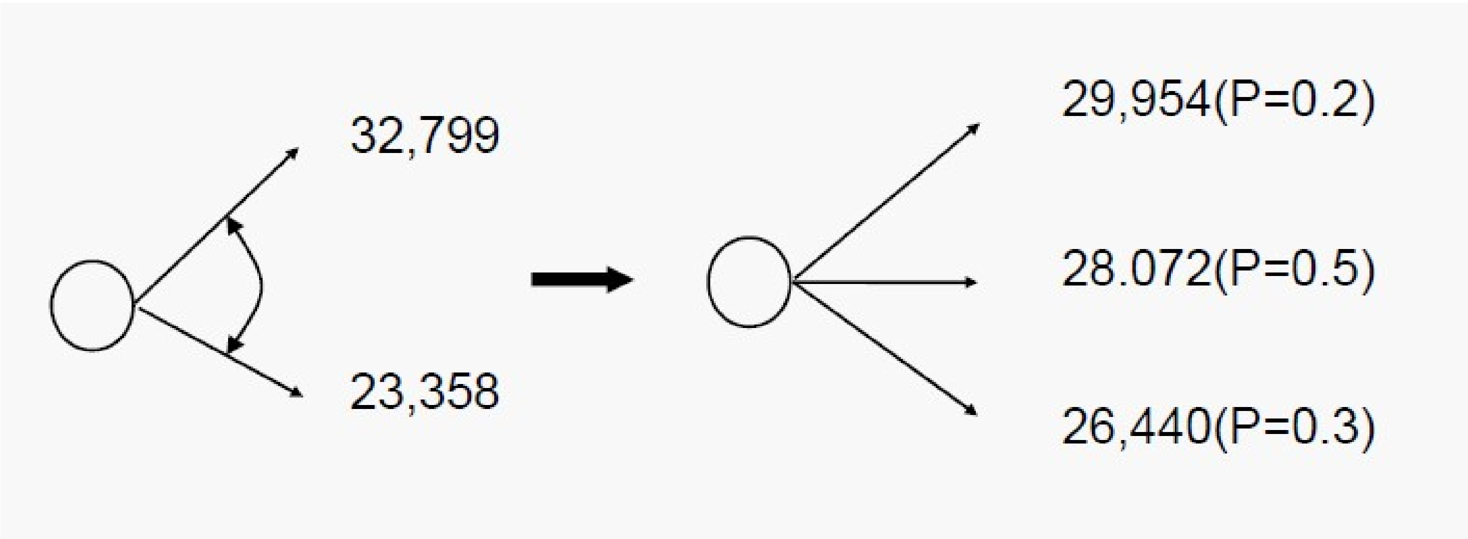
Discretizing a PDF – Bracket Median Method



6. Discrete Approximation

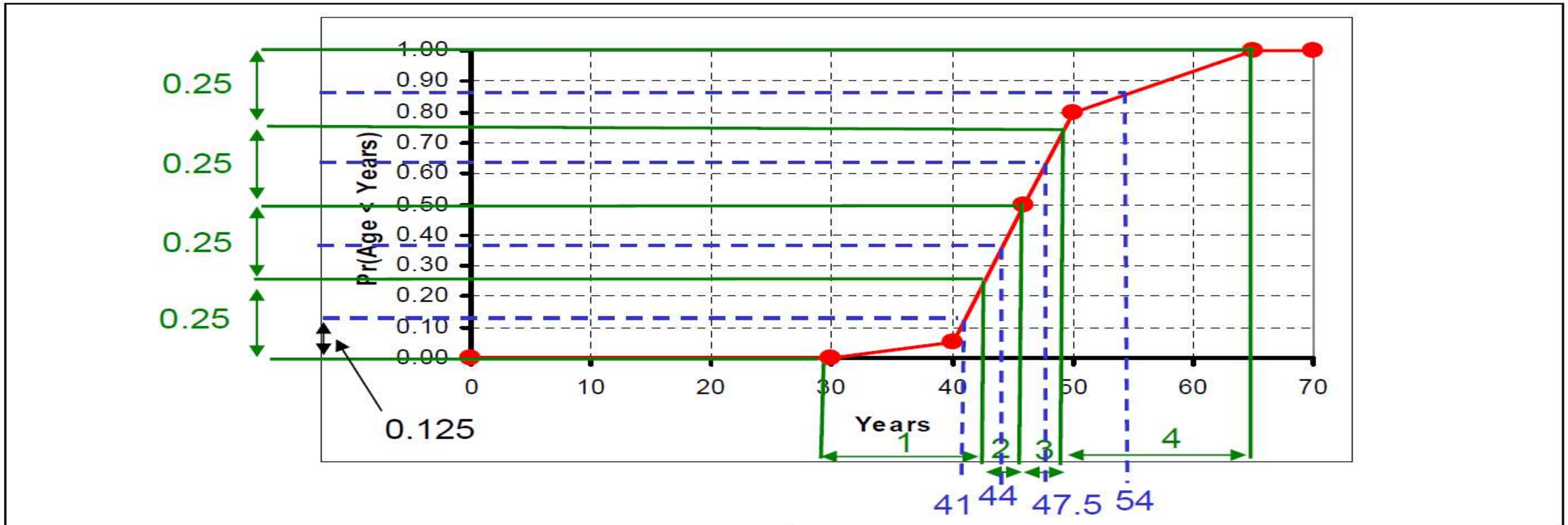
Discretizing a PDF – Bracket Median Method

