Naval Architecture & Ocean Engineering

- Ship Stability -

Ch.5 Hydrostatic Values

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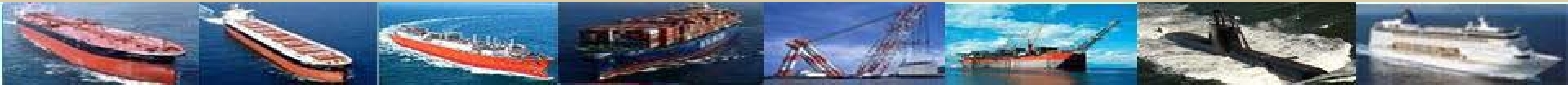
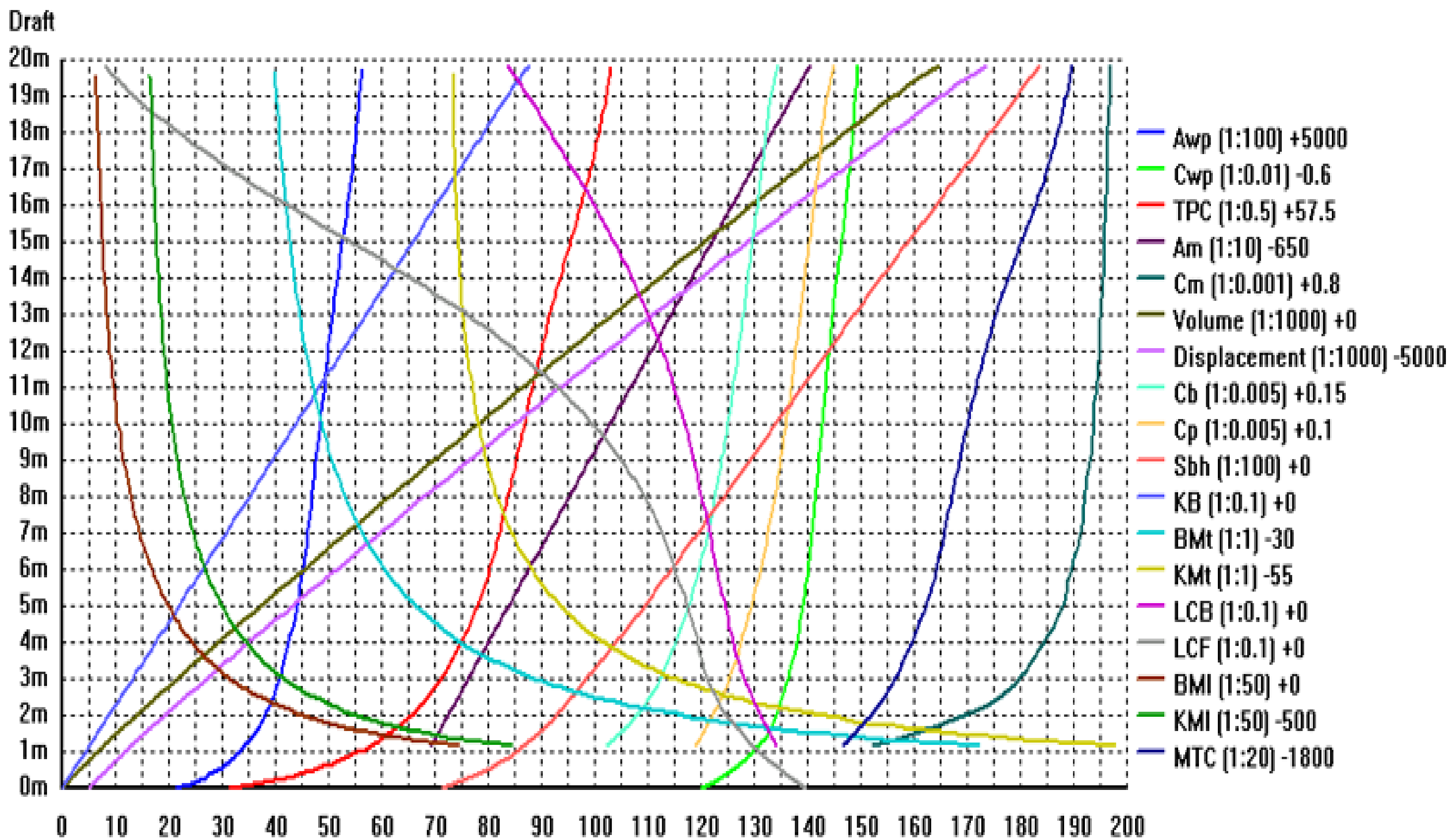


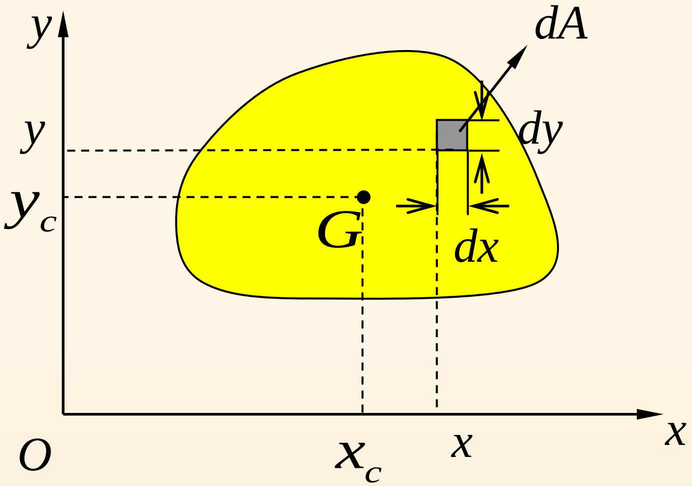
Hydrostatic Values

- ☑ **Draft Mld** : Draft from baseline , moudled (m)
- ☑ **Disp.Mld** : Displacement moulded (m^3)
- ☑ **Disp.Ext** : Displacement extreme (tonnes) S.G. = 1.025 (S.G.: Specific gravity)
- ☑ **VCB** : Vertical center of buoyancy above base line (m)
- ☑ LCB : Longi. center of buoyancy from midship (Sign : - Aft / + Forward)
- ☑ LCF : Longi. center of floatation from midship (Sign : - Aft / + Forward)
- ☑ KM_T : Trans. metarcenter height above base line (m)
- ☑ KM_L : Longi. metarcenter height above base line (m)
- ☑ MTC : Moment to change trim one centimeter (Tonnes-m)
- ☑ TPC : Increase in Disp.MLD(ton) per one centimeter immersion
- ☑ WSA : Wetted surface area (m^2)
- ☑ C_B : Block coefficient
- ☑ C_{WP} : Water plane area coefficient
- ☑ C_M : Midship section area coefficient
- ☑ C_P : Prismatic coefficient
- ☑ **Trim**

Hydrostatic Values



Area(A), 1st Moment of Area(M_A), 2nd Moment of Area(I)



