6. VTOL Rotor Aerodynamics Analysis

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Topics

1. Rotor aerodynamics
2. Cruise aerodynamics
3. How helicopters are controlled

Rotor Aerodynamics

Recommended Textbooks

A rotorcraft is a vehicle that relies on rotor or rotors to fly.

Most eVTOL aircraft rely on rotors to provide lift and propulsion, and their governing flight aerodynamics are more similar to helicopters than to airplane. Therefore, it is crucial to understand **rotor aerodynamics, helicopter aerodynamics, helicopter stability & control, and helicopter aeroelasticity & dynamics** before designing eVTOL aircraft. For safety, and to save time and money, recruit experienced rotorcraft engineers of all types and find the most experienced rotorcraft designer to help you.

It is impossible to teach an entire helicopter aerodynamics course in few hours. In this module, I will provide the basic governing aerodynamics equations that are useful for doing preliminary performance estimation and sizing of rotors. To grasp the physics and the theory behind these equations, I recommend the following 3 textbooks.

1. **"Aerodynamics of the Helicopter"** by Alfred Gessow and Garry C. Myers (a classic, easier to follow and a good introductory level college text book)
2. **"Principles of Helicopter Aerodynamics"** by J. Gordon Leishman (over 500 pages and if you can finish this text book, you are on your way to design a good rotor)
3. **"Helicopter Performance, Stability, and Control"** by Raymond W. Prouty. Good textbook on stability & control, and handling qualities.

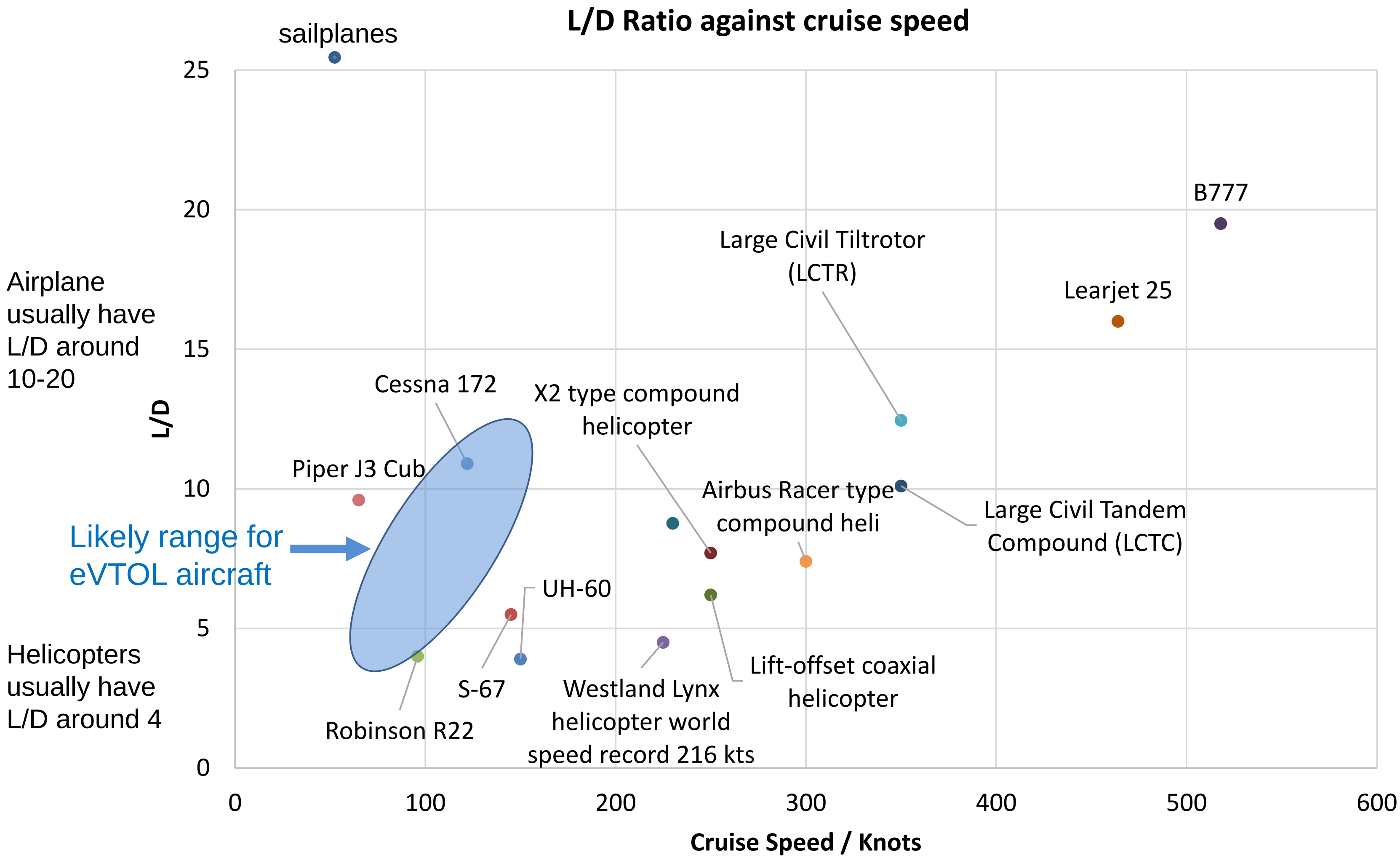
Generic Characteristics for VTOL Aircraft

For eVTOL, best to have L/D>10 for cruise, and disk loading <20 lb/ft² for hover

Configuration	V_{cruise} (mph)	$\left(\frac{L}{D}\right)_{cruise}$	Disk Loading (lb/ft^2)	Not including battery Empty weight fraction	$\overline{C_l}$ (upper limit)	N
Lift + cruise	150	10	15 and up	0.53	1.0	8
Compound helicopter	150	9	4.5	0.5	0.8	1
Tilt wing	150	12	15 and up	0.55	1.0	8
Tilt rotor	150	14	15 and up	0.55	1.0	12
Conventional helicopter	100	4.25	4.5	0.43	0.6	1
Coaxial heli	150	5.5	7	0.43	0.6	2
Multirotor	50	1.5	4 and up	0.43	0.6	8
Autogyro	100	3.5	3.75	0.5	0.8	1
Tilt duct	150	10	40	0.55	1.0	36

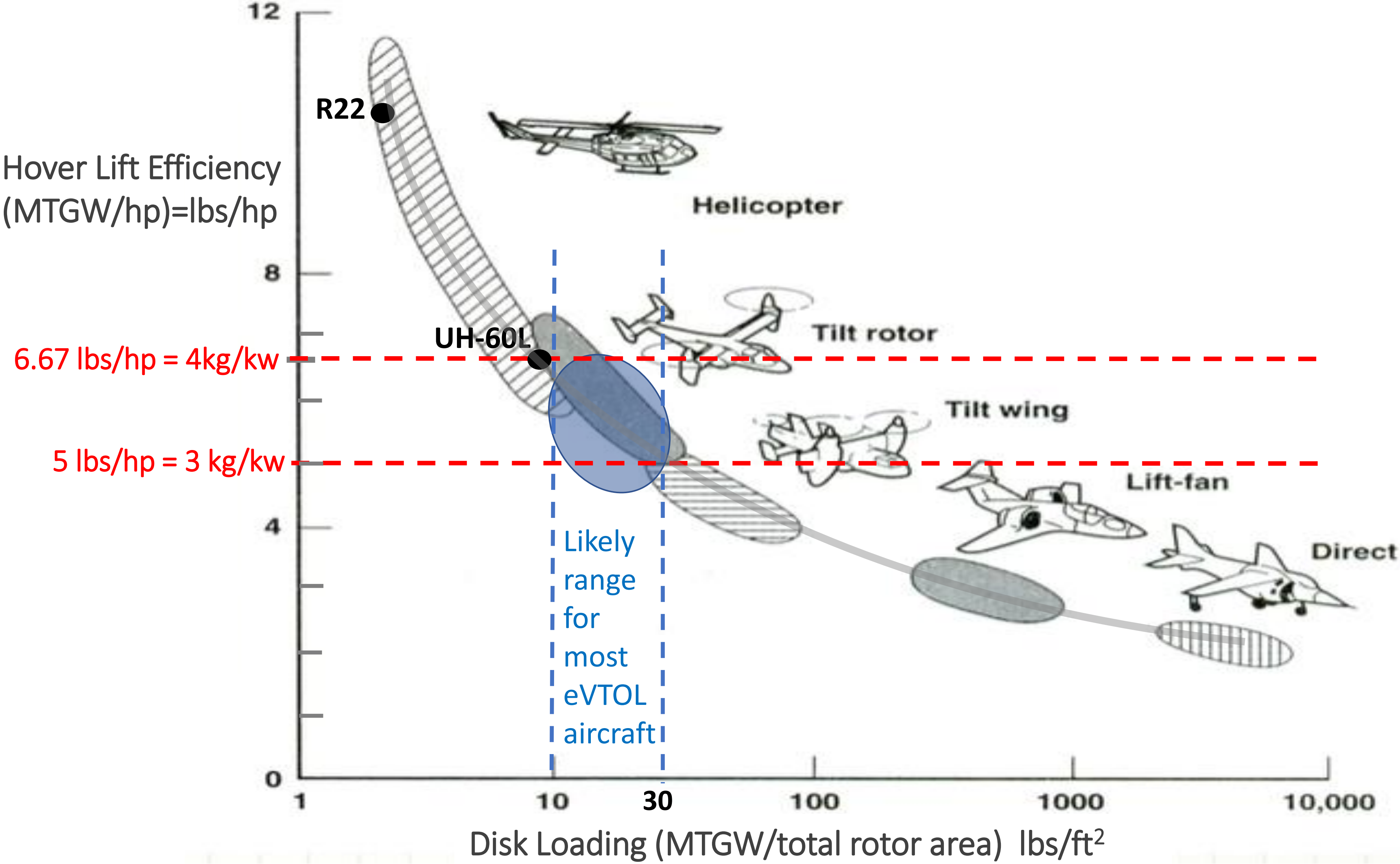
Source: https://convex.mit.edu/publications/arthur_ondemand.pdf

Total Aircraft L/D versus Cruise Speed



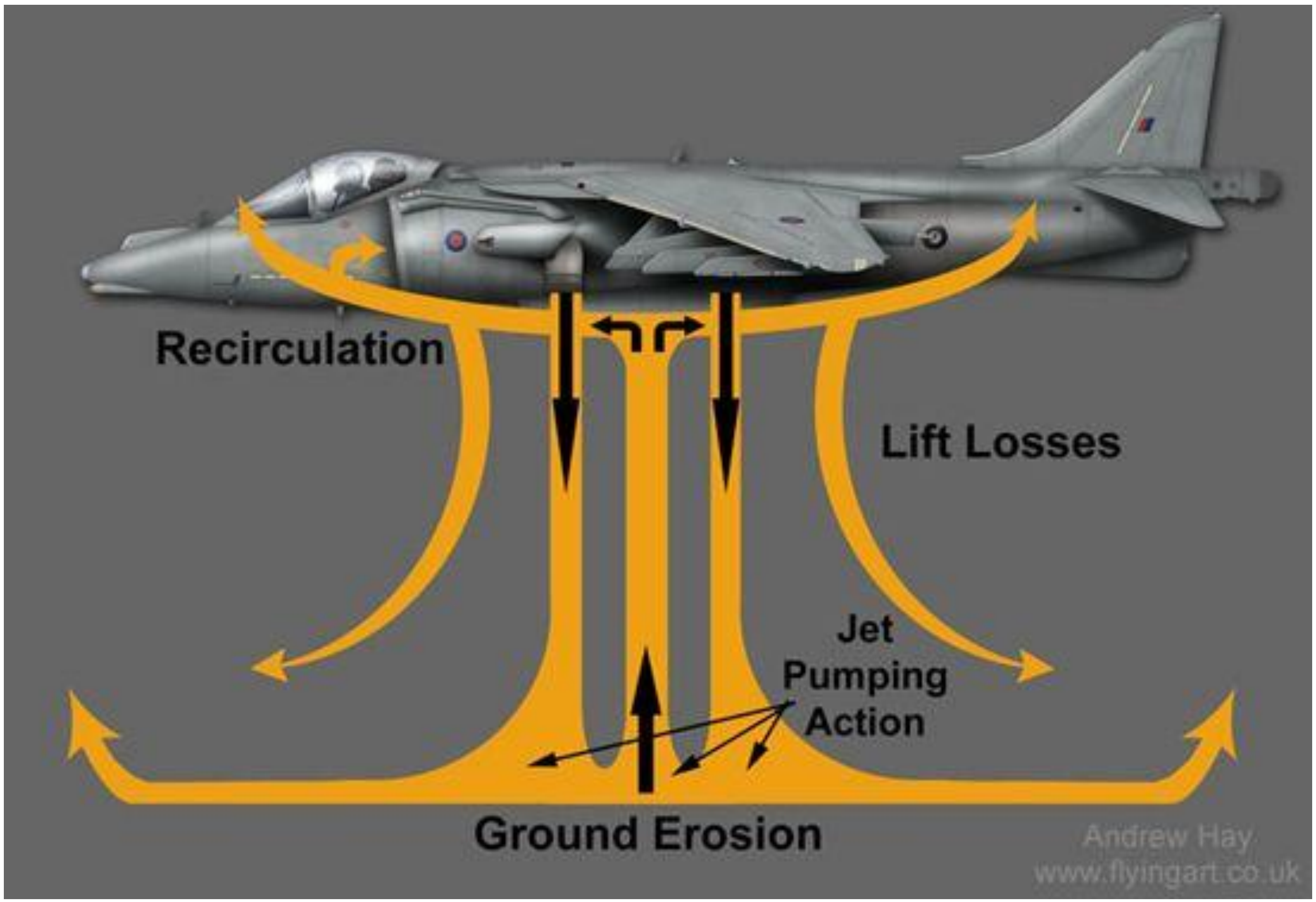
Source: Johnson, W., H. Yeo, and C. Acree Jr, *Performance of Advanced Heavy-Lift, High-Speed Rotorcraft Configurations*. 2007

High Disk Loading Reduces Hover Efficiency

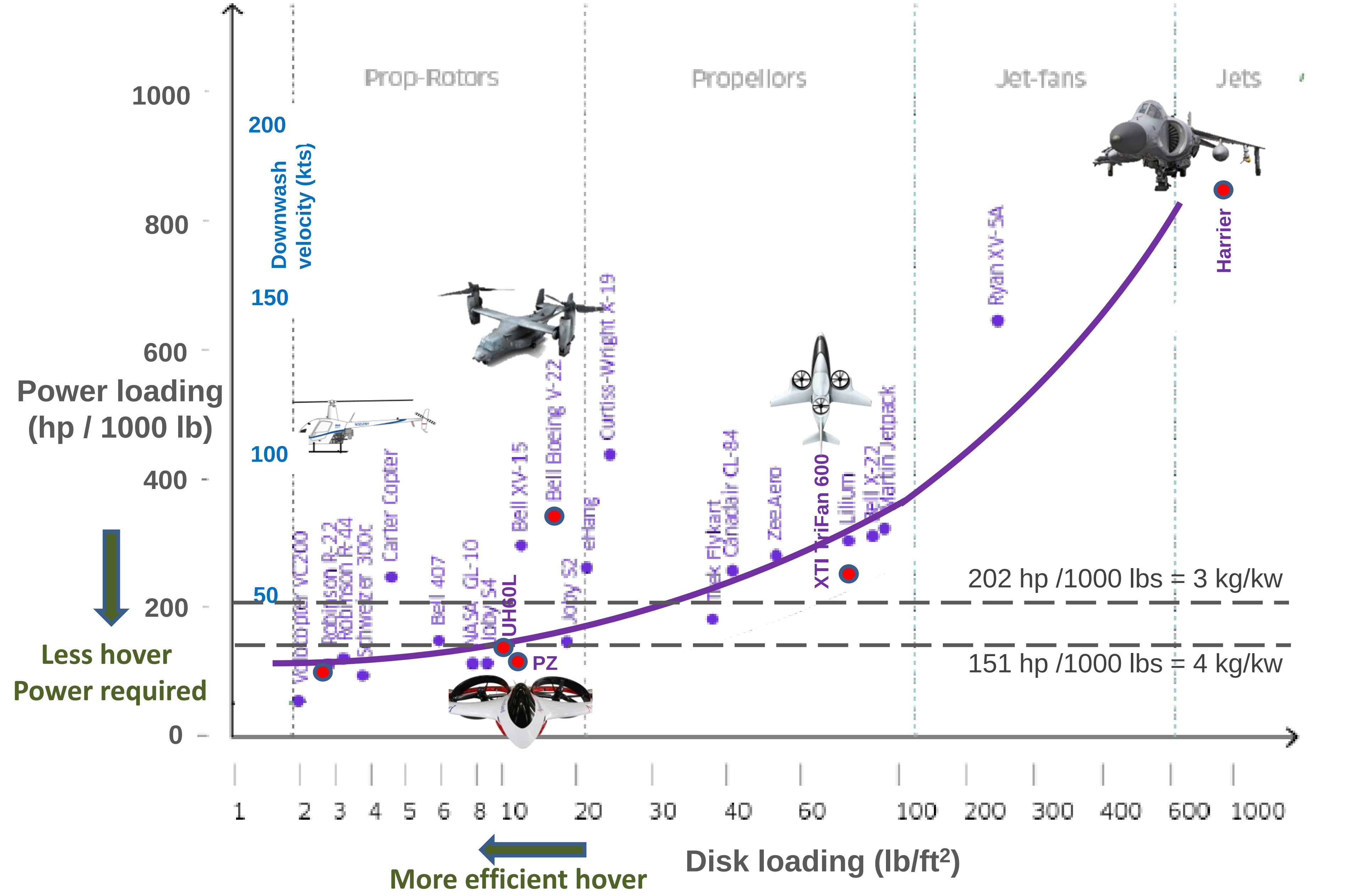


Source: https://history.nasa.gov/monograph17.pdf

> 80 lb/ft^2 disk loading causes tremandous downwash. At 80 lb/ft^2 the downwash is 100 km/hr



As Disk Loading Increases, Power Loading also Increases

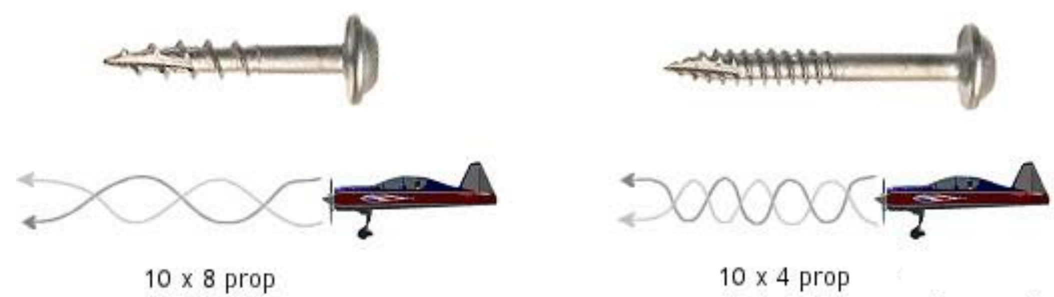


Propeller

Airplane Propeller Pitch Explained

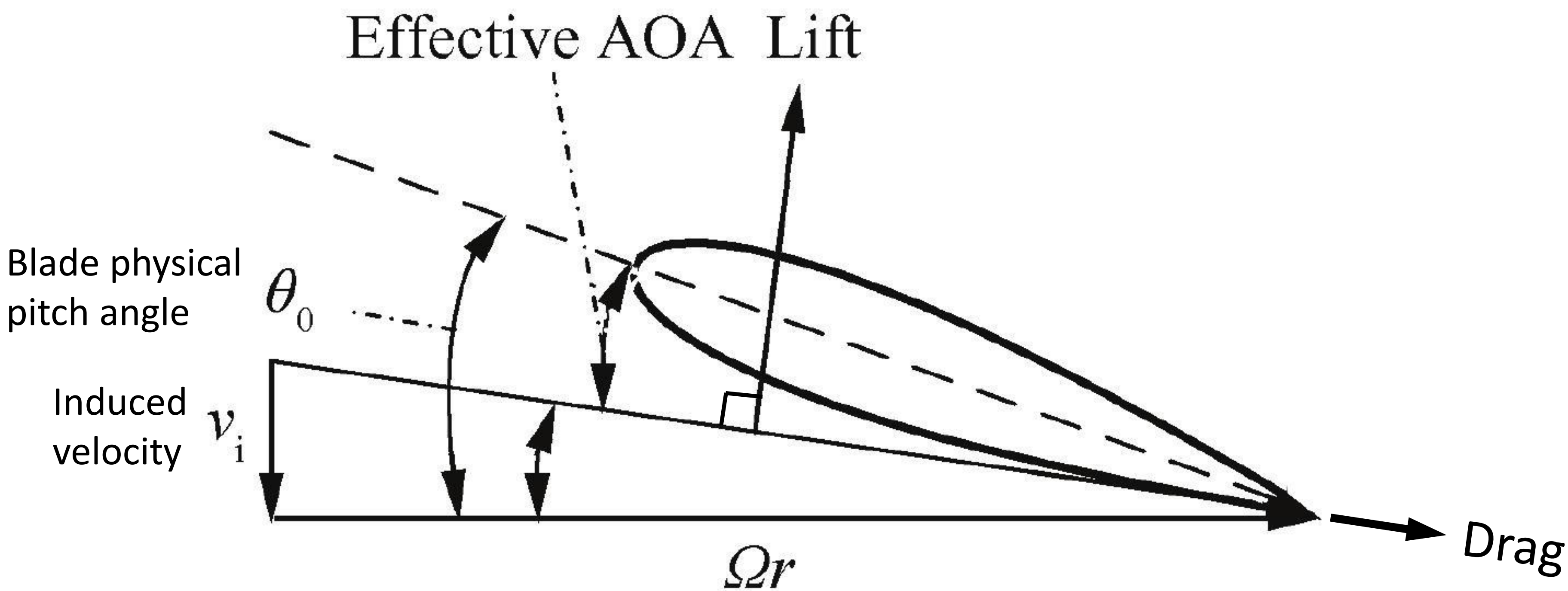
A propeller “size” is reported as a diameter × pitch value.

The first number is the diameter (measured from tip to tip) and the second is the pitch. Almost always, these numbers are in inches. The **pitch** is how much the propeller translates in one rotation. The following images illustrates this:



Plane on the left has 8 inches pitch prop; on the right 4 inches. In general, the larger the pitch, the greater the thrust force: the propeller “pulls” or “cuts through” more air per single rotation. However, a larger pitch also requires more power and is less stable. A smaller pitch can spin faster and is generally more stable, but produces less thrust force. Larger diameters increase the thrust force but take more time to change speed because of inertia.

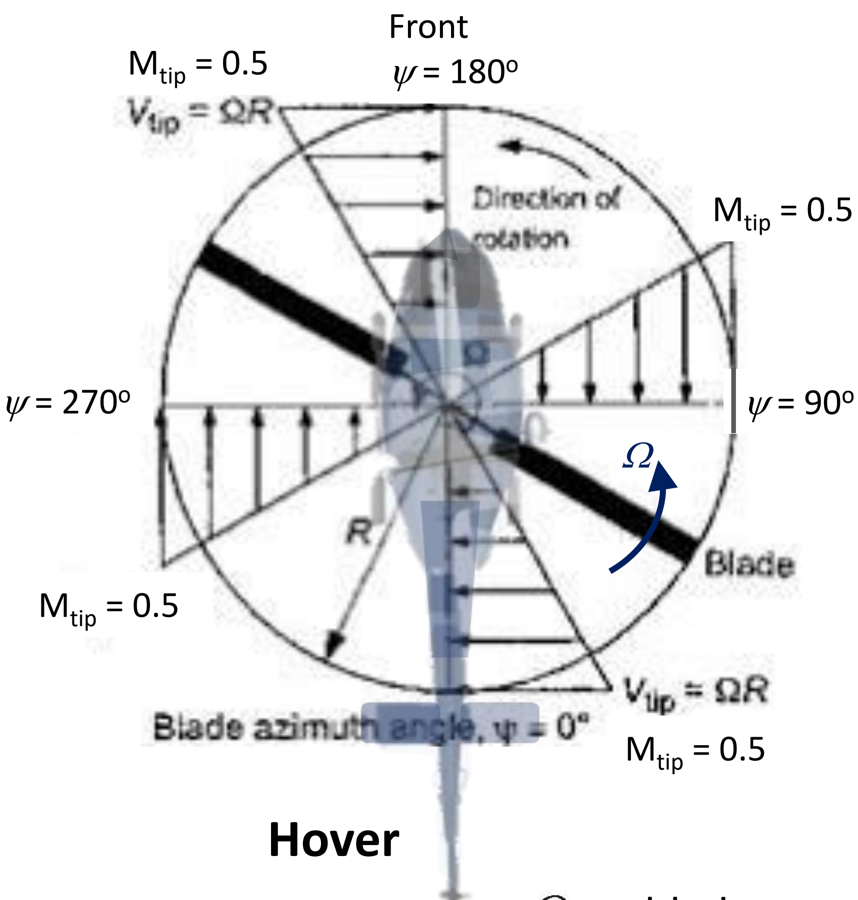
Blade Section Aerodynamics in Hover

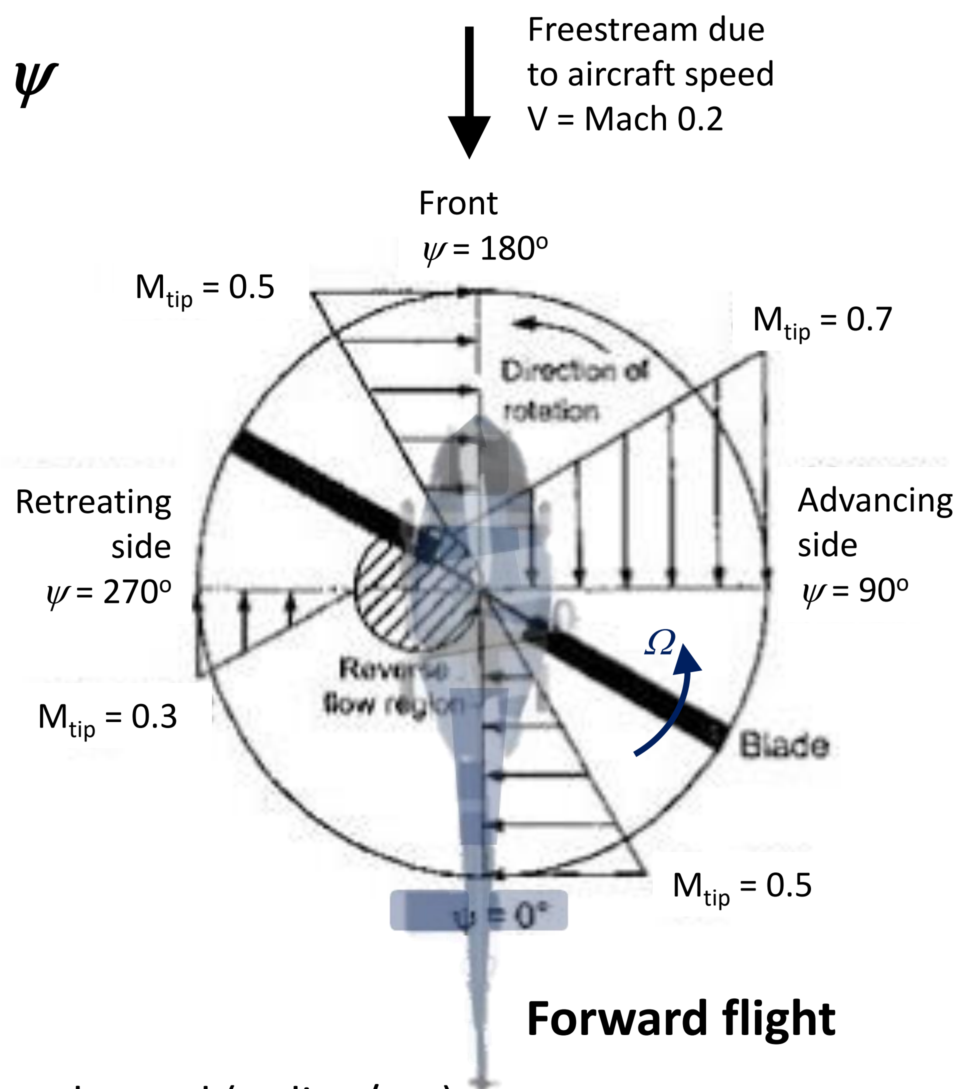


Ωr =Blade velocity at radius location r

Ω = blade rotational speed (radian/sec)

Blade Azimuth Angle ψ





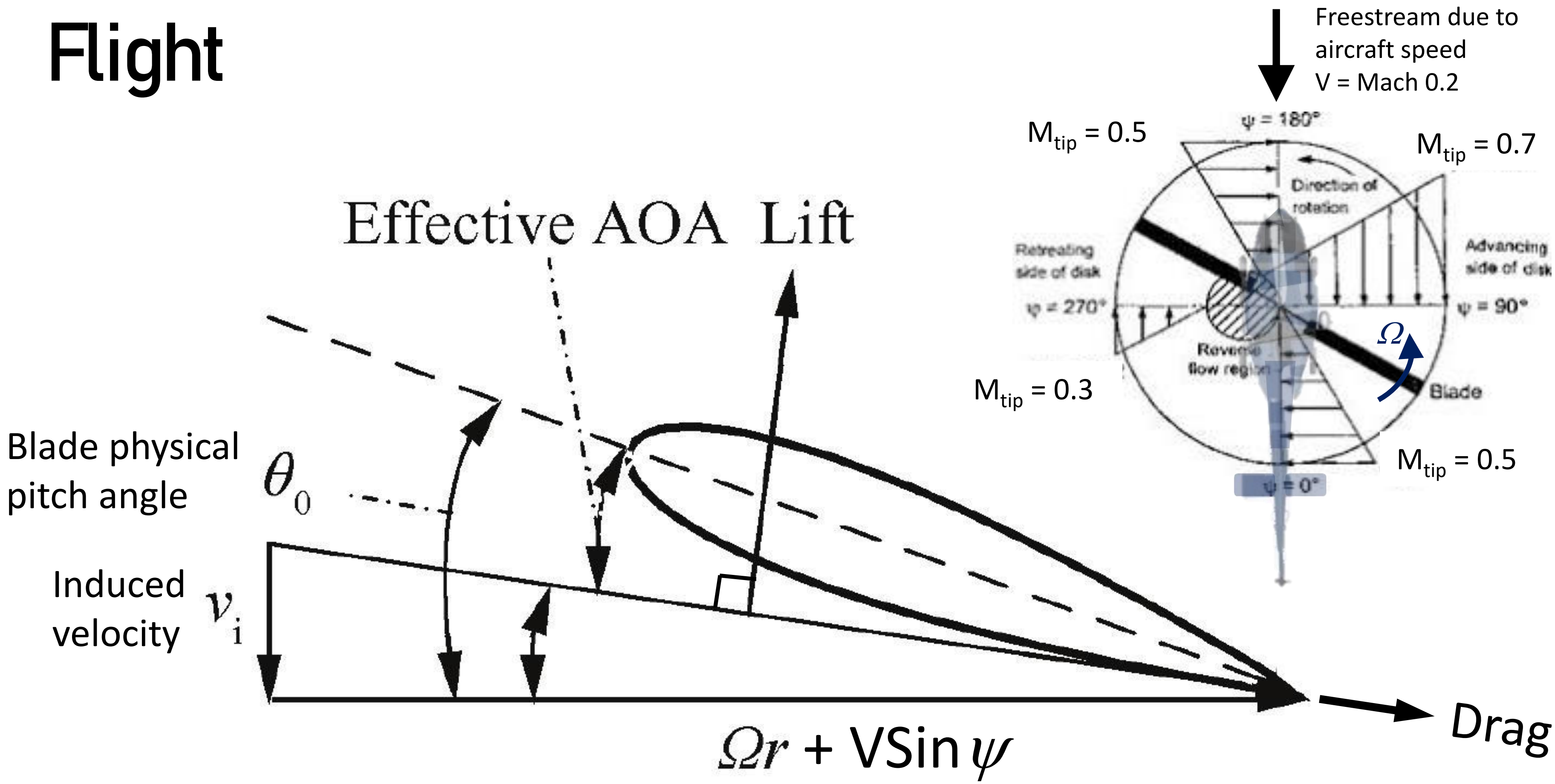
Ω = blade rotational speed (radian/sec)

$\Omega R + V\sin\psi$ = Blade tip velocity (ft/sec or m/sec)

