## (5) Nonlinear Pseudo-Elastic Model
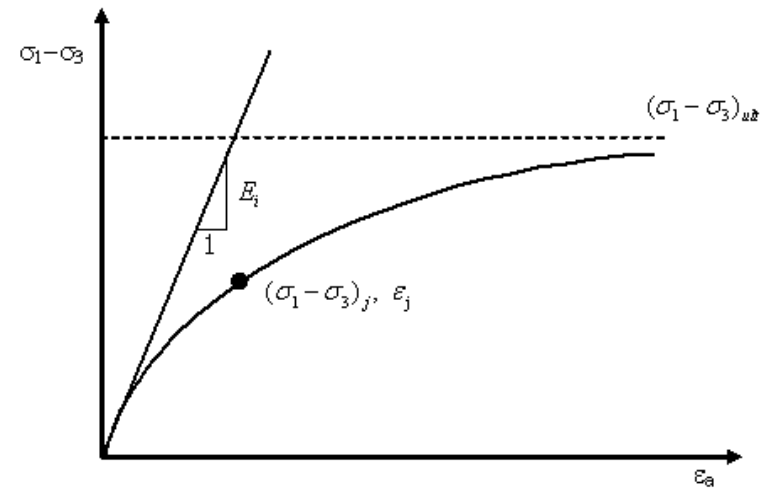
- **Duncan-Chang (1970)**

- **Incremental Stress-Strain Relations**

  i) Nonlinearity
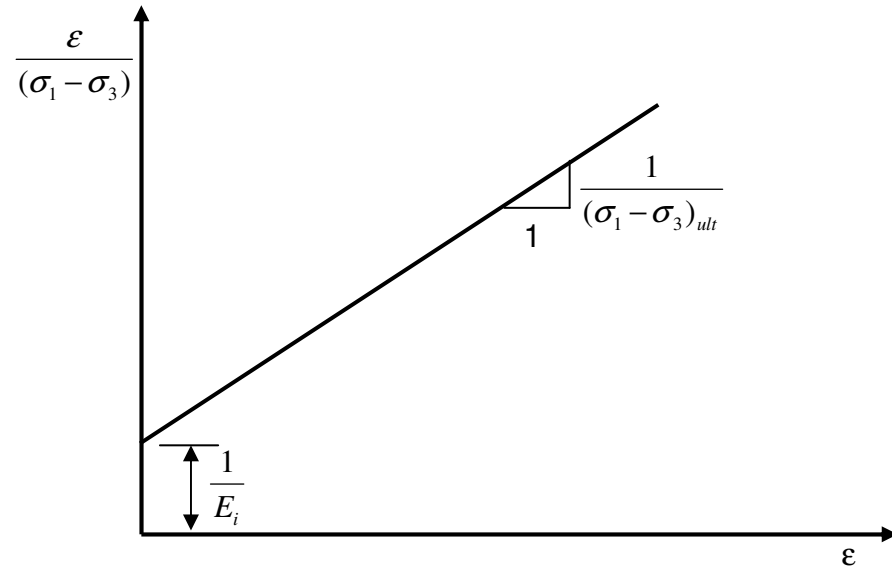
   - Kondner and Zelasko (1963)

     • σ-ε relations are approximated by a hyperbola.

$$(\sigma_1 - \sigma_3)_j = \frac{\varepsilon_j}{\dfrac{1}{E_i} + \dfrac{\varepsilon_j}{(\sigma_1 - \sigma_3)_{ult}}}$$   ------------------------------------------------------------------- (1)

$\Rightarrow$ Replot σ-ε data on :



Transformed σ-ε relations :     $\dfrac{\varepsilon}{(\sigma_1 - \sigma_3)} = \dfrac{1}{E_i} + \dfrac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}$

Physical significance

- Curve defined in terms of

   $\rightarrow E_i$, Young's modulus.

