No Class on Thursday (March 18)!

노트 제목 2010-03-16

$$\sigma_{ij} = -p\delta_{ij} + \lambda\delta_{ij}\frac{\partial u_k}{\partial x_k} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad - (*)$$

μ ↳ viscosity

λ: second viscosity coefficient

average normal stress $\bar{p} = -\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$

↳ mechanical pressure $\neq$ thermo. press

$$(*) \rightarrow -\bar{p} = \frac{1}{3}\left[-3p + 3\lambda\frac{\partial u_k}{\partial x_k} + 2\mu\frac{\partial u_k}{\partial x_k}\right]$$

$$= -p + (\lambda + \tfrac{2}{3}\mu)\frac{\partial u_k}{\partial x_k}$$

$$\therefore \underbrace{p - \bar{p}}_{\text{difference bet. thermo and mech pressures}} = (\lambda + \tfrac{2}{3}\mu)\frac{\partial u_k}{\partial x_k} \equiv K\frac{\partial u_k}{\partial x_k}$$

K ↳ bulk viscosity

$\bar{p}$: measure of translational energy of molecules

p : " " "

+ vibrational " " "

+ rotational

→ bulk viscosity : measure of energy transfer from translational mode to other modes

e.g. during the passage of shock wave, ↗ $K \neq 0$

vib. modes are excited at the expense of trans. mode

If fluid is a mono-atomic gas,
only translational mode exists. $\rightarrow p = \bar{p} \rightarrow \kappa = 0$

$\rightarrow \lambda = -\frac{2}{3}\mu$
Stokes relation

For incomp. flow, $\frac{\partial u_k}{\partial x_k} = 0 \rightarrow p = \bar{p}$

- Navier–Stokes eqs : mtm conservation + constitutive eq.

$$\rho \frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} = \frac{\partial \sigma_{ij}}{\partial x_i} + \rho f_j$$

$$\sigma_{ij} = -p\delta_{ij} + \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad \text{N-S eq.}$$

$$\rightarrow \boxed{\rho \frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial u_k}{\partial x_k}\right) + \frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \rho f_j}$$

For incomp. flow and μ = const, $\left(\partial u_k / \partial x_k = 0\right)$

$$\boxed{\rho \frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \mu \frac{\partial^2 u_j}{\partial x_i \partial x_i} + \rho f_j}$$

When $\mu = 0$ (inviscid),

$$\boxed{\rho \frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \rho f_j} \quad \text{Euler eq.}$$

- Energy equation

$$\rho \frac{\partial e}{\partial t} + \rho u_k \frac{\partial e}{\partial x_k} = \sigma_{ij} \frac{\partial u_j}{\partial x_i} - \frac{\partial q_j}{\partial x_j}$$

$$\sigma_{ij} \frac{\partial u_j}{\partial x_i} = \left[-p\delta_{ij} + \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \frac{\partial u_j}{\partial x_i}$$

$$= -p \frac{\partial u_k}{\partial x_k} + \lambda \left(\frac{\partial u_k}{\partial x_k}\right)^2 + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \frac{\partial u_j}{\partial x_i}$$

$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0$ energy transfer due to compression

Φ : dissipation (function)

For incomp. flow,

$$\Phi = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i}$$

$$= \mu\left(\quad 〃 \quad \right)\cdot\left[\frac{1}{2}\left(\; 〃 \; \right) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)\right]$$

$$= \frac{1}{2}\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 \geq 0$$

$\therefore$ Φ always works to increase irreversibly the internal energy of incomp. flow.

$$\therefore \boxed{\rho\frac{\partial e}{\partial t} + \rho u_k \frac{\partial e}{\partial x_k} = -p\frac{\partial u_k}{\partial x_k} + \frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) + \Phi}$$ energy eq.

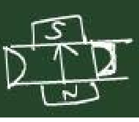
• Governing eqs. for Newtonian fluids

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k}(\rho u_k) = 0 \\ \rho\frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\lambda\frac{\partial u_k}{\partial x_k}\right) + \frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \rho f_j \\ \rho\frac{\partial e}{\partial t} + \rho u_k \frac{\partial e}{\partial x_k} = -p\frac{\partial u_k}{\partial x_k} + \frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) + \lambda\left(\frac{\partial u_k}{\partial x_k}\right)^2 + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i} \\ p = p(\rho, T) \\ e = e(\rho, T) \end{cases}$$

unit vector in z direction

f_j ? gravity $f_j = -g\,\underline{e}_z$

electric current density



Lorentz force $\underline{f} = \rho_e \underline{E} + \underline{J} \times \underline{B}$

electric field vector

magnetic field vector

relation bet. $\underline{E}$ & $\underline{B}$ → Maxwell eq.

enthalpy $h = e + p/\rho$

for $u_j = 0$ and perfect fluid, $\rho \frac{\partial e}{\partial t} = \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right)$

(p & ρ are const.) $\rightarrow \rho \frac{\partial h}{\partial t} =$ "

($h = c_p T$) $\rightarrow \rho c_p \frac{\partial T}{\partial t} =$ "

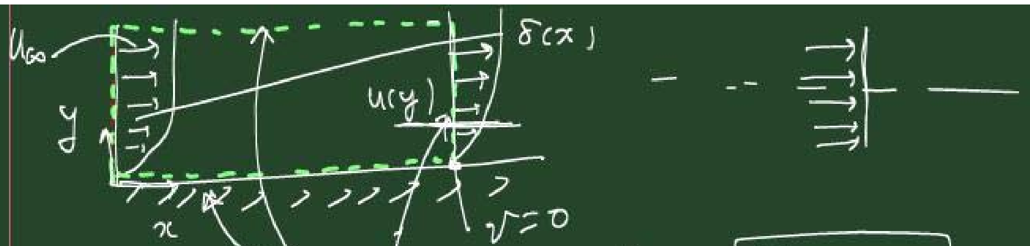
heat conduction eq.