(y_c', z_c') : center of A

✓ **Differential element of area,** dA

$$dA = dx\,dy$$

✓ **Area,** A

$$A = \int dA = \iint dx\,dy$$

✓ **1st moment of area with respect to** y **axis,** $M_{A,x}$

$$M_{A,x} = \int x\,dA = \iint x\,dx\,dy$$

✓ **1st moment of area with respect to** x **axis,** $M_{A,y}$

$$M_{A,y} = \int y\,dA = \iint y\,dx\,dy$$

✓ **Centroid** G

$$G = \left(\frac{M_{A,x}}{A}, \frac{M_{A,y}}{A}\right) = (x_c, y_c)$$

✓ **2nd moment of area with respect** y **axis,** I_x

$$I_x = \int x^2 dA = \iint x^2 dx\,dy$$

✓ **2nd moment of area with respect** y **axis,** I_y

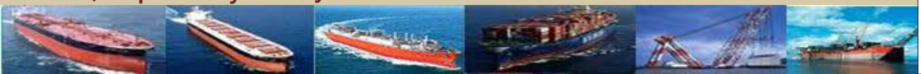
$$I_y = \int y^2 dA = \iint y^2 dx\,dy$$

✓ **Product of inertia with respect to the x & y axis**

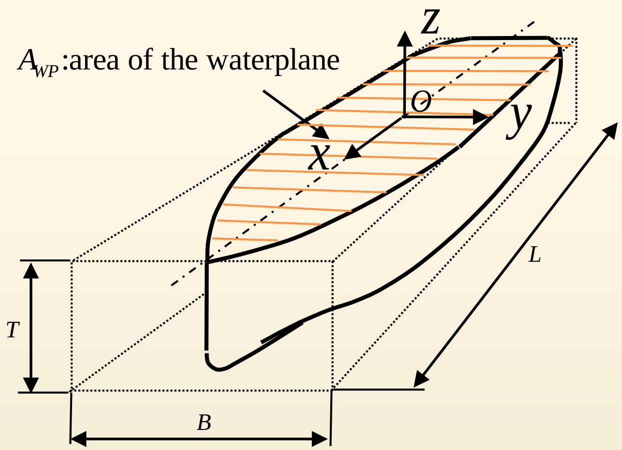
$$I_{xy} = \int xy\,dA = \iint xy\,dx\,dy$$

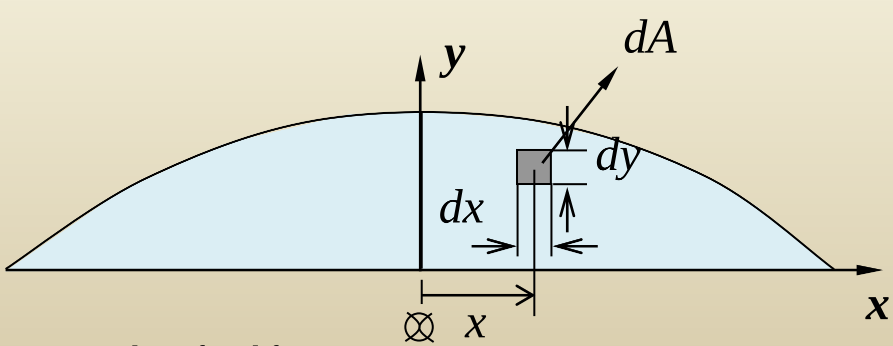
* $M_{A,x}$, $M_{A,y}$, I_x, I_y, I_{xy}

The index indicate the principal direction of the lever !



Waterplane Area(A_{WP}), Tones per 1cm Immersion(TPC), Longitudinal Area Moment(L_{WP}), Longitudinal Center of Floating(LCF)





Oxyz : Waterplane fixed frame

y : Half breadth of each section
ρ_{sw} : Density of water(1.025[ton/m3])

✓Waterplane area

$$A_{WP} = \int dA = 2 \cdot \int_0^L y\,dx$$

✓ Tones per 1cm Immersion (TPC)

$$TPC = \rho_{sw} \cdot A_{WP} \cdot \frac{1}{100}$$

ρ_{sw} : density of sea water

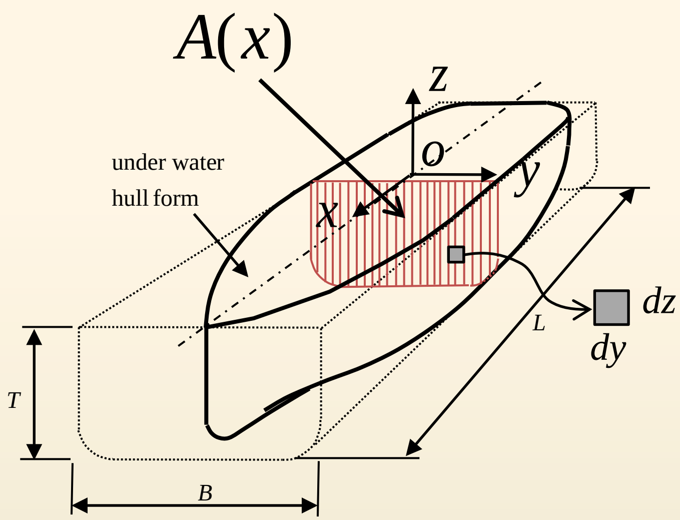
✓1st moment of waterplane area about x axis

$$L_{WP} = \int x\,dA \quad = \iint x\,dx\,dy$$

✓ Longitudinal Center of Floatation

$$LCF = \frac{L_{WP}}{A_{WP}}$$

Sectional Area(A), Displacement Volume(∇)



$Oxyz$: Waterplane fixed frame

✓ **Sectional Area**

$$A(x) = \int dA = \iint dy\,dz$$

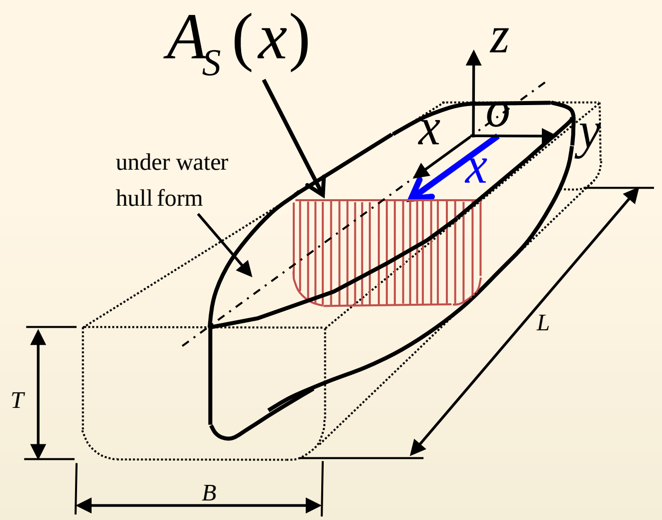
✓ **Displacement volume**

$$\nabla = \int dV = \iiint dx\,dy\,dz$$
$$= \int \left(\iint dy\,dz \right) dx$$
$$\Rightarrow A(x)$$

$$\therefore \nabla = \int A(x)\,dx$$

After calculation of each sectional area, displacement volume can be calculated by integral of section area over the length of ship

Longitudinal Moment of Volume ($M_{\nabla,x'}$), Longitudinal Center of Buoyancy (LCB)



