Naval Architecture & Ocean Engineering

- Ship Stability -

Ch.3 Hydrostatic Pressure, Force and Moment on a Floating Body

2009 Fall

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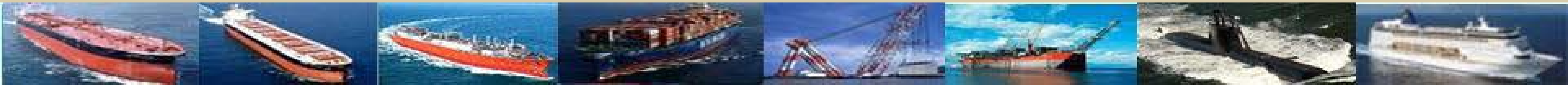


Definition of Coordinate System

6 D.O.F Equations of Ship Motions

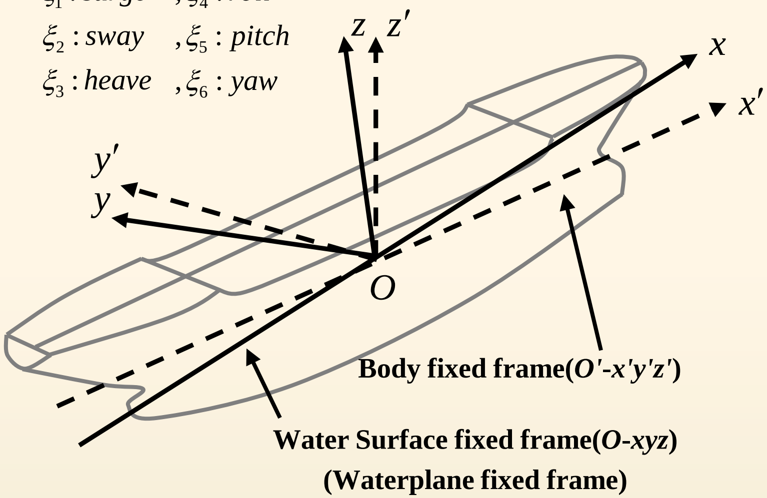
6 D.O.F Equations of Ship Motions and Coordinate System
(D.O.F : Degree Of Freedom)

x' axis – origin: Midship, (+):forward
y' axis – origin: Centerline, (+):port
z' axis – origin: waterplane,(+):upward

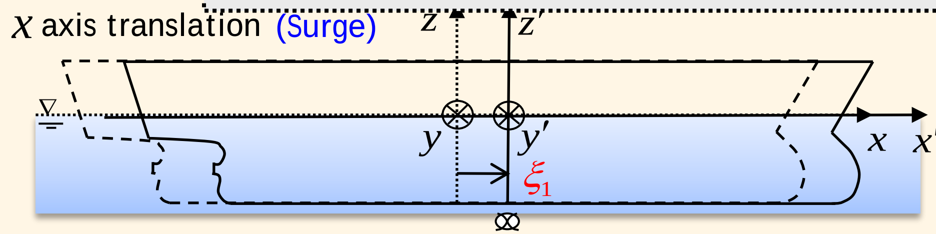
x axis – origin: Midship, (+): Intersection line between waterplane and plane which is perpendicular to waterplane and include x' axis
y axis– origin: centerline, (+): Direction of outer product between z axis and x axis
z axis – origin: waterplane, (+): Perpendicular direction to waterplane

(Displacement: $\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$)

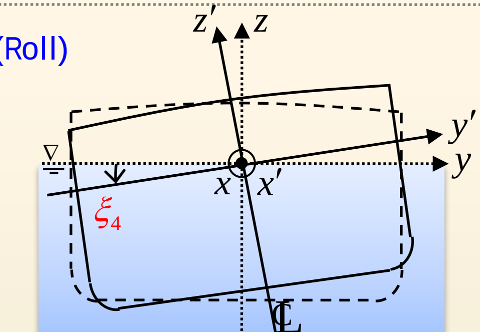
ξ_1 : surge , ξ_4 : roll
ξ_2 : sway , ξ_5 : pitch
ξ_3 : heave , ξ_6 : yaw



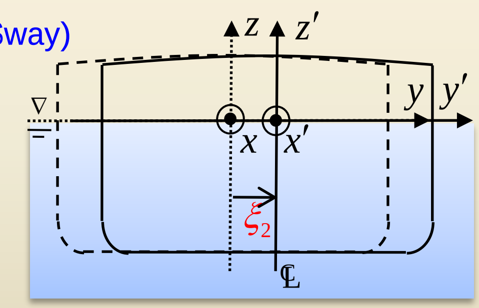
x axis translation (Surge)



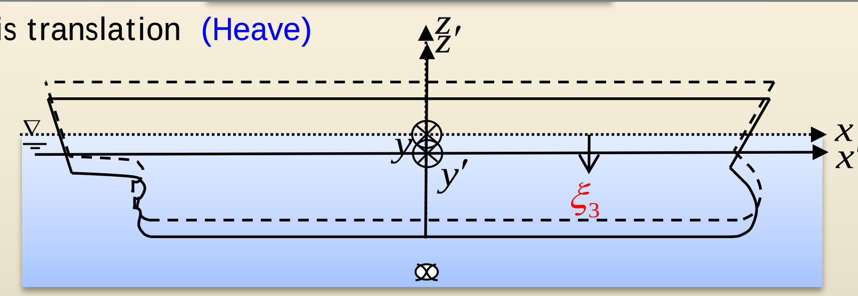
x axis rotation (Roll)



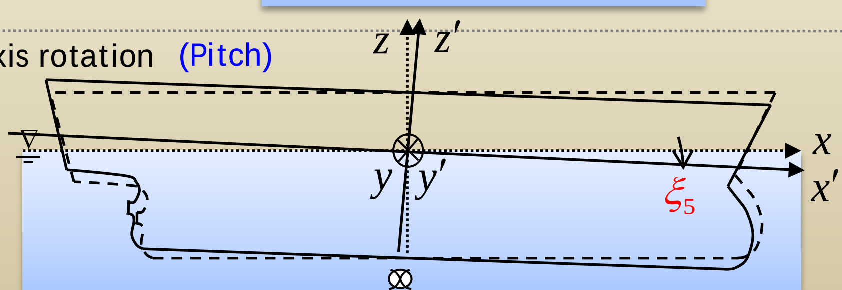
y axis translation (Sway)



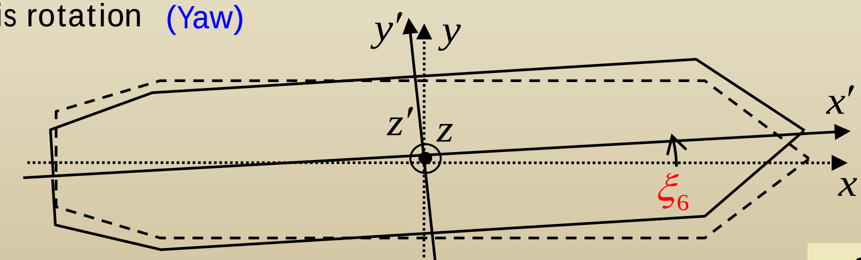
z axis translation (Heave)



y axis rotation (Pitch)



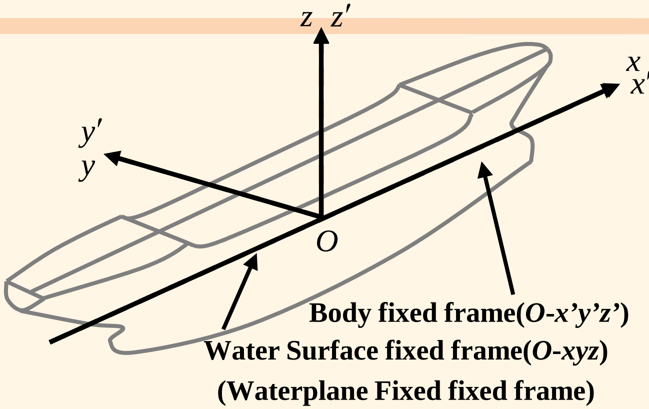
z axis rotation (Yaw)



6 D.O.F Equations of Ship Motions

xyz : Waterplane fixed frame
$x'y'z'$: Body-fixed coordinate system.

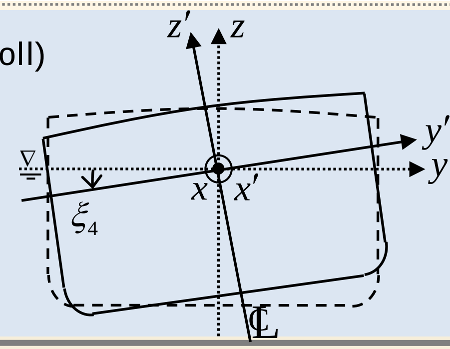
D.O.F : Degree Of Freedom



(Q) Which motion are displaced volume and center of buoyancy changed by?

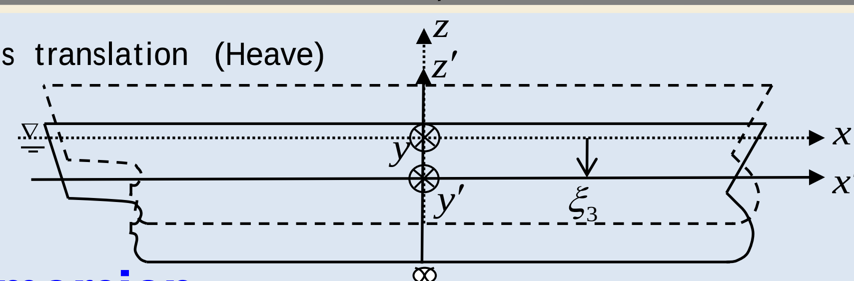
Displaced volume and center of buoyancy are changed by heel, immersion, trim only. So, they cause external force of equations of the motions (buoyant force, hydrodynamic force) to be changed.

x axis rotation (Roll)



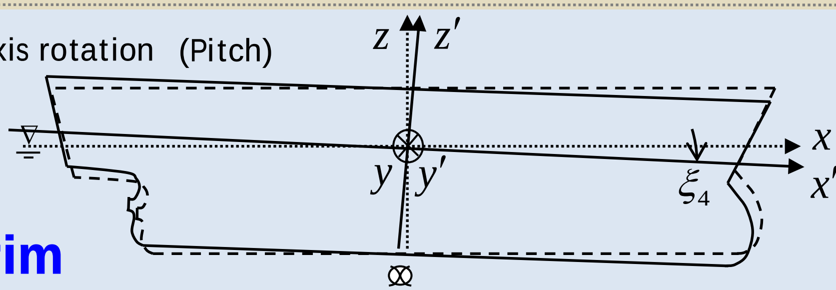
Heel

z axis translation (Heave)



Immersion

y axis rotation (Pitch)



