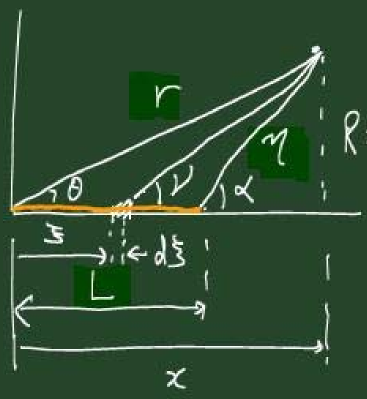
노트 제목 2010-04-27

- Line-distributed source



line source $0 \le x \le L$.

source strength q per unit length

$\therefore qL$ is the total volume.

Then, for $q\,d\xi$, $d\psi = -\frac{q\,d\xi}{4\pi}(1+\cos\nu)$

$\psi = -\frac{Q}{4\pi}(1+\cos\nu)$ $\quad \psi = -\int_0^L \frac{q\,d\xi}{4\pi}(1+\cos\nu)$

$$x-\xi = R\cot\nu \rightarrow -d\xi = -R\,\mathrm{cosec}^2\nu\,d\nu$$

$$\therefore \psi = -\frac{qR}{4\pi}\int_\theta^\alpha \mathrm{cosec}^2\nu\,(1+\cos\nu)\,d\nu$$

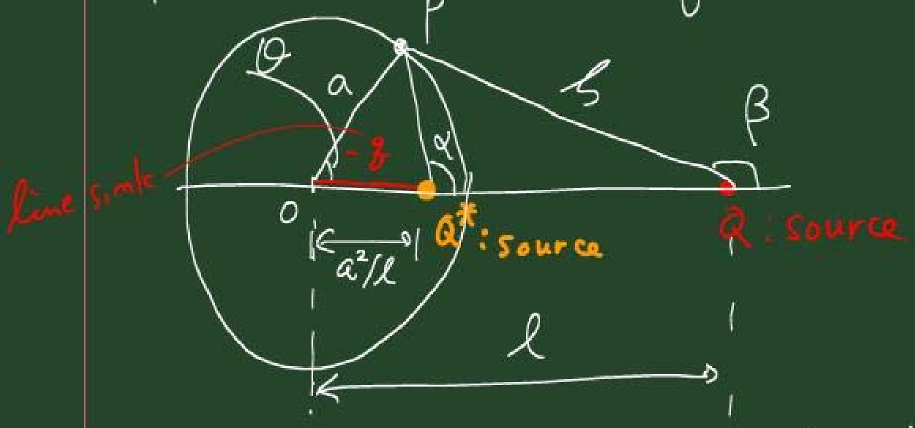
$$= -\frac{qR}{4\pi}\left(\cot\theta - \cot\alpha + \frac{1}{\sin\theta} - \frac{1}{\sin\alpha}\right)$$

$$\left(\begin{array}{ll} x = R\cot\theta, & x-L = R\cot\alpha, \\ r = R/\sin\theta, & \eta = R/\sin\alpha \end{array}\right)$$

$$= -\frac{q}{4\pi}(L+r-\eta)$$ //

Likewise, $\phi = -\int_0^L \frac{q\,d\xi}{4\pi R/\sin\nu} = -\frac{q}{4\pi}\ln\left(\frac{\tan\alpha/2}{\tan\theta/2}\right)$.

- Sphere in the flow field of source



If the surface of $r=a$ is the streamline, total sink strength inside the sphere must equal to the total source strength.

$$\rightarrow \quad q\,a^2/\ell = Q^*$$

$$\psi(r,\theta) = -\frac{Q}{4\pi}(1+\cos\beta) - \frac{Q^*}{4\pi}(1+\cos\alpha) + \frac{Q^*}{4\pi}\frac{\ell}{a^2}\left(\frac{a^2}{\ell}+r-\eta\right)$$

on the sphere $(r=a)$

$$\psi(a,\theta) = -\frac{Q}{4\pi}(1+\cos\beta) - \frac{Q^*}{4\pi}(1+\cos\alpha) + \frac{Q^*}{4\pi}\left(1+\frac{\ell}{a}-\frac{\ell\eta}{a^2}\right)$$

use the geometrical equality

⋮

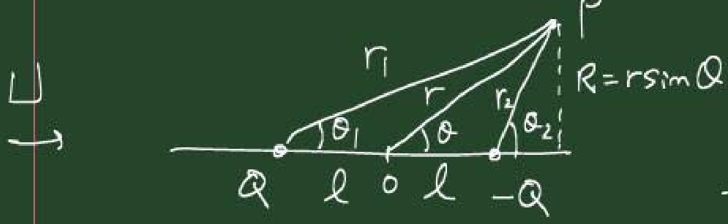
$$\rightarrow \quad \psi(a,\theta) = (1+\cos\beta)\left(-\frac{Q}{4\pi} + \frac{Q^*}{4\pi}\frac{\ell}{a}\right)$$

