Naval Architecture & Ocean Engineering

Computer Aided Ship Design Part.3 Grillage Analysis of Midship Cargo Hold

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Summary

u : Axial Displacement G : Shear Modulus A : Sectional Area G : Shear Modulus E : Young's Modulus

w : Vertical Displacement θ : Angle of Twist l : Length J : Polar Moment of Inertia I : Moment of Inertia

Element | Differential Equation → Variational Method → •Discretization •Approximation → Finite Element Method

•Superposition of Stiffness Matrix
•Coordinates Transformation

Behavior → Structure

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} \ , where\ \ddot{\mathbf{x}} = 0$$

$$\mathbf{Kd} = \mathbf{F}$$

Bar

$$EA\frac{d^2u(x)}{dx^2} + f(x) = 0 \Rightarrow \delta\int_0^l \left[\frac{EA}{2}\left(\frac{du}{dx}\right)^2 - (f\,u) \right] dx = 0$$

$$u(x) = a_0 + a_1 x$$

$$\frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Tension → **Truss**, **Frame**

Beam

$$EI\frac{\partial^4 w(x)}{\partial x^4} + f(x) = 0 \Rightarrow \delta\int_0^l \left[\frac{EA}{2}\left(\frac{d^2w}{dx^2}\right)^2 + (f\,w) \right] dx = 0$$

$$w(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3$$

$$\frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix}\begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ M_1 \\ f_2 \\ M_2 \end{bmatrix}$$

Bending → **Frame**, **Grillage**

Shaft

$$GJ\frac{\partial^2 \theta(x)}{\partial x^2} + f(x) = 0 \Rightarrow \delta\int_0^l \left[\frac{GJ}{2}\left(\frac{d\theta}{dx}\right)^2 - (f\,\theta) \right] dx = 0$$

$$\theta(x) = c_0 + c_1 x$$

$$\frac{GJ}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

Torsion → **Grillage**

Application

Midship Cargo Hold → **Grillage Modeling** → **Solution**

Engineering Concept !

Grillage Modeling
- 3D→ 2D
- Equivalent Force & Moment
- Boundary condition

Solution
- programming
- visulaization

Beam Theory : Sign Convention, Deflection of Beam

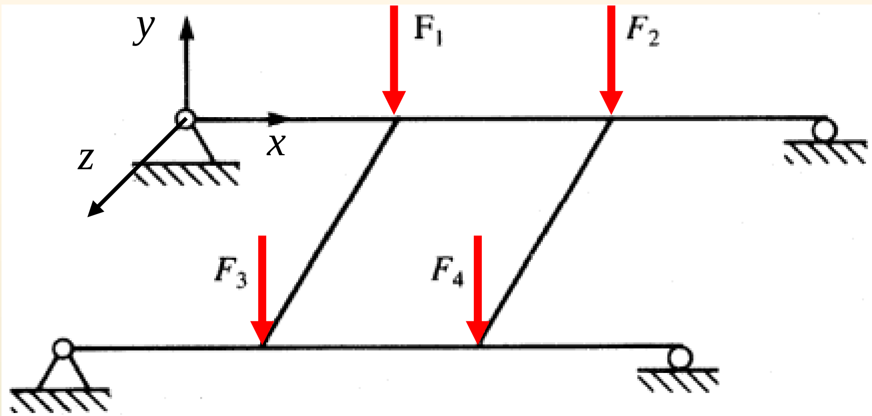
Elasticity : Displacement, Strain, Stress, Force Equilibrium, Compatibility, Constitutive Equation

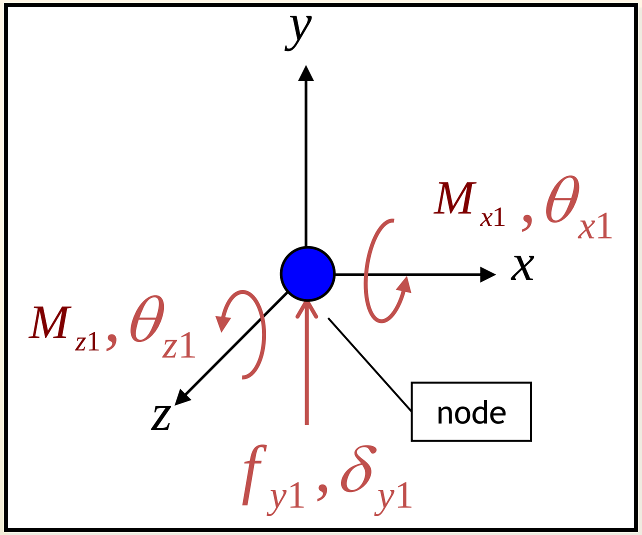
Chapter 7. Grillage

Grillage

- Grillage* (Grid Structure) : A structure that has loads applied perpendicular to its plane. The elements are assumed to be rigidly connected at the joints. The floor system shown in the figure is an example of a very common grillage.

<example of grillage : typical floor system>



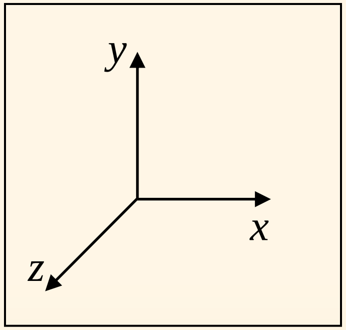


- As in the case of the beam element, we assume that axial deformation is neglected. However, in addition to bending about the horizontal axis of the cross section, the elements will also resist the loads by twisting about the axis of the element, thus developing torsional moments. Therefore, at each joint we will have a vertical displacement, a rotation about the horizontal axis of the cross section due to bending, and a rotation about the axis of the element due to torsion. There are three degrees of freedom at each node.

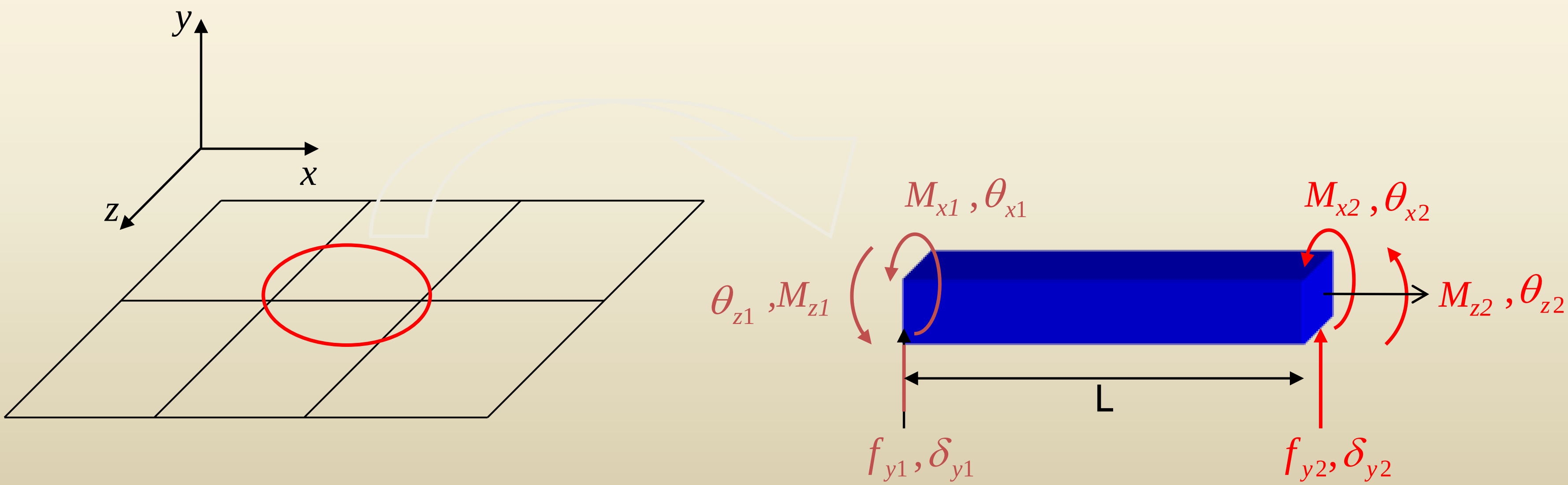
*Daryl L. Logan, A first Course in the Finite Element Method, 2nd edition, PWS Publishing,1993, p.87

Grillage : Stiffness Equations

step1. Coordinate System

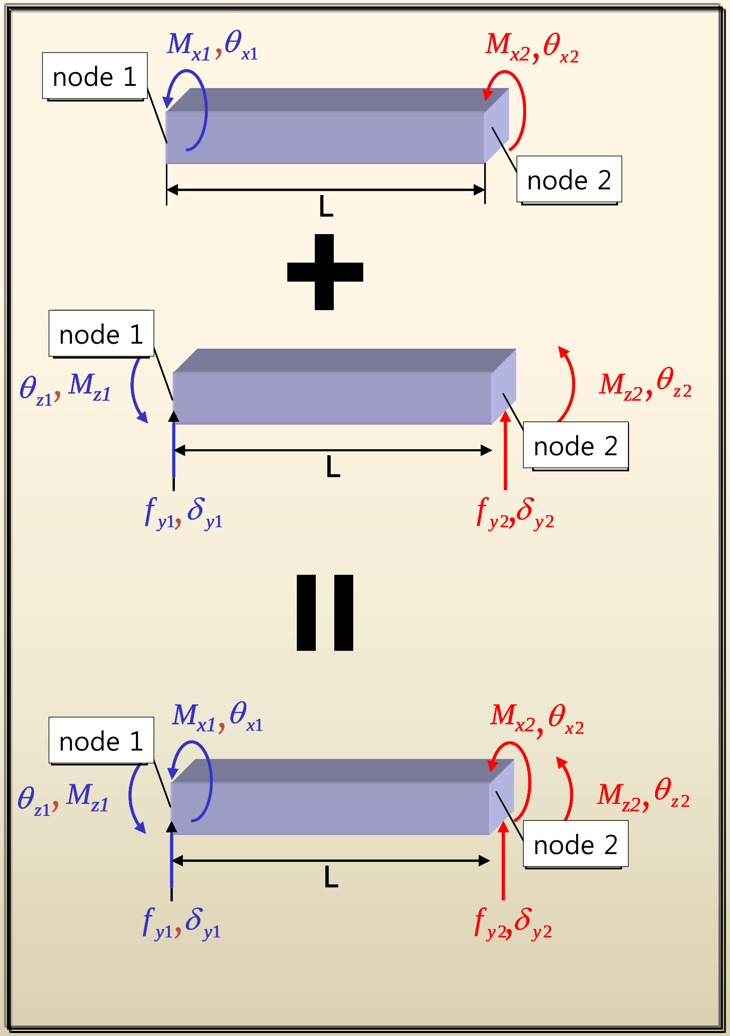


step2. Variables at each nodes



Grillage : Stiffness Equations

Step3. Stiffness Equations



shaft

$$\begin{bmatrix} M_{x1} \\ M_{x2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & -\frac{GJ}{L} \\ -\frac{GJ}{L} & \frac{GJ}{L} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \theta_{x2} \end{bmatrix}$$

+

beam

$$\begin{bmatrix} f_{y1} \\ M_{z1} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \delta_{y1} \\ \theta_{z1} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

=

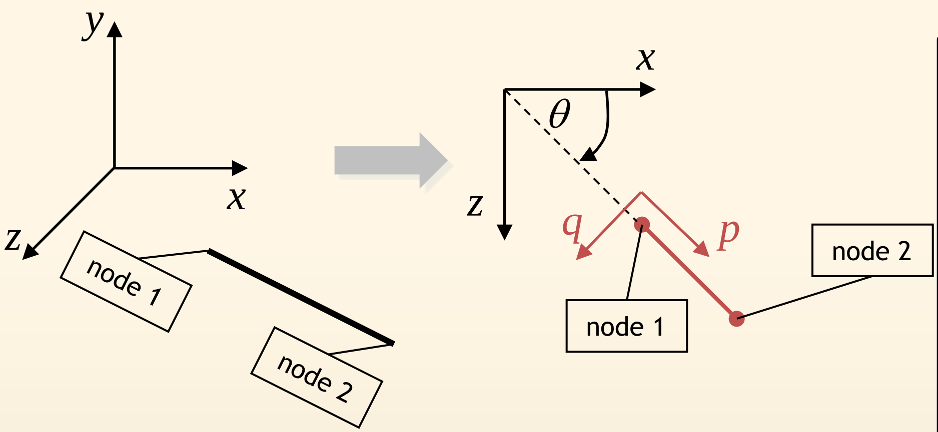
Grillage

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

① $[\mathbf{F}_{pqr}] = [\mathbf{K}_{pqr}][\delta_{pqr}]$

Grillage : Stiffness Equations

- transformation matrix between ***pq*** and ***xy*** coordinate system



rotation transformation along with y axis

$$[\mathbf{T}] = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

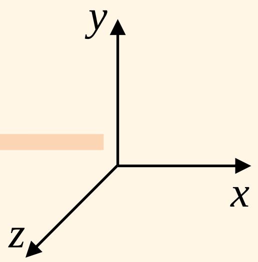
② $[\delta_{pqr}] = [\mathbf{T}][\delta_{xyz}]$

$$\begin{bmatrix} \theta_{p1} \\ \delta_{q1} \\ \theta_{r1} \\ \theta_{p2} \\ \delta_{q2} \\ \theta_{r2} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

③ $[f_{pqr}] = [\mathbf{T}][f_{xyz}]$

$$\begin{bmatrix} M_{p1} \\ f_{q1} \\ M_{r1} \\ M_{p2} \\ f_{q2} \\ M_{r2} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix}$$

Grillage : Stiffness Equations



- Stiffness Equations

① $[f_{pq}]=[\mathbf{K}_{pq}][\delta_{pq}]$ → $[\mathbf{T}][f_{xy}]=[\mathbf{K}_{pq}][\mathbf{T}][\delta_{xy}]$

② $[f_{pq}]=[\mathbf{T}][f_{xy}]$

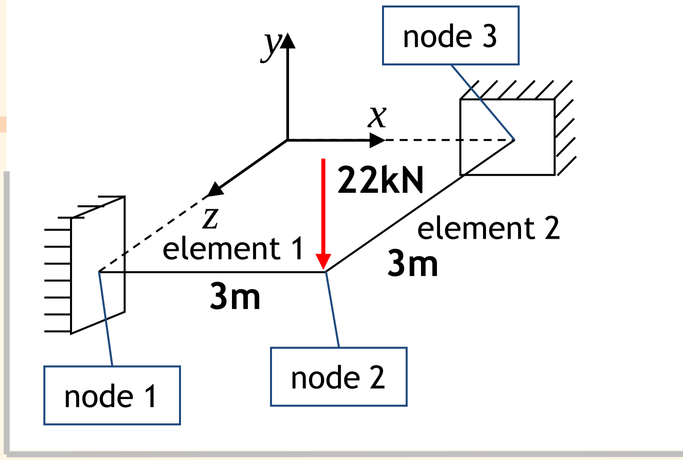
③ $[\delta_{pq}]=[\mathbf{T}][\delta_{xy}]$

multiply $[\mathbf{T}]^{-1}=[\mathbf{T}]^{T}$

$$[f_{xy}]=[\mathbf{T}]^{T}[\mathbf{K}_{pq}][\mathbf{T}][\delta_{xy}]=[\mathbf{K}_{xy}][\delta_{xy}]$$

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

Ex.) Grillage



ex.) Find displacements and reaction force at each nodes of frame in the following figure.

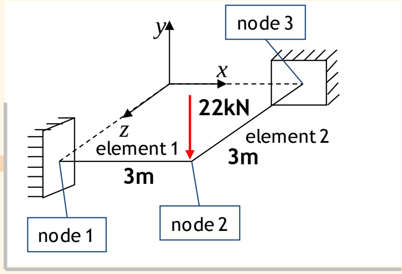
Step1. Input Data

- constants ($\theta_1 = 0$, $\theta_2 = 270°$)

element	cos θ	sin θ	length (m)	moment of inertia (m^4)	Young's moldulus (kN/m^2)	shear modulus (kN/m^2)	polar moment of inertia (m^4)
1	1	0	3	$I=16.6\times10^{-5}$	$E=210\times10^{6}$	$G=84\times10^{6}$	$J=4.6\times10^{-5}$
2	0	-1	3				

Ex.) Grillage

$$[\mathbf{K}_{pqr}] = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



Step2. Stiffness Equation

$$[\mathbf{F}_{xyz}] = [\mathbf{K}_{xyz}][\delta_{xyz}] = [\mathbf{T}]^T[\mathbf{K}_{pqr}][\mathbf{T}][\delta_{xyz}]$$

☞ **element 1**

$$[\mathbf{T}] = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{K}_{pqr}] = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 \\ -0.128 & 0 & 0 & 0.128 & 0 & 0 \\ 0 & -1.55 & -2.32 & 0 & 1.55 & -2.32 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65 \end{bmatrix}$$

$$\frac{GJ}{L} = \frac{(84\times10^6)\cdot(4.6\times10^{-5})}{3} = 0.128\times10^4$$

$$\frac{4EI}{L} = \frac{4\cdot(210\times10^6)(16.6\times10^{-5})}{3} = 4.65\times10^4$$

$$\frac{6EI}{L^2} = \frac{6\cdot(210\times10^6)(16.6\times10^{-5})}{3^2} = 2.32\times10^4$$

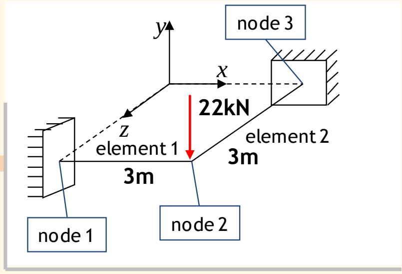
$$\frac{12EI}{L^3} = \frac{12\cdot(210\times10^6)(16.6\times10^{-5})}{3^3} = 1.55\times10^4$$

$$[\mathbf{F}_{xyz}] = [\mathbf{T}]^T[\mathbf{K}_{pqr}][\mathbf{T}][\delta_{xyz}]$$

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 \\ -0.128 & 0 & 0 & 0.128 & 0 & 0 \\ 0 & -1.55 & -2.32 & 0 & 1.55 & -2.32 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65 \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

Ex.) Grillage

$$[\mathbf{K}_{pqr}] = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



Step2. Stiffness Equation

$$[\mathbf{F}_{xyz}] = [\mathbf{K}_{xyz}][\delta_{xyz}] = [\mathbf{T}]^T[\mathbf{K}_{pqr}][\mathbf{T}][\delta_{xyz}]$$

☞ **element 2**

$$[\mathbf{T}] = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[\mathbf{K}_{pqr}] = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 \\ -0.128 & 0 & 0 & 0.128 & 0 & 0 \\ 0 & -1.55 & -2.32 & 0 & 1.55 & -2.32 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65 \end{bmatrix}$$

$$\frac{GJ}{L} = \frac{(84\times10^6)\cdot(4.6\times10^{-5})}{3} = 0.128\times10^4$$

$$\frac{4EI}{L} = \frac{4\cdot(210\times10^6)(16.6\times10^{-5})}{3} = 4.65\times10^4$$

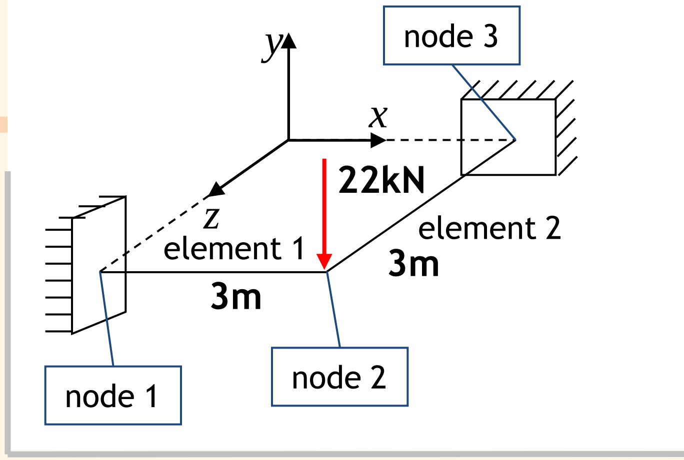
$$\frac{6EI}{L^2} = \frac{6\cdot(210\times10^6)(16.6\times10^{-5})}{3^2} = 2.32\times10^4$$

$$\frac{12EI}{L^3} = \frac{12\cdot(210\times10^6)(16.6\times10^{-5})}{3^3} = 1.55\times10^4$$

$$[\mathbf{F}_{xyz}] = [\mathbf{T}]^T[\mathbf{K}_{pqr}][\mathbf{T}][\delta_{xyz}]$$

$$\begin{bmatrix} M_{x2} \\ f_{y2} \\ M_{z2} \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = 10^4 \times \begin{bmatrix} 4.65 & 2.32 & 0 & 2.32 & -2.32 & 0 \\ 2.32 & 1.55 & 0 & 2.32 & -1.55 & 0 \\ 0 & 0 & 0.128 & 0 & 0 & -0.128 \\ 2.32 & 2.32 & 0 & 4.65 & -2.32 & 0 \\ -2.32 & -1.55 & 0 & -2.32 & 1.55 & 0 \\ 0 & 0 & -0.128 & 0 & 0 & 0.128 \end{bmatrix} \begin{bmatrix} \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \\ \theta_{x3} \\ \delta_{y3} \\ \theta_{z3} \end{bmatrix}$$

Ex.) Grillage



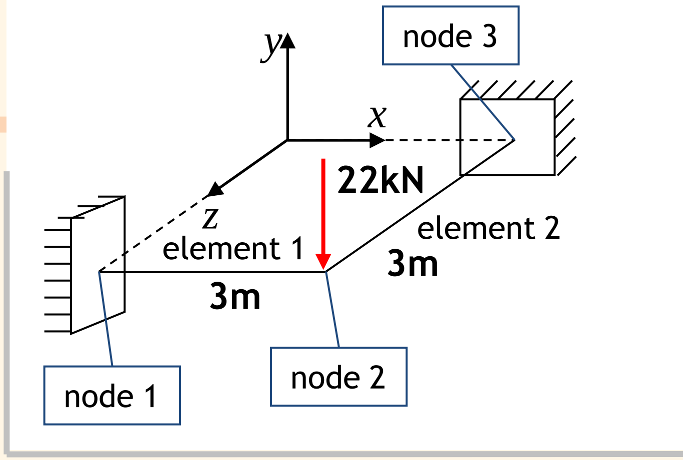
Step3. Find Displacements

- known/unknown displacements
  - ✓ known : $\theta_{x1}, \delta_{y1}, \theta_{z1}, \theta_{x3}, \delta_{y3}, \theta_{z3}(=0)$
  - ✓ unknown : $\theta_{x2}, \delta_{y2}, \theta_{z2}$
- known/unknown forces
  - ✓ known : $M_{x2}(=0)$, $f_{y2}(=-22\text{kN})$, $M_{z2}(=0)$
  - ✓ unknown : M_{x1}, f_{y1}, M_{z1}, M_{x3}, f_{y3}, M_{z3}

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 \\ -0.128 & 0 & 0 & 0.128 & 0 & 0 \\ 0 & -1.55 & -2.32 & 0 & 1.55 & -2.32 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65 \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

$$\begin{bmatrix} M_{x2} \\ f_{y2} \\ M_{z2} \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = 10^4 \times \begin{bmatrix} 4.65 & 2.32 & 0 & 2.32 & -2.32 & 0 \\ 2.32 & 1.55 & 0 & 2.32 & -1.55 & 0 \\ 0 & 0 & 0.128 & 0 & 0 & -0.128 \\ 2.32 & 2.32 & 0 & 4.65 & -2.32 & 0 \\ -2.32 & -1.55 & 0 & -2.32 & 1.55 & 0 \\ 0 & 0 & -0.128 & 0 & 0 & 0.128 \end{bmatrix} \begin{bmatrix} \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \\ \theta_{x3} \\ \delta_{y3} \\ \theta_{z3} \end{bmatrix}$$

Ex.) Grillage

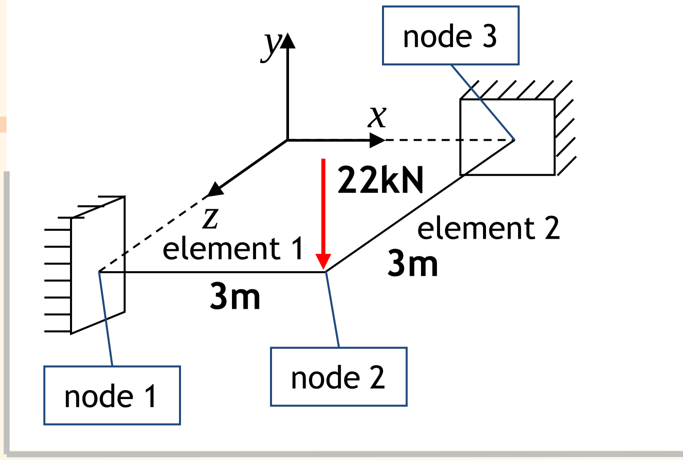


Step3. Find Displacements

- known/unknown displacements
  - ✓ known : $\theta_{x1},\delta_{y1},\theta_{z1},\theta_{x3},\delta_{y3},\theta_{z3}(=0)$
  - ✓ unknown : $\theta_{x2},\delta_{y2},\theta_{z2}$
- known/unknown forces
  - ✓ known : $M_{x2}(=0)$, $f_{y2}(=-22kN)$, $M_{z2}(=0)$
  - ✓ unknown : M_{x1}, f_{y1}, M_{z1}, M_{x3}, f_{y3}, M_{z3}