Trim

Hydrostatic Pressure, Force and Moment on a Floating Body

● Archimedes' Principle

■ "The buoyant force on an immersed body has the same magnitude as the weight of the fluid displaced by the body[1]. And the direction of the buoyancy is opposite to the gravity"

① Archimedes' Principle

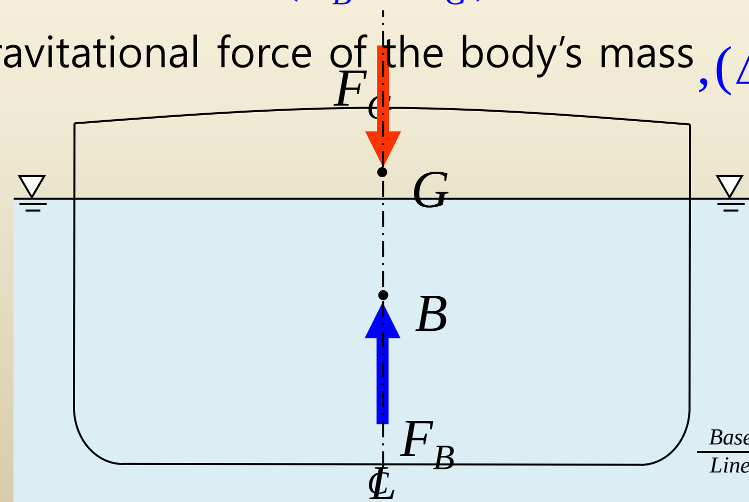
Buoyant force = Weight of displaced water by body (displacement) $,(F_B = \Delta(= \rho g V))$

② Equilibrium Condition (Sum of the forces in vertical direction is equal zero)

Buoyant force – Gravitational force of the body's mass=0 $,(F_B = F_G)$

③ $\therefore$ Weight of the displaced water by body = Gravitational force of the body's mass $,(\Delta = F_G)$

G: Center of mass
B: Center of buoyancy
F_G: Gravitational force, F_B : Buoyant force
ρ : Density of fluid
V: Volume of the body below waterplane



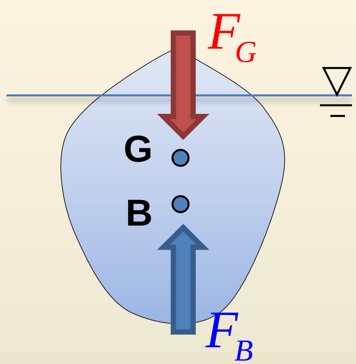
Ref 1) Ohanian, H. C., Physics (2nd ed), W. W. Norton & Company, p478.

Example 4>

Question) A chunk of ice floats in water. What percentage of the volume of ice will be above the level of the water? The density of ice is ρ_i=0.917 Mg/m^3 and the density of sea water is ρ_w=1.025 Mg/m^3. [1)]

Solution)

Gravity Force=Buoyant Force (∵ Because iceberg is 'float' in the water)



$$\text{(LHS) } M_i g = \rho_i V_i g$$

$$\text{(RHS) } F_B = \rho_w \, g \, V_w$$

Lose your g,

$$\rho_i \cancel{g} V_i = \rho_w \cancel{g} V_w$$

$$\rho_i V_i = \rho_w V_w,$$

$$\frac{\rho_i}{\rho_w} = \frac{V_w}{V_i} \Rightarrow \frac{\text{Exposured Volume}}{\text{Entire Volume}} = \frac{V_i - V_w}{V_i} = \frac{V_i - \frac{V_w}{V_i} V_i}{V_i} = \frac{V_i - \frac{\rho_i}{\rho_w} V_i}{V_i}$$

$$= \frac{1 - \frac{\rho_i}{\rho_w}}{1} = 1 - \frac{0.917}{1.025} \approx 0.105, \ 10.5\%$$

∴The fraction of ice above the water level is therefore 10.5%

M_i : Mass of iceberg
V_i : Total volume of iceberg,
V_w : Volume underwater
ρ_i : Density of the ice,
ρ_w : Density of underwater
g : Acceleration of gravity

Ref 1) Ohanian, H. C., Physics (2nd ed), W. W. Norton & Company, p478.

Pressure and Force acting on Fluid Particle

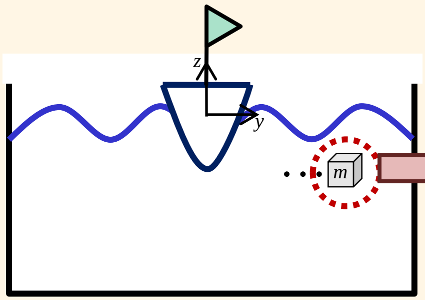
$\mathbf{r}$: displacement of particle with respect to time

$\mathbf{V} = \frac{d\mathbf{r}}{dt}, \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bendidng Moment

✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
④ Irrotational flow
⑤ Incompressible flow



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✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Shear force

Curl & Rotation

Newton's 2nd Law

$m\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body force} + \text{Surface force})$

→ Cauchy equation → ①② Navier-Stokes equation → ③ Euler equation → ④⑤ Bernoulli equation

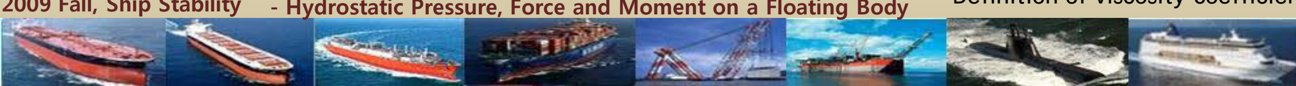
Microscopic/ Macroscopic Derivation(RTT[1])

Mass Conservation Law → ④⑤ Laplace Equation

$$\nabla^2 \Phi = 0 \quad (\mathbf{V} = \nabla \Phi)$$

$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho |\nabla \Phi|^2 + \rho g z = 0$$

* A **Newtonian fluid** : fluid whose stress versus strain rate curve is linear.
**Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion



Derivation of Buoyant Force
- Equations of Motions of Fluid Particle (Cauchy eq. ~ Bernoulli eq.)[1)]

1)Kundu,P.k.,Cohen,I.M.,Fluid Mehcanics 4th, Academic Press,2008

* A **Newtonian fluid** : fluid whose stress versus strain rate curve is linear.

**Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

Cauchy Equation : $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}+\nabla\bullet\sigma \quad ,\left(\mathbf{V}=[u,v,w]^T\right)$

① Newtonian fluid*
② Stokes assumption**

Navier-Stokes Equation : (in general form) $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}-\nabla P+\mu\left(\frac{1}{3}\nabla(\nabla\cdot\mathbf{V})+\nabla^2\mathbf{V}\right)$

$(\mu=0)$ ③ Invicid fluid

Euler Equation : $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}-\nabla P$

$\rho=\rho(P)$ ④ barotropic flow

Euler Equation : (Another form) $\frac{\partial\mathbf{V}}{\partial t}+\nabla B=\mathbf{V}\times\boldsymbol{\omega} \quad ,\left(B=\frac{1}{2}q^2+gz+\int\frac{dP}{\rho}\ ,\ q^2=u^2+v^2+w^2\right)$

$\boldsymbol{\omega}\equiv\nabla\times\mathbf{V}$
If $\boldsymbol{\omega}=0$, (irrotational flow)
the $\nabla\times\mathbf{V}=0$
If $\nabla\times\mathbf{V}=0$, then $\mathbf{V}=\nabla\Phi$.

⑤ Steady flow $\left(\frac{\partial\mathbf{V}}{\partial t}=0\right)$

along streamlines and vortex lines

Bernoulli equation (case1) $B=\text{Constant}$

$\left(\frac{1}{2}q^2+gz+\int\frac{dP}{\rho}=C\right)$

$\left(q^2=|\nabla\Phi|^2\right)$

⑥ Unsteady, Irrotational flow

Bernoulli equation (case2) $\frac{\partial\Phi}{\partial t}+\frac{1}{2}|\nabla\Phi|^2+gz+\int\frac{dP}{\rho}=F(t)$

$(\rho=constant)$ ⑦ Incompressible flow

Bernoulli equation (case3) $\frac{\partial\Phi}{\partial t}+\frac{1}{2}|\nabla\Phi|^2+gz+\frac{P}{\rho}=F(t)$

Continuity Equation $\frac{\partial\rho}{\partial t}+\nabla\bullet\rho\mathbf{V}=0$

⑦ incompressible flow $\rho=constant,\left(\frac{\partial\rho}{\partial t}=0\right)$

$\nabla\bullet\mathbf{V}=0$

⑥ irrotational flow $(\mathbf{V}=\nabla\Phi)$

Laplace Equation $\nabla^2\Phi=0$

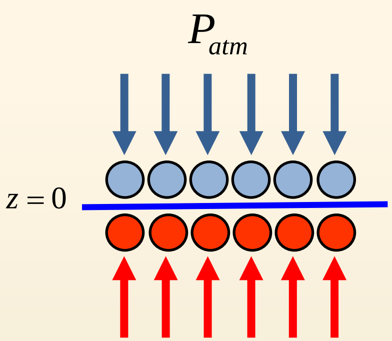
Newtonian fluid
Stokes assumption
Invicid fluid
Unsteady flow
Irrotational flow
Incompressible flow

Derivation of Buoyant Force
-Meaning of F(t) in Bernoulli Equation and Gauge Pressure

Bernoulli Equation

☞ An identical equation

$$\frac{\partial \Phi}{\partial t}+\frac{P}{\rho}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+g\,z=F(t)$$



If a fluid particle is in equilibrium condition at free surface (z=0)

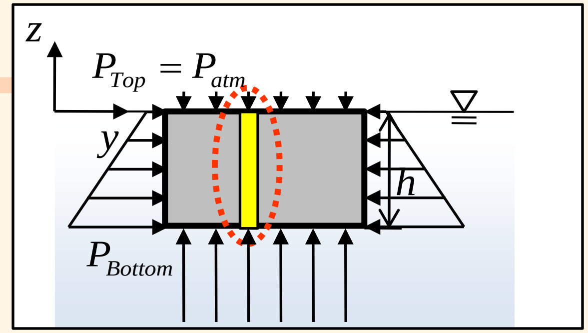
$$\frac{\partial \Phi}{\partial t}=0,\ \nabla\Phi=0,\quad P=P_{atm}$$

$$\cancel{\frac{\partial \Phi}{\partial t}}+\frac{P}{\rho}+\frac{1}{2}\left|\cancel{\nabla \Phi}\right|^{2}+g\cancel{z}=F(t)\longrightarrow \frac{P_{atm}}{\rho}=F(t)$$

(Atmospheric pressure(P_{atm})) = (Pressure at $z=0$)

$$\therefore \frac{\partial \Phi}{\partial t}+\frac{P}{\rho}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+g\,z=\frac{P_{atm}}{\rho}$$

1) Gauge pressure : The pressure result out of the difference of total pressure and atmosphere pressure



✓How is the pressure on the bottom of object expressed?

$$\frac{\partial \Phi}{\partial t}+\frac{P_{Bottom}}{\rho}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+gz=\frac{P_{atm}}{\rho}$$

$$\frac{\partial \Phi}{\partial t}+\frac{\cancel{P_{atm}}+P_{Fluid}}{\rho}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+gz=\frac{\cancel{P_{atm}}}{\rho}$$

$$\therefore \frac{\partial \Phi}{\partial t}+\frac{P_{Fluid}}{\rho}+\frac{1}{2}\left|\nabla \Phi\right|^{2}+gz=0$$

'gauge pressure'

※ In case that R.H.S of Bernoulli equation is expressed by zero, pressure P means the pressure due to fluid which exclude atmosphere pressure.