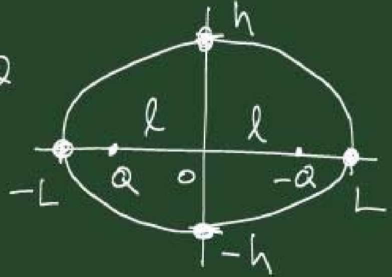
$$\rightarrow \quad \psi = 0 \quad \rightarrow \quad Q^* = Q\frac{a}{\ell}.$$

$$\therefore \psi(r,\theta) = -\frac{Q}{4\pi}(1+\cos\beta) - \frac{Q}{4\pi}\frac{a}{l}(1+\cos\alpha) + \frac{Q}{4\pi}\left(\frac{a}{l} + \frac{r}{a} - \frac{r_1}{a}\right)$$

Similarly, $\phi(r,\theta) = -\frac{Q}{4\pi\varsigma} - \frac{Qa}{4\pi\eta l} + \frac{Q}{4\pi a}\ln\left(\frac{\tan\alpha/2}{\tan\theta/2}\right)$.

- Rankine solids : uniform flow + source + sink





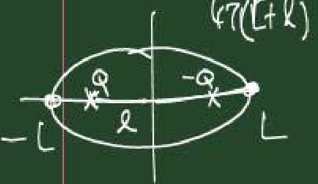
$$\psi(r,\theta) = \frac{1}{2}Ur^2\sin^2\theta - \frac{Q}{4\pi}(\cos\theta_1 - \cos\theta_2)$$

$r_0(\theta)$: radius of the surface $\rightarrow \psi = 0$

$$0 = \frac{1}{2}Ur_0^2\sin^2\theta - \frac{Q}{4\pi}(\cos\theta_1 - \cos\theta_2)$$

$$R_0 = r_0\sin\theta \rightarrow 0 = \frac{1}{2}UR_0^2 - \frac{Q}{4\pi}(\cos\theta_1 - \cos\theta_2)$$

$4\pi(L+l)^2$



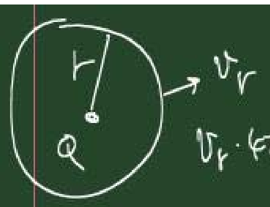
$$\rightarrow R_0^2 = \frac{Q}{2\pi U}(\cos\theta_1 - \cos\theta_2)$$

$R_0 = 0$; $\underline{\theta_1 = \theta_2 = 0,\ \pi}$

R_0 max : $\cos\theta_1 = -\cos\theta_2$; $\theta = \frac{\pi}{2}, \frac{3}{2}\pi$

L? stag pt. $u = v = 0$

$$\underset{(x=L)}{u} = U + \frac{Q}{4\pi(L+l)^2} - \frac{Q}{4\pi(L-l)^2} = 0$$

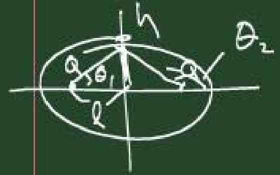


$$v_r \cdot 4\pi r^2 = Q$$

$$\Downarrow$$

$$(L^2-l^2)^2 - \frac{Ql}{\pi U}L = 0$$

h? $\rightarrow$ $\cos\theta_1 = -\cos\theta_2$ & $\tan\theta_1 = \frac{h}{l}$, $R_0 = h$.



$$h^2 = \frac{Q}{2\pi U}\left(\frac{l}{\sqrt{h^2+l^2}} + \frac{l}{\sqrt{h^2+l^2}}\right)$$

$$\rightarrow \quad h^2\sqrt{h^2+l^2} - \frac{Ql}{\pi U} = 0$$

$$\phi(r,\theta) = Ur\cos\theta - \frac{Q}{4\pi r_1} + \frac{Q}{4\pi r_2}.$$