CHAPTER 8. Transmission Lines

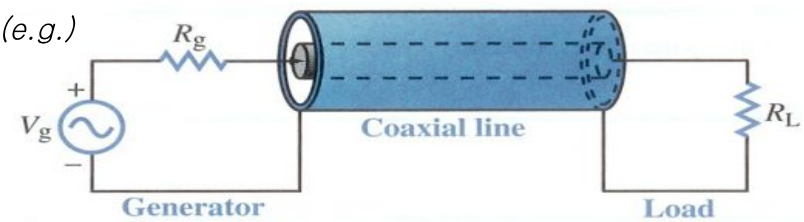
Reading assignments: Cheng Ch.8, Ulaby Ch.7, Hayt Ch.11

1. Transmission-Line Types and General Equations

A. Types of Transmission Lines

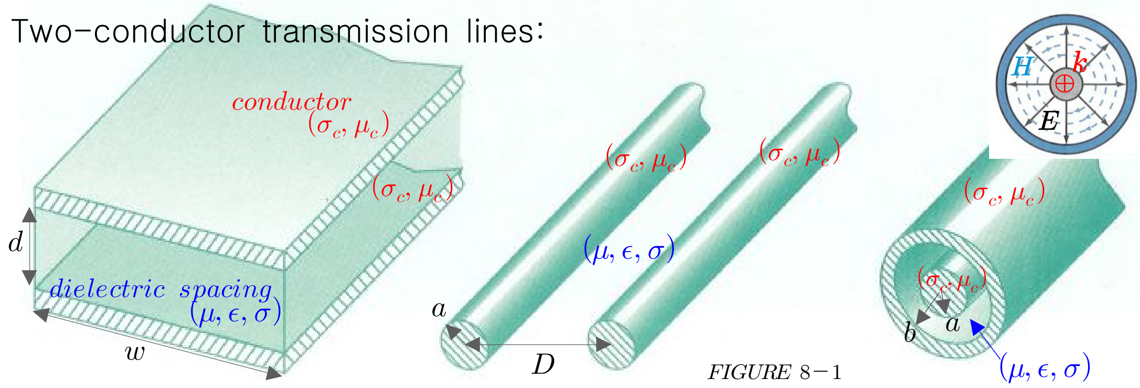
Transmission line = A four-terminal device (a two-port network) for transmitting or guiding power (for lighting, heating, or performing work) and information (audio, video, data, signal, etc) from a generator to a load

(e.g.)

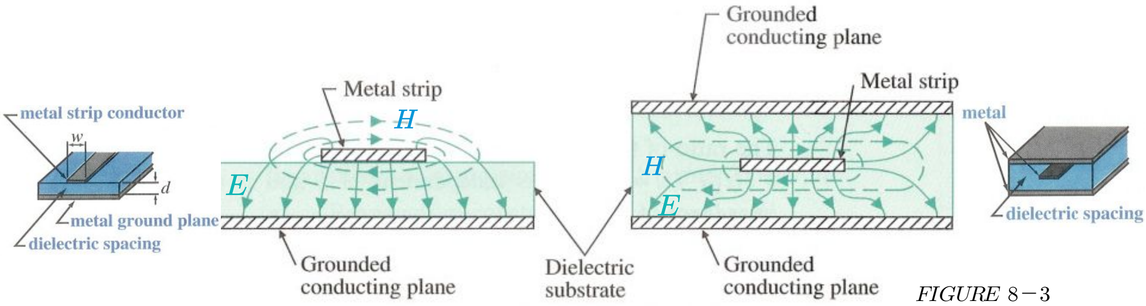


1) TEM-mode type [Both $\boldsymbol{E}$ and $\boldsymbol{H}$ are transverse ($\boldsymbol{E},\boldsymbol{H} \perp \boldsymbol{k}$ or β)}

Two-conductor transmission lines:



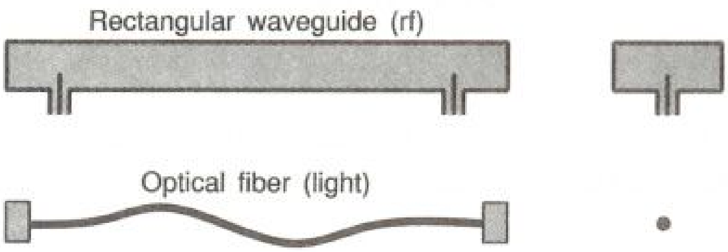
(a) Parallel-plate transmission line. (b) Two-wire transmission line. (c) Coaxial transmission line.