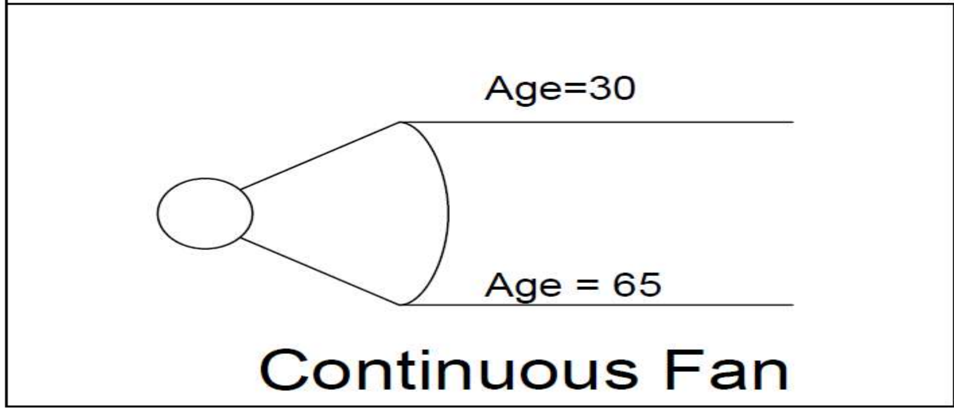
- we can approximate (discretize) the continuous risk profile through a bracket median method to assign it to the decision tree. The results are as follows:

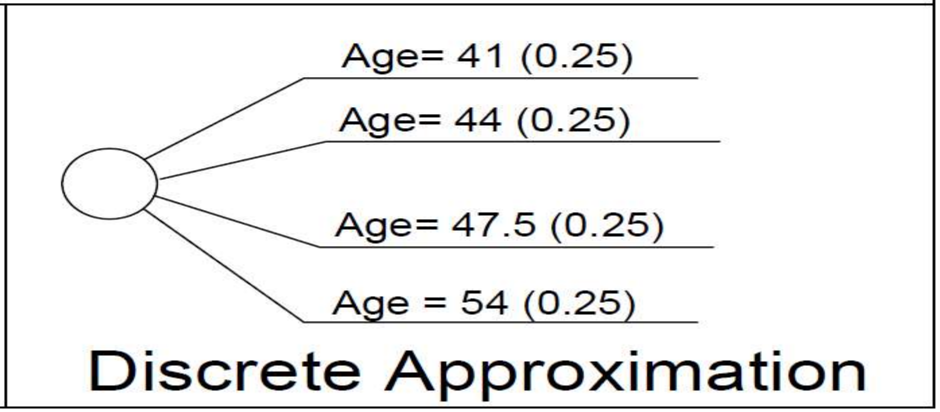


6. Discrete Approximation

Discretizing a PDF – Bracket Median Method (확률구간을 각각 25%씩 4구간으로 나눔)