V = aircraft speed

ψ = blade azimuthal angle

Blade Section Aerodynamics in Forward Flight



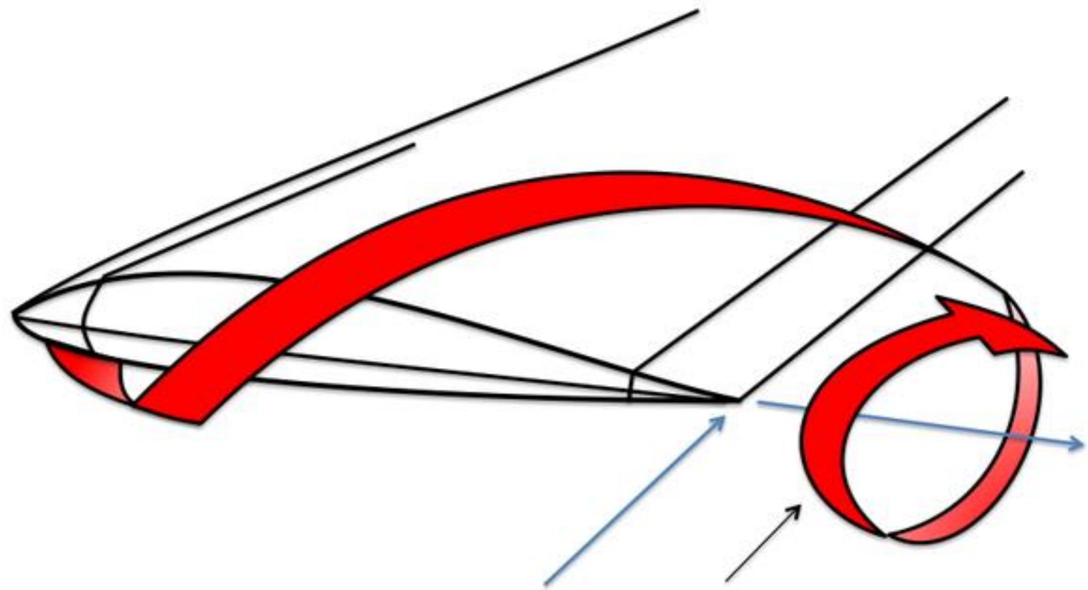
$\Omega\, r$ + sin ψ = Blade velocity at radius location r

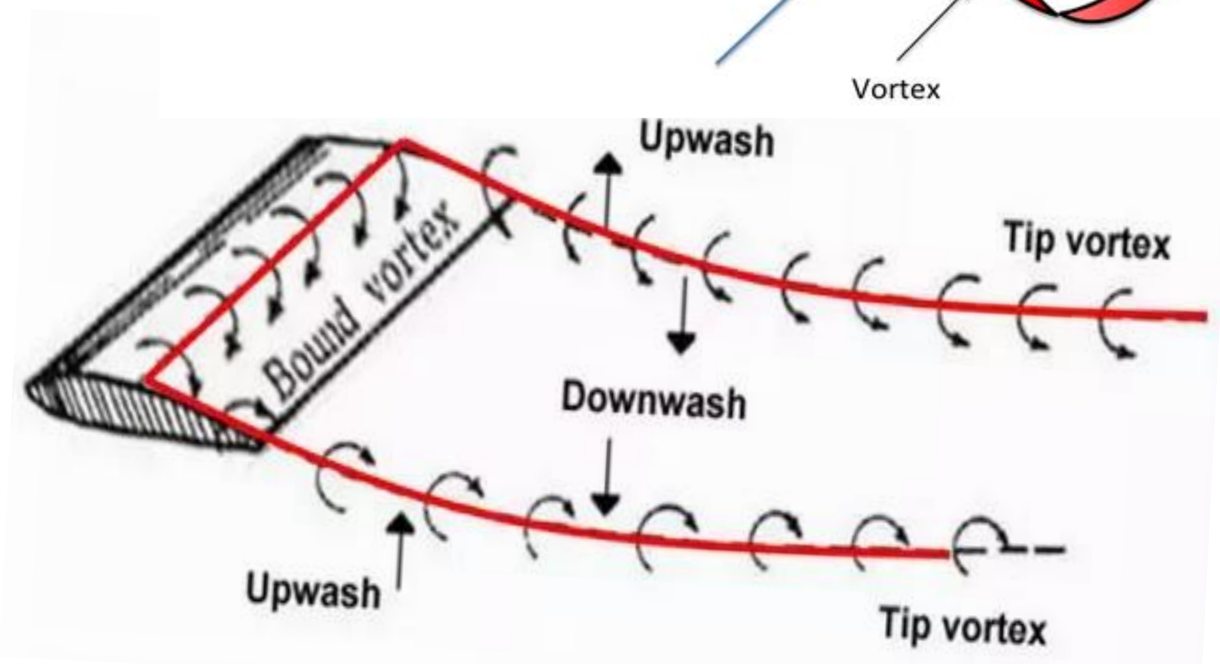
Ω = blade rotational speed (radian/sec)

V = aircraft speed

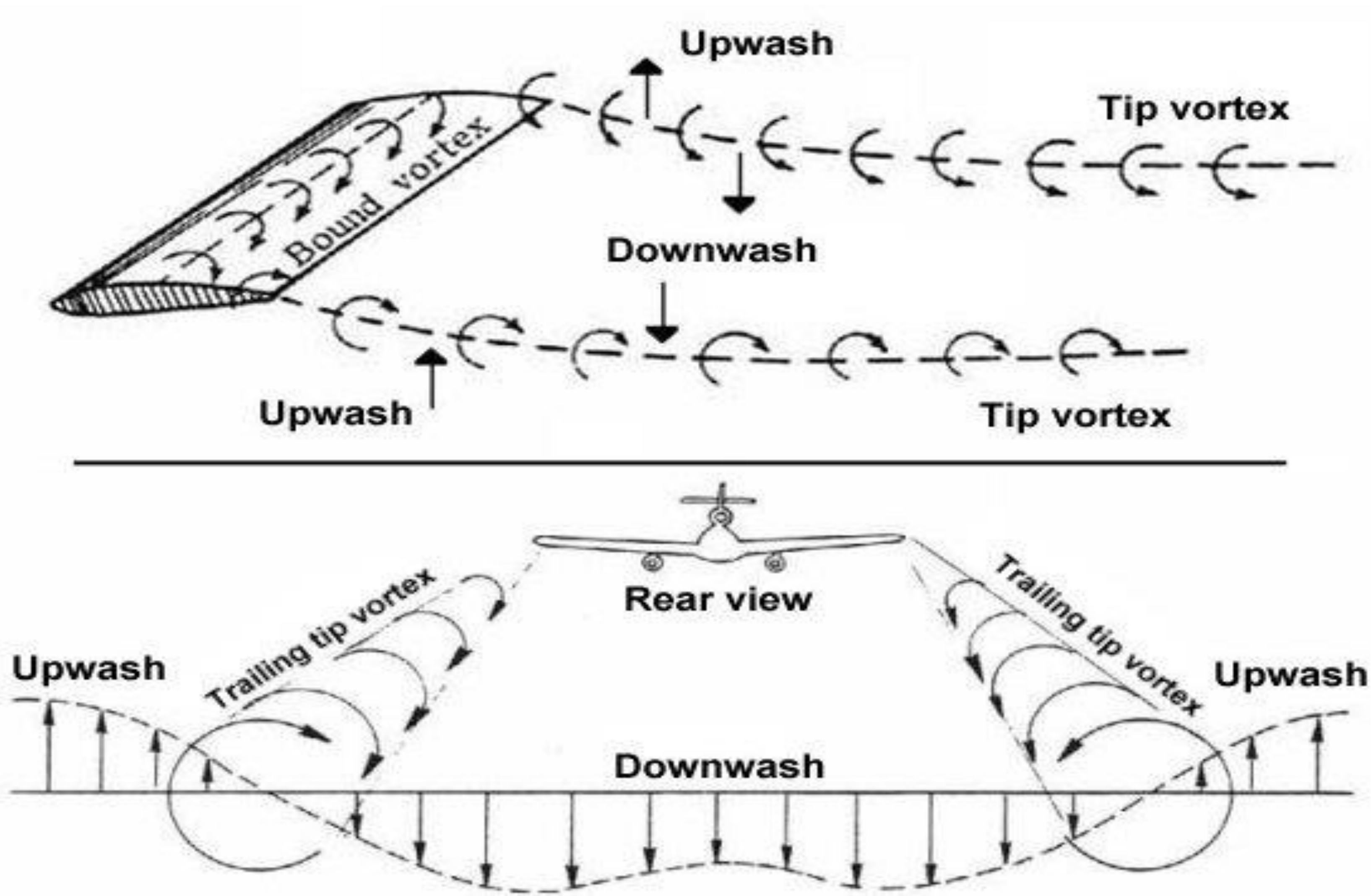
φ = blade azimuthal angle

Tip Vortex (Trailed Vortex)

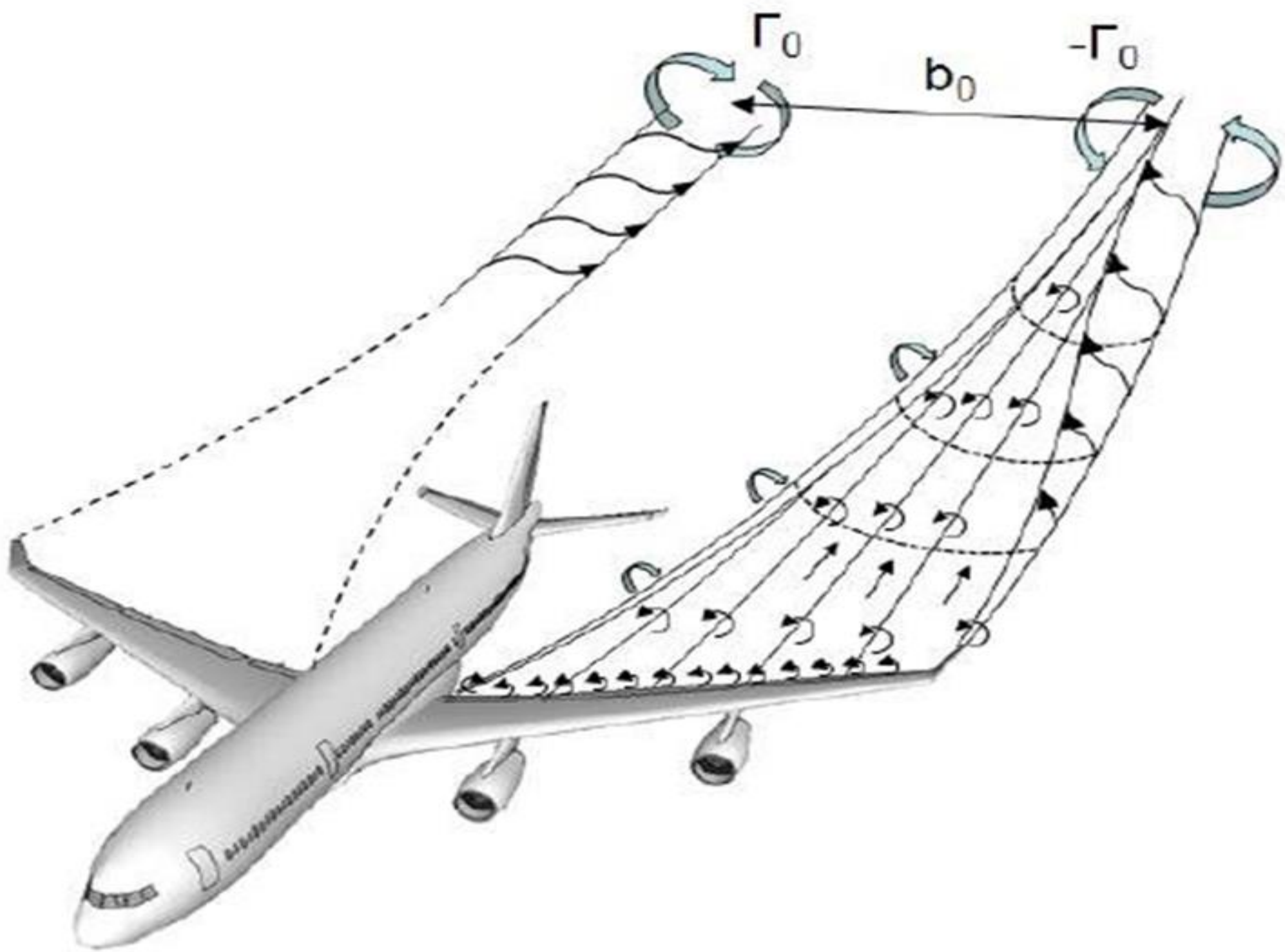




Trailed and Shed (inboard) Vortices



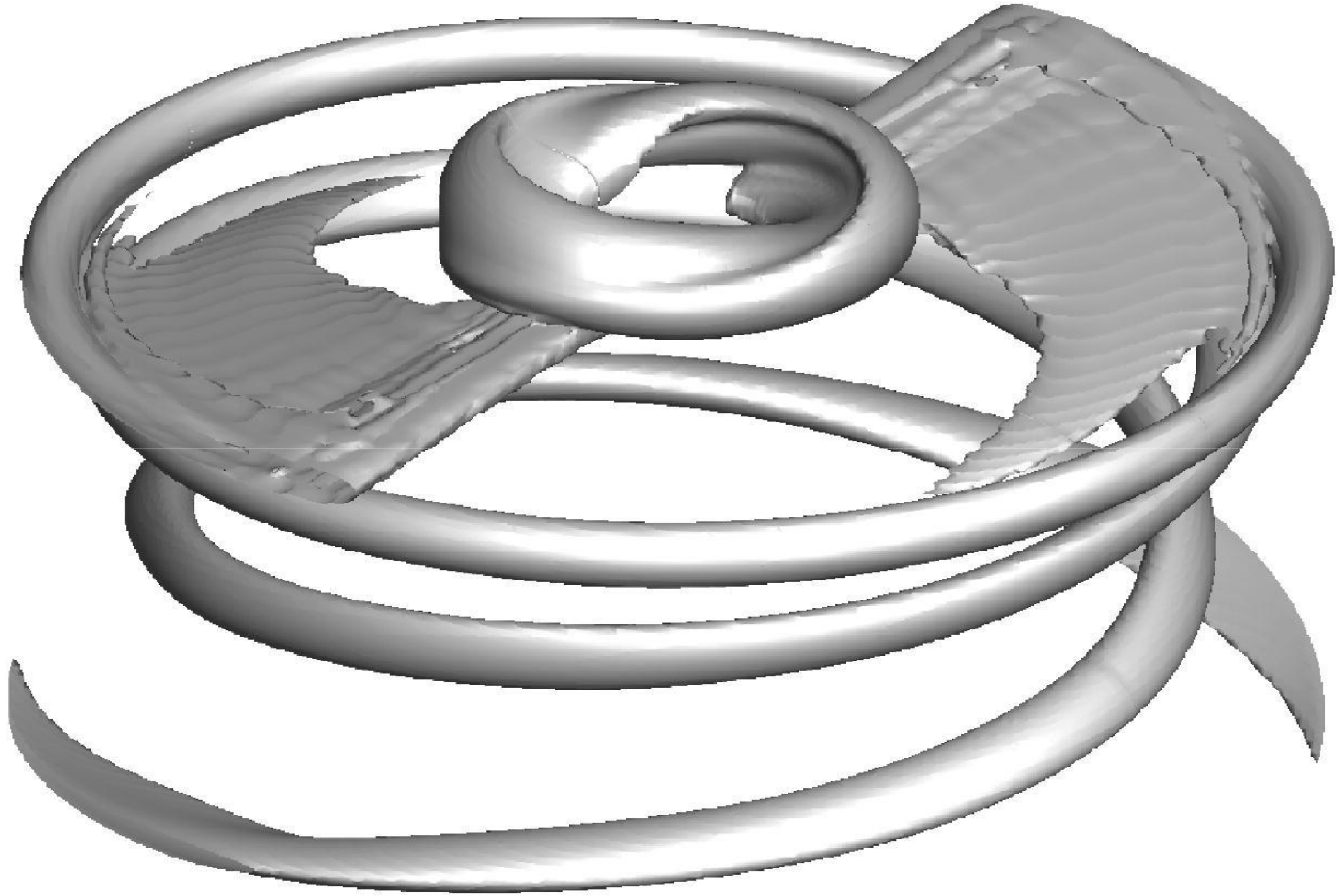
Trailed and Shed (inboard) Vortices



Source: https://www.researchgate.net/figure/Wake-vortex-roll-up-process_fig16_283090470

Helicopter Trailed and Shed Vortices

This rotor spins counter clockwise



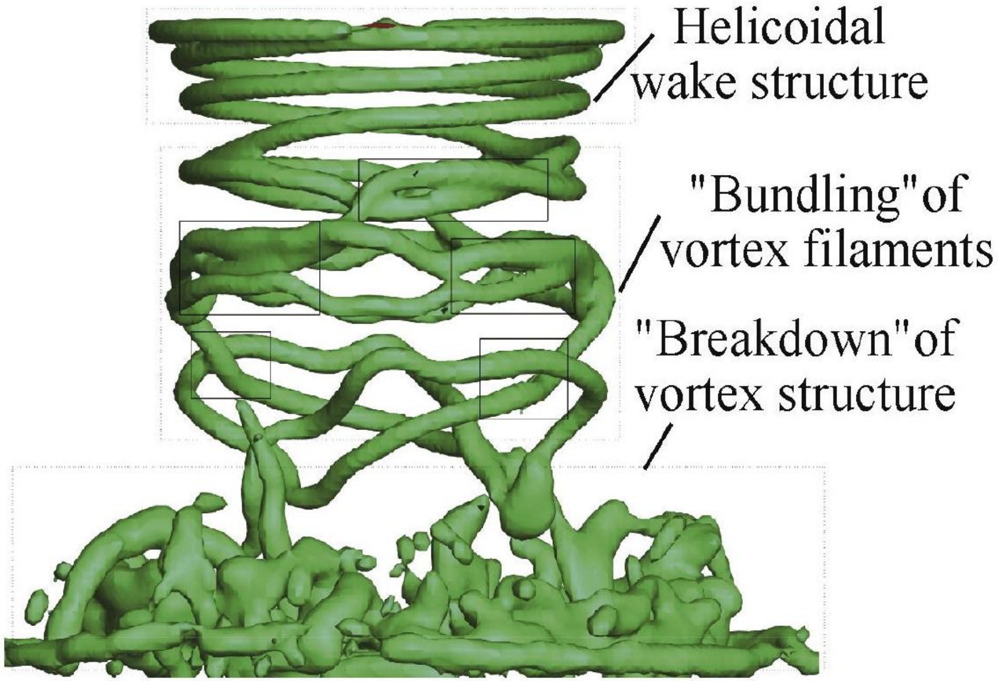
Source: An Eulerian-based CFD module is used to model the blade near body flowfield, and a Lagrangian-based VTM module is employed for vortex tracking in the far wake

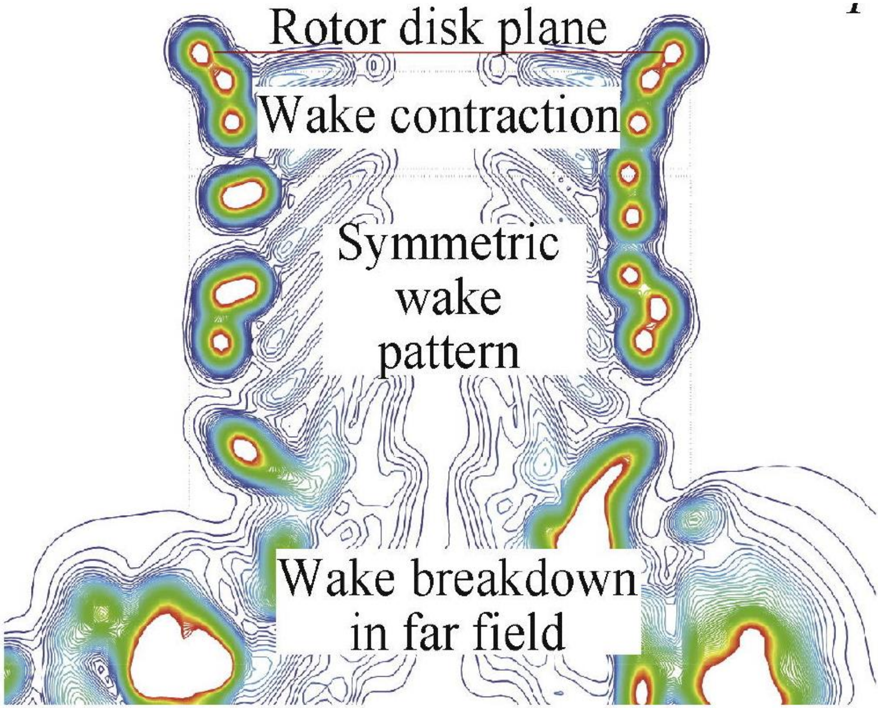
https://www.sciencedirect.com/science/article/pii/S1000936118300761

Helicopter and Propeller Trailed Vortices

Watch this very good video at https://www.youtube.com/watch?v=lslarZiRJhg

Helicopter Rotor Tip Vortex Side View





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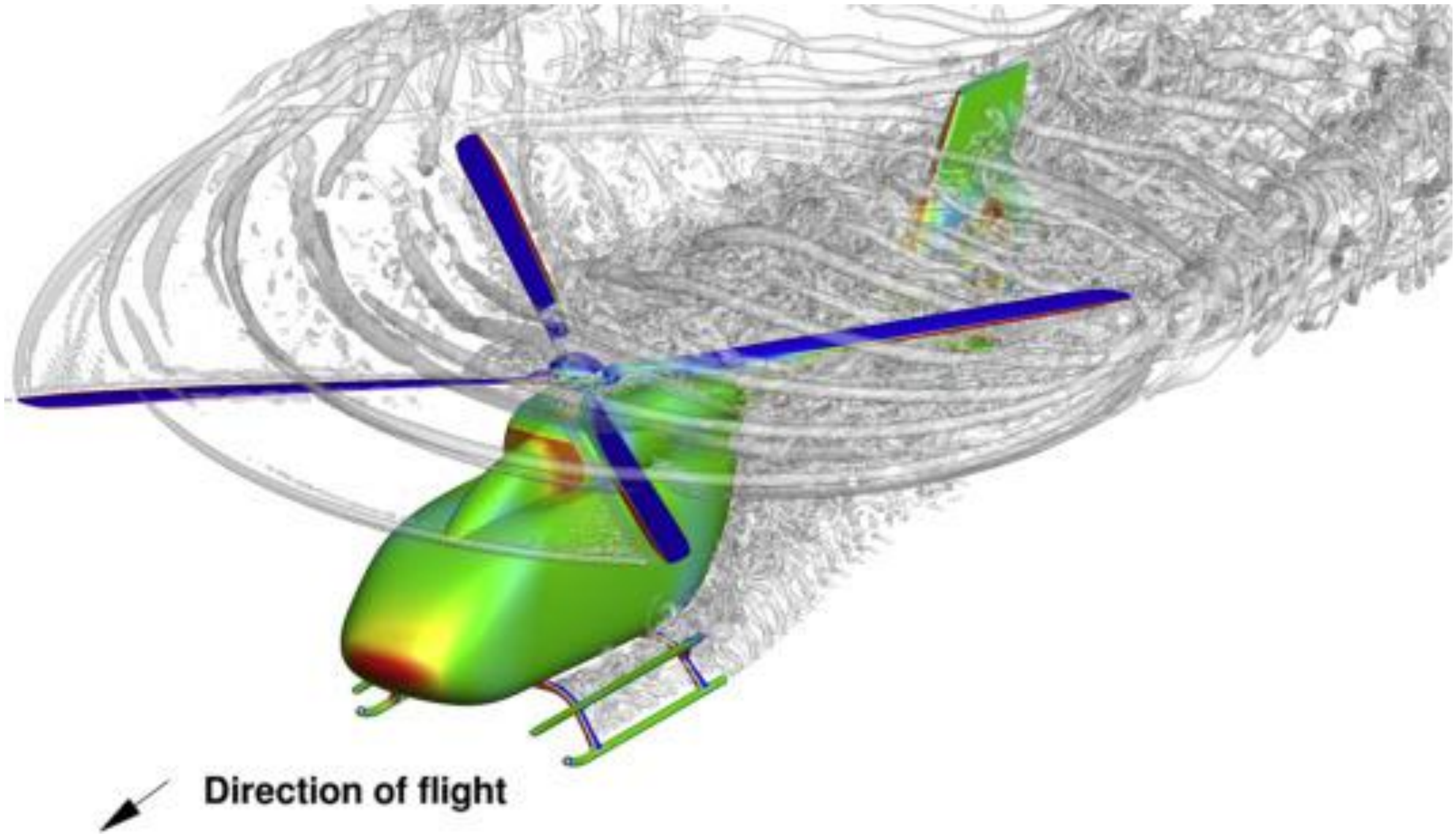
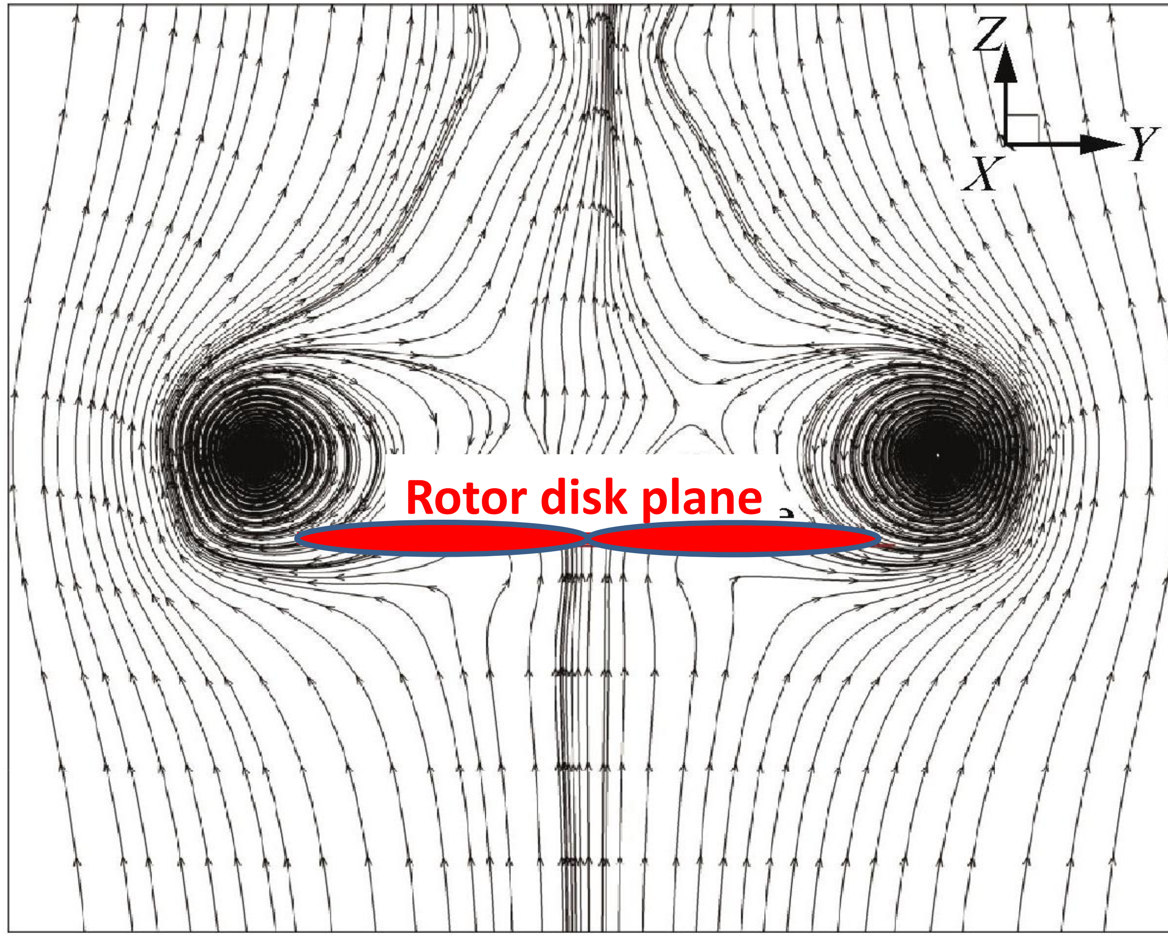
Rotor Tip Vortex in Forward Flight



Source: https://www.sciencedirect.com/topics/engineering/blade-vortex-interaction

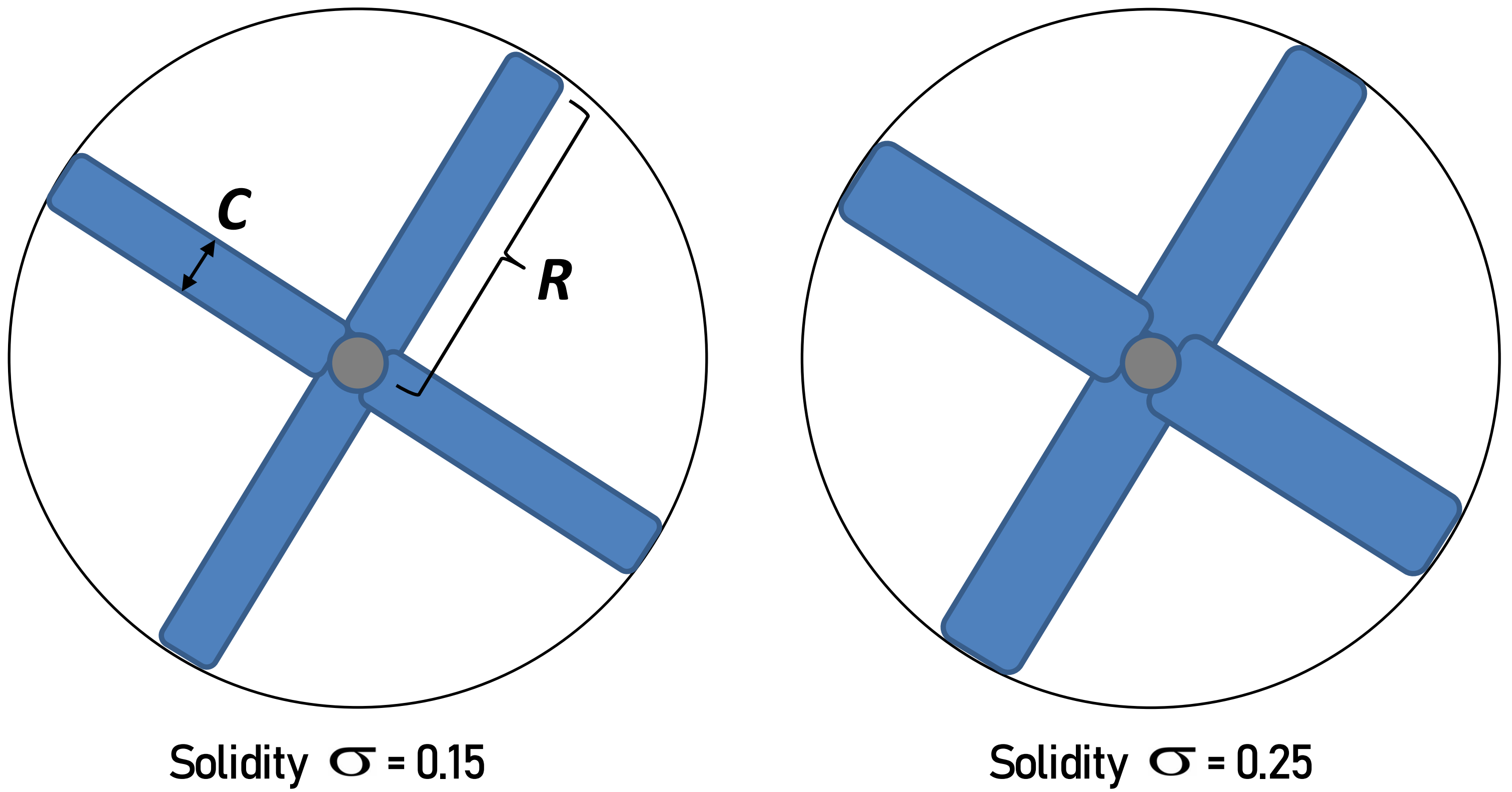
Vortex Ring State During Vertical Descent



Watch this very good video on vortex ring: https://www.youtube.com/watch?v=HjeRSDsy-nE