Oxyz : Waterplane fixed frame

✓Longitudinal moment of volume

$$M_{\nabla,x} = \int x dV$$
$$= \iiint x\, dx\, dy\, dz$$
$$= \int \left(\iint x\, dy\, dz \right) dx$$
$$\Rightarrow M_{A,x'}$$

$M_{A,x}$: Longitudinal moment of area about y axis

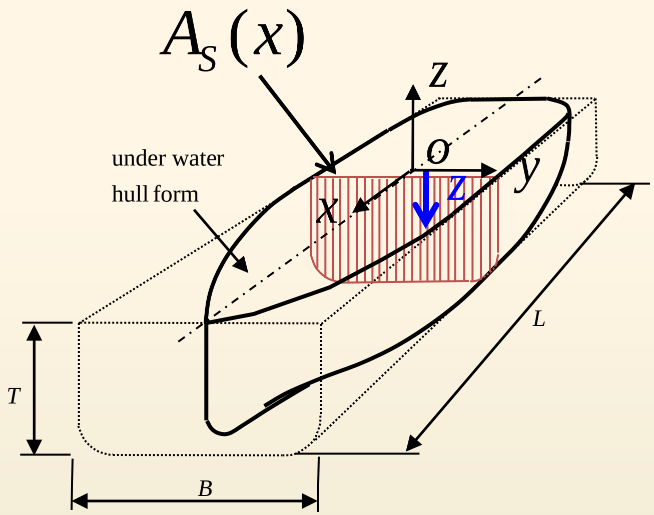
$$\therefore M_{\nabla,x} = \int M_{A,x} dx$$

After calculation of each longitudinal moment of sectional area about the y axis($M_{A,x}$), longitudinal moment of displaced volume can be calculated by integral of longitudinal moment of section area over the length of ship

✓ Longitudinal Center of Buoyancy

$$LCB = \frac{M_{\nabla,x}}{\nabla}$$

Vertical Moment of Volume ($M_{\nabla,y'}$), Vertical Center of Buoyancy (VCB)



Oxyz : Waterplane fixed frame

✓ Vertical moment of volume

$$M_{\nabla,z} = \int z\,dV$$

$$= \iiint z\,dx\,dy\,dz$$

$$= \int \left(\iint z\,dy\,dz \right) dx$$

$$\Rightarrow M_{A,z}$$

$M_{A,z}$: Vertical moment of area about y axis

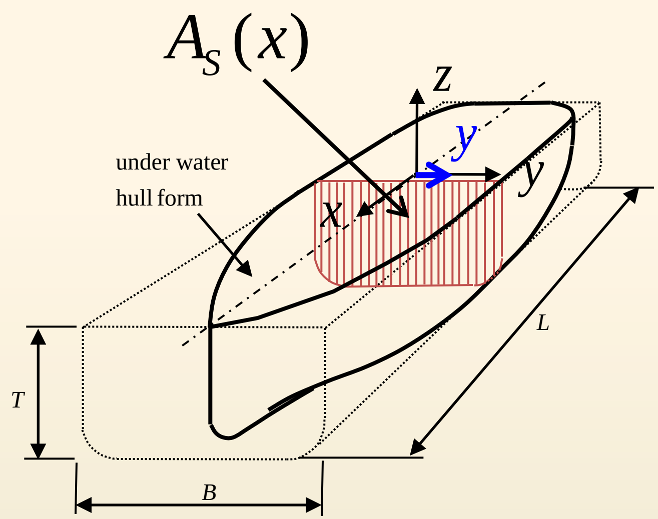
$$\therefore M_{\nabla,z} = \int M_{A,z}\,dx$$

After calculation of each vertical moment of sectional area about the y axis($M_{A,z}$), vertical moment of displaced volume can be calculated by integral of vertical moment of section area over the length of ship

✓ Vertical Center of Buoyancy

$$VCB = \frac{M_{\nabla,z}}{\nabla}$$

Transverse Moment of Volume ($M_{\nabla,z'}$), Transverse Center of Buoyancy (TCB)



Oxyz : Waterplane fixed frame

✓ Transverse moment of volume

$$M_{\nabla,y} = \int y\, dV$$

$$= \iiint y\, dx\, dy\, dz$$

$$= \int \left(\iint y\, dy\, dz \right) dx$$

$$\Rightarrow M_{A,y}$$

$M_{A,y}$: Vertical moment of area about z axis

$$\therefore M_{\nabla,y} = \int M_{A,y} dx$$

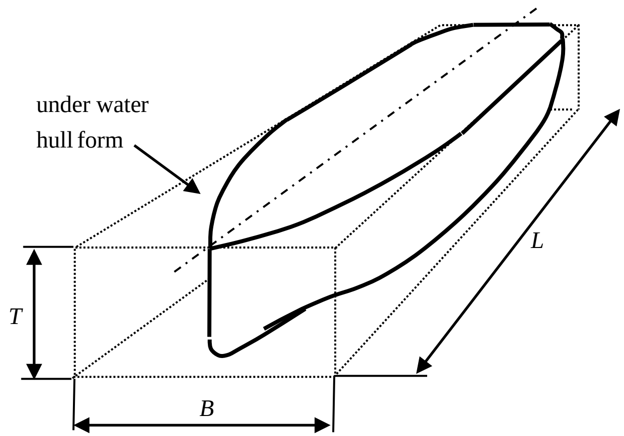
After calculation of each transverse moment of sectional area about the z axis($M_{A,y}$), transverse moment of displaced volume can be calculated by integral of transverse moment of section area over the length of ship

✓ Transverse Center of Buoyancy

$$TCB = \frac{M_{\nabla,y}}{\nabla}$$

Block Coefficient(C_B)
Prismatic Coefficient(C_{WP})

C_B : Block coefficient



$$C_B = \frac{\nabla}{L \cdot B_{mld} \cdot T}$$

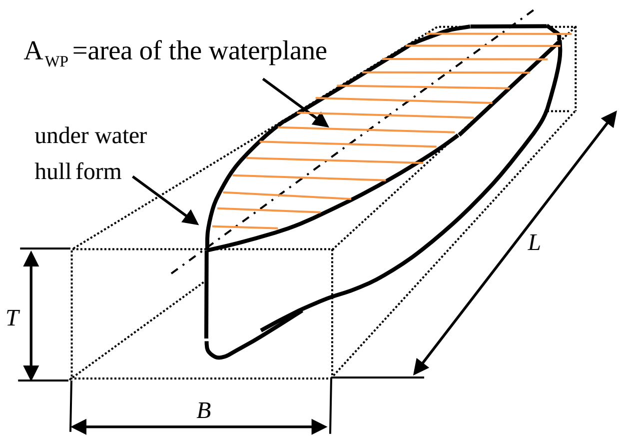
▽ : Moulded volume of displacement

L : Length (L_{WL} or L_{BP})

