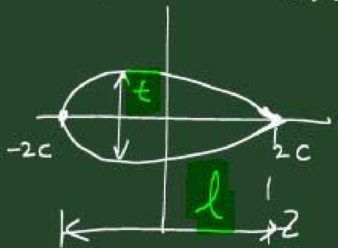
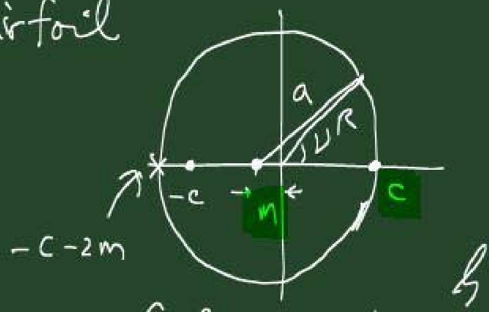
노트 제목 2010-04-15

- Symmetric Joukowski airfoil





Joukowski family of airfoils are obtained from a series of circles in the ζ plane whose centers are slightly displaced from the origin.

$z = \zeta + \frac{c^2}{\zeta}$

radius $a = c + m = c\left(1 + \frac{m}{c}\right) = c(1+\varepsilon)$

$\varepsilon \ll 1 \rightarrow$ thin airfoil

$\zeta = c \rightarrow z = 2c$

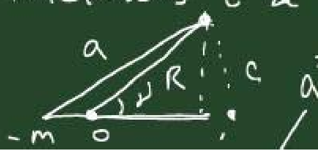
$\zeta = -(c+2m) = -c(1+2\varepsilon)$

$\rightarrow z = -c(1+2\varepsilon) - \frac{c}{1+2\varepsilon}$

$= -c(1+\cancel{2\varepsilon}) - c\left(1 - \cancel{2\varepsilon} + O(\varepsilon^2)\right) = -2c + O(\varepsilon^2)$

$\therefore l = 4c$: chord length

thickness t & locus of airfoil



$a^2 = (R\sin\nu)^2 + (m + R\cos\nu)^2$

$$(c+m)^2 = R^2 + m^2 + 2mR\cos\nu = R^2\left(1 + \cancel{\frac{m^2}{R^2}} + 2\frac{m}{R}\cos\nu\right)$$

$$\left(\frac{m}{R} \le \frac{m}{c} = \varepsilon\right)$$

$$c+m \simeq R\left(1+2\frac{m}{R}\cos\nu\right)^{\frac{1}{2}} = R\left(1+\frac{m}{R}\cos\nu + O(\varepsilon^2)\right)$$

$$c(1+\varepsilon) = R + m\cos\nu = R + c\varepsilon\cos\nu$$

$$\therefore R = c[1+\varepsilon(1-\cos\nu)]$$

On the circle $\zeta = Re^{i\nu}$

$$z = \zeta + \frac{c^2}{\zeta} = c[1+\varepsilon(1-\cos\nu)]e^{i\nu} + \frac{c\,e^{-i\nu}}{1+\varepsilon(1-\cos\nu)}$$

$$= c[1+\varepsilon(1-\cos\nu)]e^{i\nu} + c[1-\varepsilon(1-\cos\nu)+O(\varepsilon^2)]e^{-i\nu}$$

$$= c\left[\underbrace{2\cos\nu}_{x} + i\,\underbrace{2\varepsilon(1-\cos\nu)\sin\nu}_{y} + O(\varepsilon^2)\right]$$

$\rightarrow$ $x = 2c\cos\nu$

$y = 2c\varepsilon(1-\cos\nu)\sin\nu$

$\rightarrow$ $y = \pm 2c\varepsilon\left(1-\frac{x}{2c}\right)\sqrt{1-\left(\frac{x}{2c}\right)^2}$

To find max. t, $\frac{dy}{dx} = 0$ or $\frac{dy}{d\nu} = 0$

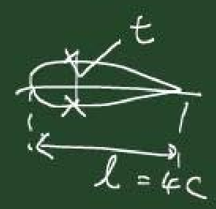
$$\frac{dy}{d\nu} = 2c\varepsilon\left(\sin^2\nu + (1-\cos\nu)\cos\nu\right) = 0$$

$$\rightarrow \cos 2\nu = \cos\nu \rightarrow \nu = 0,\ \frac{2}{3}\pi,\ \frac{4}{3}\pi$$

min max.