Source: https://www.sciencedirect.com/science/article/pii/S1000936118300761

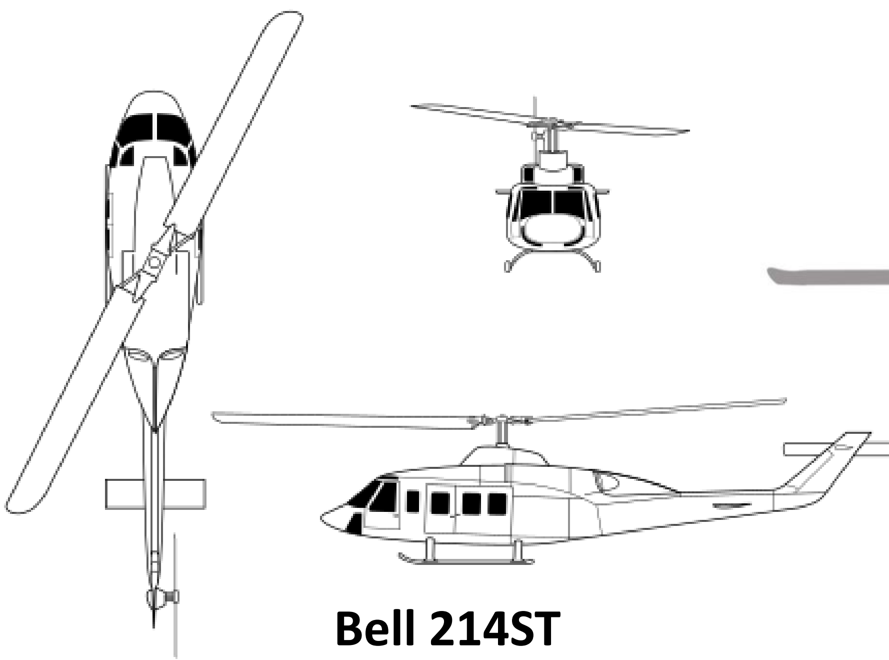
Solidity σ = (N_b x R x C) / rotor disk area
Typical = 0.15 to 0.25



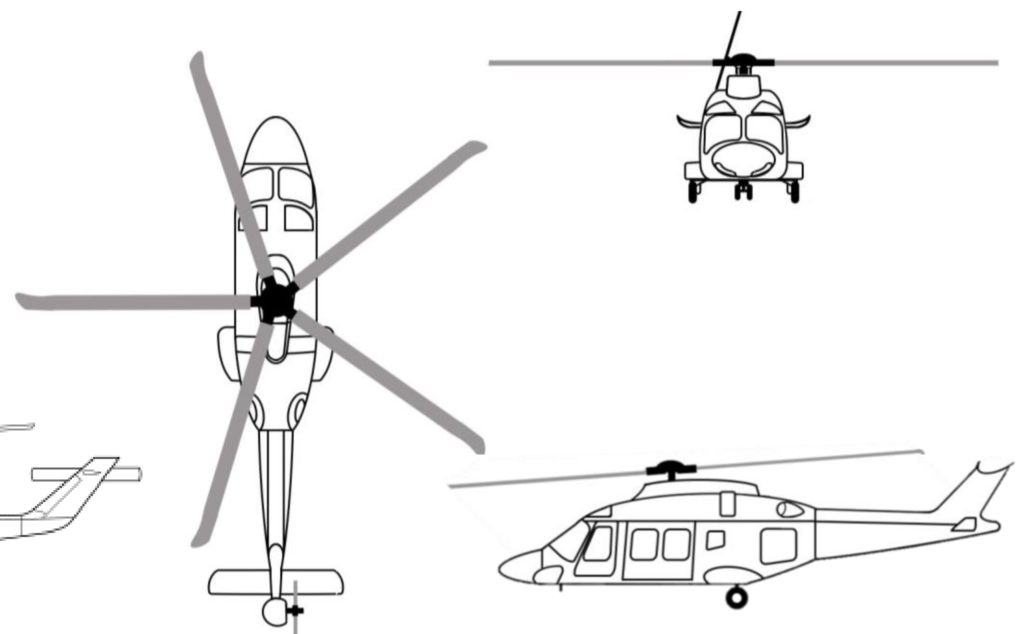
N_b = number of blades
R = radius
C = chord width

Effect of Number of Blades (N_b)

- Try keep **Solidity** constant when trying different N_b
- N_b affects vibration and perceived acoustic noise. Generally higher N_b gives smoother ride and better noise characteristics
- 2-bladed rotors are not axisymmetric and can cause unpleasant 2/rev vibration and distinct impulsive noise



Bell 214ST



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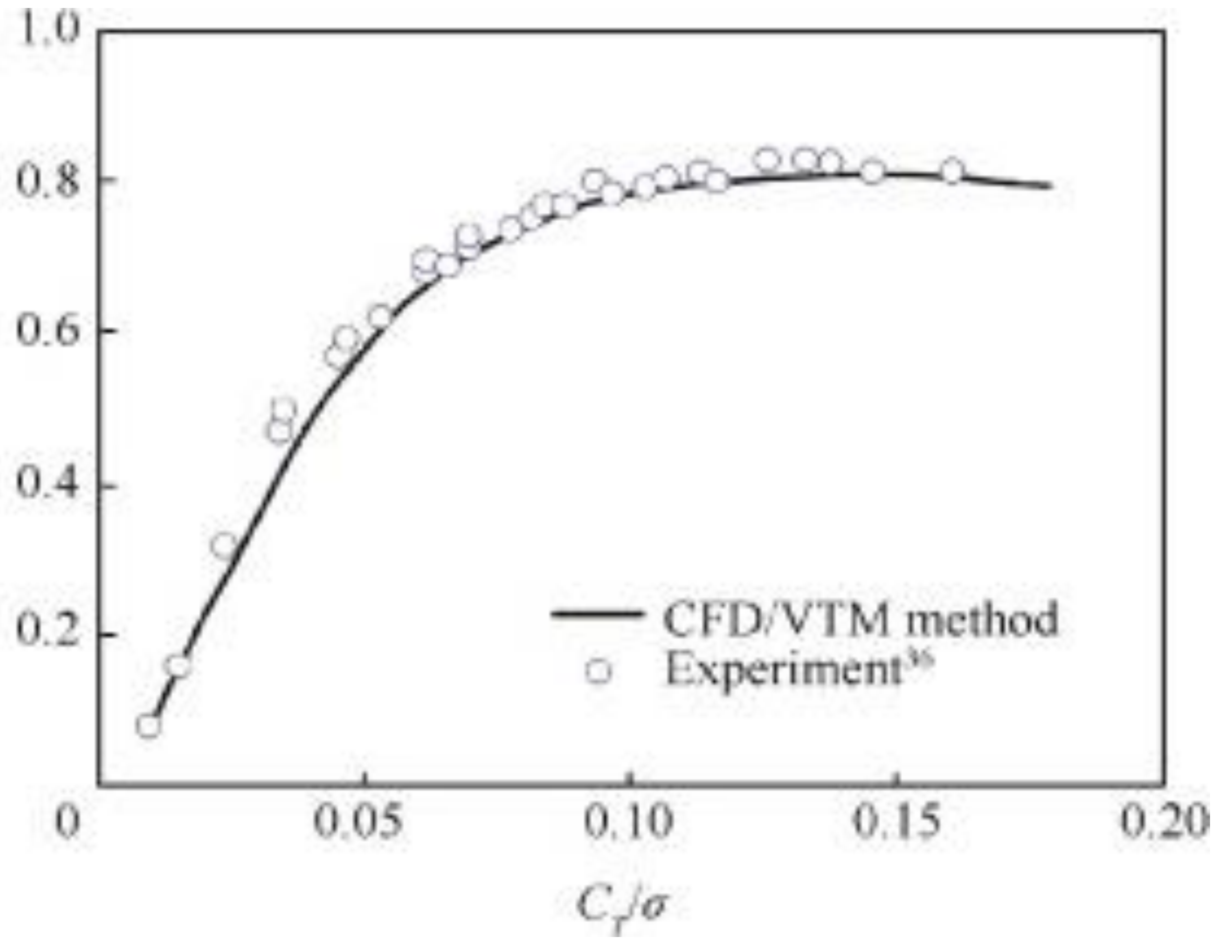
Effect of Number of Blades (N_b)

- Less N_b increases stress on blade grip, pitch link and components
- If variable pitch blade is used, then need to consider N_b carefully
- FAA and EASA regulations require blade to handle bird strike, fewer bigger blades maybe stronger than many smaller blade

Figure of Merit Measure Rotor Aerodynamic Efficiency in Hover

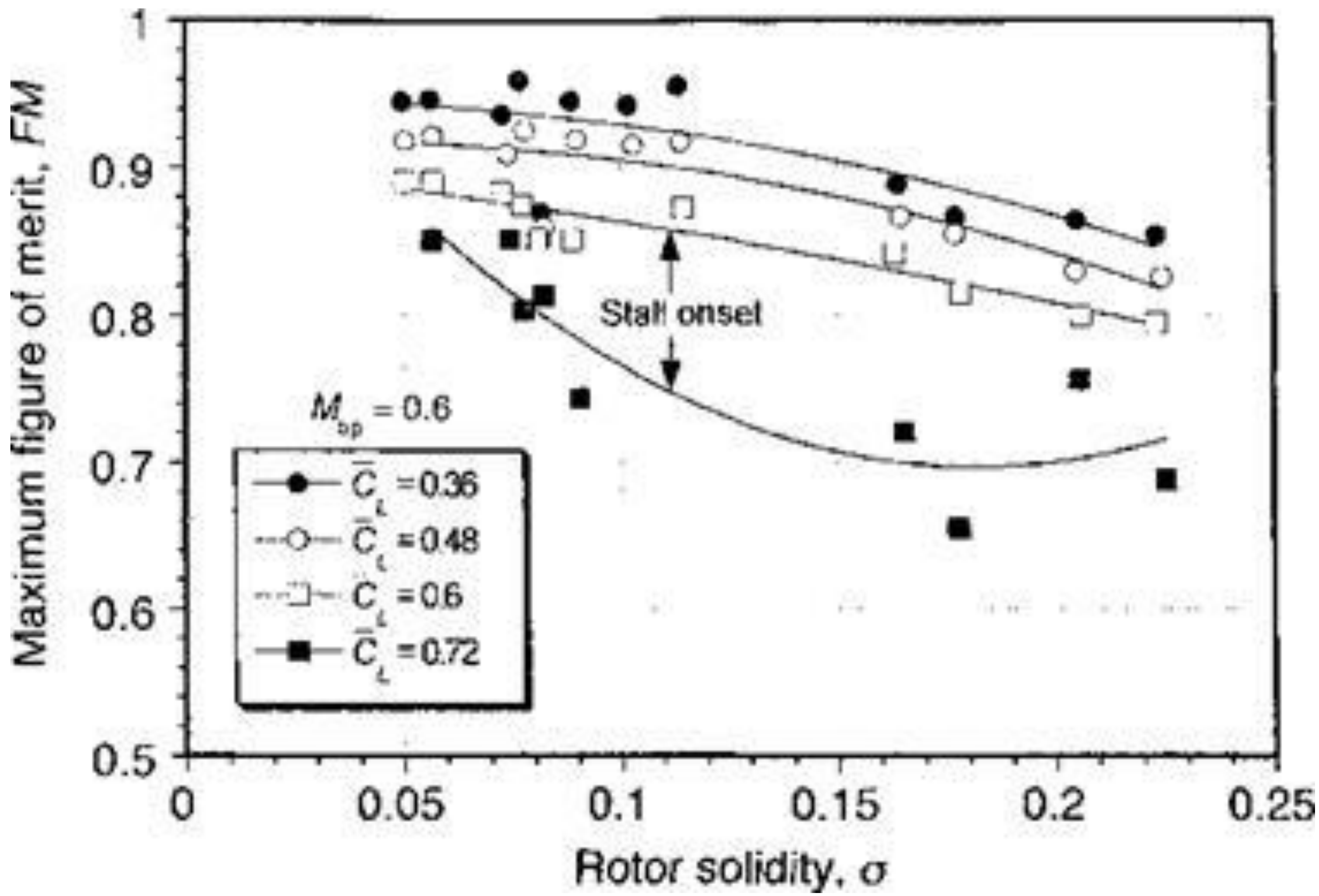
$$M = \frac{\textit{min power required to hover}}{\textit{actual power required to hover}}$$



Source: An Eulerian-based CFD module is used to model the blade near body flowfield, and a Lagrangian-based VTM module is employed for vortex tracking in the far wake

https://www.sciencedirect.com/science/article/pii/S1000936118300761

Lower Solidity Gives Higher Hover Figure of Merit (think of higher aspect ratio wing)



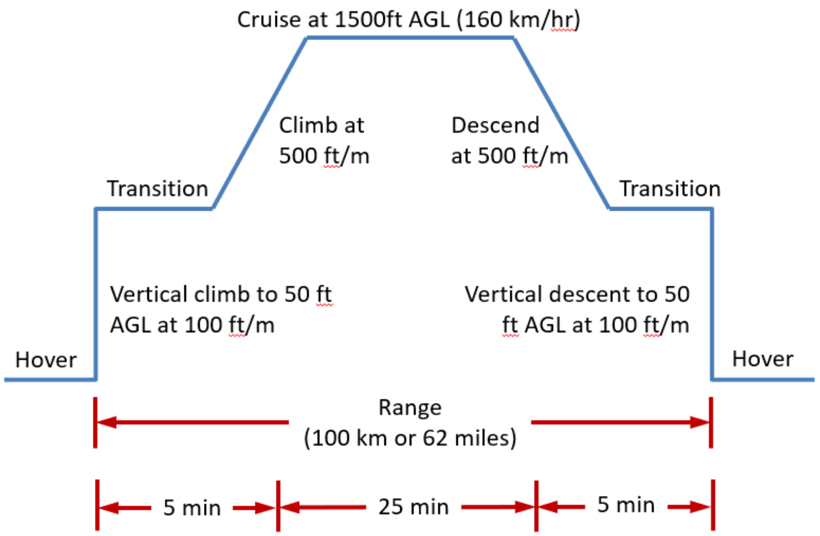
Source: Department of the Army, Engineering Design Handbook (1974)

Can Use Excel or MATLAB for Initial Trade Studies

3 Physics Theory

- Momentum Theory
- Blade Element Theory
- Conservation of Energy

Example Flight Profile



5 Flight Segments

- Power & Energy Requirements of each segment

1 Iterative Process

- Could do by convergence of the battery weight

How it works

Input → Calculations → Energy Consumption → Iteration → Output

Input

- Mission Profile
- Geometrical Parameters
- Performance Parameters

Calculations

- Power Requirement for flight segments
- Aerodynamic Relationship

Energy Consumption

- Main Mission
- Reserve Mission

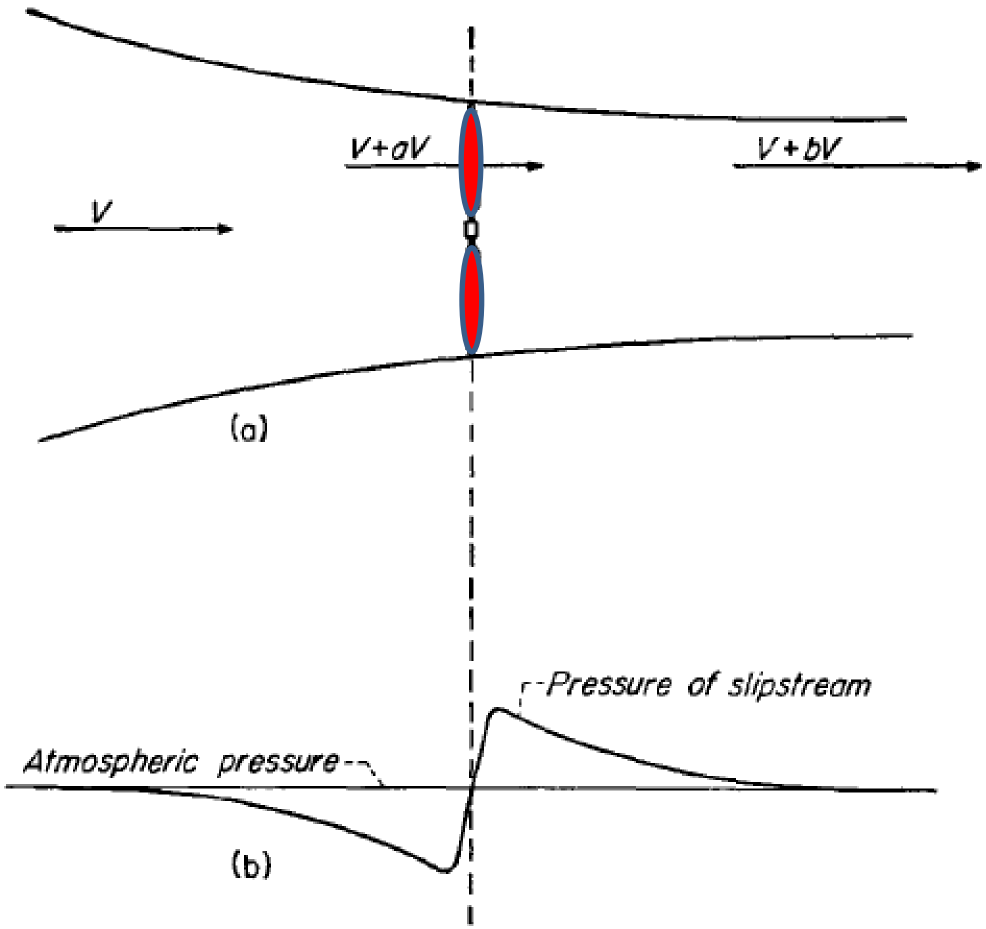
Iteration

- Converged Battery Weight

Output

- Geometrical & Performance Parameters
- Mass Fraction
- Energy Consumption Breakdown
- Battery Energy Fraction

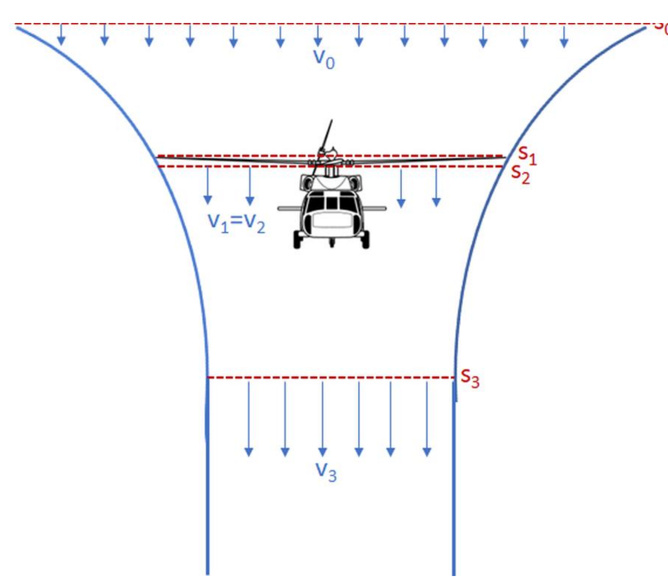
Momentum Theory



Airscrew in forward flight.
(a) Airstream velocities
(b) Pressure distribution

aV = increase in velocity at the disk

bV = increase in velocity over free air at an infinitely large distance downstream

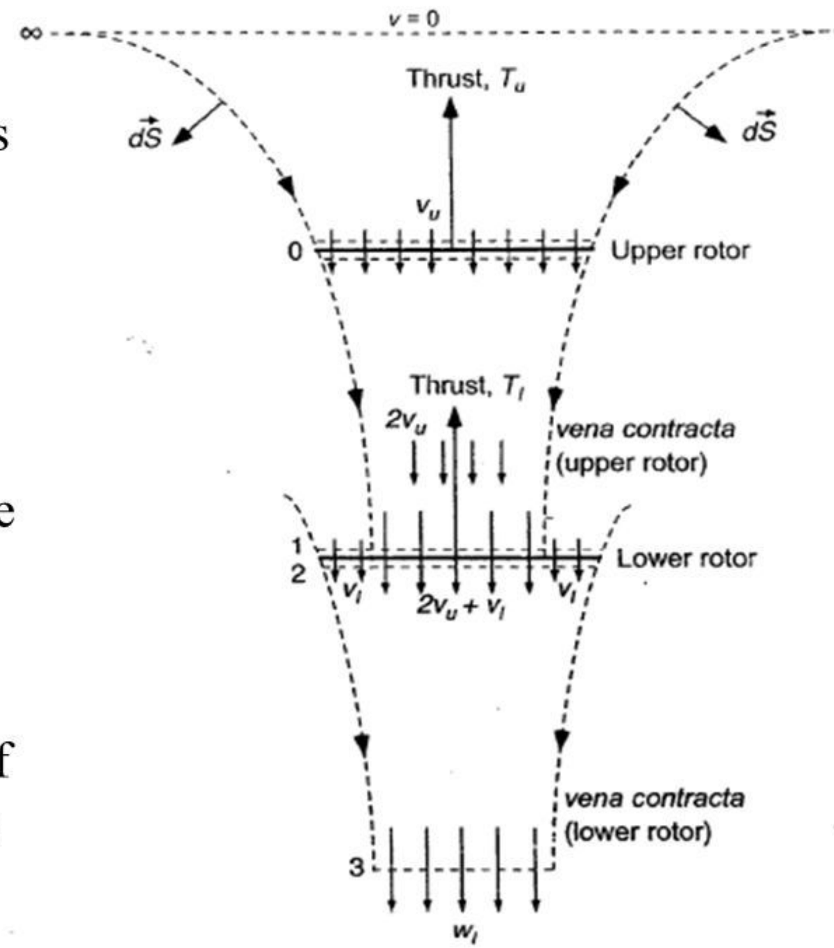


Propeller/rotor travel direction

Source: https://www.spinningwing.com/the-helicopter/momentum-theory/

Momentum Theory for Coaxial Rotors

- Based on ideal flow considerations, this means that only half of the area of the lower rotor operates in an effective climb velocity induced by the upper rotor.
- This problem can be tackled by means of the simple momentum theory and the application of the mass, momentum, and energy conservation equations in integral form.
- We will assume that the performance of the upper rotor is not influenced by the lower rotor.



Source: https://slideplayer.com/slide/11813562/

Momentum Theory

Simple momentum theory assumes:

(1) The rotor is made up of infinite number of blades and can be considered as an actuator disk, uniformly accelerating the air through the disk with no loss at the blade tips.

(2) The power required to produce thrust is represented by the axial kinetic energy imparted to the air composing the slipstream. A frictionless fluid is assumed, hence there is no blade friction, profile-drag losses and rotational energy imparted to the slipstream is ignored.

(3) The disk is infinitely thin so that no discontinuities in velocity occur on the two sides of the disk.

Since work done per unit time by the thrust of the airscrew must be equals to the increase in KE of the slipstream per unit time:

$$T(V + aV) = \Delta K.E. \text{ – } \mathbf{(1)}$$

Taking A as the area of disk and ρ as mass density of air, thrust developed by airscrew can be expressed as change of the axial momentum of air in unit time:

$$T = [\rho A(V + aV)]bV \text{ – } \mathbf{(2)}$$

The increase in KE of the air can be expressed as:

$$\Delta K.E. = \frac{1}{2}\rho A(V + aV)[(V + bV)^2 - V^2] \text{ – } \mathbf{(3)}$$

Substituting (2) and (3) into (1):

$$[\rho AV(1+a)bV][V(1+a)] = \frac{1}{2}\rho AV^3(1+a)(b^2+2b)$$

Solving the above for b in terms of a:

b = 2a **– (4)**

For hovering,

$$V = 0$$

Hence, increase in velocity at the rotor disk aV equals to the total velocity through the disk, or induced velocity υ.

From (4), the increase in velocity downstream from the disk, bV, will be 2υ.

Hence, with R being the rotor radius, the thrust T of the hovering rotor may be expressed using (2) as

$$T = (\rho \pi R^2 \upsilon) 2\upsilon \text{ – (5)}$$

Solving for υ**,**

$$\upsilon = \sqrt{\frac{T}{2\rho\pi R^2}} \text{ – (6)}$$

Rotor Figure of Merit for Hover

M is called the rotor figure of merit where

$$M = \frac{min\,power\,required\,to\,hover}{actual\,power\,required\,to\,hover} = \frac{Tv}{P} \quad \text{– (7)}$$

Substituting (6) into (7):

$$M = \frac{1}{\sqrt{2}}\frac{T}{P}\sqrt{\frac{T}{\rho\pi R^2}} \quad \text{– (8)}$$

Note T = W

$$Disk\ loading = \frac{W}{\pi R^2}$$

$$Power\ loading = \frac{W}{P}$$

In terms of disk loading, D.L., and power loading, P.L., (8) can be expressed as

$$M = \frac{1}{\sqrt{2}}P.L.\sqrt{\frac{D.L.}{\rho}} \quad \text{– (9)}$$

Nondimensional Figure of Merit

As (8) is not expressed in terms of nondimensional quantities, the following coefficients are introduced:

$$\left.\begin{aligned} T &= C_T \pi R^2 \rho (\Omega R)^2 \\ Q &= C_Q \pi R^2 \rho (\Omega R)^2 R \\ P &= C_p \pi R^2 \rho (\Omega R)^3 \end{aligned}\right\} \quad \textbf{(10)}$$

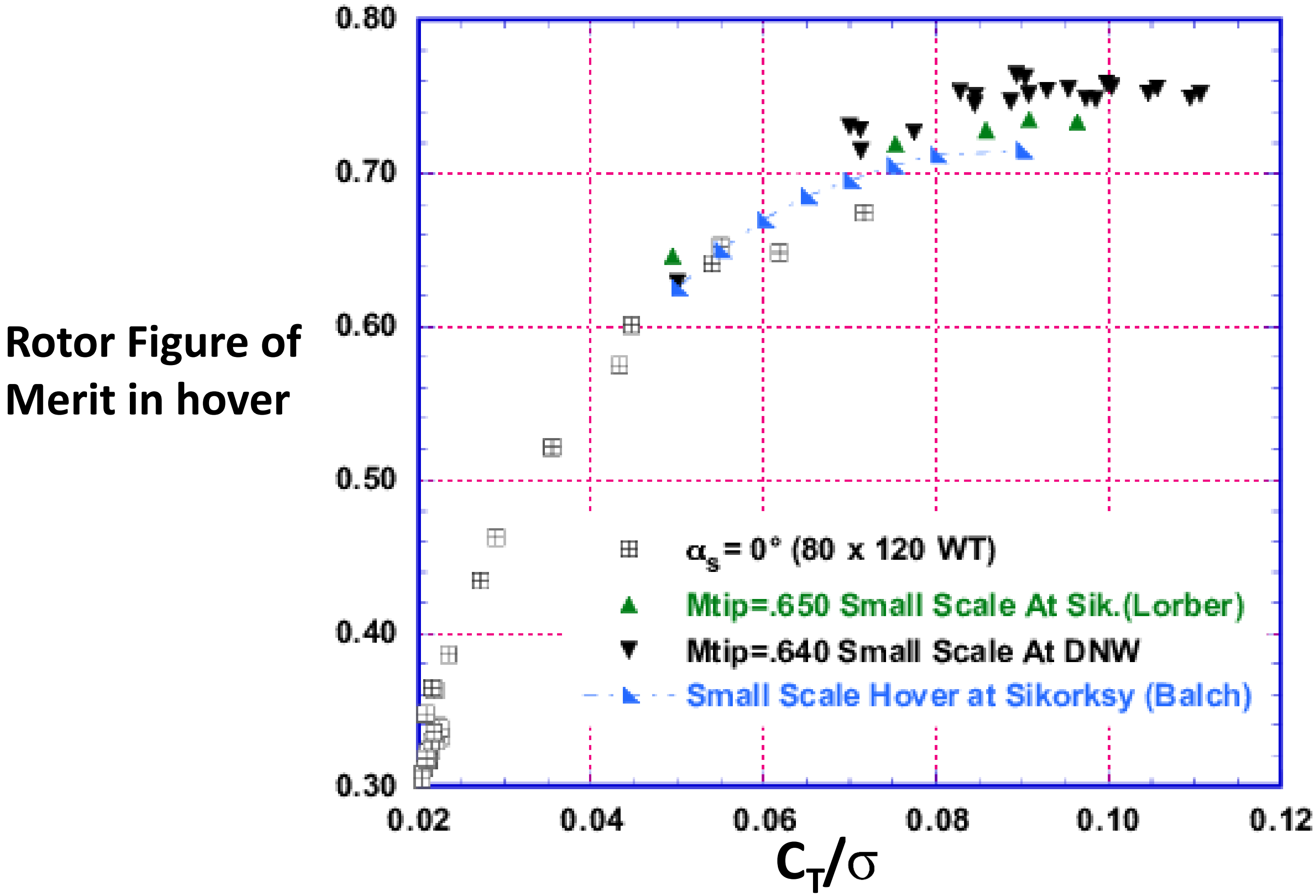
Note: $C_Q = C_p$

Substituting (10) into (8) :

$$M = \frac{1}{\sqrt{2}} \frac{C_T \pi R^2 \rho (\Omega R)^2}{C_p \pi R^2 \rho (\Omega R)^3} \sqrt{\frac{C_T \pi R^2 \rho (\Omega R)^2}{\pi R^2 \rho}} = \frac{1}{\sqrt{2}} \frac{{C_T}^{3/2}}{C_Q}$$

Example: Figure of Merit for Black Hawk Helicopter Rotor at Different C_T/σ

Today M=0.78 to 0.80 is consider good



Hovering Power Required for eVTOL

By rearranging equation **(8)**, you will get a relationship for power required to hover (P) as a function of total rotor thrust (T_{VTOL}), total rotor disk area (S_{Disk}), air density ρ, and Rotor Figure of Merit (M).

Based on previous slide, one can pick a Figure of Merit around 0.78.

$$P = \frac{(T_{VTOL})^{\frac{3}{2}}}{\sqrt{2 \cdot \rho \cdot S_{Disk}}} \cdot \frac{1}{M}$$

Very useful equation

The beauty of the momentum theory is it is very easy to use. For example, if your eVTOL aircraft has 6 hovering rotors, then S_{disk} = the total rotor disk areas for all 6 rotors.

Example, Calculate Power Required for Bell Nexus to Hover (assume no duct benefit)

G = gravity = 9.8 m/sec^2

MTGW = 3175 kg

T_{VTOL} = 3175 kg x 9.8 m/sec^2

Rotor diameter = 8 feet = 2.438m

S_{disk} = 6 x π R2 = 301.6 ft^2 = 28.02m^2

Air density ρ = 1.225 kg/m^3

M = 0.78

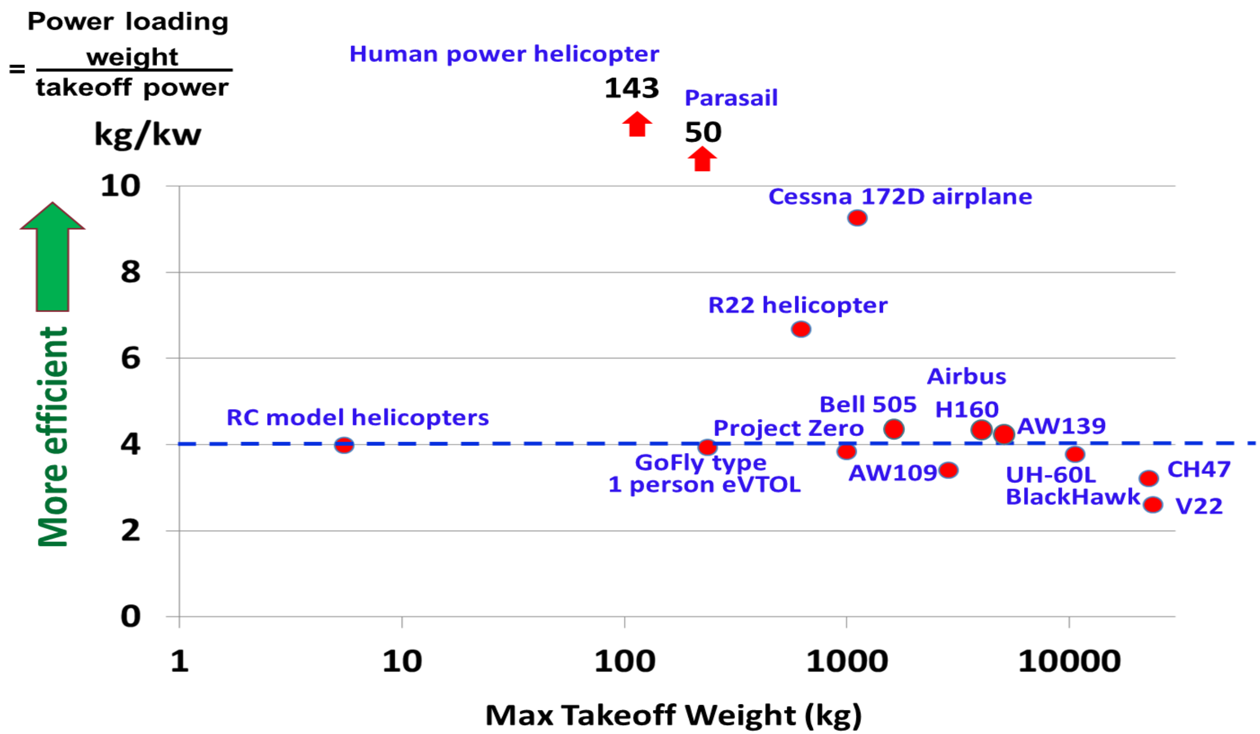
$$P = \frac{(T_{VTOL})^{\frac{3}{2}}}{\sqrt{2 \cdot \rho \cdot S_{Disk}}} \cdot \frac{1}{M}$$

= 849,262 kg·m^2/ sec^3

= **849 kw = 1138 hp**

Tip: always do a dimensional analysis

Always Do a Sanity Check of the Answer



The above chart shows most helicopters have a power loading of 4kg / kw for taking off. Therefore, a 3,175 kg VTOL aircraft would require 3175kg/(4 kg/kw) = **794 kw** power to hover.