B_{mld} : Moulded breadth

T_{mld} : Moulded draft

C_{WP} : Water plane Area Coefficient



$$C_{WP} = \frac{A_{WP}}{L \cdot B_{mld}}$$

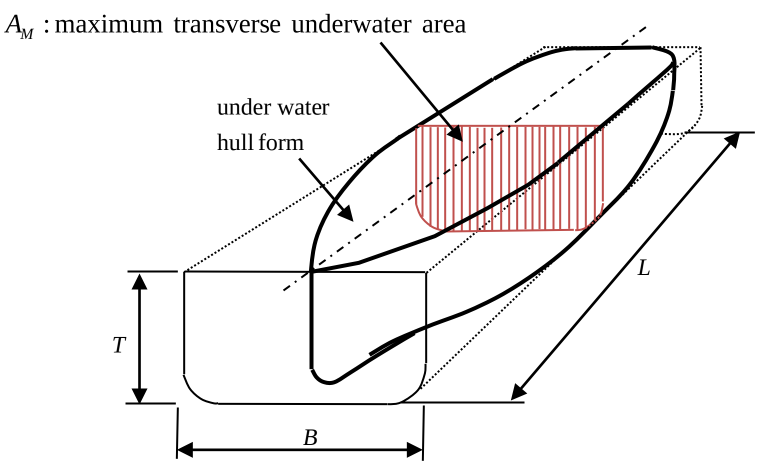
A_{WP} : Waterplane area

L : Length(L_{WL} or L_{BP})

B_{mld} : Moulded breadth

Midship Section Coefficient(C_M)
Prismatic Coefficient (C_P)

C_M (Midship section coefficient)



$$C_M = \frac{A_M}{B_{mld} \times T}$$

B_{mld} : Moulded breadth

T_{mld} : Moulded draft

A_M : Sectional area in midship

C_P (Prismatic coefficient)

$$C_P = \frac{\nabla}{L_{BP} \cdot A_M}$$

$$= \frac{\nabla}{L_{BP} \cdot B_{mld} \cdot T \cdot C_M} = \frac{C_B}{C_M}$$

▽ : Moulded volume of displacement

L : Length (L_{WL} or L_{BP})

B_{mld} : Moulded breadth

T_{mld} : Moulded draft

C_M : Midship Coefficient

C_B : Block Coefficient

Transverse Metacentric Radius(BM), Longitudinal Metacentric Radius(BM_L) Moment to change Trim 1 Cm(MTC), Trim

BM [Refer to Chap.6]

Transverse righting moment = $\Delta \cdot GZ \approx \Delta \cdot GM \cdot \sin\phi$

$$GM = KB + BM - KG$$

$$BM_0 = \frac{I_T}{\nabla}\left(1+\tan^2\phi\right)$$

,(ϕ : Angle of heel)

: In case of considering vertical displacement of V.C.G

$$BM = \frac{I_T}{\nabla}$$

: In case of ignore vertical displacement of V.C.G

GM : Transverse metacentric height

KB : Vertical center of displaced volume

BM : Transverse metacentric Radius

KG : Vertical center of gravity

BM$_L$ [Refer to Chap.7]

Longitudinal righting moment = $\Delta \cdot GZ_L = \Delta \cdot GM_L \cdot \sin\theta$

$$GM_L = KB + BM_L - KG$$

$$BM_{L0} = \frac{I_L}{\nabla}\left(1+\tan^2\theta\right)$$

,(θ : Angle of trim)

: In case of considering vertical displacement of V.C.G

$$BM_L = \frac{I_L}{\nabla}$$

: In case of ignore vertical displacement of V.C.G

GM_L : Longitudinal metacentric height

KB : Vertical center of displaced volume

BM_L : Longitudinal metacentric Radius

KG : Vertical center of gravity

MTC [Refer to Chap.7]

Moment to change Trim 1 Cm

$$MTC = \Delta \cdot GM_L \cdot \frac{1}{L_{BP}\cdot 100}$$

$$GM_L = KB + BM_L - KG$$

(Unit conversion for cm)

If we assume that KB,KG are enough small than BML

$$GM_L \approx BM_L$$

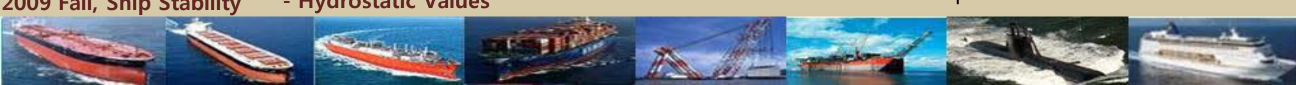
$$\therefore MTC = \Delta \cdot BM_L \cdot \frac{1}{L_{BP}\cdot 100}$$

Trim [Refer to Chap.7]

$$Trim\ Lever = LCB - LCG$$

$$Trim[m] = \frac{\Delta \cdot Trim\,Lever}{MTC \cdot 100}$$

$$MTC = \frac{\Delta \cdot GM_L}{100 \cdot L_{BP}}$$



Example of Programming for Calculation of the Hydrostatics
- Example of Hydrostatic Table and Hydrostatic Curves

Given: Formulas for calculating hydrostatic values
Find: Hydrostatic curves and Hydrostatics Table as function of draft

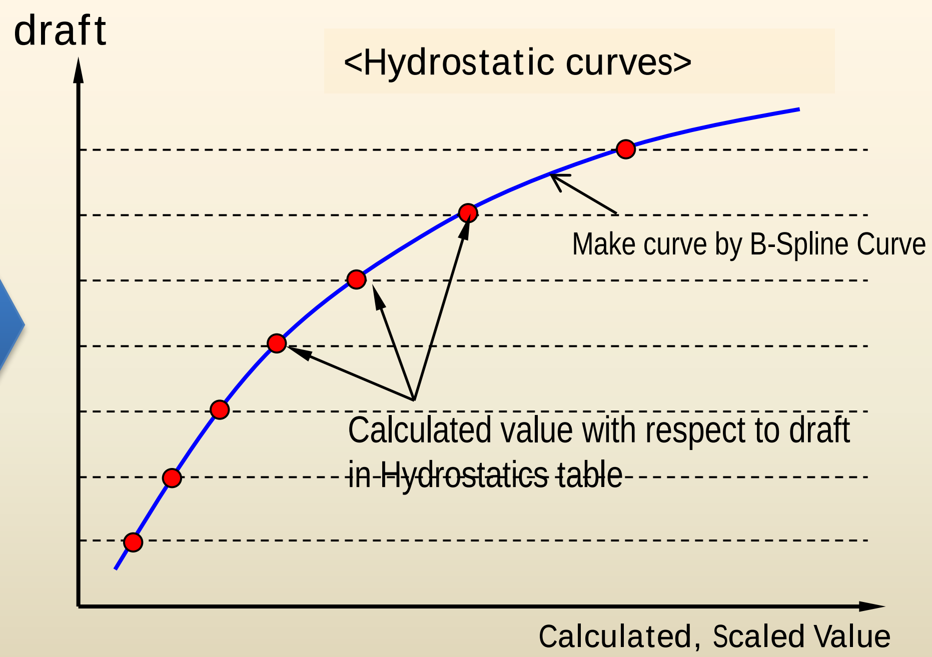
✓ Calculation of hydrostatic values as function of draft

<Hydrostatics Table>

흘수 [m]

	Δ	LCB	TCB	KB	KM_T	...
0	...	...	...	...	...	...
1	...	...	...	...	...	...
2	...	...	...	...	...	...
3	...	...	...	...	...	...
4	...	...	...	...	...	...
5	...	...	...	...	...	...
...	...	...	...	...	...	...