If the motion of fluid is small, square term could be linearized.

$$\frac{\partial \Phi}{\partial t}+\frac{P_{Fluid}}{\rho}+\cancel{\frac{1}{2}\left|\nabla \Phi\right|^{2}}+gz=0$$

$$P_{Fluid}=\underbrace{-\rho\frac{\partial \Phi}{\partial t}}_{P_{dynamic}}\underbrace{-\rho gz}_{P_{static}}=0$$

'Linearized Bernoulli Equation'

Pressure and Force acting on Fluid Particle

$\mathbf{r}$: displacement of particle with respect to time

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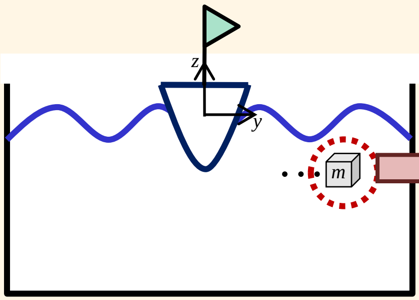

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Microscopic/ Macroscopic Derivation(RTT[1])

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$m\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body force} + \text{Surface force})$

Cauchy equation ①②

Navier-Stokes equation ③

Euler equation ④⑤

Bernoulli equation

Mass Conservation Law ④⑤

Laplace Equation

$$\nabla^2 \Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$$

Velocity potential Φ

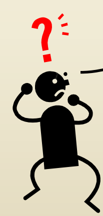
$\Phi = \Phi_I$ (Incident wave potential)
$+ \Phi_D$ (Diffraction potential)
$+ \Phi_R$ (Radiation potential)

✓Calculation of Fluid Force

Linearization

$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$$



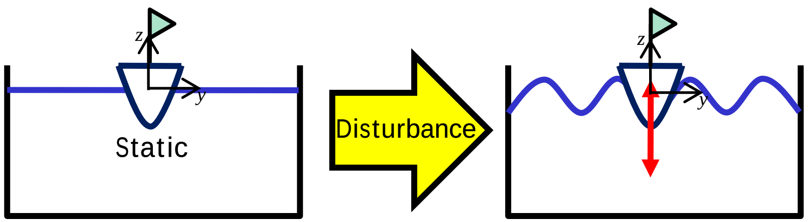
$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

(S_B: wetted surface)

* A **Newtonian fluid** : fluid whose stress versus strain rate curve is linear.
**Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

Pressure and Force acting on Fluid Particle - Fluid Force acting on ship

$$\rho\frac{\partial\Phi}{\partial t}+P+\frac{1}{2}\rho\left|\nabla\Phi\right|^2+\rho g\,z=0$$

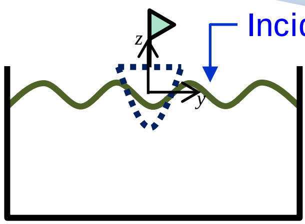


✓ **Pressure due to fluid particle around the ship in wave**

: Velocity, acceleration, pressure of the fluid particle are changed due to motion of fluid, then pressure of fluid particle over ship that acting on ship is changed.

Linearization

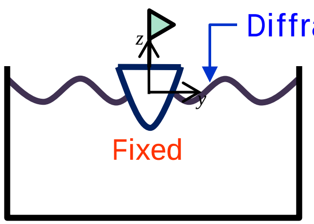
Incident wave velocity potential (Φ_I)



✓ **Velocity potential of Incident wave that is independent of the body motions and defined with the body fixed in position[1)]**

+

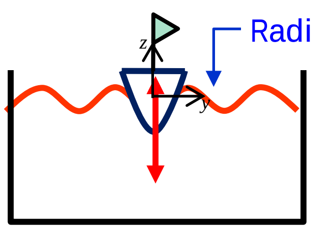
Diffraction wave velocity potential (Φ_D)



✓ **Velocity potential of disturbance of the incident waves by the fixed body[1)]**

+

Radiation wave velocity potential (Φ_R)



✓ **Velocity potential of wave which induced by rigid body motion, In the absence of the incident waves.[1)]**

✓ **Total Velocity Potential**

$$\Phi_T=\Phi_I+\Phi_D+\Phi_R$$

Superposition Theorem

For homogeneous linear PDE, superpositon of solution is again a solution of PDE[2)]

$$P \quad \rho g z \qquad -\rho\frac{\partial\Phi_T}{\partial t}$$

$$\mathbf{F}_{Fluid}=\iint_{S_B}P\mathbf{n}dS$$

$$=\mathbf{F}_{static}+\mathbf{F}_{F.K}+\mathbf{F}_D+\mathbf{F}_R$$

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 287
2) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 12.1 (pp 535)

Pressure and Force acting on Fluid Particle
- Fluid Force acting on ship

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
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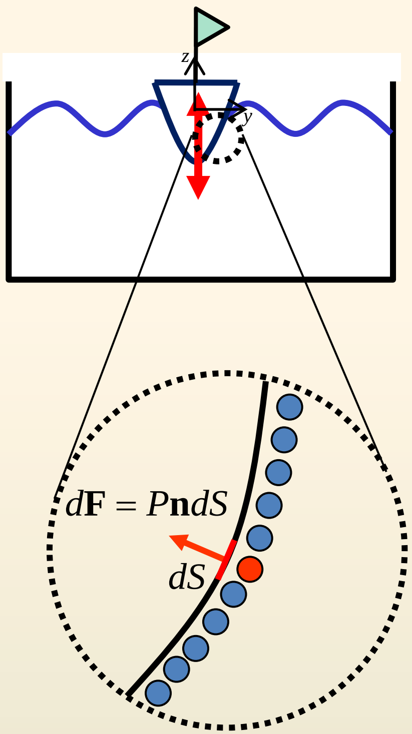
Φ_I : Incident wave velocity potential
Φ_D : Diffraction potential
Φ_R : Radiation potential



$d\mathbf{F} = P\mathbf{n}dS$

dS

✓ Bernoulli Equation

$$\rho\frac{\partial\Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho|\nabla\Phi|^2} + \rho g z = 0$$

Linearization

✓ Laplace Equation

$$\nabla^2\Phi = 0$$

$$\Phi = \Phi_I + \Phi_D + \Phi_R$$

Linear combination of Basic solutions

Basic solutions

$$P_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = -\rho g z - \rho\frac{\partial\Phi}{\partial t} = \boxed{-\rho g z} - \rho\left(\frac{\partial\Phi_I}{\partial t} + \frac{\partial\Phi_D}{\partial t} + \frac{\partial\Phi_R}{\partial t}\right)$$

Pressure of fluid particle acting on ship

$$= \boxed{P_{Buoyancy}(\mathbf{r})} + \underbrace{P_{F.K}(\mathbf{r}) + P_D(\mathbf{r}) + P_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})}_{P_{dynamic}}$$

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P_{Fluid}\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

(S_B: wetted surface)

Integral over wetted surface of ship
(Force and moment acting on ship due to fluid particle)

$d\mathbf{F}$: Force of one fluid particle acting on ship

dS : Differential Area

$\mathbf{n}$: Normal vector of differential Area

$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$

ξ_1 : surge , ξ_4 : roll
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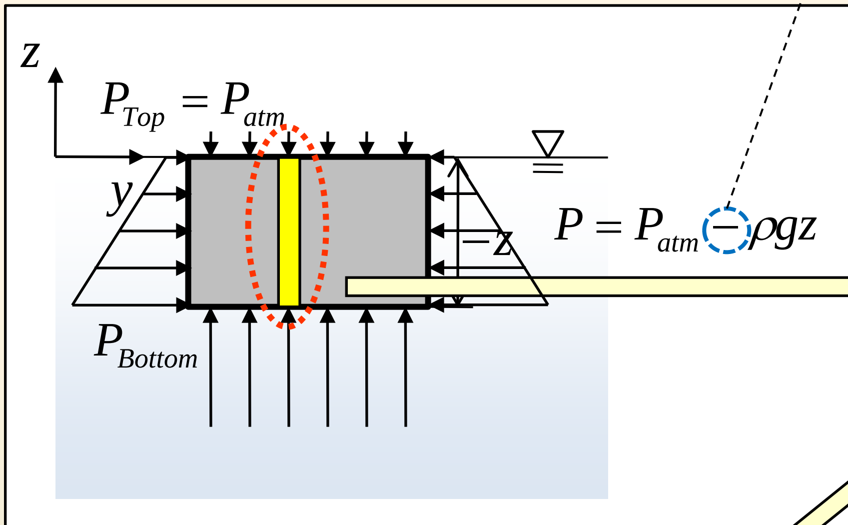
Derivation of Buoyant Force

※ Pressure : force per unit area applied in a direction perpendicular to the surface of an object
So, in order to calculate force, we should multiply pressure by area and normal vector of the area.

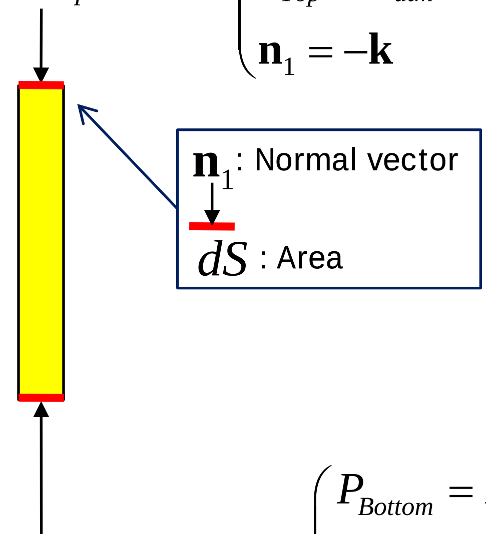
According to frame, (-) sign is added because value of z is (-).

✓ What is the force acting on the bottom of object?