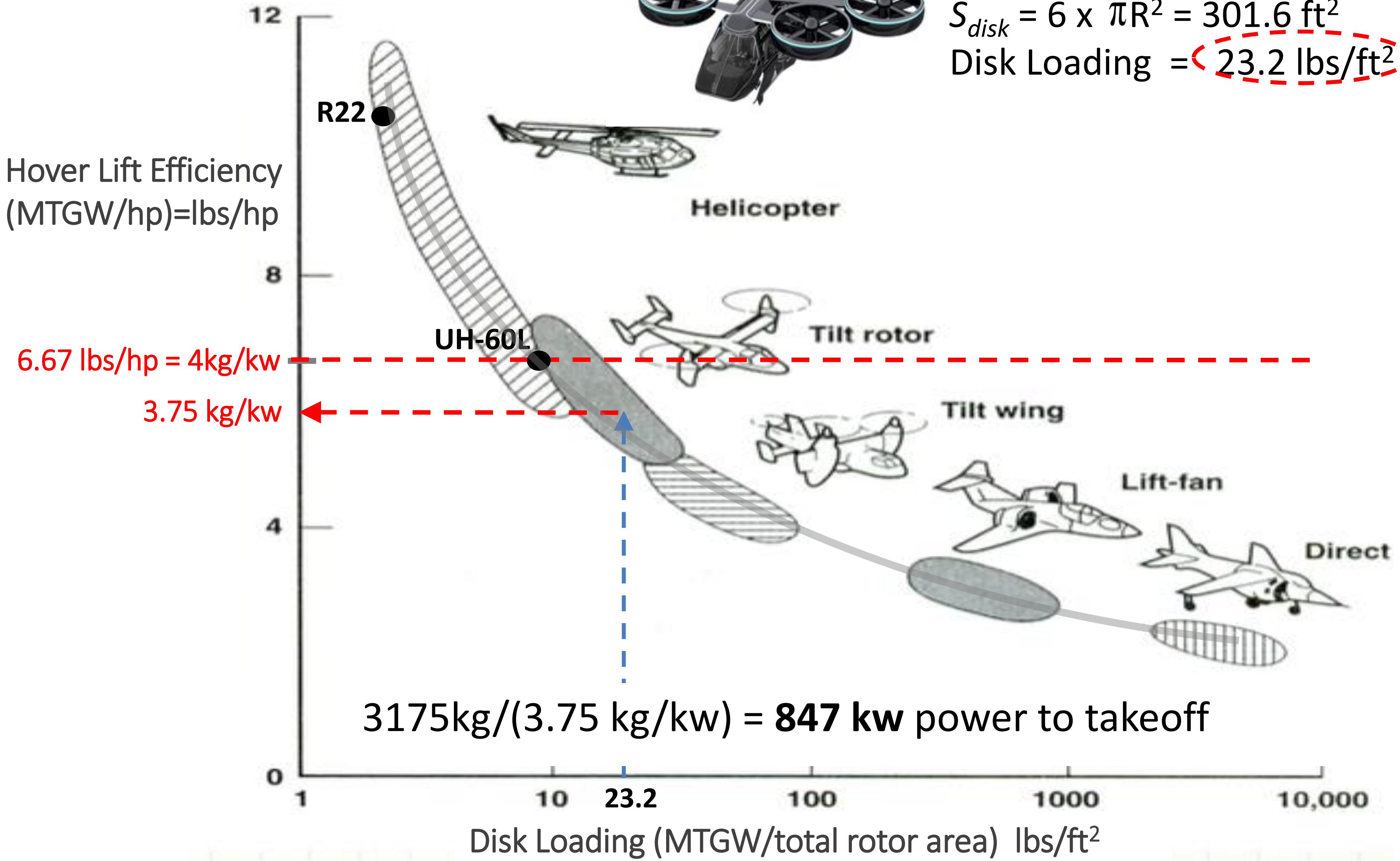
This is only true if the disk loading is around 10 lbs/ft^2 like helicopters!

Let's Check Again

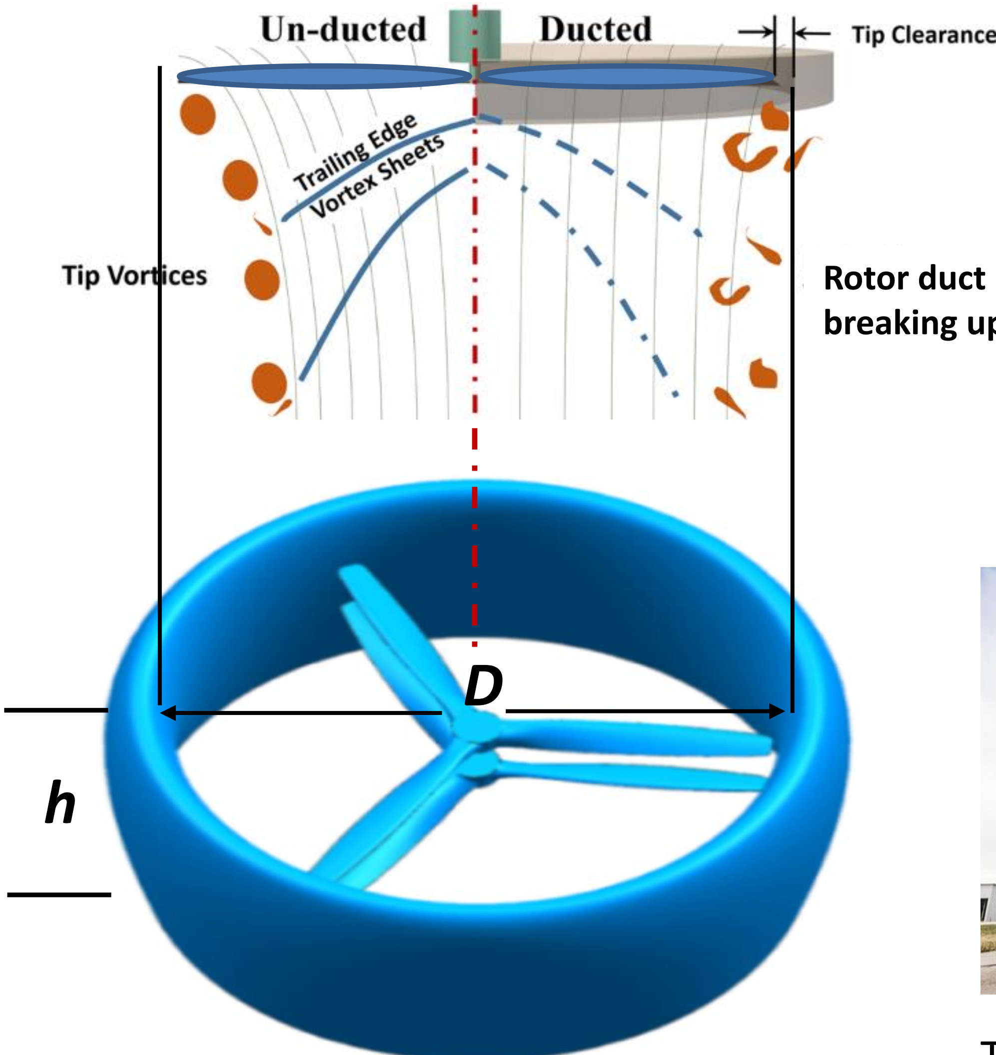
MTGW = 3175 kg = 7000 lbs

S_{disk} = 6 x πR^2 = 301.6 ft^2

Disk Loading = 23.2 lbs/ft^2



A Duct Increases Thrust in Hover ?



Rotor duct helps by breaking up tip vortices

The duct musts be tall and with a full lip to be aerodynamically beneficial.

The ducts on Airbus CityAirbus are more for safety and for supporting the motors, and not for increasing thrust.

To Add a Rotor Duct or Not?

For a properly designed duct, the height *h* of the duct should be much > 0.4*D* and the lip should be round and full, and not sharp. Then you may get >20% or more thrust for the same amount of power, or same thrust but requiring less power. A rounded lip also provides upward suction. In forward flight, a duct adds drag but can generate lift like a wing if there is an incidence angle.

I have tested with and without duct. In general, it is complex to design an excellent duct; duct adds extra weight and reverberates to create more noise. A ducts can provide safety protection or structure support. I recommend conduct a CFD trade study before making a decision.

Asymmetric Duct Airfoil for Cruise

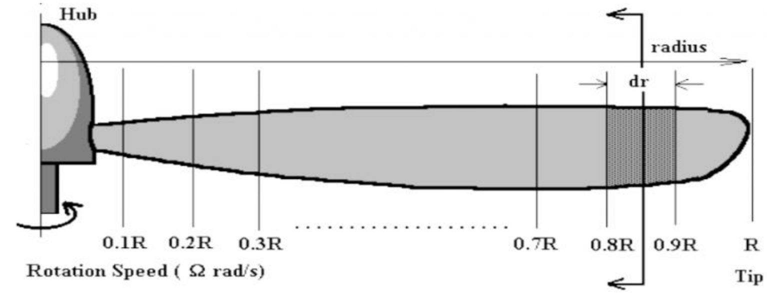
- Duct can add 700 lbs thrust out of baseline 3200 lbs in hover, But only 2% time in hover and has to carry 150 lbs of weight and very high wetted and form drag in cruise. L/D is very low.

- Could schedule the duct incidence angle to get lift during Vbr.

Momentum Theory Limitations

- Momentum Theory provides no information on how rotor blades should be designed to produce a given thrust.
- It does not account for rotor rpm, number of blades, nor blade area.
- Profile-drag losses are ignored.
- Blade Element Theory provide ways to reduce these limitations.

Blade Element Theory



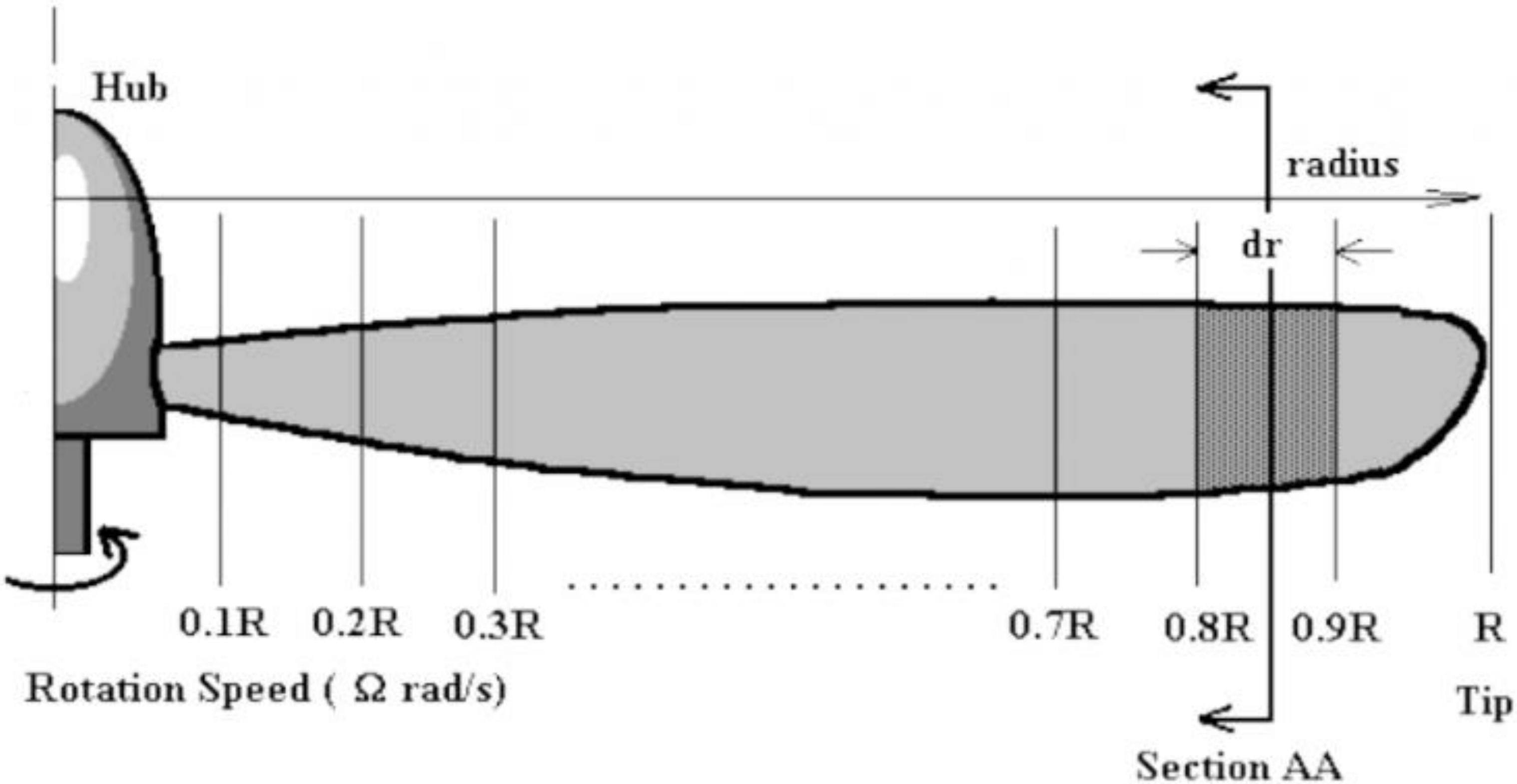
- Blade Element Theory is a relatively simple method to predict the performance of a propeller.

- Each element of a propeller is considered an airfoil segment that follows a helical path.

- Lift and drag are calculated from the resultant velocity acting on the airfoil, with each element being independent from the adjoining elements.

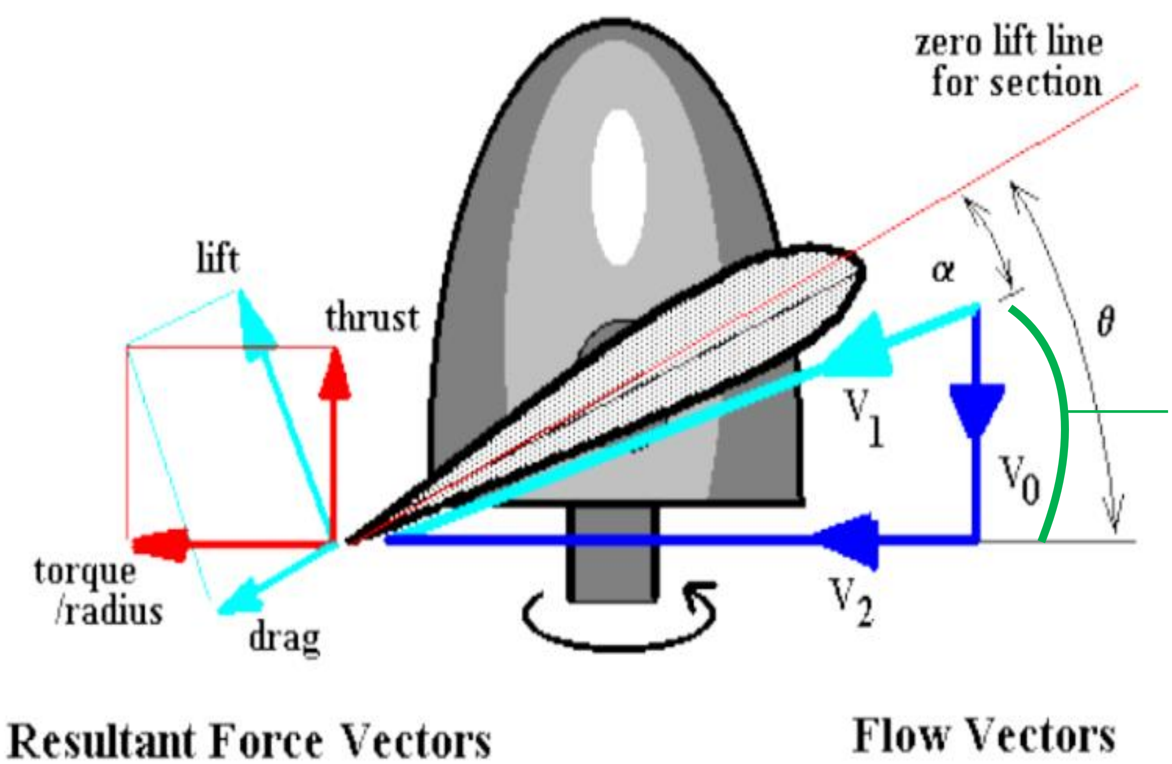
- The thrust and torque of the propeller are obtained by integrating the individual contribution of each element along the radius.

Blade Element Theory



Source: *http://www.aerodynamics4students.com/propulsion/blade-element-propeller-theory.php#:~:text=Blade%20Element%20Theory%20for%20Propellers,use%20of%20Blade%20Element%20Theory.&text=The%20resulting%20values%20of%20section,overall%20performance%20of%20the%20propeller.*

Thrust at section AA (radius = r) shown in the previous slide, the flow on the blade would consist of the following components.



The difference in angle between thrust and lift directions is defined as
$\phi = \theta - \alpha$ (inflow angle of attack)

V_0 -- axial flow at propeller disk, V_2 -- Angular flow velocity vector

V_1 -- section local flow velocity vector, summation of vectors V_0 and V_2

Source: *http://www.aerodynamics4students.com/propulsion/blade-element-propeller-theory.php#:~:text=Blade%20Element%20Theory%20for%20Propellers,use%20of%20Blade%20Element%20Theory.&text=The%20resulting%20values%20of%20section,overall%20performance%20of%20the%20propeller.*

Let's look at one rotor blade first

For differential lift dL on a rotor blade elements operating at a distance r from the axis of rotation and rotating with an angular velocity Ω :

$dL = c_l(\text{c} \;\; dr)(\frac{1}{2}\rho\, {V_R}^2)$ where V_R is the resultant velocity **– (11)**

For simplification:

$$\left.\begin{aligned} &\sin\varnothing = \varnothing \\ &\cos\varnothing = 1 \\ &V_R = \Omega r \end{aligned}\right\} \quad (12)$$

Blade-element lift coefficient may be expressed as

$c_l = a\,\alpha_r = a(\theta - \varnothing)$ **– (13)**

Where a is the slope of the curve of section lift coefficient against angle of attack

Substitution of (12) and (13) into (11) gives the differential lift or thrust expression:

$dT = dL = \frac{1}{2}\rho\,(\Omega \text{r})^2 a(\theta - \varnothing)c\; dr$ **– (14)**

For simplicity of integration, it is assumed that pitch angle of a blade element will vary with its radial position r by $\theta = \theta_t \frac{R}{r}$ **– (15)**

Where θ_t is the pitch angle at the blade tip.

The pitch distribution by (15) results in a uniform inflow distribution along the blade span, resulting in *ideal twist*, and the following variation of inflow angle

$\emptyset = \emptyset_t \frac{R}{r}$ **– (16)**

Where $\emptyset_t$ is the inflow angle at the blade tip.

Substituting (15) and (16) into (14) will give:

$$dT = \frac{1}{2}\rho\,(\Omega \mathrm{r})^2 a \frac{R}{r}(\theta_t - \emptyset_t)c\,dr \quad \textbf{– (17)}$$

Integrating (17) over the blade radius, assuming the blade chord c is constant, the thrust of the rotor is (where N_b is the number of blades):

$$T = \frac{N_b}{2}\rho\,\Omega^2 a \frac{R^3}{2}(\theta_t - \emptyset_t)c \quad \textbf{– (18)}$$

Equating (18) to the expression for thrust of (10)

$$T = \frac{N_b}{2}\rho\,\Omega^2 a \frac{R^3}{2}(\theta_t - \emptyset_t)c = C_T \pi R^2 \rho (\Omega R)^2$$

We can obtain

$$C_T = \frac{a}{4}\frac{N_b}{\pi}\frac{c}{R}(\theta_t - \emptyset_t) \quad \textbf{– (19)}$$

Solidity

Solidity, σ, of a rotor having rectangular blades, may be defined as the ratio of the total blade area to the rotor disk area.

Hence,

$$\sigma = \frac{N_b cR}{\pi R^2} = \frac{N_b c}{\pi R} \quad \textbf{– (20)}$$

Substituting (20) into (19)

$$C_T = \frac{\sigma}{4} a(\theta_t - \emptyset_t) \quad \textbf{– (21)}$$

Equation (21) expressed the thrust of an ideally twisted, constant-chord blade.

Expression for Torque for the Rotor

Drag = Profile Drag + Induced Drag

Torque about the axis of rotation results from the drag on the element is *dD times r*

$$dQ = N_b \frac{1}{2} \rho \, (\Omega r)^2 c \left(C_{d_o} + \emptyset C_l\right) r \, dr$$ **– (22)**

Expression for the blade-section angle of attack for an ideally twisted blade is

$$\alpha_r = \frac{R}{r}(\theta_t - \emptyset_t) = \frac{1}{x}(\theta_t - \emptyset_t)$$ **– (23)**

Assume that $C_{d_o} = \delta$ where δ is an average blade profile-drag coefficient.

$$\left.\begin{aligned} C_{d_o} &= \delta \\ C_l &= a\frac{R}{r}\left(\theta_t - \emptyset_t\right) \\ \emptyset &= \emptyset_t\frac{R}{r} \end{aligned}\right\} \quad \textbf{(24)}$$

Substituting (24) into (22):

$$dQ = \mathrm{N_b}\frac{1}{2}\rho\,\Omega^2 r^3 c[\delta + \emptyset_t\frac{R^2}{r^2}(\theta_t - \emptyset_t)a]dr \;\textbf{– (25)}$$

After integration over the blade, the torque equation is

$$Q = \frac{\mathrm{N_b}}{4}\rho\,\Omega^2 R^4 c[\frac{\delta}{2} + a\emptyset_t(\theta_t - \emptyset_t)] \;\textbf{– (26)}$$

Equating (28) to (10) and using the solidity equation

$$C_Q = \frac{\sigma}{4}[\frac{\delta}{2} + a\emptyset_t(\theta_t - \emptyset_t)] \text{ – (26)}$$

Substituting (21) into (26) and rearranging terms

$$C_Q = \frac{\sigma\delta}{8} + \emptyset_t C_T \text{ – (27)}$$

To use (27) easily, $\emptyset_t$ needs to be replaced.

From equation (6)

$$\upsilon = \sqrt{\frac{T}{2\rho\pi R^2}} = \sqrt{\frac{C_T \pi R^2 \rho(\Omega R)^2}{2\rho\pi R^2}} = \Omega R\sqrt{\frac{C_T}{2}} \text{ – (28)}$$

By definition,

$$\phi_t = \frac{v}{\Omega R} \quad \textbf{– (29)}$$

Combining (28) and (29)

$$\phi_t = \sqrt{\frac{C_T}{2}} \quad \textbf{– (30)}$$

Substituting (21) into (26)

$$C_Q = \frac{\sigma\delta}{8} + \frac{{C_T}^{3/2}}{\sqrt{2}} \quad \textbf{– (31)}$$

(31) expresses the hovering performance of an ideally twisted constant-chord blade of solidity σ and average profile-drag coefficient δ. The first term expresses the profile-drag loss while the second part represents induced loss.

Calculations

Power Requirements for Hover Maneuvers

- Disk Loading
- Coefficient of Thrust
- Inflow Velocity
- Coefficient of Power
- Corrected by Figure of Merit

Aerodynamics Parameters for Cruise Maneuvers

- Aspect Ratio
- Overall CD_o
- Optimised $\frac{L}{D}$
- $\frac{L}{D}$ based on Cruise Speed
- Corrected by propulsive efficient

Excel Spreadsheet: Limitations

- Relatively low-fidelity methodology
  - Unable to incorporate other numerical tools at the moment
  - Unable to automatically perform high level analysis
- Only applicable for 2 types of eVTOL- Multirotors and Lift+Cruise
- Inaccurate mass distribution for small scale prototype

Hover and Forward Flight Performance

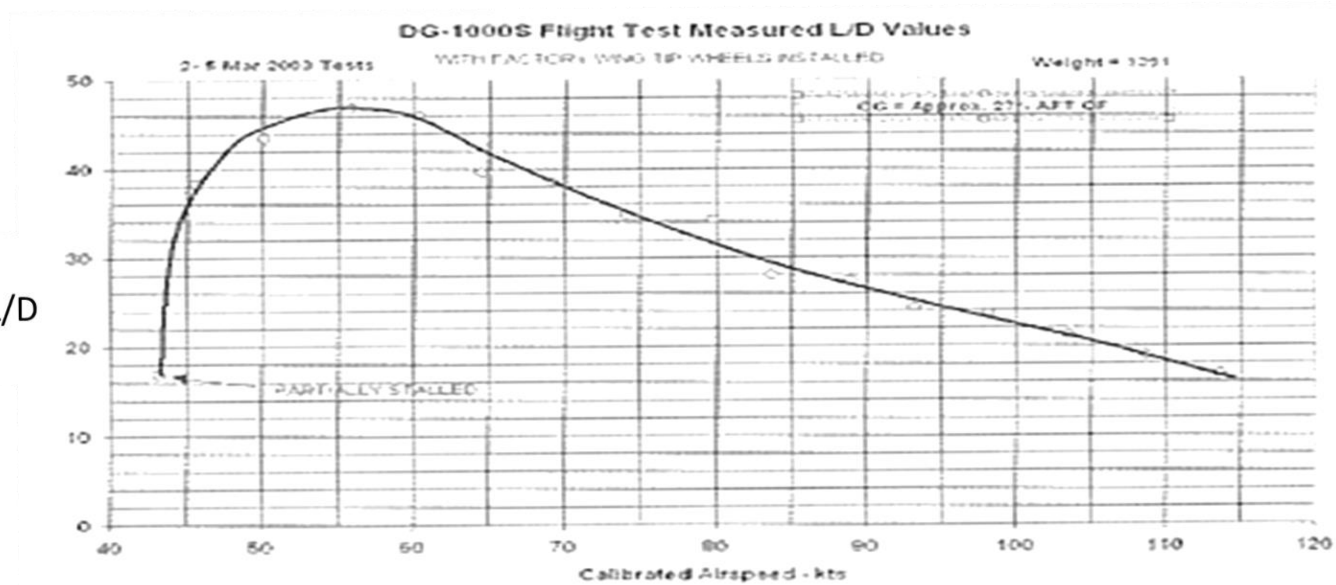
Using either an Excel Spreadsheet or writing a simple Matlab routine will help provide a feel for rotor sizing and help get you to 80 to 90% of some of the rotor performance characteristics estimations.

Later on, add aircraft L/D and other parameters to the spreadsheet to estimate performance for airplane mode cruising flight.

Cruise Aerodynamics

L/D versus Airspeed for DG-1000 Glider

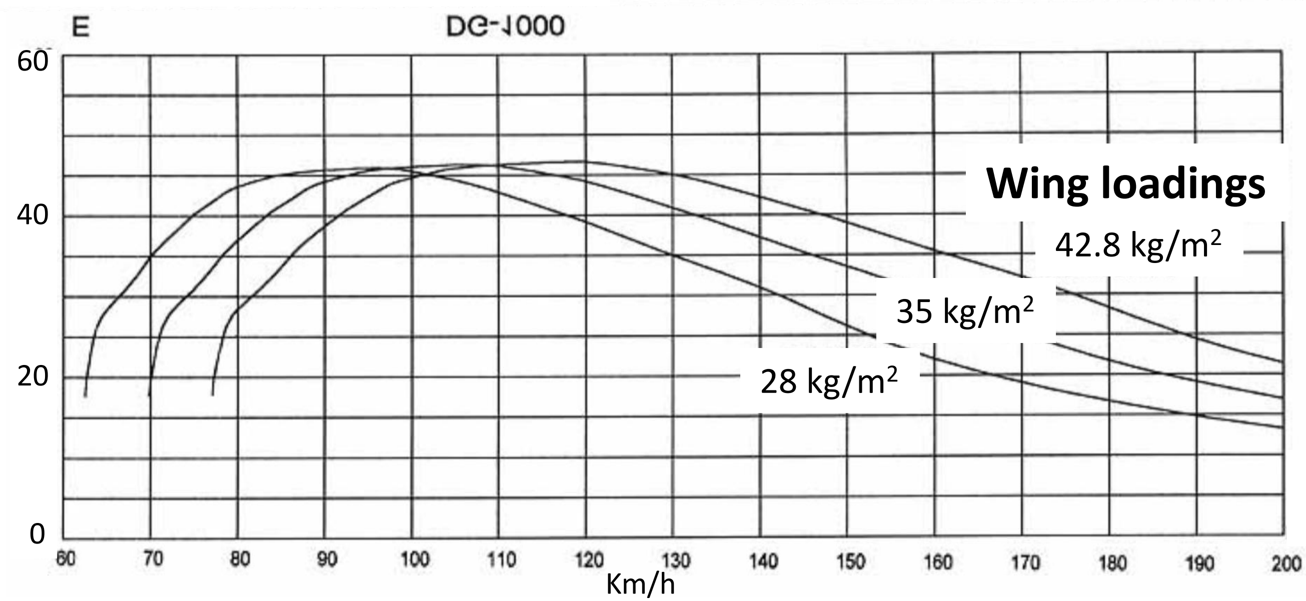
L/D



L/D drops at higher air speed because L = W = constant, but D goes up with airspeed.

Higher wing loading gives higher L/D because W is higher.

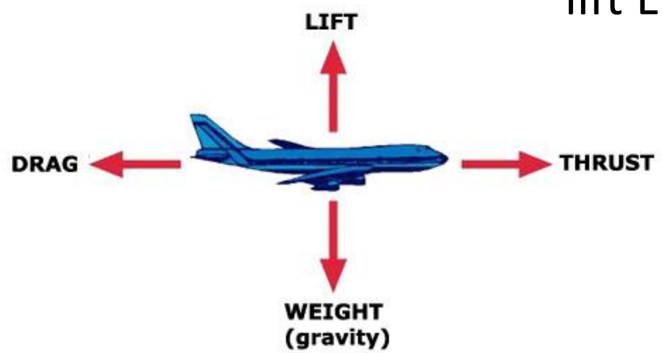
L/D



Source: https://www.dg-flugzeugbau.de/en/library/dg-sailplane-a

Power Required in Forward Flight

In steady level flight, thrust required (T_R) = drag (D) of the aircraft
lift L = Weight of the aircraft.



$$\underline{\text{Power Req}} = (force)\bullet(vel) = \text{time rate of doing work}$$

$P_R = T_R V_\infty$ since T_R, V_∞ are aligned

$$P_R = T_R V_\infty = D V_\infty = \frac{L}{(L/D)} V_\infty = \frac{W}{(C_L/C_D)} V_\infty$$

Drag of the entire aircraft, especially for a complex geometry eVTOL aircraft, is not that easy to calculate. Need to do CFD analysis or wind tunnel testing to measure.

Power Required at Airspeed

However, we know the L/D of different types of aircraft 😊
L/D = 30 - 45 for gliders
L/D = 15 for Boeing 747
L/D = 7 to 11 for Cessna 172 (L/D drops at higher speed)
L/D = 2 to 5 for helicopters

Let's do a mathematic check for Cessna 172Q aircraft, we know its MTGW=1150 kg, it's Max L/D=10.9, max speed is 302 kmh (83.9 m/sec), installed engine power = 180 hp (134 kW)

What's the power required at 302 kmh for Cessna 172Q?
Answer is:
Power = W/(L/D) x velocity
= 1150 kg x gravity (9.8 m/sec2) / (7) x 83.9 m/sec
= 135 kW

What's the power required at 226 kmh (best cruise speed) for Cessana 172Q?
Answer is:
Power = W/(L/D) x velocity
= 1150 kg x gravity (9.8 m/sec2) / (10.9) x 62.8 m/sec
= 65 kW

Blade Design

affects noise, dynamic stability, and vibratory loads

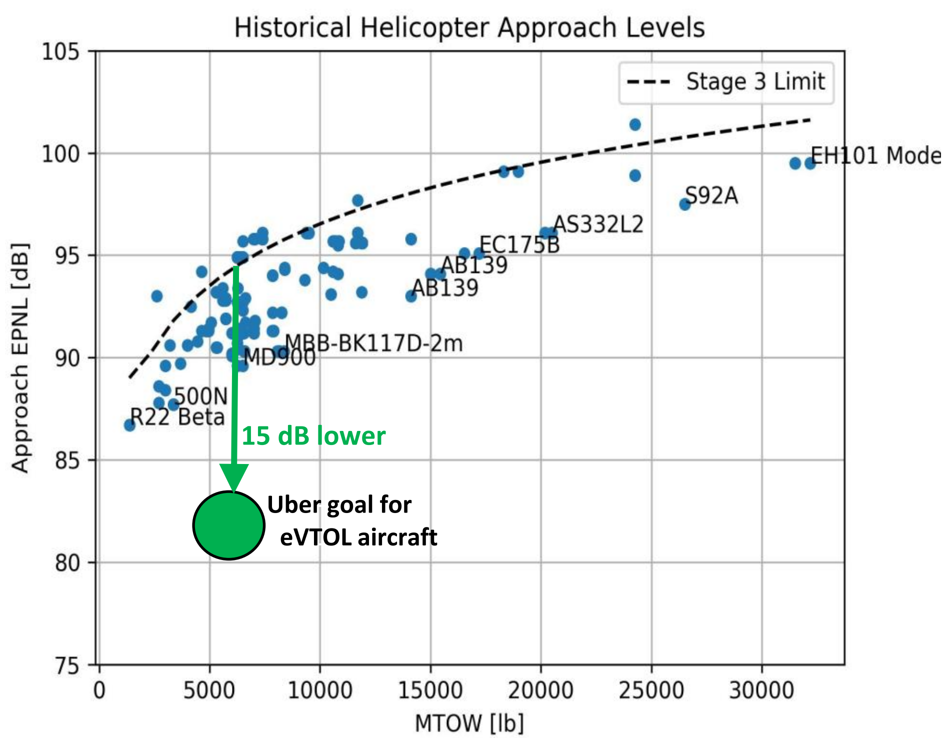
Momentum and Blade Element Methods

Momentum theory does not account for rotor rpm. Blade element theory includes effects of rotor rpm and blade surface area. Both methods are only adequate for conceptual and preliminary design and not for advanced blade design.