<Hydrostatic curves>



Example of Programming for Calculation of the Hydrostatics
- Example of Hydrostatic Table and Hydrostatic Curves

- 320K VLCC 선박의 Hydrostatics Table 예시 (1)

Hydrostatics Table

Draft	Awp	Cwp	TPC	Am	Cm	Disp. Vol.	Dispacement	Cb	Cp	
1	13969.707634	0.727589	143.189503	57.595373	0.959923	13274.704872	13606.572494	0.691391	0.720257	0
2	14665.449669	0.763826	150.320859	117.023844	0.975199	27625.670041	28316.311792	0.719418	0.737715	1
3	15077.051700	0.785263	154.539780	176.973600	0.983187	42515.292743	43578.175062	0.738113	0.750735	1
4	15357.591332	0.799875	157.415311	236.973600	0.987390	57741.104204	59184.631810	0.751837	0.761439	2
5	15581.372337	0.811530	159.709066	296.973600	0.989912	73212.579375	75042.893859	0.762631	0.770403	2
6	15749.689195	0.820296	161.434314	356.973600	0.991593	88884.693834	91106.811180	0.771569	0.778110	3
7	15875.551257	0.826852	162.724400	416.973600	0.992794	104697.883311	107315.330393	0.779002	0.784656	3
8	15995.591849	0.833104	163.954816	476.973600	0.993695	120634.354919	123650.213792	0.785380	0.790363	4
9	16108.202427	0.838969	165.109075	536.973600	0.994396	136685.843246	140102.989327	0.791006	0.795464	4
10	16220.139230	0.844799	166.256427	596.973600	0.994956	152848.654175	156669.870529	0.796087	0.800123	5
11	16334.646305	0.850763	167.430125	656.973600	0.995415	169122.501317	173350.563850	0.800769	0.804458	5
12	16456.300612	0.857099	168.677081	716.973600	0.995797	185509.431357	190147.167141	0.805162	0.808561	6
13	16586.144990	0.863862	170.007986	776.973600	0.996120	202010.815322	207061.085705	0.809338	0.812491	6
14	16733.101975	0.871516	171.514295	836.973600	0.996397	218662.950551	224129.524315	0.813478	0.816420	7
15	16880.258424	0.879180	173.022649	896.973600	0.996637	235526.994120	241415.168973	0.817802	0.820561	7
16	17033.256489	0.887149	174.590879	956.973600	0.996848	252548.055106	258861.756483	0.822097	0.824696	8
17	17190.202935	0.895323	176.199580	1016.973600	0.997033	269669.514686	276411.252553	0.826193	0.828652	8
18	17330.470220	0.902629	177.637320	1076.973600	0.997198	286937.720924	294111.163948	0.830260	0.832593	9
19	17450.827341	0.908897	178.870980	1136.973600	0.997345	304340.487982	311949.000181	0.834267	0.836487	9
20	17554.763112	0.914311	179.936322	1196.973600	0.997478	321853.728657	329900.071874	0.838161	0.840280	10
21	17654.425395	0.919501	180.957860	1256.973600	0.997598	339467.205809	347953.885955	0.841933	0.843960	10
22	17745.043330	0.924221	181.886694	1316.973600	0.997707	357175.445606	366104.831746	0.845586	0.847529	11
23	17829.121813	0.928600	182.748499	1376.973600	0.997807	374971.328289	384345.611496	0.849120	0.850986	11
24	17906.567070	0.932634	183.542312	1436.973600	0.997898	392848.739497	402669.957984	0.852536	0.854332	12
25	17977.456424	0.936326	184.268928	1496.973600	0.997982	410799.466249	421069.452905	0.855832	0.857562	12
26	18042.453063	0.939711	184.935144	1556.973600	0.998060	428815.884445	439536.281557	0.859006	0.860676	13
27	18109.462826	0.943201	185.621994	1616.973600	0.998132	446896.925743	458069.348887	0.862070	0.863683	14
28	18169.982624	0.946353	186.242322	1676.973600	0.998199	465040.875432	476666.897318	0.865031	0.866592	14
29	18227.152414	0.949331	186.828312	1736.973600	0.998261	483242.386920	495323.446593	0.867892	0.869404	15
30	18281.613265	0.952167	187.386536	1796.973600	0.998319	501498.412094	514035.872397	0.870657	0.872123	15

닫기

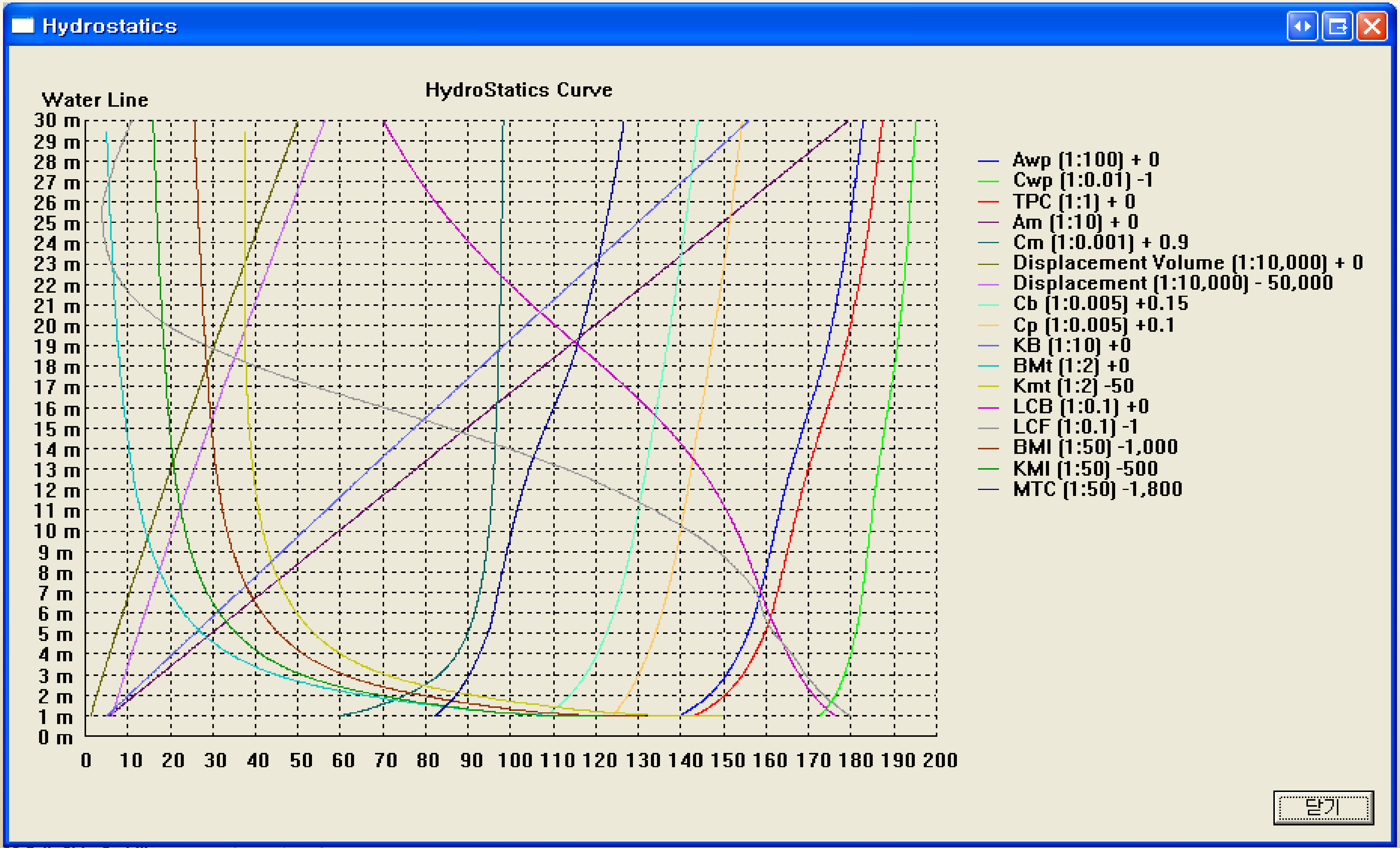
Example of Programming for Calculation of the Hydrostatics
- Example of Hydrostatic Table and Hydrostatic Curves
- Example of hydrostatic table of 320K VLCC