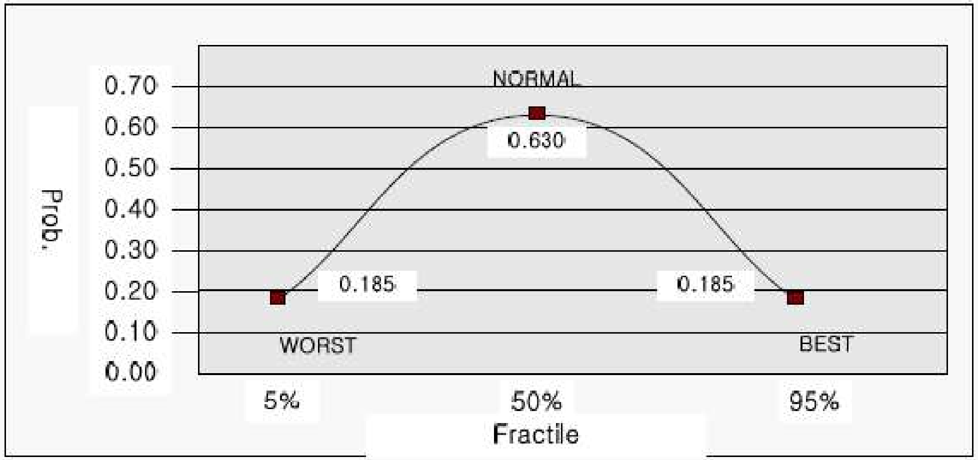


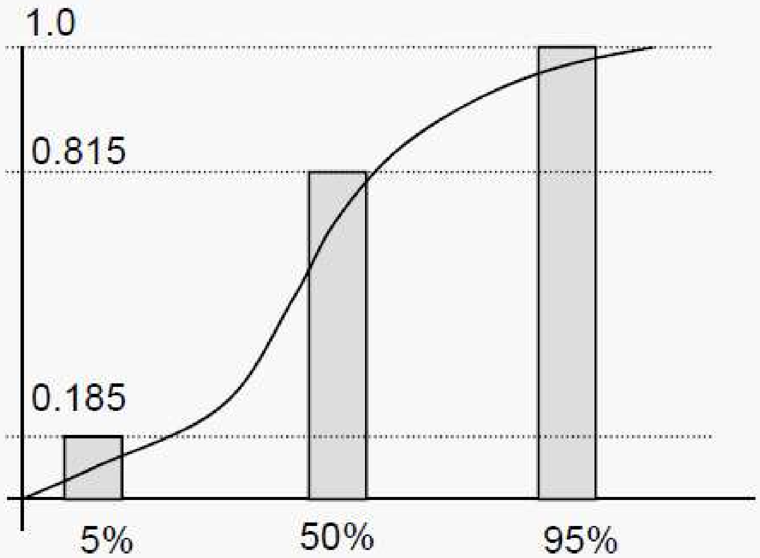


6. Discrete Approximation

Discretizing a PDF – Pearson-Tukey 3 points Method

- Developed by Keefer & Bodily(1983)
- Works best for approximating systematic distributions for easy and simple measurement
- This method adopts three-point approximation using the median and the 0.05 and 0.95 fractiles
- Based on the approximation, the probability of normal scenario case is 0.63, and the probability of worst and best scenario is 0.185, respectively