   $\rightarrow$ Asymptotic value of principal stress difference ($\sigma_1$-$\sigma_3$) at   failure.
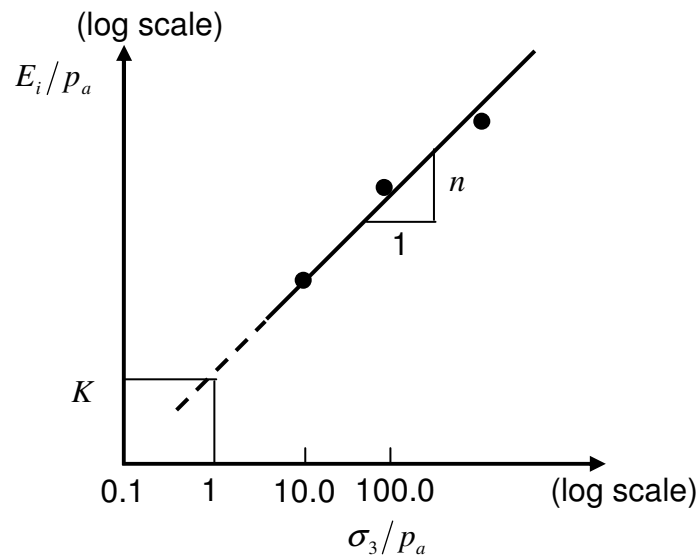
ii) Stress-dependent nature of σ-ε behavior

For most cases : an increase of $\sigma_3$ results in increases in $E_i$ & $(\sigma_1-\sigma_3)_{ult}$.

a) To account $E_i$ dependence on stress (Janbu, 1964).

$$E_i = Kp_a(\frac{\sigma_3}{p_a})^n$$   ----------------------------------------------------------------------------------   (2)

$K$ ≡ modulus number,  $n$ ≡ modulus exponent,  $p_a$ ≡ atmospheric pressure

To evaluate  $K$ ,  $n$ ; perform 3 TX tests at different values of $\sigma_3$.

(log scale)

$E_i/p_a$

$n$ ≈ 0.5          for sands

$n$ ≈ 0.0          for NC saturated clays

          under UU tests

$K$

0.1    1    10.0  100.0    (log scale)

$\sigma_3/p_a$

b) To account $(\sigma_1 - \sigma_3)_{ult}$ dependence on $\sigma_3$.

- relate $(\sigma_1 - \sigma_3)_{ult}$ to $(\sigma_1 - \sigma_3)_f$.

$$R_f = \text{failure ratio} = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} \quad (0.5 \le R_f \le 0.95)$$
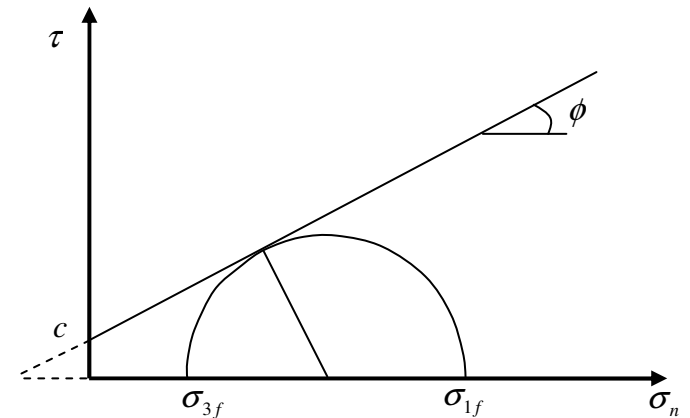
- define failure (i.e. $(\sigma_1 - \sigma_3)_f$ ).

Based on Mohr-Coulomb failure criteria.

General expression (from geometry of circle)

$$(\sigma_1 - \sigma_3)_f = \frac{2c\cos\phi + 2\sigma_3\sin\phi}{1 - \sin\phi}$$

$$(\sigma_1 - \sigma_3)_{ult} = \frac{2c\cos\phi + 2\sigma_3\sin\phi}{R_f(1 - \sin\phi)} \quad \text{-------------------------------------------------} \quad (3)$$

iii) Relate tangential modulus, $E_t$ to stress level.

Differentiate eq. (1) with respect to $\varepsilon$.

$$\frac{d(\sigma_1-\sigma_3)}{d\varepsilon} = \frac{d\left[\dfrac{\varepsilon}{\dfrac{1}{E_i} + \dfrac{\varepsilon}{(\sigma_1-\sigma_3)_{ult}}}\right]}{d\varepsilon}$$

Let $a = \dfrac{1}{E_i}$, $b = \dfrac{1}{(\sigma_1-\sigma_3)_{ult}}$

$$\frac{d(\sigma_1-\sigma_3)}{d\varepsilon} = \frac{d\left[\dfrac{\varepsilon}{a+b\varepsilon}\right]}{d\varepsilon} = \frac{a+b\varepsilon-b\varepsilon}{(a+b\varepsilon)^2} = \frac{a}{(a+b\varepsilon)^2} \text{------------------------------------------------ (i)}$$