: Force acting on upper differential area

$$dF_{Top} = P_{Top} \cdot \mathbf{n}_1 dS \quad \begin{pmatrix} P_{Top} = P_{atm} - \rho g \cdot 0 \\ \mathbf{n}_1 = -\mathbf{k} \end{pmatrix}$$



$$d\mathbf{F}_{Bottom} = P_{Bottom} \cdot \mathbf{n}_2 dS \quad \begin{pmatrix} P_{Bottom} = P_{atm} - \rho g z \\ \mathbf{n}_2 = \mathbf{k} \end{pmatrix}$$

: Force acting on lower differential area

$$\begin{aligned} d\mathbf{F} &= d\mathbf{F}_{Top} + d\mathbf{F}_{Bottom} \\ &= P_{Top} \cdot \mathbf{n}_1 dS + P_{Bottom} \cdot \mathbf{n}_2 dS \\ &= \cancel{P_{atm}(-\mathbf{k})dS} + (\cancel{P_{atm}} - \rho g z)\mathbf{k}dS \\ &= -\rho g z \mathbf{k} dS = \mathbf{k}(-\rho g z \cdot dS) \end{aligned}$$

: Force due to atmosphere pressure is vanished.

$$\mathbf{F} = \int d\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \quad ,(P = P_{static} = -\rho g z)$$

Pure static pressure that total static pressure minus atmosphere pressure.

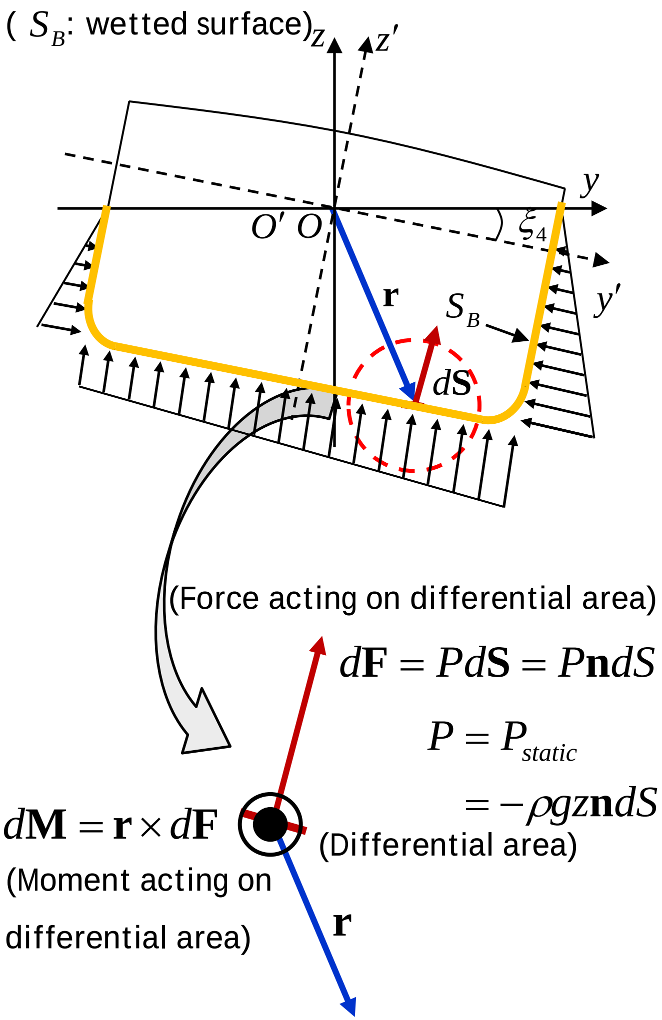
$$= -\rho g \iint_{S_B} \mathbf{n} z dS$$

Cf) Linearized Bernoulli eq.

$$P = \underbrace{-\rho g z}_{P_{static}} \underbrace{- \rho \frac{\partial \Phi}{\partial t}}_{P_{dynamic}}$$

Derivation of Buoyant Force
- Hydrostatic Pressure and Buoyant Force acting on Ship

In case that ship is heel about –x axis
(Front side view)



- Hydrostatic force (Surface force)

: Hydrostatic force is calculated from integral of force over wetted surface.

✓ Force acting on differential area :

$$d\mathbf{F} = P \cdot d\mathbf{S} = P \cdot \mathbf{n}dS$$

P is hydrostatic pressure, P_{static}.

$$P = P_{static} = -\rho gz$$

$$d\mathbf{F} = P_{static} \cdot \mathbf{n}dS = -\rho gz \cdot \mathbf{n}dS$$

✓ Total force : (S_B: wetted surface)

$$\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \Rightarrow \boxed{\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS}$$

- Hydrostatic Moment : (Moment)=(Position vector) X (Force)

✓ Moment acting on differential area :

$$d\mathbf{M} = \mathbf{r} \times d\mathbf{F} = \mathbf{r} \times P\mathbf{n}dS = P(\mathbf{r} \times \mathbf{n})dS$$

✓ Total moment :

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r} \times \mathbf{n})dS \Rightarrow \boxed{\mathbf{M} = -\rho g \iint_{S_B} z(\mathbf{r} \times \mathbf{n})dS}$$

Derivation of Buoyant Force

- Hydrostatic Pressure and Buoyant Force acting on Ship

✓ Hydrostatic force (Surface force)

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(p458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(p414~417)

$$\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS$$

(S : wetted surface)

By divergence theorem[1) ,

$$\left(\iint_S f \cdot \mathbf{n}dA = \iiint_V \nabla f dV \right)$$

$$\mathbf{F} = \rho g \iiint_V \nabla z dV \qquad \left(\nabla z \overset{2)}{=} \frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} + \frac{\partial z}{\partial z}\mathbf{k} = \mathbf{k} \right)$$

$$= \mathbf{k}\rho g \iiint_V dV$$

$$= \mathbf{k}\rho g V(t)$$

When ship moves, displacement volume(V) is changed in process of time.
So, V is the function of time, $V(t)$.

: The buoyant force on an immersed body has the same magnitude as the weight of the fluid displaced by the body1). And the direction of the buoyant force is opposite to the gravity (≒Archimedes' Principle)

※ The reason that (−) sign is disappeared

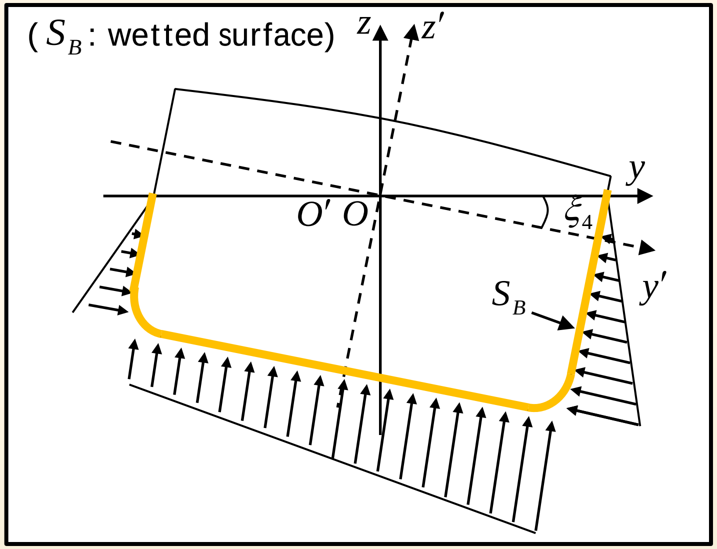
: Divergence theorem is based on outer unit vector of surface.

Normal vector for calculation of buoyant force is based on inner unit vector of surface, so (-)sign is added, then divergence theorem to be applied.

Hydrostatic Moment

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(pp.458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(pp.414~417)

✓ Hydrostatic moment



$$\mathbf{M} = -\rho g \iint_{S_B} (\mathbf{r}\times\mathbf{n}) z dS = \rho g \iint_{S_B} (\mathbf{n}\times\mathbf{r}) z dS$$

By divergence theorem[1) ,

$$\left(\iiint_V \nabla\times\mathbf{F} dV = \iint_S \mathbf{n}\times\mathbf{F} dA \right)$$

$$\mathbf{M} = -\rho g \iiint_V (\nabla\times\mathbf{r}) z dV$$

Because direction of normal vector is opposite,
(-) sign is added

$$\left(\nabla\times\mathbf{r}\,z \overset{2)}{=} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xz & yz & z^2 \end{vmatrix} = \mathbf{i}\left(\frac{\partial}{\partial y}z^2 - \frac{\partial}{\partial z}yz\right) + \mathbf{j}\left(\frac{\partial}{\partial z}xz - \frac{\partial}{\partial x}z^2\right) + \mathbf{k}\left(\frac{\partial}{\partial x}yz - \frac{\partial}{\partial y}xz\right) = -\mathbf{i}y + \mathbf{j}x \right)$$

$$\therefore \mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV$$

6 D.O.F Equation of ship motions

$\mathbf{r}$: displacement of particle with respect to time

$\mathbf{V}=\frac{d\mathbf{r}}{dt}, \mathbf{a}=\frac{d^2\mathbf{r}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
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✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
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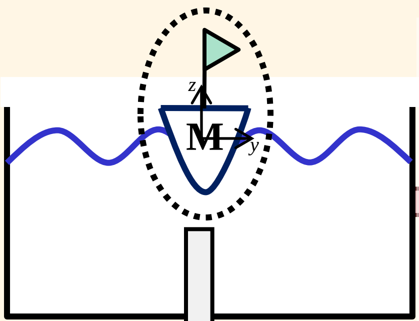


✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Newton's 2nd Law

$m\ddot{r}=\sum F=(\text{Body force}+\text{Surface force})$

Cauchy equation → ①② Shear force → Navier-Stokes equation → ③ Euler equation → ④⑤ Curl & Rotation → Bernoulli equation

Microscopic/ Macroscopic Derivation(RTT[1])

Mass Conservation Law → ④⑤ Laplace Equation

$$\nabla^2\Phi=0 \quad (\mathbf{V}=\nabla\Phi)$$

$$\rho\frac{\partial\Phi}{\partial t}+P+\frac{1}{2}\rho|\nabla\Phi|^2+\rho g z=0$$

Velocity potential Φ

$\Phi=\Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

✓Calculation of Fluid Force

Linearization

$\left(\frac{1}{2}\rho|\nabla\Phi|^2=0\right)$

$$P=-\rho gz-\rho\frac{\partial\Phi}{\partial t}$$

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})=\iint_{S_B}P\mathbf{n}dS=\mathbf{F}_{Buoyancy}(\mathbf{r})+\mathbf{F}_{F.K}(\mathbf{r})+\mathbf{F}_D(\mathbf{r})+\mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

(S_B: wetted surface)

✓6 D.O.F equations of motions

① Coordinate system
(Waterplane Fixed & Body-fixed frame)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{r}}=\sum\mathbf{F}=(Body\ Force)+(Surface\ Force)$$

$$=\mathbf{F}_{gravity}(\mathbf{r})+\boxed{\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})}$$

$$=\mathbf{F}_{gravity}+\mathbf{F}_{Buoyancy}(\mathbf{r})+\mathbf{F}_{F.K}(\mathbf{r})+\mathbf{F}_D(\mathbf{r})+\mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}})+\mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})$$