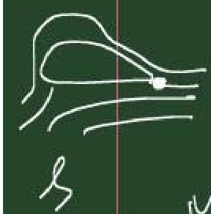
For $\nu = \frac{2}{3}\pi, \frac{4}{3}\pi \rightarrow x = -c$

$y = \pm \frac{3\sqrt{3}}{2} c\varepsilon$



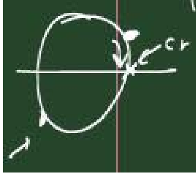
$\rightarrow t = 3\sqrt{3}\, c\varepsilon$

$\rightarrow t/l = \frac{3\sqrt{3}}{4}\varepsilon \rightarrow \varepsilon = \frac{4}{3\sqrt{3}}\frac{t}{l} = 0.77\frac{t}{l} \quad \left(\varepsilon = \frac{m}{c}\right)$



$\therefore \frac{y}{t} = \pm 0.385\left(1 - 2\frac{x}{l}\right)\sqrt{1 - \left(2\frac{x}{l}\right)^2}, \quad (l = 4c)$

Magnitude of the circulation for attack angle α (Kutta cond.)



$\sin\theta_s = -\frac{\Gamma}{4\pi U a}$

$\therefore \Gamma = 4\pi U a \sin\alpha \qquad a = c + m = c(1+\varepsilon) = \frac{l}{4}(1+\varepsilon)$

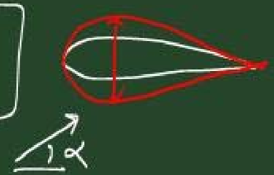
$= \pi U l\left(1 + 0.77\frac{t}{l}\right)\sin\alpha$

Then, lift force (Kutta-Joukowski law)

$Y = \rho U \Gamma$

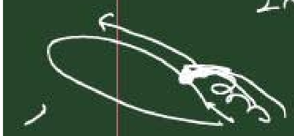
$= \pi\rho U^2 l\left(1 + 0.77\frac{t}{l}\right)\sin\alpha$

$$\boxed{C_L = \frac{Y}{\frac{1}{2}\rho U^2 l} = 2\pi\left(1 + 0.77\frac{t}{l}\right)\sin\alpha}$$



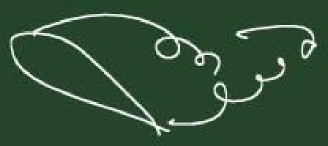
As $t\uparrow$, $C_L\uparrow$

In reality, however, separation occurs when $t\uparrow$.

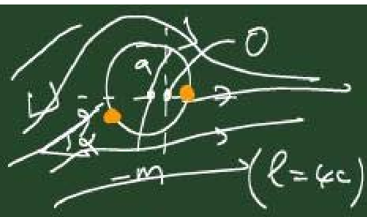


$\rightarrow$ form drag $\uparrow$

stall occurs $\longrightarrow$



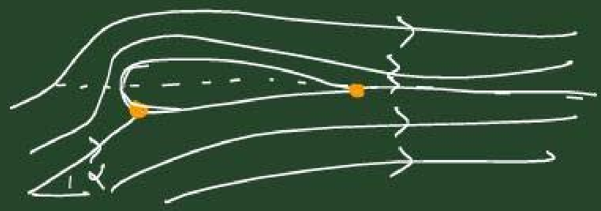
complex potential $\zeta \to \zeta + m$



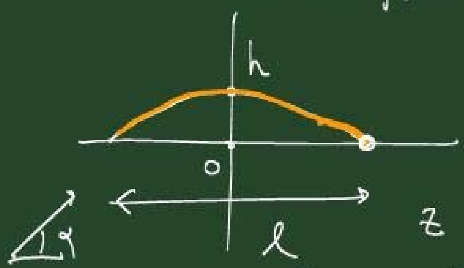
$$F(\zeta) = U\left((\zeta+m)e^{-i\alpha} + \frac{a^2}{\zeta+m}e^{i\alpha}\right) + \frac{i\Gamma}{2\pi}\ln\frac{\zeta+m}{a}$$

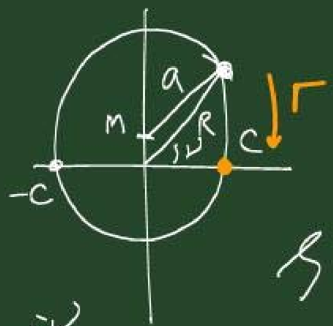
where $a = \frac{l}{4} + \underbrace{0.77\frac{tc}{l}}_{m}$

$\hookrightarrow F(z) = \cdots\cdots$



- Circular-arc airfoil





$$z = \zeta + \frac{c^2}{\zeta} = Re^{i\nu} + \frac{c^2}{R}e^{-i\nu}$$

$l = 4c$

$h = 2m$

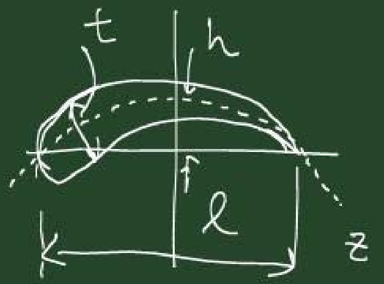
$\vdots$

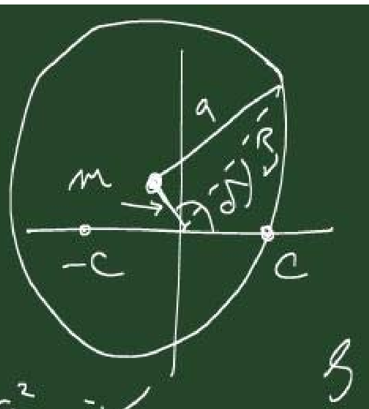
$$x^2 + \left(y + \frac{l^2}{8h}\right)^2 = \frac{l^2}{4}\left(1 + \frac{l^2}{16h^2}\right)$$