Rewrite eq. (1) in terms of strain.

$$\varepsilon = \frac{a(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \qquad \text{------------------------------------------------------------------------} \quad \text{(ii)}$$

Sub. (ii) into (i).

$$E_t = \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon} = \frac{a}{\left[ a + \dfrac{ab(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \right]^2} = \frac{\left[ 1 - b(\sigma_1 - \sigma_3) \right]^2}{a} = \left[ 1 - \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_{ult}} \right]^2 E_i \quad \text{--------------} \quad \text{(iii)}$$

Put eq. (2), (3) into (iii).

$$E_t = \left[ 1 - \frac{R_f(1 - \sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi} \right]^2 Kp_a\left(\frac{\sigma_3}{p_a}\right)^n \quad \text{-------------------------------------------} \quad \text{(4)}$$

$$\Rightarrow E_t \text{ is a } f(\phi, c, K, n, R_f)$$
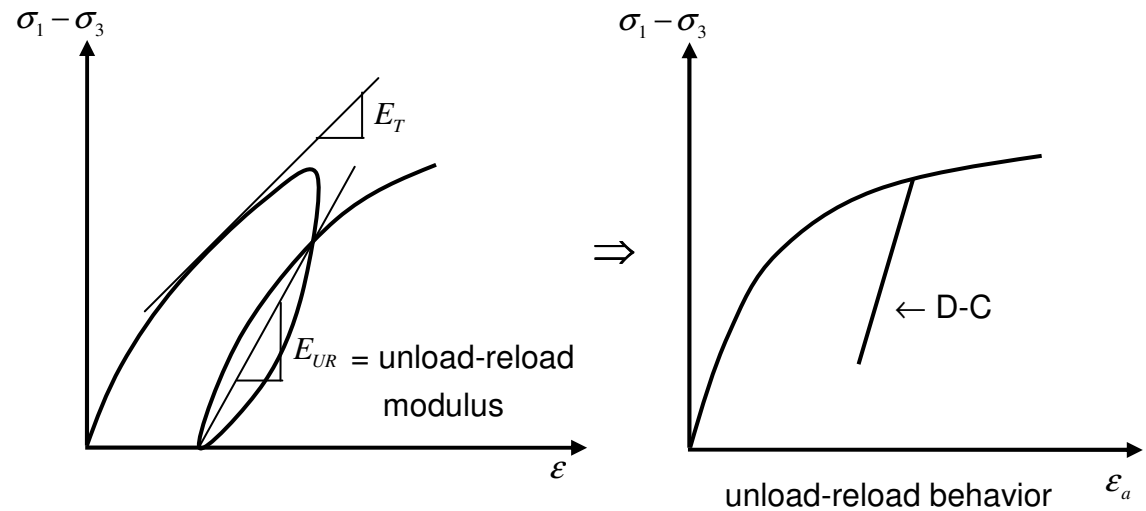
$$\nwarrow (\sigma_1 - \sigma_3)_{ult}$$

iv) Cyclic loading behavior

-   Use different E values for loading and unloading-reloading.

$$E_{UR} > E_T \quad (E_{UR} > E_i)$$

Empirically, $E_{UR} = K_{UR} p_a (\dfrac{\sigma_3}{p_a})^n$

$E_T$

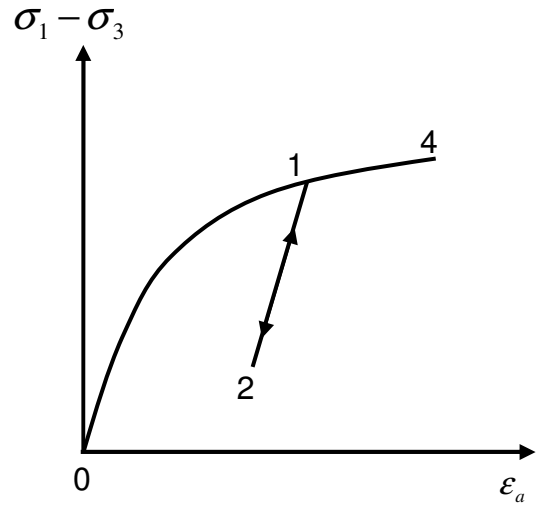$E_{UR}$ = unload-reload modulus

← D-C

unload-reload behavior

Assume that $n$ is same for unloading-reloading as for primary loading.