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 & 0 & 0 & 0 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 & 0 & 0 & 0 \\ -0.128 & 0 & 0 & 0.128+4.65 & 2.32 & 0 & 2.32 & -2.32 & 0 \\ 0 & -1.55 & -2.32 & 2.32 & 1.55+1.55 & -2.32 & 2.32 & -1.55 & 0 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65+0.128 & 0 & 0 & -0.128 \\ 0 & 0 & 0 & 2.32 & 2.32 & 0 & 4.65 & -2.32 & 0 \\ 0 & 0 & 0 & -2.32 & -1.55 & 0 & -2.32 & 1.55 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.128 & 0 & 0 & 0.128 \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \\ \theta_{x3} \\ \delta_{y3} \\ \theta_{z3} \end{bmatrix}$$

Ex.) Grillage



Step3. Find Displacements

- known/unknown displacements
  - ✓ known : $\theta_{x1}, \delta_{y1}, \theta_{z1}, \theta_{x3}, \delta_{y3}, \theta_{z3}(=0)$
  - ✓ unknown : $\theta_{x2}, \delta_{y2}, \theta_{z2}$
- known/unknown forces
  - ✓ known : $M_{x2}(=0)$, $f_{y2}(=-22kN)$, $M_{z2}(=0)$
  - ✓ unknown : M_{x1}, f_{y1}, M_{z1}, M_{x3}, f_{y3}, M_{z3}

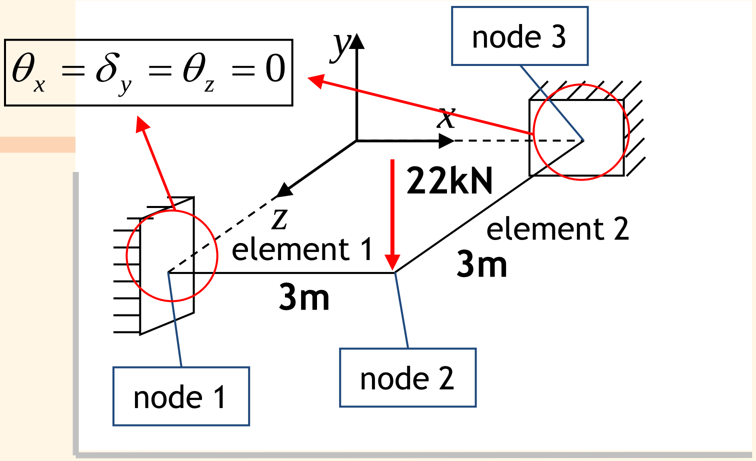
$$\begin{bmatrix} M_{x2}=0 \\ f_{y2}=-22kN \\ M_{2}=0 \end{bmatrix} = 10^4 \times \begin{bmatrix} 4.778 & 2.32 & 0 \\ 2.32 & 3.10 & -2.32 \\ 0 & -2.32 & 4.778 \end{bmatrix} \begin{bmatrix} \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

given

find

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2}=0 \\ f_{y2}=-22kN \\ M_{z2}=0 \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 & 0 & 0 & 0 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 & 0 & 0 & 0 \\ -0.128 & 0 & 0 & 0.128+4.65 & 2.32 & 0 & 2.32 & -2.32 & 0 \\ 0 & -1.55 & -2.32 & 2.32 & 1.55+1.55 & -2.32 & 2.32 & -1.55 & 0 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65+0.128 & 0 & 0 & -0.128 \\ 0 & 0 & 0 & 2.32 & 2.32 & 0 & 4.65 & -2.32 & 0 \\ 0 & 0 & 0 & -2.32 & -1.55 & 0 & -2.32 & 1.55 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.128 & 0 & 0 & 0.128 \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \\ \theta_{x3} \\ \delta_{y3} \\ \theta_{z3} \end{bmatrix}$$

Ex.) Grillage



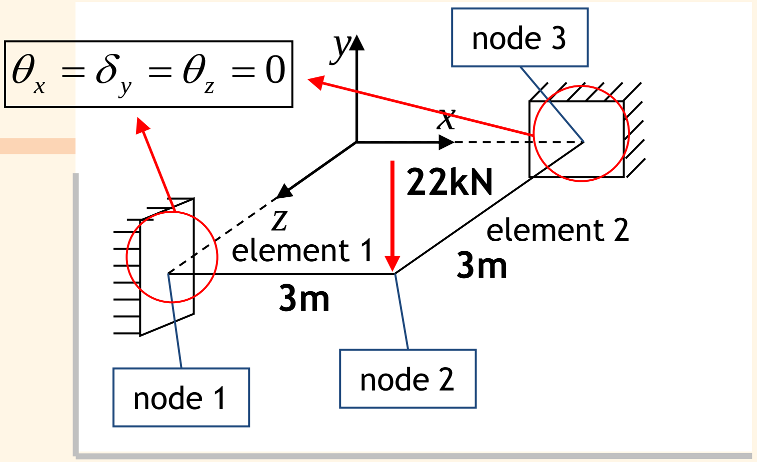
Step3. Find Displacements

- known/unknown displacements
  - ✓ known : $\theta_{x1}, \delta_{y1}, \theta_{z1}, \theta_{x3}, \delta_{y3}, \theta_{z3}(=0)$
  - ✓ unknown : $\theta_{x2}, \delta_{y2}, \theta_{z2}$
- known/unknown forces
  - ✓ known : $M_{x2}(=0)$, $f_{y2}(=-22kN)$, $M_{z2}(=0)$
  - ✓ unknown : M_{x1}, f_{y1}, M_{z1}, M_{x3}, f_{y3}, M_{z3}

$$\begin{bmatrix} M_{x2}=0 \\ f_{y2}=-22kN \\ M_2=0 \end{bmatrix} = 10^4 \times \begin{bmatrix} 4.778 & 2.32 & 0 \\ 2.32 & 3.10 & -2.32 \\ 0 & -2.32 & 4.778 \end{bmatrix} \begin{bmatrix} \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

$$\therefore \begin{bmatrix} \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix} = \frac{1}{10^4} \begin{bmatrix} 4.778 & 2.32 & 0 \\ 2.32 & 3.10 & -2.32 \\ 0 & -2.32 & 4.778 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -22 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.126\times10^{-2}\,rad \\ -0.259\times10^{-2}\,cm \\ -0.126\times10^{-2}\,rad \end{bmatrix}$$

Ex.) Grillage



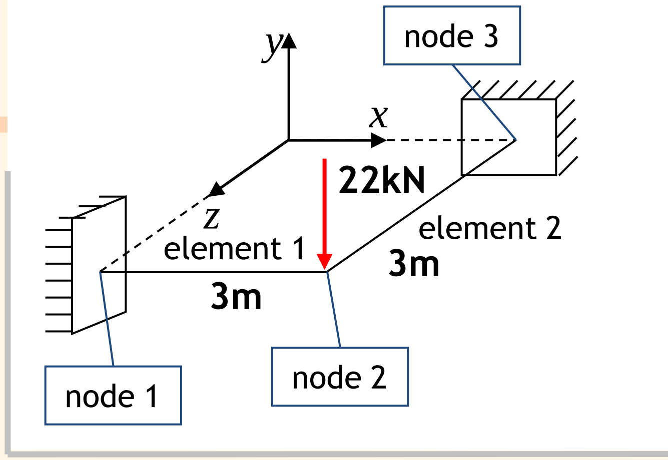
Step3. Find Displacements

- known/unknown displacements
  - ✓ known : $\theta_{x1}, \delta_{y1}, \theta_{z1}, \theta_{x3}, \delta_{y3}, \theta_{z3}(=0)$
  - ✓ unknown : $\theta_{x2}, \delta_{y2}, \theta_{z2}$
- known/unknown forces
  - ✓ known : $M_{x2}(=0)$, $f_{y2}(=-22kN)$, $M_{z2}(=0)$
  - ✓ unknown : M_{x1}, f_{y1}, M_{z1}, M_{x3}, f_{y3}, M_{z3}

find **given**

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2}=0 \\ f_{y2}=-22kN \\ M_{z2}=0 \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 & 0 & 0 & 0 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 & 0 & 0 & 0 \\ -0.128 & 0 & 0 & 0.128+4.65 & 2.32 & 0 & 2.32 & -2.32 & 0 \\ 0 & -1.55 & -2.32 & 2.32 & 1.55+1.55 & -2.32 & 2.32 & -1.55 & 0 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65+0.128 & 0 & 0 & -0.128 \\ 0 & 0 & 0 & 2.32 & 2.32 & 0 & 4.65 & -2.32 & 0 \\ 0 & 0 & 0 & -2.32 & -1.55 & 0 & -2.32 & 1.55 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.128 & 0 & 0 & 0.128 \end{bmatrix} \begin{bmatrix} \theta_{x1}=0 \\ \delta_{y1}=0 \\ \theta_{z1}=0 \\ \theta_{x2}=0.126\times10^{-2}rad \\ \delta_{y2}=-0.259\times10^{-2}cm \\ \theta_{z2}=-0.126\times10^{-2}rad \\ \theta_{x3}=0 \\ \delta_{y3}=0 \\ \theta_{z3}=0 \end{bmatrix}$$

Ex.) Grillage



Step4. Find Reaction Forces

superposition of reaction forces of each elements

- reaction forces for element 1

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = 10^4 \times \begin{bmatrix} 0.128 & 0 & 0 & -0.128 & 0 & 0 \\ 0 & 1.55 & 2.32 & 0 & -1.55 & 2.32 \\ 0 & 2.32 & 4.65 & 0 & -2.32 & 2.33 \\ -0.128 & 0 & 0 & 0.128 & 0 & 0 \\ 0 & -1.55 & -2.32 & 0 & 1.55 & -2.32 \\ 0 & 2.32 & 2.33 & 0 & -2.32 & 4.65 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.126 \times 10^{-2} rad \\ -0.259 \times 10^{-2} cm \\ -0.126 \times 10^{-2} rad \end{bmatrix} = \begin{bmatrix} -1.65 kN \cdot m \\ 11 kN \\ 31 kN \cdot m \\ 1.65 kN \cdot m \\ -11 kN \\ 1.65 kN \cdot m \end{bmatrix}$$

- reaction forces for element 2

$$\begin{bmatrix} M_{x2} \\ f_{y2} \\ M_{z2} \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = 10^4 \times \begin{bmatrix} 4.65 & 2.32 & 0 & 2.32 & -2.32 & 0 \\ 2.32 & 1.55 & 0 & 2.32 & -1.55 & 0 \\ 0 & 0 & 0.128 & 0 & 0 & -0.128 \\ 2.32 & 2.32 & 0 & 4.65 & -2.32 & 0 \\ -2.32 & -1.55 & 0 & -2.32 & 1.55 & 0 \\ 0 & 0 & -0.128 & 0 & 0 & 0.128 \end{bmatrix} \begin{bmatrix} 0.126 \times 10^{-2} rad \\ -0.259 \times 10^{-2} cm \\ -0.126 \times 10^{-2} rad \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.65 kN \cdot m \\ -11 kN \\ -1.65 kN \cdot m \\ -31 kN \cdot m \\ 11 kN \\ 1.65 kN \cdot m \end{bmatrix}$$

Ex.) Grillage

- reaction forces for element 1

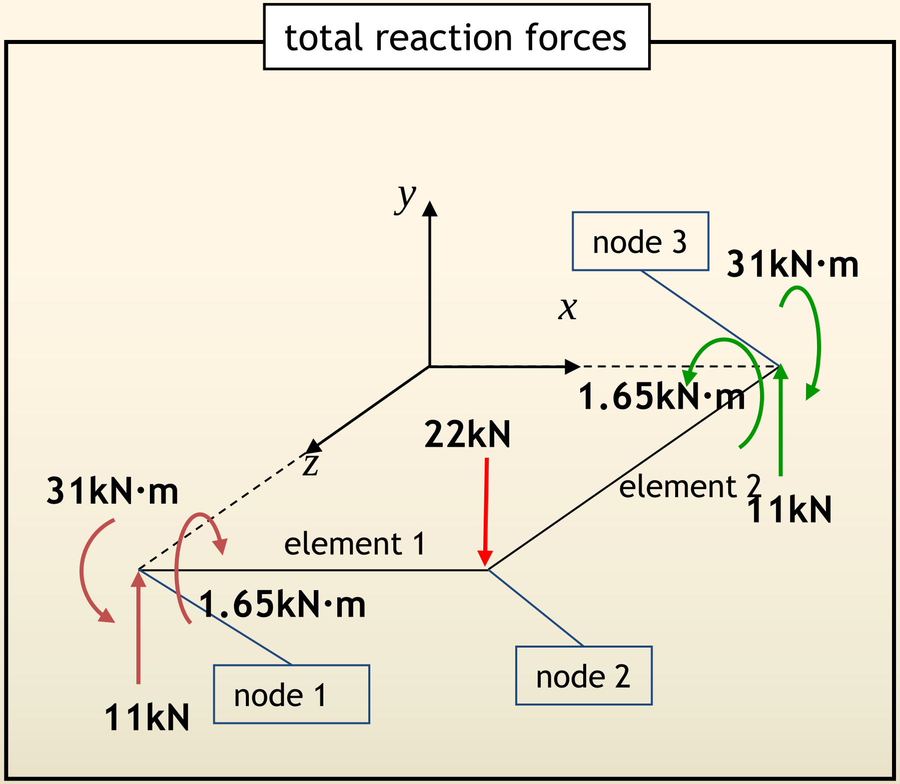
$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} -1.65kN\cdot m \\ 11kN \\ 31kN\cdot m \\ 1.65kN\cdot m \\ -11kN \\ 1.65kN\cdot m \end{bmatrix}$$

- reaction forces for element 2

$$\begin{bmatrix} M_{x2} \\ f_{y2} \\ M_{z2} \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = \begin{bmatrix} -1.65kN\cdot m \\ -11kN \\ -1.65kN\cdot m \\ -31kN\cdot m \\ 11kN \\ 1.65kN\cdot m \end{bmatrix}$$

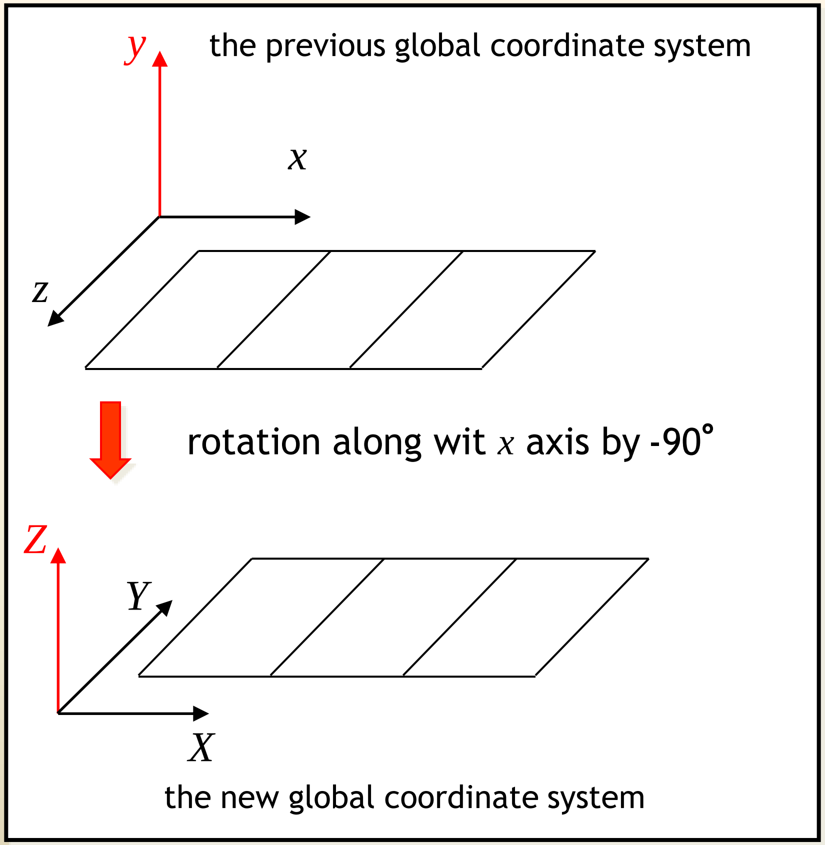
- total reaction forces

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \\ M_{x3} \\ f_{y3} \\ M_{z3} \end{bmatrix} = \begin{bmatrix} -1.65kN\cdot m \\ 11kN \\ 31kN\cdot m \\ 0 \\ -22kN \\ 0 \\ -31kN\cdot m \\ 11kN \\ 1.65kN\cdot m \end{bmatrix}$$



Grillage : New Global Coordinate System

- New Global Coordinate System : the left-handed orientation

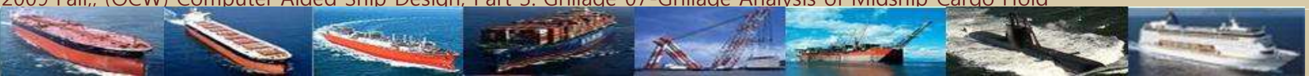


- stiffness equation in the previous global coordinate system

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

- rotation matrix along wit x axis by -90˚

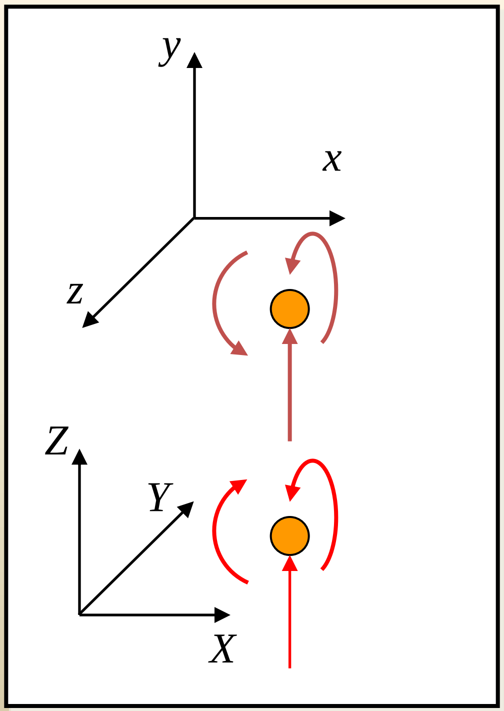
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos(-90) & \sin(-90) & 0 & 0 & 0 \\ 0 & -\sin(-90) & \cos(-90) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(-90) & \sin(-90) \\ 0 & 0 & 0 & 0 & -\sin(-90) & \cos(-90) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Grillage : New Global Coordinate System

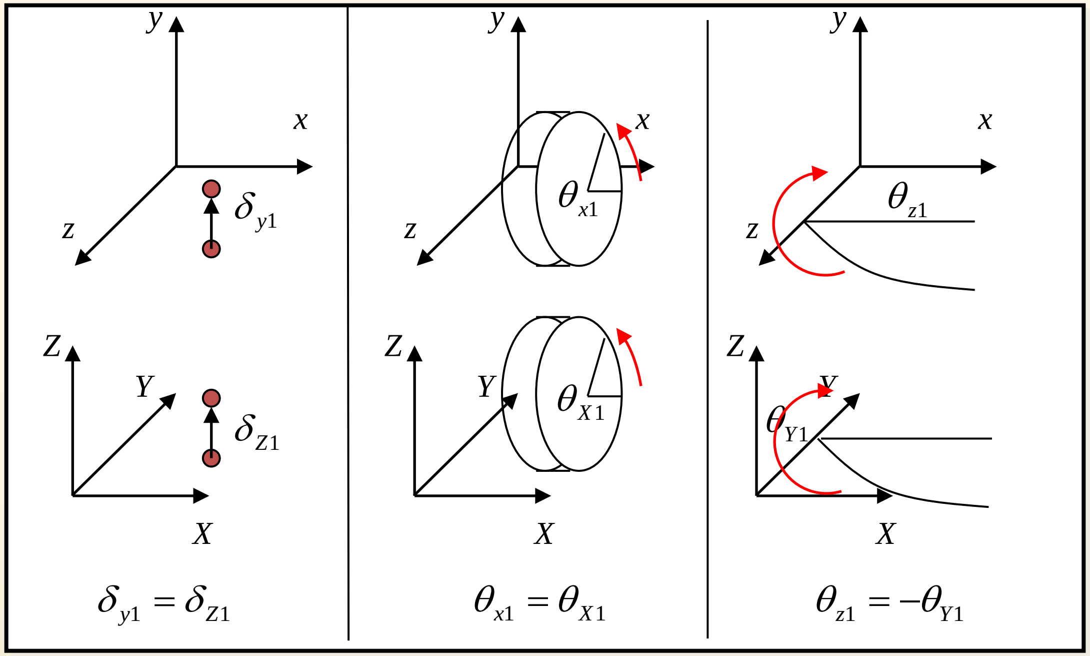
- multiply the rotation matrix both sides of the stiffness equation

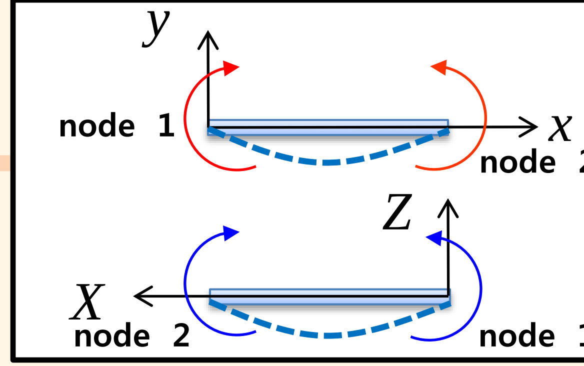
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$



$$\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix} = \begin{bmatrix} M_{x1} \\ -M_{z1} \\ f_{y1} \\ M_{x2} \\ -M_{z2} \\ f_{y2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{4EI}{L} \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

Grillage : New Global Coordinate System





$$
\begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix} = \begin{bmatrix} \theta_{X1} \\ \delta_{Z1} \\ -\theta_{Y1} \\ \theta_{X2} \\ \delta_{Z2} \\ -\theta_{Y2} \end{bmatrix}
$$

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{2EI}{L} \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{4EI}{L} \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \end{bmatrix} \begin{bmatrix} \theta_{X1} \\ \delta_{Z1} \\ -\theta_{Y1} \\ \theta_{X2} \\ \delta_{Z2} \\ -\theta_{Y2} \end{bmatrix}
$$

Grillage : New Global Coordinate System

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{2EI}{L} \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{4EI}{L} \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2}
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \delta_{Z1} \\ -\theta_{Y1} \\ \theta_{X2} \\ \delta_{Z2} \\ -\theta_{Y2} \end{bmatrix}
$$

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & -\frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & -\frac{2EI}{L} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & -\frac{4EI}{L} & \frac{6EI}{L^2} \\
0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3}
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ -\theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ -\theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

remove (−) sign of θ_Y

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} \\
0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3}
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

- stiffness equation in the new global coordinate system

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} \\
0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3}
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

Grillage Analysis : Midship Cargo Hold

- Background

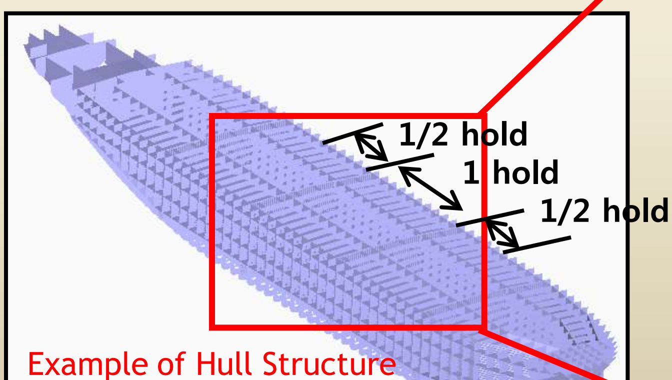
  to determine the distribution of deflection and stress

FEM Approach	Grillage Analysis Approach
- Could calculate the accurate deflection and stress distribution but - Time Consuming for Model Preparation - Analysis Model may not be available before the design completed	- Could estimate the overall deflection and stress distribution comparatively in a short time and even the design is not over - A simplified and practical approach

Midship Cargo Hold Analysis

- Analysis Region : 2 Holds (½ Hold + 1 Hold + ½ Hold)
  - ① **1 Hold : Analysis is not correct because of the boundary condition**
  - ② **All Holds : It takes much time to preparing the analysis model**
  - ③ **2 Holds : Comparatively correct considering the time for model preparation**