Hydrostatics Table

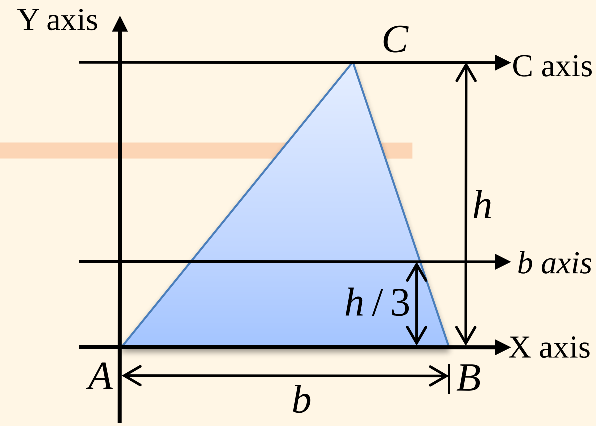
KB	BMt	KMt	LCB	LCF	BMl	KMl	MTC	Wetted Surface Area
0.509932	249.279769	249.789701	17.634696	16.988722	5579.686819	5580.196750	2314.646744	14102.067144
1.025653	131.559866	132.585519	17.124977	16.375976	2962.881019	2963.906672	2557.861669	15079.444762
1.543595	89.894069	91.437664	16.785825	15.944990	2045.756860	2047.300456	2717.998493	15882.807875
2.060474	68.385545	70.446019	16.518405	15.612685	1570.949684	1573.010157	2834.636543	16618.776733
2.576277	55.320467	57.896744	16.287570	15.207640	1281.933552	1284.509829	2932.926936	17331.697356
3.092244	46.498881	49.591125	16.069941	14.941734	1081.449552	1084.541796	3003.884761	18026.084613
3.607174	40.131690	43.738864	15.890147	14.769625	932.964856	936.572030	3052.482676	18706.387874
4.121509	35.310328	39.431836	15.716638	14.383665	824.011114	828.132622	3106.376536	19367.844148
4.635703	31.535720	36.171423	15.530695	13.873811	739.817809	744.453512	3160.081909	20026.661200
5.150036	28.499889	33.649925	15.320611	13.206166	673.530311	678.680346	3217.131299	20688.395322
5.664717	26.007295	31.672012	15.078149	12.389904	620.434826	626.099544	3279.046555	21355.594668
6.179868	23.940218	30.120085	14.798156	11.426314	577.378964	583.558831	3347.163851	22031.346533
6.695516	22.197901	28.893417	14.478059	10.313393	542.171603	548.867119	3422.641486	22719.069067
7.213571	20.701056	27.914627	14.108800	8.961314	514.225484	521.439055	3513.814422	23436.142778
7.736683	19.395506	27.132189	13.686550	7.550015	490.042460	497.779143	3606.819609	24153.666246
8.261164	18.253453	26.514617	13.221739	6.036404	469.665833	477.926997	3706.662270	24885.589906
8.784388	17.250265	26.034653	12.711991	4.427362	452.305205	461.089592	3811.653906	25648.473411
9.309007	16.358312	25.667320	12.168722	3.027873	435.400427	444.709435	3904.150199	26390.817987
9.834664	15.558514	25.393178	11.610030	1.874104	418.610230	428.444894	3981.251301	27121.767720
10.360640	14.833239	25.193879	11.052104	0.949584	402.322606	412.683246	4046.532211	27828.171680
10.886729	14.168543	25.055272	10.508656	0.314228	387.475682	398.362411	4110.477717	28519.892075
11.412880	13.555606	24.968487	9.990360	-0.119337	373.550750	384.963631	4169.473618	29205.249360
11.939003	12.987957	24.926960	9.503047	-0.379617	360.593551	372.532554	4225.382593	29882.641610
12.465035	12.463030	24.928065	9.049601	-0.523423	348.430560	360.895595	4277.515818	30554.971648
12.990852	11.977942	24.968794	8.629644	-0.588068	336.938839	349.929691	4325.446727	31223.264679
13.516351	11.528007	25.044358	8.242049	-0.578749	326.080741	339.597092	4369.643798	31887.840180
14.041601	11.109971	25.151572	7.887679	-0.442092	316.247188	330.288788	4416.559250	32557.540530
14.566638	10.721379	25.288016	7.565974	-0.286588	306.814475	321.381113	4458.789754	33226.725389
15.091404	10.360160	25.451564	7.274229	-0.103187	297.903898	312.995302	4498.743464	33896.183818
15.615903	10.023641	25.639544	7.010481	0.115336	289.495842	305.111745	4536.928276	52901.394845

닫기

Example of Programming for Calculation of the Hydrostatics
- Example of Hydrostatic Table and Hydrostatic Curves
- Example of hydrostatic table of 320K VLCC



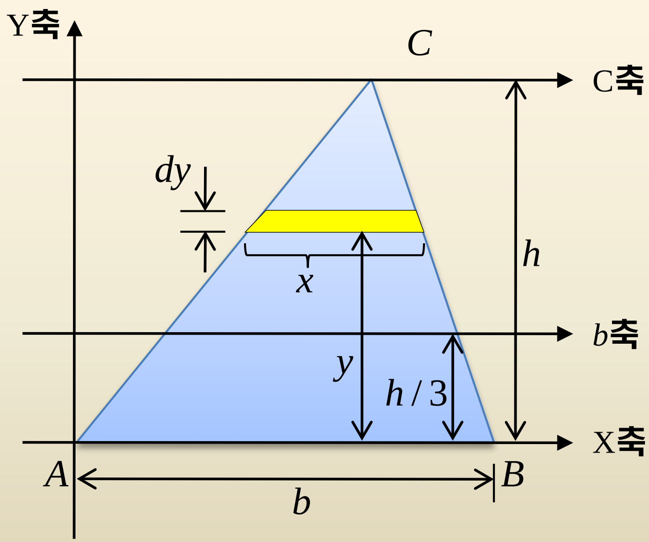
Example 1>



Question) Calculate 2nd moment of area of triangle with lower side a and height h about following axes.

① X axis : I_X=? ② b axis : I_b =? ③ C axis : I_C =?