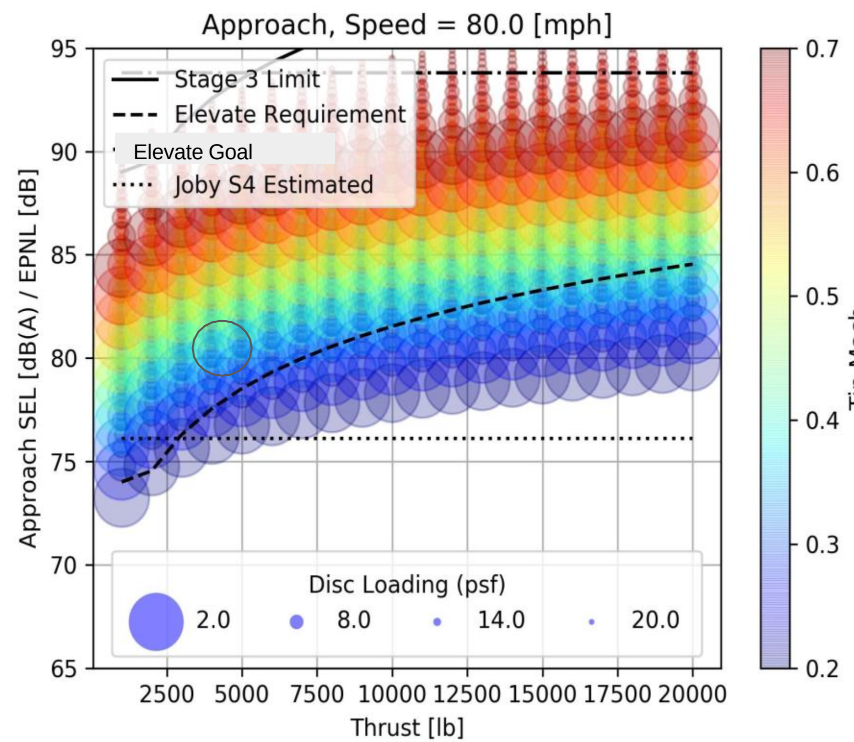
To keep rotor noise reasonable, try keeping the rotor blade tip speed under 550 ft/sec (Mach 0.49). This means bigger diameter rotors must spin slower.

External Noise Level

A nice target for eVTOL aircraft design is to have external noise 15 dB lower than the Stage 3 noise boundary for current helicopters



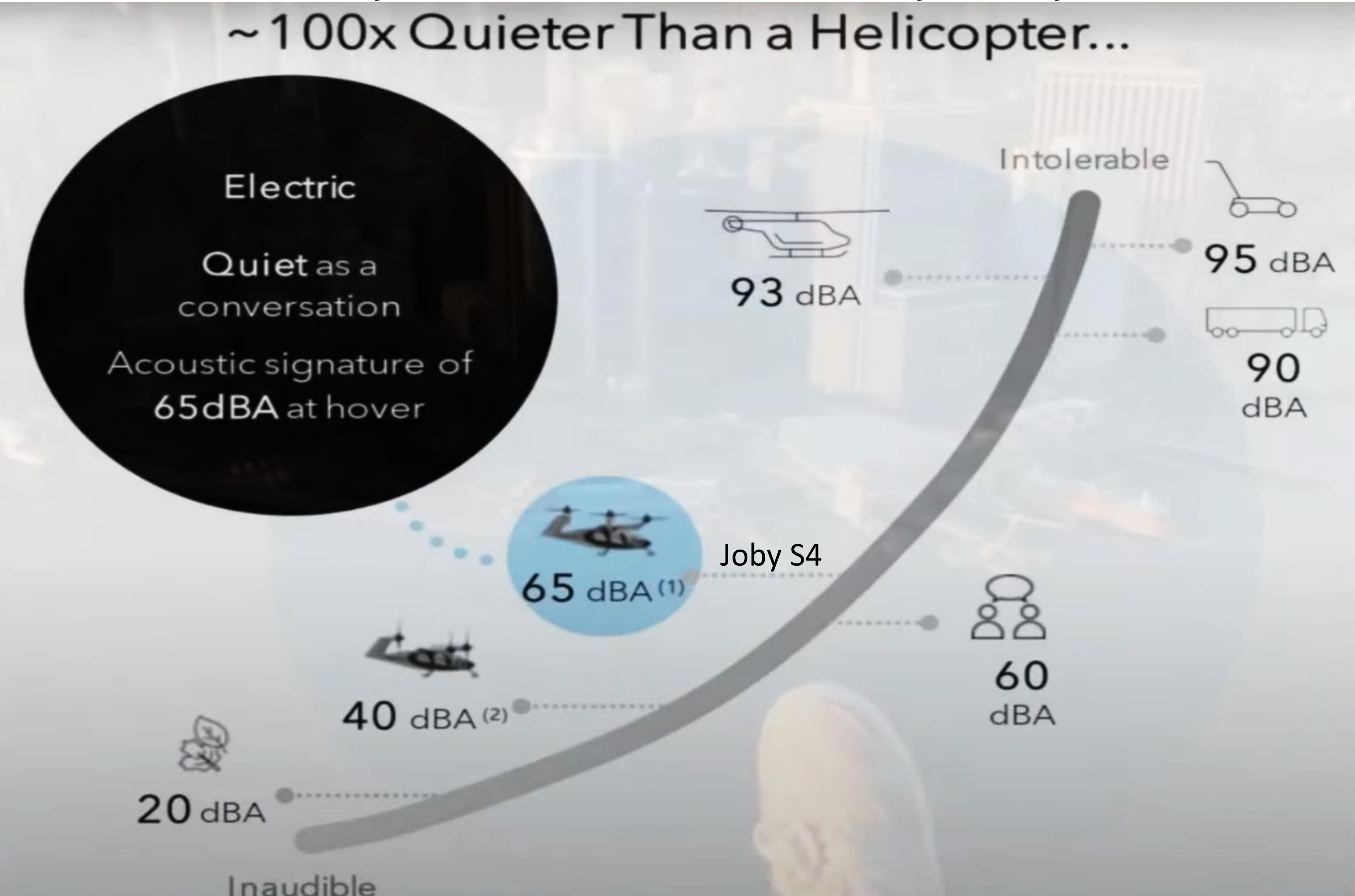
Assume an eVTOL aircraft weighs under 7000 lbs



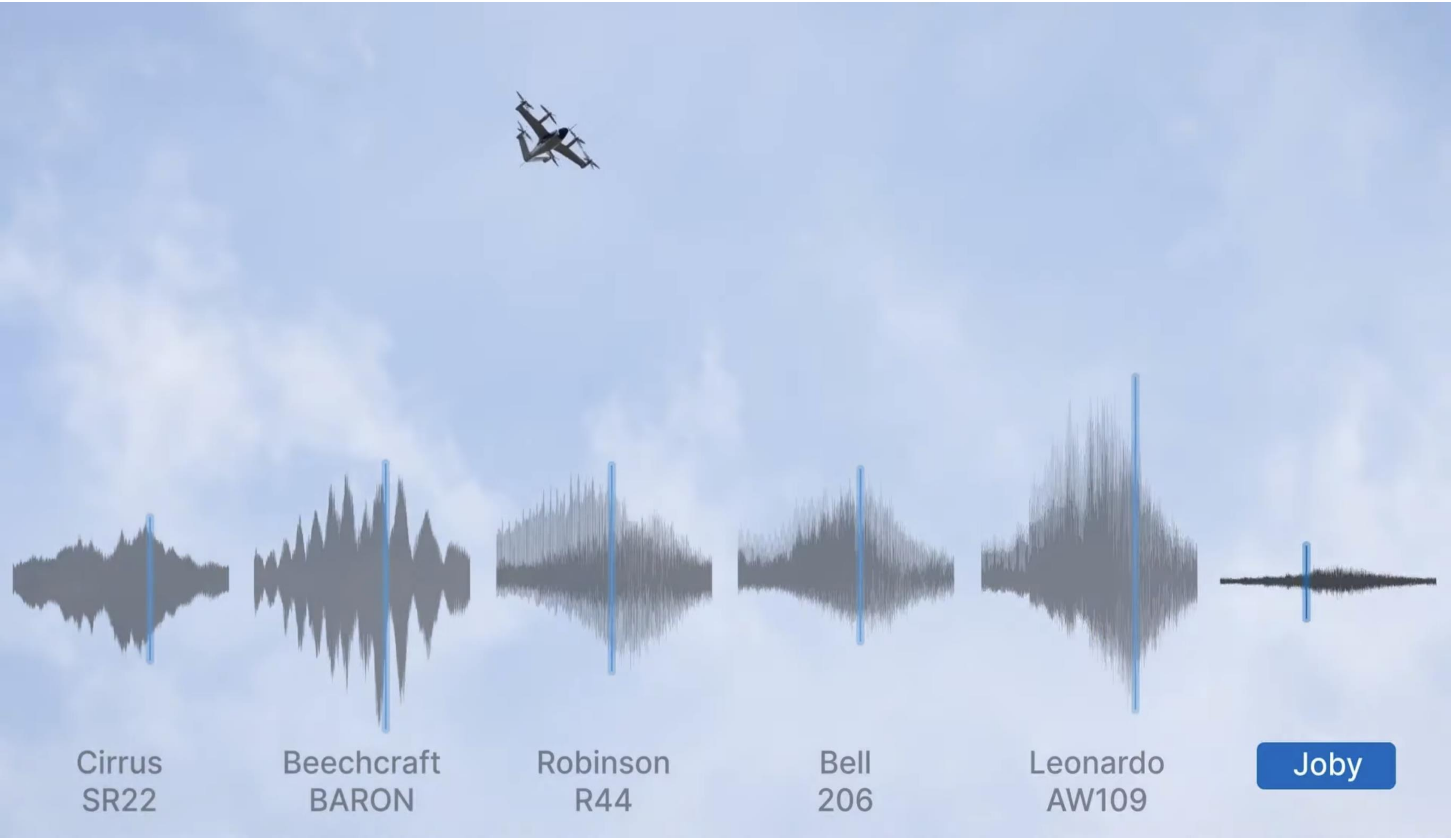
The lower the blade tip speed, the lower the rotor noise. Try keep it under 550 ft/sec (Mach 0.49)

Data from Uber presentation at VFS Forum 2019

External Fly Over Noise Given by Joby for S4

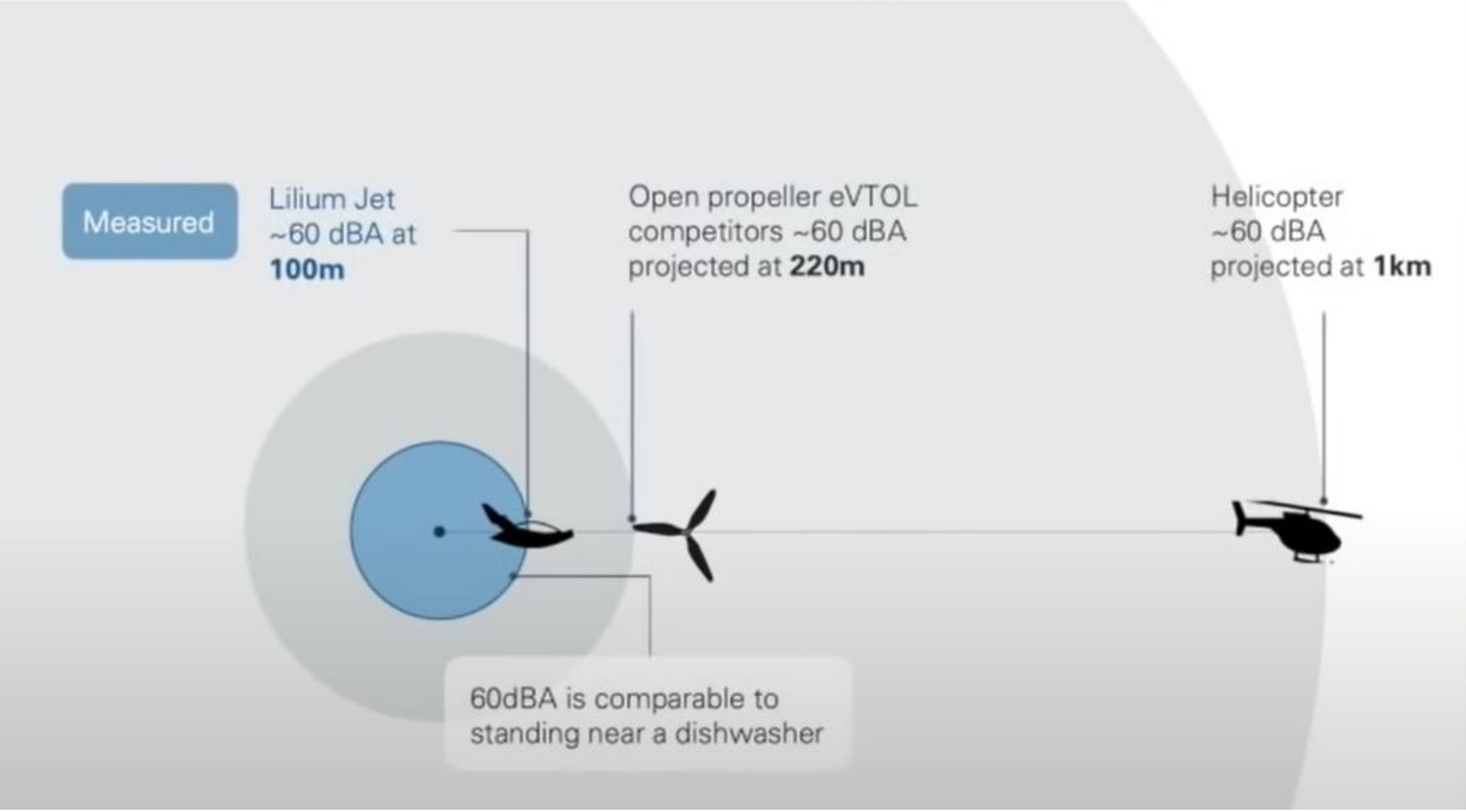


External Fly Over Noise Signature



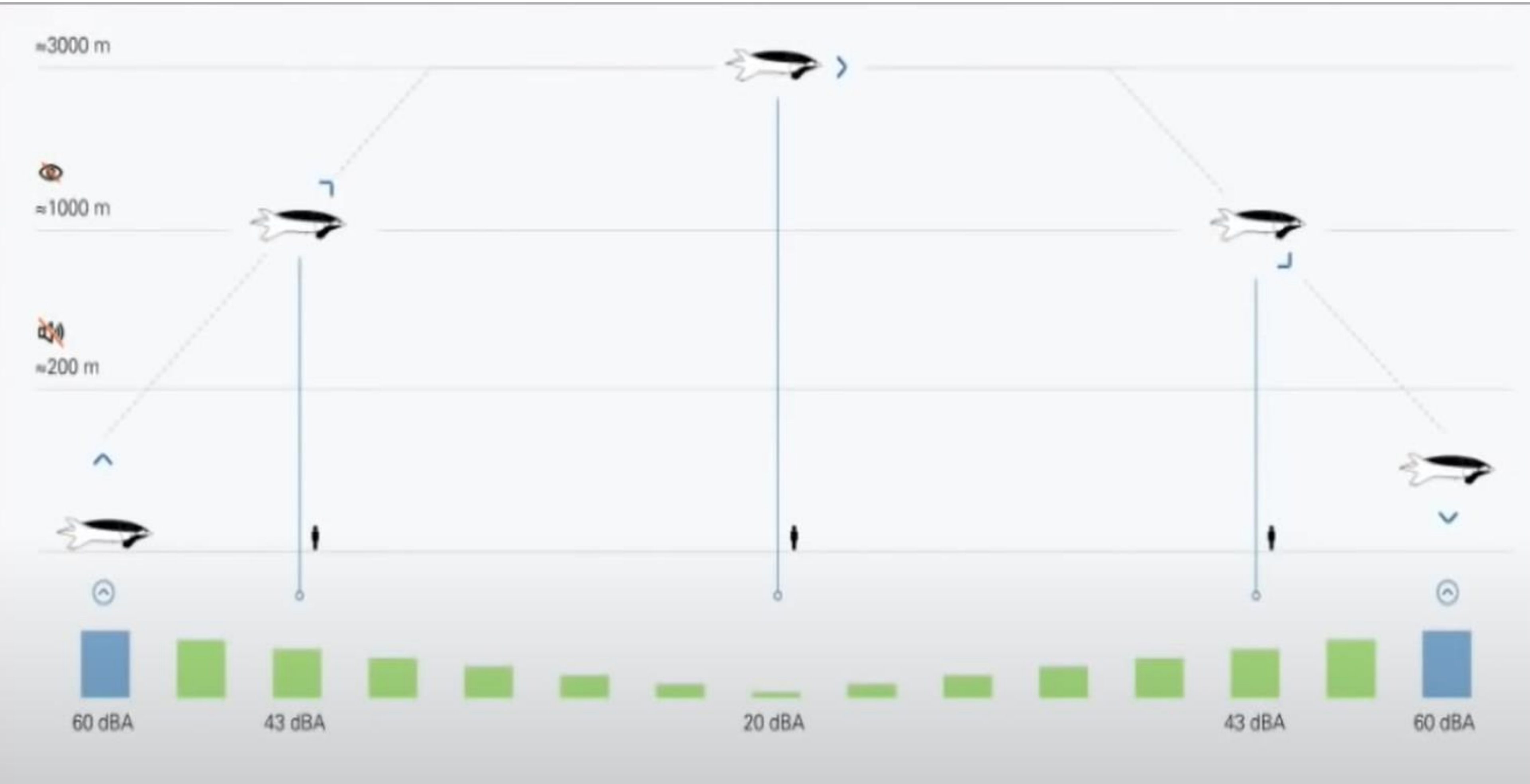
External Noise

Lilium Noise at Landing from Public Domain



Source: https://www.youtube.com/watch?v=fmiVd-CiNmw

Lilium Noise at Cruise from Public Domain

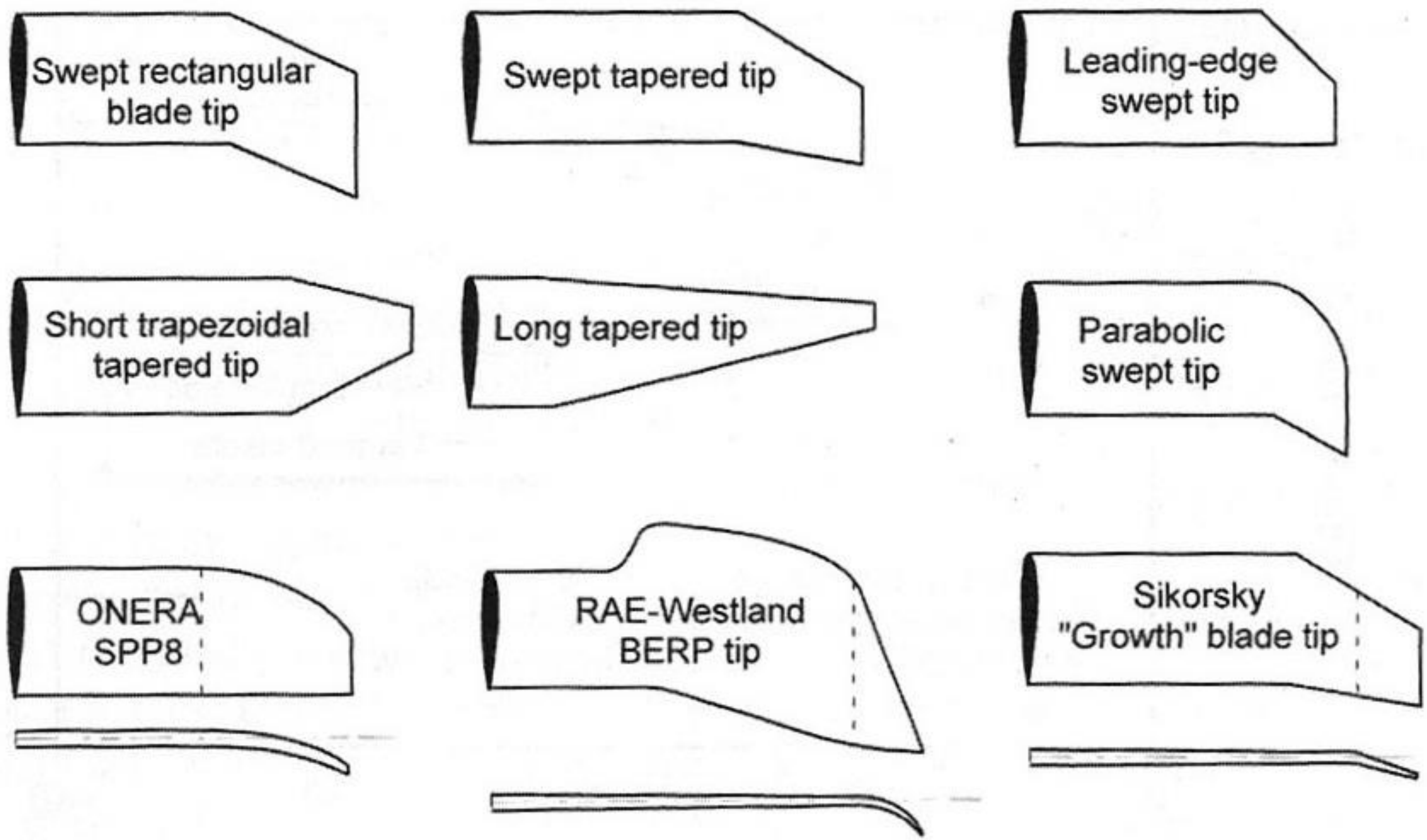


Source: https://www.youtube.com/watch?v=fmiVd-CiNmw

Chevron on Lilium Jet Duct Exhaust End

Source: https://www.youtube.com/watch?v=fmiVd-CiNmw

Blade Tips for Performance and Acoustics

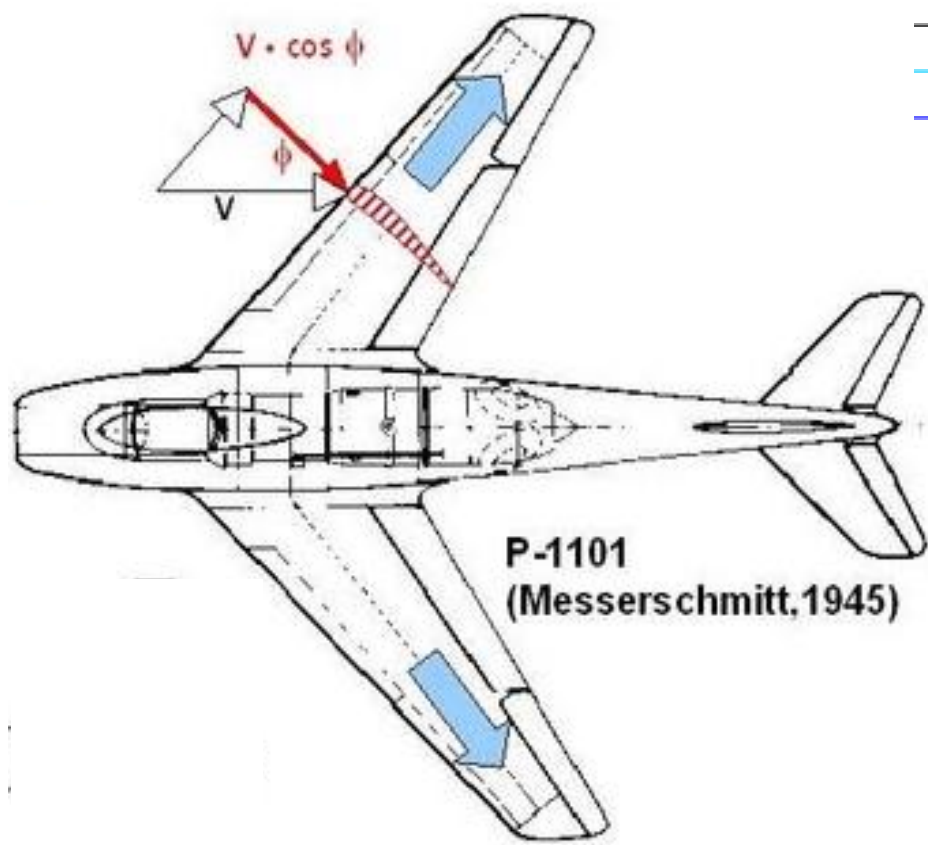


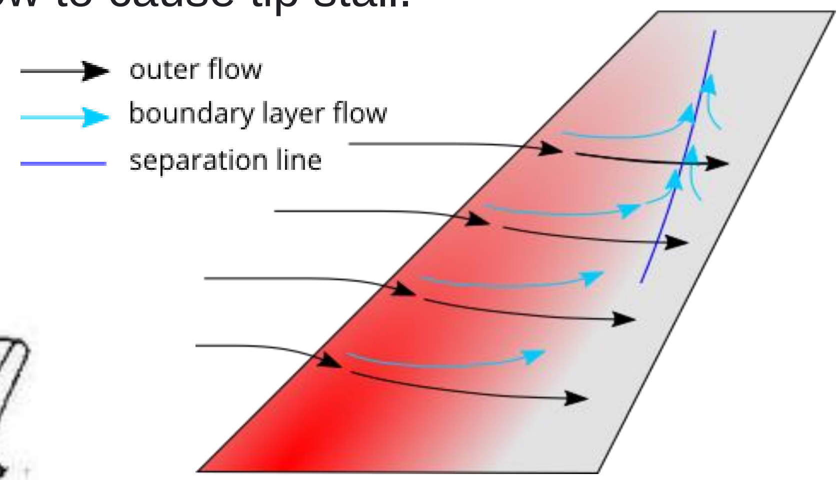
Source: https://www.sciencedirect.com/science/article/abs/pii/S0376042112000644

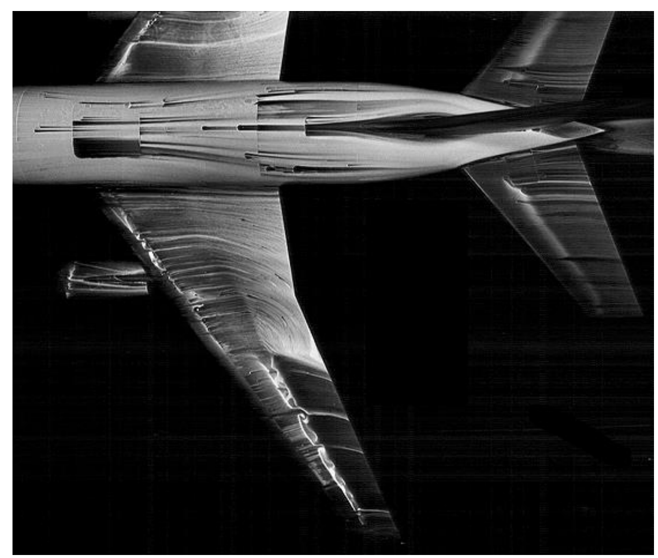
https://aviation.stackexchange.com/questions/63666/what-is-the-bend-in-the-tip-of-the-blade-of-the-new-black-hawk

Sweep the Leading Edge Like Jet Plane Wing

In transonic flight, a swept wing allows a higher Critical Mach Number than a straight wing of similar chord and camber. This results in the principal advantage of wing sweep which is to delay the onset of wave drag. But blue arrow shows it can also cause spanwise flow to cause tip stall.

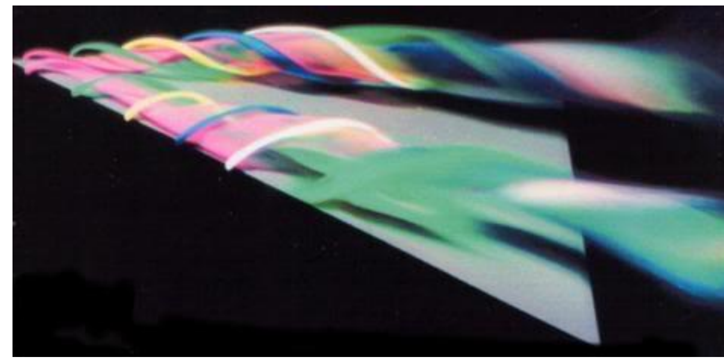


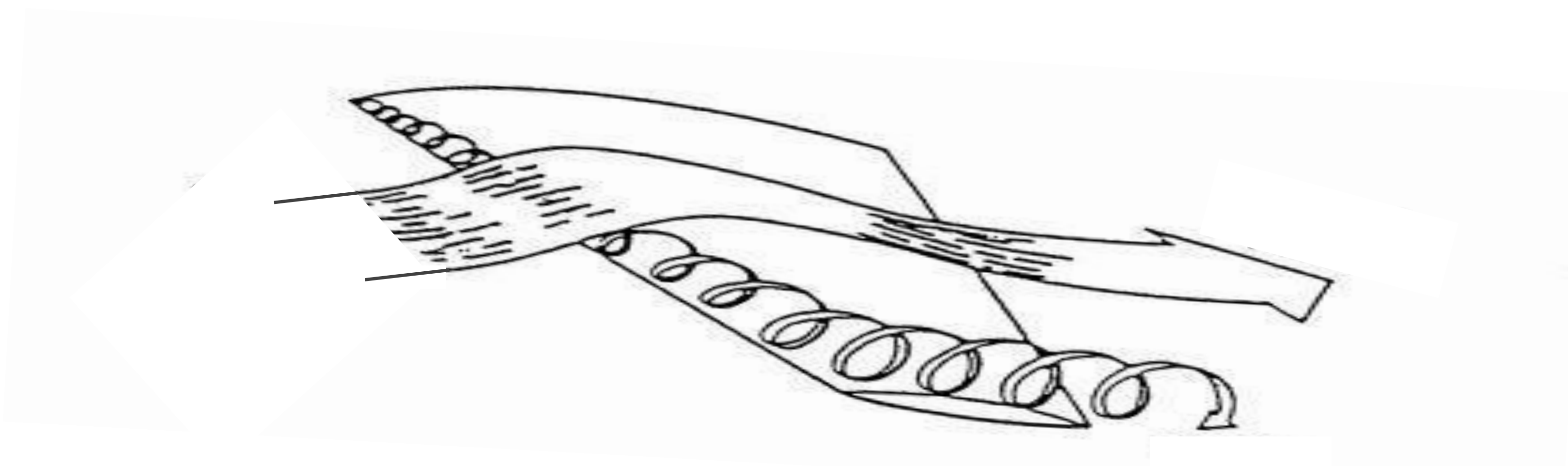




Spurce: https://www.researchgate.net/figure/Wing-sweep-for-reduced-drag_fig5_224792385
https://www.quora.com/Is-airflow-deflected-inwards-or-outwards-by-wing-sweep

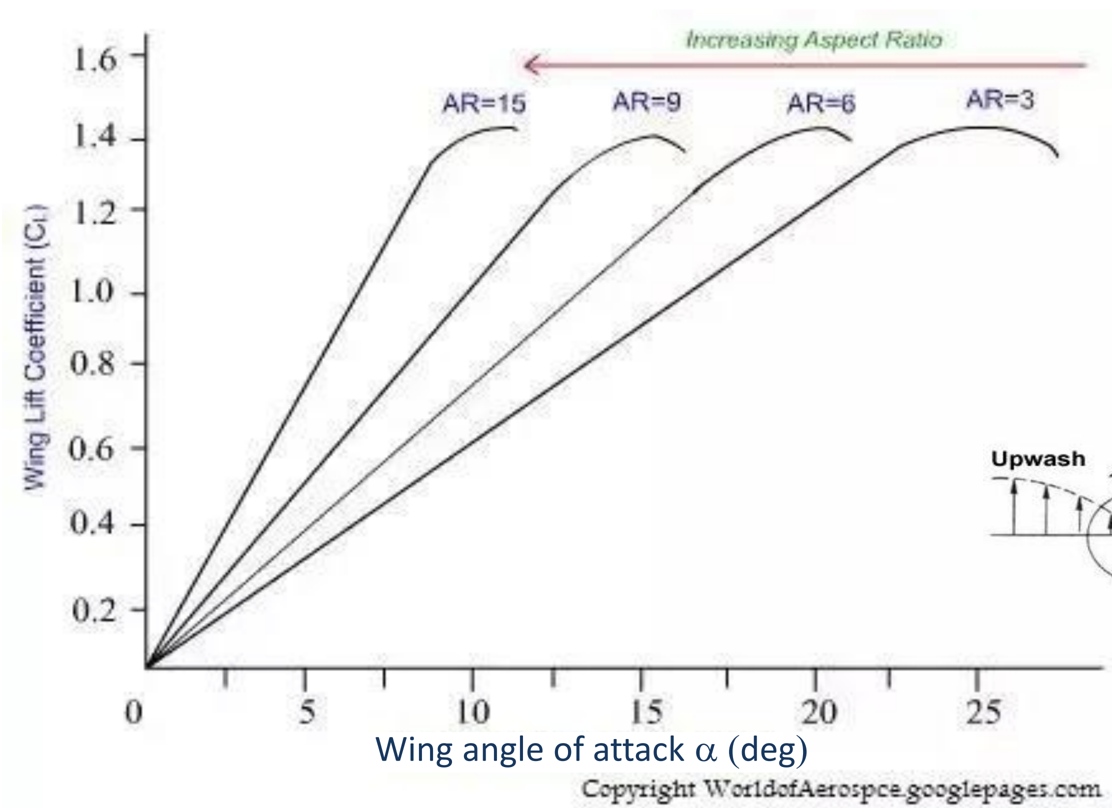
Vortex Lift on a Delta Wing

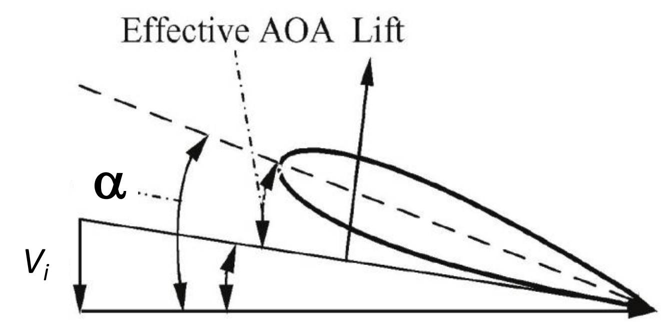


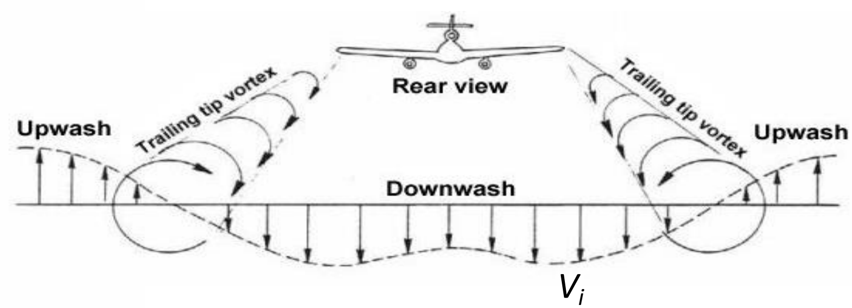


Lower Aspect Ratio Increases Wing Stall Angle

For low aspect ratio wings, there is greater downwash. This reduces the effective angle of attack on the wing sections, so you have to rotate the whole wing to a larger physical angle of attack to compensate.