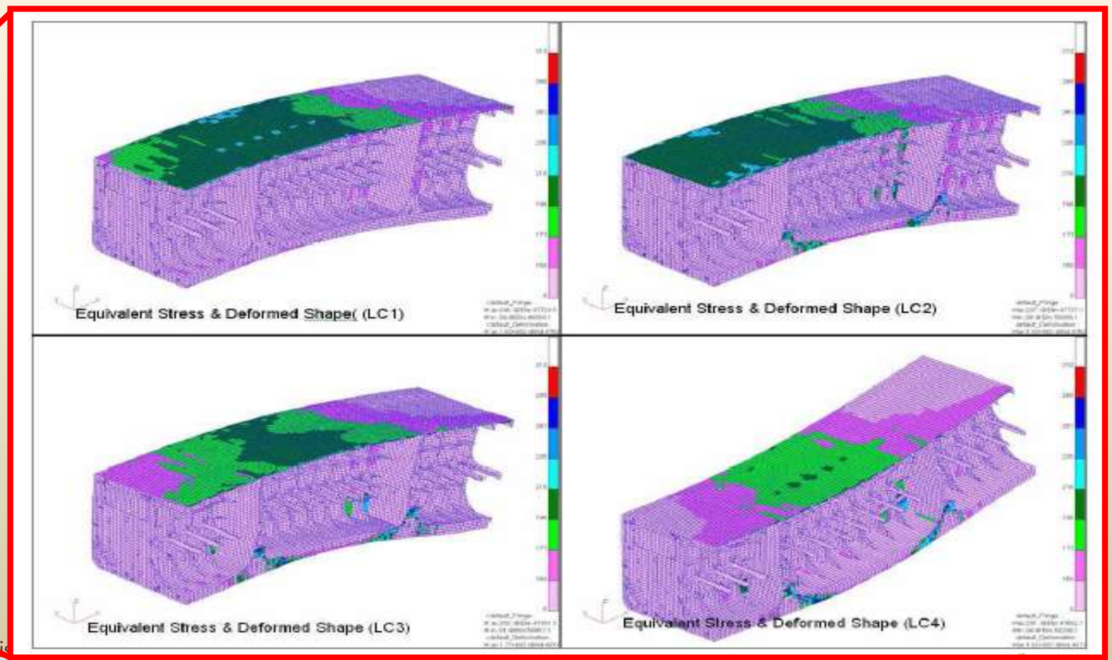
VLCC (Very Large Crude oil Carrier)



Example of Hull Structure

(EzStruct Model)

Structural Analysis Result (MSC.Patran)



Stress acting on a Ship

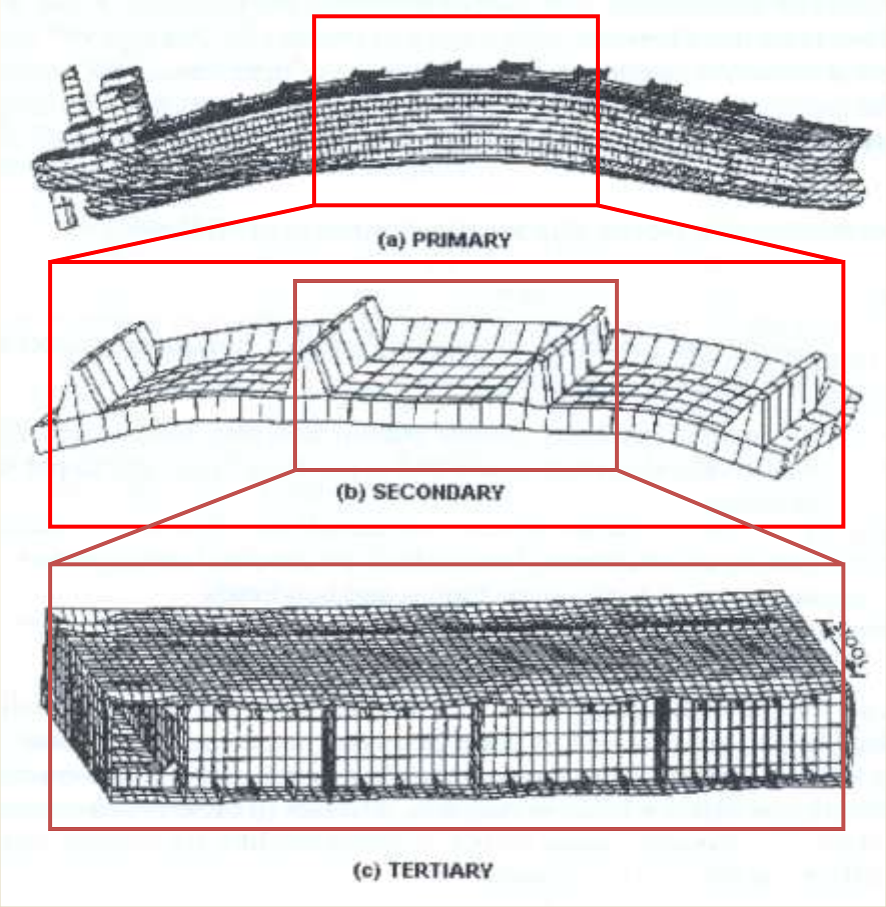
- Stress and Deflection Components

The structural response of the hull girder and the associated members can be subdivided into three components

• Primary response is the response of the entire hull, when the ship bends as a beam under the longitudinal distribution of load.

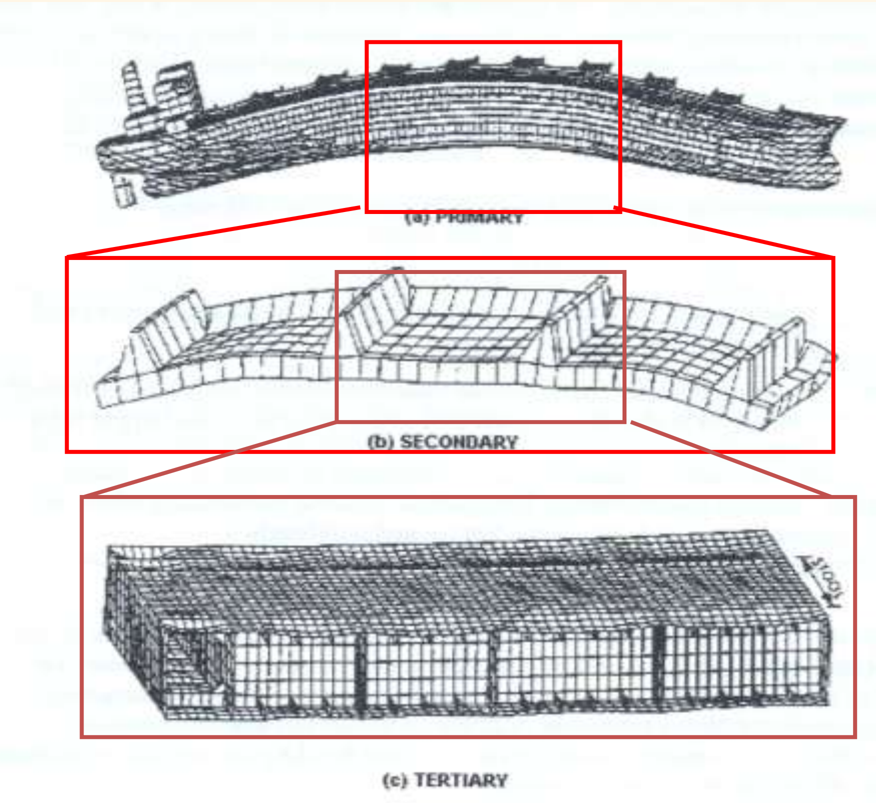
• Secondary response relates to the global bending of stiffened panels (for single hull ship) or to the behavior of double bottom, double sides, etc., for double hull ships

• Tertiary response describes the out-of-plane deflection and associated stress of an individual unstiffened plate panel included between 2 longitudinals and 2 transverse web frames.

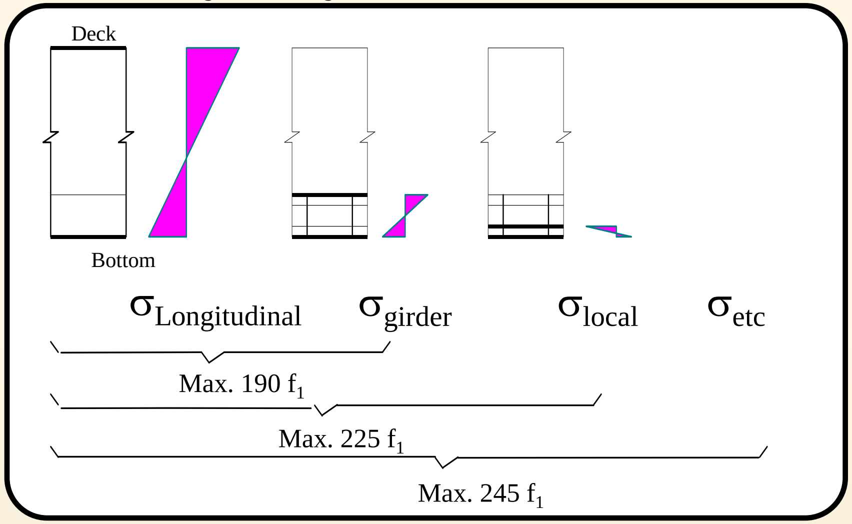


*Philippe RIGO and Enrico Rizzuto,, Ship Design and Construction,, 2002, chapter 18. Analysis and design of ship structure, p.18-21

Local Strength & Allowable Stresses



The sum of $\sigma_{Longitudinal}, \sigma_{girder}, \sigma_{local}$ is not to exceed 245 f1 N/mm2



Loads and Stresses

1) Hogging or Sagging

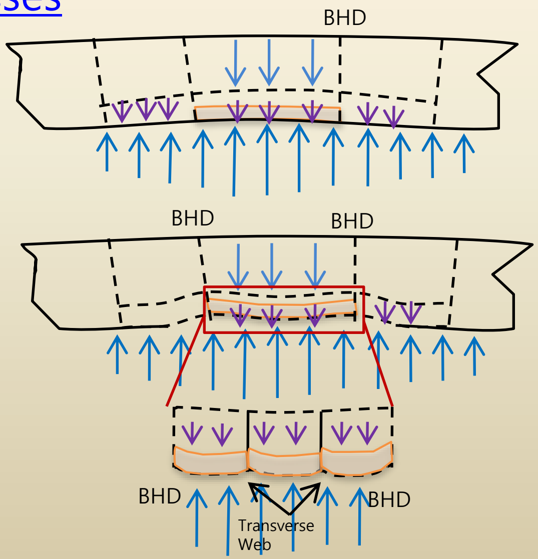
$$\sigma_1 = \sigma_{Longitudinal}$$

2) Cargo Load

$$\sigma_2 = \sigma_{girder}$$

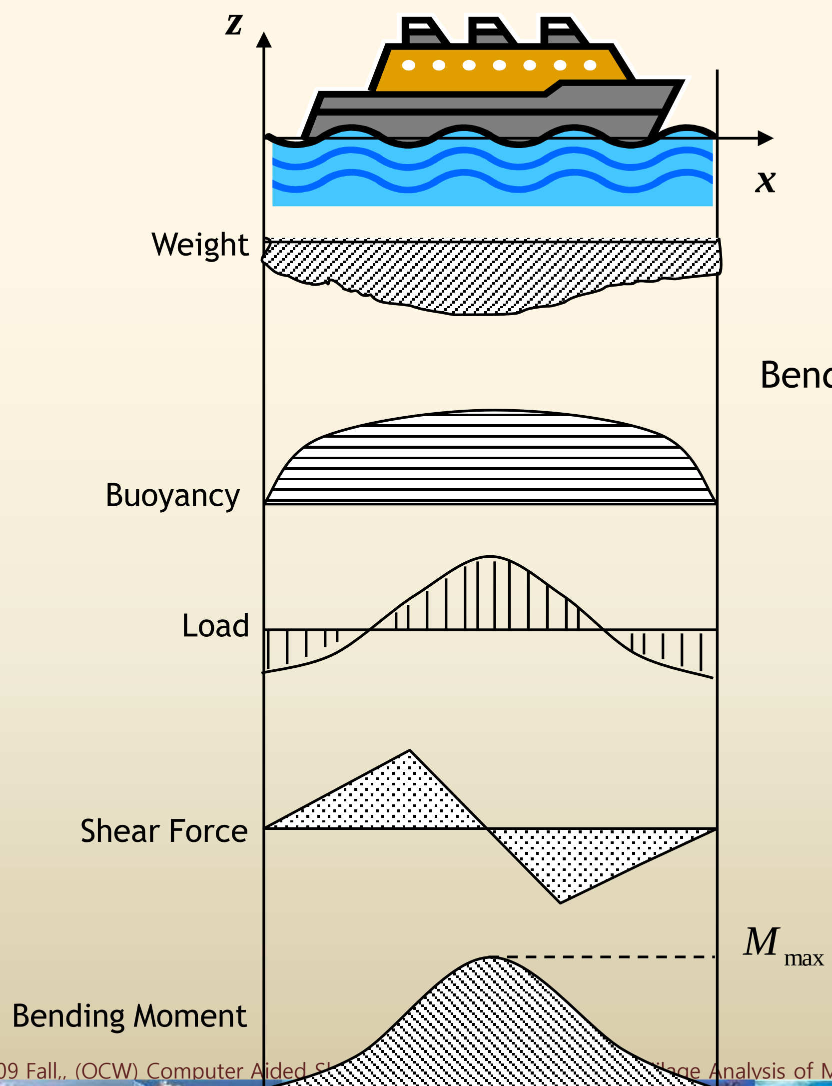
3) Ballasting Load

$$\sigma_3 = \sigma_{local}$$



Primary Stress (σ_1) and Longitudinal Strength of a Ship

- Longitudinal Stress Analysis



Load= Weight + Buoyancy

⇓

Shear Force = ∫(Load) dx

⇓

Bending Moment = ∫(Shear Force) dx

⇓

$$\sigma_L = \frac{M_{max}}{Z}$$ (Section Modulus $Z = \frac{I}{y}$)

⇓

$$\sigma_L \le 175 f_1 \ N/mm^2$$

(f_1 : Material constant
ex) mild Steel : f_1=1.0)

Beam Theory

w: Load

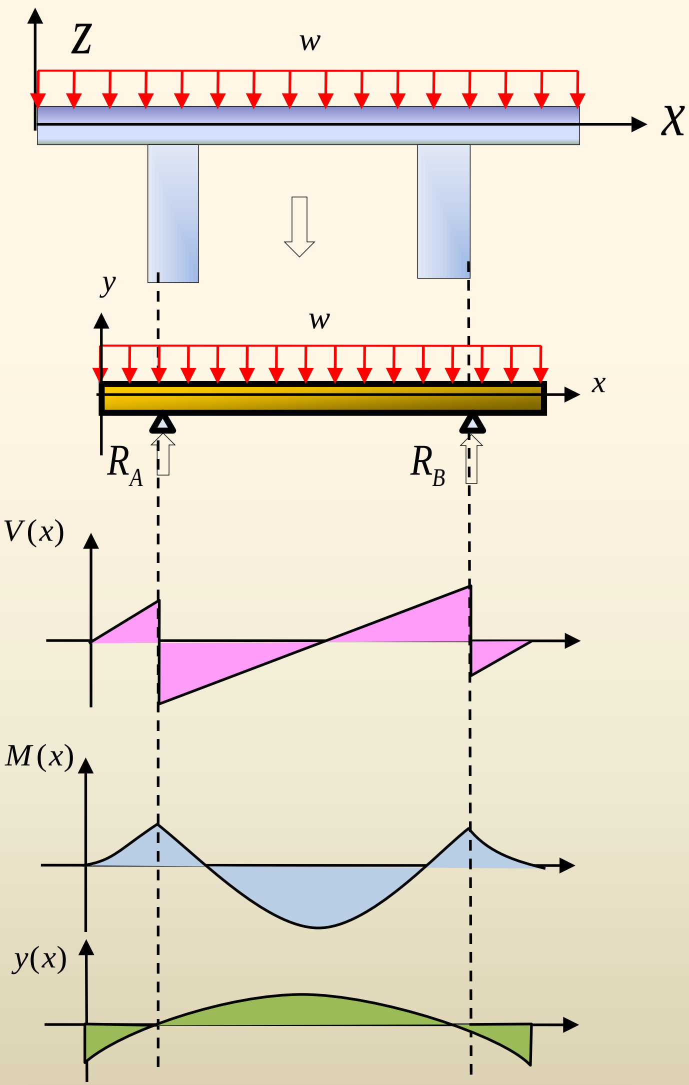
Shear Force:
$$V = -\int w dx$$

Bending Moment:
$$M = \int V dx$$

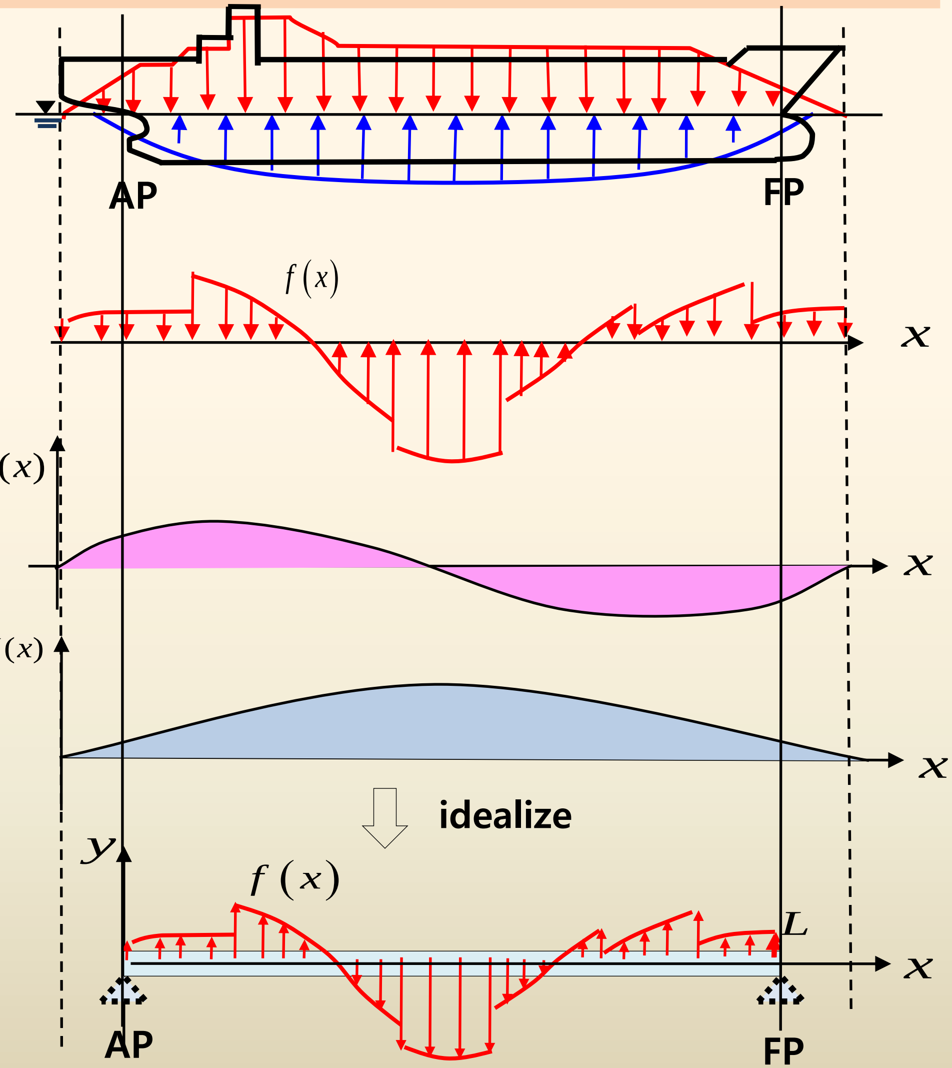
Primary Stress

☞ $\sigma_L = \sigma_1$

Applying Beam Theory on a Ship



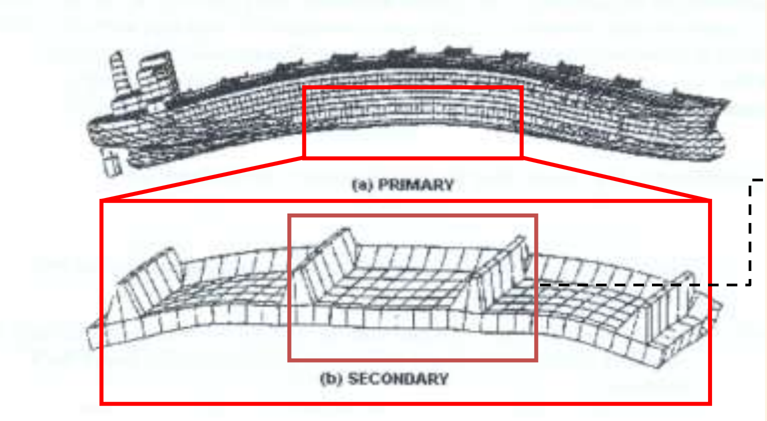
Deflection exist at the end point*

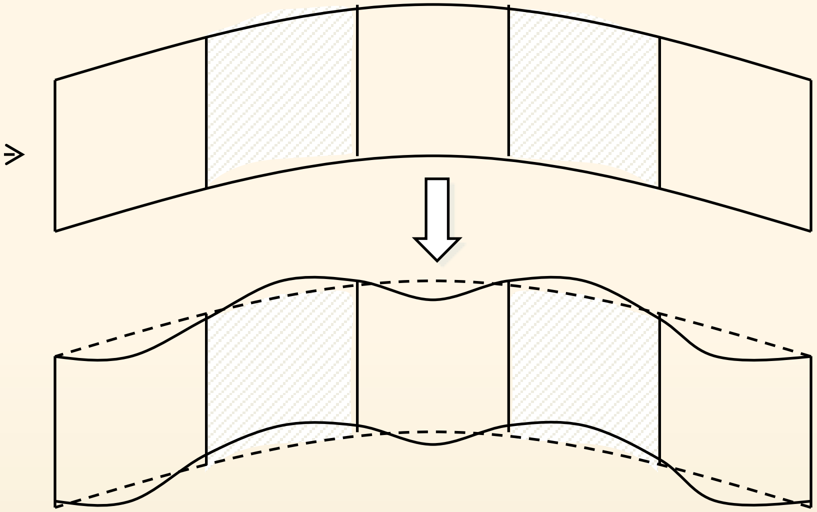


Assume : small deflection at the ends of the ship and imaginary and imaginary supports at the AP and FP

*James M. Gere, Mechanics of Materials 6th Edition, Thomson, Chap.4, p.292

Grillage Analysis and Secondary Stress (σ_2)



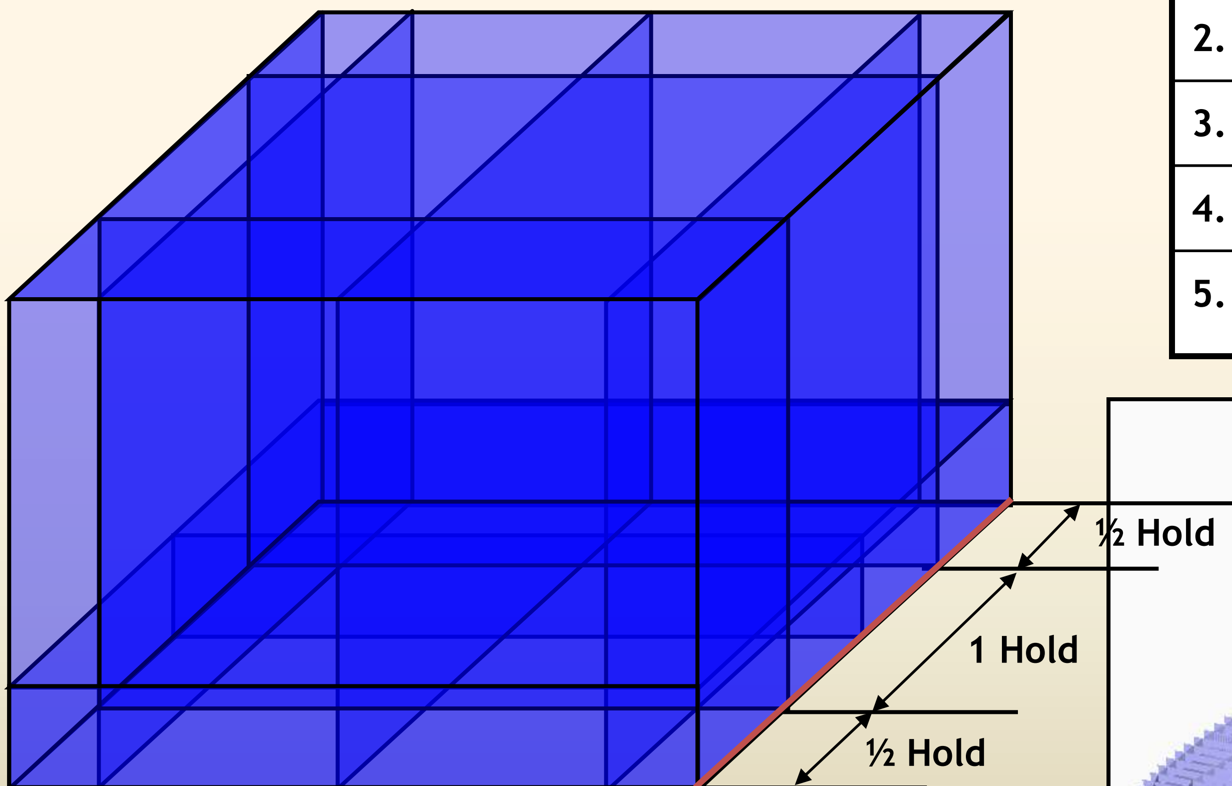


For a stiffened panel, there is the stress (σ2) and deflection of the global bending of the orthotropic stiffened panels, for example, the panel of bottom structure contained between two adjacent transverse bulkheads. The stiffener and the attached plating bend under the lateral load and the plate develops additional plane stresses since the plate acts as a flange with the stiffeners. In longitudinally framed ships there is also a second type of secondary stresses which corresponds to the bending under the hydrostatic pressure of the longitudinals between transverse frames (web frames). For transversally framed panels, this stress may also exist and would correspond to the bending of the equally spaced frames between two stiff longitudinal girder*