Non-linear terms → Non-linear equation
→ Difficulty of getting analytic solution

Numerical Method

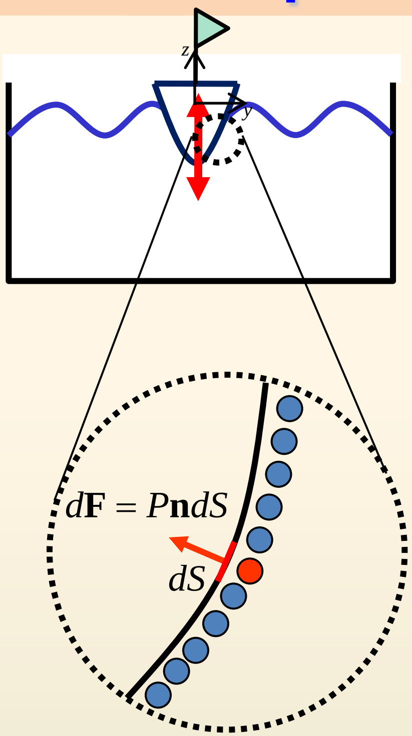
(displacement : $\mathbf{r}=[\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6]^T$)

ξ_1 : *surge* $\quad \xi_4$: *roll*
ξ_2 : *sway* $\quad \xi_5$: *pitch*
ξ_3 : *heave* $\quad \xi_6$: *yaw*

Derivation of 6 D.O.F Equations of ship motions

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_{D}$: Diffraction force
$\mathbf{F}_{R}$: Radiation force

Φ_I : Incident wave velocity potential
Φ_D : Diffraction potential
Φ_R : Radiation potential



$d\mathbf{F}$: Force of one fluid particle acting on ship

dS : Differential Area

$\mathbf{n}$: Normal vector of differential Area

$$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$$

ξ_1 : *surge* ξ_4 : *roll*
ξ_2 : *sway* ξ_5 : *pitch*
ξ_3 : *heave* ξ_6 : *yaw*

$\mathbf{M}_A$: 6×6 added mass matrix
$\mathbf{B}$: 6×6 damping coeff. matrix
$\mathbf{C}$: 6×6 restoring coeff. matrix

G : Gravity Constant

✓ Fluid force acting on ship

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{F.K}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{D}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{R}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

: Fluid force is obtain by calculating pressure acting on one fluid particle from body force and surface force, then integrating the pressure over wetted surface of ship.

✓ 6 D.O.F equations of motions

Newton's 2[nd] Law

Acting on ship as surface force

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body Force}) + \boxed{(\text{Surface Force})}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Fluid} + \mathbf{F}_{external}$$

Body force Surface force

$$\mathbf{M}\ddot{\mathbf{r}} = \boxed{\mathbf{F}_{Gravity}(\mathbf{r})} + \boxed{\mathbf{F}_{Buoyancy}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Hydrodynamic}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})} + \boxed{\mathbf{F}_{external,\,dynamic} + \mathbf{F}_{external,\,static}}$$

Assumption that forces are constant or proportional to displacement, velocity and acceleration.

$$\boxed{\mathbf{M}\ddot{\mathbf{r}} = \boxed{\mathbf{F}_{gravity}} + \boxed{\mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})} + \boxed{\mathbf{F}_{ext,\,dynamic} + \mathbf{F}_{ext,\,static}}}$$

6 D.O.F Equation of ship motions

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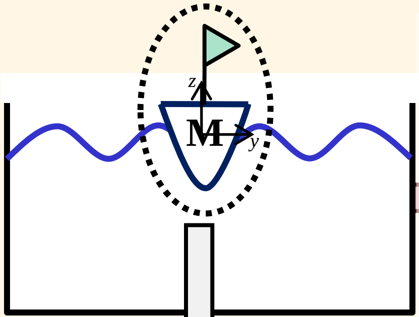

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Shear force

Curl & Rotation

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$\nabla^2 \Phi = 0$
$(\mathbf{V} = \nabla\Phi)$

$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho |\nabla\Phi|^2 + \rho g z = 0$

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$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = (Body\ \ Force) + (Surface\ \ Force)$$

$$= \mathbf{F}_{gravity}(\mathbf{r}) + \boxed{\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r})$$
$$+ \mathbf{F}_{R,Damping}(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}})$$

Non-linear terms → Non-linear equation
→ Difficulty of getting analytic solution

Numerical Method

Velocity potential Φ

✓Calculation of Fluid Force

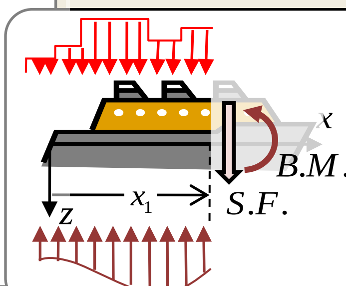
$\Phi = \Phi_I$ (Incident wave potential)
$+ \Phi_D$ (Diffraction potential)
$+ \Phi_R$ (Radiation potential)

Linearization
$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho \frac{\partial \Phi}{\partial t}$$

$$\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

(S_B: wetted surface)



$-m(x)a_z$, $f_{Gravity}$
(a_z : Acceleration of z direction by heave& pitch motion)
$f_{F.K.}$, f_D , $-a_{33}a_z$,
$-b_{33}v_z$, f_{Static}

✓Shear force(S.F.) & bending moment(B.M.)

Shear force(*S.F.*)
↓ Integral
Bending moment(*B.M.*)

-Pressure and Force acting on Fluid Particle
-6 D.O.F Equations of Ship Motions
: Relations among Undergraduate Lectures

$\mathbf{F}_{F.K}$: Froude- krylov force
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Ship Hydrodynamics, Dynamics (2nd-year undergraduate)

✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

ear stress

Curl & Rotation

Newton's 2nd Law

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Cauchy equation → ①② Navier-Stokes equation → ③ Euler equation → ④⑤ Bernoulli equation

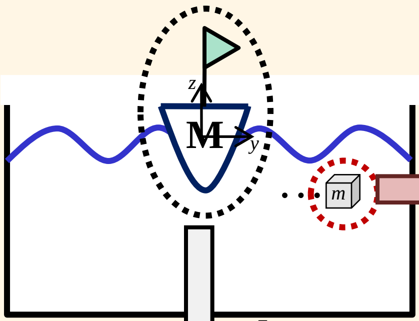
Mass Conservati... Law ④⑤ Laplace

Enigneering Math. (2nd-year undergraduate)

$(\mathbf{V} = \nabla\Phi)$

$$\nabla^2\Phi = 0$$

$$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$$



$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$

ξ_1 : surge , ξ_4 : roll
ξ_2 : sway , ξ_5 : pitch
ξ_3 : heave , ξ_6 : yaw

Microscopic/ Macroscopic Derivation(RTT[1])

✓6 D.O.F equations of motion

Behavior of ship and its control (3rd -year undergraduate)
Dynamics (2nd -year undergraduate)

① Coordinate system
(Waterplane Fixed & Body-fixed frame)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = (Body\ Force) + (Surface\ Force)$$

$$= \mathbf{F}_{gravity}(\mathbf{r}) + \mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r})$$

$$\mathbf{F}_{R,Damping}(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}})$$

Planning procedure of naval architecture and ocean engineering (2nd-year undergraduate)

...ms → Non-linear equation
Difficulty of getting analytic solution

Numerical Method

Computer aided ship design (3rd-year undergraduate)

...locity potential Φ

$\Phi = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

✓Calculation of Fluid Force

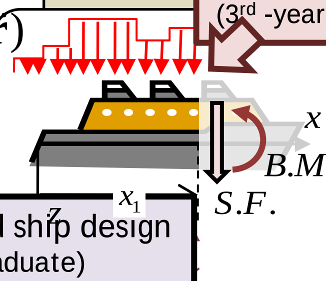
Ocean environment Information system (3rd -year undergraduate)

$$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$$

$$\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r}) + \mathbf{F}_{R}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

Ship Structural Design system (3rd -year undergraduate)

Fundamental of maritime Structural statics (2nd -year undergraduate)



$f_{Gravity}$
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$f_{F.K.}$, f_D , $-a_{33}a_z$,
$-b_{33}v_z$, f_{Static}

& bending moment(B.M.)

Shear force(S.F.)
🠗 Integral
Bending moment(B.M.)

Hydrostatic Force & Moment

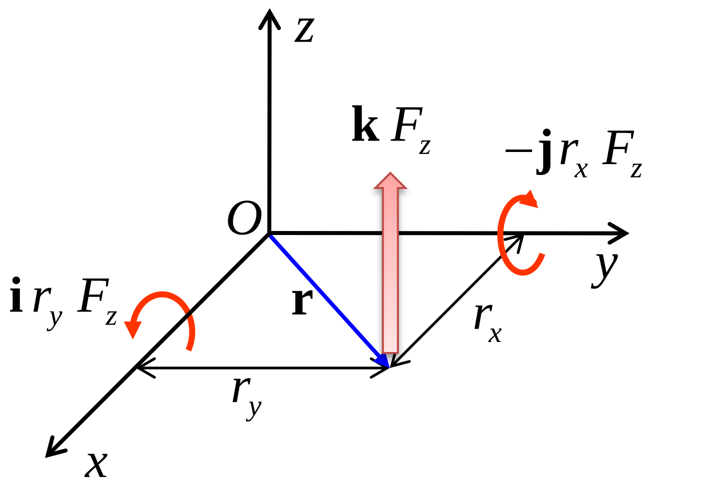
Hydrostatic Force & Moment

Hydrostatic force and gravitational force are applied in perpendicular direction($\mathbf{k}$) to waterplane

Which direction dose moment due to buoyancy and gravitational force applied in?