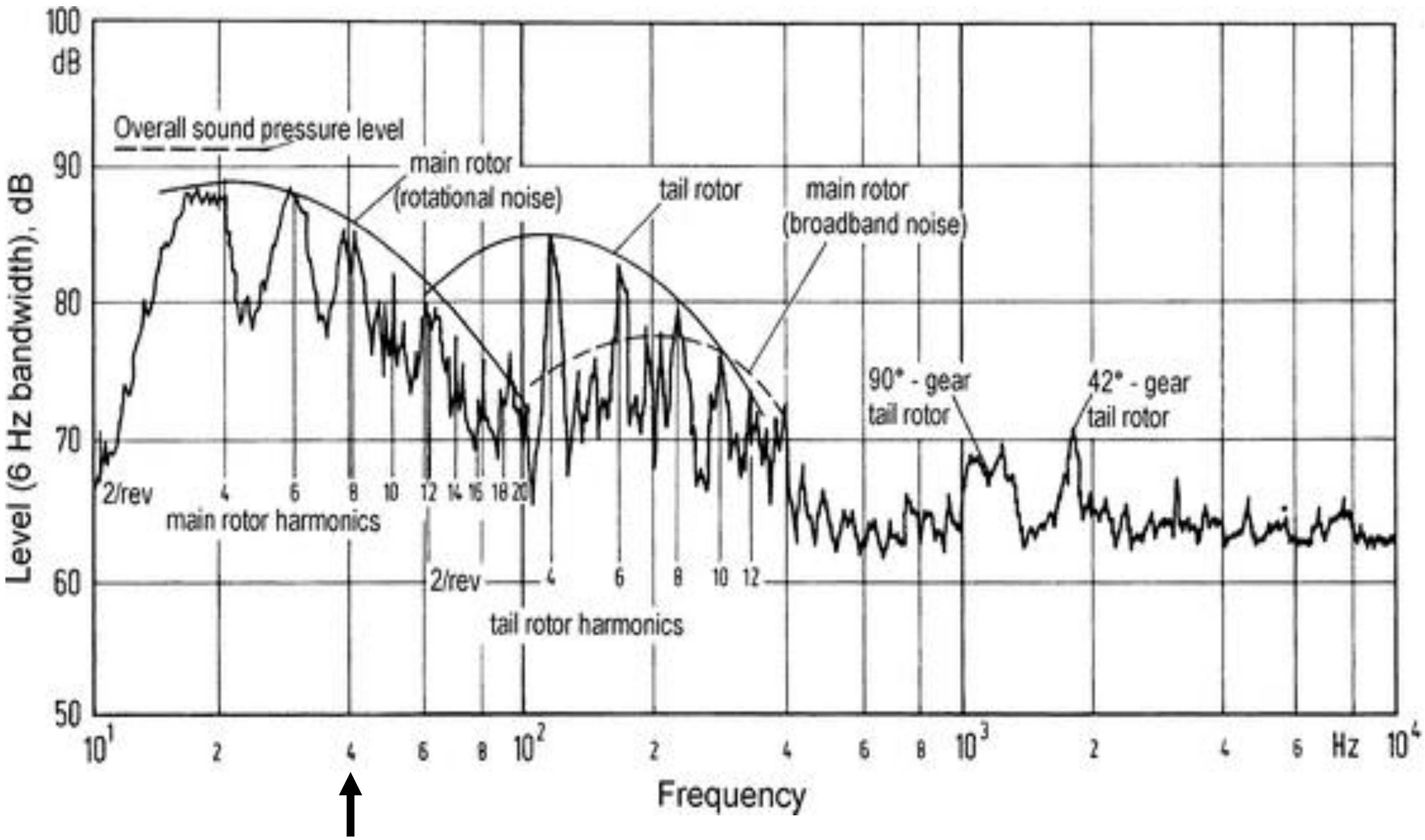




Downwash velocity *Vi* gets larger as gets farther from the wing tip

Source: https://www.quora.com/How-come-low-aspect-ratio-wings-stall-at-a-higher-angle-of-attack

External Noise of a 4-Bladed Rotor

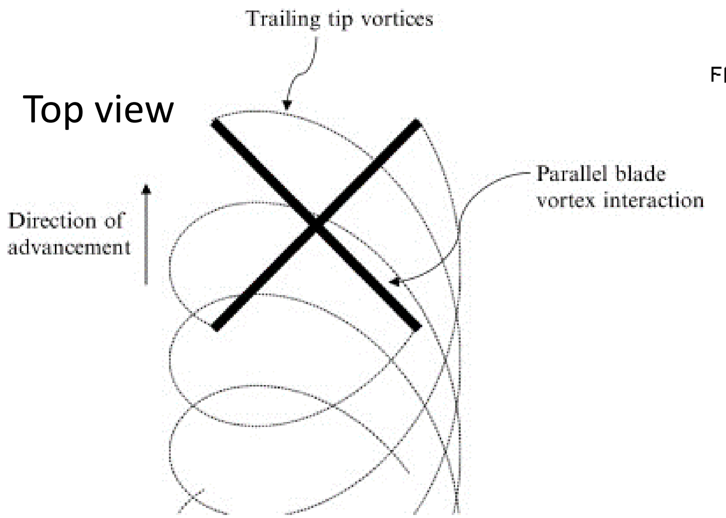


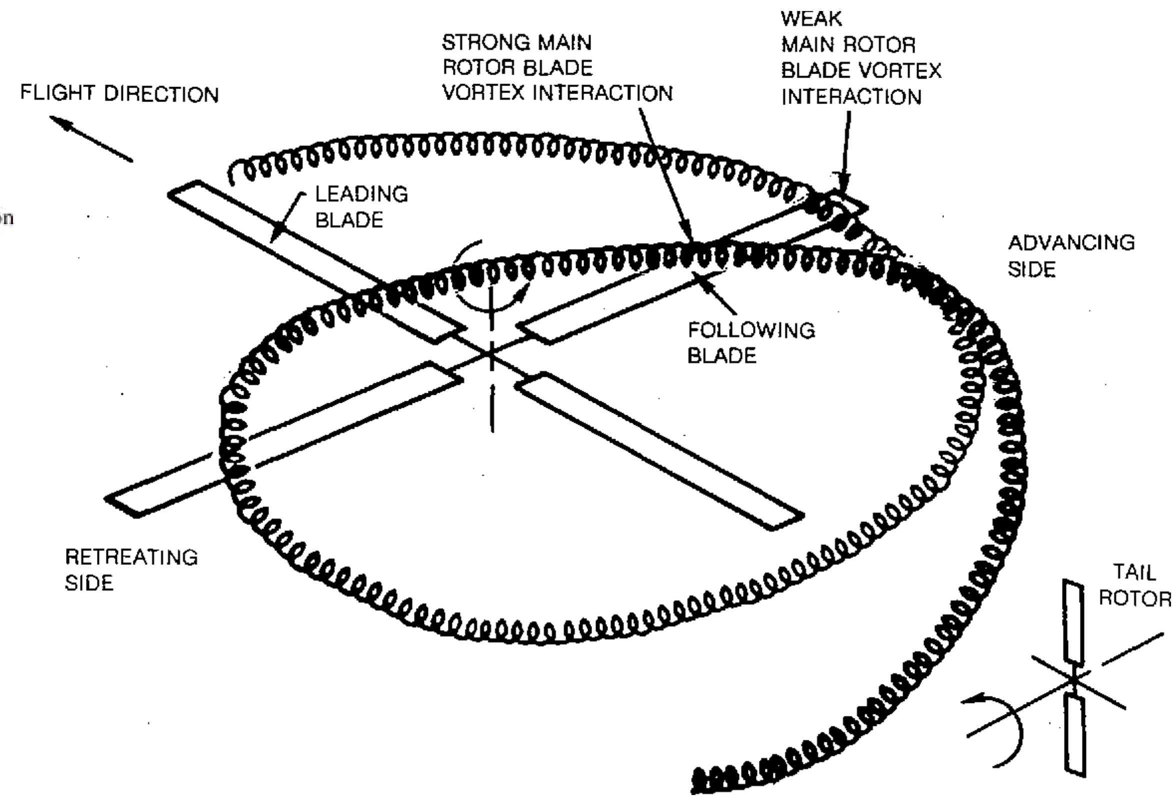
40 Hz/(8/rev) = 5 Hz, 5Hz x 60 = 300 rpm

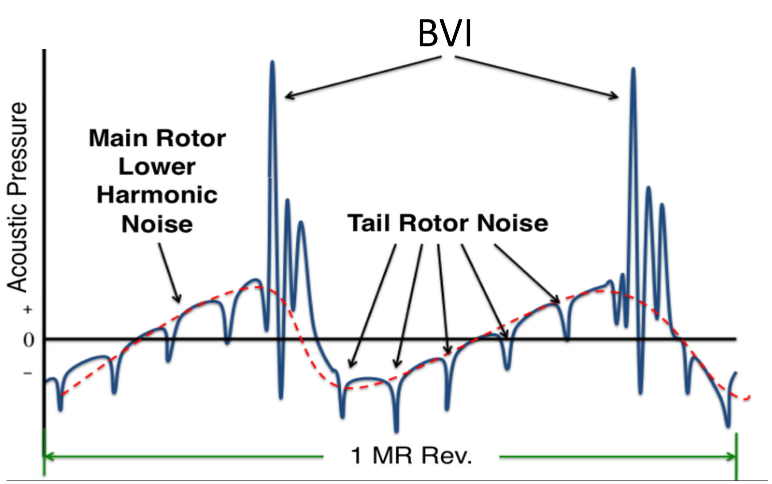
Source: https://www.wired.com/2010/02/eurocopter-moves-one-step-closer-to-whisper-mode/

Blade Vortex Interaction (BVI)

A blade vortex interaction (BVI) is an unsteady phenomenon of three-dimensional nature, which occurs when a rotor blade passes within a close proximity of the shed tip vortices from a previous blade.







Source: https://www.wikiwand.com/en/Blade-vortex_interaction
https://www.sciencedirect.com/topics/engineering/blade-vortex-interaction
https://www.semanticscholar.org/paper/Rotor-vortex-interaction-noise-Schlinker-Amiet/9287f8af716fba7034d1d7e88e50e64ca7d7648b

Blade Vortex Interaction (BVI)

Parallel BVI occurs when the vortex and the blade axes are nominally parallel. It is the BVI phenomenon that produces the largest-amplitude impulse (harmonic) noise, due to that the unsteady vortex moves towards to the downstream.

Perpendicular BVI occurs when the axes are perpendicular and in parallel planes. Due to its low unsteadiness, the noise effect of perpendicular BVI are less significant with respect to parallel BVI. It produces a continuous broadband noise characterised by a much lower intensity compared to the impulse (harmonic) noise, which caused by parallel BVI.

Oblique BVI occurs between the vortex and the blade when the axes are oblique. In helicopter research field, oblique BVI is a common phenomenon that looks like an intermediate action of parallel BVI and perpendicular BVI.

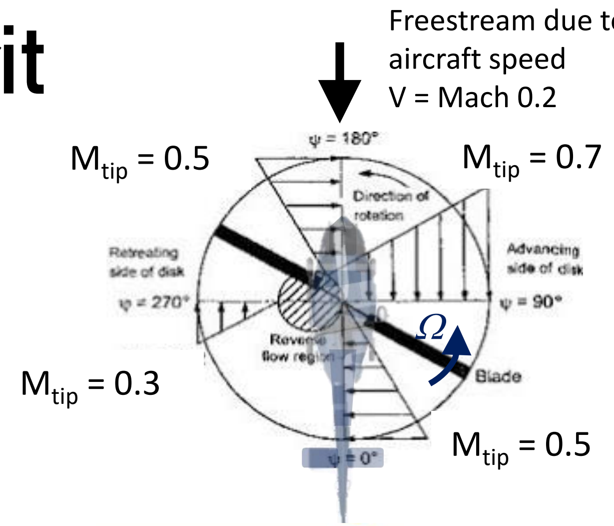
Orthogonal BVI occurs when the axes of the vortex are in orthogonal planes. In the context of helicopter application, the orthogonal interaction usually exits between the tip vortices generated by the main rotor and the blade of the tail rotor.

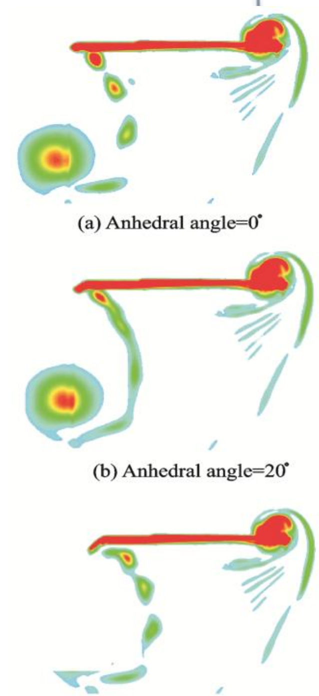
Source: https://www.wikiwand.com/en/Blade-vortex_interaction

Airbus Blue Edge Blade Tip To Reduce Noise

Source: https://www.wired.com/2010/02/eurocopter-moves-one-step-closer-to-whisper-mode/

Sikorsky S-92 Swept-Anhedral Blade Tip for Improving Hover Figure of Merit





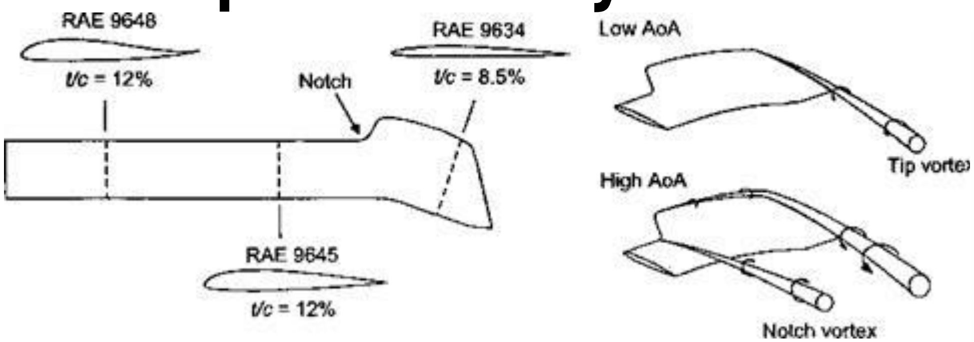
(a) Anhedral angle=0°

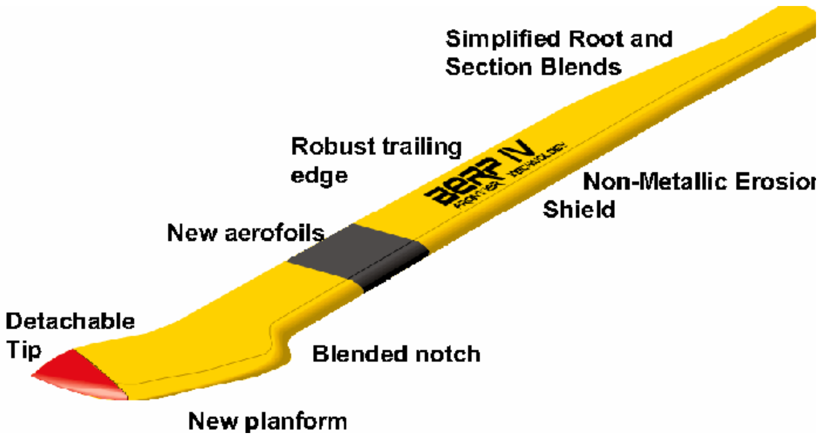
(b) Anhedral angle=20°

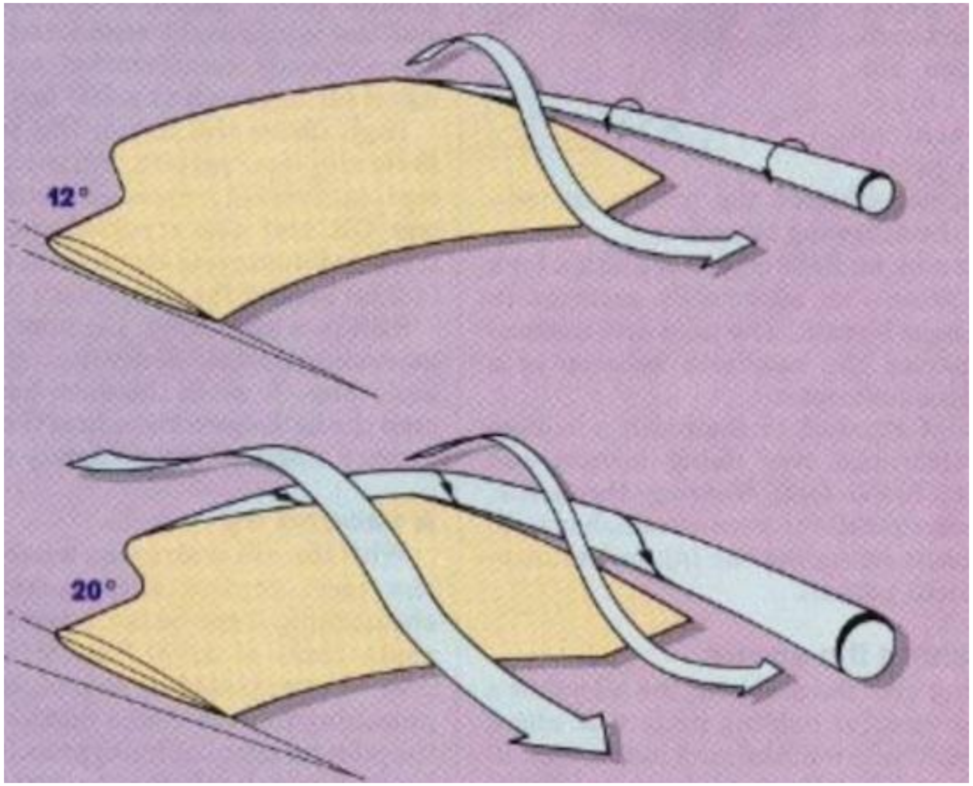
(c) Anhedral angle=45°

Sources: https://www.rotorworks.com/2019/10/helicopter-ground-school-drag/
http://tnuaa.nuaa.edu.cn/html/2018/1/E201801018.htm

AgustaWestland BERP Blade Tip Reduces Compressibility and Increases Stall Angle







Original successful BERP tip at high angle of attack

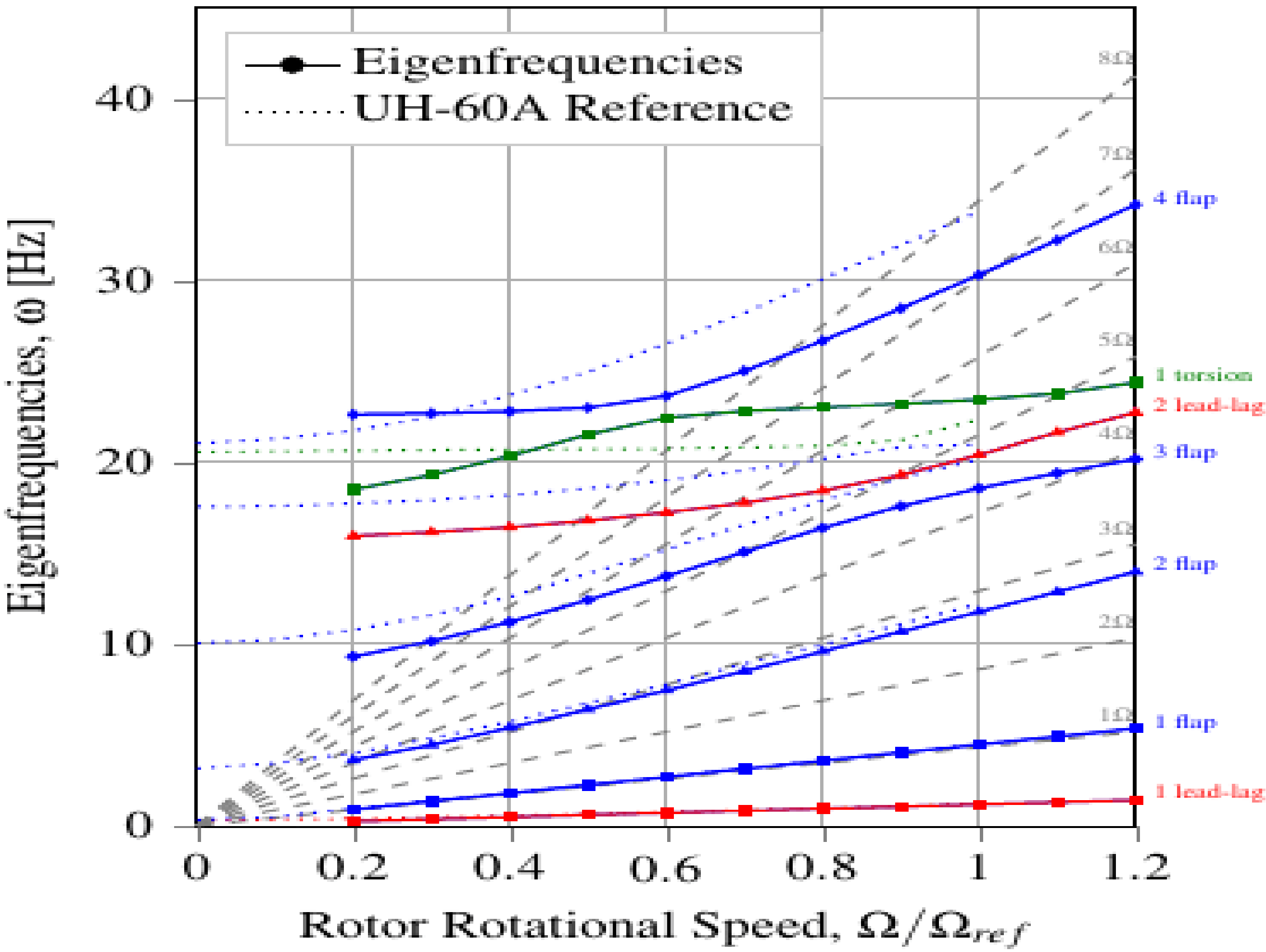
Newer generation anhedral BERP IV

Acoustics and Dynamics

Rotor acoustics and rotor dynamics are two very specialized topics and challenging to predict. An eVTOL company needs to hire helicopter specialists.

Dynamic issues include rotor stability, blade vibratory load, coupled rotor-body vibration, and whirl flutter. These are extremely difficult to analyze and even for helicopter dynamicists, it takes years of experience. Noise is a nuisance, but dynamics issues cause failure and crash.

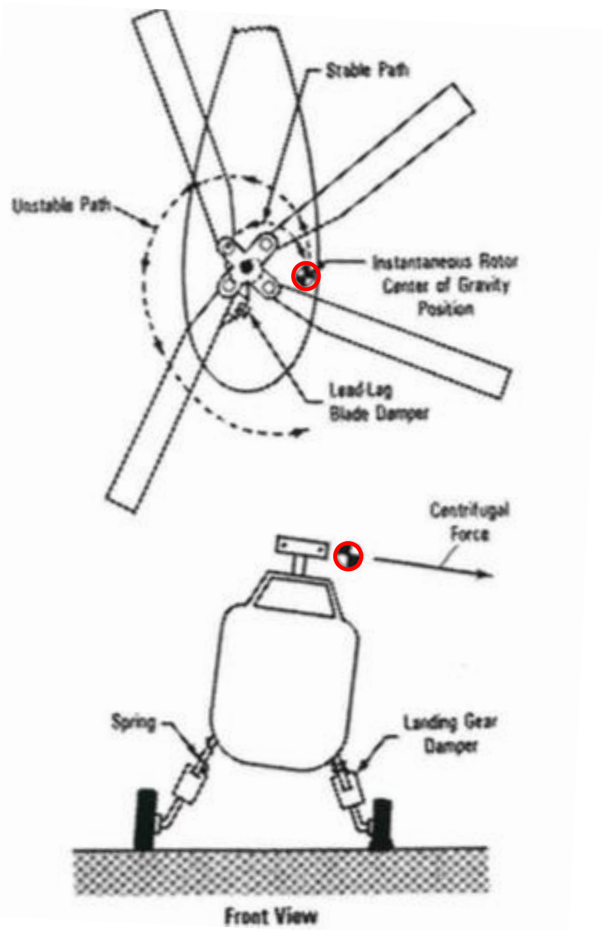
Fan Diagram Shows Blade Modal Frequencies at Different Rotor RPM

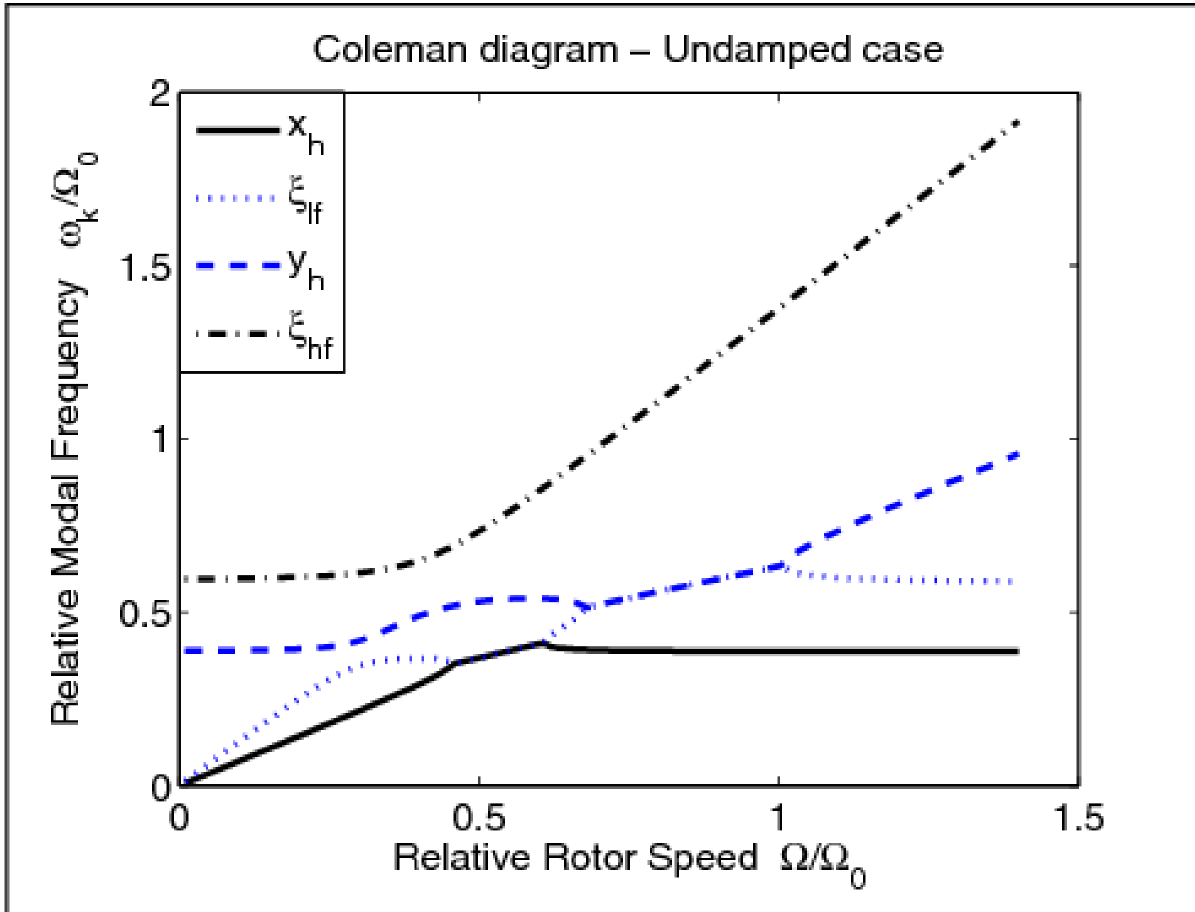


Source: https://www.researchgate.net/figure/Fan-Plot-of-the-new-rotor-blades-compared-to-the-UH-60A-Fan-Plot-from-Bowen-Davies-36_fig11_328232648