- Grillage Analysis : an analysis approach which models the cross-stiffened panel as a system of discrete intersecting beams, each beam being composed of stiffener and associated effective plating
- Object : to determine the distribution of deflection and stress over the length and width dimensions of the stiffened panel

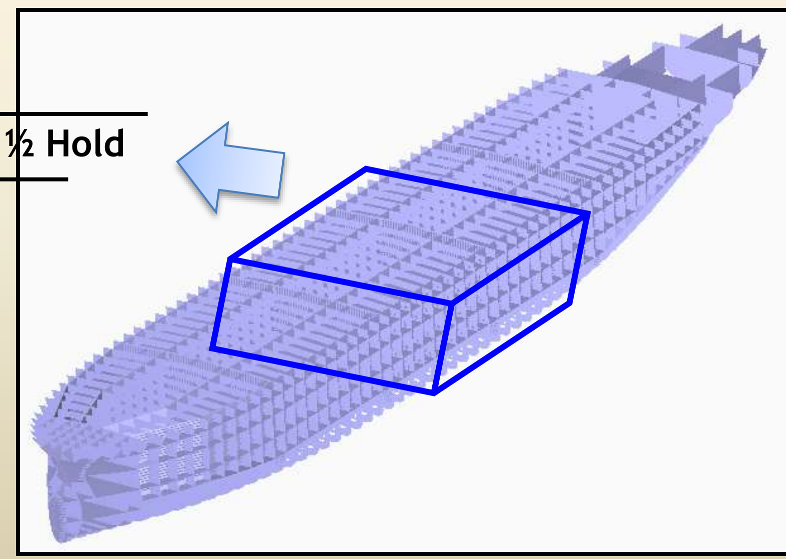
*Philippe RIGO and Enrico Rizzuto,, Ship Design and Construction,, 2002, chapter 18. Analysis and design of ship structure, p.18-21

Grillage Analysis : Midship Cargo Hold



Grillage Analysis

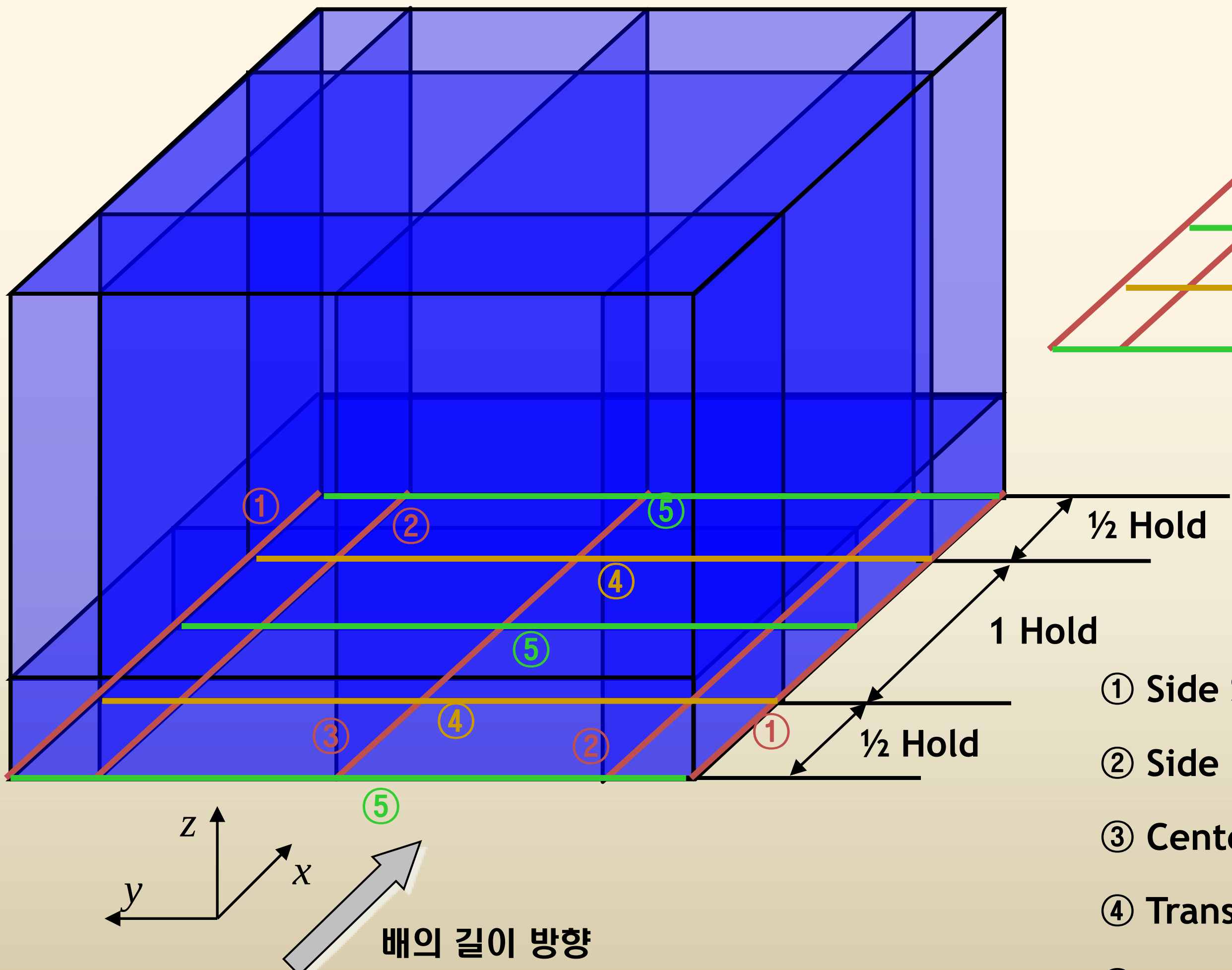
1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

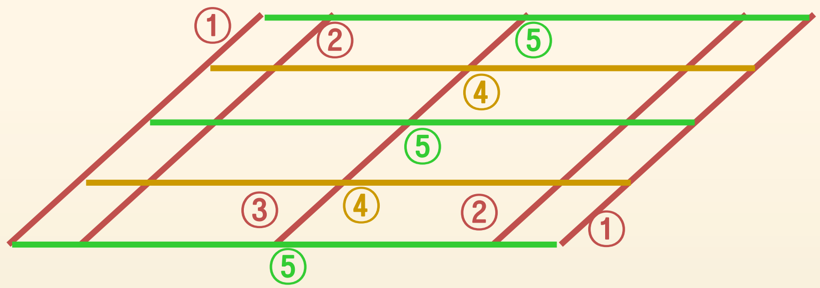


- Analysis Region : ½ Hold + 1 Hold + ½ Hold

Grillage Analysis : Midship Cargo Hold

Step1. **Grillage Model**





① **Side Shell**

② **Side Longitudinal Bulkhead**

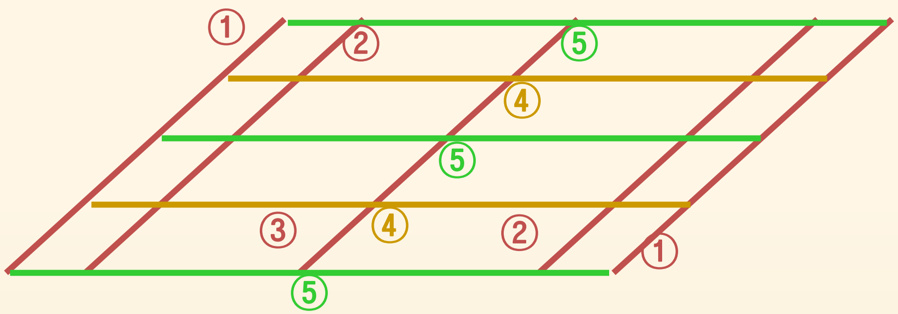
③ **Center Girder + Longitudinal Bulkhead**

④ **Transverse Bulkhead**

⑤ **Transverse Floor**

Grillage Analysis : Midship Cargo Hold

Step2. Properties for the Elements



NOMENCLATURE

D_T – Depth of Tank

$I_\otimes$ – Vertical Moment of Inertia of Full Midship Section

l – Spacing of Transverse Bulkheads

t_B – Thickness of Bottom Shell

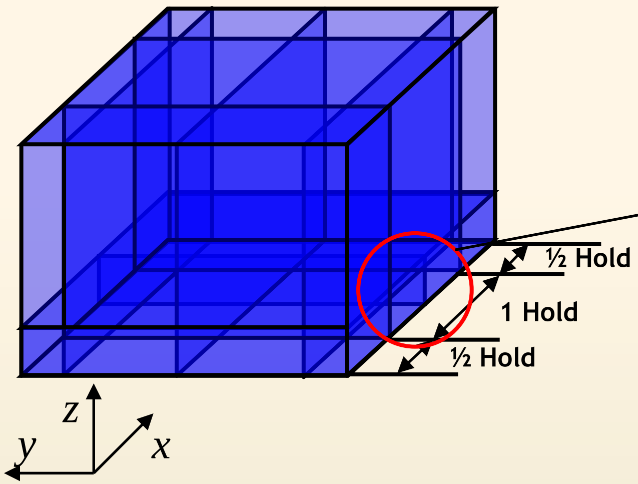
t_D – Thickness of Deck Plating

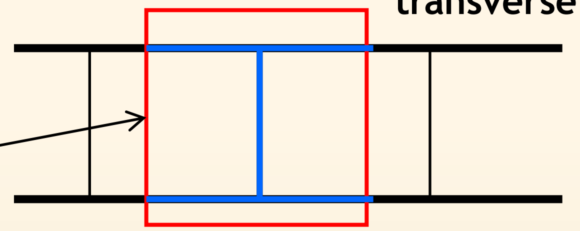
BAR TYPE (See Idealization)	TORSION CONSTANT (J)	INERTIA (I)
1.Center Longi. Bulkhead	$5 \times I_\otimes$	$0.11 \times I_\otimes$
2. Longitudinal Bulkhead	$5 \times I_\otimes$	$0.22 \times I_\otimes$
3. Side Shell	$5 \times I_\otimes$	$0.17 \times I_\otimes$
4. Bottom Transv. floor	10^{-5}	I 형 element 의 Inertia
5.Oil-tight Bulkhead	$l \cdot D_T^{\ 2} \cdot (t_B + t_D)/4$	Not less than $0.3 \times I_\otimes$

Grillage Analysis : Midship Cargo Hold

Step2. Element Properties

I : Moment of Inertia
J : Polar Moment of Inertia



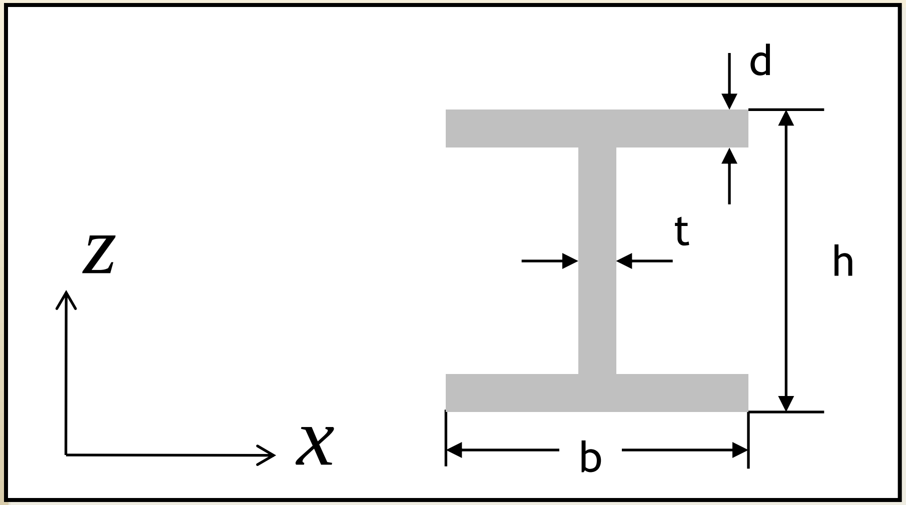


transverse floor ⟶ I-type element

$$I_x = \frac{bh^3}{12} - \frac{(b-t)(h-2d)^3}{12}$$

$$I_z = \frac{2db^3}{12} + \frac{(h-2d)t^3}{12}$$

$$J = I_x + I_z$$



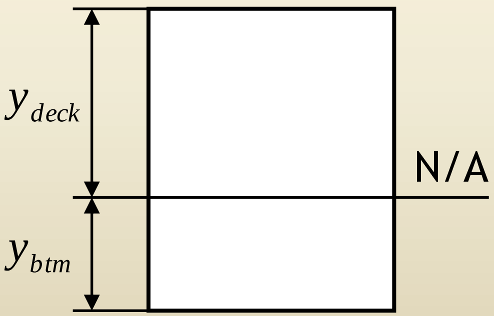
Grillage Analysis : Midship Cargo Hold

Vertical moment of inertia of the midship Section $I_{\otimes}$ is calculated by using the midship section modulus

<ex. given section modulus (cm³)>

	Rule Requirement	Design
Deck	18,274,500	22,036,400
Bottom	18,274,500	26,933,300

sol.)



① $y_{deck} + y_{btm} = Depth$

② $Z_{deck} = \dfrac{I_{\otimes}}{y_{deck}} \Rightarrow y_{deck} = \dfrac{I_{\otimes}}{Z_{deck}}$

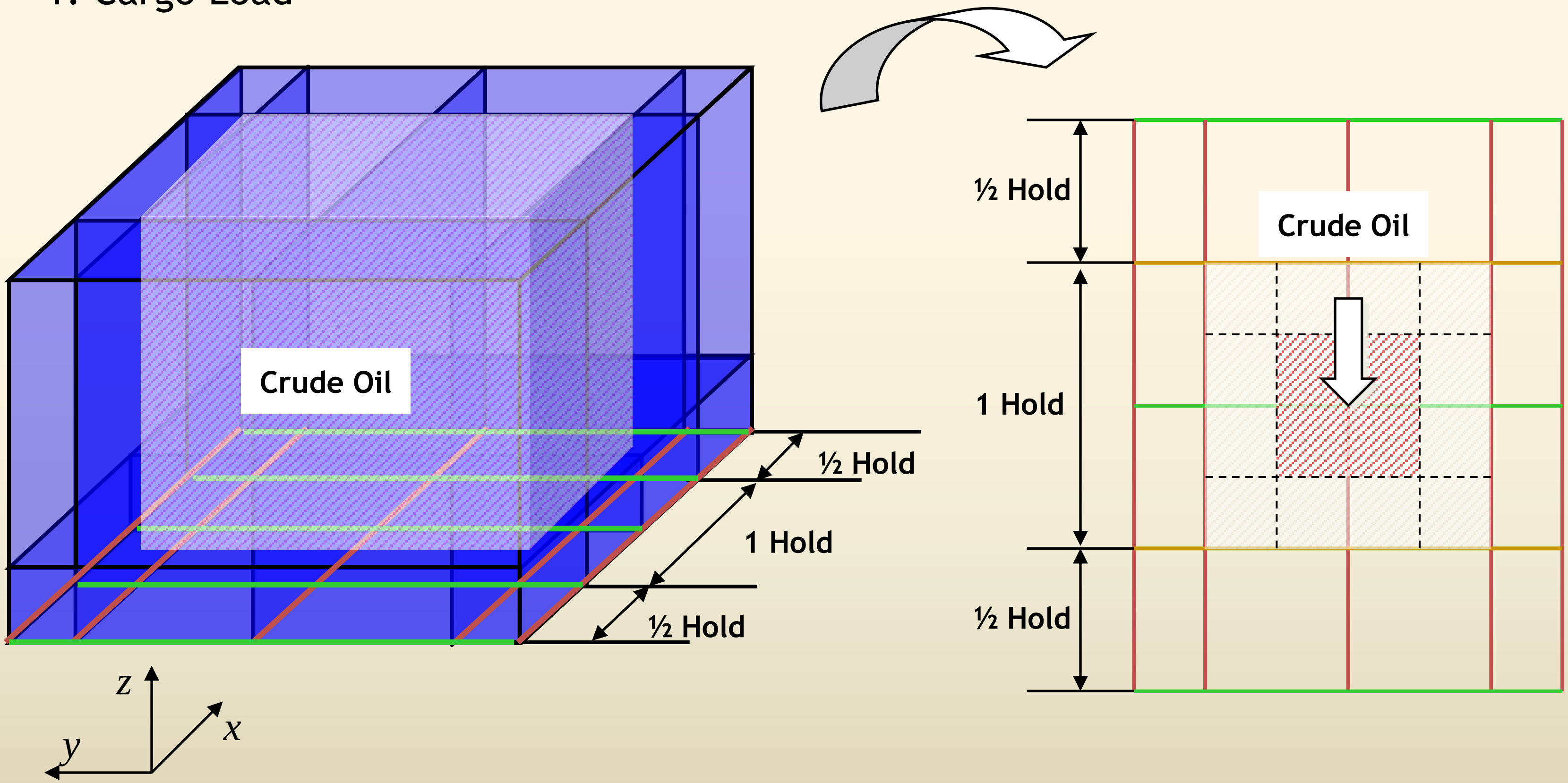
③ $Z_{btm} = \dfrac{I_{\otimes}}{y_{btm}} \Rightarrow y_{btm} = \dfrac{I_{\otimes}}{Z_{btm}}$

④ $\dfrac{I_{\otimes}}{Z_{deck}} + \dfrac{I_{\otimes}}{Z_{btm}} = Depth \Rightarrow I_{\otimes} = \dfrac{Depth \times (Z_{deck} Z_{btm})}{Z_{deck} + Z_{btm}}$