$K_{UR}$ ≡ unload-reload modulus number.

$K_{UR} > K$    for dense sand (20% greater)

for very loose sand (as much as 3 times greater)

- How long does soil stay "elastic" during unloading-reloading?

$\sigma_1 - \sigma_3$

Loading sequence

$0 - 1 - 2 - 1 - 4$

between 1-1, use $E_{UR}$

maximum shear stress level (SL$_{max}$) in loading history

- Notes

Define stress level (SL)

$$SL = \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f}$$
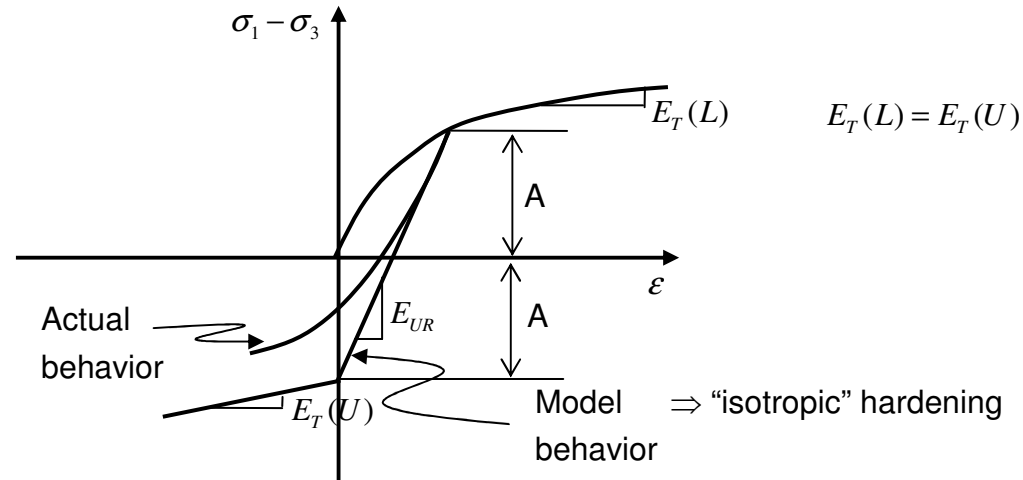
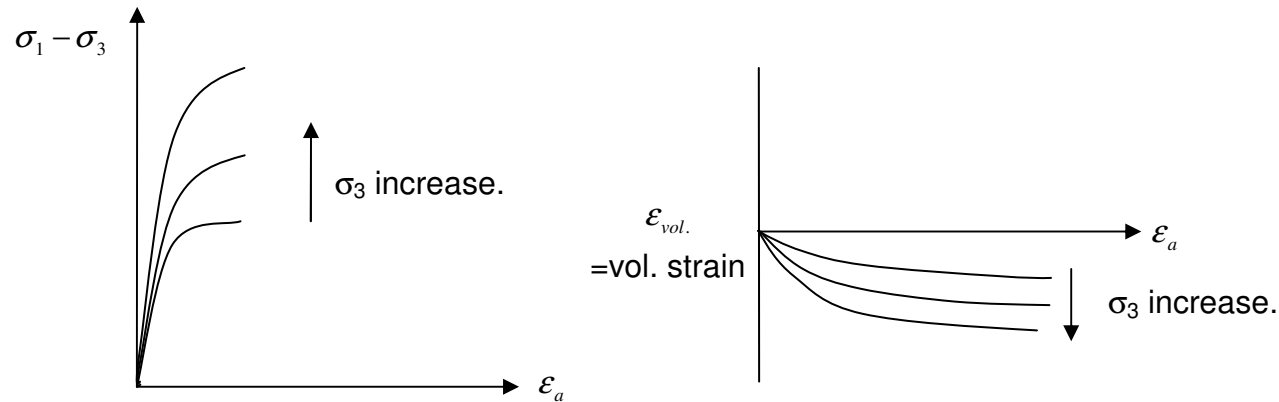$0 \le SL \le 1.0$

↑       ↑

Isotropic   failure

Loading

If $SL < SL_{max}$ , then use elastic ($E_{UR}$) for unloading-reloading.

- What does SL criteria imply about complete cycles of loading?



