

Micro Electro Mechanical Systems for mechanical engineering applications

Lecture 6: Device examples (1): piezoresistivity transduction

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Generalized Classifications

Output Input	Mechanical (mechano-)	Thermal (thermo-)	Electrical (electro-)	Magnetic (magneto-)	Radiant (photo- or Radio-)	Chemical (chemo-)
Mechanical	Acoustics fluidics	Cooling Effects	Piezoelectric Piezoresistor	Piezomagnetic	Photo Elasticity	
Thermal	Bi-metallic		Seebeck effect (TC's)			
Electrical	Piezoelectric	Peltier effect (TC's)			Electro- luminescence	
Magnetic	Magnetometer	Thermo- Magnetic				Electrolysis
Radiant		Thermo- Pile	Photo- electric			
Chemical		Calorimeter		Nuclear Magnetic resistance		

Madou, Fundamentals of Microfabrication, 2003

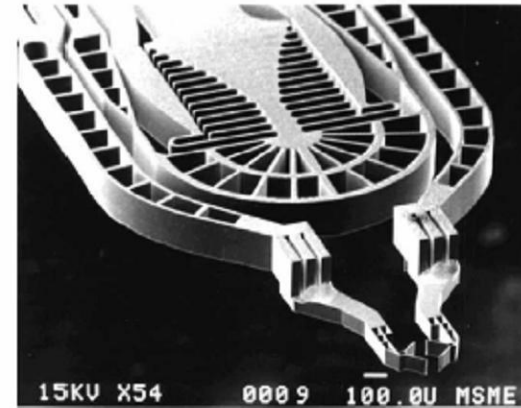


Piezoresistive Transduction

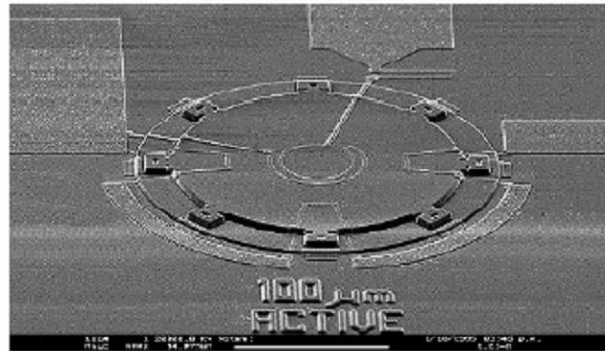
- Applied stress/strain affects resistance in piezoresistive materials
- Discovered in Si in 1954 (Bell labs)
- Physics: majority carrier mobility affected by stress:
 - In p-type Si, hole mobility decreases: R increases
 - In n-type Si, electron mobility increases: R decreases
- Advantages:
 - Simple fabrication
 - Simple interface circuits: measure change in R using a simple Wheatstone bridge topology
- Disadvantages:
 - Temperature sensitive
 - High thermal noise



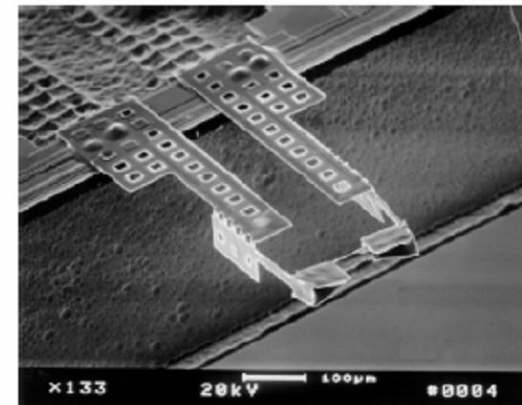
Piezoresistive MEMS Examples (1)



Force-feedback microgripper



Pressure sensors

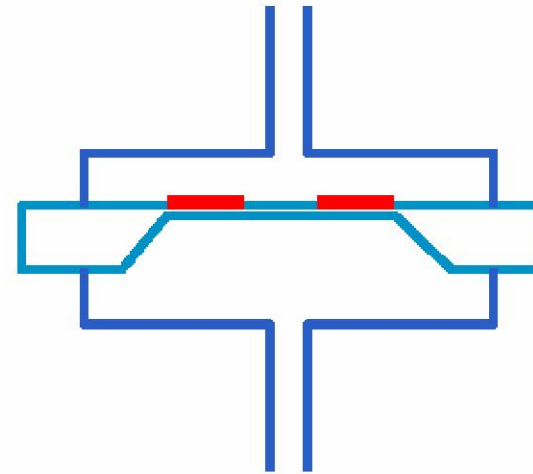
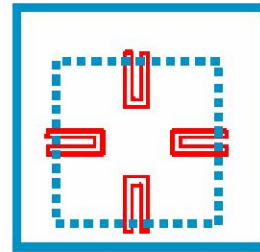


Cell force sensor

Piezoresistive MEMS Examples (2)