Grillage Analysis : Midship Cargo Hold

Step3. Loading

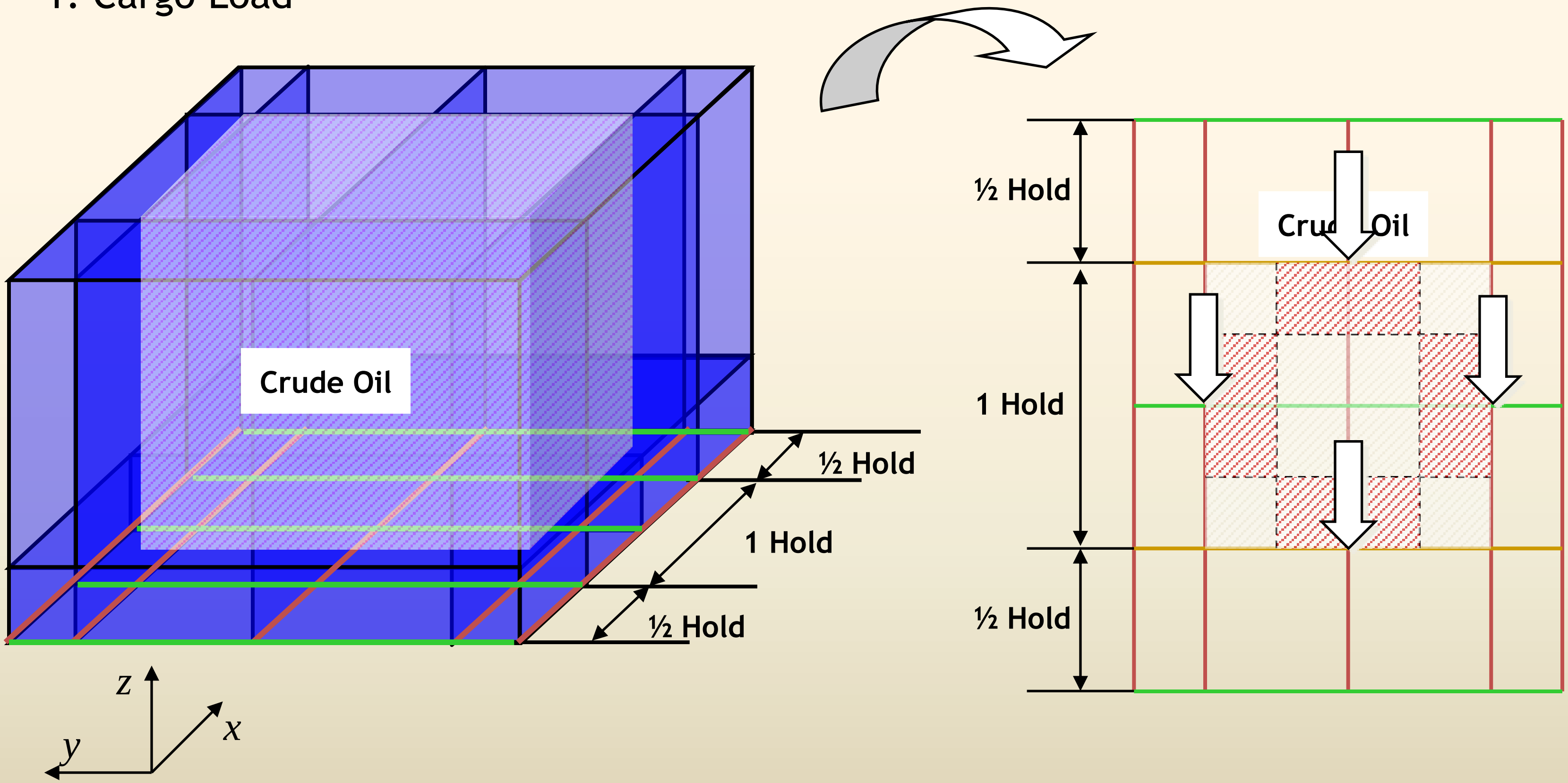
1. Cargo Load



Grillage Analysis : Midship Cargo Hold

Step3. Loading

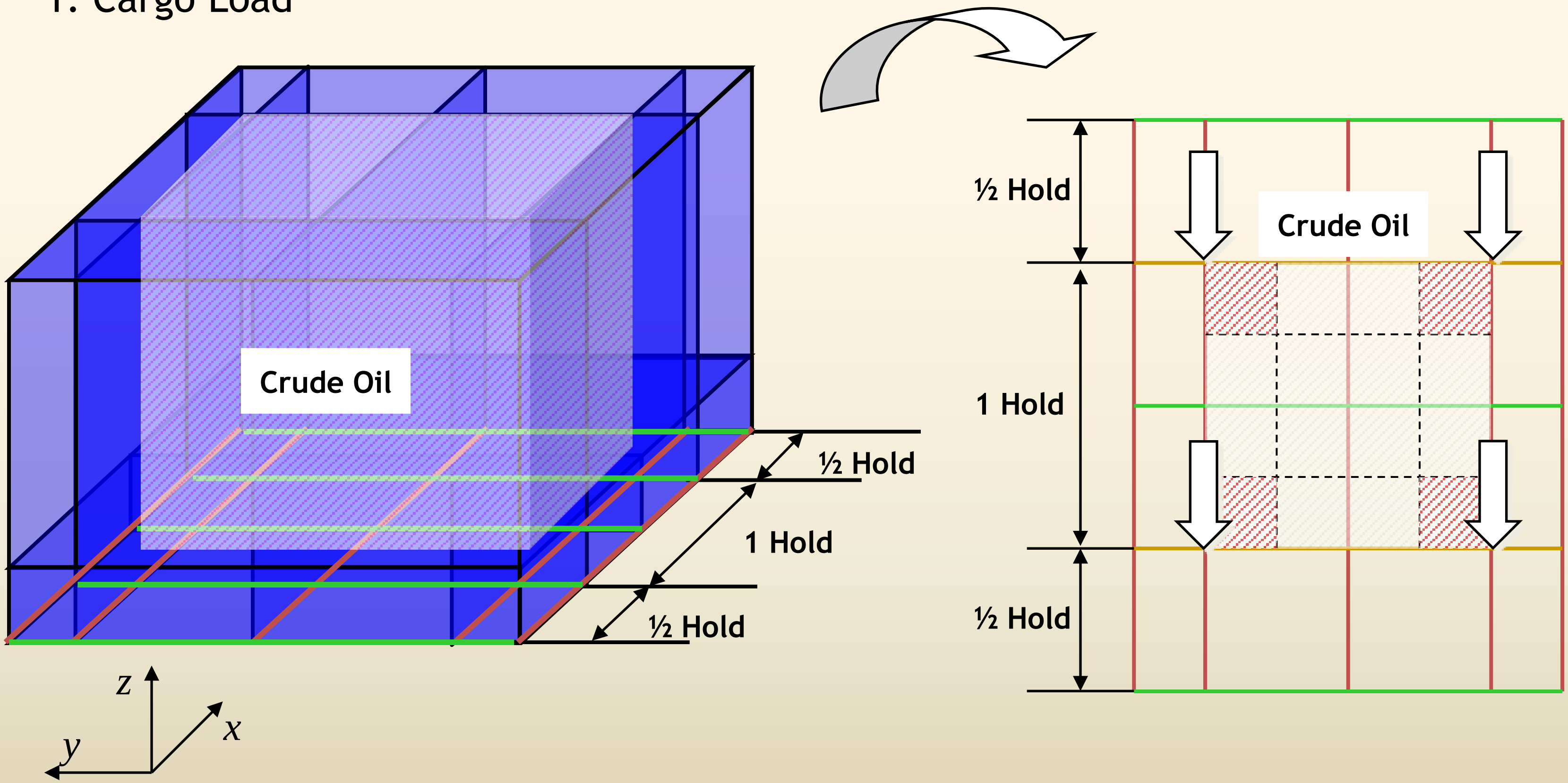
1. Cargo Load



Grillage Analysis : Midship Cargo Hold

Step3. Loading

1. Cargo Load

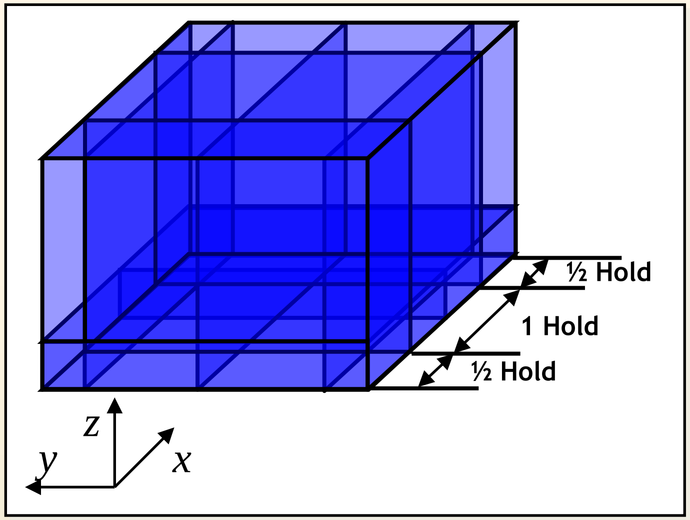


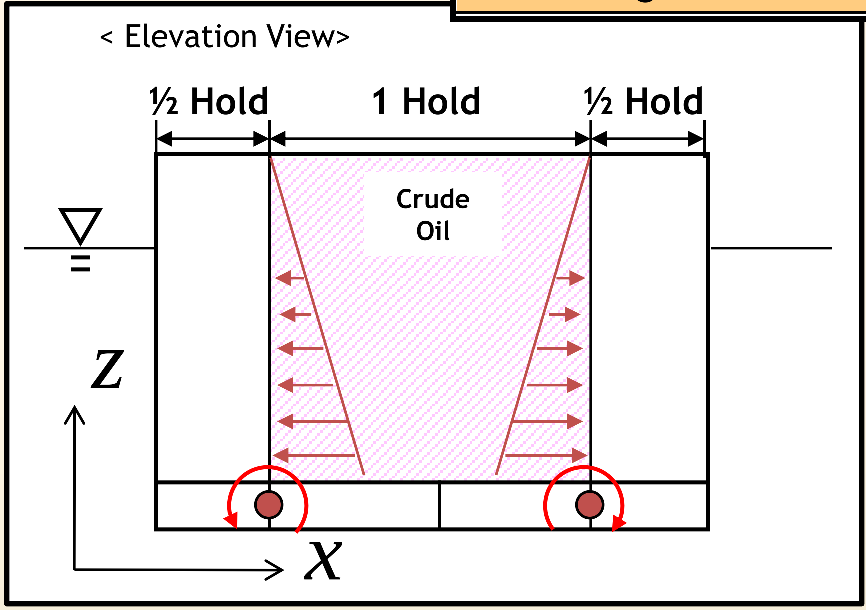
※ the sea water pressure is applied from under the bottom in the same way.

Grillage Analysis : Midship Cargo Hold

Step3. Loading

2. Moment : caused by water pressure and cargo Load



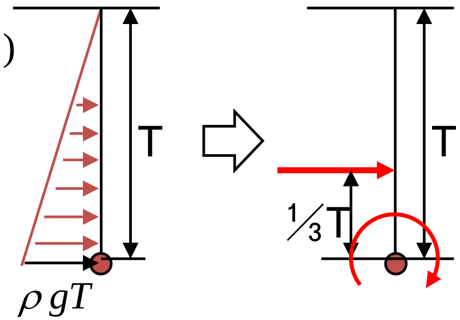


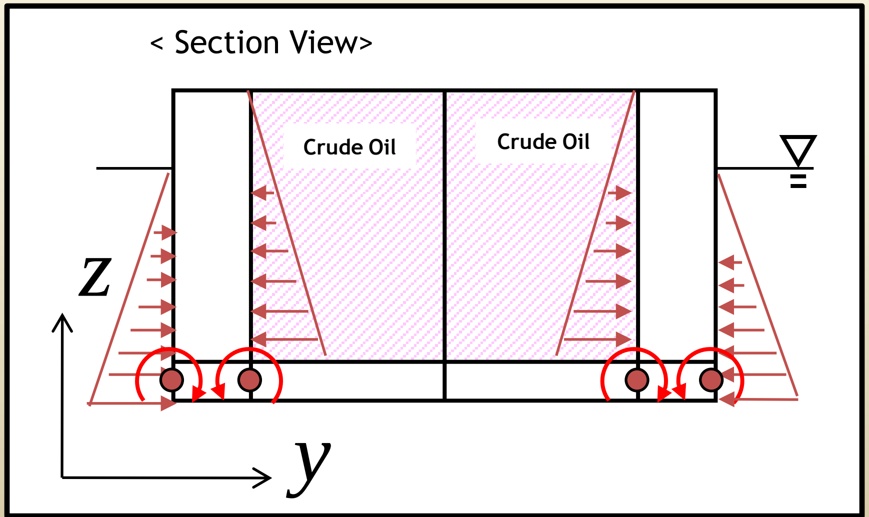
※ 모멘트 계산 방법

$$M = (Total\ Force) \times (Distance\ to\ the\ center)$$

$$= (\frac{1}{2} \times \rho g T \times T) \times (\frac{1}{3} T)$$

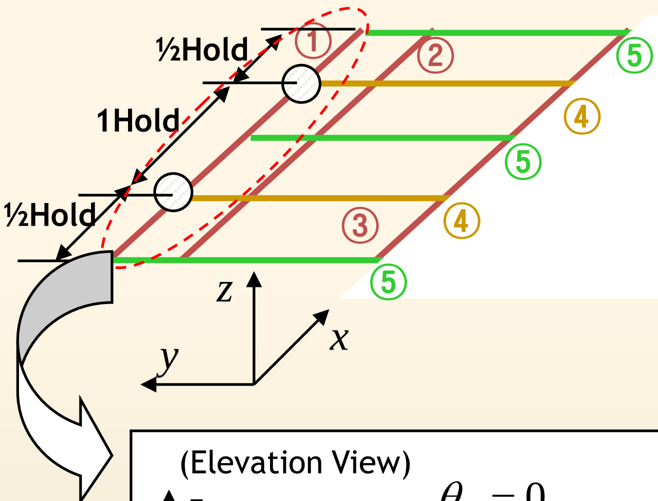
$$= \frac{1}{6} \rho g T^3$$





Grillage Analysis : Midship Cargo Hold

Step4. Boundary Conditions



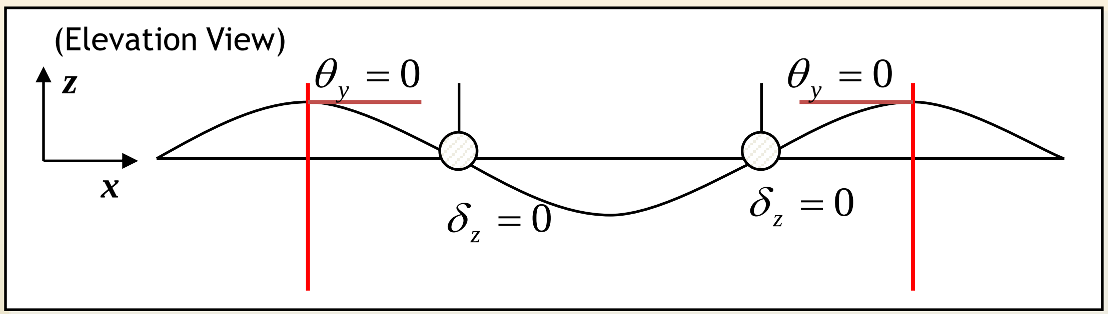
① Side Shell

② Side Longitudinal Bulkhead

③ Center Girder + Longitudinal Bulkhead

④ Transverse Bulkhead

⑤ Transverse Floor



(1) Transversal Symmetry : $\theta_x = 0$ at Center Girder since only half-width is considered:

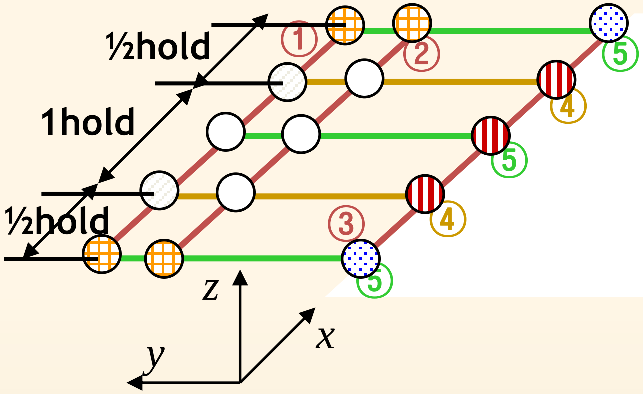
(2) Constraint : $\delta_z = 0$ at the intersection of Side Shell and T.BHD

(3) Longitudinal Symmetry : $\theta_y = 0$ at the end point of ½ Hold

※ $\theta_y = \dfrac{dz}{dx} = 0$

Grillage Analysis : Midship Cargo Hold

Step4. Boundary Conditions



① Side Shell

② Side Longitudinal Bulkhead

③ Center Girder + Longitudinal Bulkhead

④ Transverse Bulkhead

⑤ Transverse Floor

Grillage Constraints

$\delta_x = 0$

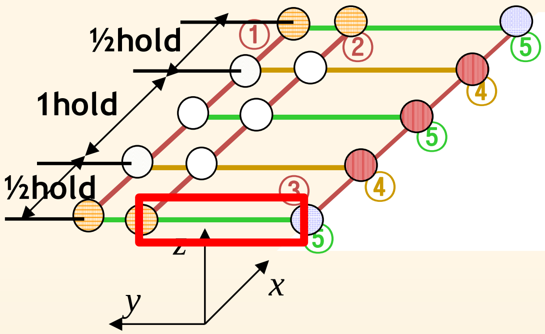
$\delta_y = 0$

$\theta_z = 0$

	Remark	θ_x	θ_y	δ_z	known (0 or Given)	unknown
	Constraints	−	−	0	M_x, M_y, δ_z	θ_x, θ_y, F_z
	Longitudinal Symmetry	−	0	−	M_x, θ_y, F_z	θ_x, M_y, δ_z
	Longitudinal and Transversal Symmetry	0	0	−	θ_x, θ_y, F_z	M_x, M_y, δ_z
	Transversal Symmetry	0	−	−	θ_x, M_y, F_z	M_x, θ_y, δ_z
	No Conditions	−	−	−	M_x, M_y, F_z	$\theta_x, \theta_y, \delta_z$

Grillage Analysis : Midship Cargo Hold

step5. Displacement



① Side Shell
② Side Longitudinal Bulkhead
③ Center Girder + Longitudinal Bulkhead
④ Transverse Bulkhead
⑤ Transverse Floor

G : Shearing Modulus
E : Modulus of elasticity
I : Moment of Inertia
J : Polar Moment of Inertia

<Stiffness Matrix of Grillage>

$$\begin{bmatrix} M_{x1} \\ M_{y1} \\ f_{z1} \\ M_{x2} \\ M_{y2} \\ f_{z2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \theta_{y1} \\ \delta_{z1} \\ \theta_{x2} \\ \theta_{y2} \\ \delta_{z2} \end{bmatrix}$$

$\Rightarrow [\mathbf{K}_{pq}]$

<Coordinates Transformation>

$$[\mathbf{T}] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

<Stiffness Equation>

$$[\mathbf{F}_{xy}] = [\mathbf{T}]^T [\mathbf{K}_{pq}][\mathbf{T}][\delta_{xy}]$$

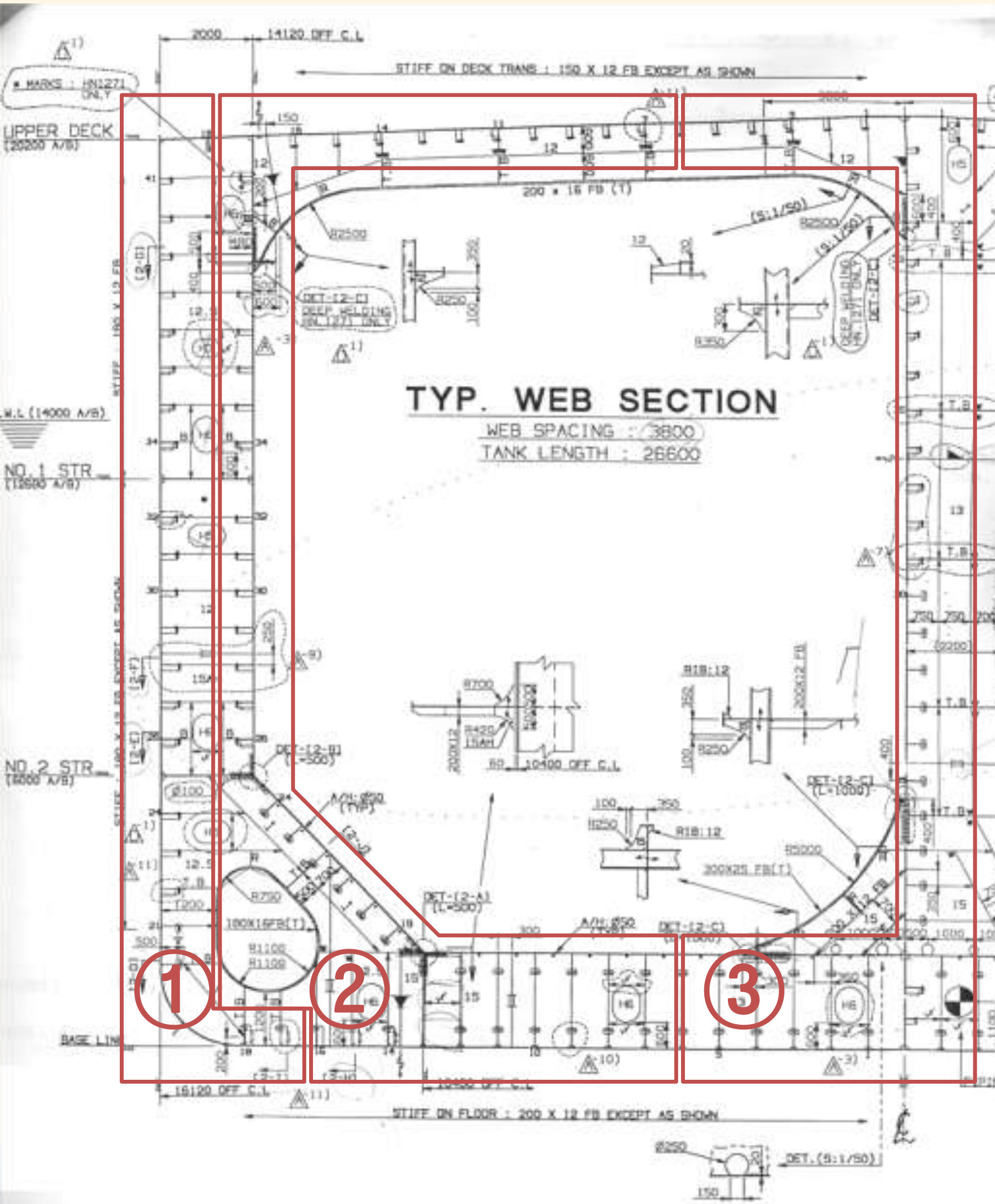
$$[\mathbf{F}_{xy}] = [\mathbf{K}_{xy}][\delta_{xy}]$$

22 equations

⇩

superposition

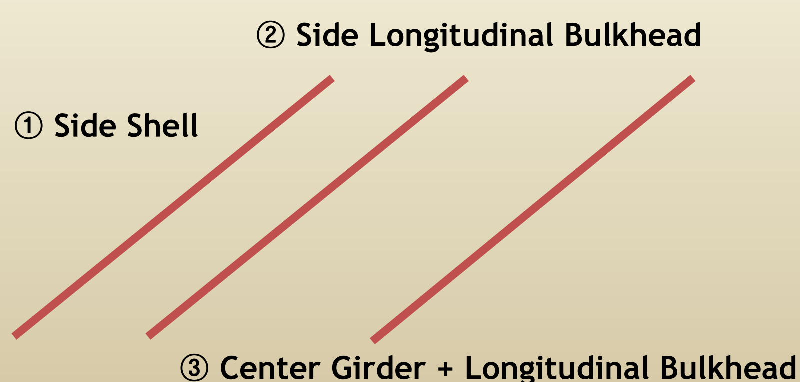
Grillage Analysis : Midship Cargo Hold



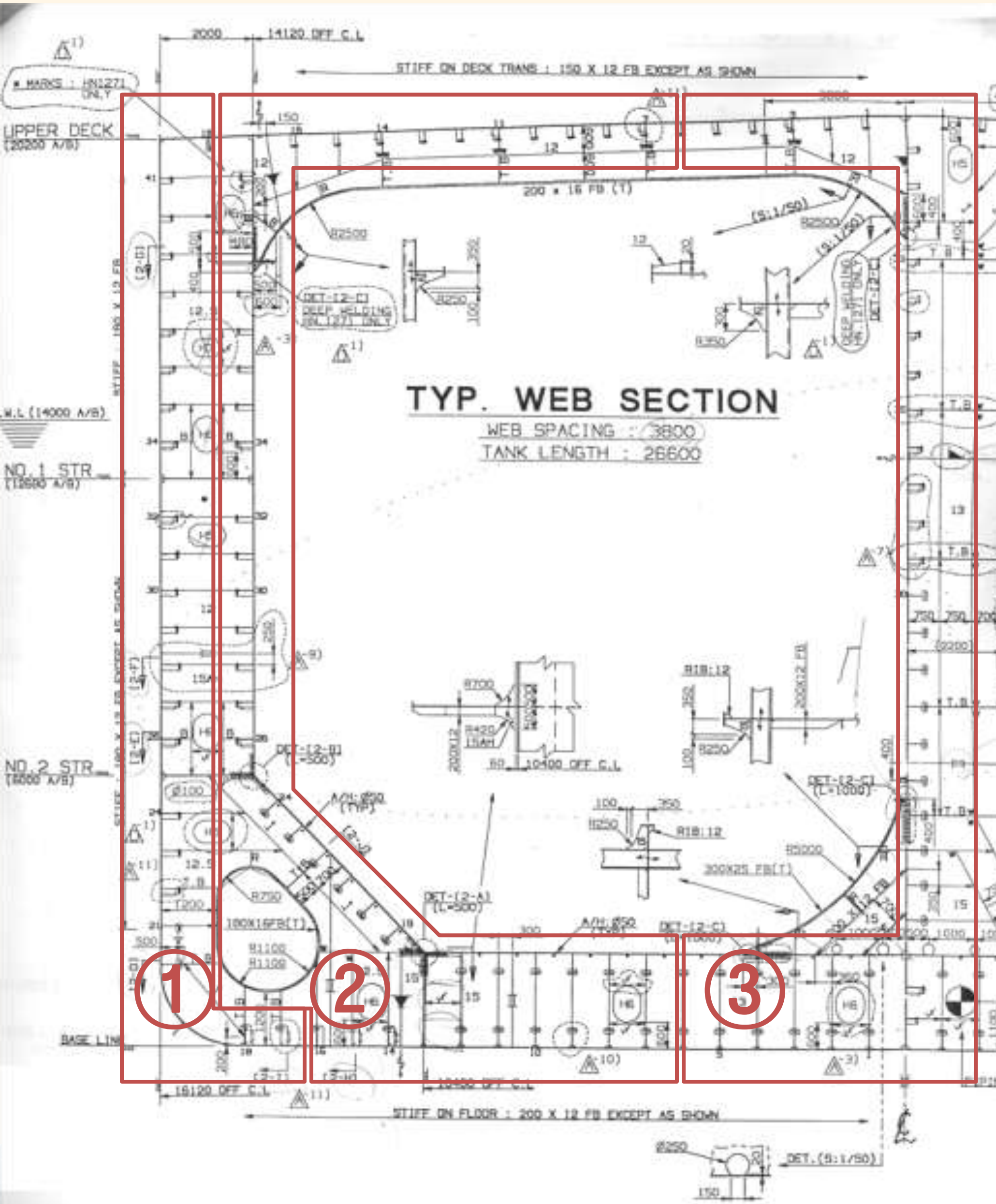
72.5K Oil Tanker
Principal Dimensions

LOA : 228.50m
LBP : 219.00m
Breadth : 32.24m
Depth : 20.20m
Draft Scantling : 14.00m
Draft Design :12.20m

Web Frame Space : 3,800mm
Cargo Tank length : 26,600mm
Number of Web between
Transverse Bulkhead : 6



Ex.) Grillage Analysis



72.5K Oil Tanker Principal Dimensions

LOA : 228.50m
LBP : 219.00m
Breadth : 32.24m
Depth : 20.20m
Draft Scantling : 14.00m
Draft Design :12.20m

Web Frame Space : 3,800mm
Cargo Tank length : 13,300mm
Number of Web between Transverse Bulkhead : 3

② Side Longitudinal Bulkhead

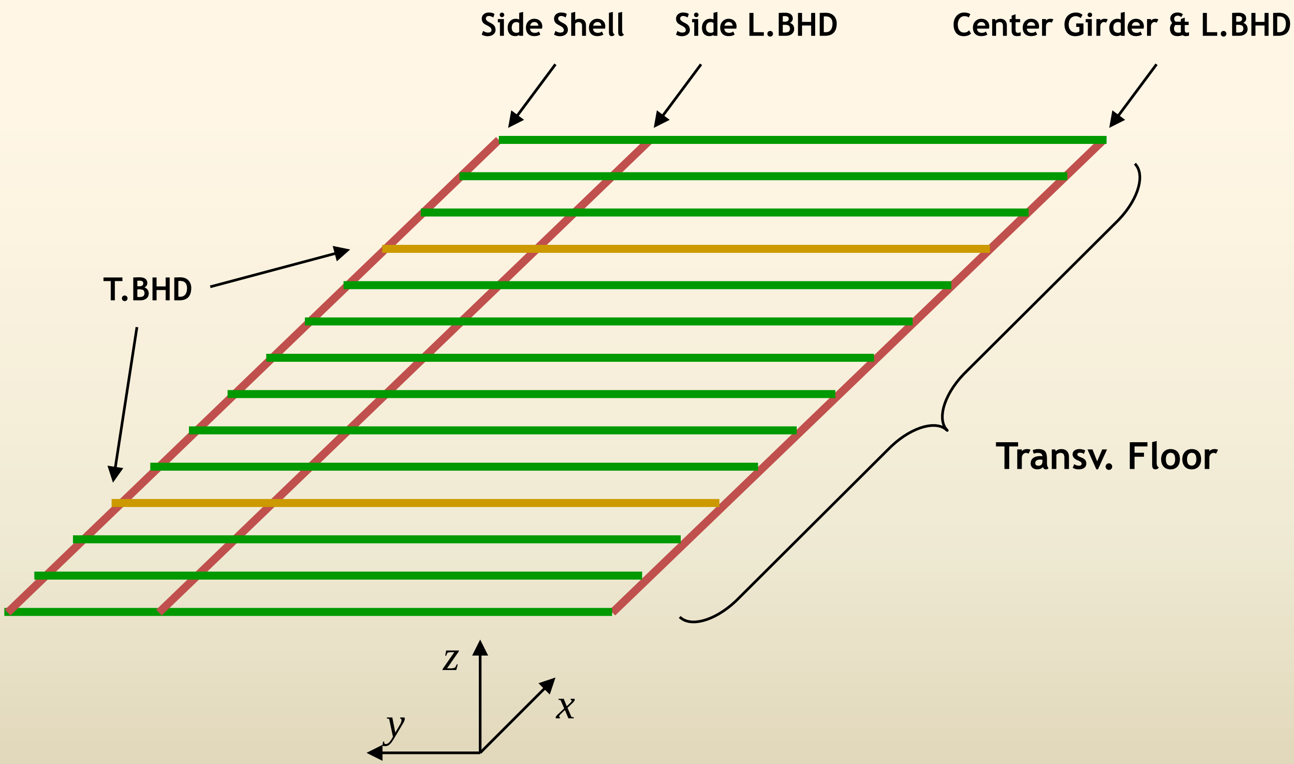
① Side Shell

③ Center Girder + Longitudinal Bulkhead

Ex.) Grillage Analysis

Step1. Grillage Model

- Analysis Region : ½ Hold + 1 Hold + ½ Hold



Ex.) Grillage Analysis

Step2. Element Properties

< Section Modulus(cm²-m) >

	Rule REQ	Design
Deck	18,274,500	22,036,400 [cm³]= 22.0364 [m³]
Bottom	18,274,500	26,933,300 [cm³]= 26.9333 [m³]

$$\therefore I_{\otimes} = \frac{Depth \times (Z_{deck} Z_{btm})}{Z_{deck} + Z_{btm}} = \frac{20.20 \times (22.3464 \times 26.9333)}{22.3464 + 26.9333} = 244.824[m^4]$$