✓ Hydrostatic Moment

1. In case that F_z is applied on $(r_x, r_y, 0)$

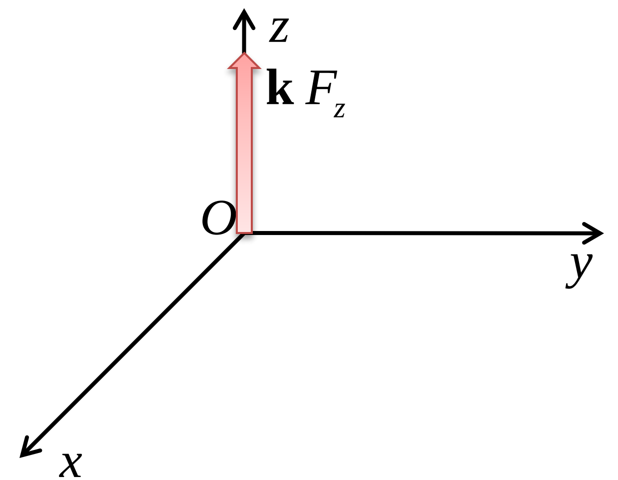


Moment about x axis $(\mathbf{i}\,r_y\,F_z)$, moment about y axis $\mathbf{j}\,r_x\,F_z$ are applied

Force in z direction dose not result in moment about z axis inherently.

$$\mathbf{r}\times\mathbf{F} = (r_x, r_y, 0)\times(0,0,F_z) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & 0 \\ 0 & 0 & F_z \end{vmatrix} = \mathbf{i}\,r_y\,F_z - \mathbf{j}\,r_x\,F_z$$

2. In case that F_z is applied on origin of the frame



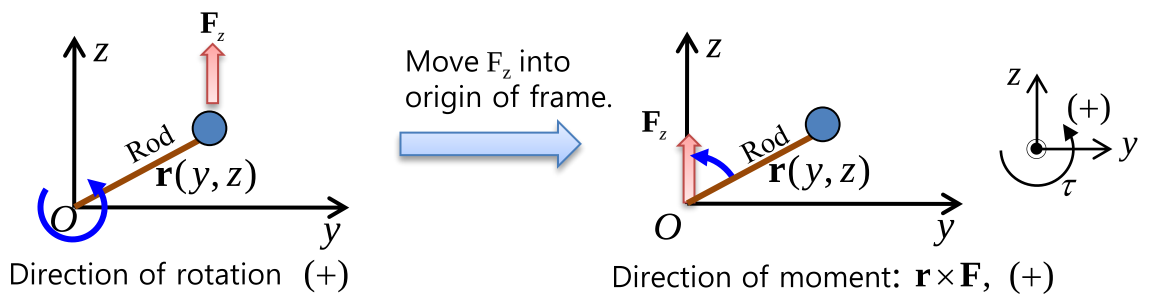
Because moment arms about x axis and y axis are zero, F_z does not result in moment.

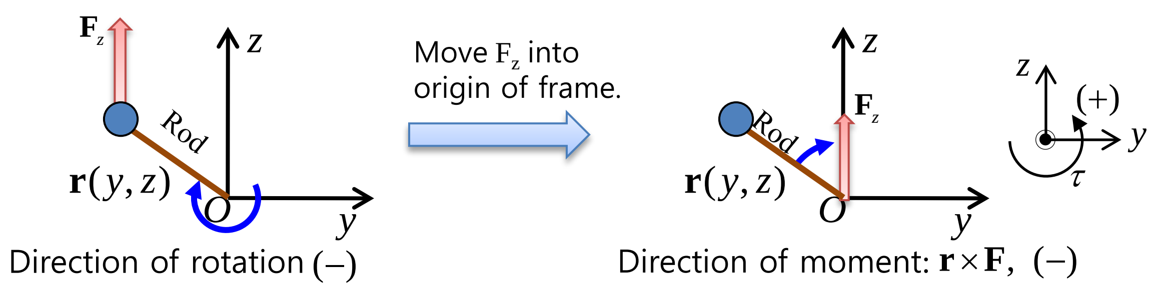
Why moment is defined as $\mathbf{F}\times\mathbf{r}$, not $\mathbf{r}\times\mathbf{F}$?

Sign Convention for Moment

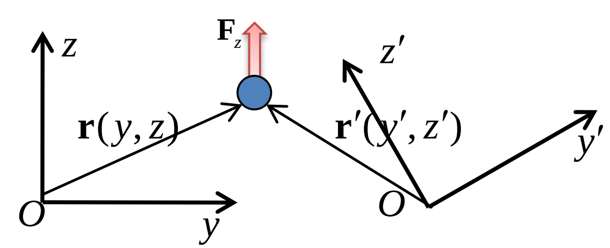
Why moment is defined as $\mathbf{F}\times\mathbf{r}$, not $\mathbf{r}\times\mathbf{F}$?

1. Consistent with reference frame





2. Convenient if we define something variant about reference frame(position vector) in the first place.

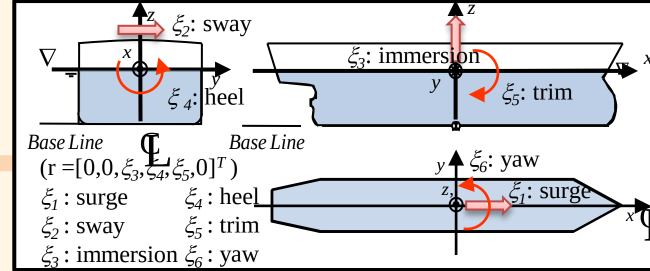


Position Vector(r) is **variant** about reference frame.

Force(F) is **invariant** about reference frame.

So, moment is consistent if we define position vector first.

Hydrostatic Moment



✓ Hydrostatic moment

$$\mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV = \rho g \iiint_V [\mathbf{i}y - \mathbf{j}x] dV$$

$$= \rho g \iiint_V \mathbf{i} y dV - \rho g \iiint_V \mathbf{j} x dV = \mathbf{i} \rho g V \cdot y_B - \mathbf{j} \rho g V \cdot x_B = \mathbf{i} M_{TB} + \mathbf{j} M_{LB}$$

Buoyancy

A moment about longitudinal axis due to buoyancy

A moment about transverse axis due to buoyancy

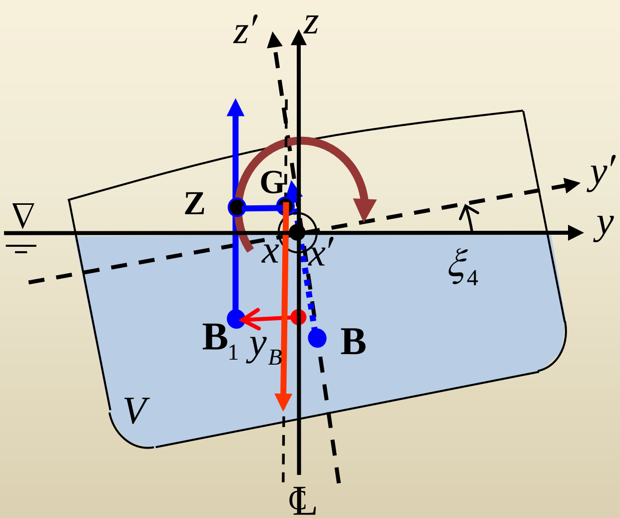
$$\left(M_{LB} = -\rho g V \cdot x_B \right)$$

$$\left(y_B = \frac{\iiint_V y dV}{\iiint_V dV} = \frac{\iiint_V y\, dV}{V}, x_B = \frac{\iiint_V x dV}{\iiint_V dV} = \frac{\iiint_V x\, dV}{V} \right)$$

Transverse center of buoyancy

Longitudinal center of buoyancy

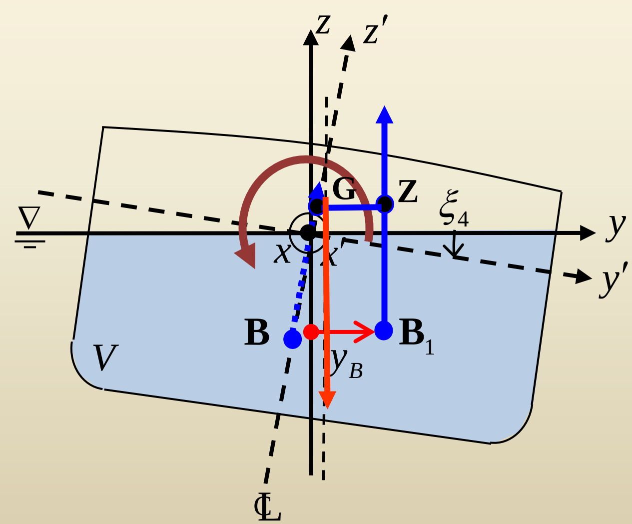
<In case that a ship is heel in positive direction about x axis(+)>



$$\mathbf{M}_{TB} < 0$$

$y_B<0$, so moment is negative.

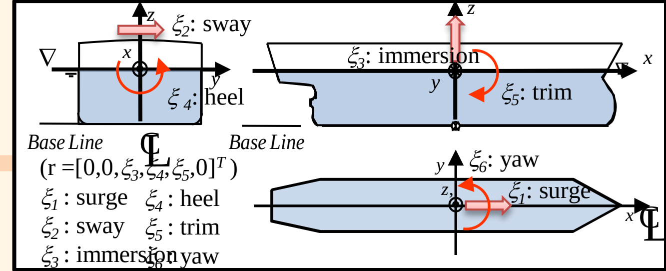
<In case that a ship is heel in negative direction about x axis(-)>



$$\mathbf{M}_{TB} > 0$$

$y_B>0$, moment is positive.

Hydrostatic Moment



✓ Hydrostatic moment

$$\mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV = \rho g \iiint_V [\mathbf{i}y - \mathbf{j}x] dV$$

$$= \rho g \iiint_V \mathbf{i} y dV - \rho g \iiint_V \mathbf{j} x dV = \mathbf{i} \boxed{\rho g V} y_B - \mathbf{j} \boxed{\rho g V} x_B = \mathbf{i} M_{BT} + \mathbf{j} M_{BL}$$

부력

A moment about longitudinal axis due to buoyancy

A moment about transverse axis due to buoyancy

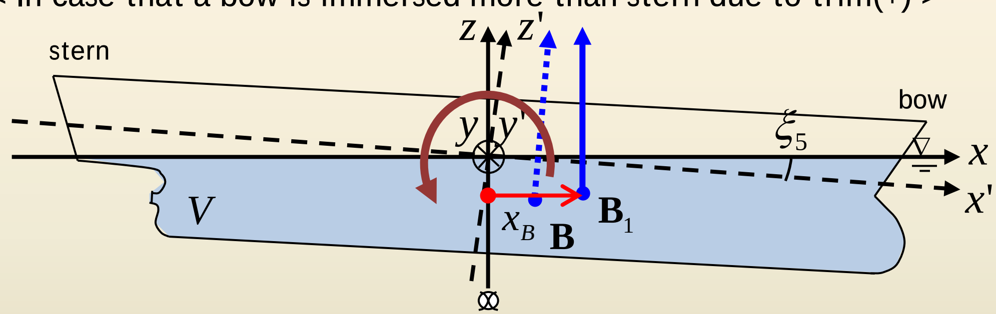
$$\left(M_{BL} = -\rho g V \cdot x_B\right)$$

$$\left(y_B = \frac{\iiint_V y dV}{\iiint_V dV} = \frac{\iiint_V y\, dV}{V}, \quad x_B = \frac{\iiint_V x dV}{\iiint_V dV} = \frac{\iiint_V x\, dV}{V} \right)$$

Transverse center of buoyancy

Longitudinal center of buoyancy

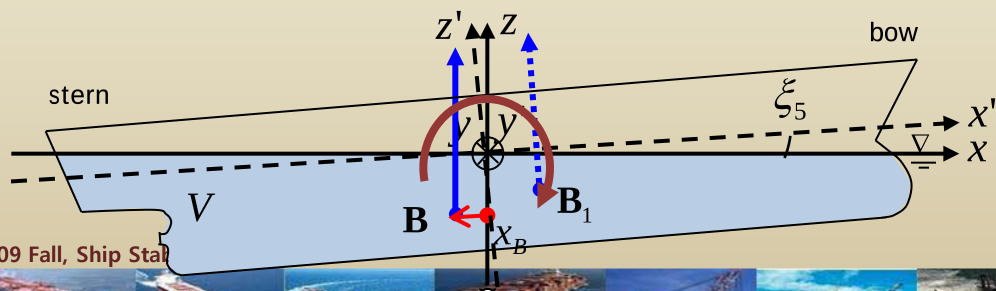
< In case that a bow is immersed more than stern due to trim(+) >



$$\mathbf{M}_{BL} < 0$$

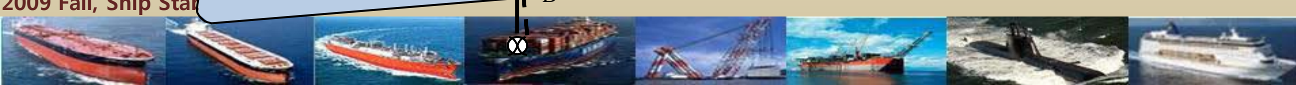
$x_B > 0$, so moment is negative.