$E_T(L) = E_T(U)$

Actual behavior

Model behavior $\Rightarrow$ "isotropic" hardening

v) Nonlinear volume change

- No expansion, only compression

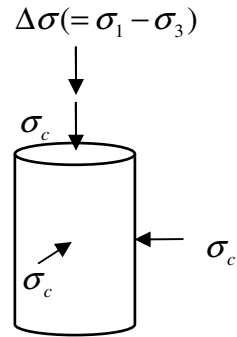- Typical triaxial compression test results



$$K_B = \text{Bulk modulus} = \frac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3\varepsilon_{vol}} \quad \rightarrow \quad \varepsilon_{vol} = \frac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3K_B}$$

$K_B$ is a function of $\sigma_3$ and it is independent of stress level $(\sigma_1 - \sigma_3)$. $K_B$ is not changed by increase of loading.

Vol. strains are a function of changes in mean normal stress.

Triaxial test

$\Delta\sigma(=\sigma_1 - \sigma_3)$

$\sigma_c$
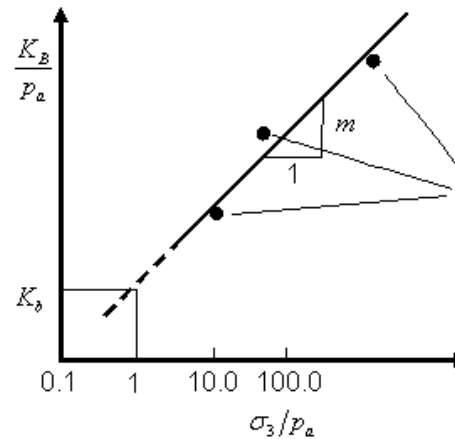
$\sigma_c$

$\sigma_c$

$$K_B = \frac{(\sigma_1 - \sigma_3)}{3\varepsilon_v}$$

$$\rightarrow \varepsilon_v = \frac{(\sigma_1 - \sigma_3)}{3K_B}$$

Use Janbu relation.

$$K_B = K_b p_a (\frac{\sigma_3}{p_a})^m$$

$K_b \equiv$ bulk modulus number,    $m \equiv$ bulk modulus exponent



results of 3 different tests

Typical soils : $0 < m < 1$

$m < 0 \Rightarrow K_B$ decreases as $\sigma_3$ increase.

i.e. : quick clay

<u>Restrictions on</u>  $K_B$

$0 < \nu < 0.5$

$$\nu = \frac{1}{2} - \frac{E_t}{6K_B} \qquad (\Leftarrow K_B = \frac{E_t}{3(1-2\nu)} )$$

for  $\nu = 0,$  $K_B = E_t / 3$

$\nu = 0.49,$  $K_B = 16.7E_t$

$E_t / 3 < K_B < 16.7E_t$

## Summary

From i) ~ v) & 8 parameters,

| Symbol | Name | Purpose |
|--------|------|---------|
| $K$ <br><br> $K_{UR}$ <br><br> $n$ | modulus # <br><br> unload-reload modulus # <br><br> modulus exponent | relate $E_i$, $E_{UR}$ to $\sigma_3$. |
| $c$, $\phi$ | Mohr-Coulomb parameters | relate $(\sigma_1-\sigma_3)_f$ to $\sigma_3$. |
| $R_f$ | failure ratio | relate $(\sigma_1-\sigma_3)_f$ to $(\sigma_1-\sigma_3)_{ult}$. |
| $K_b$ <br><br> $m$ | bulk modulus number <br><br> bulk modulus exponent | relate $K_B$ to $\sigma_3$. |

Comments

i) Limitations

a) Useful for predicting movements in stable earth masses (no plastic considerations).

b) Does not include volume changes caused by shear-induced dilatancy (only volumetric compression is allowed for monotonically-increasing loads).

c) Parameters are not fundamentally soil properties, but represent empirical parameters for a limited set of loading conditions.

d) Full stress reversal must occur before nonlinear (inelastic) strains develop for cyclic loads or load reversals.

ii) Advantages

a) Parameters come from simple TX tests.

b) Effective stress and total stress relationships are the same, but parameters different.

c) General model ; both sands (+) clays can be handled.

d) Values of parameters have been calculated for more than 150 soils.