BAR TYPE	TORSION CONSTANT (J)	INERTIA (I)
1.Center Longi. Bulkhead	$5 \times I_{\otimes}$ =1224.12 [m⁴]	$0.11 \times I_{\otimes}$ = 26.93 [m⁴]
2. Longitudinal Bulkhead	$5 \times I_{\otimes}$ =1224.12 [m⁴]	$0.22 \times I_{\otimes}$ = 53.86 [m⁴]
3. Side Shell	$5 \times I_{\otimes}$ =1224.12 [m⁴]	$0.17 \times I_{\otimes}$ = 41.62 [m⁴]
4. Bottom Transv. floor	10^{-5} [m⁴]	0.1335 [m⁴]
5.Oil-tight Bulkhead	$l \cdot D_T^{\,2} \cdot (t_B + t_D)/4$ = 65.36 [m⁴]	Not less than $0.3 \times I_{\otimes}$ = 73.45 [m⁴]

NOMENCLATURE

D_T – Depth of Tank

$I_{\otimes}$ – Vertical Moment of Inertia of Full Midship Section

l – Spacing of Transverse Bulkheads

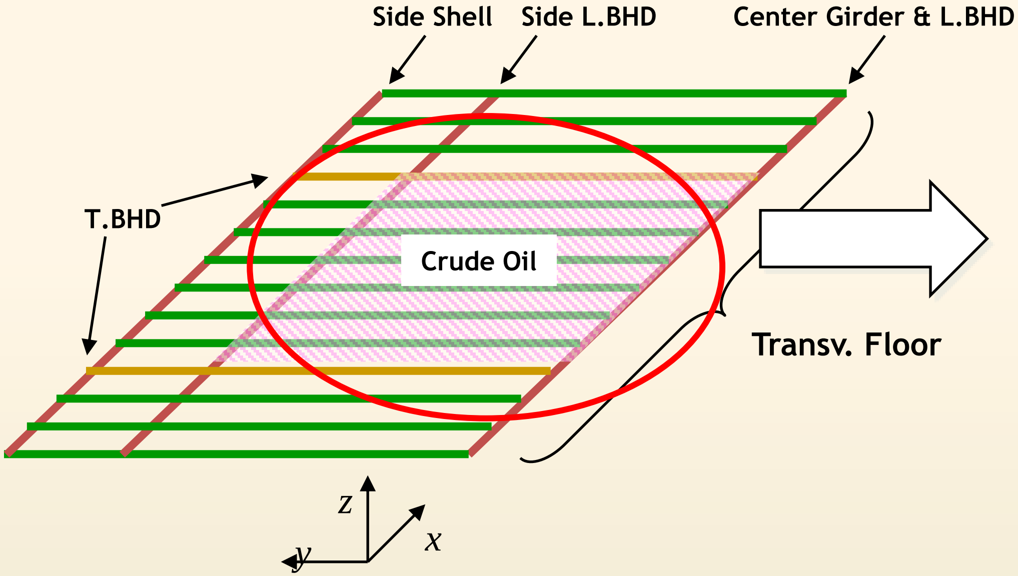
t_B – Thickness of Bottom Shell

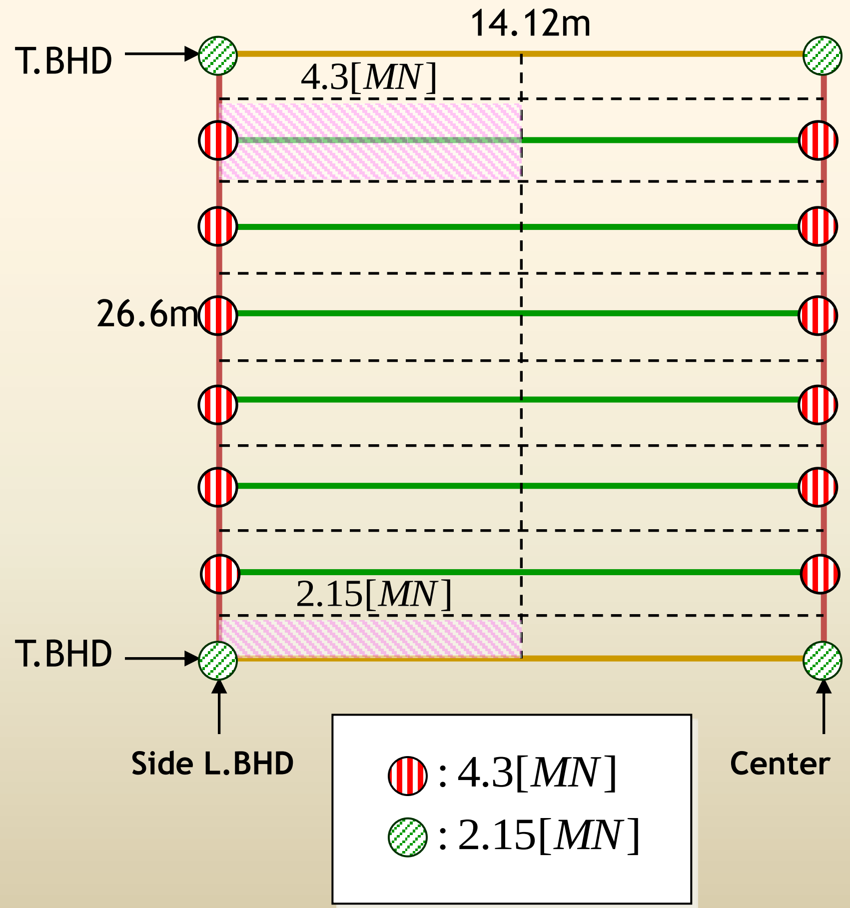
t_D – Thickness of Deck Plating

$$l \cdot D_T^{\,2} \cdot (t_B + t_D)/4$$
$$= 26.6 \cdot 18.1 \cdot (0.015 + 0.015)/4$$
$$= 65.36$$

Ex.) Grillage Analysis

Step3. Loading





- Cargo Load

$$W_{oil} = \rho_{oil} g \times (\text{Cargo Volume}) = 900 \times 9.81 \times 26.6 \times 14.12 \times 18.1$$
$$= 60021422\ [N] = 60\ [MN]$$

- Nodal Load

① at the intersection of L.BHD and Transverse Floor

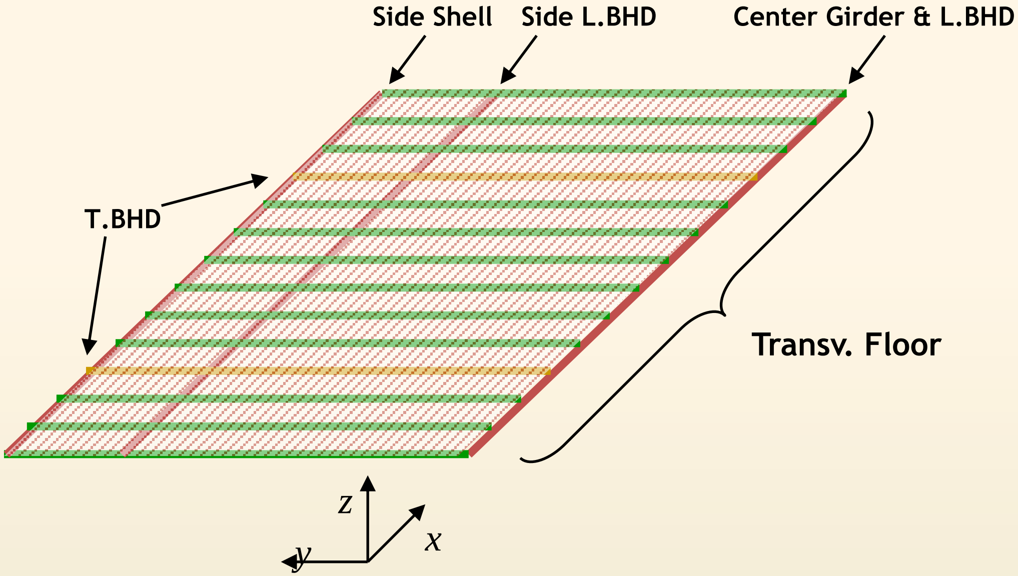
$$W_{oil}/14 \approx 4.3[MN]$$

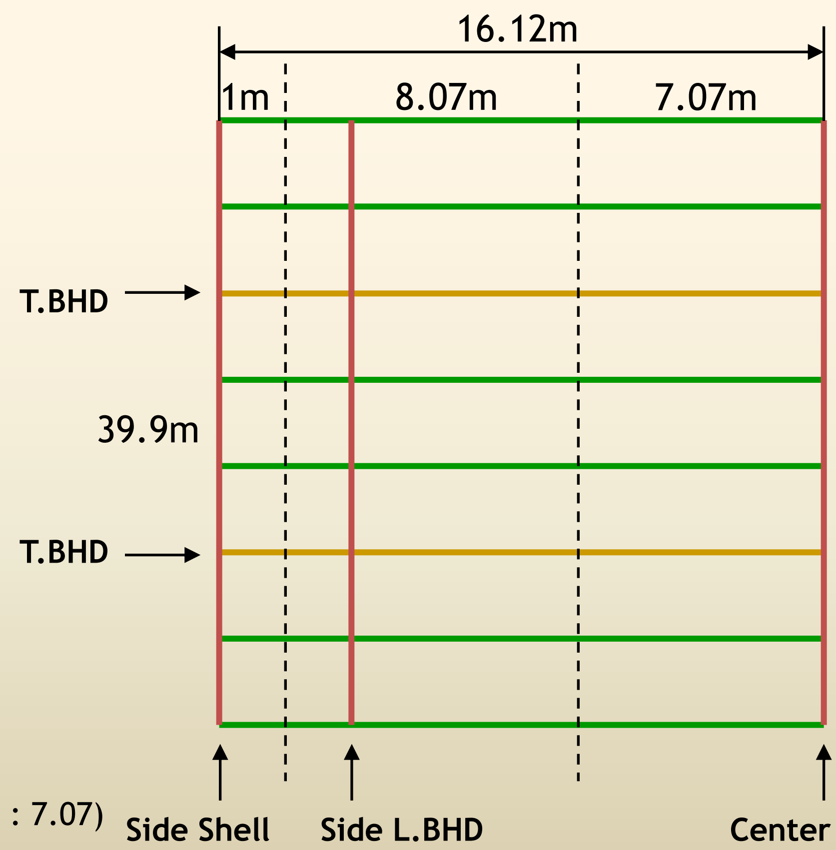
② at the intersection of L.BHD and T.BHD Floor

$$4.3/2 = 2.15[MN]$$

Ex.) Grillage Analysis

Step3. Loading





- Load by Water Pressure

$$B = \rho_{sea} g T \times (area\ of\ \ cargo\ hold\ bottom)$$
$$= 1024 \times 9.81 \times 14 \times (16.12 \times 39.9)$$
$$= 90455490[N] = 90.46[MN]$$

- Nodal Load

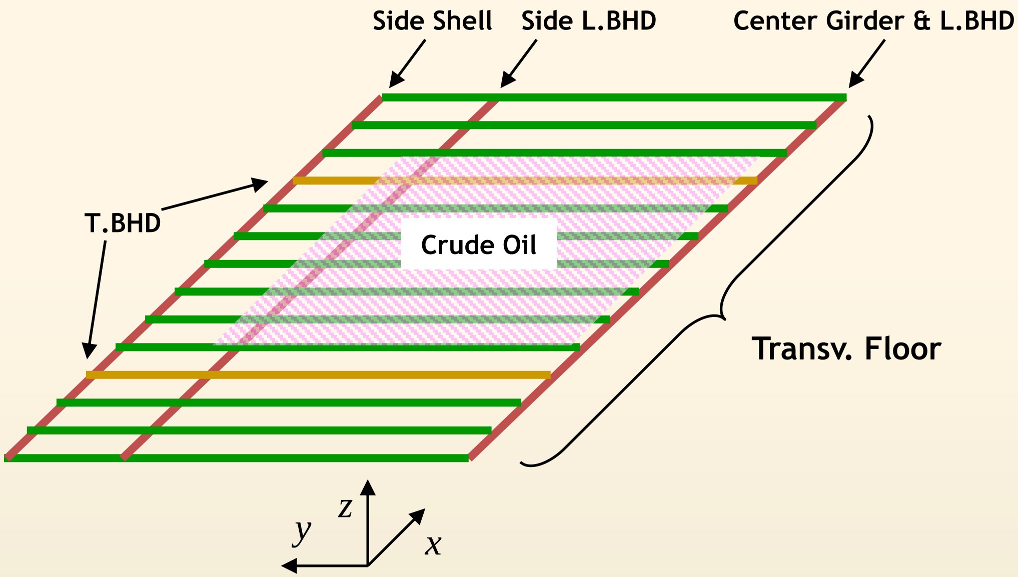
① distribute the load depend on the ratio of node width (1 : 8.07 : 7.07)

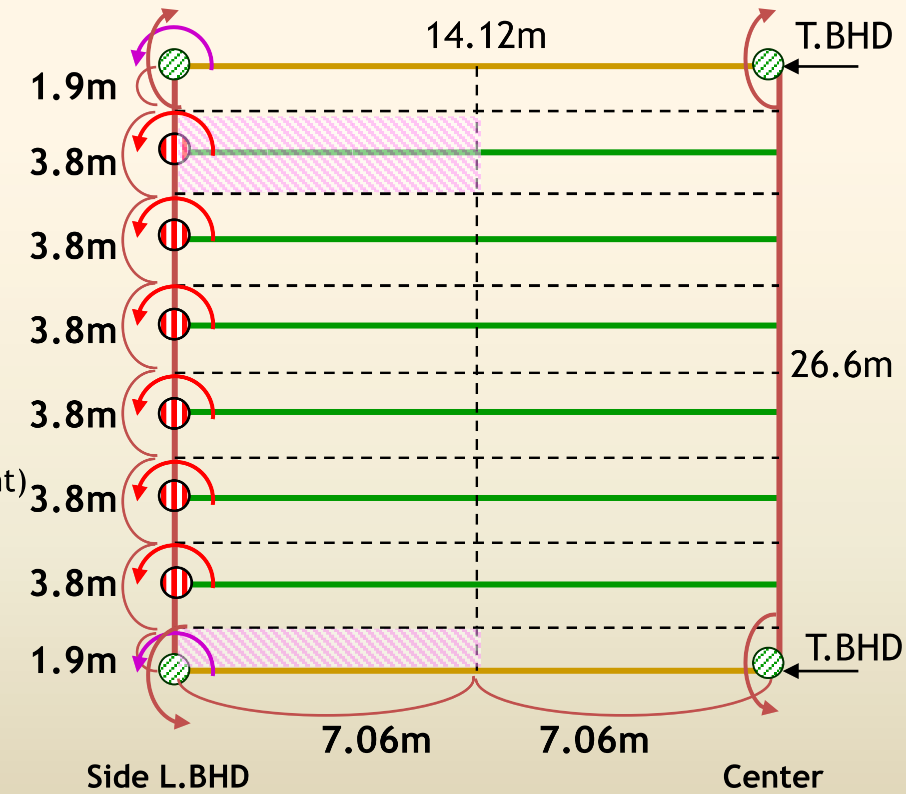
② divide the distributed load by the number of nodes(14).

Ex.) Grillage Analysis

Step3. Load

- moment by cargo load





- Moment per Length (D: cargo hold height , d: double bottom height)

$$= \rho_{oil} g \cdot \frac{D^2}{2} \cdot \left(\frac{D}{3} + \frac{d}{2} \right)$$

$$= 900 \cdot 9.81 \cdot \frac{18.1^2}{2} \left(\frac{18.1}{3} + \frac{2.1}{4} \right)$$

$$= 9,484,887 [N \cdot m / m] = 9.48 [MN \cdot m / m]$$

- Nodal Moment

$$M_{x1} = (moment\ \ per\ \ length) \times 3.8 = 35.08 [MN \cdot m]$$

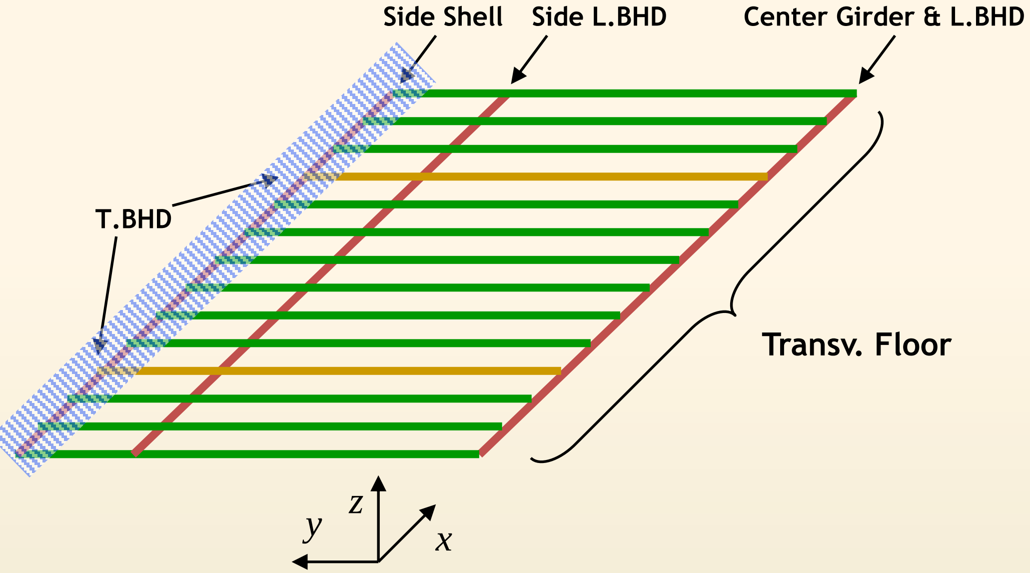
$$M_{x2} = (moment\ \ per\ \ length) \times 1.9 = 17.54 [MN \cdot m]$$

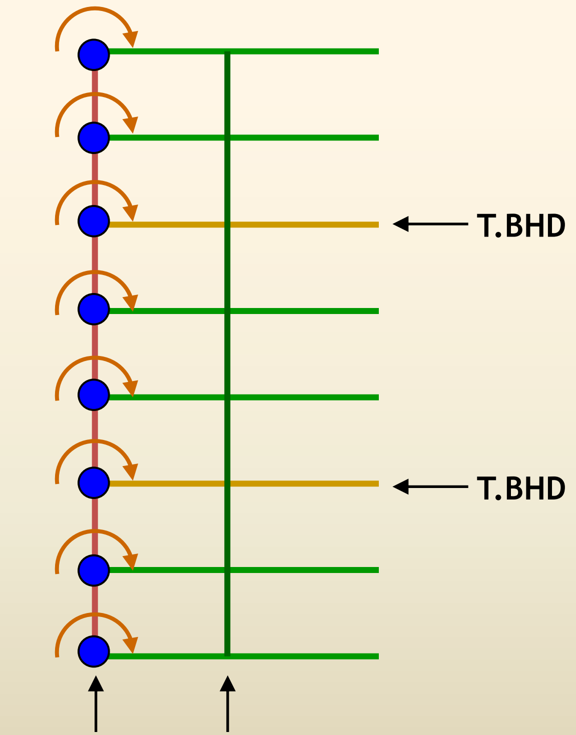
$$M_{y} = (moment\ \ per\ \ length) \times 7.06 = 65.18 [MN \cdot m]$$

Ex.) Grillage Analysis

Step3. Load

- moment by water pressure





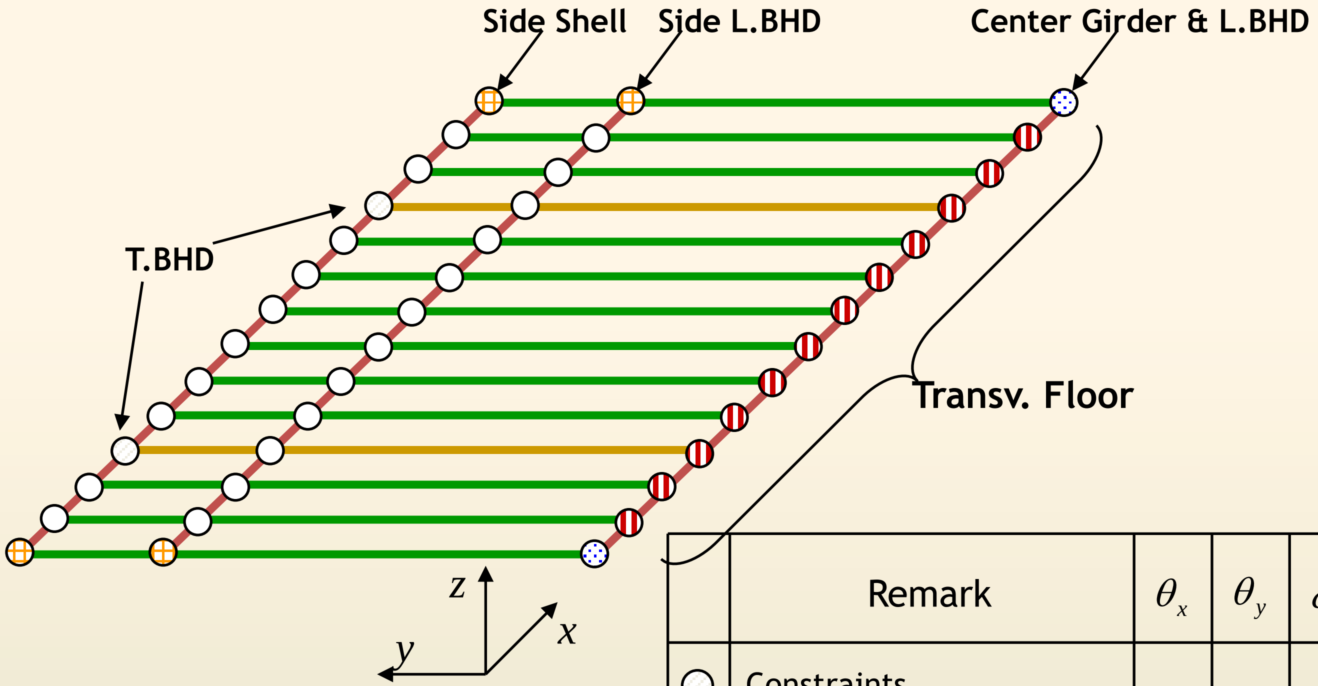
- Moment per Length(T: draft , d: double bottom height)

$$= \rho_{sea} g \cdot \frac{(T-d/2)^2}{2} \cdot \frac{(T-d/2)}{3} = \rho_{sea} g \cdot \frac{(T-d/2)^3}{6}$$