• Boundary conditions

no-slip : $\underline{u} = \underline{U}$ on solid boundaries

as $\underline{x} \rightarrow \infty$: $\underline{u} \rightarrow 0$ (far-field cond.)

$\underline{u} \rightarrow \underline{u}_\infty$



$\underline{u=0}$

$(u=v=0)$

$u = U_\infty$, $\boxed{\frac{\partial u}{\partial y} = 0}$, $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$

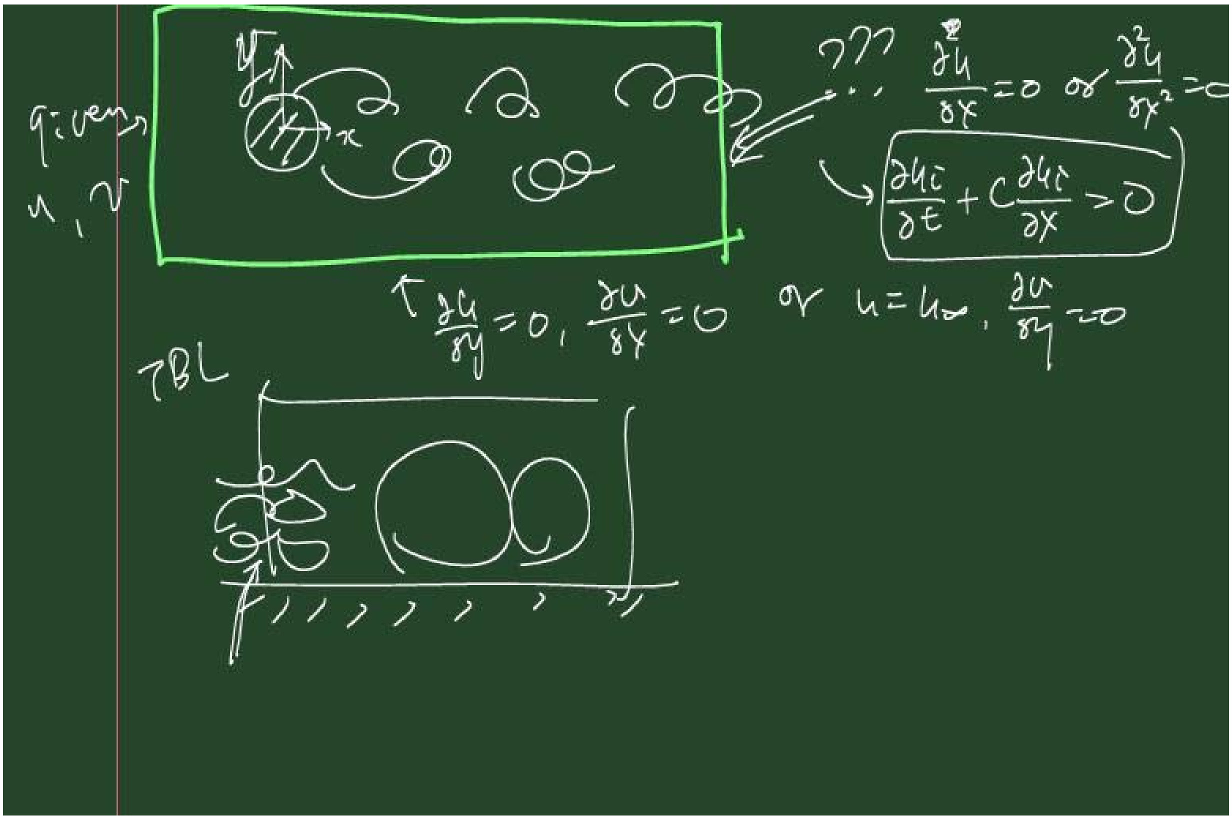
$\frac{\partial v}{\partial y} = 0$ $\quad \cancel{\frac{\partial v}{\partial y} = 0}$? $\uparrow \; \neq 0$

$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} > 0$

$u = \cancel{0}$ ← impossible

$\frac{\partial u}{\partial x} = 0 \rightarrow \frac{\partial v}{\partial y} = 0 \rightarrow v \equiv 0$ @ exit X

$\frac{\partial^2 u}{\partial x^2} = 0 \quad \frac{\partial^2 v}{\partial x^2} = 0$


given
u, v
???
$\frac{\partial u}{\partial x} = 0$ or $\frac{\partial^2 u}{\partial x^2} = 0$
$\frac{\partial u_i}{\partial t} + c\frac{\partial u_i}{\partial x} = 0$
$\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0$ or $u = u_\infty, \frac{\partial v}{\partial y} = 0$
TBL