(d) Microstrip lines

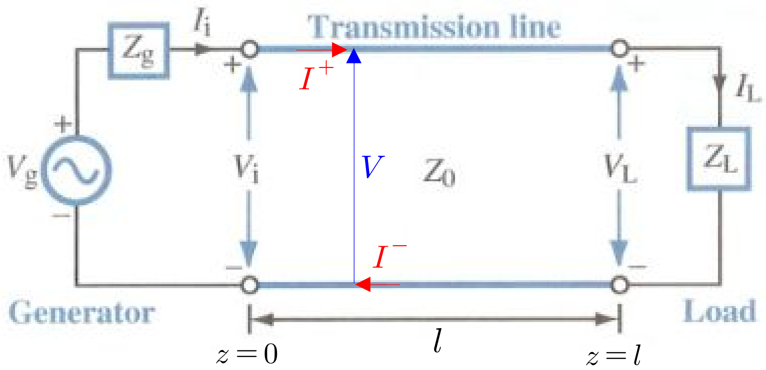
2) Higher-mode type [$\boldsymbol{E}$ and/or $\boldsymbol{H}$ have components of transmission direction
(TE or TM mode, $\boldsymbol{E}$ or $\boldsymbol{H} \parallel \boldsymbol{k}$ or β)]

Hollow single-conductor waveguides (rectangular, circular, ...),
Optical fibers, Dielectric rods



3) TEM space waves between antennas of a radio link

B. Circuit Model for a Two-Conductor Transmission Line



Voltage between conductors and current along the lines are closely related with TEM fields by

$$V = -\int \boldsymbol{E} \cdot d\boldsymbol{l} \qquad \text{(3-28)}$$

$$I = \oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int_S \boldsymbol{J} \cdot d\boldsymbol{s} \qquad \text{(5-63) (4-5)}$$

1) Lumped circuit analysis : Two-terminal systems

If $L_{syst.}$ or $l \ll \lambda$, no appreciable change of $V(z)$, $I(z)$ at any z.

no reflected signal, no standing waves

$\Rightarrow$ can be represented by discrete lumped parameters (*R, L, C*)

- localized energy dissipation (*R*)
- localized magnetic energy storage (*L*)
- localized electric energy storage (*C*)

2) Distributed circuit analysis : Four-terminal systems

If $L_{syst.}$ or $l \gg \lambda$,

$\cos(\omega t - kl) = \cos(\omega t - 2\pi \frac{l}{\lambda})$ $\Rightarrow$ appreciable phase shift at the load

Reflected signals (Standing waves), Power loss on the line,

Dispersive (distortion) effects by different u_p

$\Rightarrow$ completely described by distributed circuit parameters, whose value per unit length are constant everywhere on the line.

Transmission-line parameters:

- R = series resistance of both conductors per unit length (Ω/m)
- L = series inductance of both conductors per unit length (H/m)
- G = shunt conductance of dielectric medium per unit length (S/m)
- C = shunt capacitance of two conductors per unit length (F/m)

Equivalent circuit of a differential length $\triangle z$:

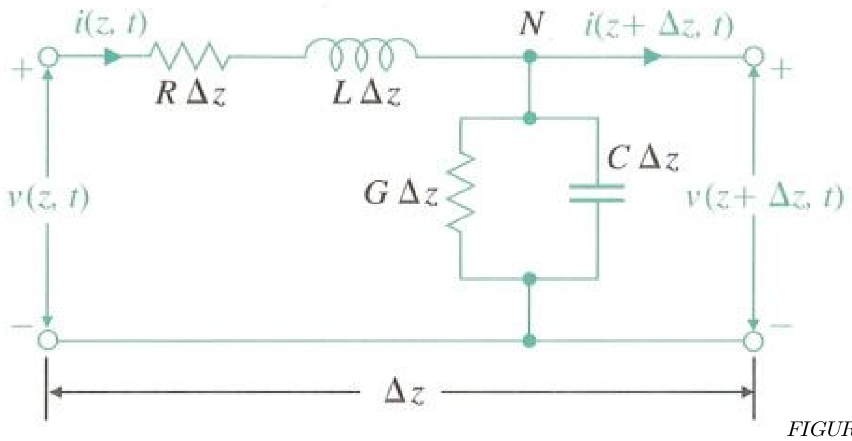


FIGURE 8−2

C. *Transmission-Line Equations*

1) General transmission-line equations

Kirchhoff's voltage law

$$\Rightarrow \quad v(z,t) - R\triangle z\, i(z,t) - L\triangle z \frac{\partial i(z,t)}{\partial t} - v(z+\triangle z, t) = 0 \qquad (8\text{-}1)$$