e) Can handle anisotropic soils with plane strain or true TX test results.
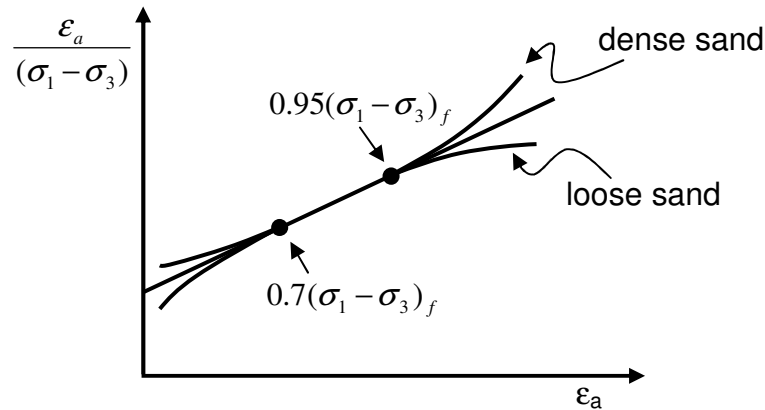
● **Parameter Identification**

$\left\{\begin{array}{l} \text{Drained conditions : 3-CID TXC tests with volume change measurements.} \\ \\ \text{Undrained conditions : 3-CIU TXC tests.} \end{array}\right.$

① Plot σ-ε data.
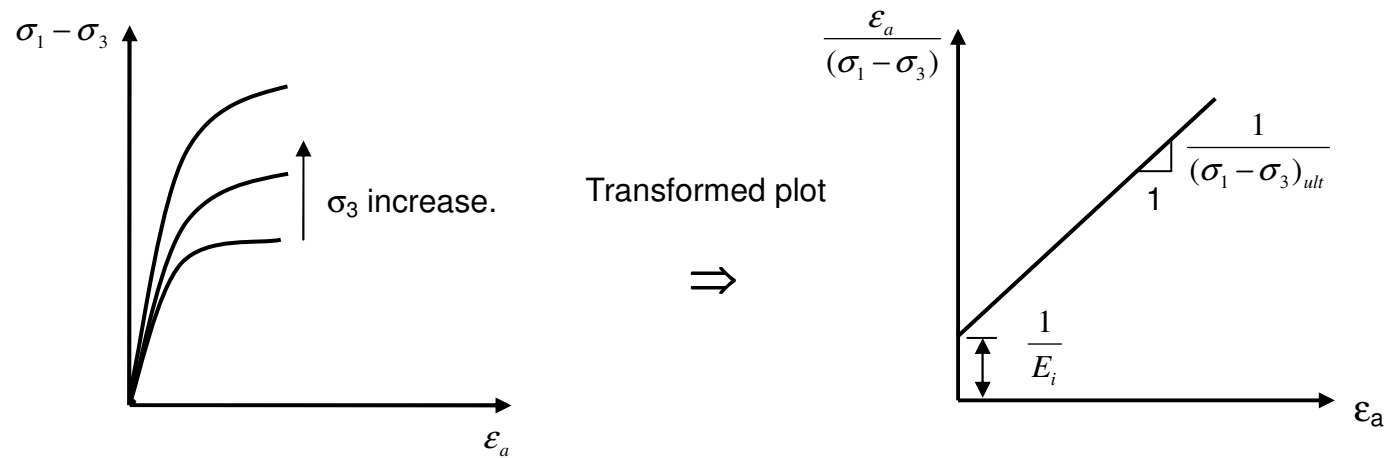


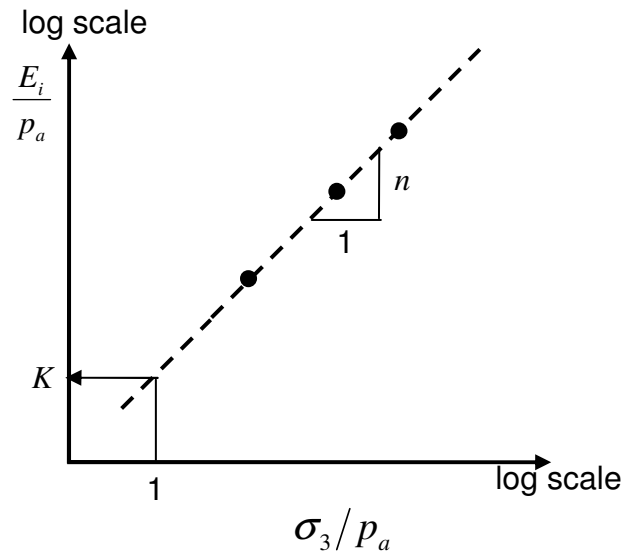3 curves are plotted.