Solution)



• Calculation of dA

$$b : h = x : (h - y)$$

$$\therefore x = \frac{b(h-y)}{h}$$

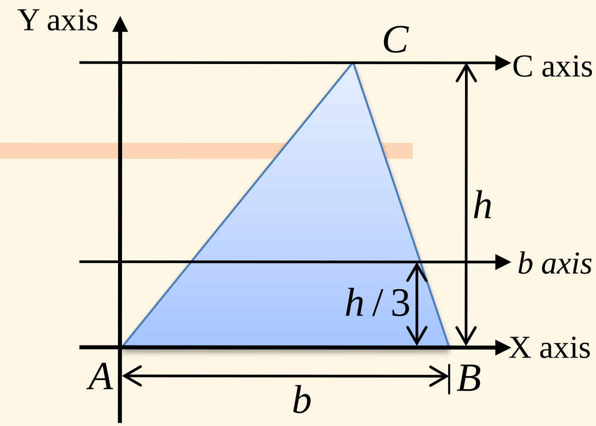
$$\therefore dA = \frac{b(h-y)}{h} dy$$

① Calculation of I_X

$$I_X = \int_0^h y^2 dA = \int_0^h y^2 \frac{b(h-y)}{h} dy$$

$$= b\int_0^h y^2 dy - \frac{b}{h}\int_0^h y^3 dy = \frac{bh^3}{12}$$

Example 1>



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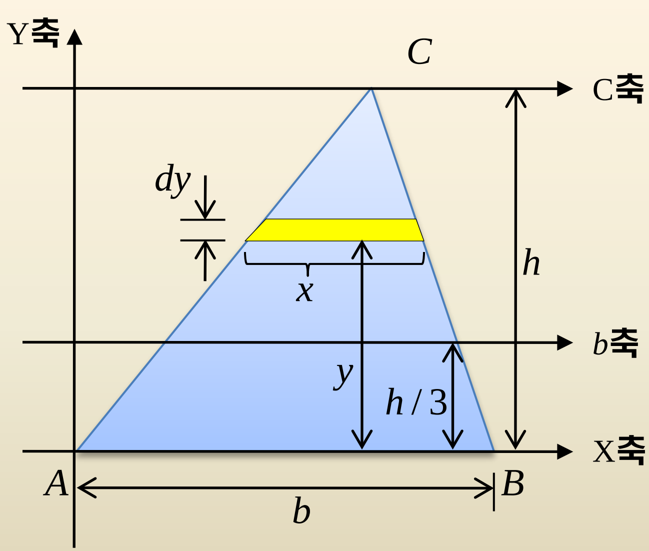
Solution)



② Calculation of I_b

by parallel axis theorem

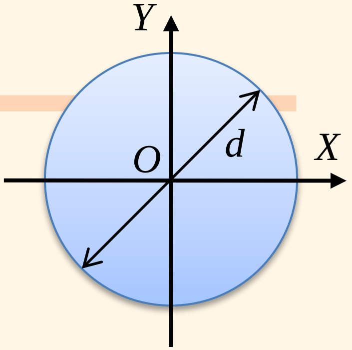
$$I_b = I_X - \left(\frac{h}{3}\right)^2 A = \frac{bh^3}{12} - \frac{h^2}{9}\frac{bh}{2} = \frac{bh^3}{36}$$

③ Calculation of I_C

by parallel axis theorem

$$I_C = I_b + \left(\frac{2h}{3}\right)^2 A = \frac{bh^3}{36} + \frac{4h^2}{9}\frac{bh}{2} = \frac{bh^3}{4}$$

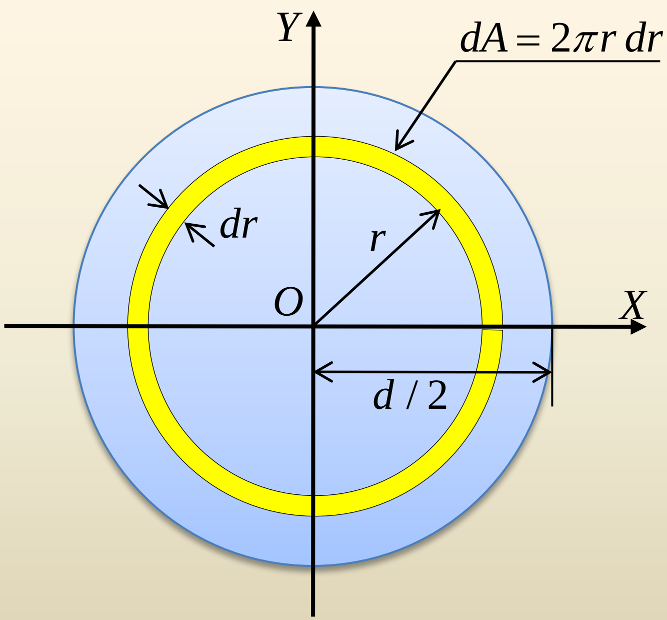
Example 2>



Question) Calculate polar moment of area of circle with diameter d about origin O.

Solution)

$$I_P = I_X + I_Y$$



① Calculation of I_P

$$I_P = \int_0^{d/2} r^2 dA = \int_0^{d/2} r^2\, 2\pi r\, dr$$

$$= 2\pi \int_0^{d/2} r^3 dr = 2\pi \left[\frac{r^4}{4} \right]_0^{d/2} = \frac{\pi d^4}{32}$$

$$= I_X + I_Y$$

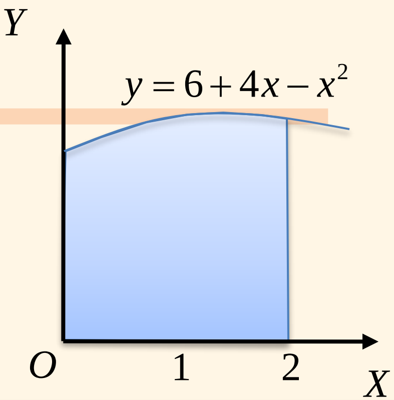
② Calculation of I_X

$I_X = I_Y$ in circle

$$\frac{\pi d^4}{32} = 2 I_X$$

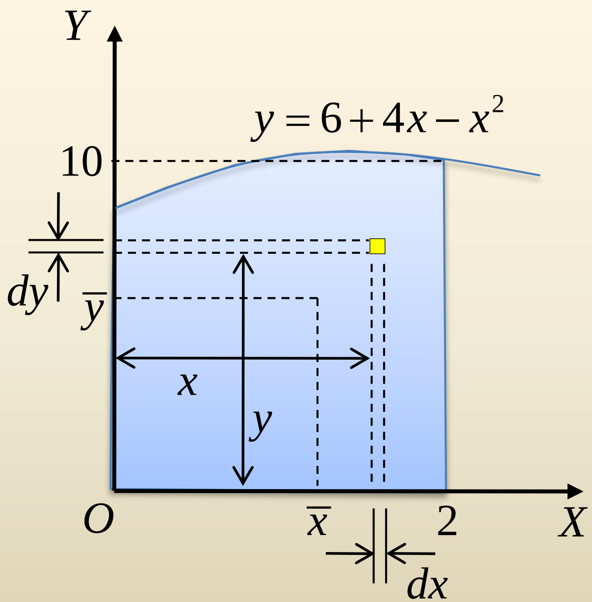
$$\therefore I_X = \frac{\pi d^4}{64} = I_Y$$

Example 3>



Question) Calculation area and centroid of bounded region by curves $x=0$, $x=2$, $y=0$ and parabolic curve $y = 6 + 4x - x^2$.