$\lim_{\triangle z \to 0} \frac{(8-1)}{\triangle z}$

$$\Rightarrow \quad -\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} \qquad (8\text{-}3)$$

Kirchhoff's current law at N

$$\Rightarrow \quad i(z,t) - G\triangle z\, v(z+\triangle z, t) - C\triangle z \frac{\partial v(z+\triangle z, t)}{\partial t} - i(z+\triangle z, t) = 0 \qquad (8\text{-}4)$$

$\lim_{\triangle z \to 0} \frac{(8-4)}{\triangle z}$

$$\Rightarrow \quad -\frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C\frac{\partial v(z,t)}{\partial t} \qquad (8\text{-}5)$$

2) Time-harmonic transmission-line equations

For cosine-reference time-harmonic dependence,

$$v(z,t) = Re[V(z)e^{j\omega t}],\ \ i(z,t) = Re[I(z)e^{j\omega t}] \qquad \text{(8-6, 8-7)}$$

$\frac{\partial}{\partial t} \rightarrow j\omega$ in (8-3) and (8-5):

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z) = ZI(z) \qquad (\leftarrow \text{Faraday's law}) \qquad \text{(8-8)}$$

$$-\frac{dI(z)}{dz} = (G + j\omega C)V(z) = YV(z) \qquad (\leftarrow \text{Ampere's law}) \qquad \text{(8-9)}$$

where

reactance

$$Z \equiv R + j\omega L = R + jX \ : \text{series impedance} \qquad (1)$$

$$Y \equiv G + j\omega C = G + jB \ : \text{shunt admittance} \qquad (2)$$

succeptance

Combining (8-8) and (8-9) yields Helmholtz-type equations:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \qquad [\leftarrow \text{(7-45b)}] \qquad \text{(8-10)}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \qquad \text{(8-11)}$$

where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$ (m^{-1})[←(7-43)] (8-12)

D. Transmission-Line Parameters

R, L, G, C in (8-12) are functions of physical dimensions (d, w, a, D, b) and constitutive parameters $(\mu, \epsilon, \sigma, \mu_c, \sigma_c)$.

For $\sigma_c \rightarrow \infty$ in usual cases, $R \rightarrow 0$ and TEM wave along the transmission line. Then, (8-12) becomes

$$\gamma = j\omega\sqrt{LC}\left(1 + \frac{G}{j\omega C}\right)^{1/2} \qquad \text{(8-13)}$$

For a TEM wave in a medium with (μ, ϵ, σ), from (7-43)

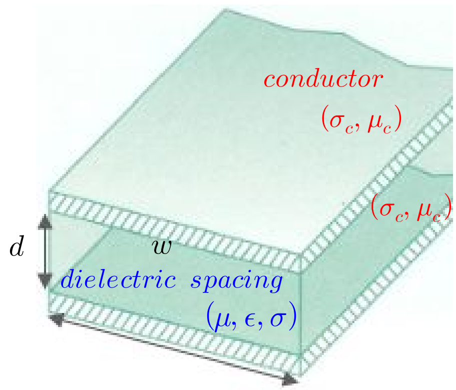
$$\gamma = j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j\omega\epsilon}\right)^{1/2} \qquad \text{(8-14)}$$

Using (4-38), $\frac{G}{C} = \frac{\sigma}{\epsilon}$ (8-15)

comparison of (8-13) with (8-14) gives

$$LC = \mu\epsilon \qquad \text{(8-16)}$$

1) Parallel-plate transmission line



(3-87) $\Rightarrow$ $C = \epsilon w/d$ (8-17)

(8-17) in (8-16, 15):

$$L = \mu d/w \quad (8\text{-}18)$$

$$G = \sigma w/d \quad (8\text{-}19)$$

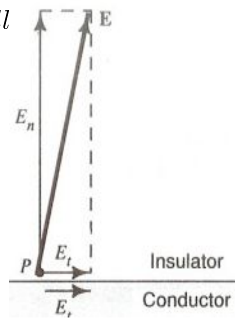
For good conductor (not strictly TEM),

(4-16) $\Rightarrow$ $R = 1/\sigma_c S$ (7-56)

