Lecture 2

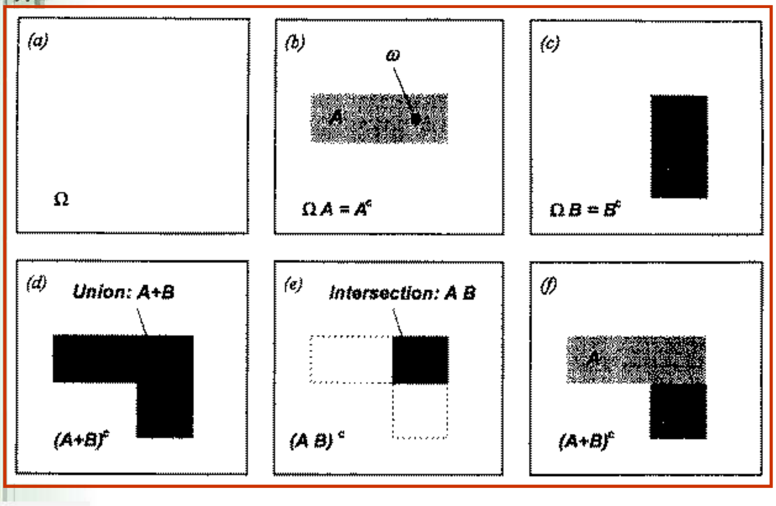
Basic Probability Concepts

Statistics for
Civil & Environmental Engineers

Venn Diagram

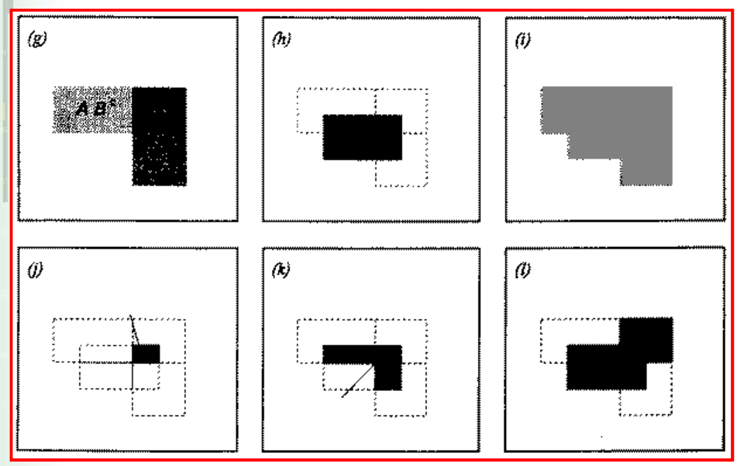
- ***Venn Diagrams*** **provide a very useful visual representation of sets and set operations such as the complement, union, intersection, and the other combinations**
- **However, a Venn diagram is not intended to be used in defining relationships among the probabilities of events**

Venn Diagram Example (1)



Venn Diagram Example (2)

- Venn Diagram(2)



Measures of Probability

❖ **Probability Axioms**

- **$Pr[\Omega]= 1$ where, Ω: sample space**
- **$0 \leq Pr[A] \leq 1$**
- **$Pr[A \cup B]= Pr[A] + Pr[B]$**

  if A&B are mutually exclusive

Measures of Probability

❖ **Addition Rules**

- **From 2nd axiom**

  $Pr[A^c]= 1 – Pr[A]$

- **From 3rd axiom**

  $Pr[A_1+A_2+ \cdots + A_k]= Pr[A_1] + Pr[A_2] + \cdots + Pr[A_k]$

  if $A_iA_j= \emptyset$ for any $i \neq j$

- **The general addition rule**

  $Pr[A \cup B] \equiv Pr[A+B]=$

  ((note)) $Pr[A \cup B \cup C]=$

Measures of Probability

- **Further Properties of Probability Functions**
  - **Property 1: Probability of null event**

    $Pr[\emptyset] = 0$

  - **Property 2: Probability of a contained event**

    $Pr[A] \leq Pr[B]$ if $A \subset B$

  - **Property 3: Boole's inequality**

    $Pr[A_1 + A_2 + \cdots + A_n] \leq Pr[A_1] + Pr[A_2] + \cdots + Pr[A_n]$

Measures of Probability

- **Conditional Probability and Multiplication Rule**
  - **Definition**

    $Pr[A|B] = Pr[A \cap B] / Pr[B]$

    $Pr[A \cap B] = Pr[A]\, Pr[B]$

    if A&B are independent

  - **Applications**

    $Pr[AB] =$

    $Pr[ABC] =$

Measures of Probability

❖ **Independence**

➢ **Definition**

Two events defined in a given probability space *A* are *independent* if either the conditional probability of one event equals its marginal probability, or their joint probability equals the product of the marginals

➢ **Events A&B independent**

- **Pr[A|B]= Pr[A] if Pr[B] > 0**

  or Pr[B|A]= Pr[B] if Pr[A] > 0

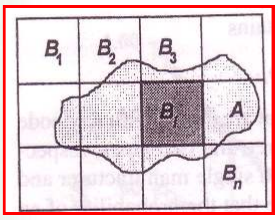
- **Pr[A∩B]≡ Pr[AB] = Pr[A] Pr[B]**

Measures of Probability

❖ **Total Probability**

➢ **Total Probability**

Pr[A]=



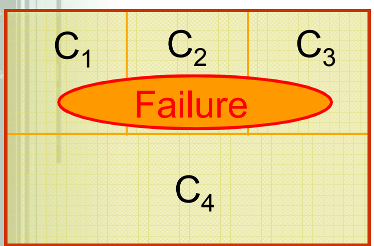
Venn diagram for the theorem of total probability. Events B_i, i = 1, … , n, are mutually exclusive and exhaustive, and some of them intersect A

Measures of Probability

- **Bayes' Theorems**
  - **Bayes' Theorems**

    $$Pr[B_j|A] = (Pr[A|B_j]\ Pr[B_j]) \;/\; (\sum_i Pr[A|B_i]\ Pr[B_i])$$

((example)) FEMA Study(1)



C_1= flood

C_2= earthquake

C_3= seepage

C_4= normal operation

- **Pr[fail]=**

((example)) FEMA Study(2)

	$Pr[C_i]$	$Pr[fail \mid C_i]$	$Pr[C_i \cap fail]$
C_1	0.03	0.10	
C_2	0.02	0.08	
C_2	0.01	0.05	
			= Pr[fail]

- Pr[flood|fail]=