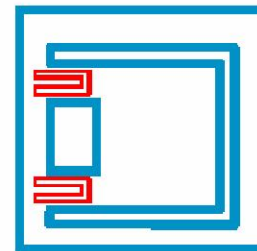
- **Membrane type**

Pressure sensor:



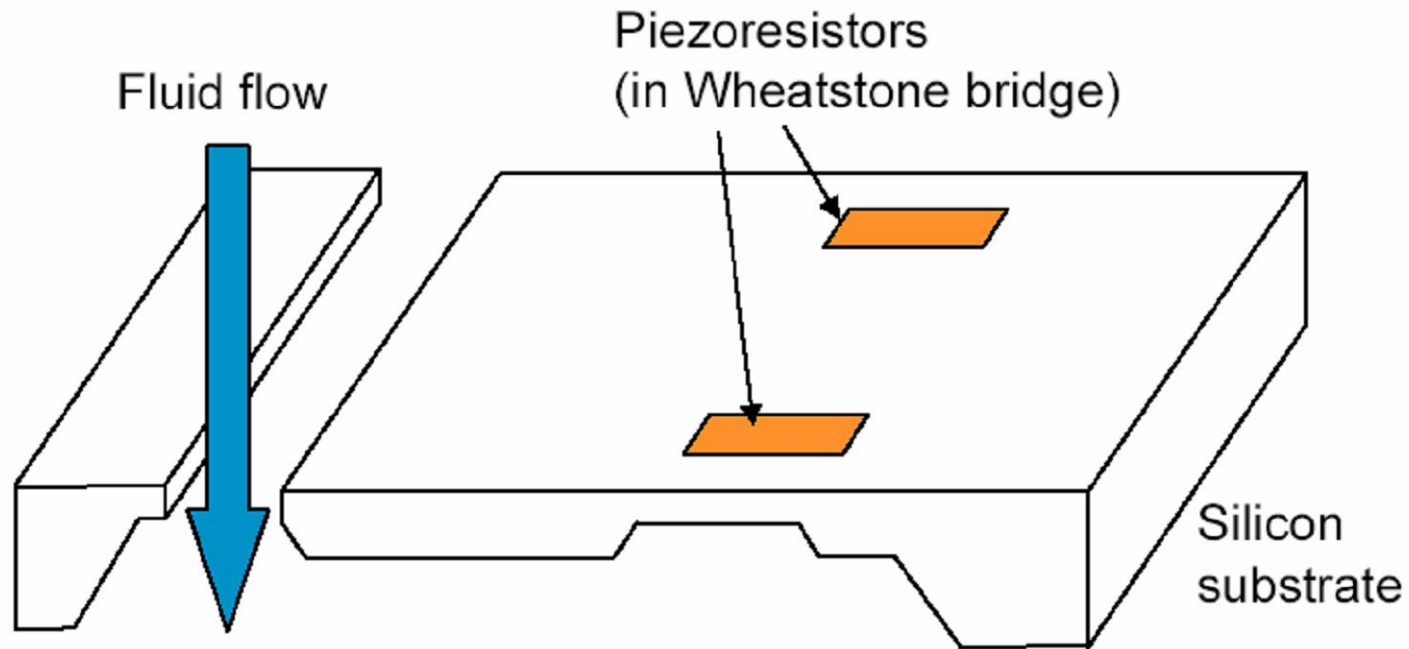
- **Cantilever type**

Accelerometer:



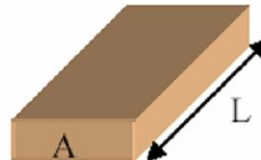
Piezoresistive MEMS Examples (3)

Piezoresistive Flow Sensing



- Cantilever deflection is translated into a voltage signal via the Wheatstone bridge.
- Voltage change is interpreted as flow rate.
- Can sense direction and magnitude of flow.

- Consider the resistance of a rectangular beam with resistivity ρ [Ω cm]:



$$R = \frac{\rho L}{A}$$

$$\ln R = \ln \rho + \ln L - \ln A$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} = \frac{d\rho}{\rho} + \varepsilon(1+\nu)$$

↑ piezoresistive effect
↑ gauge effect

- For semiconductor piezoresistors, $d\rho/\rho \gg \varepsilon(1+2\nu)$: $\frac{dR}{R} \approx \frac{d\rho}{\rho}$
- Gauge factor: $GF = \frac{1}{\varepsilon} \frac{dR}{R} \longrightarrow R(\varepsilon) = R_0 + dR = R_0(1 + GF \varepsilon)$

Material	Gauge factor (GF)
Metal foil ¹	1-5
Thin-film metals ¹	~2
Doped SCS ¹	80-200
Doped poly-Si ²	15-27