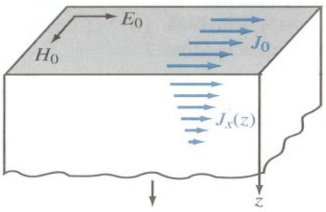
$$\Rightarrow R = \frac{2}{\sigma_c w\delta} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{w}R_s \quad (8\text{-}22)$$

from which $\eta_c = (1+j)\sqrt{\dfrac{\pi f \mu_c}{\sigma_c}} \equiv R_s + jX_s$ (7-53)

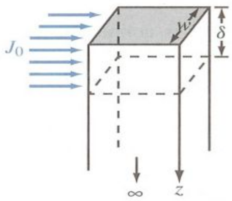
Notes) i) $E_{1n} \gg E_{1t}$ since $E_{1t} = E_{2t} = \dfrac{J_{2t}}{\sigma_2} = \dfrac{finite}{large} \rightarrow small$



ii) Skin depth δ

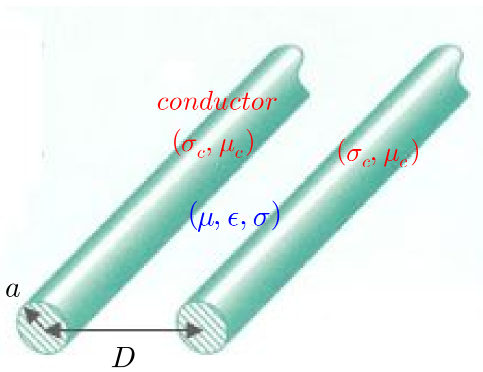


(a) Exponentially decaying $J_x(z)$



(b) Equivalent J_0 over skin depth δ

2) Two-wire transmission line



(3-165) $\Rightarrow$ $C = \dfrac{\pi\epsilon}{\cosh^{-1}(D/2a)}$ (8-23)

(8-23) in (8-16, 15):

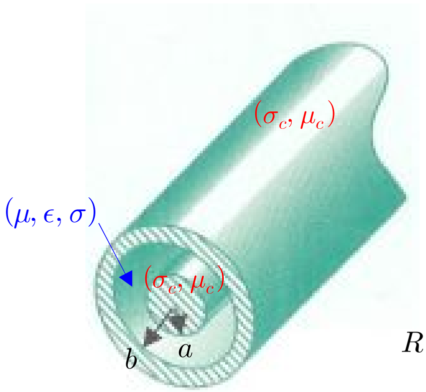
$$L = \frac{\mu}{\pi}cosh^{-1}\left(\frac{D}{2a}\right) \quad (8\text{-}24)$$

$$G = \frac{\pi\sigma}{\cosh^{-1}(D/2a)} \quad (8\text{-}25)$$

$$R = \frac{2}{\sigma_c(2\pi a\delta)} = \frac{1}{\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{R_s}{\pi a} \quad (8\text{-}27)$$

equivalent one conducting wire

3) Coaxial transmission line



(3-90) $\Rightarrow$ $$C = \frac{2\pi\epsilon}{\ln(b/a)} \quad (8\text{-}28)$$

(8-28) in (8-16, 15):

$$L = \frac{\mu}{2\pi} ln \frac{b}{a} \quad (8\text{-}29)$$

$$G = \frac{2\pi\sigma}{\ln(b/a)} \quad (8\text{-}30)$$

$$R = \frac{1}{\sigma_c(2\pi a\delta)} + \frac{1}{\sigma_c(2\pi b\delta)} = \frac{1}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)\sqrt{\frac{\pi f\mu_c}{\sigma_c}} \quad (8\text{-}32)$$

$$= \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$$

TABLE 8-1 Summary of Distributed Transmission-Line Parameters

Parameter	**Parallel-Plate Line**	**Two-Wire Line**	**Coaxial Line**	**Unit**
R	$\frac{2}{w}R_s$	$\frac{R_s}{\pi a}$	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	Ω/m
L	$\mu\frac{d}{w}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	H/m
G	$\sigma\frac{w}{d}$	$\frac{\pi\sigma}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\sigma}{\ln(b/a)}$	S/m
C	$\epsilon\frac{w}{d}$	$\frac{\pi\epsilon}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\epsilon}{\ln(b/a)}$	F/m

Note: $R_s = \sqrt{\pi f\mu_c/\sigma_c}$; $\cosh^{-1}(D/2a) \cong \ln(D/a)$ if $(D/2a)^2 \gg 1$. Internal inductance is not included.