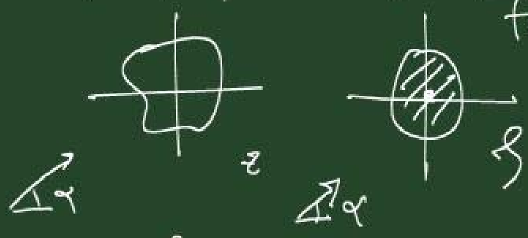
Mid-term exam : $\boxed{\text{up to ch. 4}}$ $\boxed{\text{Apr. 27}}$ 18:30 –

(Tuesday)

노트 제목 2010-04-13

- Joukowski transformation : $z = \zeta + \frac{c^2}{\zeta}$

For large $|\zeta|$, $z \to \zeta$: identity mapping far from the origin

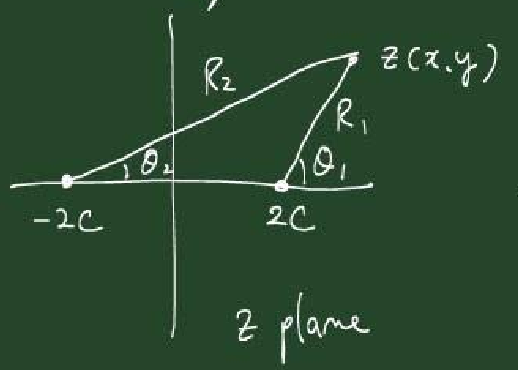


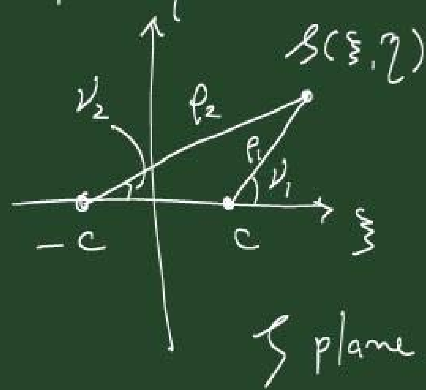
$$\frac{dz}{d\zeta} = 1 - \frac{c^2}{\zeta^2}$$

$\zeta = 0$: singular pt.

$\zeta = \pm c$; $\frac{dz}{d\zeta} = 0$; critical pts

$$z = \zeta + \frac{c^2}{\zeta}$$





$\zeta = c \to z = 2c$

$\zeta = -c \to z = -2c$

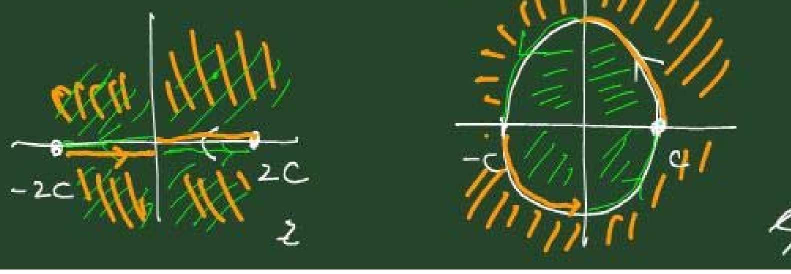
$$z + 2c = \frac{(\zeta + c)^2}{\zeta^2}, \quad z - 2c = \frac{(\zeta - c)^2}{\zeta^2}$$

$$\rightarrow \frac{z-2c}{z+2c} = \left(\frac{\zeta-c}{\zeta+c}\right)^2 \rightarrow \frac{R_1 e^{i\theta_1}}{R_2 e^{i\theta_2}} = \left(\frac{\rho_1 e^{i\nu_1}}{\rho_2 e^{i\nu_2}}\right)^2$$

$$\rightarrow \frac{R_1}{R_2} e^{i(\theta_1-\theta_2)} = \left(\frac{\rho_1}{\rho_2}\right)^2 e^{i2(\nu_1-\nu_2)}$$

$$\rightarrow \frac{R_1}{R_2} = \left(\frac{\rho_1}{\rho_2}\right)^2, \quad \theta_1-\theta_2 = 2(\nu_1-\nu_2)$$

- on $\zeta = c e^{i\nu}$, $\rightarrow z = \zeta + \frac{c^2}{\zeta} = c e^{i\nu} + c e^{-i\nu} = 2c\cos\nu$
  real



→ If a smooth curve passes through the point $\zeta = c$, the corresponding curve in the z plane will form a knife-edge.