< In case that a stern is immersed more than stern due to trim(-) >



$$\mathbf{M}_{BL} > 0$$

$x_B < 0$, so moment is positive.



Governing Equation of Ship Hydrostatic

Governing Equation of Ship Hydrostatics
: Ship Hydrodynamics vs Ship Hydrostatics

Ship Hydrodynamics

The objective of ship hydrodynamics is to find <u>force & moment and position in state of equilibrium</u> that hydrodynamic force & moment, gravitational force & moment and external force & moment are balanced

(Ship hydrodynamics is the case that acceleration and hydrodynamics forces & moments exist in 6 D.O.F equations of motions)

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = \mathbf{F}_{Body}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Surface}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{ext,\,dynamic} + \mathbf{F}_{ext,\,static}$$

$$= \mathbf{F}_{Gravity}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Bouyancy}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Hydrodynamic}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{ext,\,dynamic} + \mathbf{F}_{ext,\,static}$$

$$\boxed{\mathbf{M}\ddot{\mathbf{r}} = \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}}) + \mathbf{F}_{ext,\,dynamic} + \mathbf{F}_{ext,\,static}}$$

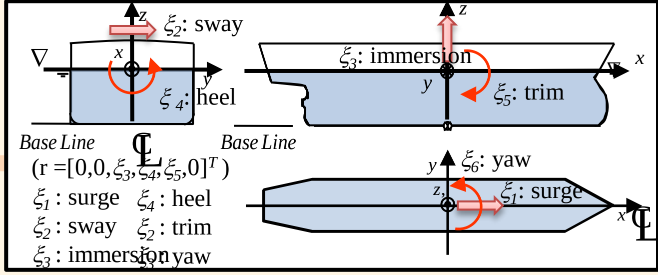
$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$

ξ_1 : surge , ξ_4 : roll

ξ_2 : sway , ξ_5 : pitch

ξ_3 : heave , ξ_6 : yaw

Governing Equation of Ship Hydrostatics
: Ship Hydrodynamics vs Ship Hydrostatics



Ship Hydrodynamics

The objective of ship hydrodynamics is to find force & moment and position in static equilibrium that hydrodynamic force & moment, gravitational force & moment and external force & moment are balanced.

(Ship hydrodynamics is the case that acceleration and hydrodynamics forces & moments exist in 6 D.O.F equations of motions)

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = \mathbf{F}_{Body}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Surface}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

Assumption: Universal Gravitational Force

$$= \mathbf{F}_{Gravity}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Wind} + \mathbf{F}_{Current} + \mathbf{F}_{Mooring} \cdots$$

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS\,,\ P = \cancel{\rho g z} \quad -\rho\frac{\partial \Phi}{\partial t}$$

$$= \iint_{S_B} -\rho g z dS + \iint_{S_B} -\rho \frac{\partial \Phi}{\partial t} dS$$

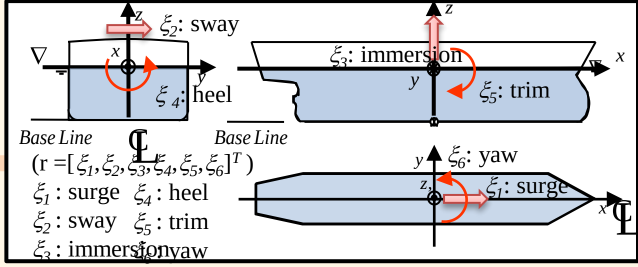
$$= \mathbf{F}_{Gravity}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Bouyancy}(\mathbf{r}) + \mathbf{F}_{Hydrodynamic}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{External}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

$$\Phi = \Phi_I + \Phi_D + \Phi_R$$

$$\iint_{S_B} -\rho \frac{\partial \Phi}{\partial t} dS = \underbrace{\iint_{S_B} -\rho \frac{\partial \Phi_I}{\partial t} dS}_{\mathbf{F}_{F.K}(\mathbf{r})} + \underbrace{\iint_{S_B} -\rho \frac{\partial \Phi_D}{\partial t} dS}_{\mathbf{F}_{D}(\mathbf{r})} + \underbrace{\iint_{S_B} -\rho \frac{\partial \Phi_R}{\partial t} dS}_{\mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})}$$

$$\mathbf{M}\ddot{\mathbf{r}} = \mathbf{F}_{Gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}}) + \mathbf{F}_{Ext,\,Dynamic} + \mathbf{F}_{Ext,\,Static}$$

Governing Equation of Ship Hydrostatics
: Ship Hydrodynamics vs Ship Hydrostatics



Ship Hydrodynamics

The objective of ship hydrodynamics is to find force & moment and position in state of equilibrium that hydrodynamic force & moment, gravitational force & moment and external force & moment are balanced.

(Ship hydrodynamics is the case that acceleration and hydrodynamic forces & moments exist in 6 D.O.F equations of motions)

$$\mathbf{M}\ddot{\mathbf{r}} = \mathbf{F}_{Gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}}) + \mathbf{F}_{Ext,\ Dynamic} + \mathbf{F}_{Ext,\ Static}$$

Ship Hydrostatics

The objective of ship hydrostatics is to find force & moment and position in static equilibrium that hydrostatic force & moment, gravitational force & moment and external force & moment are balanced.

$\ddot{\mathbf{r}} = 0$

$\mathbf{F}_{Hydrodynamic} = 0$

(Ship hydrostatics is the case that acceleration and hydrodynamic forces & moments do not exist in 6 D.O.F equations of motions)

$$\cancel{\mathbf{M}\ddot{\mathbf{r}}} = \mathbf{F}_{Gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \cancel{\mathbf{F}_{F.K}(\mathbf{r})} + \cancel{\mathbf{F}_{D}(\mathbf{r})} + \cancel{\mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}})} + \cancel{\mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})} + \cancel{\mathbf{F}_{Ext,\ Dynamic}} + \mathbf{F}_{Ext,\ Static}$$

$$0 = \mathbf{F}_{Gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{Ext,\ Static}(\mathbf{r})$$

Equilibrium condition

$$0 = \mathbf{F}_{Gravity}(\mathbf{r}) + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{Ext,\ Static}(\mathbf{r})$$

(Find) Force & moment and position in static equilibrium

$(\mathbf{r} = [0, 0, \xi_3, \xi_4, \xi_5, 0])$

(Given) External force or moment

($\mathbf{F}_{Buoyancy}(\mathbf{r})$ is changed due to Immersion(ξ_3), Heel(ξ_4), Trim(ξ_5) only.

Governing Equation of Ship Hydrostatics : Change of Position of Ship – 1. Immersion

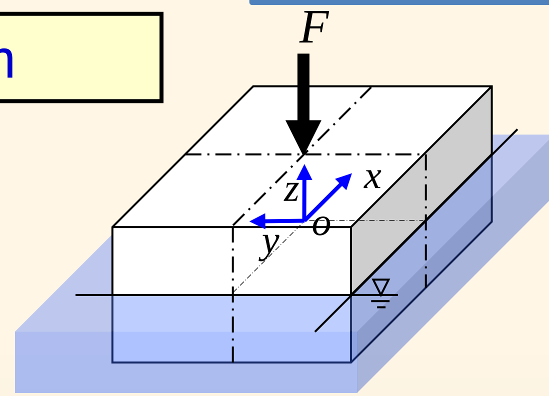
(Ship Hydrostatics) Equilibrium condition

$$0 = \mathbf{F}_{Gravity}(\mathbf{r}) + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{Ext,\,static}$$

(Find) Force & moment and position in static equilibrium $(\mathbf{r} = [0, 0, \xi_3, \xi_4, \xi_5, 0])$

(Given) External force or moment

Change of Position of Ship – 1. Immersion



$(\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T)$

ξ_1 : surge

ξ_2 : sway

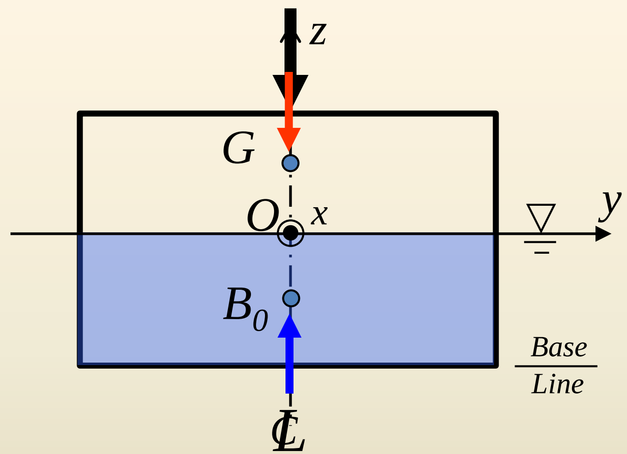
ξ_3 : heave

ξ_4 : roll

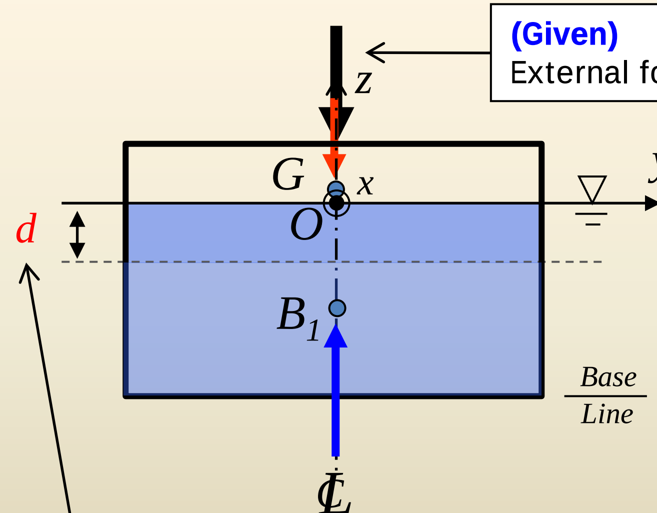
ξ_5 : pitch

ξ_6 : yaw

Immersion due to external force



(Given) External force $\mathbf{k}\, F_{Ext,static}$



G : Center of gravity
B : Center of buoyancy
F : Force
d : Immersion

(Find)
Equilibrium position(ξ_3)

$$\mathbf{k}\, F_{Gravity} + \mathbf{k}\, F_{Buoyancy} + \mathbf{k} F_{Ext,static} = 0$$

