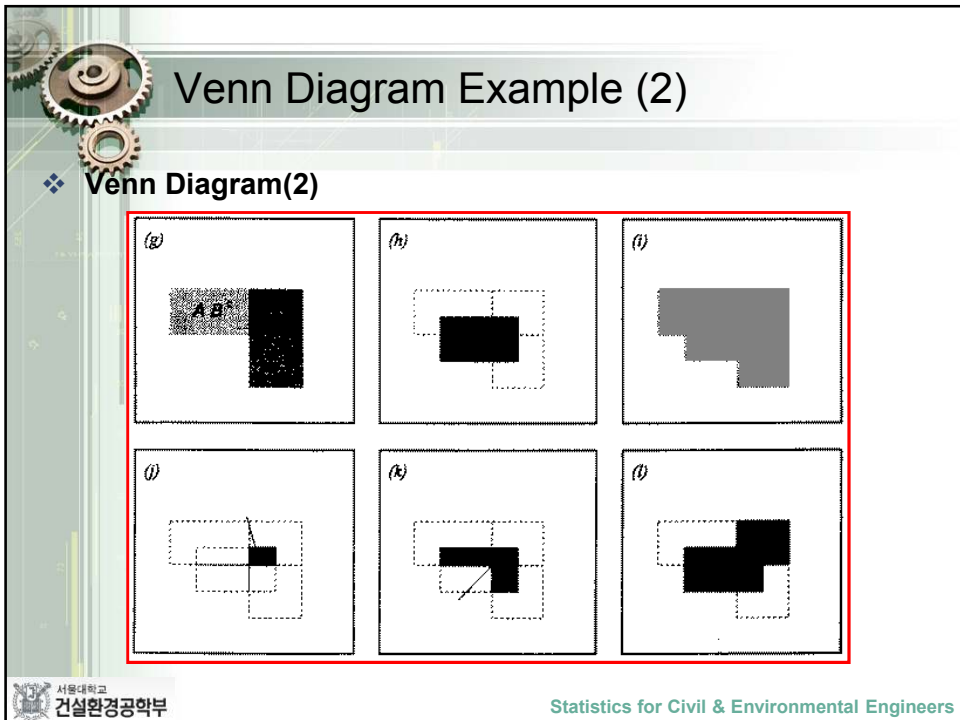
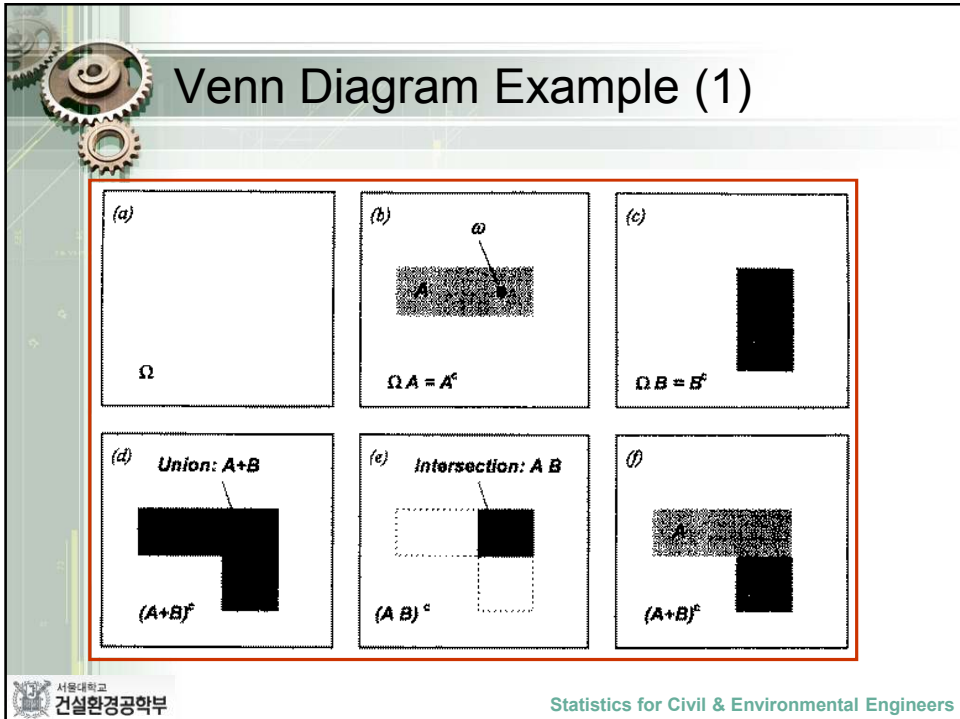
The slide content is set against a light green background with a gear icon on the left. The title "Venn Diagram" is in a large, black, sans-serif font. Below the title are two bullet points, each starting with a blue diamond symbol. The first bullet point describes the utility of Venn diagrams for visualizing set operations. The second bullet point, starting with the word "However" in red, notes that Venn diagrams are not used for defining probability relationships. At the bottom left is the Seoul National University logo and the text "서울대학교 건설환경공학부". At the bottom right is the text "Statistics for Civil & Environmental Engineers".


