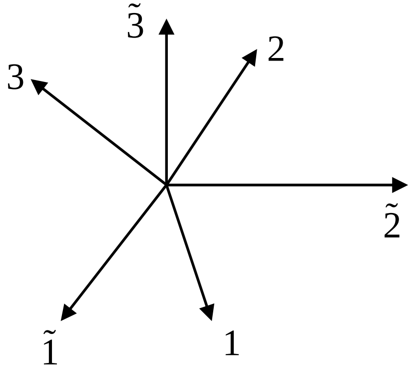
Rotation of Material Properties

❖ Rotation of Material Properties



(˜) : material coordinate system

(   ) : structural coordinate system

$$\begin{bmatrix} \tilde{D} \\ \tilde{T} \end{bmatrix} = \begin{bmatrix} \tilde{\varepsilon}^{s} & \tilde{e} \\ \tilde{e} & \tilde{c}^{E} \end{bmatrix} \begin{Bmatrix} \tilde{E} \\ \tilde{S} \end{Bmatrix}$$

In general, use tensor transformations

$$\tilde{D}_{m} = a_{mp} D_{p}$$

First order tensor transformation

(D, E)

$$\tilde{T}_{mn} = a_{\tilde{m}p} a_{\tilde{n}g} \, T_{pg}$$

$$\boxed{T_{mn} = a_{m\tilde{p}} a_{n\tilde{g}} \, \tilde{T}_{pg}}$$

Inverse transformation

- good for any tensor material property rotations

Rotation of Material Properties

- **Matrix**

$$\tilde{D} = F\ D \qquad .\ .\ .\ .\ (1)$$

$$\tilde{E} = F\ E \qquad .\ .\ .\ .\ (2)$$

	1	2	3
$\tilde{1}$	$a_{\tilde{1}1}$	$a_{\tilde{1}2}$	$a_{\tilde{1}3}$
$\tilde{2}$	$a_{\tilde{2}1}$	$a_{\tilde{2}2}$	$a_{\tilde{2}3}$
$\tilde{3}$	$a_{\tilde{3}1}$	$a_{\tilde{3}2}$	$a_{\tilde{3}3}$

$a_{\tilde{i}j}$: direction cosine $c_{\tilde{i}j}$

$$F = \begin{bmatrix} a_{\tilde{1}1} & a_{\tilde{1}2} & a_{\tilde{1}3} \\ a_{\tilde{2}1} & a_{\tilde{2}2} & a_{\tilde{2}3} \\ a_{\tilde{3}1} & a_{\tilde{3}2} & a_{\tilde{3}3} \end{bmatrix}$$

$$\tilde{T} = AT \qquad .\ .\ .\ .\ (3)$$

$$\tilde{T}_{11} = (a_{\tilde{i}j},\ \mathrm{T}_{ij})$$

$$\tilde{s}\ =\ B\ S \qquad .\ .\ .\ .\ (4)$$

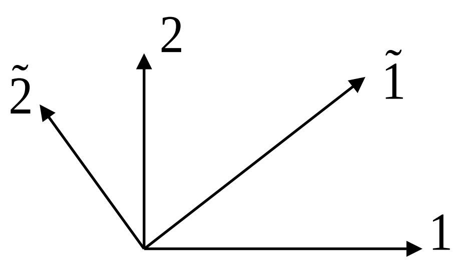
$$B = A(\beta = 2,\ \alpha = 1)$$

Rotation of Material Properties

$$\begin{bmatrix} F & 0 \\ 0 & A \end{bmatrix} \begin{Bmatrix} D \\ T \end{Bmatrix} = \begin{bmatrix} \tilde{\varepsilon}^S & \tilde{e} \\ -\tilde{e}_t & \tilde{c}^E \end{bmatrix} \begin{bmatrix} F & 0 \\ 0 & B \end{bmatrix} \begin{Bmatrix} E \\ S \end{Bmatrix}$$

$$\begin{bmatrix} \tilde{\varepsilon}^S & \tilde{e} \\ -\tilde{e}_t & \tilde{c}^E \end{bmatrix} = \begin{bmatrix} F^{-1} & 0 \\ 0 & \mathrm{A}^{-1} \end{bmatrix} \begin{bmatrix} \tilde{\varepsilon}^S & \tilde{e} \\ -\tilde{e}_t & \tilde{c}^E \end{bmatrix} \begin{bmatrix} F & 0 \\ 0 & B \end{bmatrix}$$