→ transformed area shows that the mapping is double-valued. ⇒ this does not cause any problem in fluid mechanics because $|\zeta| < c$ locates inside the body.

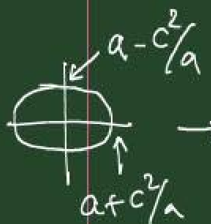
- Flow around an ellipse.

$$z = \zeta + \frac{c^2}{\zeta}$$

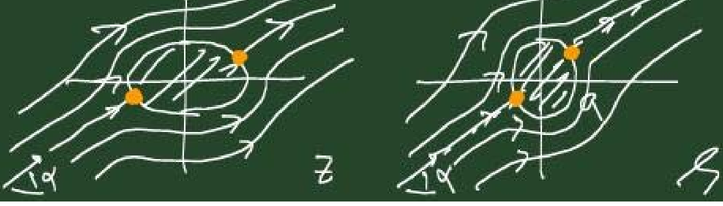
$$\zeta = a e^{i\nu} \quad (a > c)$$

$$\rightarrow z = a e^{i\nu} + \frac{c^2}{a} e^{-i\nu} = \left(a + \frac{c^2}{a}\right)\cos\nu + i\left(a - \frac{c^2}{a}\right)\sin\nu$$

$$= \quad x \quad + i\, y$$



$$\rightarrow \left(\frac{x}{a + c^2/a}\right)^2 + \left(\frac{y}{a - c^2/a}\right)^2 = 1 \quad ; \text{ellipse}$$



$$\Downarrow$$

$$F(\zeta) = U\left(\zeta e^{-i\alpha} + \frac{a^2}{\zeta} e^{i\alpha}\right)$$

$$z = \zeta + \frac{c^2}{\zeta}$$

Joukowski trans. $\zeta^2 - z\zeta + c^2 = 0$

$$\rightarrow \zeta = \frac{z}{2} \pm \sqrt{\left(\frac{z}{2}\right)^2 - c^2}$$

For large z, $\zeta \rightarrow z \Rightarrow$ eliminate '−'!

$$\therefore \zeta = \frac{z}{2} + \sqrt{\left(\frac{z}{2}\right)^2 - c^2}$$

$$F(z) = U\left[z e^{-i\alpha} + \left(\frac{a^2}{c^2} e^{i\alpha} - e^{-i\alpha}\right)\left(\frac{z}{2} - \sqrt{\left(\frac{z}{2}\right)^2 - c^2}\right)\right]$$

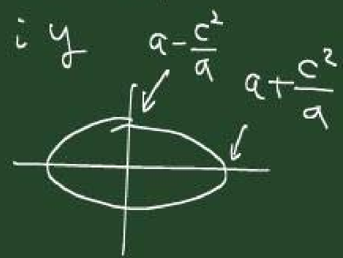
$\rightarrow$

stag. points in the ζ plane. $\zeta_s = ae^{i\alpha}$ and $ae^{i(\alpha+\pi)}$

$$\rightarrow \zeta_s = \pm a e^{i\alpha}$$

$$z = \zeta + \frac{c^2}{\zeta} = \pm\left(a+\frac{c^2}{a}\right)\cos\alpha \pm i\left(a-\frac{c^2}{a}\right)\sin\alpha$$
$$= \quad x \quad + \quad iy$$

$$\Rightarrow \begin{cases} x_s = \pm\left(a+\frac{c^2}{a}\right)\cos\alpha \\ y_s = \pm\left(a-\frac{c^2}{a}\right)\sin\alpha \end{cases}$$



⑥ Kutta condition

Potential flow sol. for flow around a sharp edge.