Venn Diagram

- ❖ **Venn Diagrams** provide a very useful visual representation of sets and set operations such as the complement, union, intersection, and the other combinations
- ❖ **However**, a Venn diagram is not intended to be used in defining relationships among the probabilities of events

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Measures of Probability

- ❖ **Probability Axioms**
 - $\Pr[\Omega] = 1$ where, Ω : sample space
 - $0 \leq \Pr[A] \leq 1$
 - $\Pr[A \cup B] = \Pr[A] + \Pr[B]$
if A&B are mutually exclusive




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Measures of Probability

- ❖ **Addition Rules**
 - From 2nd axiom
 $\Pr[A^c] = 1 - \Pr[A]$
 - From 3rd axiom
 $\Pr[A_1 + A_2 + \dots + A_k] = \Pr[A_1] + \Pr[A_2] + \dots + \Pr[A_k]$
if $A_i A_j = \emptyset$ for any $i \neq j$
 - The general addition rule
 $\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B]$
((note)) $\Pr[A \cup B \cup C] = \Pr[A] + \Pr[B] + \Pr[C] - \Pr[A \cap B] - \Pr[A \cap C] - \Pr[B \cap C] + \Pr[A \cap B \cap C]$



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Measures of Probability


- ❖ **Further Properties of Probability Functions**
 - **Property 1: Probability of null event**

$$\Pr[\emptyset] = 0$$
 - **Property 2: Probability of a contained event**

$$\Pr[A] \leq \Pr[B] \quad \text{if } A \subset B$$
 - **Property 3: Boole's inequality**

$$\Pr[A_1 + A_2 + \dots + A_n] \leq \Pr[A_1] + \Pr[A_2] + \dots + \Pr[A_n]$$


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Measures of Probability

- ❖ **Conditional Probability and Multiplication Rule**
 - **Definition**


$$\Pr[A|B] = \Pr[A \cap B] / \Pr[B]$$

$$\Pr[A \cap B] = \Pr[A] \Pr[B]$$

if A&B are independent
 - **Applications**

$$\Pr[AB] =$$


$$\Pr[ABC] =$$


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Measures of Probability

- ❖ Independence
 - Definition

Two events defined in a given probability space A are **independent** if either the conditional probability of one event equals its marginal probability, or their joint probability equals the product of the marginals
 - Events A&B independent
 - $\Pr[A|B] = \Pr[A]$ if $\Pr[B] > 0$
 or $\Pr[B|A] = \Pr[B]$ if $\Pr[A] > 0$
 - $\Pr[A \cap B] = \Pr[A] \Pr[B]$

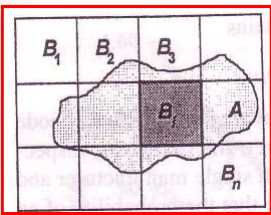
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Measures of Probability

- ❖ Total Probability
 - Total Probability

$$\Pr[A] =$$



Venn diagram for the theorem of total probability. Events $B_i, i = 1, \dots, n$, are mutually exclusive and exhaustive, and some of them intersect A


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Measures of Probability

- ❖ Bayes' Theorems
 - Bayes' Theorems

$$\Pr[B_j|A] = (\Pr[A|B_j] \Pr[B_j]) / (\sum_i \Pr[A|B_i] \Pr[B_i])$$

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((example)) FEMA Study(1)


C ₁	C ₂	C ₃
Failure		
C ₄		

- C₁= flood
- C₂= earthquake
- C₃= seepage
- C₄= normal operation

- Pr[fail]=

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((example)) FEMA Study(2)

	$\Pr[C_i]$	$\Pr[\text{fail} C_i]$	$\Pr[C_i \cap \text{fail}]$
C_1	0.03	0.10	
C_2	0.02	0.08	
C_2	0.01	0.05	
			$= \Pr[\text{fail}]$

➤ $\Pr[\text{flood} | \text{fail}] =$



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