- **2-D specialization**



	1	2	3
$\tilde{1}$	c	s	0
$\tilde{2}$	$-s$	c	0
$\tilde{3}$	0	0	1

$$\tilde{D} = R_E D = FD$$

$$\tilde{E} = R_E E = FE$$

$$R_E = \begin{bmatrix} c & \mathrm{s} & 0 \\ -s & \mathrm{c} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{S} = BS = R_S S$$

Rotation of Material Properties

$$R_S = \begin{bmatrix} c^2 & s^2 & 0 & 0 & 0 & cs \\ s^3 & c^3 & 0 & 0 & 0 & -cs \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & -s & 0 \\ 0 & 0 & 0 & s & c & 0 \\ -cs & cs & 0 & 0 & 0 & c^2 - s^2 \end{bmatrix}$$

$$R_T = (R_{s_t})^{-1}$$

$$\tilde{T} = (R_{\varepsilon_t})^{-1}$$

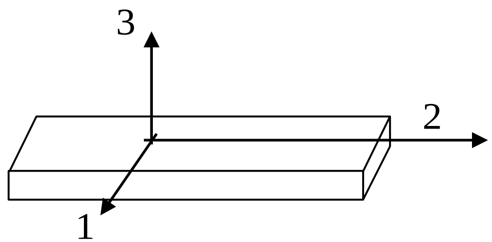
$$R_E{}^T = (R_E)^{-1}$$

$$\begin{Bmatrix} D \\ T \end{Bmatrix} = \begin{bmatrix} R_{E_t} \tilde{\varepsilon}^S R_E & R_{E_t} \tilde{e} R_S \\ -R_{S_t} \tilde{e}_t R_E & R_{S_t} \tilde{c}^E R_S \end{bmatrix} \begin{Bmatrix} E \\ S \end{Bmatrix}$$

❖ Plane stress & strain

- Plane Stress

→ reduction of material properties



i) $T_3 \ll T_1,\ T_2$

ii) Ignore shear $T_4,\ T_5 \rightarrow S_4,\ S_5 = 0$

iii) $E_3 \gg E_1,\ E_2$

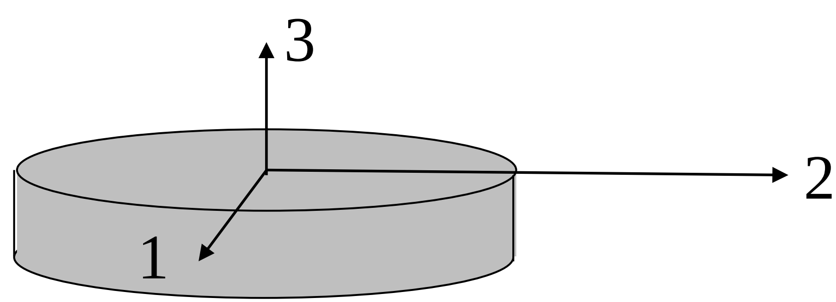
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ S_1 \\ S_2 \\ \vdots \\ S_6 \end{bmatrix} = \begin{bmatrix} \varepsilon & d \\ d_t & s \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ T_1 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \begin{matrix} = 0 \\ = 0 \\ \\ \\ = 0 \\ = 0 \\ = 0 \\ \\ \end{matrix}$$

Plane stress & strain

$$\rightarrow \begin{bmatrix} D_3 \\ S_1 \\ S_2 \\ S_6 \end{bmatrix} = \begin{bmatrix} \varepsilon_{33}{}^{T} & d_{31} & d_{31} & 0 \\ d_{31} & s_{11}{}^{E} & s_{12}{}^{E} & 0 \\ d_{31} & s_{12}{}^{E} & s_{22}{}^{E} & 0 \\ 0 & 0 & 0 & s_{66}{}^{E} \end{bmatrix} \begin{bmatrix} E_3 \\ T_1 \\ T_2 \\ T_6 \end{bmatrix}$$

$$\begin{Bmatrix} D_3 \\ T_1 \\ T_2 \\ T_6 \end{Bmatrix} = \begin{bmatrix} & \\ & c^E \end{bmatrix} \begin{Bmatrix} E_3 \\ S_1 \\ S_2 \\ S_6 \end{Bmatrix}$$

- **Plane Strain**
  - $E_3 >> E_1,\ E_2$



- $S_3,\ S_4,\ S_5$ are zero