Ref.) D-C



$\Rightarrow$ U.C. Berkeley's experience indicated using these pts to define   straight line

gives best fit of all responses.

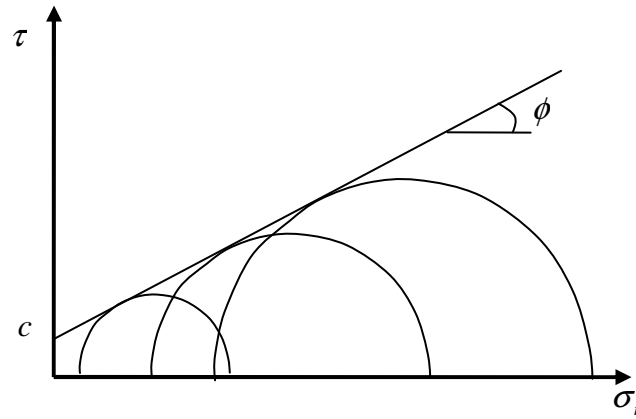$\Rightarrow$ We find $\dfrac{1}{E_i}$, $\dfrac{1}{(\sigma_1 - \sigma_3)_{ult}}$

Each test gives 1 set of values $\Rightarrow$ For $R_f$, use average value

② $E_i$.



log scale

$\dfrac{E_i}{p_a}$

$n$

$1$

$K$

$1$

$\sigma_3/p_a$

log scale

We find,  $K$ ,  $n$ .
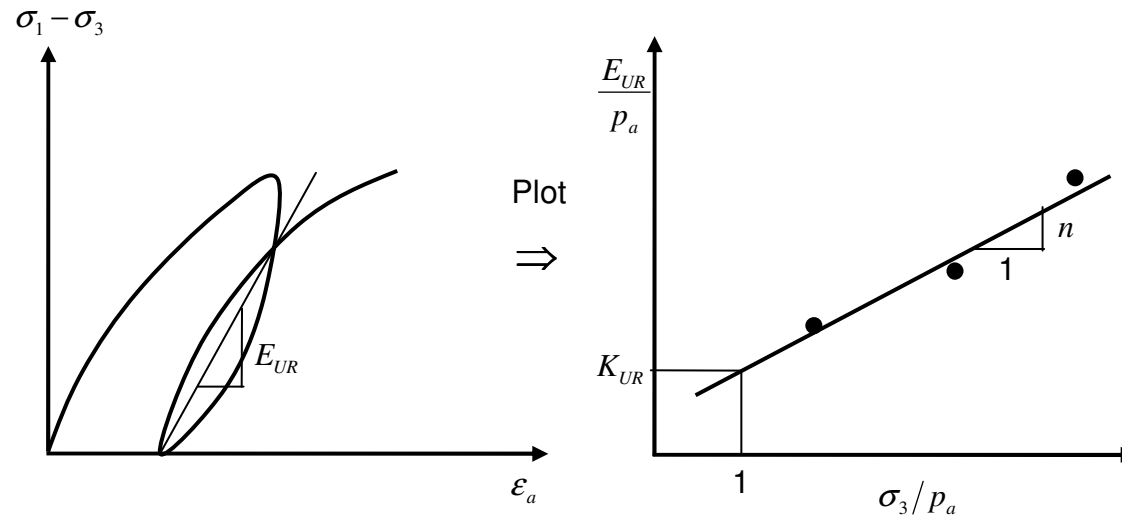
③ Plot Mohr circles at failure.



$$c, \ \phi \ \rightarrow \ (\sigma_1 - \sigma_3)_f$$

Then, we can compute $R_f = \dfrac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} = \dfrac{2c\cos\phi + 2\sigma_3 \sin\phi}{(\sigma_1 - \sigma_3)_{ult}(1 - \sin\phi)}$

↖

Use average value among values with 3

sets of $\sigma_3$ and $(\sigma_1 - \sigma_3)_{ult}$

④ To find unload-reload parameters (ideally need 1 unload-reload cycle at each of 3 confining stresses).