Governing Equation of Ship Hydrostatics : Change of Position of Ship – 2. Heel

(Ship Hydrostatics) Equilibrium condition

$$0 = \mathbf{F}_{Gravity}(\mathbf{r}) + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{Ext,\,static}$$

(Find) Force & moment and position in static equilibrium ($\mathbf{r} = [0, 0, \xi_3, \xi_4, \xi_5, 0]$)

(Given) External force or moment

Change of Position of Ship – 2. Heel

$(\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T)$

ξ_1 : surge

ξ_2 : sway

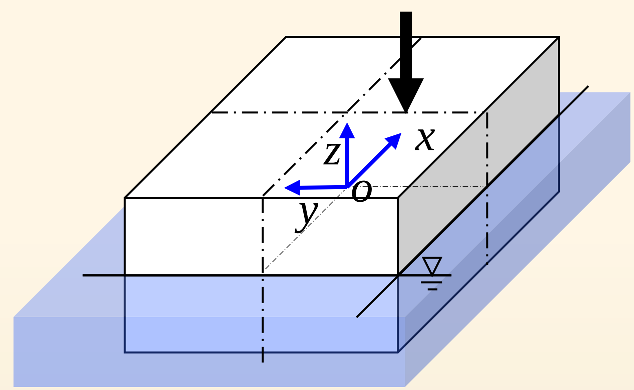
ξ_3 : heave

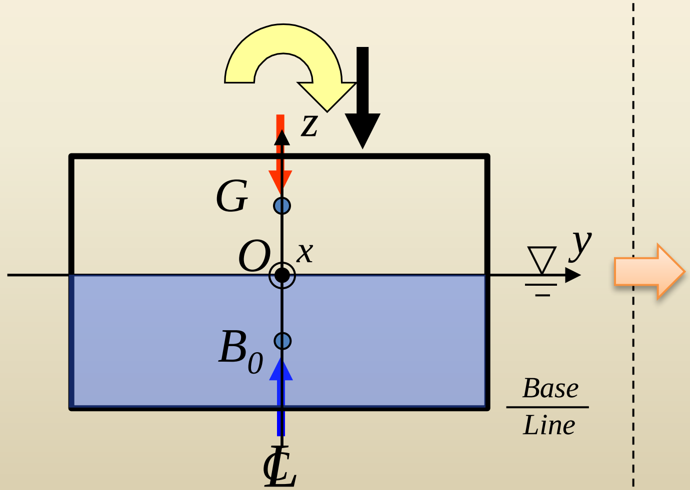
ξ_4 : roll

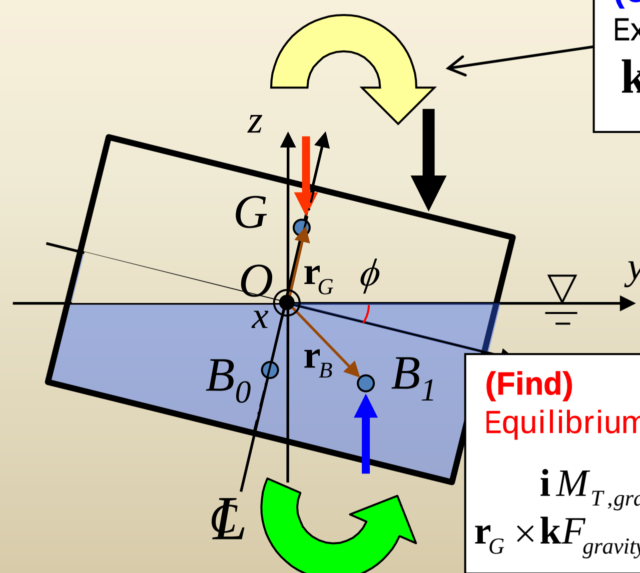
ξ_5 : pitch

ξ_6 : yaw

Heel due to external force







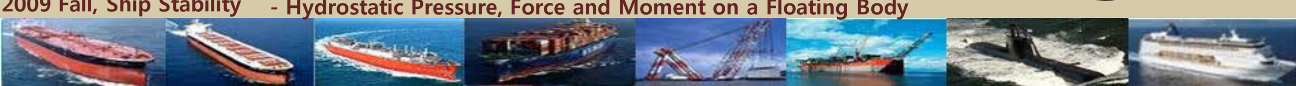
(Given)
External moment
$\mathbf{k}\, M_{T,Ext,static}$

G : Center of gravity
B : Center of buoyancy
F : Force
ϕ : Heel Angle

(Find)
Equilibrium position (ξ_4)

$$\mathbf{i}\,M_{T,gravity} + \mathbf{i}\,M_{T,Buoyancy} + \mathbf{i}\,M_{T,Ext,static} = 0$$

$$\mathbf{r}_G \times \mathbf{k}F_{gravity} + \mathbf{r}_B \times \mathbf{k}F_{Buoyancy} + \mathbf{i}\,M_{T,Ext,static} = 0$$



Governing Equation of Ship Hydrostatics : Change of Position of Ship – 3. Trim

(Ship Hydrostatics) Equilibrium condition

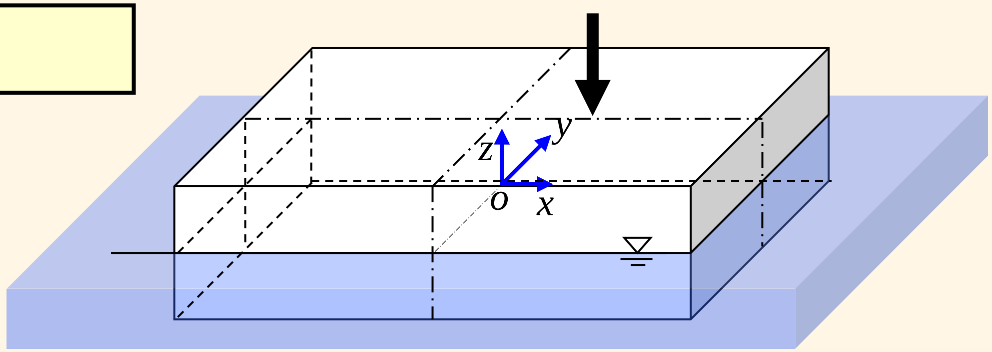
$$0 = \mathbf{F}_{Gravity}(\mathbf{r}) + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{Ext,\ static}$$

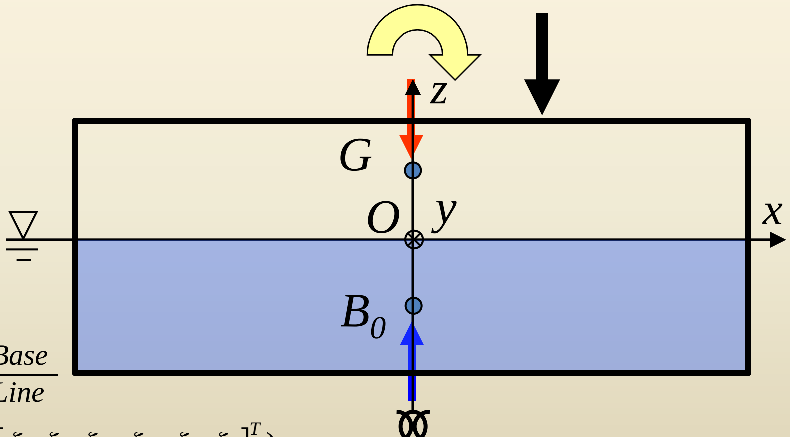
(Find) Force & moment and position in static equilibrium $(\mathbf{r} = [0, 0, \xi_3, \xi_4, \xi_5, 0])$

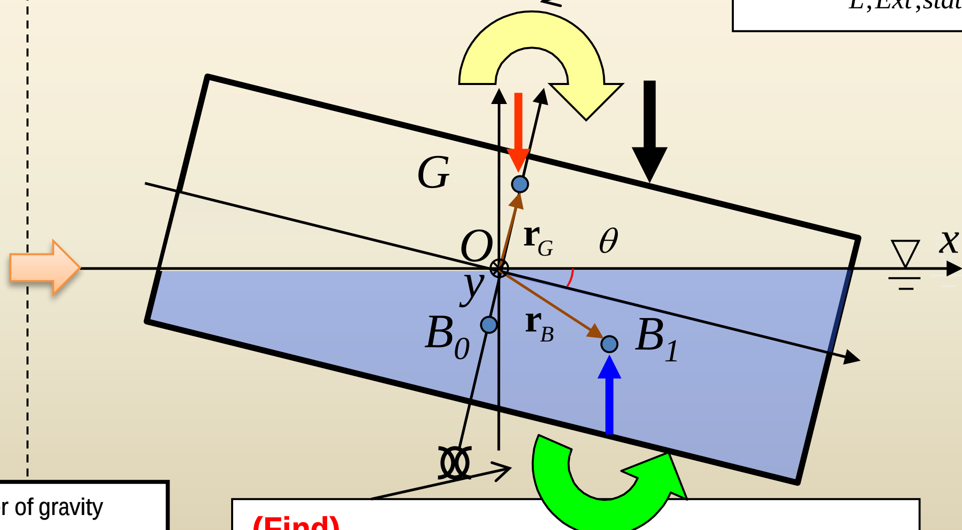
(Given) External force or moment

Change of Position of Ship – 3. Trim

Trim due to external force







(Given)
External moment
$M_{L,Ext,static}\mathbf{k}$

$(\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T)$

ξ_1 : *surge* ξ_4 : *roll*

ξ_2 : *sway* ξ_5 : *pitch*

ξ_3 : *heave* ξ_6 : *yaw*

G : Center of gravity
B : Center of buoyancy
F : Force
θ : Trim Angle

(Find)
Equilibrium position (ξ_5)

$$\mathbf{j}M_{L,Gravity} + \mathbf{j}M_{L,Buoyancy} + \mathbf{j}M_{L,Ext,static} = 0$$

$$\mathbf{r}_G \times \mathbf{k}F_{Gravity} + \mathbf{r}_B \times \mathbf{k}F_{Buoyancy} + \mathbf{j}M_{L,Ext,static} = 0$$