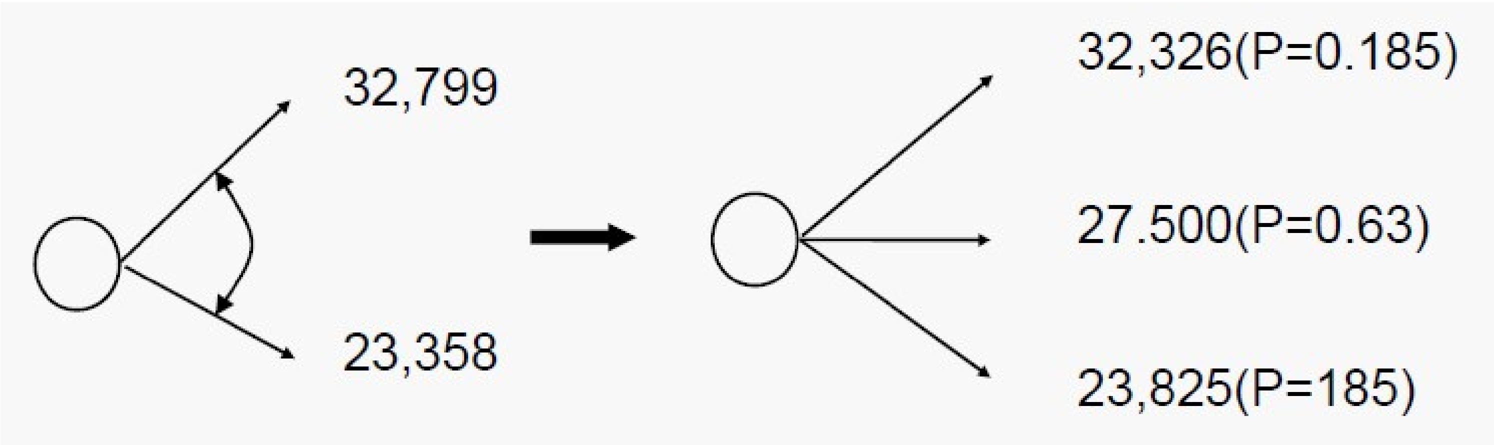




6. Discrete Approximation

Discretizing a PDF – Pearson-Tukey 3 points Method

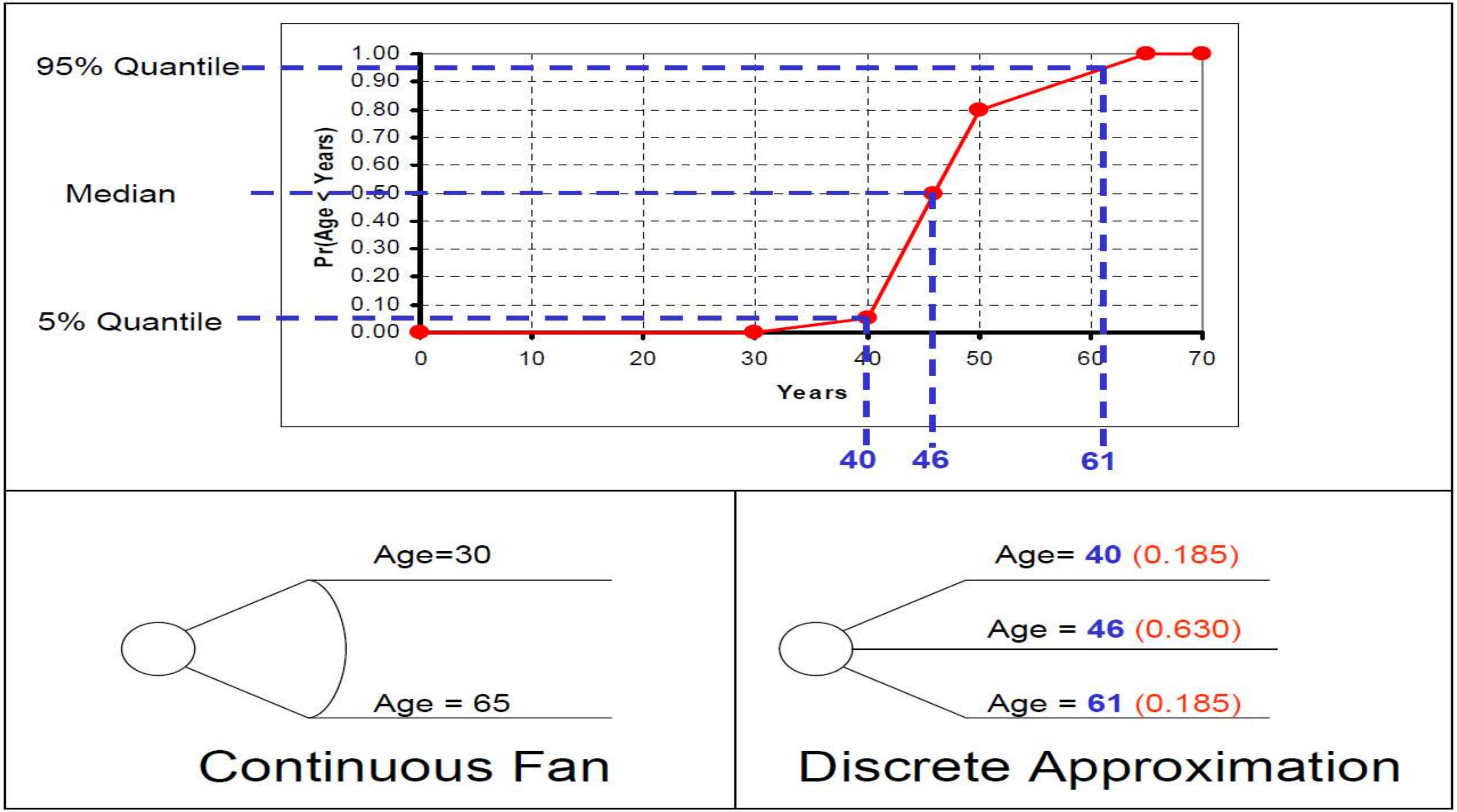
- 0.05 fractile = 23,825 (p = 0.185)
- 0.50 fractile = 27,500 (p = 0.63)
- 0.95 fractile = 32,326 (p = 0.185)



➢ The resulting approximation is reasonably accurate for a wide variety of distributions

6. Discrete Approximation

Discretizing a PDF – Pearson-Tukey 3 points Method



Q & A


Street Calculus
Risk factors
☑ black
☑ male
☐ aggressive body language
☑ baseball cap on backwards
☐ short hair
☐ baggy jacket
☐ wrong neighborhood
RF = 3
Mitigating Factors
☐ female
☐ over 40
☑ loafers
☐ tie and coat
☑ whistling Sondheim
☑ Fed Ex envelope
☑ polo shirt
MF = 4
4 > 3
Risk: Acceptable.
Risk Factors
☐ black
☑ white
☑ male
☐ long hair
☑ wrong neighborhood
☐ police officer
☐ baseball cap
Rf = 3
Mitigating factors
☐ female
☑ white
☐ briefcase
☑ groceries
☑ humming Motown
☐ tie + coat
☑ over 40
MF = 4
4 > 3
Risk: Acceptable.
Good evening.
Good evening.