$\sigma_1 - \sigma_3$

$E_{UR}$

$\varepsilon_a$

Plot

$\Rightarrow$

$\dfrac{E_{UR}}{P_a}$

$n$

1

$K_{UR}$

1

$\sigma_3 / P_a$

Assume slope of line is same as for primary loading.

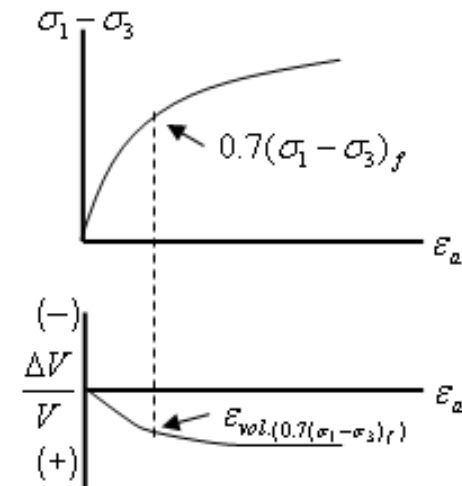⑤ For volume change parameters (only for drained tests).

- Recall : $K_B = \dfrac{\Delta \sigma_0}{\Delta \varepsilon_{vol.}}$

     for triaxial shearing, $K_B = \dfrac{\sigma_1 - \sigma_3}{3 \varepsilon_{vol.}}$
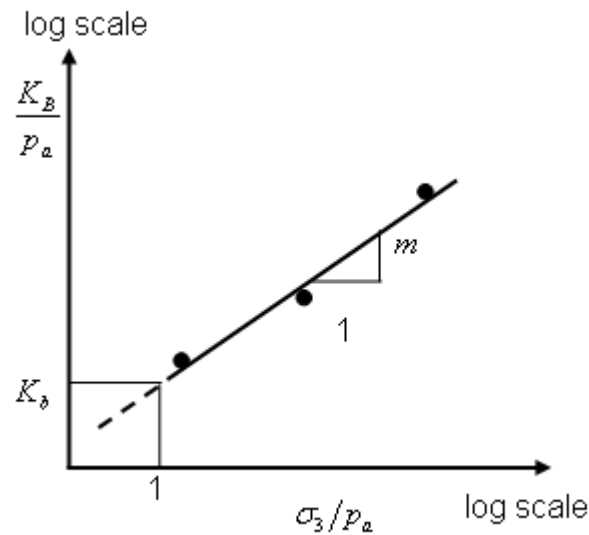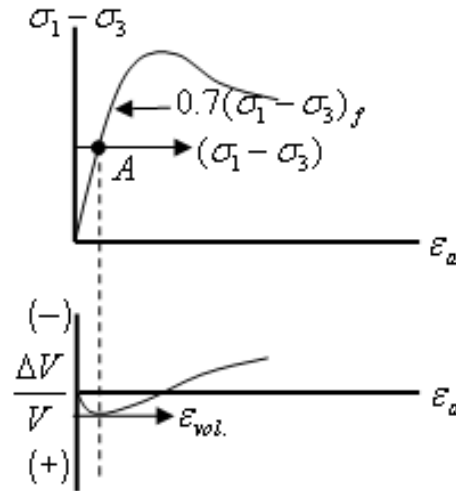
- To select $K_B$ :

   Two possibilities; (based on $0.7(\sigma_1 - \sigma_3)_f$)

     1) If the slope of volume change curve does not reach

       0 before the pt. that corresponds to $0.7(\sigma_1 - \sigma_3)_f$,

       then use $0.7(\sigma_1 - \sigma_3)_f$ and corresponding $\varepsilon_{vol.}$ to

       $K_B$.

2) If the slope of the volume change curve = 0 before $0.7(\sigma_1 - \sigma_3)_f$ stress level is reached, then use the $(\sigma_1 - \sigma_3)$ and $\varepsilon_{vol.}$ at that point to compute $K_B$.

$$\Rightarrow \quad K_B = \frac{(\sigma_1 - \sigma_3)}{3\varepsilon_{vol.}}$$

• <u>Eliminating inconsistencies from data</u>

   1) Select data appropriate for problem under consideration.

     - Natural soils : Need to run tests on high quality undisturbed samples.

     - Fills : Use specimens compacted to water content and dry density (with same

       compaction energy) as the fill was in field.

     - Use drained conditions that are expected in field loading.

2) Smooth the data.

- Stress controlled loading.