- Nodal Moment

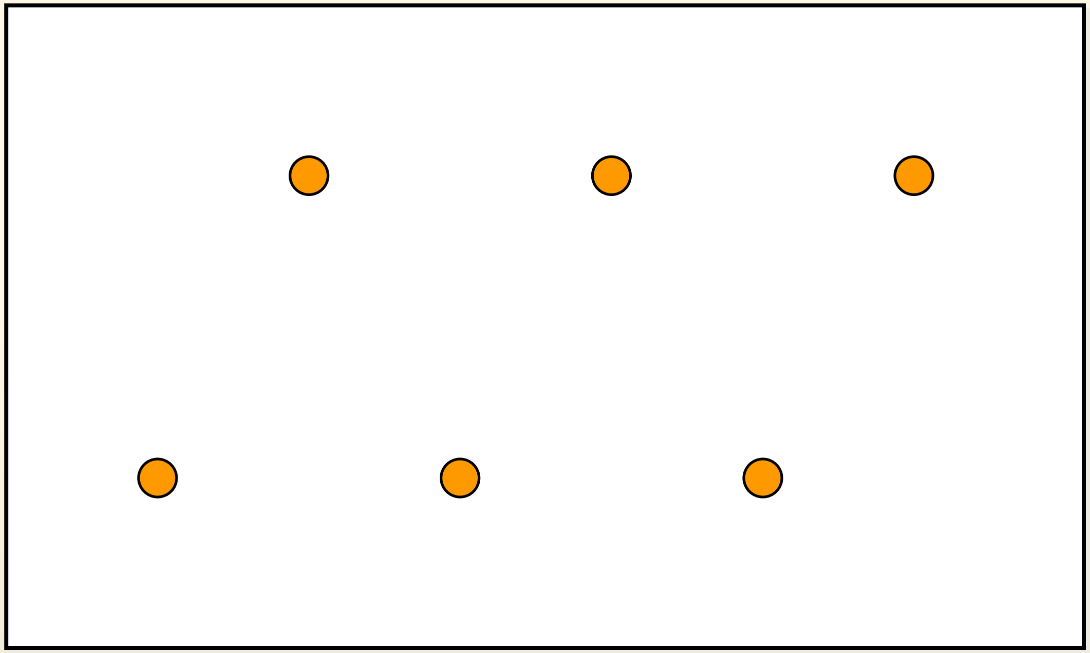
$$M_x = \rho_{sea} g \cdot \frac{(T-d/2)^3}{6} \times 3.8 = 82.90[MN \cdot m]$$

Ex.) Grillage Analysis



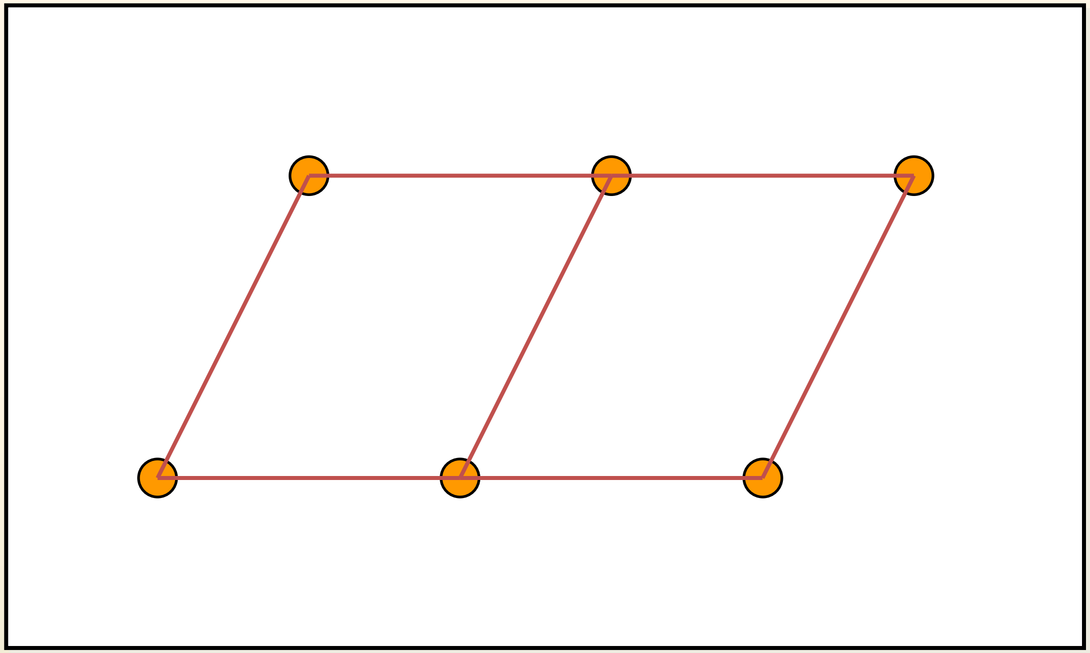
	Remark	θ_x	θ_y	δ_z	known (0 or Given)	unknown
	Constraints	–	–	0	M_x, M_y, δ_z	θ_x, θ_y, F_z
	Longitudinal Symmetry	–	0	–	M_x, θ_y, F_z	θ_x, M_y, δ_z
	Longitudinal and Transversal Symmetry	0	0	–	θ_x, θ_y, F_z	M_x, M_y, δ_z
	Transversal Symmetry	0	–	–	θ_x, M_y, F_z	M_x, θ_y, δ_z
	No Conditions	–	–	–	M_x, M_y, F_z	$\theta_x, \theta_y, \delta_z$

Grillage Analysis Program



Step1. Input : Nodes

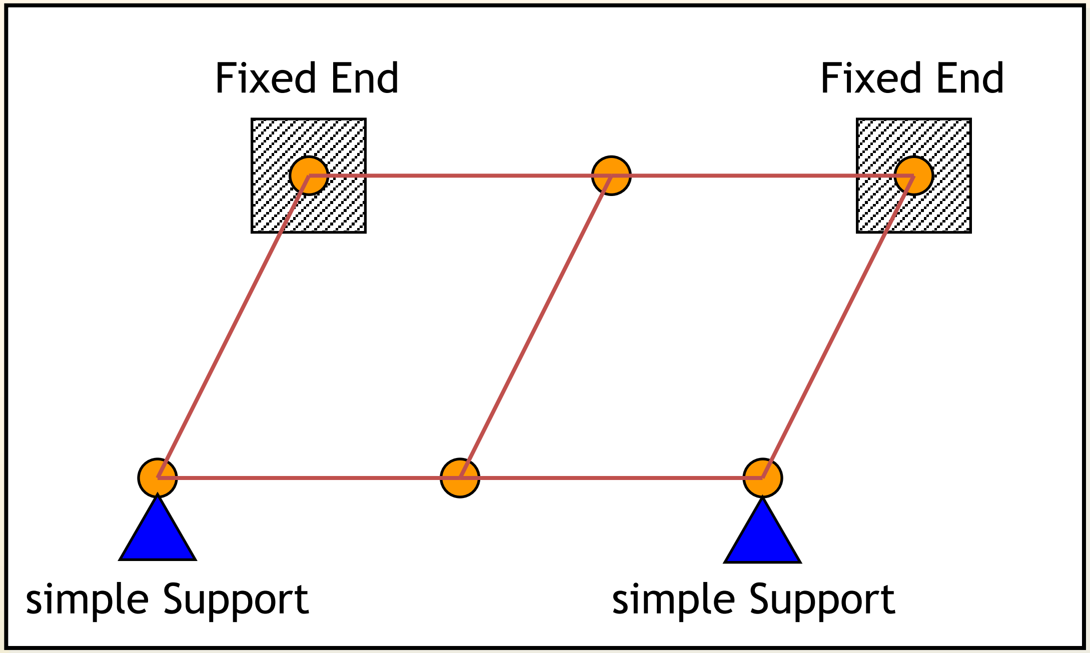
Grillage Analysis Program



Step1. Input : Nodes

Step2. Link : Between Nodes

Grillage Analysis Program

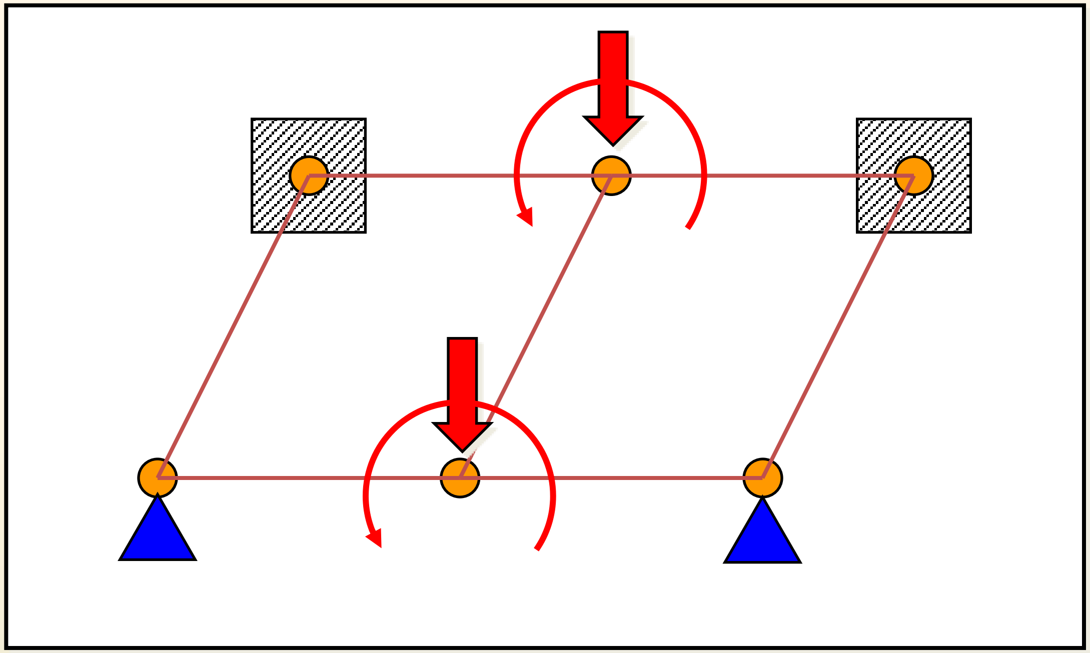


Step1. Input : Nodes

Step2. Link : Between Nodes

Step3. Input : Boundary Conditions

Grillage Analysis Program



Step1. Input : Nodes

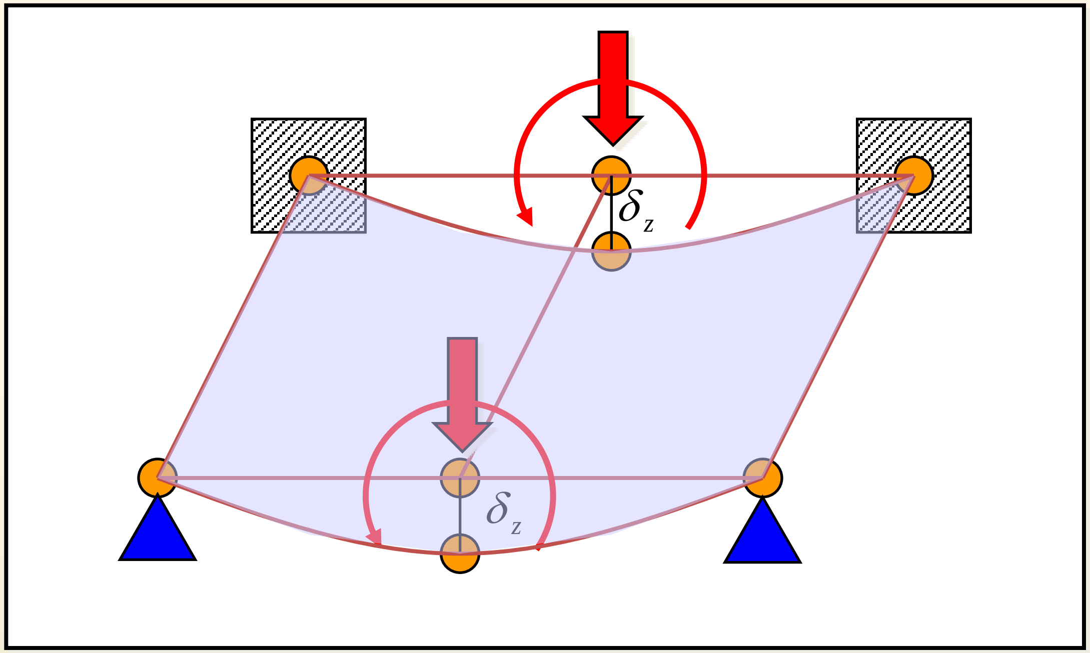
Step2. Link : Between Nodes

Step3. Input : Boundary Conditions

Step4. Input : Force and Moment

Step5. Grillage Analysis

Grillage Analysis Program



Step1. Input : Nodes

Step2. Link : Between Nodes

Step3. Input : Boundary Conditions

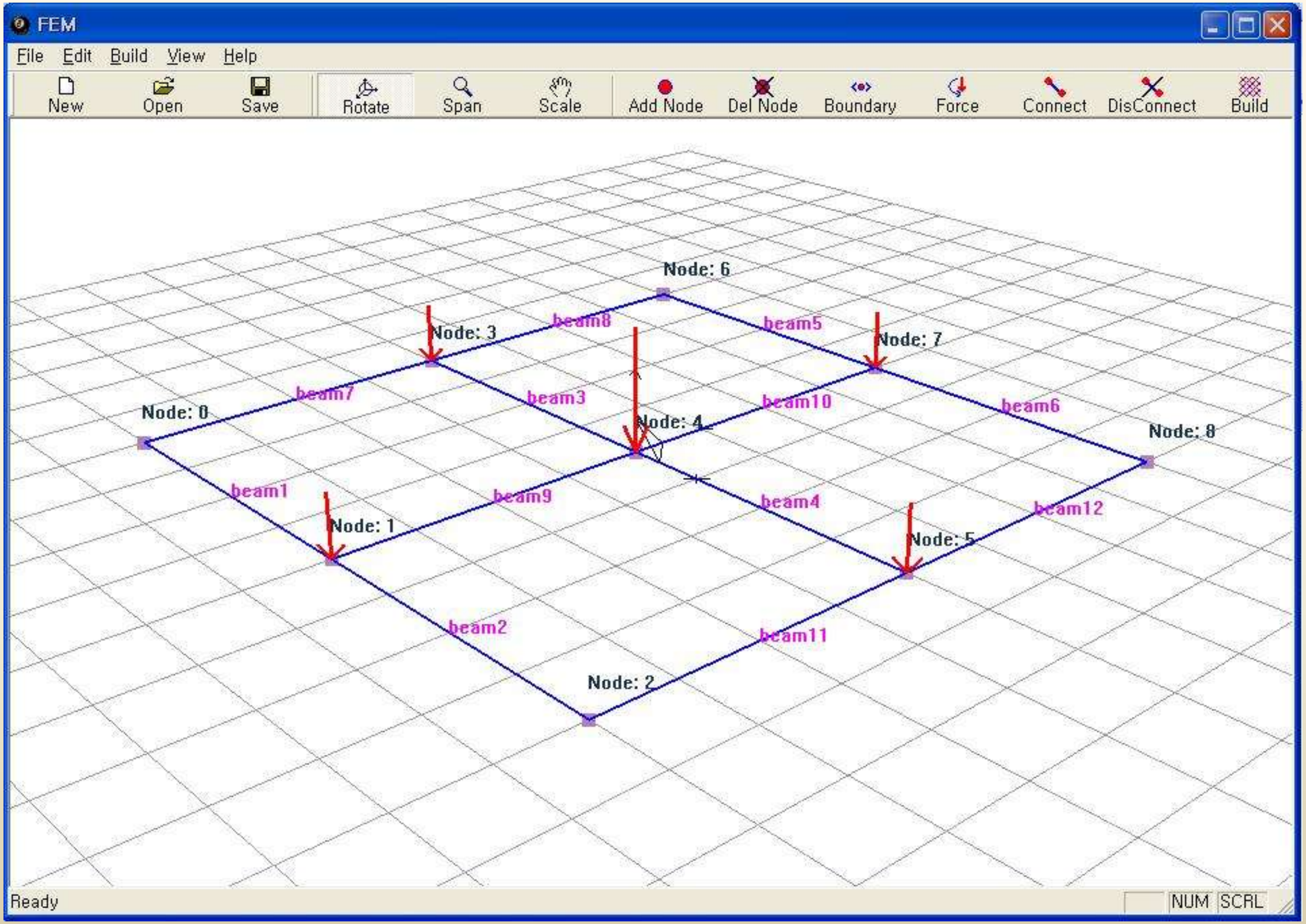
Step4. Input : Force and Moment

Step5. Grillage Analysis

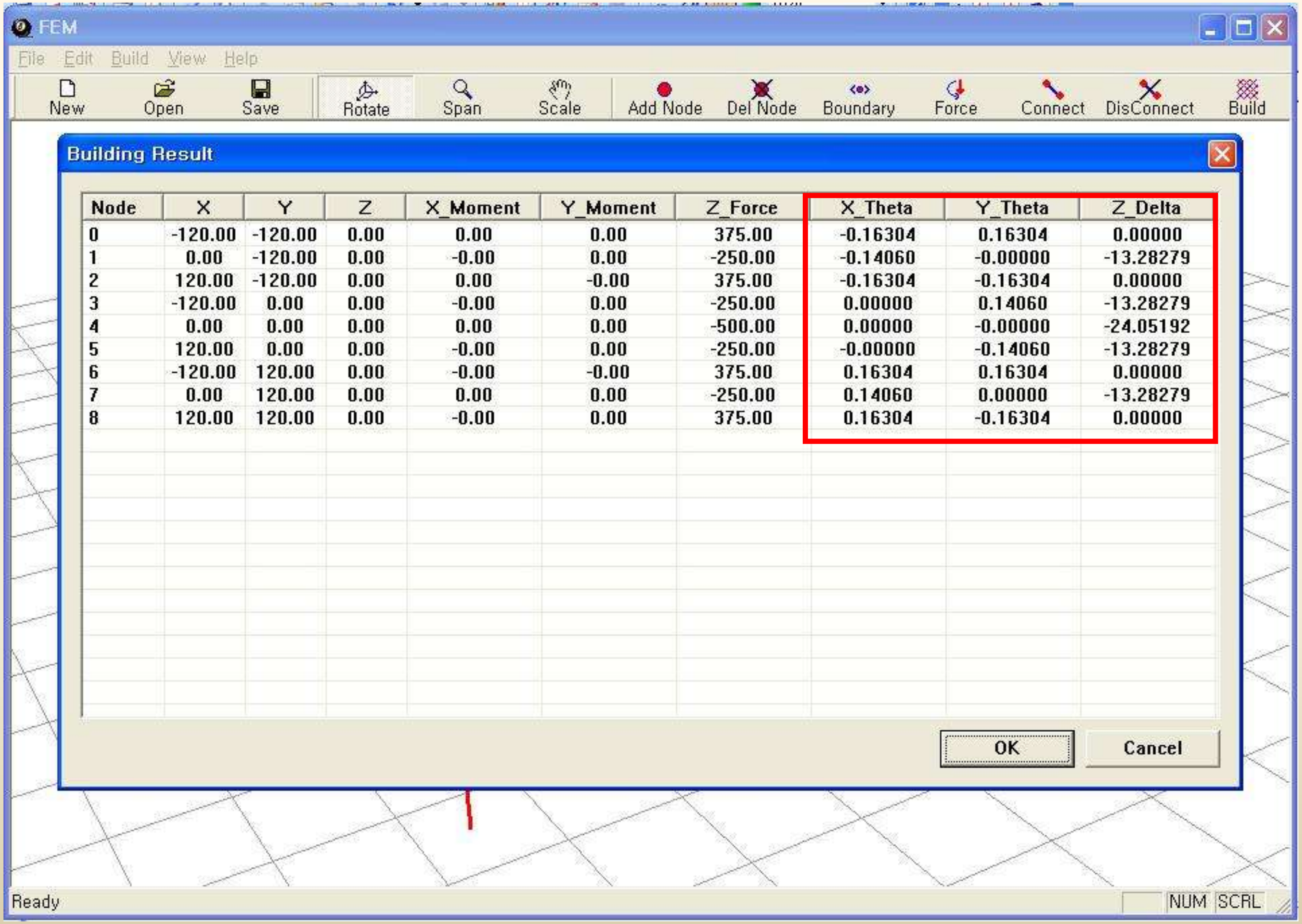
Step6. Nodal Deflection

→Visualization by B-spline Surface

Example 1 : Grillage Analysis

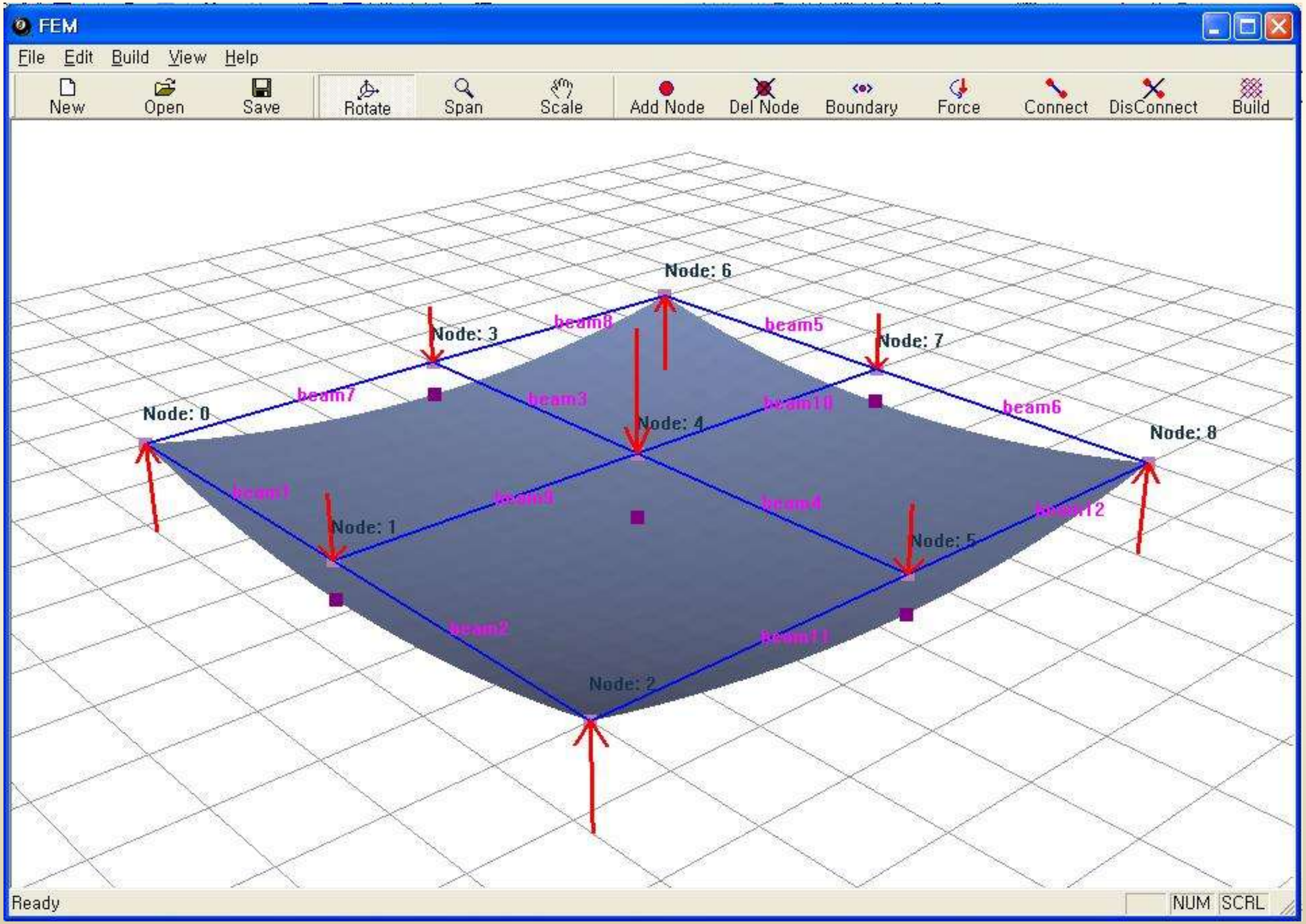


Example 1 : Grillage Analysis

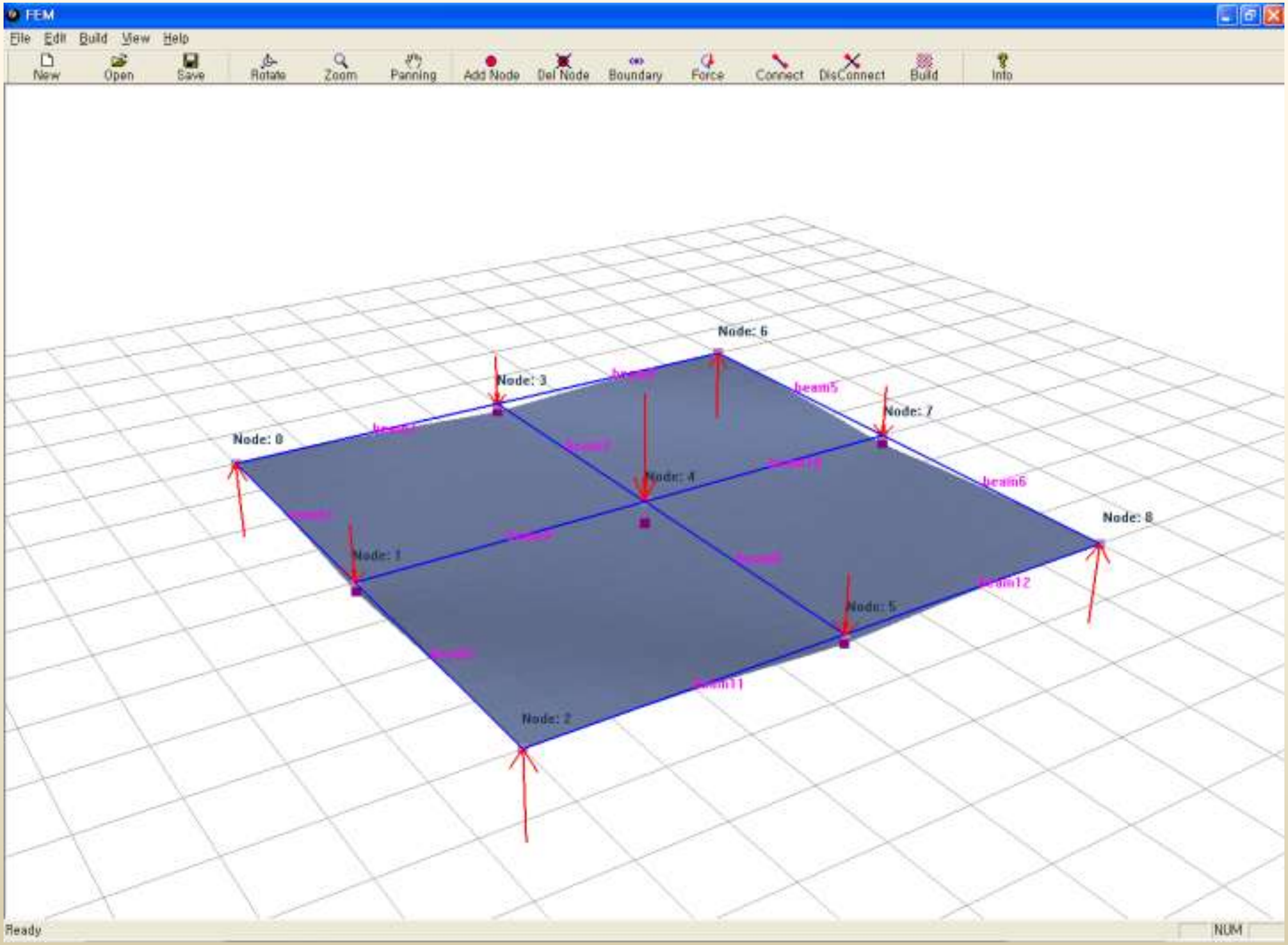


Node	X	Y	Z	X_Moment	Y_Moment	Z_Force	X_Theta	Y_Theta	Z_Delta
0	-120.00	-120.00	0.00	0.00	0.00	375.00	-0.16304	0.16304	0.00000
1	0.00	-120.00	0.00	-0.00	0.00	-250.00	-0.14060	-0.00000	-13.28279
2	120.00	-120.00	0.00	0.00	-0.00	375.00	-0.16304	-0.16304	0.00000
3	-120.00	0.00	0.00	-0.00	0.00	-250.00	0.00000	0.14060	-13.28279
4	0.00	0.00	0.00	0.00	0.00	-500.00	0.00000	-0.00000	-24.05192
5	120.00	0.00	0.00	-0.00	0.00	-250.00	-0.00000	-0.14060	-13.28279
6	-120.00	120.00	0.00	-0.00	-0.00	375.00	0.16304	0.16304	0.00000
7	0.00	120.00	0.00	0.00	0.00	-250.00	0.14060	0.00000	-13.28279
8	120.00	120.00	0.00	-0.00	0.00	375.00	0.16304	-0.16304	0.00000