Solution)



Calculations by integration are as follows

① Calculation of Area

$$A = \int_0^2 y dx = \int_0^2 (6 + 4x - x^2)\, dr$$

$$= \left[6x + 2x^2 - \frac{x^3}{3} \right]_0^2 = 17.33 \ \text{units}^2$$

② Calculation of 1st moment of Area about y axis: M_Y

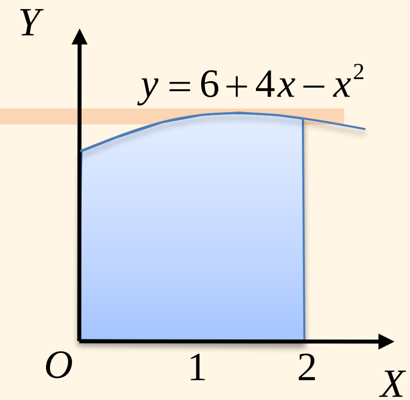
$$M_Y = \int_0^y \int_0^2 x\, dx\, dy = \int_0^2 \int_0^y dy\, x\, dx = \int_0^2 x\, y\, dx$$

$$= \int_0^2 x\, (6 + 4x - x^2)\, dx = \int_0^2 (6x + 4x^2 - x^3) dx$$

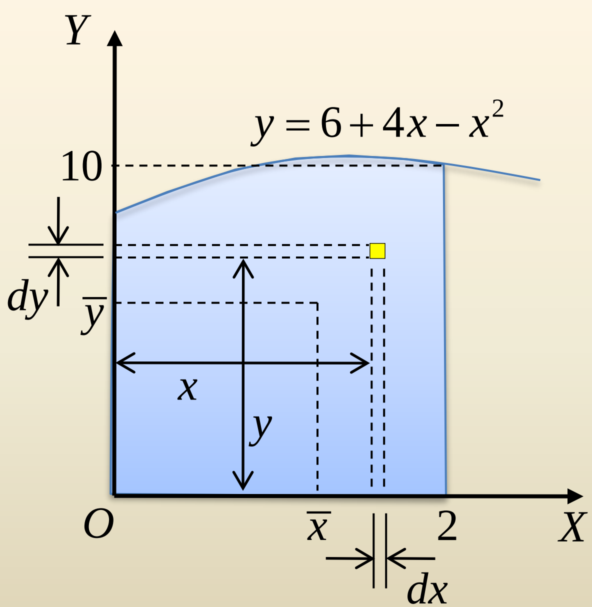
$$= \left[3x^2 + \frac{4}{3} x^3 - \frac{1}{4} x^4 \right]_0^2 = 18.67 \ \text{units}^2$$

Example 3>

Question) Calculation area and centroid of bounded region by curves $x=0$, $x=2$, $y=0$ and parabolic curve $y = 6 + 4x - x^2$.



Solution)



Calculations by integration are as follows

③ Calculation of 1st moment of Area about x axis : M_X

$$M_X = \int_0^y \int_0^2 y\,dx\,dy = \int_0^2 \int_0^y y\,dy\,dx = \frac{1}{2}\int_0^2 y^2\,dx$$

$$= \frac{1}{2}\int_0^2 (6+4x-x^2)^2\,dx = \frac{1}{2}\int_0^2 (36+24x+4x^2-8x^3+x^4)dx$$

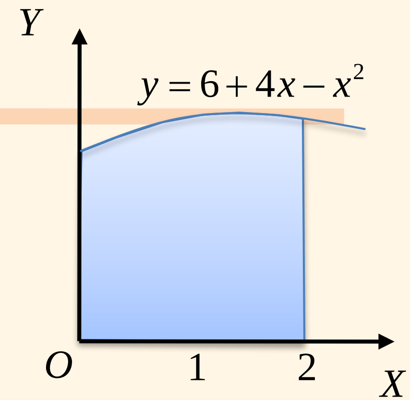
$$= \frac{1}{2}\left[36x+12x^2+\frac{4}{3}x^3-2x^4+\frac{1}{5}x^5\right]_0^2 = 76.53 \text{ units}^2$$

④ x,y coordinate of centroid

$$\bar{x} = \frac{M_Y}{A} = \frac{\int_0^2 x\,dA}{\int_0^2 y\,dx} = \frac{18.67}{17.33} = 1.1 \text{ units}$$

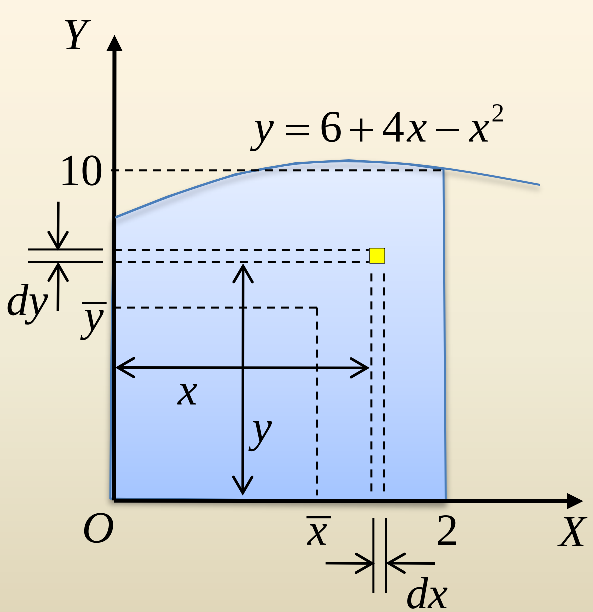
$$\bar{y} = \frac{M_X}{A} = \frac{\int_0^2 y\,dA}{\int_0^2 y\,dx} = \frac{76.53}{17.33} = 4.41 \text{ units}$$

Example 3>



Question) Calculation area and centroid of bounded region by curves $x=0$, $x=2$, $y=0$ and parabolic curve $y = 6 + 4x - x^2$.

Solution)



Calculation of 1st moment of Area about x axis : I_X

$$I_X = \int_0^y \int_0^2 y^2 dx\, dy = \int_0^2 \int_0^y y^2 dy\, dx = \frac{1}{3}\int_0^2 y^3 dx$$

$$= \frac{1}{3}\int_0^2 (6 + 4x - x^2)^3\, dx$$

$$= \frac{1}{3}\int_0^2 (216 + 432x + 180x^2 - 80x^3 - 30x^4 + 12x^5 - x^6)\, dx$$

$$= \frac{1}{3}\left[216x + 216x^2 + 60x^3 - 20x^4 - 6x^5 + 2x^6 - \frac{x^7}{7}\right]_0^2$$

$$= \frac{1}{3}(432 + 864 + 480 - 320 - 192 + 128 - 18.3)$$

$$= \frac{1}{3}(1904 - 530.3) = 457.90 \text{ units}^4$$