$\vdots$

$C_L = 2\pi \sin\left(\alpha + \frac{2h}{l}\right) \rightarrow$ The effect of positive camber is to increase the lift coeff. when $\alpha = 0$, $C_L \neq 0$ due to $\frac{2h}{l}$.

- Joukowski airfoil





$$z = \zeta + \frac{c^2}{\zeta} = Re^{i\nu} + \frac{c^2}{R}e^{-i\nu}$$

$$y = \sqrt{\frac{l^2}{4}\left(1+\frac{l^2}{16h^2}\right) - x^2} - \frac{l^2}{8h} \pm 0.385\, t \left(1-2\frac{x}{l}\right)\sqrt{1-\left(2\frac{x}{l}\right)^2}$$

$$C_L = 2\pi\left(1+0.77\frac{t}{l}\right)\sin\left(\alpha+\frac{2h}{l}\right)$$

complex potential $\zeta \rightarrow \zeta - me^{i\delta}$

$$F(\zeta) = U\left[(\zeta - me^{i\delta})e^{-i\alpha} + \frac{a^2 e^{i\alpha}}{\zeta - me^{i\delta}}\right] + \frac{i\Gamma}{2\pi}\ln\left(\frac{\zeta - me^{i\delta}}{a}\right).$$

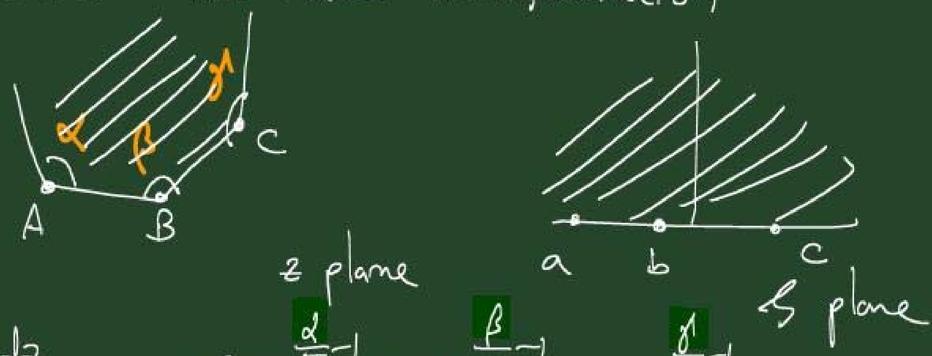
$$m\cos\delta = -0.77\frac{tc}{l}$$

$$m\sin\delta = h/2$$

$$a = l/4 + 0.77\frac{tc}{l}$$

$$\Gamma = \pi U l\left(1+0.77\frac{t}{l}\right)\sin\left(\alpha+\frac{2h}{l}\right)$$

◎ Schwarz - Christoffel Transformation



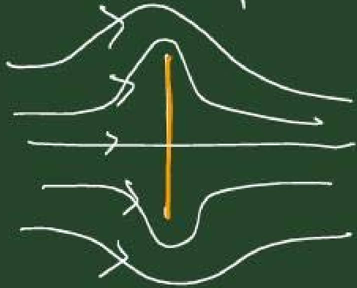
$$\frac{dz}{d\zeta} = K(\zeta - \boxed{a})^{\frac{\alpha}{\pi}-1}(\zeta - \boxed{b})^{\frac{\beta}{\pi}-1}(\zeta - \boxed{c})^{\frac{\gamma}{\pi}-1} \cdots\cdots$$

K determines the scale of the polygon and its orientation.

Constant of integration determines the location of the origin in the z plane.

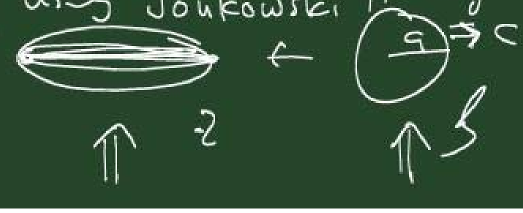
Any three constants (a, b, c, ...) may be chosen arbitrarily (-1, 0, 1) and the remaining ones will be determined by the shape of the polygon.

- Flow around a vertical flat plate (assuming non-separated flow)



vertically oriented ellipse

limiting procedure using Joukowski transf.



$$z = \zeta - \frac{b^2}{\zeta}$$

How about using Schwarz - Christoffel trans?