G. Kovacs, Micromachined Transducers Sourcebook, 1998
Seto, J. Appl. Phys., 1976

Piezoresistivity in Single Crystal Si

$$\frac{dR}{R} = \Pi_{ij} \sigma_{ij}$$

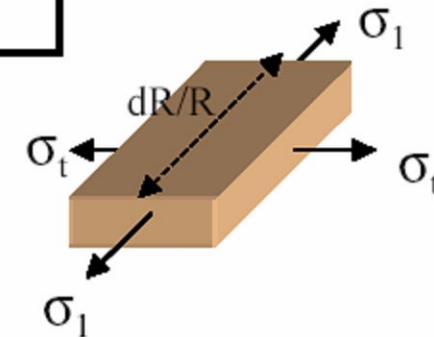
\swarrow stress tensor
 \nwarrow piezoresistive coefficient tensor

material	ρ [Ωcm]	Π_{11} [10^{-11}Pa^{-1}]	Π_{12} [10^{-11}Pa^{-1}]	Π_{44} [10^{-11}Pa^{-1}]
n-type Si	11.7	-102.2	53.4	-13.6
p-type Si	7.8	6.6	-1.12	138.1
p-type polySi	0.005	$dR/R = \Pi\sigma, \quad \Pi = 12 \times 10^{-11} \text{ Pa}^{-1}$		

- **For 1-dimensional loading:**

$$\frac{dR}{R} = \Pi_l \sigma_l + \Pi_t \sigma_t$$

Longitudinal direction \swarrow
 Transverse direction \nwarrow



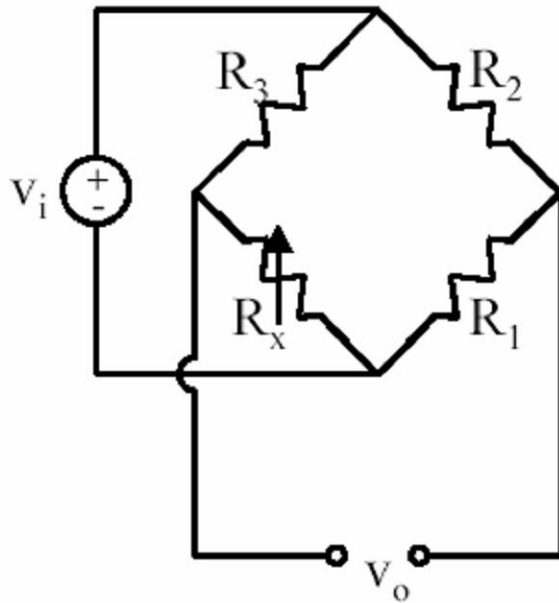
- **For the $\langle 110 \rangle$ direction:**

$$\Pi_l = \frac{1}{2} (\Pi_{11} + \Pi_{12} + \Pi_{44})$$

$$\Pi_t = \frac{1}{2} (\Pi_{11} + \Pi_{12} - \Pi_{44})$$

Piezoresistive Sensor Interfacing

- Wheatstone bridge
 - Good sensitivity
 - Temperature independent (to first order)
 - Simple implementation:



$$v_o = v_i \left(\frac{R_x}{R_x + R_3} - \frac{R_1}{R_1 + R_2} \right)$$

$$@ v_o = 0 :$$

$$\frac{R_x / R_3}{R_x / R_3 + 1} = \frac{R_1 / R_2}{R_1 / R_2 + 1} \Rightarrow \frac{R_x}{R_3} = \frac{R_1}{R_2}$$

Piezoresistive Sensor Interfacing (Con't)

- Simple case:

let $R_1=R_2=R_3=R_o$, and let $R_x=R_o(1+x)$:

$$\begin{aligned}v_o &= v_i \left(\frac{R_o(1+x)}{R_o(1+x)+R_o} - \frac{R_o}{R_o+R_o} \right) \\ &= v_i \left(\frac{(1+x)}{(2+x)} - \frac{1}{2} \right) \\ &= v_i \left(\frac{x}{4+4x} \right) \quad \xrightarrow{\text{for } x \ll 1} \quad v_o \approx v_i \frac{x}{4}\end{aligned}$$

- Output is linear for small strains
- Response in terms of GF, strain:

$$GF = \frac{1}{\varepsilon} \frac{\Delta R}{R} \quad \Delta R = R - R_o = R_o(1+x) - R_o = x$$

$$v_o \approx v_i \frac{GF}{4} \varepsilon$$

Piezoresistive Sensor Interfacing (Con't)

- 2-R case:

let $R_1=R_3=R_o$, and let $R_x=R_o(1+x)$:

$$v_o = v_i \left(\frac{R_o(1+x)}{R_o(1+x)+R_o} - \frac{R_o}{R_o+R_o(1+x)} \right)$$

$$= v_i \left(\frac{x}{(2+x)} \right)$$

for $x \ll 1$

$$v_o \approx v_i \frac{x}{2}$$

$$v_o \approx v_i \frac{GF}{2} \epsilon$$