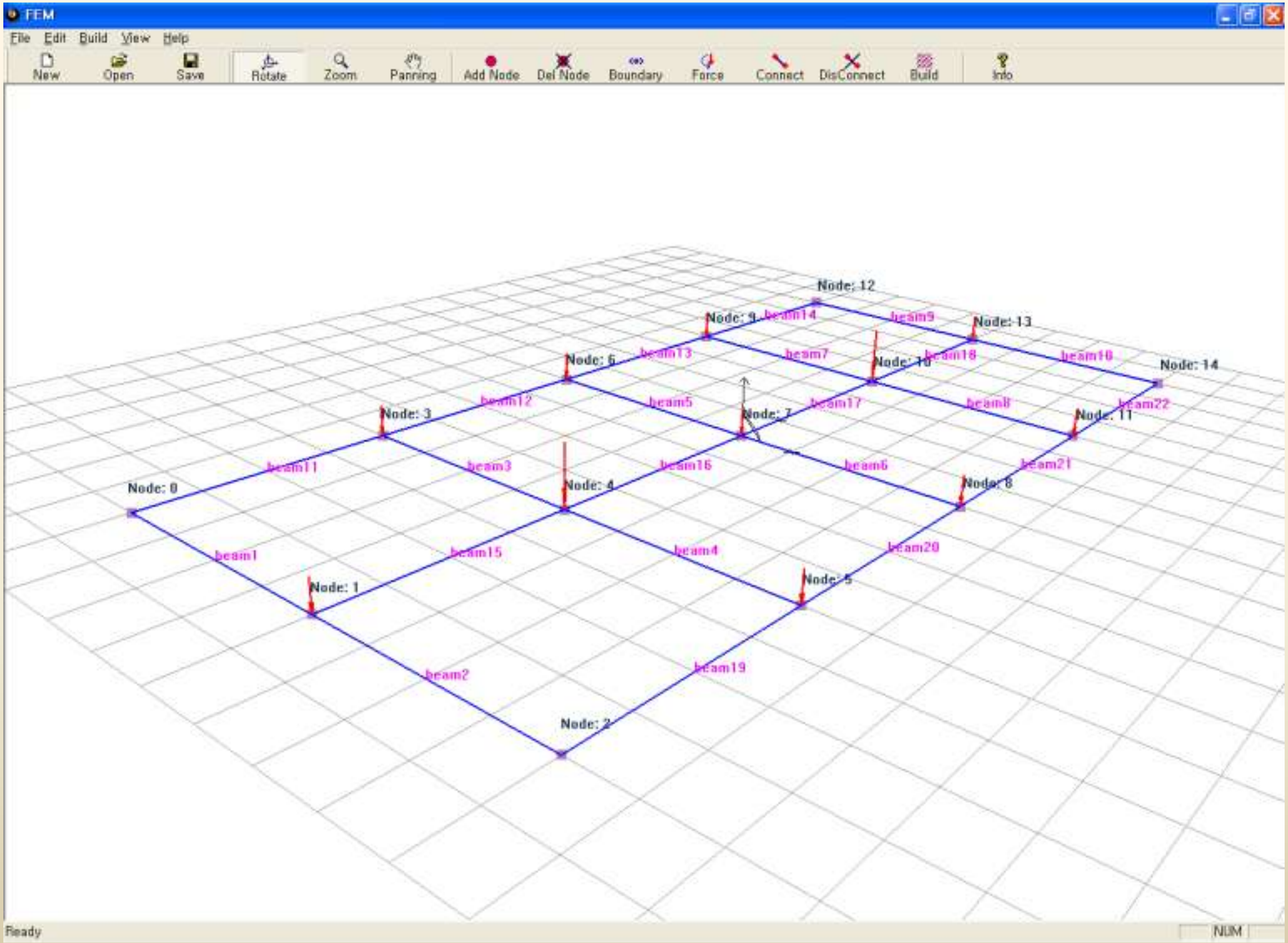
Example 1 : Grillage Analysis



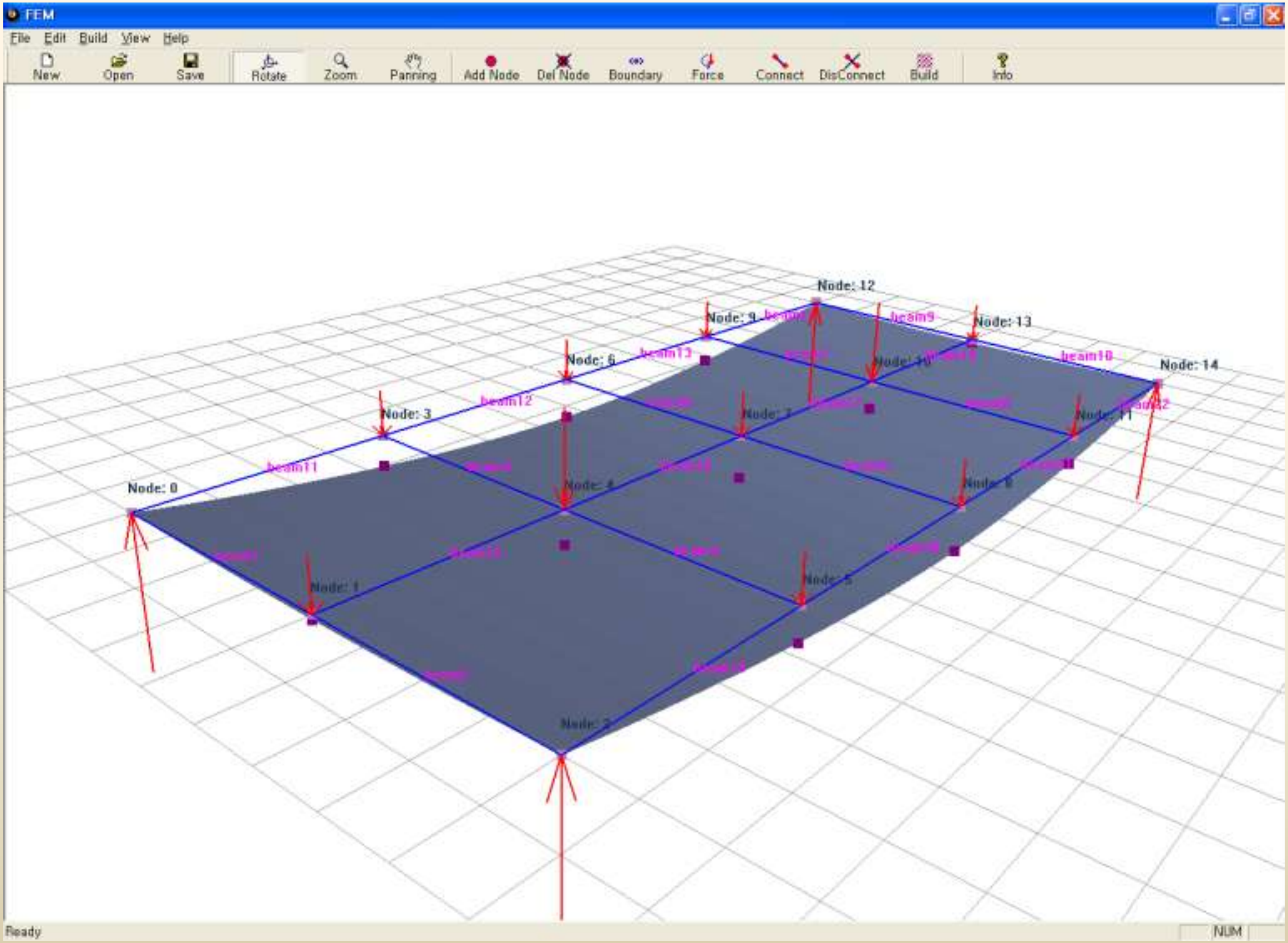
Example 1 : Grillage Analysis



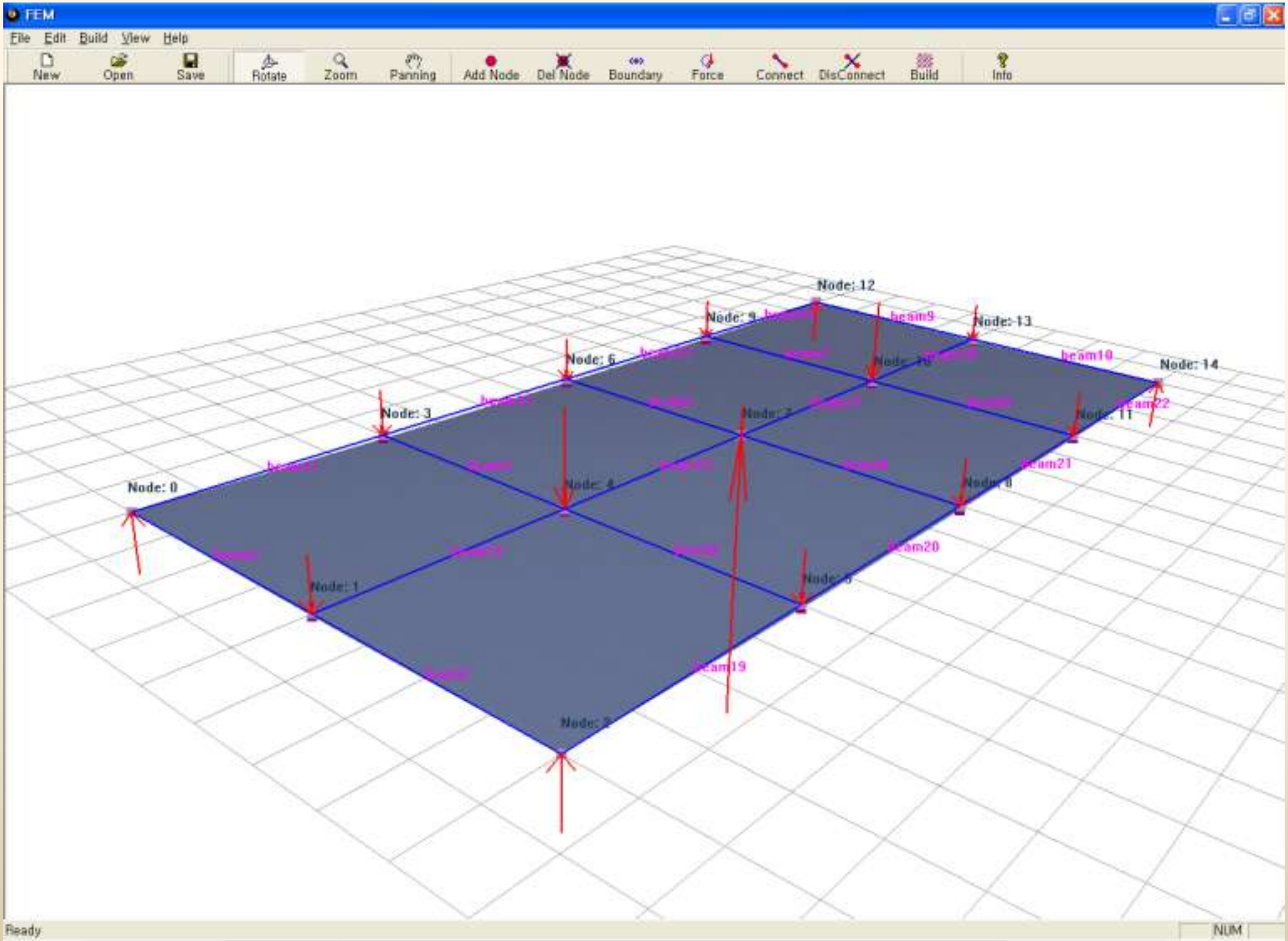
Example 1 : Grillage Analysis



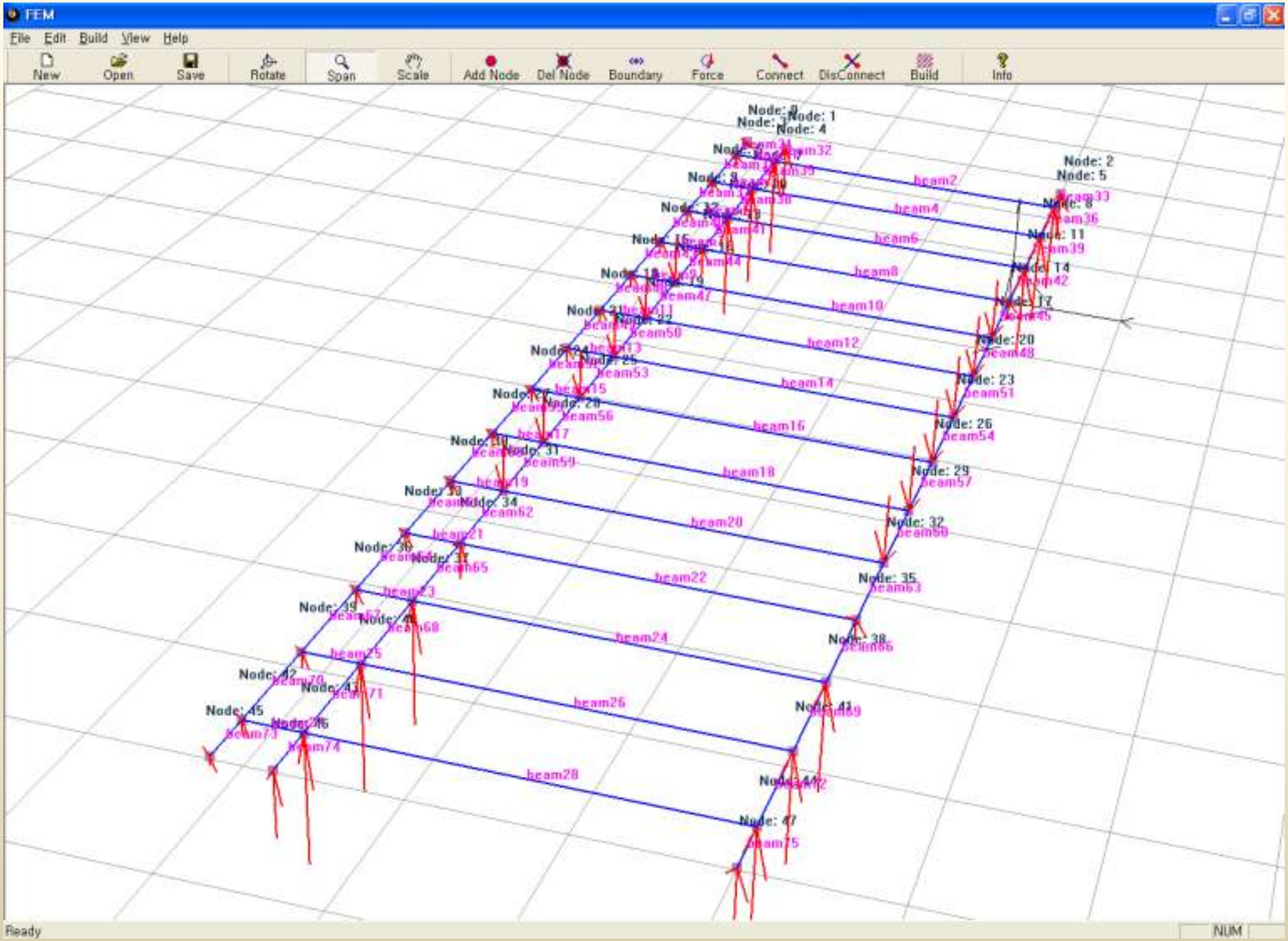
Example 1 : Grillage Analysis



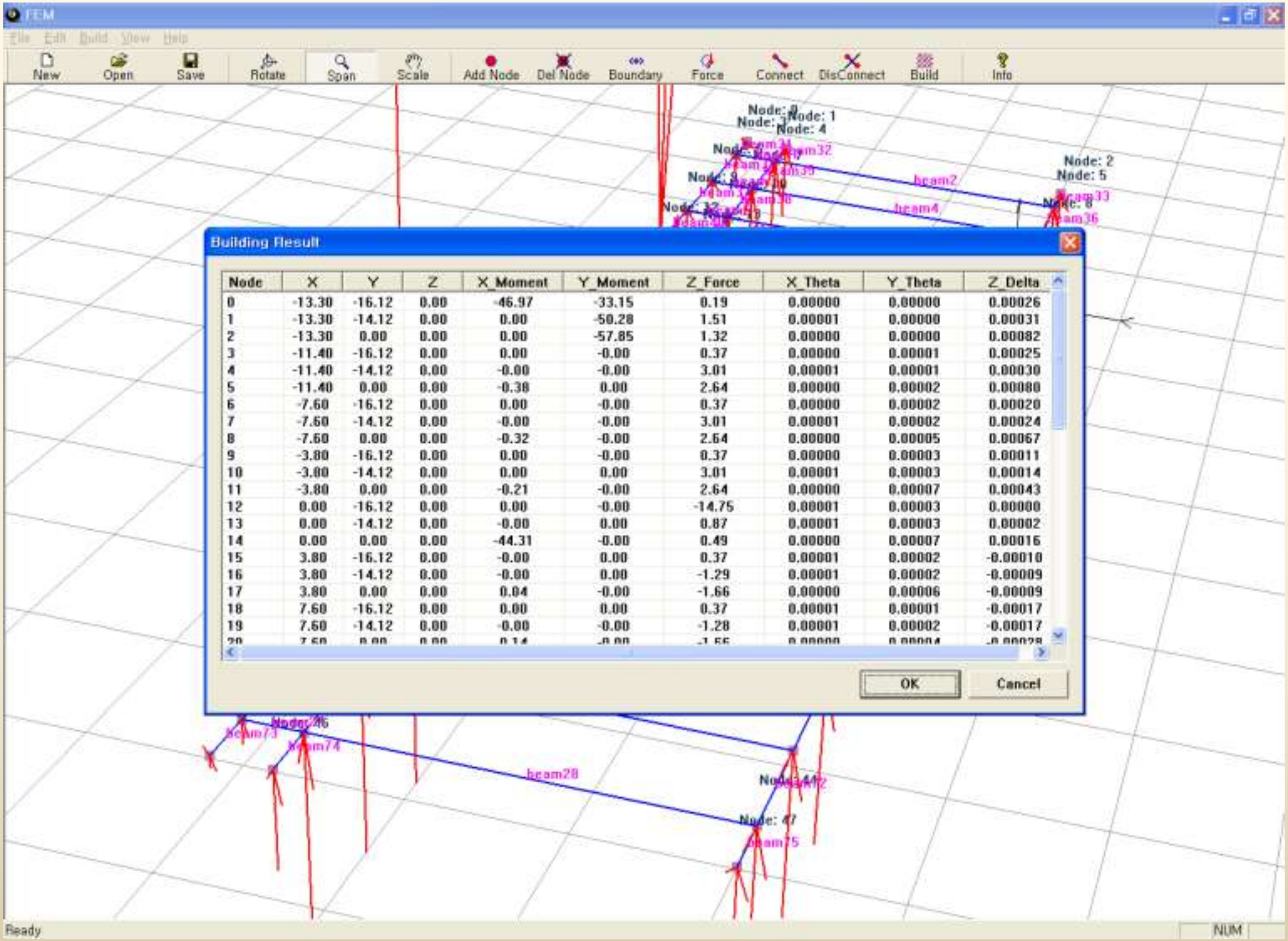
Example 2 : Grillage Analysis



Example 2 : Midship Cargo Hold Grillage Analysis



Example 2 : Midship Cargo Hold Grillage Analysis



Node	X	Y	Z	X_Moment	Y_Moment	Z_Force	X_Theta	Y_Theta	Z_Delta
0	-13.30	-16.12	0.00	-46.97	-33.15	0.19	0.00000	0.00000	0.00026
1	-13.30	-14.12	0.00	0.00	-50.28	1.51	0.00001	0.00000	0.00031
2	-13.30	0.00	0.00	0.00	-57.85	1.32	0.00000	0.00000	0.00082
3	-11.40	-16.12	0.00	0.00	-0.00	0.37	0.00000	0.00001	0.00025
4	-11.40	-14.12	0.00	-0.00	-0.00	3.01	0.00001	0.00001	0.00030
5	-11.40	0.00	0.00	-0.38	0.00	2.64	0.00000	0.00002	0.00080
6	-7.60	-16.12	0.00	0.00	-0.00	0.37	0.00000	0.00002	0.00020
7	-7.60	-14.12	0.00	-0.00	-0.00	3.01	0.00001	0.00002	0.00024
8	-7.60	0.00	0.00	-0.32	-0.00	2.64	0.00000	0.00005	0.00067
9	-3.80	-16.12	0.00	0.00	-0.00	0.37	0.00000	0.00003	0.00011
10	-3.80	-14.12	0.00	0.00	0.00	3.01	0.00001	0.00003	0.00014
11	-3.80	0.00	0.00	-0.21	-0.00	2.64	0.00000	0.00007	0.00043
12	0.00	-16.12	0.00	0.00	-0.00	-14.75	0.00001	0.00003	0.00000
13	0.00	-14.12	0.00	-0.00	0.00	0.87	0.00001	0.00003	0.00002
14	0.00	0.00	0.00	-44.31	-0.00	0.49	0.00000	0.00007	0.00016
15	3.80	-16.12	0.00	-0.00	0.00	0.37	0.00001	0.00002	-0.00010
16	3.80	-14.12	0.00	-0.00	0.00	-1.29	0.00001	0.00002	-0.00009
17	3.80	0.00	0.00	0.04	-0.00	-1.66	0.00000	0.00006	-0.00009
18	7.60	-16.12	0.00	0.00	0.00	0.37	0.00001	0.00001	-0.00017
19	7.60	-14.12	0.00	-0.00	-0.00	-1.28	0.00001	0.00002	-0.00017

Example 2 : Midship Cargo Hold Grillage Analysis