Aeromechanic Instability

- Ground Resonance
- Air Resonance

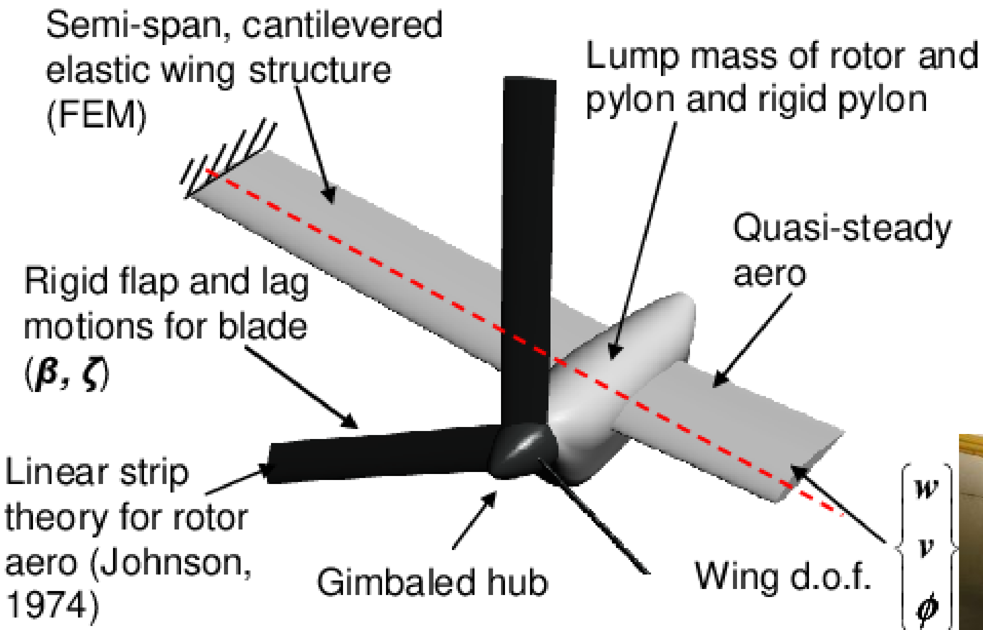




Coleman diagram for coupled rotor-fuselage motion

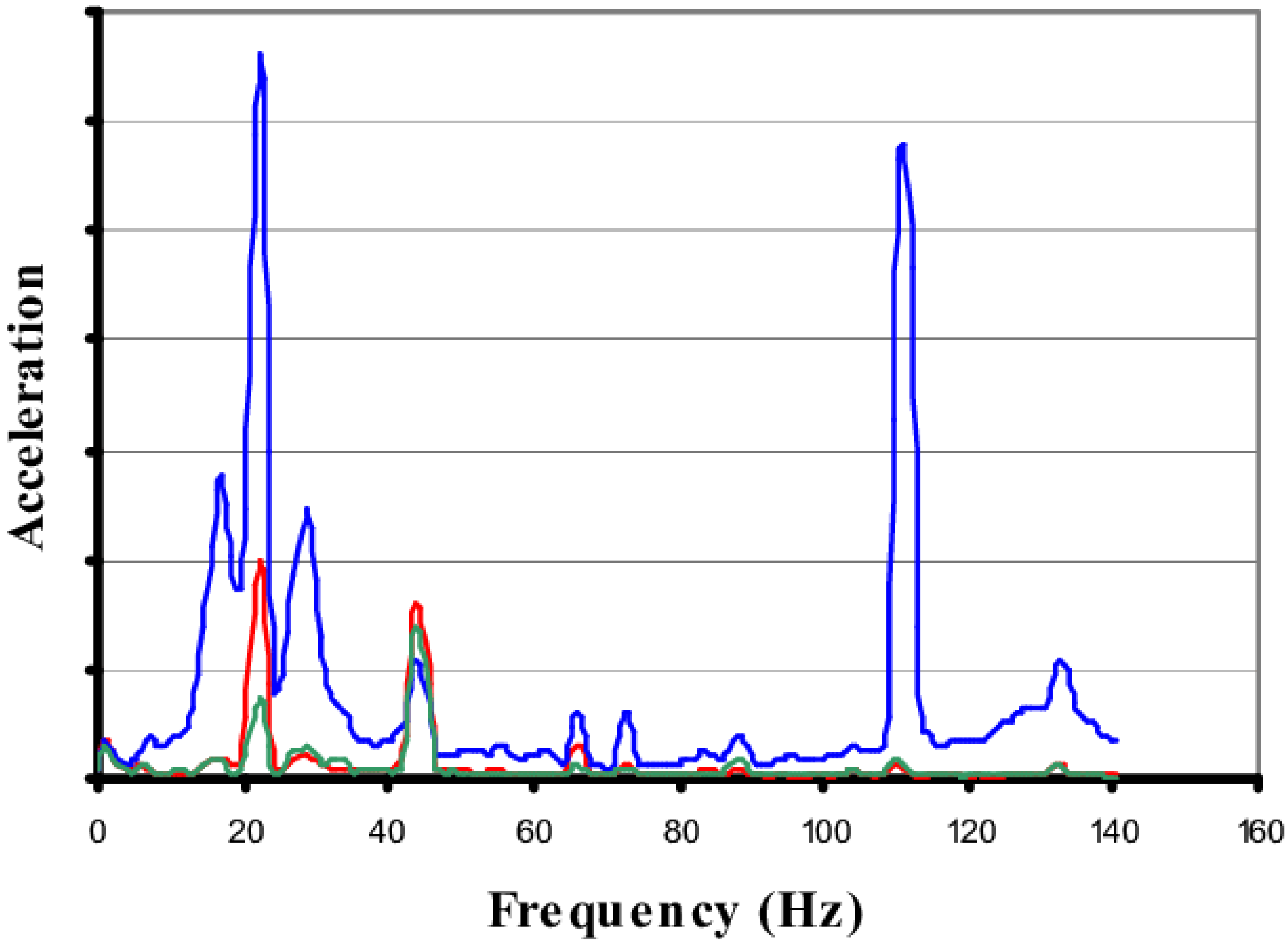
Ground Resonance

Pylon Whirl Flutter



Source: Influence of aeroelastically tailored wing extensions and winglets on whirl flutter stability
Jianhua Zhang, Edward C. Smith, Published 2013 https://www.semanticscholar.org/paper/Influence-of-aeroelastically-tailored-wing-and-on-Zhang-Smith/b703ac4dd86f515b334944e4959457f31fc97a69

Vibratory Loads Measured Inside the Cabin



List of anlytical comprehensive codes that can be used for eVTOL design and analysis

CFD Analysis for Isolated Rotor or Whole Aircraft

CHARM

FUN3D

Overflow

ICFD++

ANSYS Fluent

Open VSP

DUST

Availability of Software

- Pointwise
  - For generating CFD grids and mesh
- CHARM (Continuum Dynamic Inc. , US)
  - Anyone can buy
- FUN3D (NASA, US)
  - Free, needs to be requested directly from NASA
  - Only available to US citizens, companies or universities
- OVERFLOW (NASA, US)
  - Free, needs to be requested directly from NASA
  - Only available to US citizens, universities and companies
- ICFD++ (Metacomp Technologies, US, Distributors in Japan, China, India, Korea, UK, Spain)
  - Will have to ask company about licensing
  - Price depends on what modules are purchased
  - Personal License Price - $250
  - Professional License Price - $8000 to $12000
- ANSYS Fluent (ANSYS Inc. , US)
  - Have to pay
- OpenVSP (NASA, US)
  - Open Source software
- DUST (Milan Politecnico University)
  - Open source software, not much documentation

Features

CHARM	FUN3D
• Made specifically for rotorcraft and eVTOL aircraft • Naturally structured to capture interacting flows of multirotor-multiwing aircraft • Supports open propeller and ducted propeller and it can capture fuselage and airframe effects • Inviscid flow solver	• Unstructured Navier Stokes Solver • Contains thermodynamic models (Perfect Gas) • Allows for turbulence modelling • Grid motion (translation and rotation) is allowed • Incompressible flow to hypersonic flow covered in the code

Features

OVERFLOW	ICFD++
• 3D Navier Stokes Solver that can also solve 2D or axisymmetric flows • Full viscous flow can be solved with turbulence • Uses overset structured grids for solving to help with complex geometries • Contains Perfect Gas Model • Covers mainly compressible flow	• Handles most flow regimes • Supports all types of grids • Advanced User Interface • Allows for turbulence modelling • Meshes and grids can be moved to simulate motion

Features

ANSYS Fluent	OpenVSP
• Able to simulate a wide variety of flows – laminar, turbulent, reacting flows, multiphase flow etc • Mesh motion is possible • Comprehensive turbulence modelling including large eddy simulation modelling • Any physics which isn't built in can be simulated using user defined functions • Any kind of grid or mesh can be created with ANSYS Fluent • Wide range of numerical solvers for both steady state and transient cases	• Similar to XFLR5 • A parametric aircraft geometry tool capable of running some simple analysis • Uses vortex lattice method or panel method, same as XFLR5 • Captures entire geometry of the aircraft including fuselage, wing, tail and even propellers • Propeller flow can also be analyzed in OpenVSP – actuator disk model

Pros and Cons

Code	Pros	Cons
OVERFLOW	• Solves full viscous flow • Extensive Turbulence modelling • Overset grids can handle complex geometries	• Only available to US citizens, universities, companies • Restricted to overset grids • Hard to use for low Re flows
ICFD++	• Solves full viscous flow • Covers wide variety of flows, grids and turbulence models • Easy to use because of user interface	• Setup and solution is complicated and requires expert knowledge • Few sources available and costly to purchase

Pros and Cons

Code	Pros	Cons
CHARM	• Quick full vehicle calculations • Meshing is more straight forward than viscous flows • Hover and axial flight modelling of open and ducted rotors • Specifically suited for our use	• Inviscid flow assumption • Limited use at high AOA and sidewash condition • Reliance on airfoil data and other semi empirical models
FUN3D	• Solves full viscous flow • Grid motion is allowed • Covers a wide variety of flow regimes	• Only available to US citizens, universities, companies • Restricted to unstructured grids • Limited turbulence modelling

Pros and Cons

Code	Pros	Cons
ANSYS Fluent	• High degree of customizability of simulation • Solves full viscous flow with a wide variety of numerical solvers • Comprehensive modelling of flow physics • Comprehensive meshing and grid generation capabilities	• Difficult to setup and difficult to run – requires expert knowledge to specify every variable • Heavy computational power required for accurate results of complex analysis
OpenVSP	• Able to model the entire aircraft using potential flow methods • Propellers can be included in the analysis as well • Easy to use and easy to run	• Inviscid flow assumption • Drag calculation or viscous effects calculations is a problem • Incomplete modelling of aerodynamics interactions between the various parts of the aircraft

Other Available Sophisticated Codes

CAMRAD and RCAS – comprehensive codes for analyzing rotor dynamics (blade loads, vibratory hub loads, aeroelastic stability, aeromechanic stability, pylon whirl flutter, etc…)

FlightLab – for flight mechanics

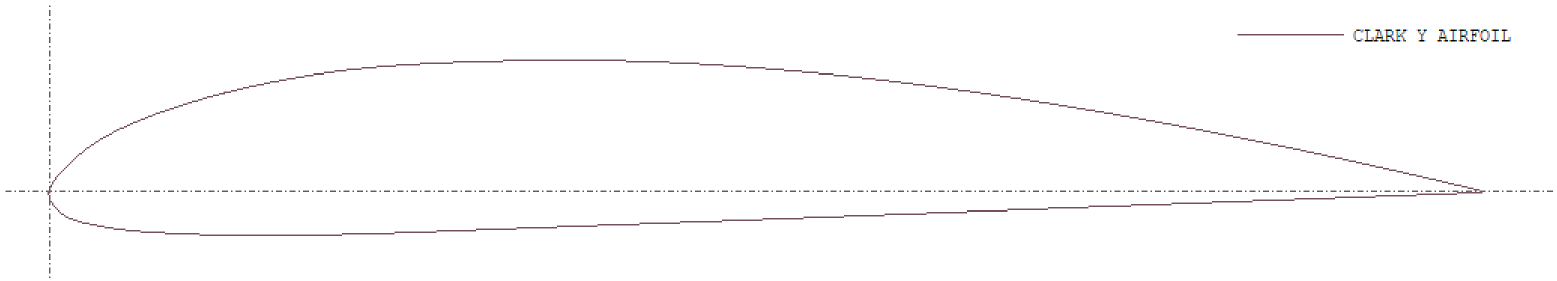
CIFER – for system identification

WOPWOP – rotor aeroacoustics

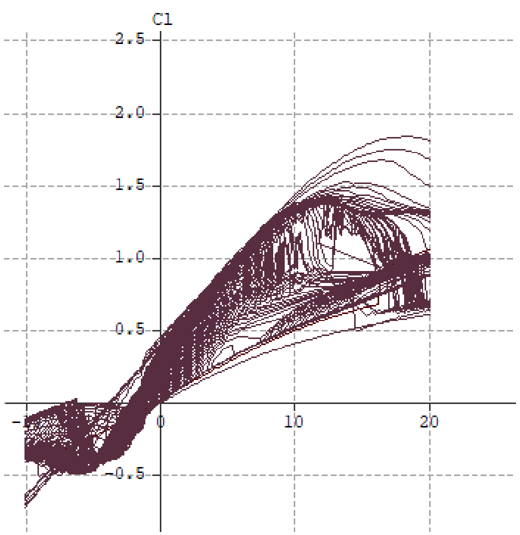
These codes require significant experience to set up inputs, run, and interpreting the results

AVL and XFLR5 are simple, no cost, off-the-shelf aerodynamics and stability analysis. XFLR5 originally developed for low Reynold Number model airplane

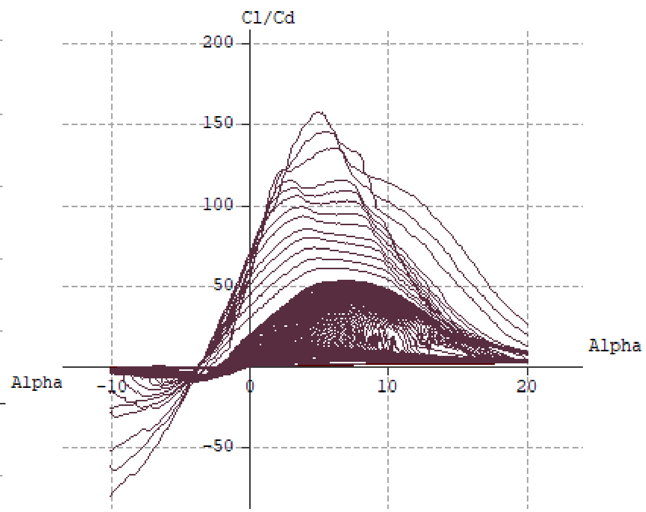
Airfoil Selection & Modelling



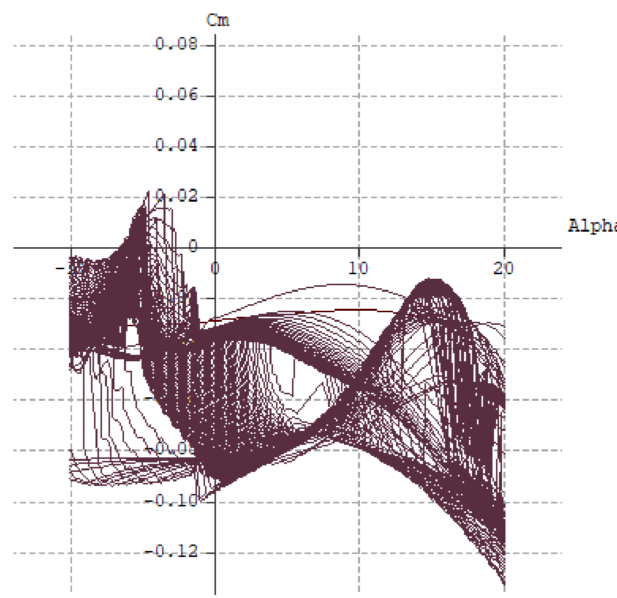
Generate Aerodynamic Curves for Various Airfoils



$C_l\ vs.\ \alpha$

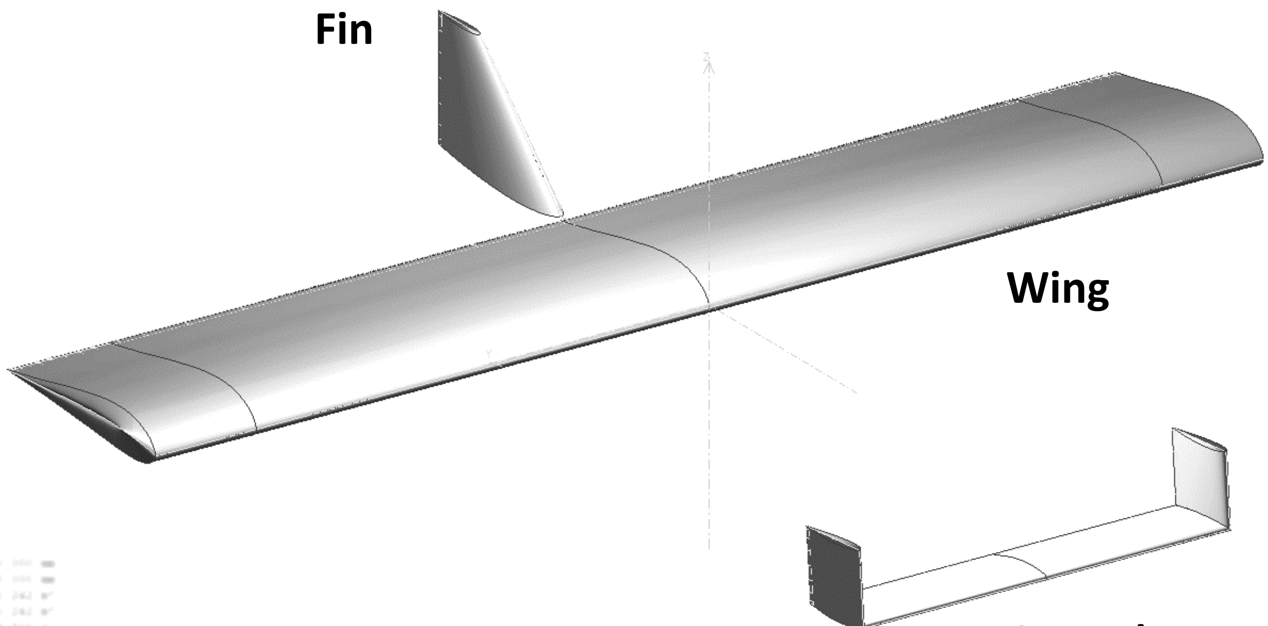


$C_l/C_d\ vs.\ \alpha$



$C_m\ vs.\ \alpha$

Aircraft Modeling & Wing Design



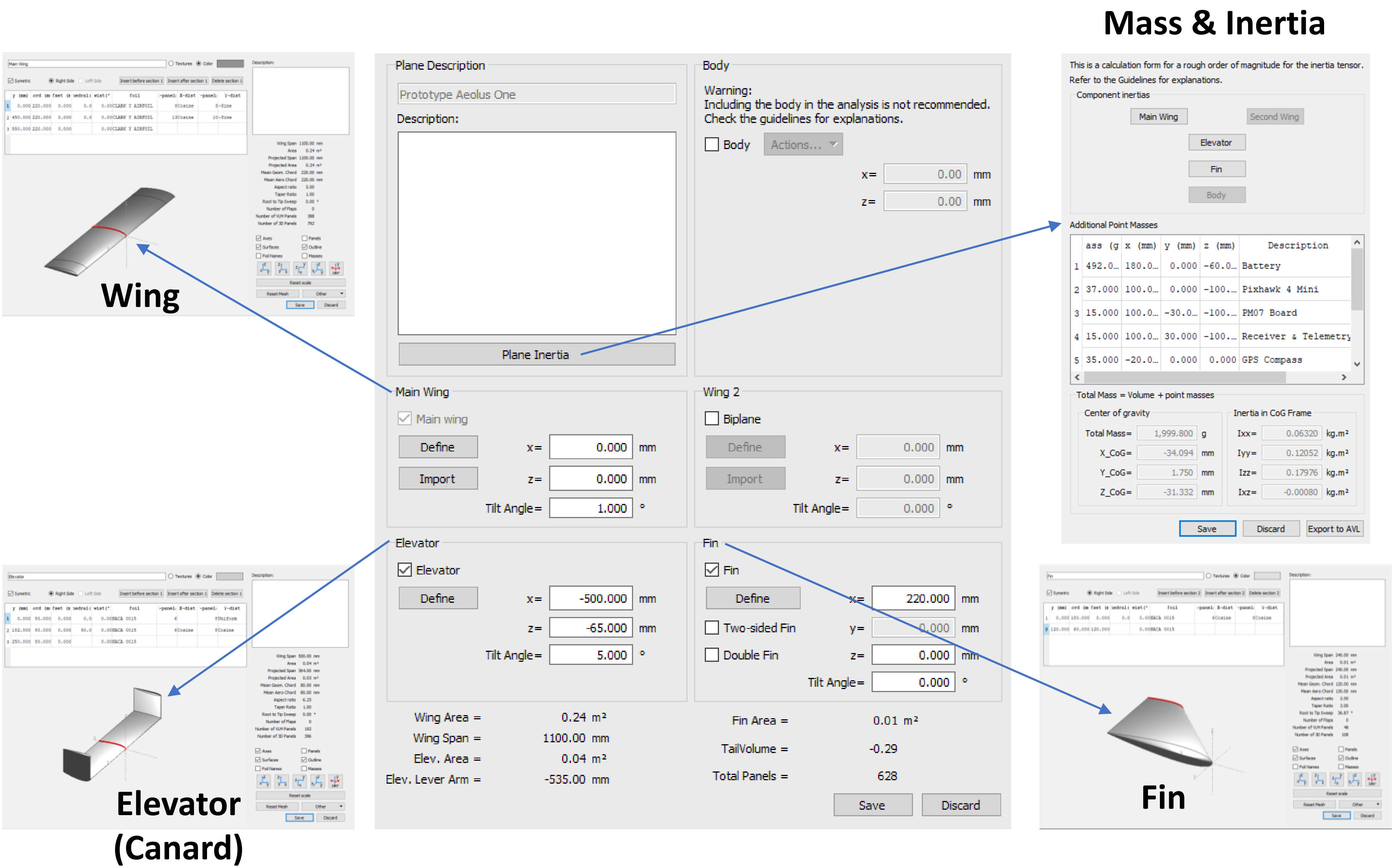

```
Prototype Aeolus One
Wing Span      =
xyProj. Span   =
Wing Area      =
xyProj. Area   =
Plane Mass     =
Wing Load      =
Tail Volume    =
Root Chord     =
MAC            =
TipTwist       =
Aspect Ratio   =
Taper Ratio    =
Root-Tip Sweep =
XNP = d(XCp.Cl)/d(
Mesh elements  =
```

Model Parameters

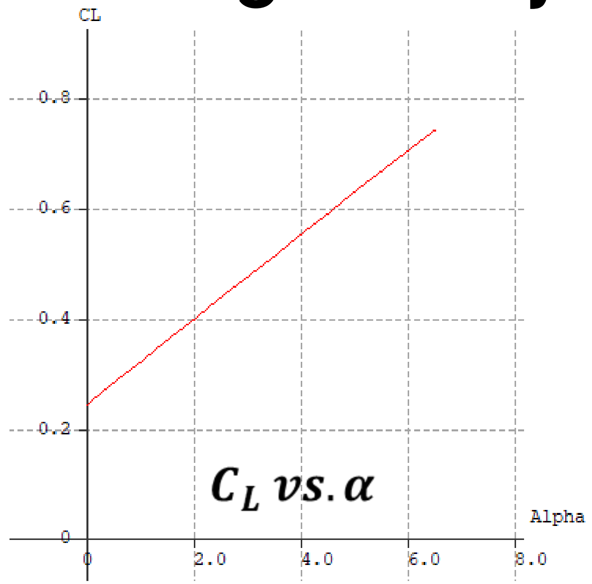
```
         V =
     Alpha =
      Beta =
        CL =
        CD =
Efficiency =
     CL/CD =
        Cm =
        Cl =
        Cn =
      X_CP =
      X_CG =
```