→ singularity at the edge itself

→ infinite velocity at the pt.

→ nonphysical.

We have to mimic the real flow situation

① finite radius of curvature at the edge

② stagnation pt. exists at the sharp edge

↳ Kutta condition

ex. flat-plate airfoil

ellipse $\zeta = a e^{i\nu}$ $(a>c)$ 

$a \to c$: ellipse $\to$ flat plate

$-2a \le x \le 2a$

stag. pts. $\to$ $x_s = \pm 2a \cos\alpha$, $y_s = 0$

infinite vel. at these pts. $(x = \pm 2a)$

$\to$ non-physical

Real airfoil 

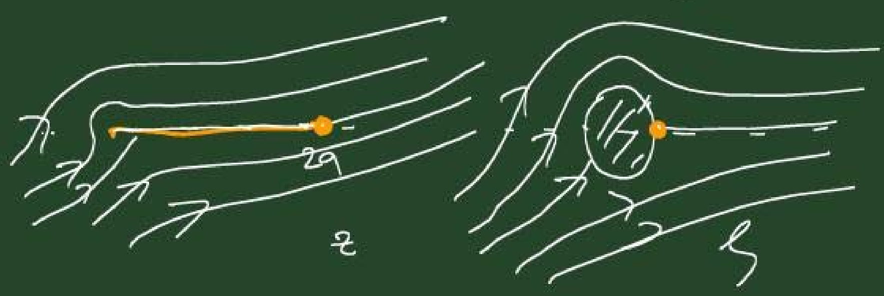
(leading edge $\to$ finite radius of curvature

(trailing " $\to$ sharp

$\to$ We have to work on trailing edge.

Kutta condition: For bodies with sharp trailing edges which are at small angles of attack to the free stream, the flow will adjust itself in such a way that the rear stag. pt. coincides with the trailing edge.

$\to$ We need a circulation to satisfy the Kutta cond.



previously stag. pt is $\zeta_s = ae^{i\alpha}$

According to Kutta cond. $z = 2a$

$\rightarrow \zeta_s = a$

$\Rightarrow$ ζ_s should be rotated clockwise through an angle α.

$\sin\theta_s = -\dfrac{\Gamma}{4\pi U a}$

$\Rightarrow \Gamma = 4\pi U a \sin\alpha$

$$\Rightarrow F(\zeta) = U\left(\zeta e^{-i\alpha} + \frac{a^2}{\zeta}e^{i\alpha}\right) + i2Ua\sin\alpha \ln\frac{\zeta}{a}$$

$\dfrac{i\Gamma}{2\pi} = 4\pi U a \sin\alpha$

$$\zeta = \frac{z}{2} + \sqrt{\left(\frac{z}{2}\right)^2 - a^2}$$

$$\Rightarrow F(z) = U\left\{\left[\frac{z}{2} + \sqrt{\left(\frac{z}{2}\right)^2 - a^2}\right]e^{-i\alpha} + \frac{a^2 e^{i\alpha}}{\frac{z}{2} + \sqrt{\left(\frac{z}{2}\right)^2 - a^2}} + i\,2a\sin\alpha \ln\left[\frac{1}{a}\left(\frac{z}{2} + \sqrt{\left(\frac{z}{2}\right)^2 - a^2}\right)\right]\right\}$$

Kutta-Joukowski Law

$Y = \rho U \Gamma = 4\pi\rho U^2 a \sin\alpha$: lift force

$$C_L = \frac{Y}{\frac{1}{2}\rho U^2 (4a)} = 2\pi\sin\alpha \simeq 2\pi\alpha$$

lift coeff.

very close to exp. results.