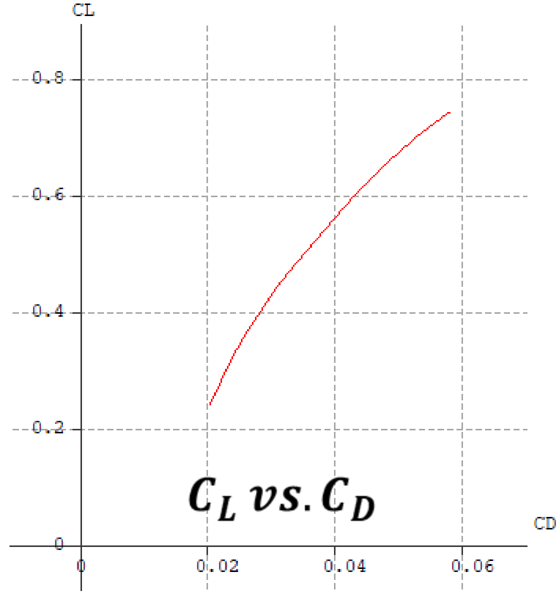
Flight Parameters

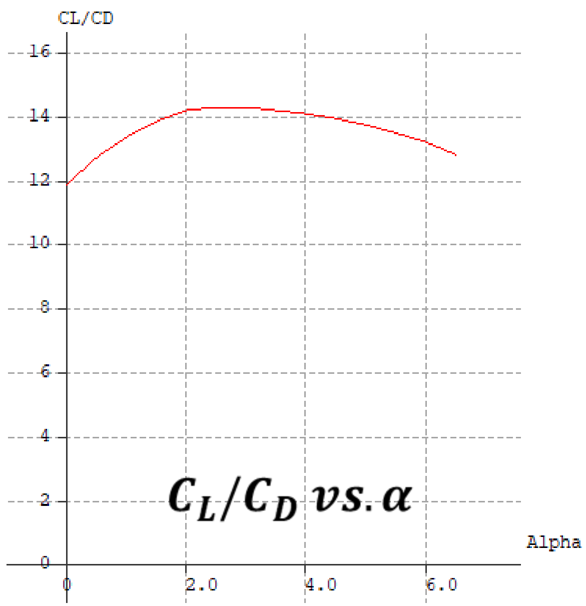
Simple Interface for Modeling

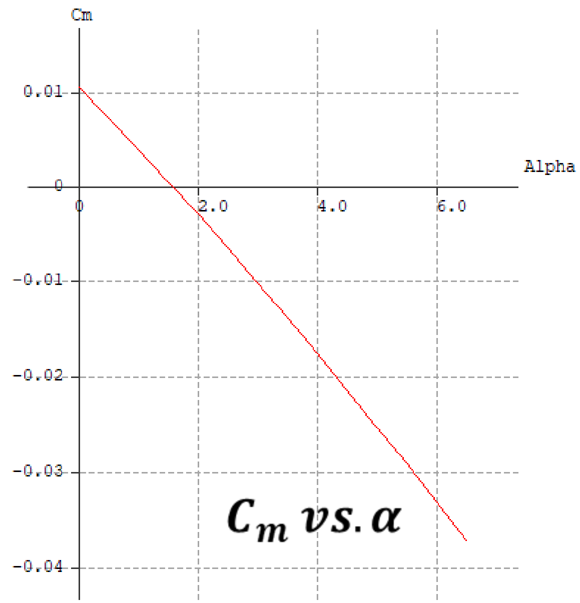


Generating Aerodynamic Coefficients

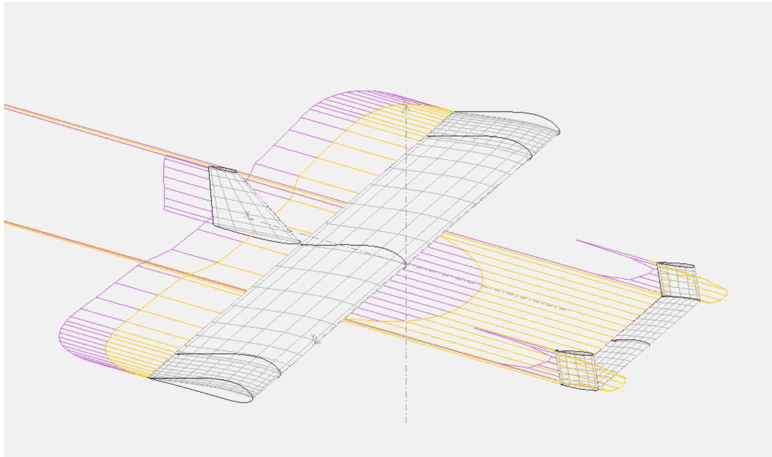




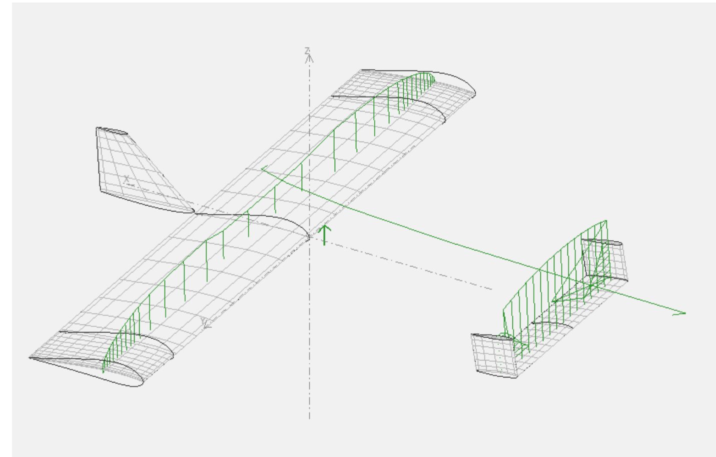




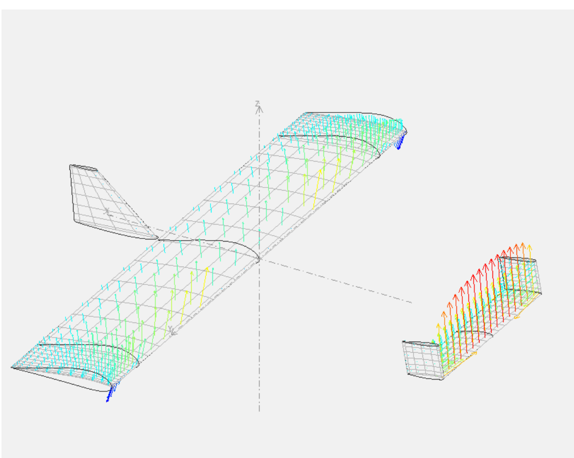
Visualization Tools



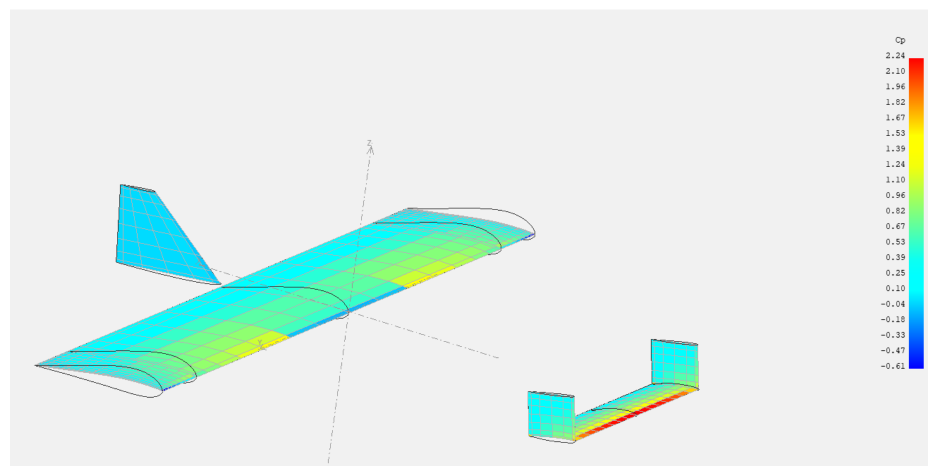
Induced & Vicious Drag



Lift Generated

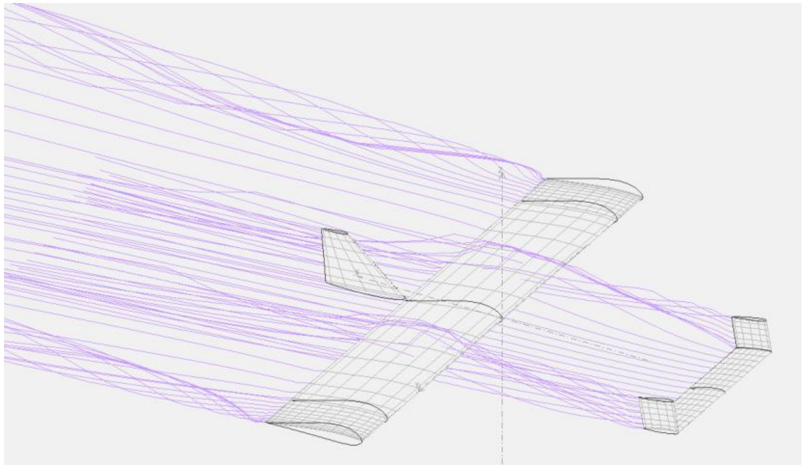


Forces on Surfaces

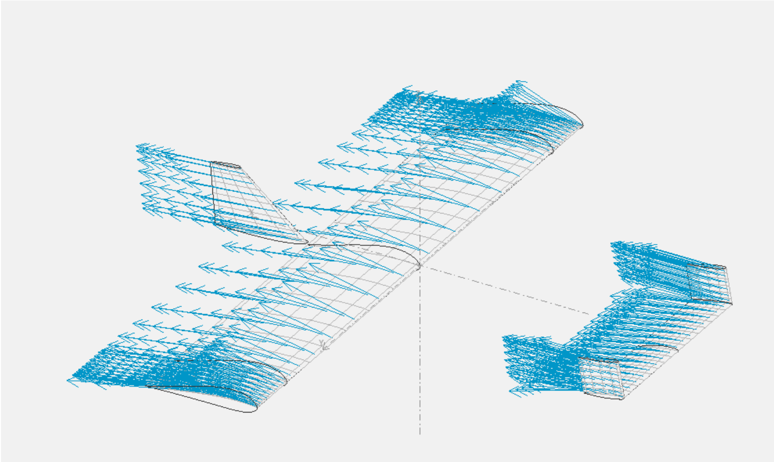


Pressure Coefficient

Visualization Tools



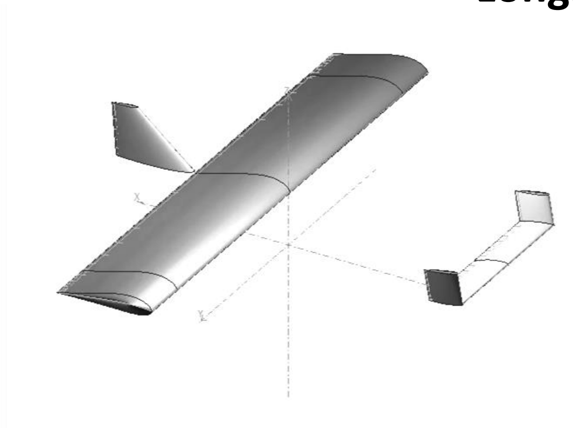
Streamlines



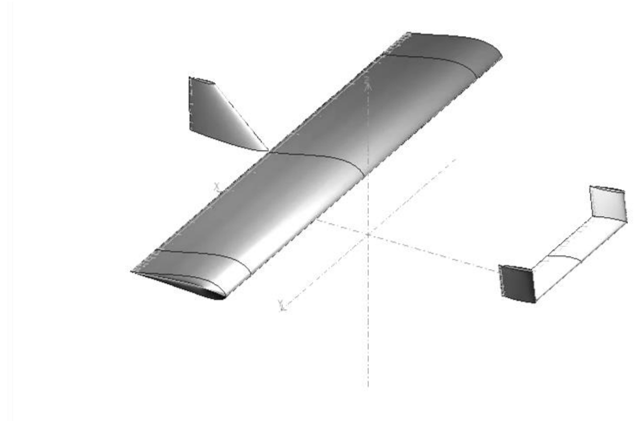
Surface Velocities

Stability Analysis

Longitudinal Modes

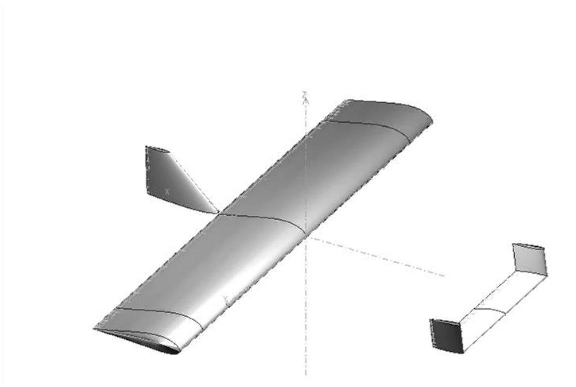


Short Period Mode

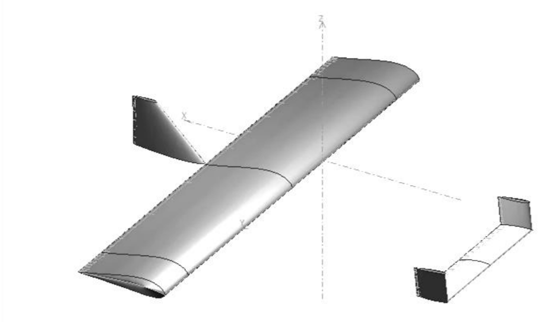


Long Period (Phugoid) Mode

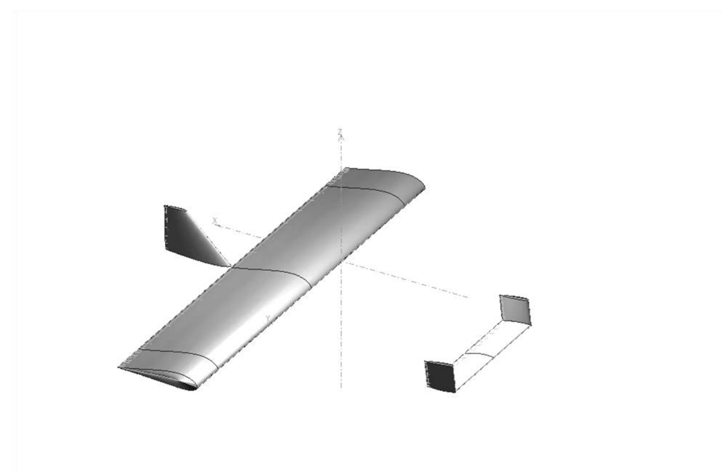
Lateral Modes



Roll Mode

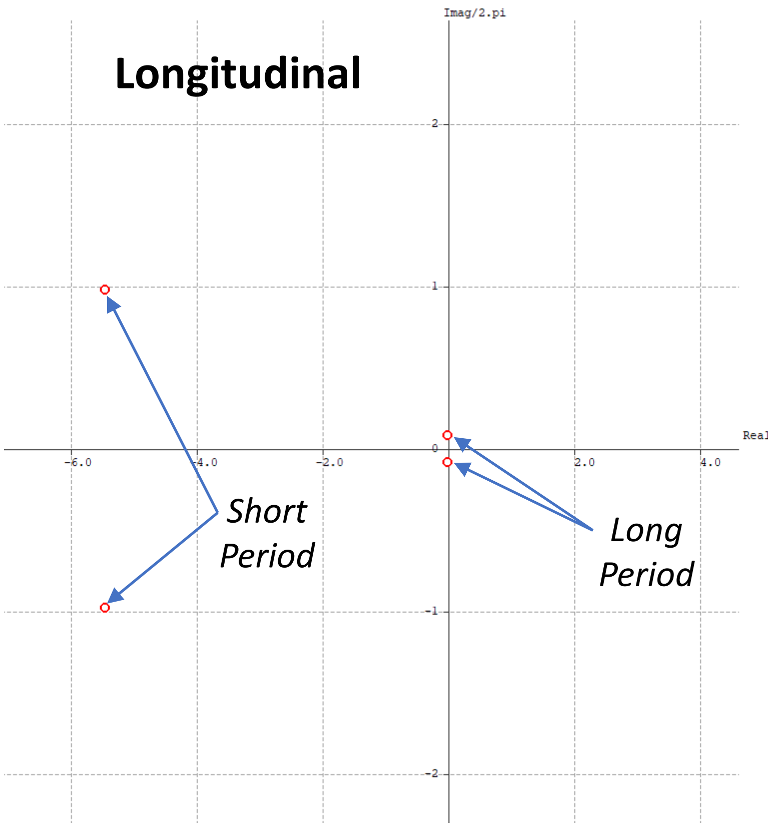


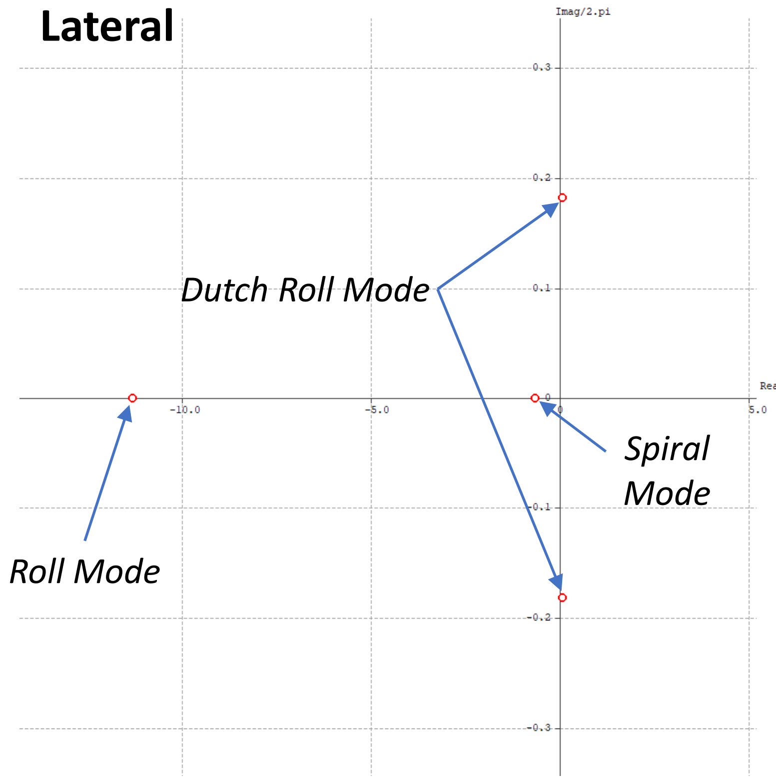
Dutch Roll Mode



Spiral Mode

Root Locus Plots & Stability Derivatives





Longitudinal derivatives

Xu=	-0.11958	Cxu=	-0.041399
Xw=	0.50005	Cxa=	0.17312
Zu=	-1.9924	Czu=	0.0072866
Zw=	-12.841	CLa=	4.4455
Zq=	-1.7272	CLq=	5.4358
Mu=	-0.0016525	Cmu=	-0.0026004
Mw=	-0.25068	Cma=	-0.39448
Mq=	-0.53511	Cmq=	-7.6551

Neutral Point position= -0.01457 m

Lateral derivatives

Yv=	-0.51864	CYb=	-0.17955
Yp=	0.0052937	CYp=	0.0033321
Yr=	0.02543	CYr=	0.016007
Lv=	-0.12236	Clb=	-0.038508
Lp=	-0.70224	Clp=	-0.40184
Lr=	0.09371	Clr=	0.053624
Nv=	-0.021586	Cnb=	-0.0067937
Np=	-0.09568	Cnp=	-0.054751
Nr=	-0.074203	Cnr=	-0.042461

Example of a Sophisticated Off-the-Shelf Rotorcraft Conceptual Design Tool from NASA

(For US use only)

NDARC

NASA Design and Analysis of Rotorcraft

NDARC Usage

Principal Tasks:

- Facilitate design of new rotorcraft concepts
  - Synthesis: Create new concepts from library of components
  - Size: Parametrically vary components to meet specified requirements

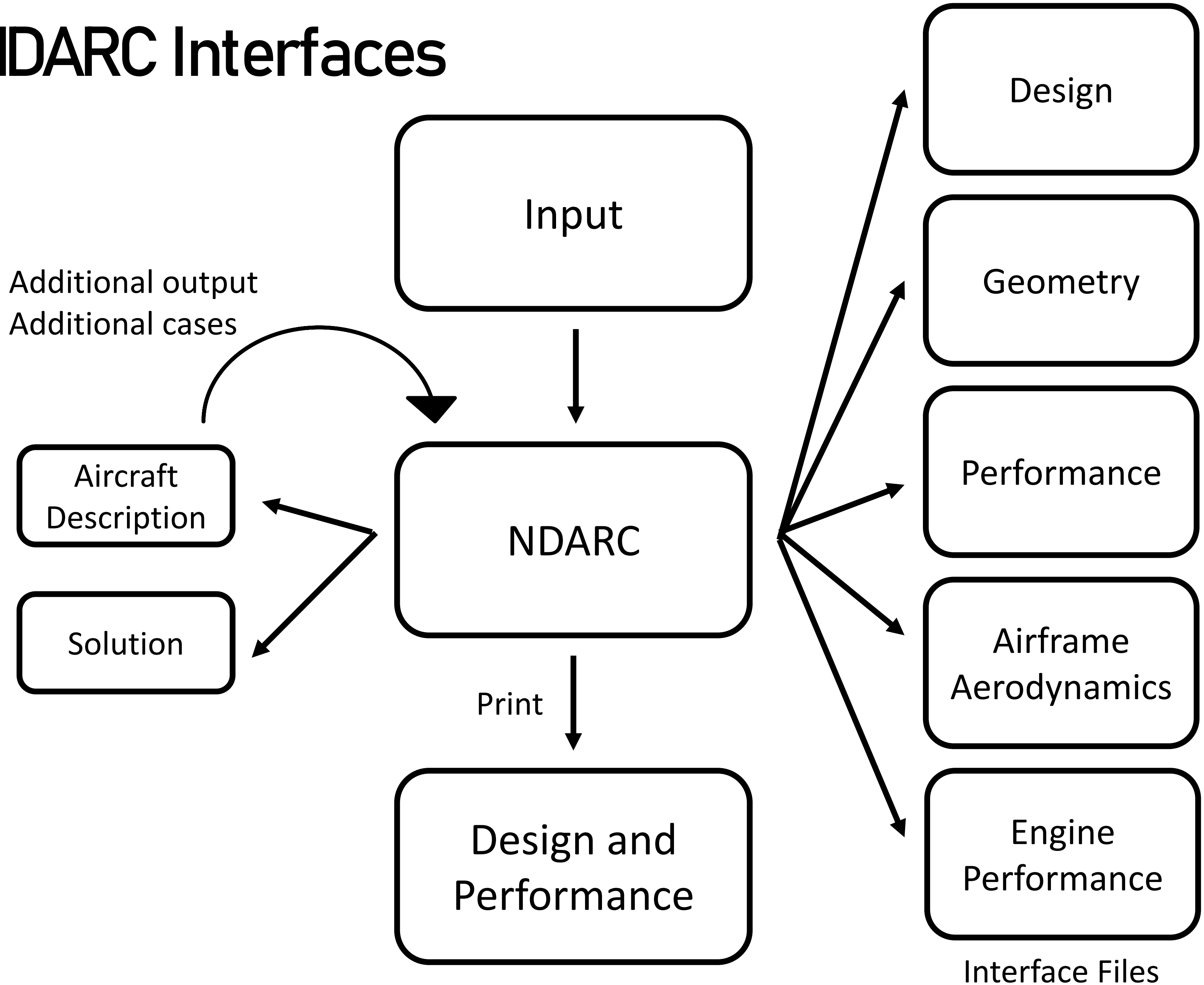
- Analyze rotorcraft air vehicle systems
  - Calculate performance at specified flight conditions
  - Mission

NDARC Features

Critical Attributes:

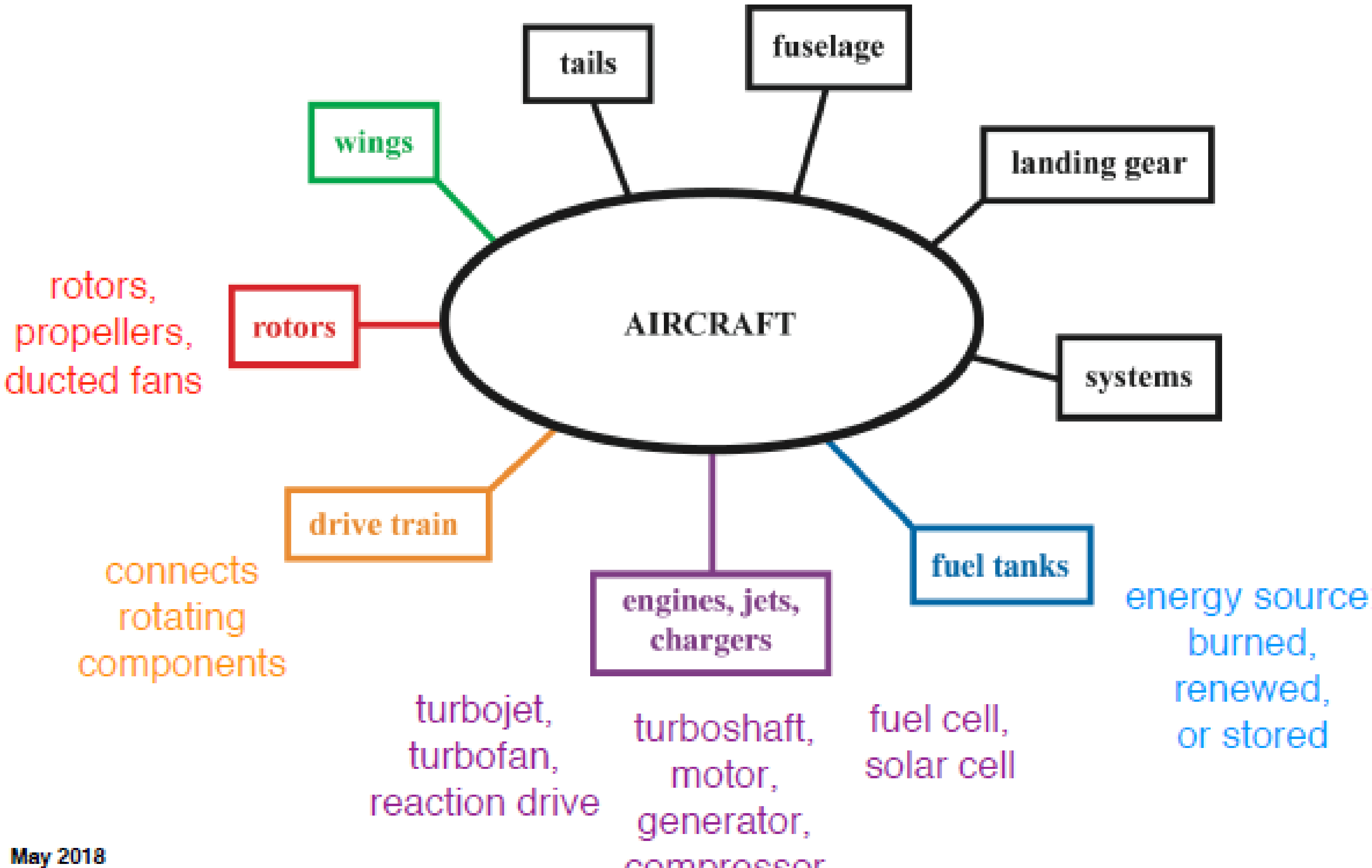
- Rapid Turnaround – Execute case on order of minutes
- Flexible Sizing Constraints – Sized based on multiple missions and performance points
- Configuration Generality – ability to model broad array of rotorcraft concepts
- Capture Technology Impact – able to consider new technology at a system and component level
- Extensible – Analysis capability and code architecture should not inhibit creativity, easy modification for individual projects
- Documentation – Complete documentation of theory and code

NDARC Interfaces

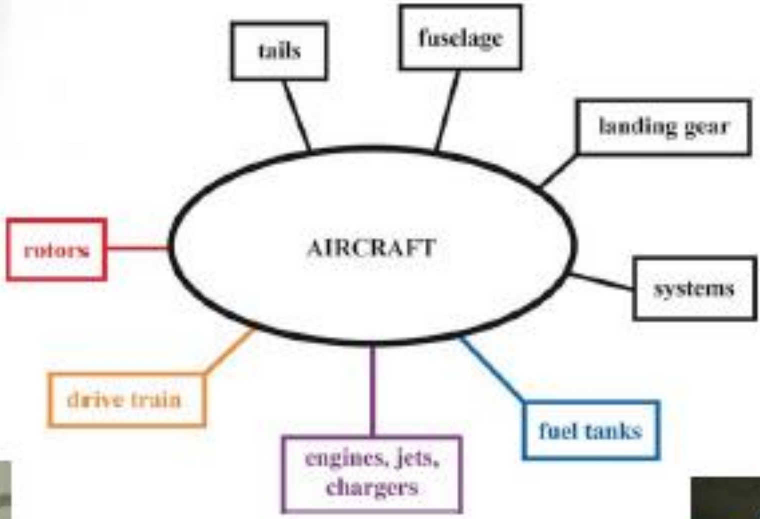


NDARC Components to Construct Aircraft

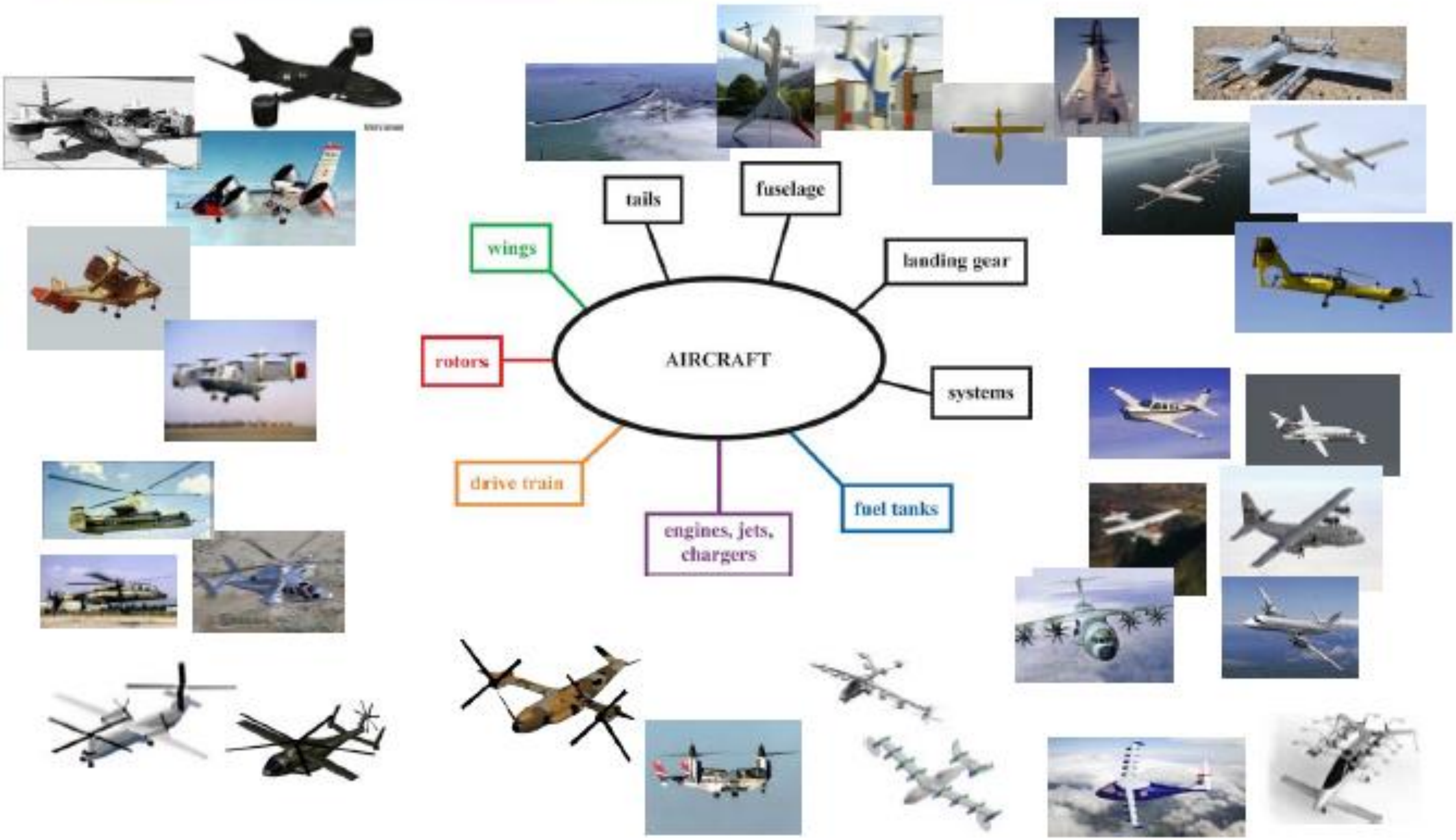
Critical to achieving capability to model wide array of rotorcraft concepts is decomposition of aircraft into set of fundamental components



Lots of Rotors

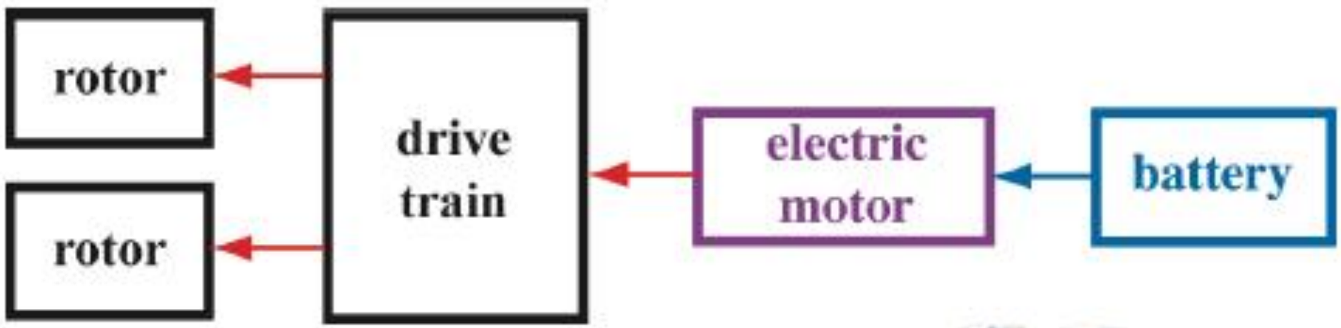


Rotors and Wings



Electric Motor

Majority of eVTOL aircraft designs in the world (90%) are of this type of propulsion



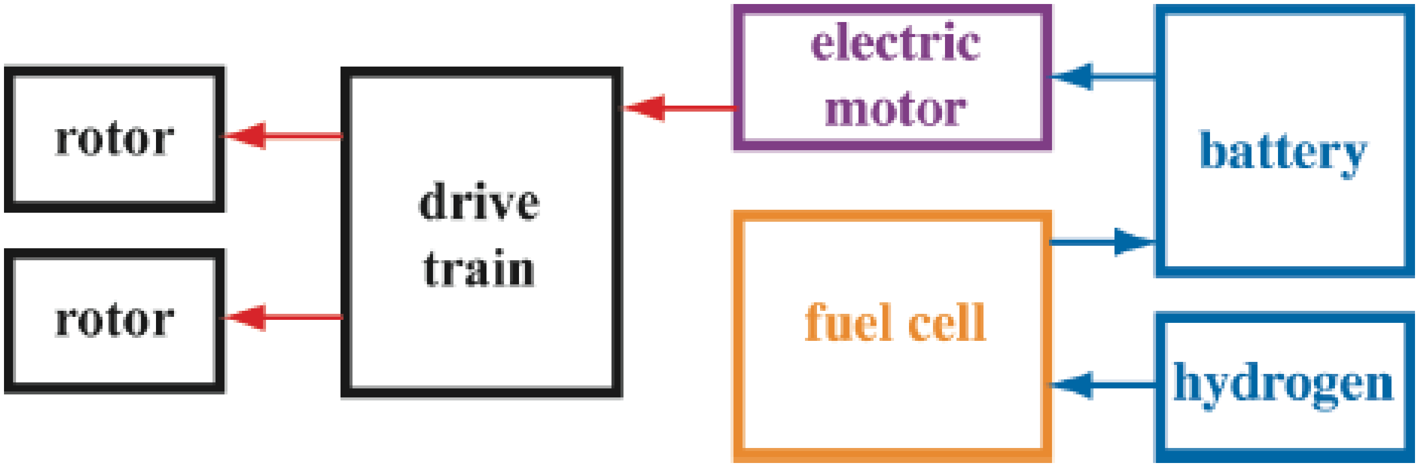






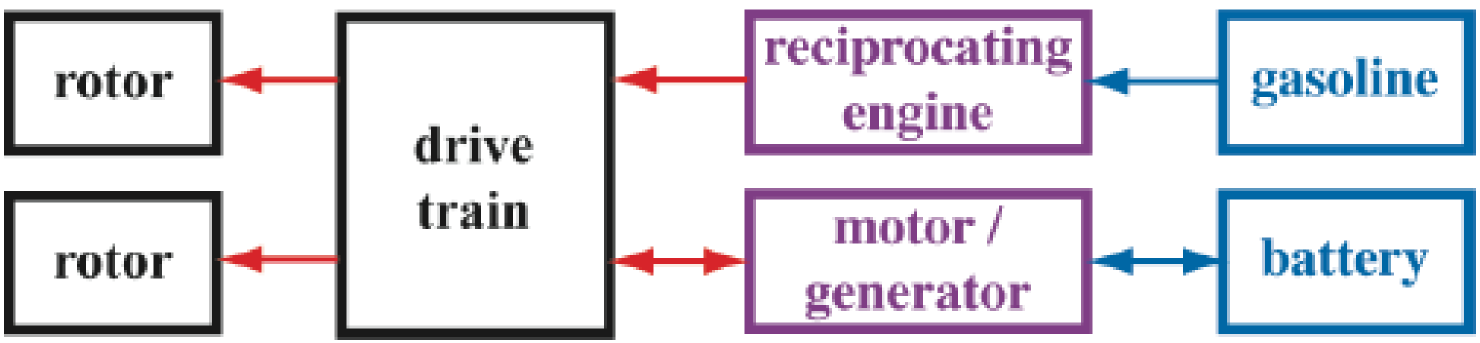



Fuel Cell



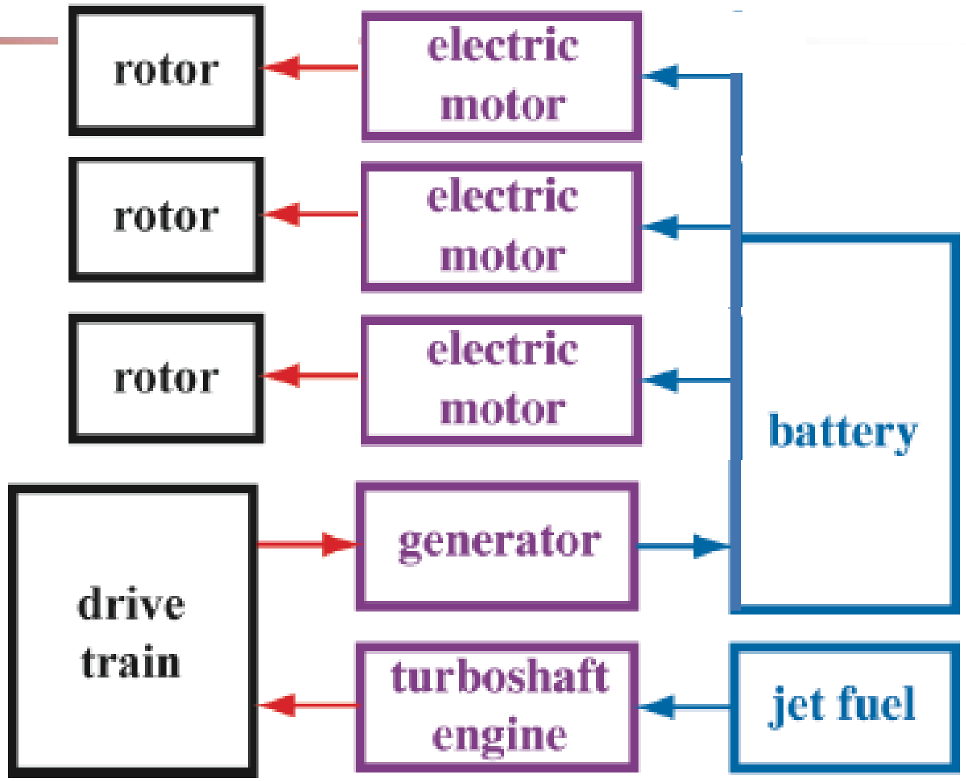
So far no one is utilizing this scheme for eVTOL aircraft

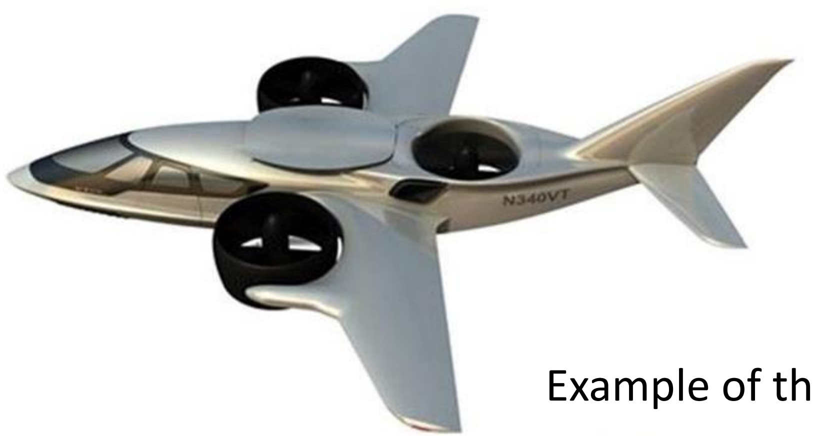
Hybrid



So far no one is utilizing this scheme for eVTOL aircraft

Turbo-Electric





Example of this type: XTI Trifan 600

Control, Trim, Power, Size

- **Depend on aircraft and propulsion configuration**

- **Conventional helicopter: single main rotor and tail rotor, with turboshaft propulsion**
  - **Control:** 4 pilot's controls (coll, cyclic, pedal) connected to 4 rotor controls (mr collective and cyclic, tr collective)
  - **Trim:** Zero net force and moment on aircraft obtained using 4 pilot's controls plus 2 aircraft attitude angles (pitch and roll)
  - **Power required:** engine power = rotor power + transmission loss + accessory power (perhaps distributed to multiple engines)
  - **Engine size:** power = maximum required over all conditions
  - **Fuel tank size:** maximum fuel required over all missions

6. VTOL Rotor Aerodynamics Analysis

by Dr. James Wang

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